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Adaptive Variable Structure Controller Design for Uncertain Switched Systems With Unknown Time-varying Delay

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Abstract: In this paper, an innovative sliding mode framework is used to propose an observer-based adaptive control scheme for uncertain switched systems with unknown time-varying delay. First, a state observer with no input information is built incorporating a new sliding manifold (SM) to rebuild the unmeasured state variables, which does not rely on accurate information of the time-delay. By picking a suitable Lyapunov function, using the average dwell time (ADT) approach and linear matrix inequality (LMI) tool, a novel stability criterion for the resultant sliding motion is devised. In addition, to meet the defined SM's arrival condition, a new adaptive variable structure controller is constructed. Finally, two illustrative examples are given to demonstrate the efficacy of the control method.

Keywords: Average dwell time, sliding mode control, uncertain switched systems, unknown time-varying delay.

1. INTRODUCTION

Sliding mode control (SMC) is an effective nonlinear control strategy that has been applied successfully to a wide range of complex systems and engineering application [1-6] due to its excellent transient performance, insensitivity to unknown uncertainties and external disturbances. For instance, an SMC strategy was proposed for stochastic Markov jump cyber-physical systems in [4]. The event-triggered SMC scheme for the discrete-time two-dimensional Roesser systems was proposed in [5].

Switched systems, which play an important role in variety of practical applications, including power systems, chemical progress, have been extensively probed in recent years and a significant deal of research has been done in stability and stabilization problems [7-10]. In addition, SMC has been utilized for switched systems to some extent [11-15]. As a further extension of switched systems, switched time-delay systems (STDS) have attracted extensive attention in past decades. Time-delay might not only affect the response speed of the system [16-18], but also cause the system to be unstable. Extensive researches have been conducted to improve STDS performance, and numerous contributions have been made in this field [19-22]. To name a few, the global sampled-data control problem was studied for STDS in [19]. A novel control handling the impulsive behavior for STDS subject to both states and time-varying delays was developed in [21].

It should be pointed out that the above literature are based on scenarios that system state variables are accessible. From the point of view of practical engineering, state variables are often unknown due to various environments such as incapable measurability as well as sensor constraints. Therefore, estimating state information from state observer is an effective way [23-26]. When the system is subjected to control channel uncertainties and external disturbances, conventional observers cannot guarantee the estimation of unknown system state well [27]. At this point, sliding mode observer (SMO), in which state estimation via the observer can be effectively realized by SMC's anti-interference capability, has been put forward for uncertain nonlinear plants [28,29]. The design of SMO based on quantitative measure for a class of Markovian jump systems with actuator faults was studied in [29]. What is noteworthy, there are few studies based on SMO of STDS with uncertainty, which is the initial motivation of the present research. Accordingly, SMO has been applied to systems with time-delay gradually [30,31]. In [30], Markov jump systems with time-delay and Itô stochastic process was studied based on SMO. Reference [31] investigated the robust estimation and observer-based finite time SMC problems for STDS.

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As seen, the above results achieve the system stability by constructing SMO in view of precise information of time-delay. However, the complexity of the industrial environment, the time-delay information may not be correctly predicted in practical applications, but the upper bound of time-delay may be straightforward to be estimated. Thus, the design of SMO in the absence of accurate time-delay information has certain practical significance for STDS, which forms the crucial motivation of the study.

Based on the above discussion, this paper is to investigate adaptive robust SMC of uncertain STDS in the occurence of unmeasured states, unknown delay and parametric uncertainties under a modified observer framework. Illustrative examples are discussed to validate the efficacy of the current method. The main contributions are listed below:

1) Compared to the observer designs in [31-33], a smooth observer is set up without any control inputs, which can generate the state variables without precise knowledge of the time-delay.
2) In contrast to the designs in [28,34], a novel linear SM is developed, from which a new exponential stability criterion of the dynamics in sliding phase is derived combining the multiple Lyapunov method and ADT approach.
3) By comparison of the design of controller in [35,36], a novel adaptive variable structure control signal is synthesized to adapt unknown boundaries of potential uncertainties and solve the reachability problem of the designated SM.

