


Optimal Regulation Performance of MIMO Networked Time-delay Systems With Limited Bandwidth and Interference Signals

Qianhao Li, Qingsheng Yang, Xisheng Zhan* , and Jie Wu

Abstract: In this paper, we investigate the optimal regulation performance of networked time delay systems with limited bandwidth and interference signals. Communication networks are primarily influenced by parameters including bandwidth, packet dropouts, coding and decoding, interference signals, and channel noise. For a given system, non-minimum phase zeros, unstable poles, and time delay are considered. The corresponding regulation performance expressions are derived using coprime decomposition, spectral decomposition techniques, and norm correlation theory in the frequency domain. Results indicate that regulation performance is dependent on the location and direction of non-minimum phase zeros and unstable poles of a given system, as well as the internal time delay of the controlled plant. In addition, network communication parameters such as bandwidth, channel noise, packet dropouts, and external interference signals influence the performance of the regulation. Finally, simulation examples are provided to demonstrate the theory's validity.

Keywords: Bandwidth limitation, incomplete information, interference signal, packet dropouts, regulation performance, time-delay system.

1. INTRODUCTION

It is well-known that the problem of studying the stability of control systems has always been a control classic [1-5]. However, in recent years, there has been a growing interest in studying the performance of control systems [6,7], especially the regulation performance and tracking performance [8-11]. In contrast to studies that only examine the relationship between external environmental influences and the performance of the controlled plant, an increasing number of researchers have shifted their attention to the influence of internal plant characteristics on performance. The results indicate that the non-minimum phase (NMP) zeros and unstable poles (UPs) of the controlled object, as well as their direction, impact the system's regulation performance. Using the regulation performance as an example, the research on the regulation performance of the system is evaluated primarily by the response limit of the system output to the disturbance after the controlled system has been disturbed.

Networked control systems (NCS) are widely used in smart grids [12], environmental monitoring [13], aerospace [14], and other fields due to the continuous advancement of Internet technology and network commu-

nication technology. In contrast to conventional control systems, it is commonly assumed that the information transfer between the controller and the controlled system is ideal when analyzing its regulation performance. However, in reality, the information interaction between the controlled objects cannot be so idealized. For instance, although NCS offers advantages such as high flexibility, high reliability, greater resource sharing, easy installation and maintenance, and low cost compared to traditional systems, with the intervention of communication networks, factors such as packet dropouts [15], time delay [16], bandwidth [17], quantization [18], and channel noise [19] are also affecting the controlled system, resulting in the information interaction between the controller and the controlled object being often missing. Therefore, the study of NCS performance under incomplete information is receiving more and more attention.

Currently, there are many works on the stability analysis of NCS communication channels with incomplete information [20-22]. The paper [20] analyzed the stability of control systems with non-periodic sampling and time-varying time-lag networks. Xiao *et al.* [21] examines the robust stability of networked linear control systems with asynchronous continuous and discrete-time event-

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triggered schemes. In [22], the robust stability and stability conditions of nonlinear NCS with network-induced delays based on the T-S fuzzy model are analyzed. More related work is also available in [23,24]. However, it is not sufficient to just examine the stability of NCS. For performance research, the main focus nowadays is on its tracking performance, which is evaluated by the error between the system output and the reference signal. It is known that the paper [25] discussed robust optimal tracking control for multiple-input multiple-output (MIMO) discrete-time systems with unknown uncertainties based on neural networks and presented an adaptive estimator design scheme. Jiang *et al.* [26] studied the output tracking control of a SIMO system under fading channels. Cheng *et al.* [27] analyzed the optimal tracking performance of a MIMO control system under multiple constraints. Less research has been conducted on the performance aspects of regulation. Li *et al.* [28] explored the problem of event-triggered output regulation of a networked flight control system using a switching system approach and designed an error feedback controller. Model-free optimal output regulation for linear discrete-time systems was discussed in [29]. Performance limitations in tracking and regulation problems for discrete-time systems were investigated in [30]. It is worth noting that the performance of system regulation is primarily investigated in this paper using a frequency domain approach. This differs from the published literature [31]. Most scholars have mainly focused on network-induced time delay in nowadays, and few have considered time-delay systems. However, the time delay phenomenon is pervasive in all types of practical systems, and the presence of time delay is a significant cause of system instability and performance deterioration. As a result, people have begun to pay extensive attention to the impact of the time delay phenomenon on system performance and how to eliminate or exploit this impact, and stability analysis and feedback control of time delay systems have become a hot topic in the study of automatic control theory. In addition, due to the limitations of the communication channel. Bandwidth limitations can cause channel data transmission delays or data loss. Interference signals can also pose a significant obstacle to message transmission. Consequently, bandwidth restriction and interference signal have become crucial research variables for optimal regulation performance today. This paper focuses on the relationship between MIMO networked time-delay systems (NTDSs) and regulation performance based on limited bandwidth and interference signal, as opposed to traditional studies of regulation performance. The contributions are as follows:

- 1) This paper employs a frequency domain approach to investigate the regulation performance of a newly constructed MIMO NTDS.
- 2) A single-degree-of-freedom (SDOF) controller is

proposed to achieve optimal regulation performance. A controller-based Youla parameterization technique that fully considers multiple communication channel constraints and derives the corresponding explicit expressions.

