International Journal of Control, Automation, and Systems 22(3) (2024) 902-911
http://dx.doi.org/10.1007/s12555-022-0459-z
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Controller Design Based on Interval-observer for Switched Systems With Observer-state-dependent Switching

Chuanjing Wu, Yue-E Wang*, and Di Wu

Abstract: In this paper, we study the problem of interval-observer based controller design for switched linear systems with disturbance and measurement noise. First, we propose the definition of finite-time interval-observers for switched systems and construct finite-time interval-observers for switched linear systems with observer-state-dependent switching strategy by giving the ranges of the external disturbance and the measurement noise. Then, by using multiple Lyapunov functions method, some sufficient conditions are provided to guarantee the existence of the finite-time interval-observers under maximum constraints, and at the same time, based on the proposed sufficient conditions, the observer gains are given and the corresponding sliding motion issue is also solved. After that, we design an interval-observer-based feedback controller for the switched systems. Different from the traditional observer-based controller with state-dependent switching signal designed in most literatures, the controller designed in the paper is based on interval-observer with observer-state-dependent switching signal. Compared with the traditional observers, the interval-observers are robust to large uncertainties and can provide the ranges of the system states in any time. Finally, the feasibility of the proposed method is verified by two numerical examples.

Keywords: Multiple Lyapunov functions, observer-state-dependent switching, state observer, switched systems.

1. INTRODUCTION

Because both discrete and continuous dynamics are involved, the dynamic behavior of switched systems is very complex and attracts much attention [1-5]. Just a couple of examples, reference [1] gave stability and L2 gain analysis for switched nonlinear systems. Reference [2] designed a dwell-time-based observer for unknown input switched linear systems without requiring strong detectability of the subsystems. Reference [4] addressed the problem of robust switching design, which sought a switching signal that makes the switched system exponentially stable and robust against switching perturbations. The stability for a class of systems with time-varying delay subject to controller failure was discussed in [5]. Generally speaking, the research on switched system related problems mainly involves switching signals dependent on time or state. The signal for switching that is dependent on time refers to the resident time switching signal and its various more optimized extension forms [6-8]. State-dependent switching signals mainly involve maximum switching [9] and minimum switching [10]. For these two kinds of switching just mentioned, the multiple Lyapunov function method is the most important research tool. Many literatures prove their conclusions by using multiple Lyapunov function, such as reference [11] investigated a new approach for the bipartite (cooperative competitive) consensus control design for a class of nonlinear agents with Lipschitz dynamics under directed switching topologies by utilizing multiple Lyapunov functions, and reference [12] studied the problem of H_∞ control of switched nonlinear systems in p-normal form in this technical note under using the generalized multiple Lyapunov functions method.

Often, however, the state information of a switched system cannot be measured directly and needs to be estimated. Interval-observer is a recently proposed robust observation method for estimating the state of a system [13]. When the uncertain ranges are known, the interval-observer can monitor the upper and lower bounds of the state at any time. Therefore, the interval observer can estimate the state of systems with large delays better than the general state observer. Due to this advantage, interval-observer has got more and more scholars' attention [14-17]. However, due to the complexity of switched system structure and the requirement of upper and lower observer and system positivity, few scholars pay attention

Manuscript received May 27, 2022; revised November 27, 2022; accepted March 3, 2023. Recommended by Associate Editor Jun Cheng under the direction of Senior Editor PooGyeon Park. This work was supported by the National Natural Science Foundation of China under Grants 62273218, the National Key R&D Program of China under Grant 2018YFB1700100, and Funded Projects for the Academic Leaders and Academic Backbones, Shaanxi Normal University: 18QNGG005.

Chuanjing Wu and Yue-E Wang are with the School of Mathematics and Statistics, Shaanxi Normal University, Xi'an 710119, China (e-mails: wuchuanjing63@163.com, baihewye@snnu.edu.cn). Di Wu is with the School of Control Science and Enginering, Dalian University of Technology, Dalian 116024, China (e-mail: Diwu@dlut.edu.cn).
* Corresponding author.

to the construction of switched system interval observer [18-20].

In addition, we are concerned with the dynamic behavior of a system in a fixed time interval in many practical issues. For instance, we need to observe that some values cannot exceed the preset parameters during rocket launching; or in chemical experiments, the temperature and pressure shall not exceed the preset value [21]. For this purpose, in [22], the relevant definitions and conclusions of both finite-time stability and finite-time boundedness were given. While FTS is not concerned with disturbances, FTB is used to analysis the transient performances of a system involving external disturbances within a finite time interval. In recent years, both FTS and FTB have gained a lot of achievements [23-29]. For example, the FTS for nonlinear multiagent systems with input delay was studied in [30]. Reference [31] dealt with finite-time obsrevers for time-varying switched systems. What we want to emphasize here is that no literatures have been reported for finite-time interval-observers of switched systems with observer-state-dependent switching scheme.

In this paper, we study the design of controller based on interval-observer for a class of switched systems subject to observer-state-dependent switching. By giving the range of external disturbances and measurement noise, we propose the design method of finite-time interval-observers for the considered switched systems with observer-statedependent switching. Then, by using multiple Lyapunov functions method, some criteria are given and used to derive the observer gains and the corresponding sliding motion issue is also solved. After that, the feedback controller based on the designed interval-observer is further designed. Finally, two examples are given to demonstrate the effectiveness of the proposed method. A preliminary version of part of the results was presented in [32].

The remainder of the paper is organized as follows: Section 2 gives problem formulation, followed by main results in Section 3. Two examples are presented to illustrate the results in Section 4. Section 5 ends the paper.

Notations: $\mathrm{E_p}$ is the vector of $(p\times 1)$ whose elements are all 1. Matrix $P>0$ means that matrix P is positive definite, $\lambda_{\max}(P)(\lambda_{\min}(P))$ denotes the maximum (minimum) eigenvalue of matrix P. $\underline{x}$ and $\bar{x}$ are used to represent the lower and the upper bounds of a variable x and satisfy $\underline{x}\leq x\leq \bar{x}$. $|\cdot|$ denotes the element-wise absolute value of a vector $x\in\mathbb{R}^n$. For a matrix $A\in\mathbb{R}^{m\times n}$, define $A^+=\max\{0,A\}$ and $A^-=A^+-A$. Let $M=\{1,\ 2,\ \cdots,\ N\}$. $*$ means the elements below the main diagonal of a symmetric matrix. $He(A)=A^{\mathrm{T}}+A$. We assume that the state of the systems considered in the paper do not jump at the switching instants and that only finitely many switches can occur in any finite interval.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the switched linear systems as follows:

$$\begin{cases}\dot{x}(t)=A_\sigma x(t)+E_\sigma u(t)+k(t),\\ y(t)=C_\sigma x(t)+o(t),\end{cases}\tag{1}$$

where $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^p$, $u(t)\in\mathbb{R}^q$, $k(t)\in\mathbb{R}^n$, $o(t)\in\mathbb{R}^p$ are the state vector, the measurement output, the control input, the exogenous disturbance, and the measurement noise, respectively. Switching signal $\sigma(t):\mathbb{R}^+\to M$ is a piecewise constant function. For any $i\in M$, A_i, E_i, C_i are constant real matrices with appropriate dimensions. In addition, for any $i\in M$, we presume that the pair (A_i,C_i) are observable.

Assumption 1: The disturbance and the measurement noise are assumed to be unknown but bounded with priori known bounds such that $-\bar{k}(t)\leq k(t)\leq\bar{k}(t)$, $|o(t)|\leq \bar{O}\mathrm{E_p}$, are verified for $\forall t\in\mathbb{R}^+$, where, $\bar{O}>0$ and $\bar{k}(t)\in\mathbb{R}^n$ is a non-negative vector and is assumed to satisfy $\int_0^T\ \bar{k}^{\mathrm{T}}(s)\bar{k}(s)\,ds\leq d_1$, $d_1\geq 0$ for a fixed $T>0$.

Assumption 2: There exist gains L_i such that the matrices $\mathcal{A}_i=A_i-L_iC_i$ are Metzler for all $i\in M$. The matrices $L_i(i\in M)$ denote the observer gains associated with each subsystem i.

