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Nonsingular Fast Terminal Sliding Mode Control for Uncertain Nonlinear Systems Based on Adaptive Super-twisting Sliding Mode Disturbance Observer

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Abstract: This paper presents a new nonsingular fast terminal sliding mode back-stepping control (BSC) for uncertain nonlinear systems subjected to unknown mismatched disturbance based on an adaptive super-twisting sliding mode nonlinear disturbance observer (ASTSM-NDO). The proposed algorithms utilize BSC technique to manage high-order uncertainty systems by compounding the dynamic surface control (DSC) architecture to get rid of 'complexity explosion'. To cope with the unknown upper-bound mismatched disturbance, an adaption law is devised by finite time stability ASTSM-NDO designation. Besides, in the last step, the actual control scheme is designed by an integral nonsingular fast terminal sliding mode control algorithms combined with disturbance estimation and uncertainty adaption law to eliminate the influence of modeling error and mismatched interference on systems. Lyapunov stability theory is applied to prove that the tracking deviation of the whole system is uniformly and ultimately bounded. Finally, two examples are simulated by comparing the derived outcomes with existing method to verify the effectiveness and feasibility of the devised methodology.

Keywords: Adaptive super-twisting sliding mode observer, dynamic surface control, filtered back stepping control, integral nonsingular sliding mode control, mismatched uncertainty, nonlinear systems.

1. INTRODUCTION

1.1. Background and motivation

Sliding mode control (SMC) is well known features characterizing of robustness to handle nonlinear systems suffered from matched bounded external disturbances, system uncertainties and perturbations [1,2]. Because of good transient control performance such as strong robustness to parameter variations, order reduction and insensitivity to disturbance, SMC has been widely studied and successfully used in plenty of engineering application [3-6]. While the most commonly used sliding surface is linear in conventional SMC, which can only guarantee asymptotic stability of the system in the sliding phase that the system states cannot converge to the equilibrium in finite time [7]. In order to achieve finite time convergence of the system states in the sliding phase, terminal sliding mode control (TSMC) based on nonlinear sliding surface has been proposed by Venkataramanet *et al*. and Man *et al*. [8,9]. However, the conventional TSMC has two main drawbacks: 1) TSMC suffers from a singularity problem which can cause the unboundedness of the control input, 2) when the system states are far away from the equilibrium, TSMC has slower convergence rate than the conventional SMC [10-13].

The super-twisting control law is one of the most powerful second order continuous sliding mode control algorithms that handle systems of a relative degree equal to one. It generates the continuous control function that drives the sliding variable and its derivative to zero in a finite time. A novel adaptive super-twisting control law that continuously drives the sliding variable and its derivative to zero in the presence of the bounded disturbance with the unknown boundary had been proposed in [14]. In [15], the authors presented a novel strategy regarding the stabilization control problem for plants with unmatched uncertainties based on adaptive smooth super twisting sliding mode control. While in many real engineering applications, the unknown disturbance boundary cannot be easily estimated, there exsits effected control methods to cope with it, such as predefined-time nonsingular TSM C approach [16,17], fractional-order sliding mode approach

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of mismatched disturbances and so on [18,19]. Therefore, the adaptive-gain control law was developed, which handled the perturbed plant dynamics with the additive disturbance/uncertainty of certain class with the unknown boundary [20].

To deal with mismatched uncertainty of the disturbance item in the high-order model plant, BSC method is a powerful technique which is based on the Lyapunov stabilization theory. While in conventional BSC technique, there exists a 'deferential complexity explosion' drawback, which restricts its application in the high order system. However, Dynamic surface control (DSC) technology can manage handling this problem with the aid of a first-order low-pass filter (FOLPF) [21-24]. Morever, the signal BSC method has not been able to ensure the robust performance to counteract lumping perturbations and uncertainties, the most effective methods are adding the robust performance control technique such as SMC, ASMC and nonlinear disturbance observer to cope with the uncertainness item [25,26].

Motivations: From the above discussions, in the area of disturbance observer on SMC technique, the ASTSM-NDO is designed for a class of nonlinear system in the present work. The ASTSM-NDO is used in adaptive manner, in which observer gains are able to adapt themselves to various uncertainties online. Compared with the existing pieces of reference, the upper-bound of the interference signal and its first derivative are unnecessary. The presented method synthesizes BSC technique by compounding dynamical surface control (DSC) architecture to avoid the defect of 'complexity explosion' concerned in BSC approach for mismatched uncertain high-order nonlinear systems.

1.2. Contributions of the paper

The main contributions regarding this paper are summarized as follows:

1) A new ASTSM-NDO is used to accurately estimate the unmatched uncertain external disturbance. Compared with the traditional STSM-NDO, the observer-gain does not depend on the disturbace's first derivative upper-bound, and the gain adaption law can achieve the effect of accurate estimation of the disturbance signal in a finite time.

2) A nonsingular integral terminal sliding mode surface is constructed to design the actual control law and the modeling error adaption law in order to make the state tracking deviation converge to zero in a finite time. Besides, FOLPF is used to eliminate the complexity defect of 'differential explosion' caused by the high-order derivation of virtual control law in the BSC algorithm to solve the influence of mismatch uncertainty.

1.3. Structure of the paper

The rest of this paper is organized as follows: In Section 2, the control problems are described, some assumptions and lemmas are proposed. Section 3 gives the main methodological results concerning a new ASTSM-NDO. The validation of the devised algorithm is expounded in Section 4. Simulation studies verify the effectiveness of the proposed controller in Section 5. Some conclusions are drawn in Section 6.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem description

Consider a class of n-dimensional high-order uncertain nonlinear systems with modeling errors and unknown disturbances described by

$$\begin{cases} \dot{x}_i = x_{i+1} + d_i(t),\ i = 1,\ 2,\ \cdots,\ n-1, \\ \dot{x}_n = f(\bar{x}_n) + g(\bar{x}_n)u + F(\bar{x}_n) + d_n(t), \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{x}_n = (x_1,\ x_2,\ \cdots,\ x_{n-1},\ x_n)^T \in \mathbb{R}^n$ is the system state vector(assumed available for measurement); $u \in \mathbb{R}$, $y \in \mathbb{R}$ denote the control input and the system output variables, respectively; $f(\bar{x}_n)$ is sufficiently smooth nonlinear function vector fields; $g(\bar{x}_n)$ is the control input gain and satisfies $g(\bar{x}_n) \neq 0$, which $[g(\bar{x}_n)]^{-1}$ is existing. $F(\bar{x}_n) = \Delta f(\bar{x}_n) + \Delta g(\bar{x}_n)u$ is the lumped system modeling error function, $\Delta f(\bar{x}_n)$, $\Delta g(\bar{x}_n)u$ are the system states modeling error function and the control input modeling error function, respectively; $d_i(t)$, $i = 1, 2, \cdots, n$ is the external mismatched unknown disturbance items. Make the system output reference track signal as x_d.

Remark 1: System in (1) is practical systems in the engineering project, such as dynamic equation of angular displacement tracking of multi joint robot system suffered by the comprehensive disturbance and magnetic suspension systems and so on.

Remark 2: Unlike previous research literature, it is generally assumed that the external mismatched uncertain disturbance is continuous and the first derivative of the disturbance exists the prior known boundary, i.e., there are unknown positive constants D_i and L_i, which has

$$|d_i(t)| < D_i,\ \left|\dot{d}_i(t)\right| < L_i,\ i = 1,\ 2,\ \cdots,\ n. \tag{2}$$

And it generally assumed that when the interference term $d_i(t)$ is a slow time-varying signal, it supposed that $\dot{d}_i \approx 0$ [27], but which is not satisfied in many cases.

Remark 3: This paper further relaxes the interference condition in Remark 2, meanwhile it also discusses the high order mismatched uncertain nonlinear systems, so it has more universal value.

Assumption 1: The lumped system modeling error function $F(\bar{x}_n)$ is continuous and derivable $\dot{F}(\bar{x}_n) \approx 0$.

Assumption 2: The reference trajectory signal x_d is bounded and exists two order derivative with respect to t.

Control objective: The control target is to design a nonsingular fast terminal adaptive BSC based on a finite time stable ASTSM-NDO for a class of n-dimensional high-order nonlinear systems with mismatched uncertain external disturbances and modeling errors shown in (1), so as to eliminate the influence of disturbances and modeling errors on the systems and make the systems output signal y track the given reference trajectory signal x_d quickly and accurately. To this end, the following preliminaries are presented in the design procedure.