- 3) This study quantitatively reveals the effects of the inherent characteristics of the controlled object (the direction and location of the UPs and NMP zeros) and channel constraints on the regulation performance of NTDSs. In addition, suggestions for the design of optimal controllers and NTDSs for practical applications are presented. The aforementioned results are indicative of the optimal controller design and system regulation performance index.

The present work is organized as follows: The preparatory work is presented in Section 2. Section 3 investigates the regulation performance of NTDSs under incomplete information and gives the corresponding expressions. Section 4 provides simulation examples to demonstrate the method's accuracy. In Section 5, the paper's contributions and directions for future research are summed up.

2. PRELIMINARIES

All proper, stable, and rational transfer function (TF) matrices are denoted as $\mathbb{R}H_\infty$. Given any complex number z , \bar{z} is its complex conjugate. For any vector u and matrix A , u^T and A^T are represented as their transposes, and u^H and A^H as their conjugate transposes. The open left and the open right planes are denoted by $C_- := \{s : \text{Re}(s) < 0\}$, $C_+ := \{s : \text{Re}(s) > 0\}$. In addition, define $\|\cdot\|_2$ as the Euclidean norm, and the space L_2 is a Hilbert space with the inner product

$$\langle F, G \rangle := \frac{1}{2\pi} \int_{-\pi}^{\pi} [F^H(e^{j\omega_1}) G(e^{j\omega_1})] d\omega_1,$$

where H_2 and H_2^\perp are a pair of orthogonal subspaces of the Hilbert space L_2 whose expressions are

$$H_2 := \left\{ f : f(s) \text{ analytic in } \mathbb{C}_+, \right. \\ \left. \|f\|_2^2 := \sup_{\theta > 0} \frac{1}{2\pi} \int_{-\infty}^{\infty} \|f(\theta + j\omega_1)\|_F^2 d\omega_1 < \infty \right\}, \\ H_2^\perp := \left\{ f : f(s) \text{ analytic in } \mathbb{C}_-, \right. \\ \left. \|f\|_2^2 := \sup_{\theta < 0} \frac{1}{2\pi} \int_{-\infty}^{\infty} \|f(\theta + j\omega_1)\|_F^2 d\omega_1 < \infty \right\}.$$

In this paper, we consider a block diagram of multi-channel NTDSs as shown in Fig. 1. G represents the controlled plant, and K is an SDOF controller. $G(s)$ and $K(s)$ are their TF matrices, respectively. H is expressed as bandwidth, $H(s)$ is represented as TF matrix, define $H(s)$ as

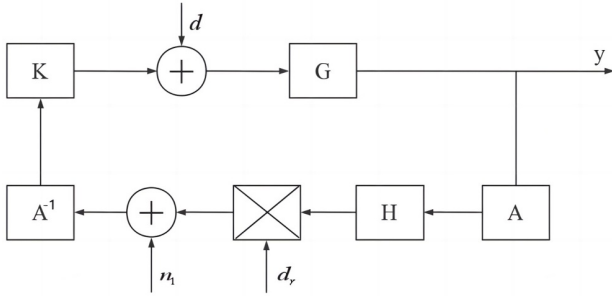


Fig. 1. NTDSs with communication constraints.

$H(s) = \text{diag}(h_1, h_2, \dots, h_n)$. A and A^{-1} represent the multichannel coding and decoding of the system. y represents system output, d denotes multichannel interference signals, n_1 denotes white Gauss noise, and d_r denotes packet dropouts.

In this paper, we set the MIMO NTDSs regulation performance metrics as: $J = E \left\{ \|y\|_2^2 \right\}$.

From Fig. 1, we obtain

$$\begin{cases} u = KA^{-1}n_1 + Kd_rHG(u+d), \\ y = G(u+d), \end{cases}$$

$$\begin{aligned} u &= (I - Kd_rHG)^{-1} (KA^{-1}n_1 + Kd_rHGd), \\ y &= G(u+d) \\ &= G(I - Kd_rHG)^{-1}d + G(I - Kd_rHG)^{-1}KA^{-1}n_1 \\ &= T_1d + T_2n_1, \end{aligned} \quad (1)$$

where $T_1 = G(I - Kd_rHG)^{-1}$, $T_2 = G(I - Kd_rHG)^{-1}KA^{-1}$.

Next, let's introduce some important decomposition. The given plant is described as: $G(s) = G_1(s)e^{-\tau s}$, where $G_1(s)$ denotes the rational TF matrix, τ represents the time delay. For any left-integrable and right-integrable TF matrix G , the following double coprime factorization is defined as follows:

$$d_rHG = e^{-\tau s}NM^{-1} = e^{-\tau s}\tilde{M}^{-1}\tilde{N}, \quad (2)$$

where $N, \tilde{N}, M, \tilde{M} \in \mathbb{RH}_\infty$, and satisfy the double Bezout identity