The rest of the article has the following structure: Section 2 presents the system description. In Section 3, the observer-based adaptive SMC scheme is introduced. Two simulations are provided in Section 4 to confirm the effectiveness of the control scheme. Finally, Section 5 gives a brief summary of this paper.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following STDS given by

$$\begin{aligned}\dot{z}(t) &= (A(\text{ß})+\Delta A(\text{ß},t))z(t)\\ &\quad +[A_d(\text{ß})+\Delta A_d(\text{ß},t)]z(t-d(t))\\ &\quad +B(\text{ß})(u(t)+g(\text{ß},t,z)),\\ y(t) &= C(\text{ß})z(t),\\ z(t) &= \psi(t),\ t\in[-\hat{\tau},\ 0],\end{aligned}\tag{1}$$

in which $z(t)\in\Re^n$ denotes state vector, and $u(t)\in\Re^m$ and $y(t)\in\Re^p$ denote control input and measured output. The initial continuous function defined on the interval $[-\hat{\tau}, 0]$ is denoted by $\psi(t)$. $\text{ß}(t):[0,\infty)\dashrightarrow \text{Æ}$ is switching signal,which is a piecewise constant function. $\text{Æ}=\{1,2,\ldots,$ æ$\}$, where æ denotes the current subsystem under operation, then plant (1) becomes

$$\begin{aligned}\dot{z}(t) &= (A_i+\Delta A_i(t))z(t)\\ &\quad +[A_{di}+\Delta A_{di}(t)]z(t-d(t))\\ &\quad +B_i[u(t)+g_i(t,z)],\\ y(t) &= C_iz(t),\\ z(t) &= \psi(t),\ t\in[-\hat{\tau},\ 0].\end{aligned}\tag{2}$$

Furthermore, $\Delta A_i(t)$ and $\Delta A_{di}(t)$ are parameter uncertainties, and satisfy $[\Delta A_i(t)\quad \Delta A_{di}(t)] = M_iF_i(t)[N_i\quad N_{di}]$, where M_i, N_i and N_{di} are known constant matrices, $F_i(t)$ is an unknown vector-valued function and $F_i^T(t)F_i(t)\le I$ for $t\ge 0$. $d(t)$ is unknown time-vary delay but the upper bound is given by $\hat{\tau}$, and satisfies $\dot{d}(t)\le\theta<1$. $g_i(t,z(t))$ is an unknown matching nonlinearity which is the lumped perturbation in the range of the input channel satisfying $\|g_i(t,z(t))\|\le\alpha_1\|y(t)\|+\alpha_2$, where α_1, $\alpha_2>0$ are unknown scalars.

Definition 1 [37]**:** For $\forall 0\le t\le T_\sigma$, $N_{\text{ß}}(t,T_\sigma)$ signifies the switching number of $\text{ß}(t)$ over (t,T_σ). If $N_{\text{ß}}(t,T_\sigma)\le N_0+\dfrac{T_\sigma-t}{T_a}$, where $T_a>0$ and $N_0\ge 0$, T_a is called ADT. As used in [37], let $N_0=0$.

3. MAIN RESULTS

3.1. Design of state observer

A modified state observer with no inputs is constructed in the following to estimate the state variables

$$\begin{aligned}\dot{\hat{z}}(t) &= A_i\hat{z}(t)+A_{di}\hat{z}(t-\hat{\tau})+L_i(y(t)-C_i\hat{z}(t)),\\ \hat{y}(t) &= C_i\hat{z}(t),\\ \hat{z}(t) &= \hat{\psi}(t),\ t\in[-\hat{\tau},\ 0],\end{aligned}\tag{3}$$

where $\hat{z}(t)\in\Re^n$ indicates the estimation of the state $z(t)$, $\hat{y}(t)$ indicates output of the observer dynamics, and $L_i\in\mathbb{R}^{n\times p}$ is an observer gain to be designed. The associated error dynamics are obtained by subtracting (3) from (2) as below

$$\begin{aligned}\dot{\tilde{z}}(t) &= (A_i-L_iC_i)\tilde{z}(t)+A_{di}z(t-d(t))+\Delta A_i(t)z(t)\\ &\quad +\Delta A_{di}(t)z(t-d(t))+B_i[u(t)+g_i(t,z)]\\ &\quad -A_{di}\hat{z}(t-\hat{\tau}),\end{aligned}\tag{4}$$

where $\tilde{z}(t)$ denotes the estimation error and $\tilde{z}(t)=z(t)-\hat{z}(t)$.