A candidate observer for the estimation of $\bar{x}$ and $\underline{x}$ is expressed as

$$\begin{cases}\dot{\bar{x}}=\mathcal{A}_i\bar{x}+E_iu+\mathcal{K}_i+L_iy,\\ \dot{\underline{x}}=\mathcal{A}_i\underline{x}+E_iu-\mathcal{K}_i+L_iy,\end{cases}\tag{2}$$

where $\mathcal{K}_i=\bar{k}+|L_i|\bar{O}\mathrm{E_p}$.

Let $\eta=\begin{bmatrix}\bar{x}^{\mathrm{T}} & \underline{x}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$. We construct the feedback controller

$$u=\begin{bmatrix}\overline{K}_i & \underline{K}_i\end{bmatrix}\eta,\tag{3}$$

where $\overline{K}_i$ and $\underline{K}_i$ are controller gains.

Let $\bar{e}(t)=\bar{x}-x$ and $\underline{e}(t)=x-\underline{x}$ be the upper estimation error and the lower estimation error, respectively. Therefore, the error systems can be established as

$$\begin{cases}\dot{\bar{e}}(t)=\mathcal{A}_i\bar{e}(t)+\varpi_{ui},\\ \dot{\underline{e}}(t)=\mathcal{A}_i\underline{e}(t)+\varpi_{li},\end{cases}\tag{4}$$

where $\varpi_{ui}=\mathcal{K}_i-k+L_io(t)$ and $\varpi_{li}=\mathcal{K}_i+k-L_io(t)$.

Let $\xi=\begin{bmatrix}\eta^{\mathrm{T}} & x^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$. We have

$$\dot{\xi}=\breve{A}_i\xi+\tilde{k}_i,\tag{5}$$

where

$$\breve{A}_i=\begin{bmatrix}\bar{A}_i & \bar{C}_i\\ \tilde{K}_i & A_i\end{bmatrix},\ \bar{A}_i=\begin{bmatrix}\mathcal{A}_i+E_i\overline{K}_i & E_i\underline{K}_i\\ E_i\overline{K}_i & \mathcal{A}_i+E_i\underline{K}_i\end{bmatrix},$$

$$\tilde{k}_i=\begin{bmatrix}\bar{k}_i\\ k\end{bmatrix},\ \bar{C}_i=\begin{bmatrix}L_iC_i\\ L_iC_i\end{bmatrix},$$

$$\tilde{K}_i = \begin{bmatrix} E_i\overline{K}_i & E_i\underline{K}_i \end{bmatrix}, \ \bar{k}_i = \begin{bmatrix} \mathcal{K}_i + L_i o(t) \\ -\mathcal{K}_i + L_i o(t) \end{bmatrix}.$$

According to Assumption 1,

$$\begin{aligned} \int_0^T o^{\mathrm{T}}(s)o(s)\,ds &\le \int_0^T p\bar{O}^2\,ds \\ &= p\bar{O}^2 T. \end{aligned}$$

Let $d_2 = p\bar{O}^2 T$. Thus $\int_0^T o^{\mathrm{T}}(s)o(s)\,ds \le d_2$ and

$$\begin{aligned} &\int_0^T (\mathcal{K}_i + L_i o(s))^{\mathrm{T}}(\mathcal{K}_i + L_i o(s))\,ds \\ &= \int_0^T \varsigma^{\mathrm{T}}(s) S \varsigma(s)\,ds \\ &\le 3\int_0^T (\bar{k}^{\mathrm{T}}\bar{k} + \mathrm{E}_{\mathrm{p}}^{\mathrm{T}}\bar{O}|L_i|^{\mathrm{T}}|L_i|\bar{O}\mathrm{E}_{\mathrm{p}} + o^{\mathrm{T}}(s)L_i^{\mathrm{T}}L_i o(s))\,ds \\ &\le 3\int_0^T (\bar{k}^{\mathrm{T}}\bar{k} + c_3 + c_4 o^{\mathrm{T}}(s)o(s))\,ds \\ &= 3d_1 + 3c_3 T + 3c_4 d_2, \end{aligned}$$

where $\varsigma^{\mathrm{T}}(s) = \begin{bmatrix} \bar{k}^{\mathrm{T}} & \mathrm{E}_{\mathrm{p}}{}^{\mathrm{T}}\bar{O}|L_i|^{\mathrm{T}} & o^{\mathrm{T}}(s)L_i^{\mathrm{T}} \end{bmatrix}$, $S = \begin{bmatrix} 1 & 1 & 1 \\ * & 1 & 1 \\ * & * & 1 \end{bmatrix}$, $c_3 = \max_{i\in M}\{E_p^{\mathrm{T}}\bar{O}|L_i|^{\mathrm{T}}|L_i|\bar{O}\mathrm{E}_{\mathrm{p}}\}$, $c_4 = \max_{i\in M}\{\lambda_{\max}(L_i^{\mathrm{T}}L_i)\}$.

Similarly, we can derive that

$$\begin{aligned} &\int_0^T (-\mathcal{K}_i + L_i o(s))^{\mathrm{T}}(-\mathcal{K}_i + L_i o(s))\,ds \\ &\le 3d_1 + 3c_3 T + 3c_4 d_2. \end{aligned}$$

Thus

$$\int_0^T \tilde{k}_i^{\mathrm{T}}\tilde{k}_i ds \le 7d_1 + 6c_3 T + 6c_4 d_2 = d. \tag{6}$$

Set

$$\mathbf{W} = \left\{\tilde{k}_i(t) \middle| \int_0^T \tilde{k}_i^{\mathrm{T}}(t)\tilde{k}_i(t)\,dt \le d\right\}. \tag{7}$$

Define the switching law in this paper based on the following largest region function strategy:

$$\sigma(t) = \arg\max_{i\in M} \xi^{\mathrm{T}}(t)\tilde{B}_i\xi(t), \tag{8}$$

where $\tilde{B}_i = \begin{bmatrix} B_i & 0 \\ 0 & P \end{bmatrix}$, with $B_i^{\mathrm{T}} = B_i \in \mathbb{R}^{2n\times 2n}$ and $P > 0, P \in \mathbb{R}^{n\times n}$.

It is supposed that there are N regions Ψ_i and the regions correspond to the observer subsystems one by one, that is, when the observer subsystem i is activated, the observer system state is in the region Ψ_i, and the regions Ψ_{ij} represent the set of jumping from the observer subsystem i to the observer subsystem j. The following two properties can meet our well-defined switched system.

Covering property: $\Psi_1 \bigcup \Psi_2 \bigcup \ldots \bigcup \Psi_N = \mathbb{R}^{3n}$.

Switching property: $\Psi_{ij} \subseteq \Psi_i \bigcap \Psi_j, i, j \in M$.

The first property means that each region has a corresponding active observer subsystem. The second property says that the switching from the observer subsystem i to the observer subsystem j occurs only at the intersection of the region Ψ_i and the region Ψ_j. Therefore, based on the matrices $\tilde{B}_i$ mentioned above, region Ψ_i and region Ψ_{ij} are defined as follows:

$$\Psi_i = \{\xi \in \mathbb{R}^{3n} | \xi^{\mathrm{T}}\tilde{B}_i\xi \ge 0\}, \ i \in M, \tag{9}$$

$$\Psi_{ij} = \{\xi \in \mathbb{R}^{3n} | \xi^{\mathrm{T}}\tilde{B}_i\xi - \xi^{\mathrm{T}}\tilde{B}_j\xi = 0\}, \ i, \ j \in M. \tag{10}$$

The description of the main result requires the following lemmas and definitions.

Lemma 1 (Covering property) [9]**:** If for every $\xi \in \mathbb{R}^{3n}$, $\theta_1\xi^{\mathrm{T}}\tilde{B}_1\xi + \theta_2\xi^{\mathrm{T}}\tilde{B}_2\xi + \ldots + \theta_N\xi^{\mathrm{T}}\tilde{B}_N\xi \ge 0$, where $\theta_i > 0$, $i \in M$, then $\Psi_1 \bigcup \Psi_2 \bigcup \ldots \bigcup \Psi_N = \mathbb{R}^{3n}$.

Lemma 2 [19]**:** The system $\dot{x}(t) = Ax(t) + u(t), x(0) = x_0$ is said to be cooperative if A is a Metzler matrix and $u(t) \ge 0$, and for any initial condition $x_0 \ge 0$ and $t \ge 0$, its solution satisfies $x(t) \ge 0$.