$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -e^{-\tau s}\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & Y \\ e^{-\tau s}N & X \end{bmatrix} = I, \quad (3)$$

where $X, \tilde{X}, Y, \tilde{Y} \in \mathbb{RH}_\infty$. For a stable plant G , an arbitrary SDOF controller could be expressed by the Youla parameterization as

$$\mathcal{K} = \left\{ K : K = (\tilde{X} - e^{-\tau s}Q\tilde{N})^{-1}(\tilde{Y} - Q\tilde{M}), \right. \\ \left. Q \in \mathbb{RH}_\infty \right\}. \quad (4)$$

Remark 1: The Youla parameterization is applied in this paper. The Youla parameterization allows us to transform a design problem into an almost unconstrained optimization problem. Besides, all closed-loop sensitivity

functions are affine w.r.t. the so-called Q parameter and hence, it can be employed for \mathcal{H}_∞ controller design in a convex optimization problem. Further more, comparing with the Lyapunov method in solving the stability problem, the construction of Lyapunov function is no need in this paper. Since the Lyapunov function is not established, the amount of calculation is greatly reduced.

It is well known that an NMP TF matrix can be factorized into a minimum phase part and an all-pass factor.

$$N = d_rHL_zN_m, \quad (5)$$

where N_m represents the minimum phase fraction and L_z is the all-pass factor. L_z contains the NMP zeros $z_i \in \mathbb{C}_+$, $i = 1, 2, 3, \dots, n$, and for a given system, is expressed as

$$L_z(s) = \prod_{i=1}^n L_i(s), \quad L_i(s) = I - \frac{2\text{Re}(z_i)}{s + \bar{z}_i} \eta_i \eta_i^H,$$

where η_i is denoted as the unit vector in the direction of the NMP zeros.

In addition, the data packet dropouts process in this article can be simulated by a Bernoulli distributed random process d_r .

$$d_r = \begin{cases} 0, & \text{when a data loss occurs,} \\ 1, & \text{when no data loss occurs.} \end{cases}$$

The probability distribution of this d_r is $P\{d_r = 1\} = 1 - \alpha$, $P\{d_r = 0\} = \alpha$, with $0 \leq \alpha \leq 1$, α represents the probability of packet dropouts.

It is assumed that both the forward multichannel interference signal d and the feedback multichannel white noise n_1 considered here are random signals with zero mean and variance of v_i^2 and σ_i^2 , and that the noise in each channel is independent of each other. Therefore, the NTDSs regulation performance index can describe as

$$J = \|T_1U\|_2^2 + \|T_2V\|_2^2,$$

where $U = \text{diag}(v_1, v_2, \dots, v_n)$, $V = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_n)$.

Finally, we define the optimal regulation performance of this paper as J^* , i.e., the optimal regulation performance achieved by the set \mathcal{K} of all stable controllers, is described as

$$J^* = \inf_{K \in \mathcal{K}} J. \quad (6)$$

3. OPTIMAL REGULATION PERFORMANCE WITH LIMITED BANDWIDTH AND INTERFERENCE SIGNALS

From (1)-(4), we obtain

$$T_1 = G(I - Kd_rHG)^{-1}$$

$$\begin{aligned}
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} \left[I - (\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} \right. \\
&\quad \times (\tilde{Y} - Q \tilde{M}) e^{-\tau s} \tilde{M}^{-1} \tilde{N} \left. \right]^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} \left[(\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} \right. \\
&\quad \times (\tilde{X} - e^{-\tau s} \tilde{Y} \tilde{M}^{-1} \tilde{N}) \left. \right]^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} \left[(\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} M^{-1} \right]^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} M (\tilde{X} - e^{-\tau s} Q \tilde{N}) \\
&= e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{X} - e^{-\tau s} Q \tilde{N}).
\end{aligned}$$

Similar to T_1 ,

$$\begin{aligned}
T_2 &= G(I - K d_r H G)^{-1} K A^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} \left[I - (\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} \right. \\
&\quad \times (\tilde{Y} - Q \tilde{M}) e^{-\tau s} \tilde{M}^{-1} \tilde{N} \left. \right]^{-1} (\tilde{X} - Q \tilde{N})^{-1} \\
&\quad \times (\tilde{Y} - Q \tilde{M}) A^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} \left[(\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} M^{-1} \right]^{-1} \\
&\quad \times (\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} (\tilde{Y} - Q \tilde{M}) A^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N M^{-1} M (\tilde{X} - e^{-\tau s} Q \tilde{N}) \\
&\quad \times (\tilde{X} - e^{-\tau s} Q \tilde{N})^{-1} (\tilde{Y} - Q \tilde{M}) A^{-1} \\
&= e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{Y} - Q \tilde{M}) A^{-1}.
\end{aligned}$$

So, we can get

$$\begin{aligned}
J &= E \left\{ \|y\|_2^2 \right\} \\
&= \|e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{X} - e^{-\tau s} Q \tilde{N}) U\|_2^2 \\
&\quad + \|e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{Y} - Q \tilde{M}) A^{-1} V\|_2^2. \quad (7)
\end{aligned}$$

From (6) and (7) we have

$$\begin{aligned}
J^* &= \inf_{K \in \mathcal{K}} J \\
&= \inf_{Q \in \mathbb{R}H_\infty} \|e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{X} - e^{-\tau s} Q \tilde{N}) U\|_2^2 \\
&\quad + \inf_{Q \in \mathbb{R}H_\infty} \|e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{Y} - Q \tilde{M}) A^{-1} V\|_2^2. \quad (8)
\end{aligned}$$

In order to obtain J^* , any suitable Q can be chosen.