Remark 1: It is worth noting that most previous observer designs such as [32-34] require accurate time-delay information, which may be hard to obtain in practical engineering applications, while the proposed observer that only needs the upper bound information of time-delay has more practical significance from the perspective of the application.

3.2. Linear sliding manifold design

Based on the state estimation data, we develop a novel SM function of the following linear-type

$$\sigma(t) = Y_i[2y(t) - C_i\hat{z}(t)], \tag{5}$$

with parameter $Y_i \in \mathbb{R}^{m\times p}$ to be given later.

Remark 2: Compared with the widely used integral SM [3,11,24], the novel SM has a simpler structure. More specifically, the design of SM can effectively reduce the technical difficulty in deducing the stability of the STDS, and there is no need to discuss the influence of matching disturbance on stability analysis by taking the attainability of the SM, see Subsection 3.4.

3.3. Adaptive controller synthesis and reachability analysis

In this section, an adaptive SM controller is developed to ensure that the system trajectory can arrive at the predefined SM in finite-time. To aid in the design of a controller, a reasonable assumption is offered here.

Assumption 1 [38]**:** Unknown scalars $c_1 > 0$, $c_2 > 0$ may exist to satisfy $\|z(t-d(t))\| \le c_1\|z(t)\| + c_2$.

Typically, the error term $\tilde{z}(t)$ might not be properly estimated since the state variables are not entirely accessible. However, on the basis of the connection from $z(t)$ and $y(t)$, it can be given that $\|z(t)\| \le d_1\|y(t)\| + d_2$ for some unknown positive scalars d_i, $i = 1, 2$. The following reasonable estimate can then be given:

There are two unknown positive constants ℓ_1, ℓ_2 satisfying

$$\begin{aligned}
&(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i[2A_i + 2\Delta A_i(t)) - L_iC_i]z(t)\\
&\quad + (B_i^\top Z_iB_i)^{-1}B_i^\top Z_i[2A_{di} + 2\Delta A_{di}(t)]z(t-d(t))\\
&\quad + 2g_i(t,z)\\
&\le \max_{i\in Æ}\{\|(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i(2A_i - L_iC_i)z(t)\|\}\\
&\quad + 2\max_{i\in Æ}\{\|(B_i^\top Z_iB_i)^{-1}B_i^\top Z_iM_i\|\|N_i\|\}\|z(t)\|\\
&\quad + 2\max_{i\in Æ}\{\|(B_i^\top Z_iB_i)^{-1}B_i^\top Z_iA_{di}\|\}\|z(t-d(t))\|\\
&\quad + 2\max_{i\in Æ}\{\|(B_i^\top Z_iB_i)^{-1}B_i^\top Z_iM_i\|\|N_{di}\|\}\|z(t-d(t))\|\\
&\quad + 2\max_{i\in Æ}\{\|g_i(t,z)\|\}\\
&\le \ell_1\|y(t)\| + \ell_2,\ t \ge 0,
\end{aligned} \tag{6}$$

some parameters Z_i, P_i, Q_i, R_i, T_i, X_i are defined and computed in Subsection 3.4. Let $\hat{\ell}_1(t)$ and $\hat{\ell}_2(t)$ be the estimations of unknown data ℓ_1 and ℓ_2 with the estimation errors represented by $\tilde{\ell}_1(t) = \hat{\ell}_1(t) - \ell_1$ and $\tilde{\ell}_2(t) = \hat{\ell}_2(t) - \ell_2$, then the new adaptive SM controller for the STDS is proposed by

$$\begin{aligned}
u(t) = &\ 0.5[(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i(A_i - L_iC_i)\hat{z}(t)\\
&+ (B_i^\top Z_iB_i)^{-1}B_i^\top Z_iA_{di}\hat{z}(t-\hat{\tau})]\\
&- 0.5[\hat{\ell}_1(t)\|y(t)\| + \hat{\ell}_2(t) + \delta]\mathrm{sgn}(\sigma(t)),
\end{aligned} \tag{7}$$

furthermore, the updating laws are designed by

$$\dot{\hat{\ell}}_1(t) = a_1\|y(t)\|\|\sigma(t)\|, \dot{\hat{\ell}}_2(t) = a_2\|\sigma(t)\|, \tag{8}$$

and $a_i > 0$, $i = 1, 2$, are scalars, and δ is a small positive constant.