Lemma 3 [18]**:** Let $x \in \mathbb{R}^n$ be a vector satisfying $\underline{x} \le x \le \bar{x}$ and $A \in \mathbb{R}^{m\times n}$ be a constant matrix, then $A^+\underline{x} - A^-\bar{x} \le Ax \le A^+\bar{x} - A^-\underline{x}$.

Definition 1 [19]**:** Consider a switched system

$$\dot{x} = f_\sigma(t, x(t), k_\sigma(t)), \ x \in \mathbb{R}^n, \ k_i \in \mathbb{R}^l, \tag{11}$$

with f_i, $i \in M$ of class C^1. The disturbance k_i are Lipschitz continuous and such that there exist $k_u(t), k_l(t) \in \mathbf{W}$, Lipschitz continuous, and such that, for all $t \ge 0$, $k_l(t) \le k_i(t) \le k_u(t)$. Moreover, the initial condition $x(0) = x_0$ is assumed to be bounded by known bounds: $\underline{x}_0 \le x_0 \le \bar{x}_0$.

Then, the dynamical system

$$\dot{Z} = \varphi_\sigma(t, Z, \tilde{k}(t)), \ Z_0 = G(t_0, \bar{x}_0, \underline{x}_0) \in \mathbb{R}^{n_z}, \tag{12}$$

associated with $\tilde{k}(t) = (k_u(t), k_l(t)) \in \mathbb{R}^{2l}$, and bounds for the solution x: $\bar{x} = H_u(t, Z)$, $\underline{x} = H_l(t, Z)$ with φ_i $(i \in M)$, H_u, H_l, G Lipschitz continuous of appropriate dimension, is called a FTS interval-observer for (11) if

(i) for all $\tilde{k}(t)$, all the solutions of (12) are defined over $\mathbb{R}^+$;

(ii) the solutions $x(t)$ and $Z(t)$ of the systems (11) and (12) satisfy $\underline{x} = H_l(t, Z) \le x(t) \le H_u(t, Z) = \bar{x}, \forall t \ge 0$;

(iii) the system (12) is FTS with respect to (c_1, c_2, T, R, d) under the switching signal σ when $\tilde{k} \equiv 0$.

3. MAIN RESULTS

This section gives the sufficient conditions of the existence of the FTS interval-observer subject to observer-state-dependent switching strategy, and then designs the state feedback controllers based on the constructed interval-observer.

Theorem 1: Let Assumptions 1 and 2 be satisfied. For any initial condition $\underline{x}_0 \le x_0 \le \bar{x}_0$ and any $i, j \in M, i \neq j$, if there exist matrices $P > 0, P_{ii} > 0, \tilde{P}_i > 0, \tilde{P}_j > 0, \tilde{B}_i = \tilde{B}_i^{\mathrm{T}}$, X_i, and constants $\theta_i > 0$, $\eta_{ij} > 0$, $\alpha > 0$, $\delta > 0$, such that

$$\begin{bmatrix} \Sigma_i & 0 & X_iC_i \\ * & \Sigma_i & X_iC_i \\ * & * & \Upsilon_i \end{bmatrix} < 0, \tag{13}$$

$$\tilde{P}_i = \tilde{P}_j + \eta_{ij}(\tilde{B}_j - \tilde{B}_i), \tag{14}$$

$$\sum_{i=1}^{N} \theta_i \tilde{B}_i \ge 0, \tag{15}$$

where $\Upsilon_i = He(PA_i) + \frac{1}{\delta}P - \alpha \min_{i\in M}\{\lambda_{\min}(P)\}I$, $\Sigma_i = He(P_{ii}\mathcal{A}_i) - He(X_iC_i) + \frac{1}{\delta}P_{ii} - \alpha \min_{i\in M}\{\lambda_{\min}(P_{ii})\}I$.

Then, the system (2) is a FTS interval-observer of the switched system (1) with respect to $(c_1, c_2, T, \lambda, R)$ under the switching signal (8), where $\lambda = \max_{i\in M}\{\lambda_{\max}(\tilde{P}_i)\}$ with $P_i = \mathrm{diag}\{P_{ii}, P_{ii}\}$, $\tilde{P}_i = \mathrm{diag}\{P_i, P\}$. Moreover, the interval-observer gains are given by $L_i = P_{ii}^{-1}X_i$.

Proof: The proof of this result takes the following three steps.

Step 1: We show that $\underline{x}(t) \le x(t) \le \bar{x}(t)$ for any $t \ge 0$. From Assumption 2 and the definitions of ϖ_{li} and ϖ_{ui}, it is obvious that $\varpi_{li} \ge 0$ and $\varpi_{ui} \ge 0$. Since the matrices $\mathcal{A}_i$ are Metzler and $\varpi_{li} \ge 0$, $\varpi_{ui} \ge 0$, by Lemma 3 and the continuity of the states $\bar{e}(t)$ and $\underline{e}(t)$ of the system (16) at the switching instants, for all $t \ge 0$, $\bar{e}(0) \ge 0$ and $\underline{e}(0) \ge 0$ imply that $\bar{e}(t) \ge 0$ and $\underline{e}(t) \ge 0$, which gives $\underline{x}(t) \le x(t) \le \bar{x}(t)$.

Step 2: Since we don't care about the control input at this stage, we can set $u = 0$ and the system (5) becomes

$$\dot{\xi} = \tilde{A}_i\xi + \tilde{k}_i, \tag{16}$$

where $\tilde{A}_i = \begin{bmatrix} \bar{A}_i & \bar{C}_i \\ 0 & A_i \end{bmatrix}$, $\bar{A}_i = \begin{bmatrix} \mathcal{A}_i & 0 \\ 0 & \mathcal{A}_i \end{bmatrix}$.

Now, we prove the FTS of the system (16). For this purpose, we consider the multiple Lyapunov functions $V(\xi) = V_\sigma(\xi)$ with $V_i(\xi) = \xi^{\mathrm{T}}\tilde{P}_i\xi$.

For any $i \in M$, we deduce

$$\begin{aligned} \dot{V}_i(\xi) &= \xi^{\mathrm{T}} He(\tilde{P}_i\tilde{A}_i)\xi + 2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_i \\ &< \xi^{\mathrm{T}}(He(\tilde{P}_i\tilde{A}_i) + \frac{1}{\delta}\tilde{P}_i)\xi + \delta\tilde{k}_i^{\mathrm{T}}\tilde{P}_i\tilde{k}_i. \end{aligned} \tag{17}$$

According to (13), it has

$$\begin{aligned} \dot{V}_i(\xi) &< \alpha\xi^{\mathrm{T}} \min_{i\in M}\{\lambda_{\min}(\tilde{P}_i)\}I\xi + \delta\lambda\tilde{k}_i^{\mathrm{T}}\tilde{k}_i \\ &\le \alpha V_i(\xi) + \delta\lambda\tilde{k}_i^{\mathrm{T}}\tilde{k}_i. \end{aligned} \tag{18}$$

Let t_k denote the switching instant, for any $t \in [t_k, t_{k+1})$, integrating from t_k to t on both sides of (18), we have

$$\begin{aligned} V(\xi(t)) &< e^{\alpha(t-t_k)}V_{\sigma(\xi(t_k))}(\xi(t_k)) \\ &\quad + \delta\lambda\int_{t_k}^{t} e^{\alpha(t-s)}\tilde{k}_i^{\mathrm{T}}\tilde{k}_i\, ds. \end{aligned} \tag{19}$$

Note that $\xi(t_k) = \xi(t_k^-)$ then $V_{\sigma(\xi(t_k))}(\xi(t_k)) = V_{\sigma(\xi(t_k^-))}(\xi(t_k^-))$ from (14). Therefore, it follows from (19) that

$$\begin{aligned} V(\xi(t)) &\le e^{\alpha t}V_{\sigma(\xi(0))}(\xi(0)) + \delta\lambda\int_0^t e^{\alpha t}\tilde{k}_i^{\mathrm{T}}\tilde{k}_i\, ds \\ &\le e^{\alpha T}V(\xi(0)) + \delta\lambda e^{\alpha T}\int_0^T \tilde{k}_i^{\mathrm{T}}\tilde{k}_i\, ds. \end{aligned} \tag{20}$$