Theorem 1: Consider the given system as shown in Fig. 1, define z_i ($i = 1, 2, 3, \dots, n$) as the NMP zeros of the controlled plant and p_j ($j = 1, 2, 3, \dots, m$) as UPs, then the optimal regulation performance can be expressed as follows:

$$\begin{aligned}
J^* &= J_1^* + J_2^* \\
&\geq \sum_{i,j=1}^n \frac{4\text{Re}(z_i)\text{Re}(\bar{z}_j)}{z_i + \bar{z}_j} e^{\tau(z_i + \bar{z}_j)} [\xi_i^H E_i^H A^{-H}(z_i) \\
&\quad \times U \gamma_i^H \gamma_j A^{-1}(z_i) U E_j \xi_j \xi_j^H L_j L_i^H \xi_i]
\end{aligned}$$

$$\begin{aligned}
&+ \sum_{k,j=1}^m \frac{4\text{Re}(p_j)\text{Re}(p_k)}{p_j + \bar{p}_k} [\omega_j^H F_j^H V A^{-H}(p_j) \lambda_j^H \lambda_k \\
&\quad \times A^{-1}(p_j) V F_k \omega_k \omega_k^H O_k O_j^H \omega_j],
\end{aligned}$$

where

$$\begin{aligned}
E_i &= \left(\prod_{k=1}^{i-1} D_{\Lambda k}(z_i) \right)^{-1}, \quad L_i = \left(\prod_{k=i+1}^n D_{\Lambda k}(z_i) \right)^{-1}, \\
\gamma_i &= N_m(z_i) M^{-1}(z_i), \\
F_j &= \left(\prod_{k=1}^{j-1} T_{\Lambda k}(p_j) \right)^{-1}, \quad O_j = \left(\prod_{k=j+1}^m T_{\Lambda k}(p_j) \right)^{-1}, \\
\lambda_j &= -e^{\tau p_j} d_r^{-1} L_z^{-1}(p_j) H^{-1}(p_j).
\end{aligned}$$

Proof: First, from (8), we can obtain

$$\begin{aligned}
J^* &= \inf_{K \in \mathcal{K}} J \\
&= \inf_{Q \in \mathbb{R}H_\infty} \|e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{X} - e^{-\tau s} Q \tilde{N}) U\|_2^2 \\
&\quad + \inf_{Q \in \mathbb{R}H_\infty} \|e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{Y} - Q \tilde{M}) A^{-1} V\|_2^2 \\
&= \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{X} - e^{-\tau s} Q \tilde{N}) U \\ e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{Y} - Q \tilde{M}) A^{-1} V \end{bmatrix} \right\|_2^2.
\end{aligned}$$

Substituting (5) into (8), since L_z and $e^{-\tau s}$ are all-pass factors, we achieve

$$\begin{aligned}
J^* &= \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{X} - e^{-\tau s} Q \tilde{N}) U \\ e^{-\tau s} d_r^{-1} H^{-1} N (\tilde{Y} - Q \tilde{M}) A^{-1} V \end{bmatrix} \right\|_2^2 \\
&= \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} e^{-\tau s} L_z N_m (\tilde{X} - e^{-\tau s} Q \tilde{N}) U \\ e^{-\tau s} L_z N_m (\tilde{Y} - Q \tilde{M}) A^{-1} V \end{bmatrix} \right\|_2^2 \\
&= \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} N_m (\tilde{X} - e^{-\tau s} Q \tilde{N}) U \\ (\tilde{Y} - Q \tilde{M}) A^{-1} V \end{bmatrix} \right\|_2^2 \\
&= \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} N_m \tilde{X} U \\ N_m \tilde{Y} A^{-1} V \end{bmatrix} - \begin{bmatrix} e^{-\tau s} N_m Q \tilde{N} U \\ N_m Q \tilde{M} A^{-1} V \end{bmatrix} \right\|_2^2.
\end{aligned}$$

Further definition is

$$\begin{aligned}
\tilde{N} U &= \tilde{N}_\Lambda L_{\Lambda p}, \quad \tilde{M} A^{-1} V = \tilde{M}_\Lambda B_{\Lambda p}, \\
L_{\Lambda p}(s) &= \prod_{i=1}^n D_{\Lambda i}(s), \quad B_{\Lambda p}(s) = \prod_{j=1}^m T_{\Lambda j}(s),
\end{aligned}$$

where $D_{\Lambda i}(s) = I - \frac{2\text{Re}(z_i)}{s + \bar{z}_i} \xi_i \xi_i^H$, $T_{\Lambda j}(s) = I - \frac{2\text{Re}(p_j)}{s + \bar{p}_j} \omega_j \omega_j^H$, ξ_i and ω_j are unit vector. From the above equation we have

$$J^* = \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} N_m \tilde{X} U \\ N_m \tilde{Y} A^{-1} V \end{bmatrix} - \begin{bmatrix} e^{-\tau s} N_m Q \tilde{N}_\Lambda L_{\Lambda p} \\ N_m Q \tilde{M}_\Lambda B_{\Lambda p} \end{bmatrix} \right\|_2^2.$$