Theorem 1: Consider the STDS (2) with observer (3) and SM variable designed in (5). Given the control signal and updating laws as (7) and (8), the state trajectory will be moved to the designed SM in limited time.

Proof: Choose the following Lyapunov function

$$\begin{aligned}
\hat{V}_1(t) = &\ 0.5[\sigma^\top(t)(B_i^\top Z_iB_i)^{-1}\sigma(t) + a_1^{-1}\tilde{\ell}_1^2(t)\\
&+ a_2^{-1}\tilde{\ell}_2^2(t)].
\end{aligned}$$

Performing the time derivative of $\hat{V}_1(t)$ becomes

$$\begin{aligned}
\dot{\hat{V}}_1(t) = &\ \sigma^\top(t)(B_i^\top Z_iB_i)^{-1}\dot{\sigma}(t)\\
&+ a_1^{-1}\tilde{\ell}_1(t)\dot{\hat{\ell}}_1(t) + a_2^{-1}\tilde{\ell}_2(t)\dot{\hat{\ell}}_2(t)\\
= &\ 2\sigma^\top(t)u(t)\\
&+ \sigma^\top\{-(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i(A_i - L_iC_i)\hat{z}(t)\\
&- (B_i^\top Z_iB_i)^{-1}B_i^\top A_{di}\hat{z}(t-\hat{\tau})\\
&+ (B_i^\top Z_iB_i)^{-1}B_i^\top Z_i[2A_i + 2\Delta A_i(t) - L_iC_i]z(t)\\
&+ (B_i^\top Z_iB_i)^{-1}B_i^\top Z_i2(A_{di} + \Delta A_{di}(t))z(t-d(t))\\
&+ 2g(t,z)\} + a_1^{-1}\tilde{\ell}_1(t)\dot{\hat{\ell}}_1(t) + a_2^{-1}\tilde{\ell}_2(t)\dot{\hat{\ell}}_2(t).
\end{aligned} \tag{9}$$

According to (6)-(9), it holds

$$\begin{aligned}
\dot{\hat{V}}_1(t) = &\ \sigma^\top(t)[(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i(A_i - L_iC_i)\hat{z}(t)\\
&+ (B_i^\top Z_iB_i)^{-1}B_i^\top Z_iA_{di}\hat{z}(t-\hat{\tau})]\\
&- \sigma^\top(t)[\hat{\ell}_1(t)\|y(t)\| + \hat{\ell}_2(t) + \delta]sgn(\sigma(t))\\
&- \sigma^\top(t)[(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i(A_i - L_iC_i)\hat{z}(t)\\
&+ (B_i^\top Z_iB_i)^{-1}B_i^\top Z_iA_{di}\hat{z}(t-\hat{\tau})]\\
&+ \sigma^\top\{(B_i^\top Z_iB_i)^{-1}B_i^\top Z_i[2A_i + 2\Delta A_i(t)\\
&- L_iC_i]z(t) + (B_i^\top Z_iB_i)^{-1}B_i^\top Z_i[2A_i\\
&+ 2\Delta A_{di}(t)]z(t-d(t)) + 2g(t,z)\}\\
&+ a_1^{-1}\tilde{\ell}_1(t)\dot{\hat{\ell}}_1(t) + a_2^{-1}\tilde{\ell}_2(t)\dot{\hat{\ell}}_2(t)\\
\le &-\|\sigma(t)\|\|y(t)\|\tilde{\ell}_1(t) - \|\sigma(t)\|\hat{\ell}_2(t)\\
&+ \|\sigma(t)\|\ell_2 + a_1^{-1}\tilde{\ell}_1(t)a_1\|y(t)\|\|\sigma(t)\|\\
&- \delta\|\sigma(t)\| + a_2^{-1}\tilde{\ell}_2(t)a_2\|\sigma(t)\|\\
= &-\delta\|\sigma(t)\| < 0, \text{ if } \sigma(\mathrm{t}) \ne 0.
\end{aligned} \tag{10}$$