2.2. Preliminaries

Lemma 1: For n-th order controlled systems described by

$$\dot{\hat{x}}_i = u_{sti}(t) + d_i(t),\ i = 1,\ 2,\ \cdots,\ n-1, \tag{3}$$

where $\hat{x}_i$ is the system states vector, d_i is the unknown mismatched disturbance satisfying the condition of Remark 2, when the super twisting sliding mode control law is designed as (4), then the system (6) would be stability and the states would converge to equilibrium in a finite time.

$$\begin{cases} u_{sti}(t) = u_{1i}(t) + u_{2i}(t), \\ u_{1i}(t) = -\gamma_i \left|\hat{x}_i\right|^{1/2} \mathrm{sgn}(\hat{x}_i), \\ u_{2i}(t) = \displaystyle\int_0^t (-\delta_i)\mathrm{sgn}(\hat{x}_i)d\tau, \end{cases} \tag{4}$$

where $\mathrm{sgn}(\cdot)$ is the symbolic function, $\gamma_i > 0$, $\delta_i > 0$ and they satisfy the condition of

$$\begin{cases} \gamma_i > 2, \\ \delta_i > \dfrac{\gamma_i}{4-8\gamma_i^{-1}} + L_i^2\gamma_i^{-1}. \end{cases} \tag{5}$$

where L_i is prior to known shown in (2). From (5), we can easily get that the traditional super-twisting observer gain values are depended on the upper-bound of the disturbance's derivation, but the exact upper-bound value in the real engineering environment is hard to get it, so we will explore the ASTSM-NDO shown in Lemma 4.

$$\begin{cases} \dot{\hat{x}}_i = u_{1i}(t) + \chi_i, \\ \dot{\chi}_i = \dot{u}_{2i}(t) + \dot{d}_i(t). \end{cases} \tag{6}$$

The proof of Lemma 1 is provided in Appendix A.

Lemma 2 [28]**:** If the continuous function $V(t)$ satisfies the following conditions as

$$\dot{V}(t) \le -aV(t) - bV^{\varsigma}(t), \forall V(t_0) \ge 0,\ t \ge t_0. \tag{7}$$

$V(t)$ would converge to zero in a finite time and the convergence time is

$$t = t_0 + \frac{1}{a(1-\varsigma)} \ln \frac{aV^{1-\varsigma}(t_0) + b}{b}. \tag{8}$$

If $a = 0$, then $\dot{V}(t) \le -bV^{\varsigma}(t)$, $V(t)$ will also converge to zero in a finite time.

Lemma 3 [29]**:** If $p \ge 1$, $q \ge 1$ and $\frac{1}{p} + \frac{1}{q} = 1$, for any real vector $\mathbf{x}$, $\mathbf{y}$ and $\beta > 0$. Thus,

$$\mathbf{x}^{\mathrm{T}}\mathbf{y} \le \beta \left\|\mathbf{x}\right\|^p + \frac{1}{q}(p\beta)^{-\frac{q}{p}} \left\|\mathbf{y}\right\|^q. \tag{9}$$

For example, when $p = q = 2$, $\beta = 1$, (9) is expressed as $\mathbf{x}^{\mathrm{T}}\mathbf{y} \le \left\|\mathbf{x}\right\|^2 + \frac{1}{4}\left\|\mathbf{y}\right\|^2$.

3. DESIGNATE OF ASTSM-NDO

It is obvious that the mismatched unknown disturbance is a great challenge in the control filed. Therefore, the dynamics and steady performance of the observer-designing play a significant role in the control process. STSM-NDO is a continuous second-order sliding mode observer. The observer not only has integral of the signum function instead of discontinuous signum function but also consists of continuous switching function, which gives the guarantee of nonchattering effect. Therefore, the unwanted chattering effect can be reduced and it promise accurate robust control in the presence of mismatches and external disturbances. The designation of the ASTSM-NDO is stated by Lemma 4.

Lemma 4: An nth order ASTSM-NDO in (10) is proposed and the observer gain adaption law is adopted shown in (11), then the ASTSM-NDO would be stability.

$$\begin{cases} \hat{d}_1(t) = \gamma_1 \left|x_1 - \hat{x}_1\right|^{\frac{1}{2}} \mathrm{sgn}(x_1 - \hat{x}_1) - \upsilon_1, \\ \dot{\upsilon}_1 = -\delta_1 \mathrm{sgn}(x_1 - \hat{x}_1), \\ \qquad \vdots \\ \hat{d}_i(t) = \gamma_i \left|x_i - \hat{x}_i\right|^{\frac{1}{2}} \mathrm{sgn}(x_i - \hat{x}_i) - \upsilon_i, \\ \dot{\upsilon}_i = -\delta_i \mathrm{sgn}(x_i - \hat{x}_i),\ i = 2,\ 3,\ \cdots,\ n-1, \\ \hat{d}_n(t) = \gamma_n \left|x_n - \hat{x}_n\right|^{\frac{1}{2}} \mathrm{sgn}(x_n - \hat{x}_n) - \upsilon_n, \\ \dot{\upsilon}_n = -\delta_n \mathrm{sgn}(x_n - \hat{x}_n), \end{cases} \tag{10}$$

where x_i, $\hat{x}_i$ are the original system states and the observer system states, respectively; $\hat{d}_i$ is the estimation of d_i, $\tilde{d}_i = \hat{d}_i - d_i$ is the deviation of the ASTSM-NDO; the NDO gain-adaption laws are designed as

$$\begin{cases} \dot{\gamma}_i = \begin{cases} \kappa_i\sqrt{\varepsilon_i/2}, & x_i \ne 0, \\ 0, & x_i = 0, \end{cases} \\ \delta_i = \rho_i + \dfrac{\theta_i^2 + \gamma_i\theta_i}{4}, \end{cases} \tag{11}$$

where $\theta_i > 4$, ε_i, ρ_i, κ_i are positive constants. Therefore, in order to tackle with the mismatched unknown disturbance, when the ASTSM-NDO is adopted, then the estim-ation systems will be stability and $\tilde{d}_i$ would converge to zero exponentially, it is proved in Appendix B. Also the proof of Lemma 4 is provided in Appendix B.

Remark 4: The ASTSM-NDO proposed in this paper can be used to precisely estimate the mismatched unknown upper-bound disturbance. Compared with the existing pieces of literatures, ASTSM-NDO gain-adaption laws can further relaxes the condition of Remark 2, which has great practical significance in engineering application. It is presented as the Theorem 1.

Theorem 1: For the mismatched unknown upper-bound disturbance nonlinear system in (1), the ASTSM-NDO and the n-dimensional observer are designed as (10), (11) and (12), then the estimated disturbance $\hat{d}_i$ and the observer states variables $\hat{x}_i$ would fast precisely track the actual signals d_i, x_i, respectively and all the estimation deviation would converge to zero in a finite time.

$$\begin{cases} \dot{\hat{x}}_1 = x_2 + \hat{d}_1(t), \\ \dot{\hat{x}}_2 = x_3 + \hat{d}_2(t), \\ \qquad \vdots \\ \dot{\hat{x}}_n = f(\bar{x}_n) + g(\bar{x}_n)u + F(\bar{x}_n) + \hat{d}_n(t). \end{cases} \tag{12}$$

Proof: The states deviation between the disturbance observer and the original system is defined as

$$\tilde{x}_i = x_i - \hat{x}_i, \; i = 1,\, 2,\, \cdots,\, n. \tag{13}$$

Substituting the first subsystem of (1), (10) and (12) into (13), one can get

$$\begin{aligned} \dot{\tilde{x}}_1 &= d_1(t) - \hat{d}_1(t) \\ &= d_1(t) - \gamma_1 |\tilde{x}_1|^{\frac{1}{2}} \operatorname{sgn}(\tilde{x}) + \int_0^t -\delta_1 \operatorname{sgn}(\tilde{x}) d\tau. \end{aligned} \tag{14}$$

γ_1, δ_1 can be obtained according to the adaption law shown in (11). According to Assumption 1, one can get

$$\begin{cases} \dot{\tilde{x}}_1 = -\gamma_1 |\tilde{x}_1|^{\frac{1}{2}} \operatorname{sgn}(\tilde{x}_1) + \Xi_1, \\ \dot{\Xi}_1 = -\delta_1 \operatorname{sgn}(\tilde{x}_1) + \dot{d}_1(t). \end{cases} \tag{15}$$

According to Lemma 1, it can be known that the estimation error $\tilde{x}_1$ and its derivation $\dot{\tilde{x}}_1$ will be convergence to zero in a finite time.

Similarity, the deviation of i-th, ($i = 2,\, 3,\, \cdots,\, n-1$) subsystem is

$$\begin{cases} \dot{\tilde{x}}_i = -\gamma_i |\tilde{x}_i|^{\frac{1}{2}} \operatorname{sgn}(\tilde{x}_i) + \Xi_i, \\ \dot{\Xi}_i = -\delta_i \operatorname{sgn}(\tilde{x}_i) + \dot{d}_i(t). \end{cases} \tag{16}$$

γ_i, δ_i can be also obtained according to the gain adaption law shown in (11). According to Lemma 1, it can be known that the estimation error $\tilde{x}_i$, $\dot{\tilde{x}}_i$ would be convergence to zero in a finite time.

Make recursion according to this method, until the n-th subsystem, one can obtain

$$\begin{cases} \dot{\tilde{x}}_n = -\gamma_n |\tilde{x}_n|^{\frac{1}{2}} \operatorname{sgn}(\tilde{x}_n) + \Xi_n, \\ \dot{\Xi}_n = -\delta_n \operatorname{sgn}(\tilde{x}_n) + \dot{d}_n(t). \end{cases} \tag{17}$$

Then the deviation of the last subsystem $\tilde{x}_n, \dot{\tilde{x}}_n$ will converge to zero in a finite time. So the whole system states would converge to zero in a finite time.