Let $\bar{P}_i = R^{-\frac{1}{2}}\tilde{P}_iR^{-\frac{1}{2}}$, we have

$$\begin{aligned} V(\xi(0)) &= \xi^{\mathrm{T}}(0)\tilde{P}_{\sigma(\xi(0))}\xi(0) \\ &< \lambda_{\max}(\bar{P}_{\sigma(\xi(0))})\xi^{\mathrm{T}}(0)R\xi(0) \\ &< \lambda_1\xi^{\mathrm{T}}(0)R\xi(0), \end{aligned} \tag{21}$$

and

$$\begin{aligned} V(\xi(t)) &= \xi^{\mathrm{T}}(t)\tilde{P}_{\sigma(\xi(t))}\xi(t) \\ &= \xi^{\mathrm{T}}(t)R^{\frac{1}{2}}\bar{P}_{\sigma(\xi(t))}R^{\frac{1}{2}}\xi(t) \\ &\ge \lambda_{\min}(\bar{P}_{\sigma(\xi(t))})\xi^{\mathrm{T}}(t)R\xi(t) \\ &\ge \lambda_2\xi^{\mathrm{T}}(t)R\xi(t), \end{aligned} \tag{22}$$

where $\lambda_1 = \max_{i\in M}\{\lambda_{\max}(\bar{P}_i)\}$, $\lambda_2 = \min_{i\in M}\{\lambda_{\min}(\bar{P}_i)\}$.

Then, combining (20), (21) with (22), it follows that

$$\xi^{\mathrm{T}}(t)R\xi(t) < \frac{e^{\alpha T}}{\lambda_2}\Big(\lambda_1\xi^{\mathrm{T}}(0)R\xi(0) + \delta\lambda\int_0^T \tilde{k}_i^{\mathrm{T}}\tilde{k}_i\, ds\Big). \tag{23}$$

From $\xi^{\mathrm{T}}(0)R\xi(0) \le c_1$ and $\tilde{k}_i \in \mathbf{W}$, we can derive that $\xi^{\mathrm{T}}(t)R\xi(t) < \frac{e^{\alpha T}}{\lambda_2}(\lambda_1 c_1 + \delta\lambda d)$, $\forall t \in (0, T]$. Let $c_2 = \frac{e^{\alpha T}}{\lambda_2}(\lambda_1 c_1 + \delta\lambda d)$, it is clear that $c_2 > c_1$, and thus the system (16) is FTB and when $\tilde{k}_i = 0$, the system (16) is FTS, that is to say the system (2) is a FTS interval-observer for the switched system (1) under the switching signal (8).

Step 3: Under the observer-state-dependent switching strategy, the sliding motion may occur at the switching surface Ψ_{ij}, that is, $\xi^{\mathrm{T}}\tilde{B}_i\xi = \xi^{\mathrm{T}}\tilde{B}_j\xi \ge 0$. So, what we're going to do is show that the FTB of the system (16) still holds.

Using Filippov's convex combination in [37], we can describe the system on the switching surface as

$$\dot{\xi} = \rho(\tilde{A}_i\xi + \tilde{k}_i) + (1-\rho)(\tilde{A}_j\xi + \tilde{k}_j), 0 \le \rho \le 1. \tag{24}$$

When the sliding motion occurs along Ψ_{ij}, it has

$$\xi^{\mathrm{T}}He(\tilde{A}_i^{\mathrm{T}}(\tilde{B}_i - \tilde{B}_j))\xi + 2\xi^{\mathrm{T}}(\tilde{B}_i - \tilde{B}_j)\tilde{k}_i < 0, \tag{25}$$

$$\xi^{\mathrm{T}}He(\tilde{A}_j^{\mathrm{T}}(\tilde{B}_i - \tilde{B}_j))\xi + 2\xi^{\mathrm{T}}(\tilde{B}_i - \tilde{B}_j)\tilde{k}_j > 0. \tag{26}$$

From (14) and $\eta_{ij} > 0$, we have

$$\xi^{\mathrm{T}}He(\tilde{A}_i^{\mathrm{T}}(\tilde{P}_j - \tilde{P}_i))\xi + 2\xi^{\mathrm{T}}(\tilde{P}_j - \tilde{P}_i)\tilde{k}_i < 0, \tag{27}$$

$$\xi^{\mathrm{T}}He(\tilde{A}_j^{\mathrm{T}}(\tilde{P}_j-\tilde{P}_i))\xi+2\xi^{\mathrm{T}}(\tilde{P}_j-\tilde{P}_i)\tilde{k}_j>0. \tag{28}$$

According to (13),

$$\begin{aligned}\xi^{\mathrm{T}}He(\tilde{A}_i^{\mathrm{T}}\tilde{P}_j)\xi+2\xi^{\mathrm{T}}\tilde{P}_j\tilde{k}_i&<\xi^{\mathrm{T}}He(\tilde{A}_i^{\mathrm{T}}\tilde{P}_i)\xi+2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_i\\&<\alpha\lambda_3\xi^{\mathrm{T}}\xi+\delta\lambda\tilde{k}_i^{\mathrm{T}}\tilde{k}_i, \qquad (29)\\ \xi^{\mathrm{T}}He(\tilde{A}_j^{\mathrm{T}}\tilde{P}_i)\xi+2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_j&<\xi^{\mathrm{T}}He(\tilde{A}_j^{\mathrm{T}}\tilde{P}_j)\xi+2\xi^{\mathrm{T}}\tilde{P}_j\tilde{k}_j\\&<\alpha\lambda_3\xi^{\mathrm{T}}\xi+\delta\lambda\tilde{k}_j^{\mathrm{T}}\tilde{k}_j, \qquad (30)\end{aligned}$$

which gives that

$$\begin{aligned}&\xi^{\mathrm{T}}[\rho He(\tilde{A}_i^{\mathrm{T}}\tilde{P}_i)+(1-\rho)He(\tilde{A}_j^{\mathrm{T}}\tilde{P}_i)]\xi+2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_i\\&<\alpha\lambda_3\xi^{\mathrm{T}}\xi+\delta\lambda\tilde{k}_i^{\mathrm{T}}\tilde{k}_i, \qquad (31)\\&\xi^{\mathrm{T}}[(1-\rho)He(\tilde{A}_j^{\mathrm{T}}\tilde{P}_j)+\rho He(\tilde{A}_i^{\mathrm{T}}\tilde{P}_j)]\xi+2\xi^{\mathrm{T}}\tilde{P}_j\tilde{k}_j\\&<\alpha\lambda_3\xi^{\mathrm{T}}\xi+\delta\lambda\tilde{k}_j^{\mathrm{T}}\tilde{k}_j, \qquad (32)\end{aligned}$$

where $\lambda_3=\min_{i\in M}\{\lambda_{\min}(\tilde{P}_i)\}$.

Hence, we can deduce that

$$\begin{aligned}&\xi^{\mathrm{T}}He[(\rho\tilde{A}_i^{\mathrm{T}}+(1-\rho)\tilde{A}_j^{\mathrm{T}})\tilde{P}_i]\xi+2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_i\\&<\alpha\xi^{\mathrm{T}}\tilde{P}_i\xi+\delta\lambda\tilde{k}_i^{\mathrm{T}}\tilde{k}_i, \qquad (33)\\&\xi^{\mathrm{T}}He[(\rho\tilde{A}_i^{\mathrm{T}}+(1-\rho)\tilde{A}_j^{\mathrm{T}})\tilde{P}_j]\xi+2\xi^{\mathrm{T}}\tilde{P}_j\tilde{k}_j\\&<\alpha\xi^{\mathrm{T}}\tilde{P}_j\xi+\delta\lambda\tilde{k}_j^{\mathrm{T}}\tilde{k}_j. \qquad (34)\end{aligned}$$

Then, the conditions of (18) and (20) are satisfied for the system (24) with Lyapunov functional $\xi^{\mathrm{T}}\tilde{P}_i\xi$ and $\xi^{\mathrm{T}}\tilde{P}_j\xi$, and the switched system (16) is FTS even if the sliding motion occurs under given conditions. This completes the proof. □

Remark 1: In [38], observer-based control for uncertain switched systems under time-dependent switching signal was investigated. However, the designed observer in [38] can not give the range of the system state in real time. Reference [39] investigated the design of hybrid state observer-based event-triggered controller for switched linear systems subject to time-dependent switching signal. In the paper, we investigate the interval-observer based controller design for switched linear systems under observer-state-dependent switching signals. Different from [38,39], interval-observers for switched systems with observer-state-dependent switching signals is discussed in the paper. Through interval-observers, we can give the range of the system state in real time, that is, get the upper and the lower bounds of the system state. Although work [19] studied the design of interval-observer for switched linear systems, it did not discuss the stabilization problem of the system, that is, it did not design the controller. In addition, the conditions in [19] are difficult to verify, and the inequality is dependent on the switching times.