Since $e^{-\tau s}$, $L_{\Lambda p}$ and $B_{\Lambda p}$ are all-pass factors, so that

$$J^* = \inf_{Q \in \mathbb{R}H_\infty} \left\| \begin{bmatrix} e^{\tau s} N_m \tilde{X} U L_{\Lambda p}^{-1} \\ N_m \tilde{Y} A^{-1} V B_{\Lambda p}^{-1} \end{bmatrix} - \begin{bmatrix} N_m Q \tilde{N}_\Lambda \\ N_m Q \tilde{M}_\Lambda \end{bmatrix} \right\|_2^2.$$

Following a partial fraction procedure, it is obtained that

$$\begin{aligned} & e^{\tau s} N_m \tilde{X} U L_{\Lambda p}^{-1} \\ &= \sum_{i=1}^n e^{\tau z_i} N_m(z_i) \tilde{X}(z_i) U E_i (D_{\Lambda i}^{-1} - I) L_i + R_1, \end{aligned}$$

where $R_1 \in \mathbb{R}H_{\infty}$, $E_i = \left(\prod_{k=1}^{i-1} D_{\Lambda k}(z_i) \right)^{-1}$, $L_i = \left(\prod_{k=i+1}^n D_{\Lambda k}(z_i) \right)^{-1}$.

Similarly,

$$\begin{aligned} & N_m \tilde{Y} A^{-1} V B_{\Lambda p}^{-1} \\ &= \sum_{j=1}^m N_m(p_j) \tilde{Y}(p_j) A^{-1}(p_j) V F_j (T_{\Lambda j}^{-1} - I) O_j + R_2, \end{aligned}$$

where $R_2 \in \mathbb{R}H_{\infty}$, $F_j = \left(\prod_{k=1}^{j-1} T_{\Lambda k}(p_j) \right)^{-1}$, $O_j = \left(\prod_{k=j+1}^m T_{\Lambda k}(p_j) \right)^{-1}$.

Thus, we can obtain

$$\begin{aligned} J^* &= \inf_{Q \in \mathbb{R}H_{\infty}} \left\| \begin{bmatrix} \sum_{i=1}^n e^{\tau z_i} N_m(z_i) \tilde{X}(z_i) U E_i (D_{\Lambda i}^{-1} - I) L_i \\ \sum_{j=1}^m N_m(p_j) \tilde{Y}(p_j) A^{-1}(p_j) \\ \times V F_j (T_{\Lambda j}^{-1} - I) O_j \end{bmatrix} \right\|_2 \\ &+ \left\| \begin{bmatrix} R_1 - N_m Q \tilde{N}_{\Lambda} \\ R_2 - N_m Q \tilde{M}_{\Lambda} \end{bmatrix} \right\|_2. \end{aligned}$$

In addition, because $\sum_{i=1}^n e^{\tau z_i} N_m(z_i) \tilde{X}(z_i) U E_i (D_{\Lambda i}^{-1} - I) L_i$

and $\sum_{j=1}^m N_m(p_j) \tilde{Y}(p_j) A^{-1}(p_j) V F_j (T_{\Lambda j}^{-1} - I) O_j \in H_2^{\perp}$, $R_1 - N_m Q \tilde{N}_{\Lambda}$ and $R_2 - N_m Q \tilde{M}_{\Lambda} \in H_2$.

Then, it follows that

$$\begin{aligned} J^* &= \left\| \begin{bmatrix} \sum_{i=1}^n e^{\tau z_i} N_m(z_i) \tilde{X}(z_i) U E_i (D_{\Lambda i}^{-1} - I) L_i \\ \sum_{j=1}^m N_m(p_j) \tilde{Y}(p_j) A^{-1}(p_j) \\ \times V F_j (T_{\Lambda j}^{-1} - I) O_j \end{bmatrix} \right\|_2 \\ &+ \inf_{Q \in \mathbb{R}H_{\infty}} \left\| \begin{bmatrix} R_1 - N_m Q \tilde{N}_{\Lambda} \\ R_2 - N_m Q \tilde{M}_{\Lambda} \end{bmatrix} \right\|_2. \end{aligned}$$

Next, we define

$$J_1^* = \left\| \begin{bmatrix} \sum_{i=1}^n e^{\tau z_i} N_m(z_i) \tilde{X}(z_i) U E_i (D_{\Lambda i}^{-1} - I) L_i \\ \sum_{j=1}^m N_m(p_j) \tilde{Y}(p_j) A^{-1}(p_j) \\ \times V F_j (T_{\Lambda j}^{-1} - I) O_j \end{bmatrix} \right\|_2,$$

$$J_2^* = \inf_{Q \in \mathbb{R}H_{\infty}} \left\| \begin{bmatrix} R_1 - N_m Q \tilde{N}_{\Lambda} \\ R_2 - N_m Q \tilde{M}_{\Lambda} \end{bmatrix} \right\|_2.$$