Until now, the proof of Theorem 1 is finished. □

4. CONTROL DESIGN METHODOLOGY AND STABILITY ANALYSIS

4.1. Proposed procedure

The proposed control technique is devised mainly by BSC strategy compounding the finite time convergence features of a nonsingular fast integration terminal sliding mode control, mismatched uncertainty disturbance estimation of ASTSM-NDO without pre-acknowledging upper-bound of its first-derivation. Besides, in order to overcome the defect of "differential explosion" caused by mult-iple derivation of virtual control quantity in BSC strategy, DSC method through a FOLPF is utilized. According to the existing pieces of literatures [21-24], DSC method incorporates a FOLPF with an aid of properly chosen bandwidth to accommodate the influence of mismatched unknown disturbance.

Control design procedure is divided into n-steps, before initiating the designate procedure; let us define the states tracing deviation of system (1)

$$\begin{cases} e_1 = x_1 - x_d, \\ e_i = x_i - \alpha_{i-1}, \; i = 2,\, 3,\, \cdots,\, n, \end{cases} \tag{18}$$

where α_i is the ideal virtual control of i-th subsystem.

Step 1: The first subsystem virtual control schemes designate and stability analysis.

From (18), taking derivation of $e_1 = x_1 - x_d$ with t, substituting (1), one can get

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_d = x_2 + d_1 - \dot{x}_d \\ &= e_2 + \alpha_1 + d_1 - \dot{x}_d. \end{aligned} \tag{19}$$

The virtual control law of the first subsystem is designed as

$$\beta_1 = -(\kappa_1 e_1 + \hat{d}_1 - \dot{x}_d), \tag{20}$$

where $\kappa_1 > 0$, $\hat{d}_1$ is the estimation of d_1 and is obtained by ASTSM-NDO. In order to overcome the defect of 'differential explosion' phenomenon, the FOLPF is introduced as

$$\tau_1 \dot{\alpha}_1 + \alpha_1 = \beta_1, \; \alpha_1(0) = \beta_1(0), \tag{21}$$

where τ_1 is the time parameter of FOLPF. Define the first subsystem filter error of FOLPF as

$$\omega_1 = \alpha_1 - \beta_1. \tag{22}$$

Substituting (20) and (22) into (19), it can be obtained

$$\dot{e}_1 = -\kappa_1 e_1 + e_2 + \omega_1 - \tilde{d}_1. \tag{23}$$

From (21) and (22), one can get $\dot{\alpha}_1 = -\omega_1/\tau_1$. Taking derivation of (22) with time, it gets

$$\dot{\omega}_1 = \frac{-\omega_1}{\tau_1} - \dot{\beta}_1 = \frac{-\omega_1}{\tau_1} + (\kappa_1 \dot{e}_1 + \dot{\hat{d}}_1 - \ddot{x}_d). \tag{24}$$

According to (24), one can obtain

$$\left| \dot{\omega}_1 + \frac{\omega_1}{\tau_1} \right| \le \hbar_1(\dot{e}_1,\, \hat{d}_1,\, \dot{x}_d,\, \ddot{x}_d), \tag{25}$$

where $\hbar(e_1, \hat{d}_1, \dot{x}_d, \ddot{x}_d)$ is an unknown continuous bounded function.

$$\omega_1 \dot{\omega}_1 \le \frac{\omega_1^2}{\tau_1} + |\omega_1||\hbar_1| \le \left(1 - \frac{1}{\tau_1}\right)\omega_1^2 + \frac{\hbar_1^2}{4}. \tag{26}$$

Then, we define the first subsystem stability Lyapunov function as

$$V_1 = \frac{1}{2}e_1^2 + \frac{1}{2}\omega_1^2. \tag{27}$$

Taking derivation of V_1 with t, it can be obtained

$$\begin{aligned} \dot{V}_1 &= e_1 \dot{e}_1 + \omega_1 \dot{\omega}_1 \\ &= -\kappa_1 e_1^2 + e_1 e_2 + e_1 \omega_1 - e_1 \tilde{d}_1 + \omega_1 \dot{\omega}_1. \end{aligned} \tag{28}$$

Substituting (26) into (28), using the absolute inequality scaling technique, it leads

$$\begin{aligned} \dot{V}_1 &\le -\kappa_1 e_1^2 + e_1 e_2 + e_1 \omega_1 - e_1 \tilde{d}_1 (1 - \frac{1}{\tau_1})\omega_1^2 + \frac{\hbar_1^2}{4} \\ &\le -\kappa_1 e_1^2 + e_1 e_2 + |e_1||\omega_1| + |e_1||\tilde{d}| + (1 - \frac{1}{\tau_1})\omega_1^2 \\ &\quad + \frac{\hbar_1^2}{4}. \end{aligned} \tag{29}$$

According to Lemma 3, the items $|e_1||\omega_1|$, $|e_1||\tilde{d}|$ in (29) can be amplified to the following inequality expressions $|e_1||\omega_1| \le e_1^2 + \omega_1^2/4$, $|e_1||\tilde{d}| \le e_1^2 + \tilde{d}^2/4$ respectively, then (29) can be expressed as

$$\begin{aligned} \dot{V}_1 &\le -\kappa_1 e_1^2 + e_1 e_2 + |e_1||\omega_1| + |e_1||\tilde{d}| + (1 - \frac{1}{\tau_1})\omega_1^2 \\ &\quad + \frac{\hbar_1^2}{4} \\ &\le -\kappa_1 e_1^2 + e_1 e_2 + (e_1^2 + \omega_1^2/4) + (e_1^2 + \tilde{d}^2/4) \\ &\quad + (1 - \frac{1}{\tau_1})\omega_1^2 + \frac{\hbar_1^2}{4} \\ &\le -(\kappa_1 - 2)e_1^2 + e_1 e_2 + \frac{\tilde{d}_1^2}{4} + \frac{5\tau_1 - 4}{4\tau_1}\omega_1^2 + \frac{\hbar_1^2}{4}, \end{aligned} \tag{30}$$

where $\kappa_1 > 2$.

Step *i*: The virtual control schemes designate and stability analysis of i-th subsystem.

Taking derivation of the i-th subsystem states tracking error with t, substituting the i-th subsystem states dynamical equation in (1), it leads

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{\alpha}_{i-1},\ (i = 2,\ 3,\ \cdots,\ n-1) \\ &= x_{i+1} + d_i - \dot{\alpha}_{i-1} \\ &= e_{i+1} + \alpha_i + d_i - \dot{\alpha}_{i-1}. \end{aligned} \tag{31}$$

The virtual control law of the i-th subsystem is designed as

$$\beta_i = -(\kappa_i e_i + \hat{d}_i - \dot{\alpha}_{i-1}), \tag{32}$$

where $\kappa_i > 0$, $\hat{d}_i$ is the estimation of d_i and obtained by ASTSM-NDO. In order to overcome the defect of 'differential explosion' phenomenon in (32), according to FOLPF, thus $\dot{\alpha}_{i-1}$ is acquired by $\dot{\alpha}_{i-1} = -\omega_{i-1}/\tau_{i-1}$. The i-th subsystem FOLPF is introduced to get β_i.

$$\tau_i \dot{\alpha}_i + \alpha_i = \beta_i,\ \alpha_i(0) = \beta_i(0), \tag{33}$$

where τ_i is the time parameter of FOLPF. Define the i-th subsystem filter error of FOLPF as

$$\omega_i = \alpha_i - \beta_i. \tag{34}$$

Substituting (32) and (34) into (31), it can be obtained

$$\dot{e}_i = -\kappa_i e_i + e_{i+1} + \omega_i - \tilde{d}_i. \tag{35}$$

From (33) and (34), one can get $\dot{\alpha}_i = -\omega_i/\tau_i$. Taking derivation of (34) with time, it gets

$$\dot{\omega}_i = \frac{-\omega_i}{\tau_i} - \dot{\beta}_i = \frac{-\omega_i}{\tau_i} + (\kappa_i \dot{e}_i + \dot{\hat{d}}_i - \ddot{\alpha}_{i-1}). \tag{36}$$

According to (36), one can obtain

$$\left| \dot{\omega}_i + \frac{\omega_i}{\tau_i} \right| \le \hbar_i(\dot{e}_i, \hat{d}_i, \dot{\alpha}_{i-1}, \ddot{\alpha}_{i-1}), \tag{37}$$

where $\hbar_i(\dot{e}_i,\ \hat{d}_i,\ \dot{\alpha}_{i-1},\ \ddot{\alpha}_{i-1})$ is an unknown continuous bounded function.

$$\omega_i \dot{\omega}_i \le \frac{\omega_i^2}{\tau_i} + |\omega_i||\hbar_i| \le (1 - \frac{1}{\tau_i})\omega_i^2 + \frac{\hbar_i^2}{4}. \tag{38}$$

Then, i-thsubsystem stability Lyapunov function is defined as

$$V_i = V_{i-1} + \frac{1}{2}e_i^2 + \frac{1}{2}\omega_i^2 = \sum_{j=1}^{i}(\frac{1}{2}e_j^2 + \frac{1}{2}\omega_j^2). \tag{39}$$