Remark 2: The current literature on event-triggered control of switched linear systems either directly assume that the state of the system can be directly obtained, or are based on the traditional observer to design the controller under the time-dependent switching signal. In the paper, the controller is designed based on the interval-observer, which can provide the state estimation information at any instant. In addition, since the state of the system cannot be directly obtained, we design the observer-state-dependent switching signal.

Next, we will prove the interval-observer based feedback controller (3) can stabilize the system (1).

Theorem 2: Let Assumptions 1 and 2 be satisfied. For any initial condition $\underline{x}_0\le x_0\le\bar{x}_0$ and any $i,\ j\in M,\ i\neq j$, if there exist matrices $P>0$, $P_{ii}>0$, $\tilde{B}_i=\tilde{B}_i^{\mathrm{T}}$, X_i, $\overline{K}_i$, $\underline{K}_i$, and constants $\theta_i>0$, $\eta_{ij}>0$, $\alpha>0$, $\delta>0$, $\epsilon>0$, such that

$$\begin{bmatrix}\Sigma_i & 0 & X_iC_i & P_{ii} & \overline{K}_i^{\mathrm{T}}E_i^{\mathrm{T}}\\ * & \Sigma_i & X_iC_i & P_{ii} & \underline{K}_i^{\mathrm{T}}E_i^{\mathrm{T}}\\ * & * & \Upsilon_i & P_{ii} & 0\\ * & * & * & -\epsilon I & 0\\ * & * & * & * & -1/\epsilon I\end{bmatrix}<0, \tag{35}$$

$$\tilde{P}_i=\tilde{P}_j+\eta_{ij}(\tilde{B}_j-\tilde{B}_i), \tag{36}$$

$$\sum_{i=1}^{N}\theta_i\tilde{B}_i\ge 0, \tag{37}$$

where Υ_i and Σ_i are given in Theorem 1.

Then with the interval-observer based feedback controller (3), the switched system (1) is FTS with respect to (c_1,c_2,T,λ,R) under the switching signal (8), where, $\lambda=\max_{i\in M}\{\lambda_{\max}(\tilde{P}_i)\}$ with $P_i=\mathrm{diag}\{P_{ii},\ P_{ii}\}$, $\tilde{P}_i=\mathrm{diag}\{P_i,\ P\}$. The controller gains are given by $\overline{K}_i$ and $\underline{K}_i$.

Proof: The proof of this result takes three steps. The first step is similar to the first step in proving Theorem 1.

Step 2: Choose the Lyapunov function $V(\xi)=V_\sigma(\xi)$ with $V_i(\xi)=\xi^{\mathrm{T}}\tilde{P}_i\xi$ for the switched system (5). For any $i\in M$, by Lemma 2, we deduce

$$\begin{aligned}\dot{V}_i(\xi)&=\xi^{\mathrm{T}}He(\tilde{P}_i\breve{A}_i)\xi+2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_i\\&<\xi^{\mathrm{T}}[He(\tilde{P}_i\breve{A}_i)+\frac{1}{\delta}\tilde{P}_i]\xi+\delta\tilde{k}_i^{\mathrm{T}}\tilde{P}_i\tilde{k}_i. \qquad (38)\end{aligned}$$

Further, in order to get (18), we need to ensure the following inequality

$$\begin{bmatrix}\Sigma_i+He(\Xi_i) & P_{ii}E_i\underline{K}_i+\Xi_i^{\mathrm{T}} & X_iC_i+\Xi_i^{\mathrm{T}}\\ * & \Sigma_i+He(P_{ii}E_i\underline{K}_i) & X_iC_i+(P_{ii}E_i\underline{K}_i)^{\mathrm{T}}\\ * & * & \Upsilon_i\end{bmatrix}<0, \tag{39}$$

which is equal to

$$\begin{bmatrix}\Sigma_i & 0 & X_iC_i\\ * & \Sigma_i & X_iC_i\\ * & * & \Upsilon_i\end{bmatrix}+2\tilde{x}^{\mathrm{T}}\tilde{y}<0, \tag{40}$$

where $\Xi_i=P_{ii}E_i\overline{K}_i$, $\tilde{x}=\begin{bmatrix}P_{ii} & P_{ii} & P_{ii}\end{bmatrix}$, $\tilde{y}=\begin{bmatrix}E_i\overline{K}_i & E_i\underline{K}_i & 0\end{bmatrix}$.

The inequality (40) is equivalent to the following one

$$\begin{bmatrix}\Sigma_i & 0 & X_iC_i\\ * & \Sigma_i & X_iC_i\\ * & * & \Upsilon_i\end{bmatrix}+1/\epsilon\tilde{x}^{\mathrm{T}}\tilde{x}+\epsilon\tilde{y}^{\mathrm{T}}\tilde{y}<0. \tag{41}$$

By using Schur Complement Lemma and (35), we can prove the inequality (39) holds.

Then, according to the steps of Theorem 1, we can get similar formulas from (19)-(23). Therefore, it can be concluded that the system (2) is FTB interval-observer for the switched system (1).

Step 3: We consider the case of sliding motion problem, the step is the same as Theorem 1, and the following inequalities can be obtained

$$\begin{aligned}&\xi^{\mathrm{T}}[\rho(\breve{A}_i^{\mathrm{T}}\tilde{P}_i+\tilde{P}_i\breve{A}_i)+(1-\rho)(\breve{A}_j^{\mathrm{T}}\tilde{P}_i+\tilde{P}_i\breve{A}_j)]\xi+2\xi^{\mathrm{T}}\tilde{P}_i\tilde{k}_i\\&<\alpha\xi^{\mathrm{T}}\tilde{P}_i\xi+\delta\lambda\tilde{k}_i^{\mathrm{T}}\tilde{k}_i, \qquad (42)\\&\xi^{\mathrm{T}}[(1-\rho)(\breve{A}_j^{\mathrm{T}}\tilde{P}_j+\tilde{P}_j\breve{A}_j)+\rho(\breve{A}_i^{\mathrm{T}}\tilde{P}_j+\tilde{P}_j\breve{A}_i)]\xi+2\xi^{\mathrm{T}}\tilde{P}_j\tilde{k}_j\\&<\alpha\xi^{\mathrm{T}}\tilde{P}_j\xi+\delta\lambda\tilde{k}_j^{\mathrm{T}}\tilde{k}_j. \qquad (43)\end{aligned}$$

Therefore, the switched system (5) is FTS even if the sliding motion occurs under given conditions. This completes the proof. □

4. NUMERICAL SIMULATION

Example 1: Consider the system (1) without control input, the system matrices are given as follows:

$$A_1=\begin{bmatrix}-1.01 & 0.48\\ 0 & -1.1\end{bmatrix},\ A_2=\begin{bmatrix}-0.61 & 0.3\\ 0.32 & -1.07\end{bmatrix},$$
$$k(t)=\begin{bmatrix}\sin(t)\\ \cos(t)\end{bmatrix},\ C_1=\begin{bmatrix}0 & 0.5\end{bmatrix},\ C_2=\begin{bmatrix}0.5 & 0\end{bmatrix},$$
$$o(t)=\sin(t)\bar{O}.$$

It is obvious that $\bar{O}=1$ and $\bar{k}=\begin{bmatrix}1 & 1\end{bmatrix}^{\mathrm{T}}$. And Assumption 2 is fulfilled due to the matrices $A_1-L_1C_1$ and $A_2-L_2C_2$ are Metzler, where $L_1=\begin{bmatrix}-0.0969 & -0.0492\end{bmatrix}^{\mathrm{T}}$, $L_2=\begin{bmatrix}-0.0256 & 0.166\end{bmatrix}^{\mathrm{T}}$.