After calculation, we have

$$\begin{aligned} & \left\| \begin{bmatrix} \sum_{i=1}^n e^{\tau z_i} N_m(z_i) \tilde{X}(z_i) U E_i (D_{\Lambda i}^{-1} - I) L_i \end{bmatrix} \right\|_2 \\ &= \sum_{i,j=1}^n \frac{4 \operatorname{Re}(z_i) \operatorname{Re}(\bar{z}_j)}{z_i + \bar{z}_j} e^{\tau(z_i + \bar{z}_j)} \\ &\quad \times [\xi_i^H E_i^H U \gamma_i^H \gamma_j U E_j \xi_j \xi_j^H L_j L_i^H \xi_i], \end{aligned}$$

where $\gamma_i = N_m(z_i) \tilde{X}(z_i)$. According to (3) and $N(z_i) = 0$, we can get $N_m(z_i) \tilde{X}(z_i) = N_m(z_i) M^{-1}(z_i)$.

Similarly, we can obtain

$$\begin{aligned} & \left\| \sum_{j=1}^m N_m(p_j) \tilde{Y}(p_j) A^{-1}(p_j) V F_j (T_{\Lambda j}^{-1} - I) O_j \right\|_2 \\ &= \sum_{k,j=1}^m \frac{4 \operatorname{Re}(p_j) \operatorname{Re}(p_k)}{p_j + \bar{p}_k} [\omega_j^H F_j^H V A^{-H}(p_j) \lambda_j^H \lambda_k \\ &\quad \times A^{-1}(p_j) V F_k \omega_k \omega_k^H \times O_k O_j^H \omega_j], \end{aligned}$$

where $\lambda_j = N_m(p_j) \tilde{Y}(p_j)$. According to (3) and $M(p_j) = 0$, we can get $N_m(p_j) \tilde{Y}(p_j) = -e^{\tau p_j} d_r^{-1} L_z^{-1}(p_j) H^{-1}(p_j)$.

Therefore, it is possible to obtain

$$\begin{aligned} J_1^* &= \sum_{i,j=1}^n \frac{4 \operatorname{Re}(z_i) \operatorname{Re}(\bar{z}_j)}{z_i + \bar{z}_j} e^{\tau(z_i + \bar{z}_j)} \\ &\quad \times [\xi_i^H E_i^H U \gamma_i^H \gamma_j U E_j \xi_j \xi_j^H L_j L_i^H \xi_i] \\ &+ \sum_{k,j=1}^m \frac{4 \operatorname{Re}(p_j) \operatorname{Re}(p_k)}{p_j + \bar{p}_k} [\omega_j^H F_j^H V A^{-H}(p_j) \lambda_j^H \lambda_k \\ &\quad \times A^{-1}(p_j) V F_k \omega_k \omega_k^H O_k O_j^H \omega_j]. \end{aligned}$$

And since N_m , \tilde{N}_{Λ} , \tilde{M}_{Λ} belong to the minimum phase function matrix, we can choose appropriate Q to make

$$\begin{aligned} \inf_{Q \in \mathbb{R}H_{\infty}} \|R_1 - N_m Q \tilde{N}_{\Lambda}\|_2 &= 0, \\ \inf_{Q \in \mathbb{R}H_{\infty}} \|R_2 - N_m Q \tilde{M}_{\Lambda}\|_2 &= 0, \end{aligned}$$

which means

$$J_2^* = \inf_{Q \in \mathbb{R}H_{\infty}} \left\| \begin{bmatrix} R_1 - N_m Q \tilde{N}_{\Lambda} \\ R_2 - N_m Q \tilde{M}_{\Lambda} \end{bmatrix} \right\|_2 \geq 0.$$

In summary,

$$\begin{aligned} J^* &= J_1^* + J_2^* \\ &\geq \sum_{i,j=1}^n \frac{4 \operatorname{Re}(z_i) \operatorname{Re}(\bar{z}_j)}{z_i + \bar{z}_j} e^{\tau(z_i + \bar{z}_j)} \\ &\quad \times [\xi_i^H E_i^H A^{-H}(z_i) U \gamma_i^H \gamma_j A^{-1}(z_i) U E_j \xi_j \xi_j^H L_j L_i^H \xi_i] \\ &+ \sum_{k,j=1}^m \frac{4 \operatorname{Re}(p_j) \operatorname{Re}(p_k)}{p_j + \bar{p}_k} [\omega_j^H F_j^H V A^{-H}(p_j) \lambda_j^H \lambda_k \end{aligned}$$

$$\times A^{-1}(p_j)VF_k\omega_k\omega_k^H O_k O_j^H \omega_j],$$

where

$$E_i = \left(\prod_{k=1}^{i-1} D_{\Lambda k}(z_i) \right)^{-1}, \quad L_i = \left(\prod_{k=i+1}^n D_{\Lambda k}(z_i) \right)^{-1},$$

$$\gamma_i = N_m(z_i)M^{-1}(z_i),$$

$$F_j = \left(\prod_{k=1}^{j-1} T_{\Lambda k}(p_j) \right)^{-1}, \quad O_j = \left(\prod_{k=j+1}^m T_{\Lambda k}(p_j) \right)^{-1},$$

$$\lambda_j = -e^{\tau p_j} d_r^{-1} L_z^{-1}(p_j) H^{-1}(p_j).$$

The proof is completed. \square

4. ILLUSTRATIVE EXAMPLE

In this section, we will examine the effect of various conditions on the regulation performance of NTDSs and provide simulation examples to illustrate and validate the theoretical results.