Under the fact $\dot{\hat{\ell}}_i(t) = \dot{\tilde{\ell}}_i(t) > 0$, $i = 1, 2$, then an instant $T_\Theta > 0$ can be found to ensure both $\tilde{\ell}_i(t) > 0$ and $a_i^{-1}\tilde{\ell}_i(t)\dot{\tilde{\ell}}_i(t) > 0$ hold for $t > T_\Theta$. Then

$$\sigma^\top(t)(B_i^\top Z_iB_i)^{-1}\dot{\sigma}(t) \le -\delta\|\sigma(t)\| < 0, \text{ for } t \ge T_\Theta. \tag{11}$$

Introducing an auxiliary function $W(t)=0.5\sigma^{\mathsf{T}}(t)(B_i^{\mathsf{T}}Z_i B_i)^{-1}\sigma(t)$, one can further has

$$\dot{W}(t)\leq-\delta\|\sigma(t)\|\leq-\text{Ø}W^{\frac{1}{2}}(t),\ \text{for } t\geq T_{\Theta},\qquad(12)$$

where $\text{Ø}=\delta\sqrt{2/\lambda_{max}(B_i^{\mathsf{T}}Z_iB_i)^{-1}}$. Thus, we can find an instant $T^{\star}=2\sqrt{W(T_{\Theta})}/\text{Ø}+T_{\Theta}$ to meet $W(t)=0$, $\sigma(t)=0$ as $t\geq T^{\star}$, the SM's reaching condition has been satisfied, thereby ending the proof. □

3.4. Stability analysis

An auxiliary function is introduced here, i.e., $\tilde{g}_i(t,z)=g_i(t,x(t))+K_iz(t)$, where matrix K_i is given to satisfy $A_i-B_iK_i$ is Hurwitz. The following form will then be used to express system (2) and error system (4)

$$\begin{aligned}\dot{z}(t)&=(A_i-B_iK_i+\Delta A_i(t))z(t)\\&\quad+[A_{di}+\Delta A_{di}(t)]z(t-d(t))\\&\quad+B_i(u(t)+\tilde{g}_i(t,z)),\\\dot{\tilde{z}}(t)&=(A_i-L_iC_i)\tilde{z}(t)+A_{di}z(t-d(t))\\&\quad+(\Delta A_i(t)-B_iK_i)z(t)-A_{di}\hat{z}(t-\hat{\tau})\\&\quad+\Delta A_{di}(t)z(t-d(t))+B_i[u(t)+\tilde{g}_i(t,z)].\end{aligned}\qquad(13)$$

Theorem 2: Given a scalar $\varrho>0$ and the SM variable defined in (5), if there exist matrices $Z_i>0$, $P_i>0$, $Q_i>0$, $R_i>0$, $T_i>0$, X_i and positive scalars ε_i, $i=1,2$ such that there hold the following conditions

$$\begin{bmatrix}\gamma_{11}&\gamma_{12}&0&\gamma_{14}&\Gamma_{1i}\\ *&\gamma_{22}&0&(Z_iA_{di})^{\mathsf{T}}&0\\ *&*&\gamma_{33}&-(Z_iA_{di})^{\mathsf{T}}&0\\ *&*&*&\gamma_{44}&\Gamma_{2i}\\ *&*&*&*&\Gamma_{3i}\end{bmatrix}<0,\qquad(14)$$