Choose $c_1=1$, $c_2=20$, $T=10$, $R=I$, let $\lambda_2=0.3981$, and fix $\alpha=0.1$, $\eta=1$, $\delta=1$, then apply Theorem 1 we obtain the following feasible solutions

$$P_{11}=\begin{bmatrix}0.7272 & 0.1226\\ * & 0.5405\end{bmatrix},\ P_{22}=\begin{bmatrix}1.0135 & -0.0288\\ * & 0.3994\end{bmatrix},$$
$$P=\begin{bmatrix}0.8443 & 0.1179\\ * & 0.6211\end{bmatrix},\ B_{11}=P_{22},\ B_{22}=P_{11},$$
$$\tilde{P}_1=\mathrm{diag}\{P_{11},\ P_{11},P\},\ \tilde{P}_2=\mathrm{diag}\{P_{22},\ P_{22},\ P\},$$
$$tildeB_1=\mathrm{diag}\{B_{11},\ B_{11},0\},$$
$$\tilde{B}_2=\mathrm{diag}\{B_{22},\ B_{22},0\}.$$

Figs. 1 and 2 show the state estimations of variables x_1 and x_2, where the exogenous disturbance of the system (1) is $k^{\mathrm{T}}(t)=\begin{bmatrix}\sin(t) & \cos(t)\end{bmatrix}$.

The simulations with the disturbance $k^{\mathrm{T}}(t)=\begin{bmatrix}e^{-t} & e^{-t}\end{bmatrix}$ are provided in Figs. 3 and 4. Fig. 5 shows the system is FTB with the initial condition $\xi^{\mathrm{T}}(0)R\xi(0)\leq 1$.

Then, according to Fig. 5, we find that the interval-observer in this paper reaches FTB at $t=6.478$ s. By using the simulation example in [40], it can be seen that the FTB is achieved at $t=15$ s. Therefore, the result in this paper may be less conservative.

Example 2: Consider the system (1) with the same ex-

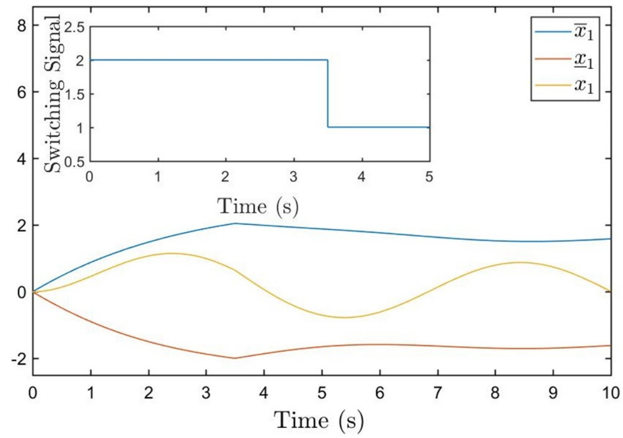

Fig. 1. The state estimation for variable x_1.

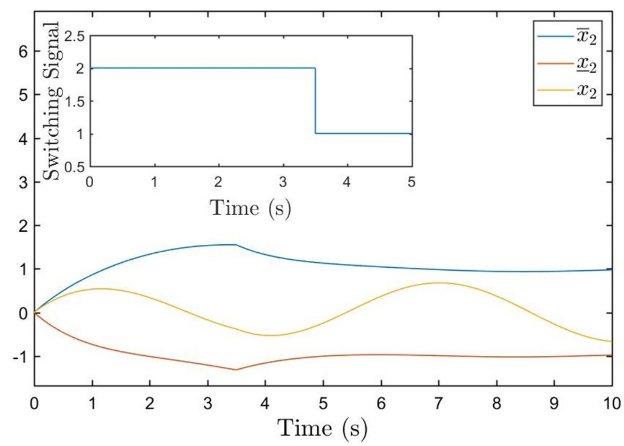

Fig. 2. The state estimation for variable x_2.

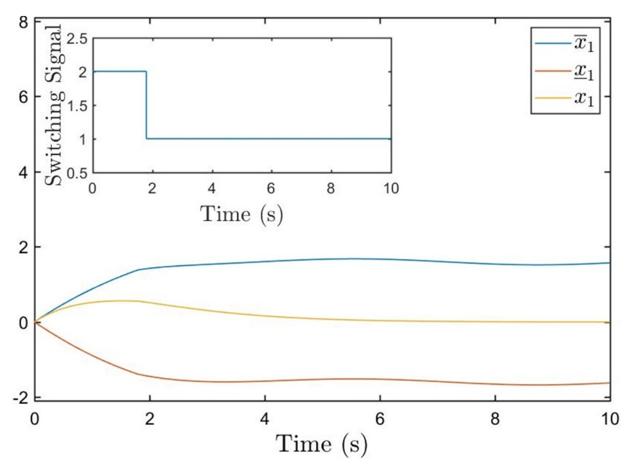

Fig. 3. The state estimation for variable x_1.

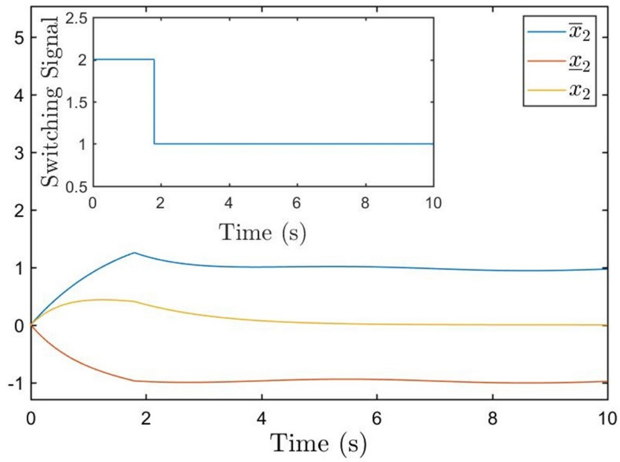

Fig. 4. The state estimation for variable x_2.

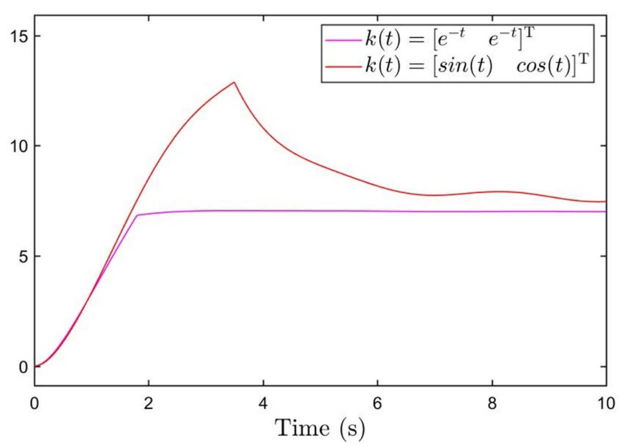

Fig. 5. $\xi^{\mathrm{T}}(t)R\xi(t)$.

ternal disturbance and measurement noise as Example 1,

$$A_1 = \begin{bmatrix} -0.74 & 1.28 \\ 1.12 & -1.82 \end{bmatrix}, A_2 = \begin{bmatrix} -2.8 & 0.48 \\ 0.8 & -2.7 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, E_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix}.$$

According to Assumption 2, $A_1 - L_1C_1$ and $A_2 - L_2C_2$ are Metzler, where $L_1 = \begin{bmatrix} -1.1925 & 1.803 \end{bmatrix}^{\mathrm{T}}$, $L_2 = \begin{bmatrix} -1.0659 & -0.1364 \end{bmatrix}^{\mathrm{T}}$. Let $c_1 = 1, c_2 = 10, T = 10, R = I$, $\lambda_2 = 0.3649$, and fix $\alpha = 0.9$, $\eta = 1$, $\epsilon = 1$, $\delta = 2$, then apply Theorem 2, we obtain the controller gains

$$\overline{K}_1 = \begin{bmatrix} -0.2704 & 0.0171 \\ * & -0.4298 \end{bmatrix},$$
$$\overline{K}_2 = \begin{bmatrix} -0.4182 & 0.0207 \\ * & -0.2818 \end{bmatrix},$$
$$\underline{K}_1 = \begin{bmatrix} -0.2598 & 0.0098 \\ * & -0.2478 \end{bmatrix},$$

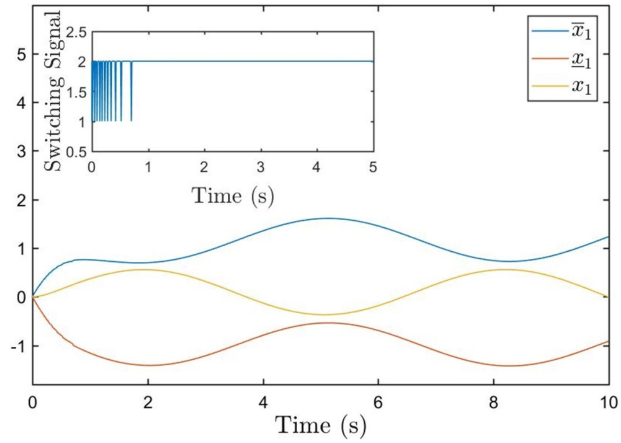

Fig. 6. The state estimation for variable x_1.