Consider the following expression for the TF matrix in the given system

$$G(S) = \begin{pmatrix} \frac{s-3}{s+2} & 0 \\ 0 & \frac{s+3}{(s-k)(s+5)} \end{pmatrix} e^{-\tau s}.$$

For the given system, it can be known that NMP zeros is $z_i = 3$, which direction vector is $\eta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, the UPs is $p_j = k$. We assume that $U = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$, $V = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $\alpha = 0.5$. Similarly, it can be selected $\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\omega = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. In addition, we assume that the codec and bandwidth TF matrices are $A(s) = \begin{pmatrix} \frac{s+1}{s-2} & 0 \\ 0 & \frac{s+2}{s-2} \end{pmatrix}$ and $H(s) = \begin{pmatrix} \frac{\mu}{s+\mu} & 0 \\ 0 & \frac{\mu}{s+\mu} \end{pmatrix}$, where μ is the bandwidth rate.

Assuming $\mu = 5$, then from Theorem 1 we obtain

$$J^* = 0.96e^{6\tau} + 1.28ke^{2\tau k} \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2}.$$

First, in order to prove that the system optimal regulation performance will be affected by the time delay, three groups of different length data $\tau_1 = 0.2$, $\tau_2 = 0.5$ and $\tau_3 = 0.8$ are selected here. Then, we can get

$$J^* = 0.96e^{1.2} + 1.28ke^{0.4k} \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2},$$

$$J^* = 0.96e^3 + 1.28ke^k \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2},$$

$$J^* = 0.96e^{4.8} + 1.28ke^{1.6k} \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2}.$$

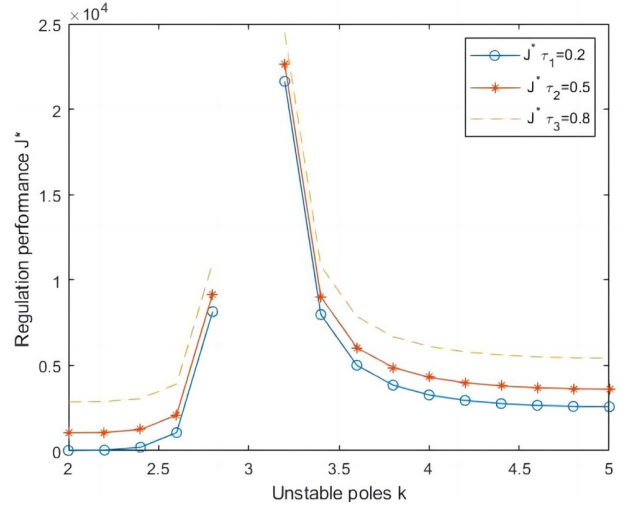


Fig. 2. Optimal regulation performance with different time delay.

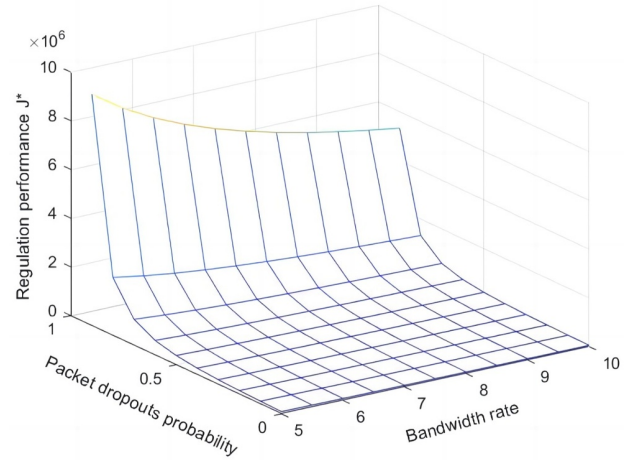


Fig. 3. Optimal regulation performance based on packet dropouts and limited bandwidth.

Fig. 2 demonstrates that as the time delay increases, the performance of the system's regulation degrades. In addition, when the NMP zeros close to the UPs, the system regulation performance will tend to infinity.

The following discussion focuses on the effect of bandwidth and packet loss on the regulation performance limits of NTDSs. With the previous data settings unchanged, assuming $\tau = 0.5$, $k = 5$, $\alpha \in (0, 1)$, $\mu \in [5, 10]$ then according to Theorem 1 it is obtained that

$$J^* = 0.96e^3 + \frac{160e^5}{(1-\alpha)^2} \frac{(\mu+5)^2}{\mu^2}.$$

Fig. 3 depicts the regulation performance limitations of NTDSs under the constraints of bandwidth and packet dropouts. According to Fig. 3, bandwidth and packet dropouts have a significant impact on the performance of