Taking derivation of V_i with t, using the absolute inequality scaling technique, it leads

$$\dot{V}_i = \sum_{j=1}^{i}(e_j \dot{e}_j + \omega_j \dot{\omega}_j)$$

$$= \sum_{j=1}^{i}(-\kappa_j e_j^2 + e_j e_{j+1} + e_j\omega_j - e_j\tilde{d}_j + \omega_j\dot{\omega}_j)$$
$$\le \sum_{j=1}^{i}(-\kappa_j e_j^2 + e_j e_{j+1} + e_j\omega_j - e_j\tilde{d}_j(1-\frac{1}{\tau_j})\omega_j^2 + \frac{\hbar_j^2}{4}$$
$$\le \sum_{j=1}^{i}(-\kappa_j e_j^2 + e_j e_{j+1} + |e_j||\omega_j| + |e_j||\tilde{d}_j| + (1-\frac{1}{\tau_j})\omega_j^2 + \frac{\hbar_j^2}{4}). \tag{40}$$

According to Lemma 3, when $i = n-1$, (40) can be expressed as

$$\dot{V}_{n-1} \le e_{n-1}e_n - \sum_{i=1}^{n-1}\left(\kappa_i e_i^2 + |e_i||\omega_i| + |e_i||\tilde{d}_i| + \left(1-\frac{1}{\tau_i}\right)\omega_i^2 + \frac{\hbar_i^2}{4}\right)$$
$$\le e_{n-1}e_n - \sum_{i=1}^{n-1}(\kappa_i - 2)e_i^2 + \sum_{i=1}^{n-1}\left[\frac{\tilde{d}_i^2}{4} + \frac{5\tau_i - 4}{4\tau_i}\omega_i^2 + \frac{\hbar_i^2}{4}\right], \tag{41}$$

where $\kappa_i > 2$, if we make e_n convergence to zero, then it should get

$$\dot{V}_{n-1} \le -\xi V_{n-1} + \Theta, \tag{42}$$

where μ, Θ are positive constants chosen by

$$\xi = \min_{1\le i\le n-1}\left\{2(\kappa_i - 2),\ -\frac{(5\tau_i - 4)}{2\tau_i}\right\},$$
$$\Theta = \sum_{i=1}^{n-1}\left[\frac{\tilde{d}_i^2}{4} + \frac{\hbar_i^2}{4}\right].$$

Step *n*: The actual control schemes designate and stability analysis of n-th subsystem.

In order to make e_n converge to zero in a finite time and avoid the singular defects of traditional terminal sliding mode control, a nonsingular integral terminal sliding mode surface is utilized in this study, which is designed as

$$\begin{cases} \sigma = \int_0^t e_n d\tau, \\ s = \sigma + \eta\dot{\sigma}^{p/q}, \end{cases} \tag{43}$$

where $\eta > 0$, p, q are positive odd constants and $1 < p/q < 2$. Supposing that the sliding surface shown in (43) converges to zero at t_r, it can be seen from (43) that the integral and derivative of tracking error will converge to zero in a finite time, and the convergence time is

$$t_s = t_r + \eta^{q/p}\frac{p}{p-q}|\sigma(t_r)|^{(p-q)/p}. \tag{44}$$

Design the actual control law as

$$\begin{cases} u_{eq} = -[g(\bar{x}_n)]^{-1}\left[f_n(\bar{x}_n) + \hat{F} + \hat{d}_n - \dot{\alpha}_{n-1} + \frac{p}{\eta q}\dot{\sigma}^{2-p/q}\right], \\ u_{sw} = -[g(\bar{x}_n)]^{-1}[h_1\tanh(\mu s) + h_2 s], \\ u = u_{eq} + u_{sw}. \end{cases} \tag{45}$$

Meanwhile, design the system modeling error uncertainties estimation law as

$$\dot{\hat{F}} = s\zeta\eta\frac{p}{q}\dot{\sigma}^{p/q-1}, \tag{46}$$

where $h_1 > |\tilde{d}_n|$, $h_2 > 0$, $\mu > 0$ are predesigned parameters, u_{eq} is equivalent control and $u_{sw}$is the switching control. $\hat{F}$ is the estimation of F.

Remark 5: In order to suppress the chattering defect in the sliding mode control law, the continuous hyperbolic tangent function $\tanh(\mu s)$ is used to replace the symbolic function $\mathrm{sgn}(s)$ in the traditional sliding mode control law, when regulation parameter $\mu \to \infty$, then $\tanh(\mu s) \to \mathrm{sgn}(s)$, so as to ensure the smoothness of the sliding mode control signal and weaken the chattering defect. Similar to the previous steps, FOLPF technique is used to generate $-\omega_{n-1}/\tau_{n-1}$to replace $\dot{\alpha}_{n-1}$ in the actual control law.

4.2. Stability proof

The actual control scheme of the nonsingular fast terminal sliding mode control based on ASTSM-NDO is stated as Theorem 2.

Theorem 2: For a class of n-dimensional high-order uncertain nonlinear systems with modeling errors and mismatched unknown disturbances, satisfying Assumptions 1 and 2, design the ASTSM-NDO as (10) and (11) to tackle with the nonlinear disturbance and design an adaptive nonsingular integral terminal sliding mode control scheme as (45) and (46), then the tracking deviation e_n would convergence to zero in a finite time and all systems states error $e_1, e_2, \cdots, e_{n-1}$ would be uniformly ultimately bounded.

Proof: Mark the estimated value of the system unmolded error item F as $\hat{F}$, $\tilde{F}$ is the deviation between the estimated value and the real value, and then it has $\tilde{F} = F - \hat{F}$. According to Assumption 2, one can get $\dot{\tilde{F}} \approx -\dot{\hat{F}}$. Taking derivative of (43) with t, it leads

$$\dot{s} = \dot{\sigma} + (\eta p/q)\dot{\sigma}^{p/q-1}\ddot{\sigma}$$
$$= \frac{\eta p}{q}\dot{\sigma}^{p/q-1}\left(\frac{q}{\eta p}\dot{\sigma}^{2-p/q} + \ddot{\sigma}\right). \tag{47}$$

Define the n-th subsystem Lyapunov function as

$$V_n = \frac{1}{2}s^2 + \frac{1}{\eta}\tilde{F}^2. \tag{48}$$

Taking derivation of (48) with t, one can obtain

$$\begin{aligned}\dot{V}_n &= s\dot{s} - \tilde{F}\dot{\hat{F}}/\eta \\ &= s\eta\frac{p}{q}\dot{\sigma}^{p/q-1}\left(\frac{q}{\eta p}\dot{\sigma}^{2-p/q} + \ddot{\sigma}\right) - \tilde{F}\dot{\hat{F}}/\eta. \end{aligned} \tag{49}$$

From (43), it has $\dot{\sigma} = e_n$, $\ddot{\sigma} = \dot{e}_n$, with

$$\begin{aligned}\ddot{\sigma} &= \dot{e}_n = \dot{x}_n - \dot{\alpha}_{n-1} \\ &= f(\bar{x}_n) + g(\bar{x}_n)u + F(\bar{x}_n) + d_n(t) - \dot{\alpha}_{n-1}. \end{aligned} \tag{50}$$

Substituting (45), (46) and (50) into (49), one can get

$$\begin{aligned}\dot{V}_{n-1} &= \eta\frac{p}{q}\dot{\sigma}^{p/q-1}(-h_1|s| - h_2 s^2 - s\tilde{d}_n) \\ &\le \eta\frac{p}{q}\dot{\sigma}^{p/q-1}[-(h_1 - |\tilde{d}_n|)|s| - h_2 s^2]. \end{aligned} \tag{51}$$

1) When $\dot{\sigma} \neq 0$, it also leads $\dot{V}_{n-1} = 0$ in a finite time from (51), because $1 < p/q < 2$ and p, q are positive odd constants.
2) When $\dot{\sigma} = 0$ but $s \neq 0$, reference [30] has proved that the system states converge to the sliding mode surface $s = 0$. When the system states converge to sliding mode surfaces$s = 0$, σ and $\dot{\sigma}$would converge to zero, then e_n would converge to zero in a finite time.

When e_n converges to zero, by multiplying $e^{\eta t}$ on both sides of (42) and making integration on $[0, t]$, one can obtain

$$\begin{aligned}&\frac{d(V_{n-1}e^{\eta t})}{dt} \le e^{\eta t}\Theta \Rightarrow \int_0^t \frac{d(V_{n-1}e^{\eta t})}{dt} \le \int_0^t e^{\eta t}\Theta \\ &\Rightarrow V_{n-1} \le \frac{\Theta}{\eta} + \left[V_{n-1}(0) - \frac{\Theta}{\eta}\right]e^{-\eta t}. \end{aligned} \tag{52}$$

Then the whole system Lyapunov function satisfies the following equations

$$\begin{aligned}V &= V_{n-1} + V_n \\ &= \sum_{i=1}^{n-1}\left(\frac{1}{2}e_i^2 + \frac{1}{2}\omega_i^2\right) + \frac{1}{2}s^2 + \frac{1}{\eta}\tilde{F}^2 \le 0. \end{aligned} \tag{53}$$

From the above proof process, it can obtain the following conclusions:

1) Under the proposed control algorithms, all the states deviation e_i $(i = 1, 2, \cdots, n)$ can converge to zero in a finite time and all the system states are uniformly ultimately bounded, because the virtual control law α_i $(i = 2, 3, \cdots, n-1)$ and β_i $(i = 1, 2, \cdots, n)$ are bounded. Meanwhile, the tracking error $e_1 = x_1 - x_d$ is uniformly ultimately bounded obviously, because the reference tracking signal x_d is bounded, thus the system state x_1 is bounded.
2) ASTSM-NDO can fast accurately estimate the unknown upper-bound mismatched disturbance, and the estimation error $\tilde{d}_i$ $(i = 1, 2, \cdots, n)$ would converge to zero in a finite time.