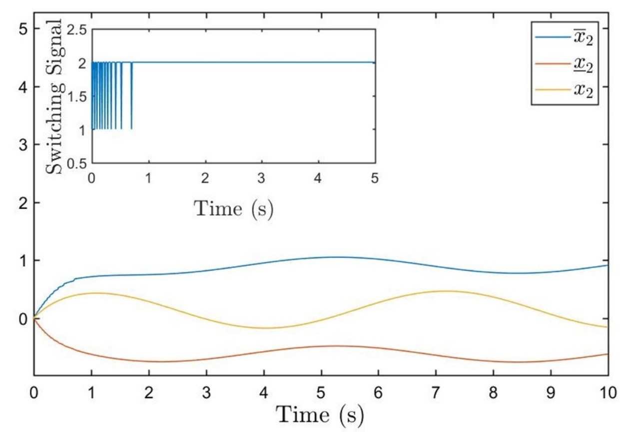

Fig. 7. The state estimation for variable x_2.

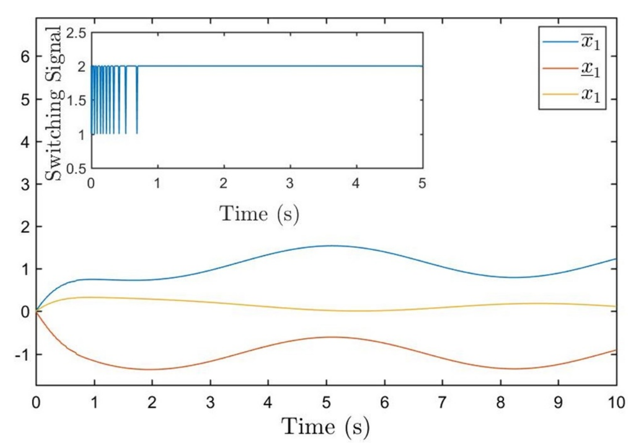

Fig. 8. The state estimation for variable x_1.

$$\underline{K}_2 = \begin{bmatrix} -0.8727 & 0 \\ * & -0.8727 \end{bmatrix}.$$

Figs. 6 and 7 show the state estimations of the variables x_1 and x_2 with $k^{\mathrm{T}}(t) = \begin{bmatrix} \sin(t) & \cos(t) \end{bmatrix}$. The simulations with the disturbance $k^{\mathrm{T}}(t) = \begin{bmatrix} e^{-t} & e^{-t} \end{bmatrix}$ are provided in Figs. 8 and 9. Fig. 10 shows the open-loop system is not

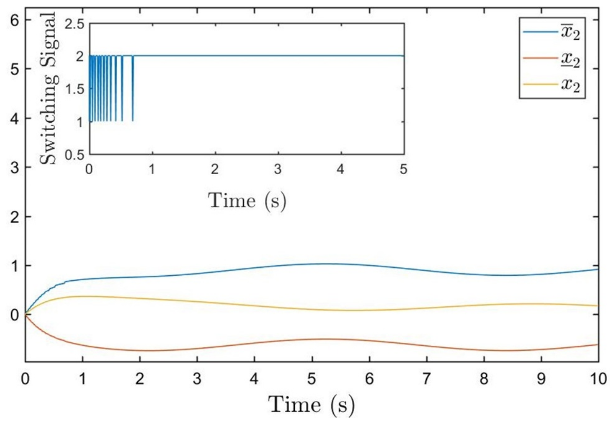

Fig. 9. The state estimation for variable x_2.

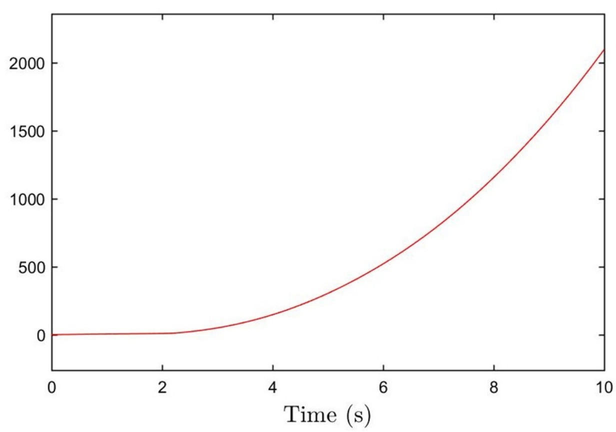

Fig. 10. $\xi^{\mathrm{T}}(t)R\xi(t)$.

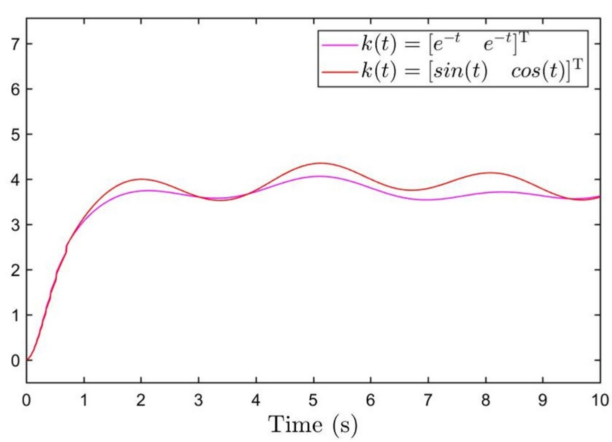

Fig. 11. $\xi^{\mathrm{T}}(t)R\xi(t)$.

FTB, while Fig. 11 shows the system is FTB with the initial condition $\xi^{\mathrm{T}}(0)R\xi(0) \leq 1$.

Then, according to Fig. 11, the switched system (1) with the interval-observer based feedback controller (3) reaches FTB at $t = 1.801$ s, however, reference [39] needs $t = 3.82$ s to reach FTB. Therefore, the result in this paper may be less conservative.

5. CONCLUSION

In the paper, the design of interval-observer based controller for switched systems has been discussed. We have constructed interval-observers and designed controller based on the proposed interval-observers for the considered switched systems with observer-state-dependent switching strategy. Then, some criteria characterizing the FTB of the observer systems have been derived. The event-triggered controller design based on intervalobserver for switched linear systems is a interesting issue that is worth further investigating.

CONFLICT OF INTERESTS

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

[1] J. Zhao and D. Hill, "On stability L_2-gain and H_∞ control for switched systems," *Automatica*, vol. 44, no. 5, pp. 1220-1232, 2008.

[2] R. Ma, J. Fu, and T. Chai, "Dwell-time-based observer design for unknowninput switched linear systems without requiring strong detectability of subsystems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4215-4221, 2017.

[3] J. Cheng, Y. Wu, H. Yan, and Z. Wu, "Protocol-based filtering for fuzzy Markovaffine systems with switching chain," *Automatica*, vol. 141, pp. 110321, 2022.

[4] Z. Sun, "Robust switching of discrete-time switched linear systems," *Automatica*, vol. 48, no. 1, pp. 239-242, 2012.

[5] X. Sun, G. Liu, D. Rees, and W. Wang, "Stability of systemswith controller failure and time-varying delay," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 941-953, 2008.

[6] J. Hespanha and A. Morse, "Stability of switched systems with average dwell-time," *Proc. of the 38th IEEE Conference on Decision and Control*,vol. 3, pp. 2655-2660, 2002.

[7] X. Zhao, L. Zhang, S. Peng, and M. Liu, "Stability and stabilization of switchedlinear systems with mode-dependent average dwell time," *IEEE Transactions on AutomaticControl*, vol. 57, no. 7, pp. 1809-1815, 2012.