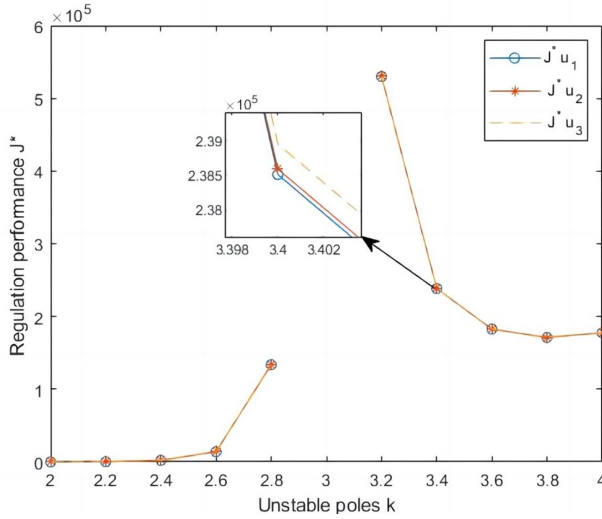


Fig. 4. Optimal regulation performance with different interference signals.

the controlled plant's regulation system. The regulation performance limit is negatively correlated with the packet dropout rate, and the greater the packet dropout rate, the worse the regulation performance. The bandwidth rate is positively correlated with the regulation performance limit, such that the greater the bandwidth, the higher the regulation performance.

Finally, assuming $\tau = 0.5$, $k = 3$, $\alpha = 0.5$ and $\mu = 5$. Select three groups of interference signals $U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$, $U_2 = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$ and $U_3 = \begin{pmatrix} 8 & 0 \\ 0 & 10 \end{pmatrix}$ with different variances to verify the regulation performance of NTDSs under different interference signals. From Theorem 1, we can obtain

$$J^* = 2.16e^3 + 1.28ke^{0.4k} \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2},$$

$$J^* = 6e^3 + 1.28ke^{0.4k} \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2},$$

$$J^* = 24e^3 + 1.28ke^{0.4k} \frac{(k^3 + 6k^2 - k - 30)^2}{(k^2 - 2k - 3)^2}.$$

Fig. 4 illustrates the regulation performance limits of the controlled system in the presence of various interference signals. It can be seen that under the influence of interference signals with different variances, the regulation performance of the controlled system is not very different, so it can be seen that the NTDSs designed in this paper have strong robustness.

In addition, the regulation performance limit under channel noise is examined in [30]. Using this information, [9] added the effects of packet dropouts and quantization noise. In this paper, we examine the constraints of band-

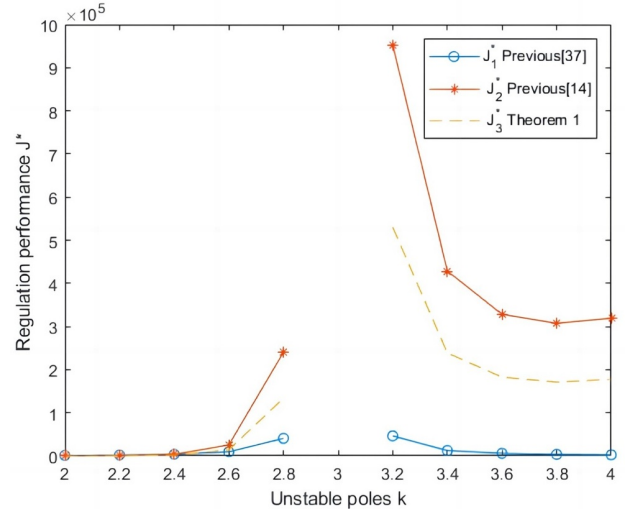


Fig. 5. Optimal regulation performance with different UPs.

width and packet dropouts. Therefore, according to [30], [9] and Theorem 1 can be obtained.

Fig. 5 depicts the optimal regulation performance under various UPs. Since the controlled system in [30] only considers channel noise interference and this paper introduces various constraints such as bandwidth limitation and packet dropouts, the results of Theorem 1 are inferior to those of [30]. However, compared to [9], the results obtained in this paper with the same introduction of communication constraints are superior. In conclusion, the results obtained in this paper are superior in terms of their regulation performance while ensuring that they are realistic.

5. CONCLUSIONS

This paper investigates the limits of the regulation performance of MIMO NTDSs under limited bandwidth and interference signals. The optimal regulation performance expression of the system is deduced using the frequency domain method, double coprime decomposition technique and H_2 norm. According to the results, the optimal regulation performance depends on the location and direction of NMP zeros and UPs, as well as the influence of inherent properties such as system delay. In addition, the effects of network channel parameters such as packet dropouts, codec, bandwidth, channel noise, and interference signals on the optimal regulation performance of MIMO NTDSs are also revealed. Finally, the optimal controller parameters are derived, and the effectiveness of the proposed method is demonstrated by some simulation examples.

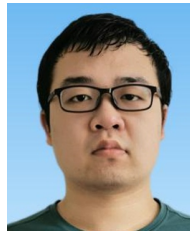
In future work, we will generalize the present results to more general models. For example, due to the limitation of network resources nowadays, to save network resources, the input energy of the system is often limited. In

addition, the influence of the external environment will not only have interference signals, but also network attacks in the network communication channel. Moreover, due to the poor external environment, we will take quantitative measures to ensure the integrity of information transmission as far as possible. Therefore, the introduction of these factors may be considered in subsequent studies. A more in-depth study of system performance based on this will be conducted.

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