Until now, the control algorithm proof is finished. □

5. NUMERICAL EXAMPLE AND SIMULATION STUDY

In this section, two examples are taken to verify the priority and effectiveness of the proposed control technique in this paper: the classical example in many pieces of literature and the real engineering, respectively.

Example 1: Consider the following three-order nonlinear system in forms of system (1) as

$$\begin{cases}\dot{x}_1 = x_2 + 0.5\sin(2t) + \sin(t), \\ \dot{x}_2 = x_3 + te^{-0.5t}, \\ \dot{x}_3 = x_1x_2x_3 + x_1\sin(x_1) + 5u + 0.01x_2e^{-0.5x_1} \\ \qquad + 0.8\sin(t), \\ y = x_1, \end{cases} \tag{54}$$

where $d_1(t) = 0.5\sin(2t) + \sin(t)$, $d_2(t) = te^{-0.5t}$, $d_3(t) = 0.8\sin(t)$, $f(x) = x_1x_2x_3 + x_1\sin(x_1)$, $g(x) = 5$, $F(x) = 0.01x_2e^{-0.5x_1}$. $x_d(t) = 0.5\sin(t) + 0.5\sin(0.5t)$ is the reference signal. The main control object of Example 1 is to use the proposed control technique to achieve the following target: 1) All the signals in closed-loop system of (54) are bounded; 2) $\hat{d}_i$ can precisely estimate the unknown mismatched disturbance d_i by ASTSM-NDO; 3) The system output tracking error converges to zero in a finite time.

The system intimal values are $x_1(0) = 0.5$, $x_2(0) = 0.5$, $x_3(0) = 0.5$. ASTSM-NDO parameters are $\kappa_1 = 0.5$, $\kappa_2 = 0.5$, $\kappa_3 = 0.5$, $\varepsilon_1 = 2$, $\varepsilon_2 = 2$, $\varepsilon_3 = 2$, $\rho_1 = 1$, $\rho_2 = 1$, $\rho_3 = 1$, $\theta_1 = 5$, $\theta_2 = 5$, $\theta_3 = 5$. The parameters of FOLPF are $\tau_1 = 0.01$, $\tau_2 = 0.01$. The parameters of controller in (45) (46) are $p = 7$, $q = 5$, $\eta = 0.01$, $\xi = 10$, $h_1 = 0.5$, $h_2 = 8$, $\mu = 500$. With the MATLAB routine, the simulation step is 0.001 and simulation results are illustrated in Figs. 1-7.

As revealed in Fig. 1, the system output signal $y = x_1$can trace the desired trajectory$x_d$with a good tracking performance. Fig. 2 reflects the system states deviation under the action of the controller. From the enlarged scope, it can be seen that the system tracing error e_1 is within the desired steady preset and the dynamic quantity is ideal, which verifies the robustness of the controller. The BSC method compounding with the DSC technique can make

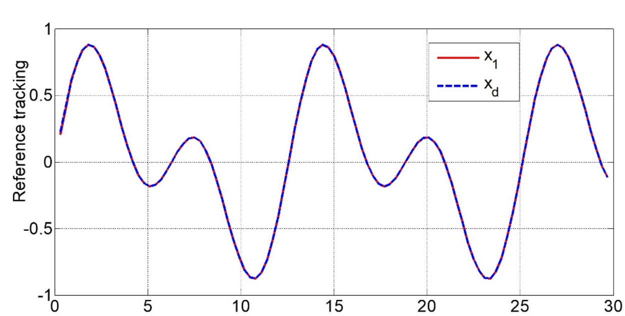


Fig. 1. System trajectory under of the controller.

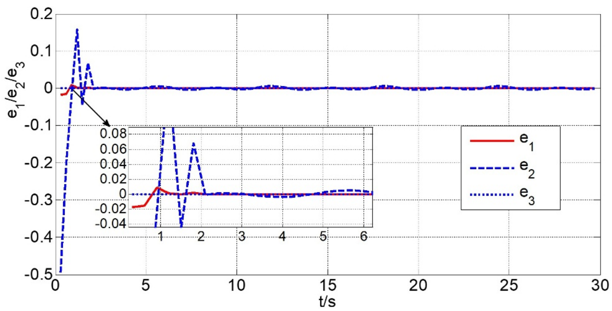


Fig. 2. System states deviation of the controller.

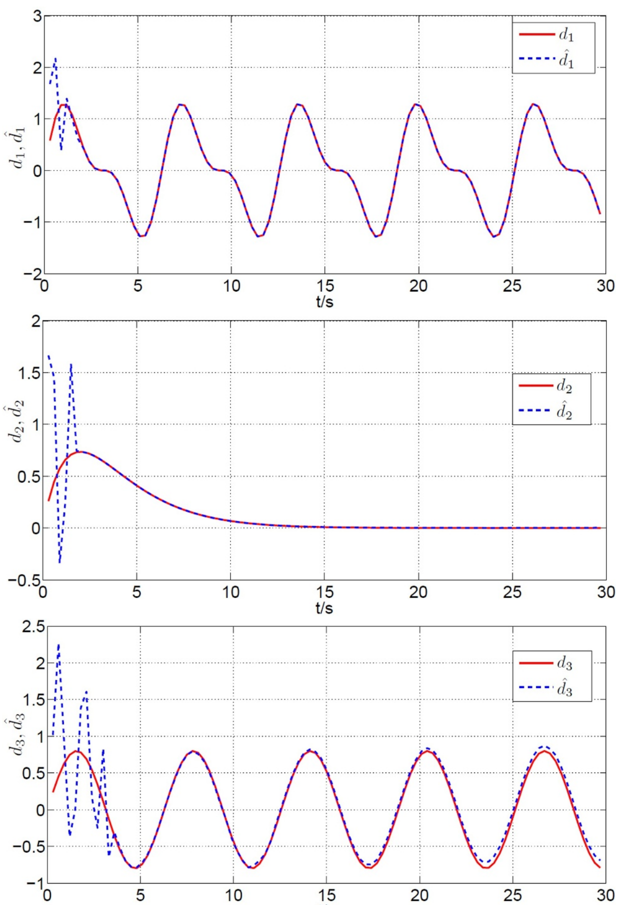


Fig. 3. Mismatched disturbance and ASTSM-NDO estimation disturbance.

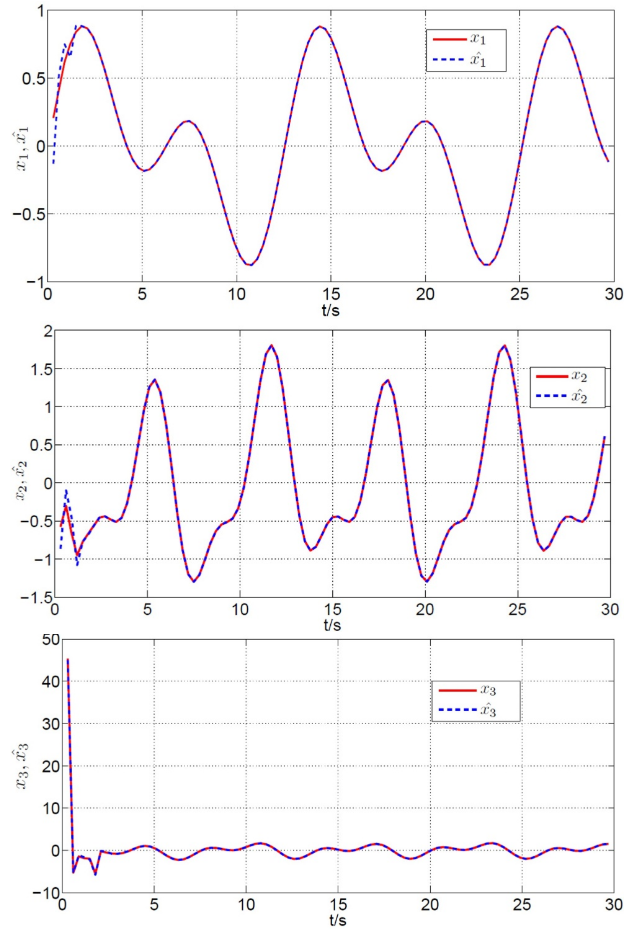


Fig. 4. System states and estimation states of ASTSM-NDO.

the whole system states bounded and trace the actual states under the proposed control schemes.

Fig. 3 demonstrates the estimation effects of unknown mismatched disturbance under the ASTSM-NDO. As discussed in Lemma 4, without the exact upper-bound value of the disturbance's derivation, the ASTSM-NDO can effectively trace the disturbance signals in a finite time and the estimation deviation will converge to zero in a finite time. Fig. 4 exhibits the evolution of ASTSM-NDO states. This figure suggests that the estimation states of ASTSM-NDO can accurately track the actual states with the reference signal. As stated in Section 3, the main contribution of the ASTSM-NDO is to suppress the mismatched disturbance.