[8] Y. Wang, B. Wu, and R. Wang, "Asynchronous stabilization of switched systems: Adjacent mode-dependent average dewll time," *Acta Physica Sinica*, vol. 64, no. 5, pp. 50201-050201, 2015.

[9] H. Liu, S. Yi, and X. Zhao, "Finite-time stabilization and boundedness of switched linear system under state-dependent switching," *Journal of the Franklin Institute*, vol. 350, no. 3, pp. 541-555, 2013.

[10] V. Phat, T. Botmart, and P. Niamsup, "Switching design for exponential stability of a class of nonlinear hybrid time-delay systems," *Nonlinear analysis: Hybrid systems*, vol. 3, pp. 1-10, 2009.

[11] S. Amina, R. Muhammad, A. Muhammad, and T. Muhammad, "A new approach using multiple Lyapunov functionsfor bipartite consensus of multi-agents over directed switching signed graphs," *Nonlinear Analysis: Hybrid Systems*, vol. 44, pp. 101143,2020.

[12] L. Long and J. Zhao, "H_∞ control of switched nonlinear systems in p-normal formusing multiple Lyapunov functions," *IEEE Transactions on Automatic Control*, vol. 57, no. 5,pp. 1285-1291, 2012.

[13] J. Gouze, A. Rapaport, and M. Hadj-Sadok, "Interval observers for uncertain biological systems," *Ecological Modelling*, vol. 133, no. 1-2, pp. 45-56, 2000.

[14] F. Mazenc and T. Dinh, "Construction of interval observers for continuous-time systems with discrete measurements," *Automatica*, vol. 50, no. 10, pp. 2555-2560, 2014.

[15] F. Mazenc and O. Bernard, "Interval observers for linear time-invariant systems with disturbances," *Automatica*, vol. 47, pp. 140-147.

[16] Z. Zhang and G. Yang, "Fault detection for discrete-time LPV systems using interval observers," *International Journal of Systems Science*, vol. 48, pp. 2921-2935, 2017.

[17] Y. Wang, M. David, and R. Rajesh, "Interval observer design for LPV systems with parametric uncertainty," *Automatica*, vol. 60, pp. 79-85, 2015.

[18] H. Ethabet, D. Rabehi, D. Efimov, and T. Raissi, "Interval estimation for continuous-time switched linear systems," *Automatica*, vol. 90, pp. 230-238, 2018.

[19] Y. Wang, H. Karimi, and D. Wu, "Construction of hybrid interval observers for switched linear systems," *Information Sciences*, vol. 454-455, pp. 242-254, 2018.

[20] J. Huang, X. Ma, X. Zhao, H. Che, and L. Chen, "Interval observer design method for asynchronous switched systems," *IET Control Theory and Applications*, vol. 14, pp. 1082-1090, 2020.

[21] Y. Wang, G. Wang, X. Shi, and Z. Zuo, "Finite-time stability analysis of impulsive switched discrete-time linear systems: The average dwell time approach," *Circuits, Systems, and Signal Processing*, vol. 31, no. 5, pp. 1877-1886, 2012.

[22] P. Dorato, "Short time stability in linear time-varying systems," *Proc. of IRE International Convention Record*, vol. 4, pp. 83-87, 1961.

[23] H. Du, X. Lin, and S. Li, "Finite-time stability and stabilization of switched linear systems," *IEEE Conference on Decision Control*, pp. 1938-1943, 2010.

[24] Y. Wang, G. Wang, X. Shi, and Z. Zuo, "Finite-time stability analysis of impulsive switched discrete-time linear systems: The average dwell time approach," *Circuits Systems and Signal Processing*, vol. 31, pp. 1877-1886, 2012.

[25] X. Lin and C. Chen, "Finite-time output feedback stabilization of planar switched systems with/without an output constraint," *Automatica*, vol. 131, pp. 109728, 2021.

[26] J. Cheng, W. Huang, J. Park, and J. Cao, "A hierarchical structure approach to finite-time filter design for fuzzy Markov switching systems with deception attacks," *IEEE Transactions on Cybernetics*, vol. 52, no. 8, pp. 7254-7264, 2022.

[27] G. Zong, R. Wang, W. Zheng, and L. Hou, "Finite-time H_∞ control for discrete-time switched nonlinear systems with time delay," *International Journal of Robust & Nonlinear Control*, vol. 25, no. 6, pp. 914-936, 2015.

[28] H. Wang, Y. Tong, D. Xu, and W. Song, "Finite-time H_∞ control for discrete-time switched systems with admissible edge-dependent average dwell time," *International Journal of Systems Science*, vol. 52, no. 3, pp. 525-540, 2021.

[29] N. Xu, Y. Chen, A. Xue, and G. Zong, "Finite-time stabilization of continuous-time switched positive delayed systems," *Journal of the Franklin Institute*, vol. 359, no. 1, pp. 255-271, 2022.

[30] K. Li, C. Hua, X. You, and X. Guan, "Finite-time observer-based leader-following consensus for nonlinear multiagent systems with input delays," *IEEE Transactions on Cybernetics*, vol. 99, no. 12, pp. 1-9, 2021.

[31] M. Iqbal, J. Xiao, and W. Xiang, "Parameter-dependent finite-time observer design for time-varying polytopic uncertain switched systems," *Journal of the Franklin Institute*, vol. 351, no. 3, pp. 1657-1672, 2014.

[32] C. Wu, Y. Wang, and D. Wu, "Finite-time robust observers for switched linear systems with state-dependent switching," *Proc. of 4th International Conference on Industrial Artificial Intelligence (IAI)*, pp. 1-6, 2022.

[33] J. Cheng, L. Xie, J. Park, and H. Yan, "Protocol-based output feedback control for semi-Markov jump systems," *IEEE Transactions on Automatic Control*, vol. 67, no. 8, pp. 4346-4353, 2022.

[34] S. Hu, D. Yue, and Z. Cheng, "Co-design of dynamic event-triggered communication scheme and resilient observer-basedcontrol under aperiodic DoS attacks," *IEEE Transactions on Cybernetics*, vol. 51, no. 9, pp. 4591-4601, 2021.

[35] S. Ma, W. Che, C. Deng, and Z. Wu, "Observer-based event-triggered containment control for MASs under DoS attacks," *IEEE Transactions on Cybernetics*, pp. 1-12, 2021.

[36] X. Lin, H. Du, and S. Li, "Finite-time boundedness and L_2-gain analysis for switched delay systems with norm-bounded disturbance," *Applied Mathematics and Computation*, vol. 217, no. 12, pp. 5982-5993, 2011.

[37] A. Filippov, "Differential equations with discontinuous righthand sides," *Academic Press*, vol. 32, pp. 312-315, 1988.

[38] Y. Dong, M. Deng, and M. Ding, "Observer-based finite-time event-triggered H_∞ control for nonlinear switched systems with interval time-varying delay," *Asian Journal of Control*, pp. 1-12, 2021.

[39] Y Chen, Y Wang, and D Wu, "Hybrid state observer-based event-triggered control for switched linear systems with quantized input," *Journal of the Franklin Institute*, vol. 358, no. 17, pp. 9086-9109, 2021.

[40] Y Chen, Y Wang, and D Wu, "Improved results on state estimation for switched continuous-time linear systems," *Asian Journal of Control*, vol. 24, no. 5, pp. 2478-2489, 2022.

Chuanjing Wu is an M.S. candidate in Shaanxi Normal University, Xi'an, China. She received her B.S. degree in Weifang University in 2020. Her research interests include stability analysis and controller design for switched systems.

Yue-E Wang received her M.S. degree from Shaanxi Normal University, Xi'an, China, in 2010, and a Ph.D. degree from Northeastern University, China, in 2014. She is currently an associate professor in School of Mathematics and Statistics, Xi'an, China. Her research interests include switched systems and time delay systems.

Di Wu received his B.S., M.S., and Ph.D. degrees from the Dalian University of Technology, Dalian, China, in 2006, 2012, and 2015, respectively. He was a Research Fellow at Rolls-Royce@NTU Corporate Lab in Nanyang Technological University, Singapore from 2015 to 2017. He is an associate professor with Dalian University of Technology. His research interests include networked control systems, event-triggered control, and industrial networks.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.