The adaption laws are illustrated in Fig. 5, from the adaption law curves, it can get the conclusion that the adaptive-gain control law of ASTSM-NDO can adapt it due to the actual control effects, which intelligently handled the perturbed plant dynamics with the unknown boundary. The designed adaptive parameters have realized the actual application with the disturbance and have a real practice engineering meanings. Fig. 6 reveals the adaption law of modeling error of the system. The curve manifests the mismatched uncertainty of the system is estimated by the designed adaption law precisely, which realizes the estimation deviation stability and boundeness.

Fig. 7 presents the actual control signal u under the proposed control technique. The figure implies that the control signal is bounded and totally smooth. From (45), one can get that the control scheme is constructed by the equivalent control item and switching control item for disturbance compensation, which are both bounded

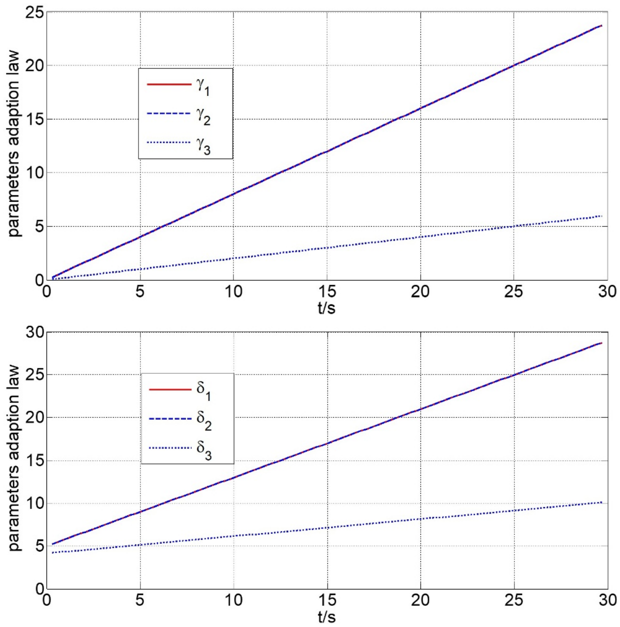


Fig. 5. Adaptive-parameters law of ASTSM-NDO.

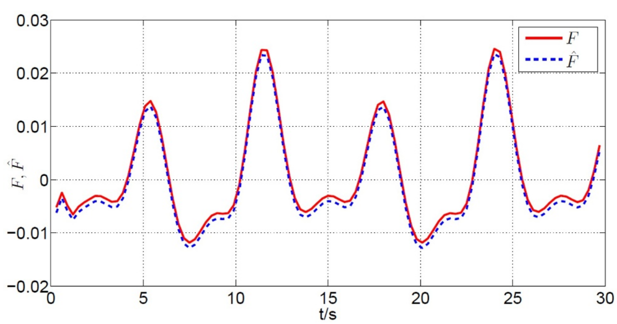


Fig. 6. Matched uncertainness of the modeling error.

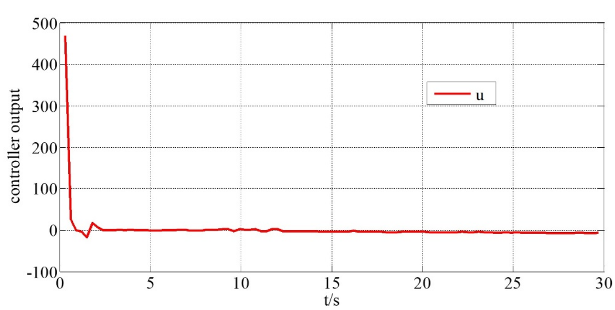


Fig. 7. Actual control signal of the proposed controller.

and stability. Moreover, the $\tanh(\mu s)$ function is utilized to replace the traditional symbolic function $sign(s)$in the switching control item. That is the main reason why the actual control law signal is smooth in such the sophisticated systems control, which contributes to reduce the energy-consumption of the controller. Thus, it especially has a greater practical application value.

Example 2 [31]**:** In order to test the engineering application value of the designed control algorithm, the magnetic levitation model is verified. The magnetic levitation dynamic model given in reference [31] is shown as

$$\begin{cases} \dot{x}_1 = x_2 + d_1, \\ \dot{x}_2 = x_3 + d_2, \\ \dot{x}_3 = f(x,t) + g(x,t)u + d_3, \end{cases} \tag{55}$$

where x_1, x_2, x_3 denote the system's position, the system velocity, and the current in the coil of the electromagnet, respectively; $f(x,t) = -6x_1 - 2.92x_2 - 1.2x_3 + x_1^2$ is the sy-stem function, $g(x,t) = 1$ is the control input gain. u denotes the input variable. d_1, d_2, d_3 indicate the mismatched unknown disturbance marked in (56). The objective of the control schemes is to drive the states x_1, x_2 and x_3 to their desired constant values x_{1d}, x_{2d}, x_{3d}, respectively.

$$\begin{cases} d_1 = 0.04\sin(t), \\ d_2 = 0.03\cos(2t) + 0.01, \\ d_3 = 0.02\sin(2t) - 0.01\cos(t). \end{cases} \tag{56}$$

In order to verify the progressiveness of the control algorithm, the TSMC algorithm in reference [32] is used for comparative experiments. Control algorithms in [32] were based on two-order sliding surface with integral function designed as (57) and the conventional nth order super-twisting disturbance observer was utilized shown in (5) and (10).

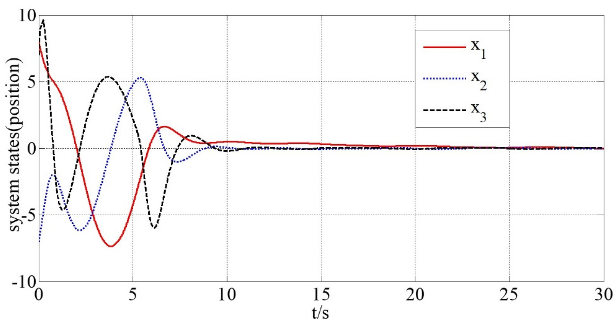
$$\begin{cases} \begin{cases} s = x_3 + \int_0^{20} (x_1 + x_2 + 1.5x_3)d\tau, \\ \sigma = \dot{s} + 2s + 0.5s^{0.6}, \end{cases} \\ \dot{u} = -\frac{1}{c}\Big\{ (\alpha + \mu\beta s^{\beta-1})(cu(t) + \sum_{i=1}^{3} a_i x_i) + a_n cu(t) \\ \quad + \sum_{i=1}^{2} a_i x_{i+1} + \phi\xi + k_2 \mathrm{sgn}(\sigma)\,|\sigma|^{\beta} + k_1\sigma \Big\}, \end{cases} \tag{57}$$

where

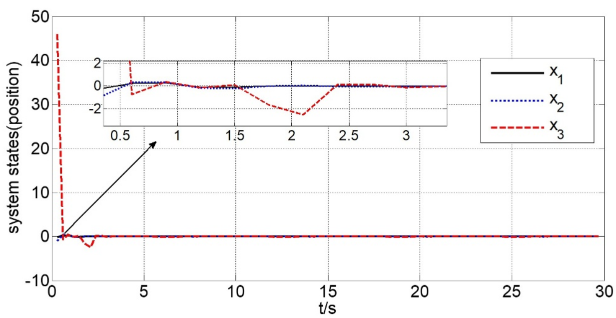
$$\begin{aligned} \phi = & \sum_{i=1}^{3} a_i \hat{d}_i + \dot{f}(x,t) + f(x,t)(\alpha + a_n + \mu\beta s^{\beta-1}) \\ & + \dot{\hat{d}}_n + a_n \hat{d}_n + \mu\beta \hat{d}_n s^{\beta-1}. \end{aligned} \tag{58}$$

$\alpha = 2$, $\beta = 0.6$, $\mu = 0.5$, $a_1 = 1$, $a_2 = 1$, $a_3 = 1.5$, $k_1 = 0.1$, $k_2 = 6$,$c = 1$. Conventional super twisting sliding mode observer parameters are selected as $\gamma = 2.5$, $\delta = 3.5$. The simulation results are exhibited in Figs. 8-10.

From Fig. 8, we can clearly see that the traditional control algorithm in [32] takes a long time for the system states to converge to the equilibrium position, and the stabilization adjustment time is about 10 seconds. Moreover, the system overshoot is large; there is a certain degree

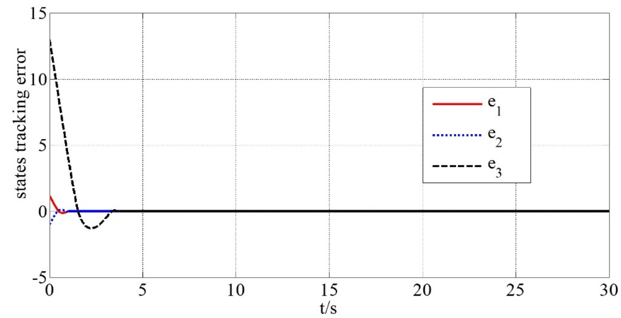


(a) The states stabilization effect in [32].

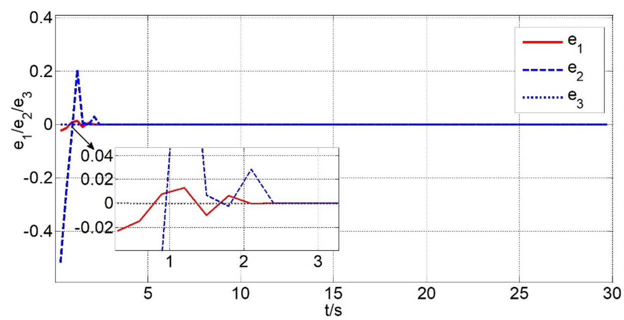


(b) States stabilization effect of the proposed method.

Fig. 8. The system states stabilization effects under two control method.



(a) States stability deviation in [32].



(b) States deviation of the proposed control method.

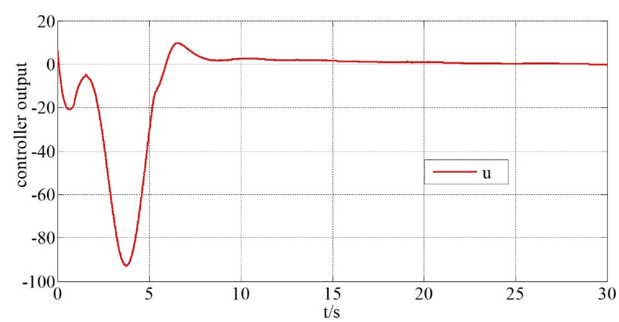
Fig. 9. States stability deviation of two control methods.

of steady-state error. Compared with the proposed control method in this paper, it has a better control effects in each control aspects. Fig. 9 reveals the system states stabilization error between the two different control methods. From two type curves, it can be seen clearly that the states stability deviation performance is much better in the dynamic and steady aspects compared with the conventional method. The tracking error converges to zero in a finite time and it has not existed steady error.

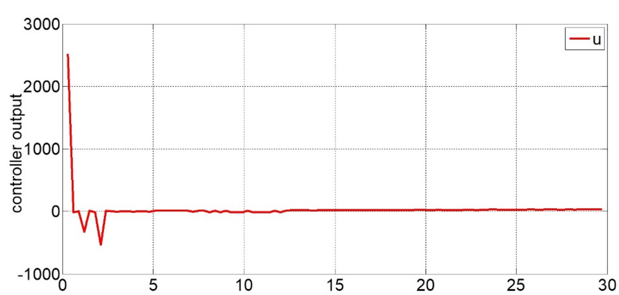
Fig. 10 displays the controller output signals of the two methods. Although chattering effect of the controller output signals are eliminated and they are entirely smooth, there exists a higher peak point in conventional method in [32], which suggests that the proposed control method has achieved a better control performance. This case implies that the controller designed in this paper has a good engineering case in application background.

From the simulation process, the main conclusion can be summarized as follows:

1) The performance of the ASTSM-NDO is not affected by the exact value of mismatched disturbance derivation as seen in (10) and (11), and it can estimate the ideal mismatched disturbance signals (Fig. 3) and the observer states is in the finite time tracing the real systems states (Fig. 4). Meanwhile, the designed observer can adapt the gain value (Fig. 5) without preassigning, but it is affect by the parameters of θ, ρ.



(a) Controller output in [32].



(b) Controller output under the proposed control method.

Fig. 10. Controller signal u of two control methods.

2) The proposed control algorithm can guarantee the fast convergence performance (Figs. 1, 2, and 8) and the smooth control signals (Figs. 7 and 10).

6. CONCLUSION

A new fast integral nonsingular terminal sliding mode control algorithm is constructed to accurately track the desired reference signals and ASTSM-NDO is proposed to solve the influence of the unmatched unknown upper-bound disturbance. Meanwhile, the BSC technique compounded with FOLPF is utilized to cope with the high-order nonlinear systems subjected to unknown mismatched disturbance. Moreover, hyperbolic tangent function is used to replace the traditional symbolic function to reduce chattering in SMC technique. Finally, the effectiveness and feasibility of the proposed control method are verified with two examples, the derived results have ren-dered much better performance. In the future, the proposed control method will be improved from two aspects. First, it is a very worthy research direction to study the influence of parameter selection on control accuracy. Second, FOLPF is an effective technique to eliminate the defect of "differential explosion", but it is a low-pass filter, which generates an equivalent signal of the virtual control law. It also produce an equivalent error and the cumulative error is getting bigger and bigger and it should not be ignored to affect the control accuracy, so we will explore a new error compensation DSC technology to compensate the dynamic surface error effect on the systems. Thus, we will focus on the improvement of the future work in the next research.

APPENDIX A: PROOF OF LEMMA 1

Subsituting (4) into (6) and omitting states varaiable subscript and parameters subscript, one can obtain

$$\begin{cases}\dot{\hat{x}} = -\gamma|\hat{x}|^{\frac{1}{2}}\mathrm{sgn}(\hat{x}) + \chi, \\ \dot{\chi} = -\delta\mathrm{sgn}(\hat{x}) + \dot{d}.\end{cases} \tag{A.1}$$

Let the symmetric positive matrix as

$$P = \frac{1}{2}\begin{bmatrix}4\delta+\gamma^2 & -\gamma \\ -\gamma & 2\end{bmatrix}. \tag{A.2}$$

Construct a real vector described by

$$\varsigma^{\mathrm{T}} = \begin{bmatrix}|\hat{x}|^{\frac{1}{2}}\mathrm{sgn}(\hat{x}) & \chi\end{bmatrix}. \tag{A.3}$$

Then quasi quadratic Lyapunov function is tranformed as

$$V(\hat{x},\chi) = \varsigma^{\mathrm{T}}P\varsigma. \tag{A.4}$$

Let $A = \begin{bmatrix}-\gamma & 1/2 \\ -\delta & 0\end{bmatrix}$, $B = \begin{bmatrix}0 & 1\end{bmatrix}^{\mathrm{T}}$, $C = \begin{bmatrix}1 & 0\end{bmatrix}$, $\bar{\dot{d}} = |\varsigma|\dot{d}$. Take derivation of ς with time, it can get

$$\dot{\varsigma} = \begin{bmatrix}\frac{1}{2|\hat{x}|^{1/2}}(-\gamma|\hat{x}|^{1/2}\mathrm{sgn}(\hat{x})) + \chi \\ -\delta\mathrm{sgn}(\hat{x})\end{bmatrix} = \frac{1}{|\varsigma_1|}(A\varsigma + B\bar{\dot{d}}). \tag{A.5}$$

Take derivation of V along the system trajectory, one can obtain

$$\begin{aligned}&\dot{V}(\hat{x},\chi) \\ &= \frac{1}{|\varsigma_1|}\begin{bmatrix}\varsigma \\ \bar{\dot{d}}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}A^{\mathrm{T}}P+PA & PB \\ B^{\mathrm{T}}P & 0\end{bmatrix}\begin{bmatrix}\varsigma \\ \bar{\dot{d}}\end{bmatrix} \\ &\le \frac{1}{|\varsigma_1|}\left\{\begin{bmatrix}\varsigma \\ \bar{\dot{d}}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}A^{\mathrm{T}}P+PA & PB \\ B^{\mathrm{T}}P & 0\end{bmatrix}\begin{bmatrix}\varsigma \\ \bar{\dot{d}}\end{bmatrix} + L^2\varsigma^2 - (\bar{\dot{d}})^2\right\} \\ &\le \frac{1}{|\varsigma_1|}\varsigma^{\mathrm{T}}(A^{\mathrm{T}}P+PA+L^2C^{\mathrm{T}}+PBB^{\mathrm{T}}P)\varsigma.\end{aligned} \tag{A.6}$$

Let $A^{\mathrm{T}}P+PA+L^2C^{\mathrm{T}}+PBB^{\mathrm{T}}P = -Q < 0$, then it has $\dot{V}(\hat{x},\chi) \le \frac{1}{|\varsigma_1|}\varsigma^{\mathrm{T}}Q\varsigma$, where

$$Q = \begin{bmatrix}\gamma\delta + \frac{\gamma^2}{2} - L^2 - \frac{\gamma^2}{4} & \frac{\gamma-\gamma^2}{2} \\ \frac{\gamma-\gamma^2}{2} & \frac{\gamma-2}{2}\end{bmatrix}. \tag{A.7}$$

Substituting (5) back into (A.7), it obviously get Q is symmetric positive.

This is the end of proof of Lemma 1. □

APPENDIX B: PROOF OF LEMMA 4

Using inequality $d|\hat{x}|/dt = \dot{\hat{x}}\mathrm{sgn}(\hat{x})$, From (A.1) and (A.3), let $\dot{\varsigma}_1 = \big(-\gamma|\hat{x}|^{\frac{1}{2}}\mathrm{sgn}(\hat{x}) + \chi\big)/\big(2|\hat{x}|^{\frac{1}{2}}\big)$, $\dot{\varsigma}_2 = -\delta\mathrm{sgn}(\hat{x}) + \dot{d}(t)$, $\overset{\frown}{\dot{d}} = |\hat{x}|^{\frac{1}{2}}\dot{d}$, $A = \begin{bmatrix}-\gamma & 1/2 \\ -\delta & 0\end{bmatrix}$, $B = \begin{bmatrix}0 & 1\end{bmatrix}^{\mathrm{T}}$, $C = \begin{bmatrix}1 & 0\end{bmatrix}$, respectively, we will get

$$\dot{\varsigma} = \frac{1}{|\hat{x}|^{\frac{1}{2}}}(A\varsigma + B\overset{\frown}{\dot{d}}). \tag{B.1}$$

Consider the following candidate Lyapunov function

$$\begin{aligned}V(\varsigma,\gamma,\delta) = V(\hat{x},\chi) + (\gamma-\gamma^*)^2/2\beta_1 \\ + (\delta-\delta^*)^2/2\beta_2,\end{aligned} \tag{B.2}$$

where $V(\hat{x},\chi) = \varsigma^{\mathrm{T}}P\varsigma$, β_1, β_2, γ^*, δ^* are positive constants, P is shown as (A.2) and noting that P is a symmetric positive matrix. Take derivation of $V(\hat{x},\chi)$ with t, it get

$$\begin{aligned}&\dot{V}(\hat{x},\chi) \\ &= 2\frac{1}{|\chi|^{\frac{1}{2}}}(\varsigma^{\mathrm{T}}A^{\mathrm{T}} + \overset{\frown}{\dot{d}}B^{\mathrm{T}})P\varsigma \\ &\le \frac{1}{|\hat{x}|^{\frac{1}{2}}}(2\varsigma^{\mathrm{T}}A^{\mathrm{T}}P\varsigma + 2\overset{\frown}{\dot{d}}B^{\mathrm{T}}P\varsigma + L|\hat{x}| - \overset{\frown}{\dot{d}}^2) \\ &= \frac{1}{|\hat{x}|^{\frac{1}{2}}}(2\varsigma^{\mathrm{T}}A^{\mathrm{T}}P\varsigma + 2\overset{\frown}{\dot{d}}B^{\mathrm{T}}P\varsigma + L^2\varsigma^{\mathrm{T}}C^{\mathrm{T}}C\varsigma - \overset{\frown}{\dot{d}}^2) \\ &\le \frac{1}{|\hat{x}|^{\frac{1}{2}}}(2\varsigma^{\mathrm{T}}A^{\mathrm{T}}P\varsigma + L^2\varsigma^{\mathrm{T}}C^{\mathrm{T}}C\varsigma + \varsigma^{\mathrm{T}}PBB^{\mathrm{T}}P\varsigma)\end{aligned}$$

$$= \frac{1}{|\hat{x}|^{\frac{1}{2}}} \varsigma^{\mathrm{T}} (A^{\mathrm{T}} P + PA^{\mathrm{T}} + L^2 C^{\mathrm{T}} C + PBB^{\mathrm{T}} P) \varsigma. \tag{B.3}$$

Let $Q = A^{\mathrm{T}} P + PA^{\mathrm{T}} + L^2 C^{\mathrm{T}} C + PBB^{\mathrm{T}} P$, then (B.3) can be expressed as

$$\dot{V}(\hat{x}, \chi) \leq -\frac{1}{|\hat{x}|^{\frac{1}{2}}} \varsigma^{\mathrm{T}} Q \varsigma. \tag{B.4}$$

Substituting A, B, C, P, one can get

$$Q = \begin{bmatrix} 2\gamma\rho + \frac{2\gamma\theta^2 - \theta^2}{4} - \delta\theta - L^2 & -\rho - \frac{\theta^2 + \gamma\theta - 2\theta}{4} + \delta \\ -\rho - \frac{\theta^2 + \gamma\theta}{4} & \frac{\theta - 2}{2} \end{bmatrix}.$$

In order to make Q positive, let $\delta = \rho + \frac{\theta^2 + \gamma\theta}{4}$ and substitute it back to Q, it can get

$$Q - \frac{\theta I}{4} = \begin{bmatrix} 2\gamma\rho + \frac{\gamma\theta^2 - \theta^3 - \theta^2 - \theta}{4} - \rho\theta - L^2 & \frac{\theta}{2} \\ \frac{\theta}{2} & \frac{\theta - 4}{4} \end{bmatrix}. \tag{B.5}$$

According to Shur's complement character, in order to get $Q - \frac{\theta I}{4} > 0$, it meets the condition $\lambda_{\min}(Q) > \frac{\theta}{4}$, then, it get

$$\begin{cases} \gamma > \dfrac{(\frac{\theta - 4}{4})(\frac{\theta^3 + \theta^2 + \theta}{4} + \rho\theta + L^2) + \frac{\theta^2}{4}}{(2\rho + \frac{\theta^2}{4})(\frac{\theta}{4} - 1)}, \\ \theta > 4. \end{cases} \tag{B.6}$$

From (B.4), it can obtain

$$\begin{aligned} \dot{V}(\hat{x}, \chi) &\leq -\frac{1}{|\hat{x}|^{\frac{1}{2}}} \varsigma^{\mathrm{T}} Q \varsigma \leq -\frac{\theta}{4|\varsigma_1|} \varsigma^{\mathrm{T}} \varsigma \\ &= -\frac{\theta}{4|\varsigma_1|} \|\varsigma\|^2 = -\frac{\theta \|\varsigma\|}{4|\varsigma_1|} \|\varsigma\|. \end{aligned} \tag{B.7}$$

From $\|\varsigma\|_2^2 = \varsigma_1^2 + \varsigma_2^2 = |\hat{x}| + \varsigma_2^2$, it can get $\|\varsigma\|_2 \geq |\varsigma_1|$, then (B.7) can be written as

$$\dot{V}(\hat{x}, \chi) \leq -\frac{\theta}{4} \|\varsigma\|_2. \tag{B.8}$$

According positive definite quadratic function

$$\lambda_{\min}(P) \|\varsigma\|_2^2 \leq V(\hat{x}, \chi) = \varsigma^{\mathrm{T}} P \varsigma \leq \lambda_{\max}(P) \|\varsigma\|_2^2, \tag{B.9}$$

it will get $(\frac{V(\hat{x},\chi)}{\lambda_{\max}(P)})^{\frac{1}{2}} \leq \|\varsigma\|_2$, combining (B.8), then we will obtain

$$\dot{V}(\varsigma) \leq -rV^{\frac{1}{2}}(\varsigma), \tag{B.10}$$

where $r = \frac{\theta}{4\lambda_{\max}^{\frac{1}{2}}(P)}$, then taking derivation of (B.2), it leads

$$\begin{aligned} &\dot{V}(\varsigma, \gamma, \delta) \\ &= -rV^{\frac{1}{2}}(\varsigma) + \frac{(\gamma - \gamma^*)}{\beta_1} \dot{\gamma} + \frac{(\delta - \delta^*)}{\beta_2} \dot{\delta} \\ &= -rV^{\frac{1}{2}}(\varsigma) - \frac{\kappa_1}{\sqrt{2\beta_1}} |\gamma - \gamma^*| - \frac{\kappa_2}{\sqrt{2\beta_2}} |\delta - \delta^*| \\ &\quad + \frac{(\gamma - \gamma^*)\dot{\gamma}}{\beta_1} + \frac{(\delta - \delta^*)\dot{\delta}}{\beta_2} + \frac{k_1}{\sqrt{2\beta_1}} |\gamma - \gamma^*| \\ &\quad + \frac{k_2}{\sqrt{2\beta_2}} |\delta - \delta^*| \\ &\leq -\min(r, k_1, k_2) \left(V(\varsigma) + \frac{(\gamma - \gamma^*)^2}{2\beta_1} + \frac{(\delta - \delta^*)^2}{2\beta_2} \right)^{\frac{1}{2}} \\ &\quad + \frac{(\gamma - \gamma^*)}{\beta_1} \dot{\gamma} + \frac{(\delta - \delta^*)}{\beta_2} \dot{\delta} + \frac{k_1}{\sqrt{2\beta_1}} |\gamma - \gamma^*| \\ &\quad + \frac{k_2}{\sqrt{2\beta_2}} |\delta - \delta^*|. \end{aligned} \tag{B.11}$$

If we choose (11) in Lemma 4, subsisting it in (B.11), it will exist positive constants γ^*, δ^*, which make $(\gamma - \gamma^*) < 0$, $(\delta - \delta^*) < 0$. Then (B.11) can be expressed as

$$\dot{V}(\varsigma, \gamma, \delta) = -\min(r, k_1, k_2) V^{\frac{1}{2}}(\varsigma) + \xi, \tag{B.12}$$

where $\xi = -(\frac{\dot{\gamma}}{\beta_1} - \frac{k_1}{\sqrt{2\beta_1}}) |\gamma - \gamma^*| - (\frac{\dot{\delta}}{\beta_2} - \frac{k_2}{\sqrt{2\beta_2}}) |\delta - \delta^*|$.

According to Lemma 2, $V(\varsigma, \gamma, \delta)$will converge to zero in a finite time.

This is the end of proof of Lemma 4. □

DECLARATION OF CONFLICTING INTERESTS

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