$$Y_iC_i={B_i}^{\mathsf{T}}Z_i,\qquad(15)$$

where $\gamma_{11}=\text{sym}[Z_i(A_i-B_iK_i)]+P_i+Q_i+\varrho Z_i+(\varepsilon_1+\varepsilon_2)N_i^{\mathsf{T}}N_i$, $\gamma_{12}=Z_iA_{di}+(\varepsilon_1+\varepsilon_2)N_i^{\mathsf{T}}N_{di}$, $\gamma_{14}=-(Z_iB_iK_i)^{\mathsf{T}}$, $\gamma_{22}=-(1-\theta)e^{-\varrho\hat{\tau}}P_i+(\varepsilon_1+\varepsilon_2)N_{di}^{\mathsf{T}}N_{di}$, $\Gamma_{1i}=[0\ \ 0\ \ \Omega_1]$, $\gamma_{33}=-e^{-\varrho\hat{\tau}}Q_i$, $\gamma_{44}=\text{sym}(Z_iA_i-X_iC_i)+R_i+T_i+\varrho Z_i$, $\Gamma_{2i}=[0\quad Z_iA_{di}\quad\Omega_2]$, $\gamma_{55}=-(1-\theta)e^{-\varrho\hat{\tau}}R_i$, $\gamma_{66}=-e^{-\varrho\hat{\tau}}T_i$, $\Gamma_{3i}=\text{diag}\{\gamma_{55},\gamma_{66},\Omega_3\}$, $\Omega_{1i}=[Z_iM_i\quad 0]$, $\Omega_{2i}=[0\quad Z_iM_i]$, $\Omega_{3i}=\text{diag}\{-\varepsilon_1I,-\varepsilon_2I\}$. Additionally, the gain matrix of the observer can be computed by $L_i={Z_i}^{-1}X_i$. Then the exponential stability of the STDS (2) restricted on the SM is guaranteed for arbitrary switching signal under $T_a>\dfrac{\ln\kappa}{\varrho}$, where κ satisfies the following

$$P_i\leq\kappa P_j,\ Q_i\leq\kappa Q_j,\ R_i\leq\kappa R_j,\ T_i\leq\kappa T_j,\qquad(16)$$

where $\forall i,\ j\in$ Æ. Further, the convergence of state is estimated as

$$\|\xi(t)\|\leq\varpi e^{-\eta(t-t_0)}\|z(t_0)\|_{c^1},\qquad(17)$$

where $\xi^{\mathsf{T}}(t)\triangleq[z^{\mathsf{T}}(t)\ \ z^{\mathsf{T}}(t-d(t))\ \ z^{\mathsf{T}}(t-\hat{\tau})\ \ \tilde{z}^{\mathsf{T}}(t)\ \ \tilde{z}^{\mathsf{T}}(t-d(t))\ \ \tilde{z}^{\mathsf{T}}(t-\hat{\tau})]$ and

$$\begin{aligned}&\varpi=\sqrt{\frac{q}{p}}\geq1,\ \eta=\frac{1}{2}(\varrho-\frac{\ln\kappa}{T_a}),\ p=\min_{i\in\text{Æ}}\lambda_{\min}(Z_i),\\&q=2\max_{i\in\text{Æ}}\lambda_{\max}(Z_i)+\hat{\tau}\max_{i\in\text{Æ}}\lambda_{\max}(R_i)+\hat{\tau}\max_{i\in\text{Æ}}\lambda_{\max}(P_i)\\&\quad+\hat{\tau}\max_{i\in\text{Æ}}\lambda_{\max}(Q_i)+\hat{\tau}\max_{i\in\text{Æ}}\lambda_{\max}(T_i).\end{aligned}\qquad(18)$$

Proof: Consider the following Lyapunov candidate

$$V(i,t)=V_1(i,t)+V_2(i,t),$$

where

$$\begin{aligned}V_1(i,t)&=z^{\mathsf{T}}(t)Z_iz(t)+\tilde{z}^{\mathsf{T}}(t)Z_i\tilde{z}(t),\\V_2(i,t)&=\int_{t-d(t)}^{t}e^{\varrho(s-t)}z^{\mathsf{T}}(s)P_iz(s)ds\\&\quad+\int_{t-\hat{\tau}}^{t}e^{\varrho(s-t)}z^{\mathsf{T}}(s)Q_iz(s)ds\\&\quad+\int_{t-d(t)}^{t}e^{\varrho(s-t)}\tilde{z}^{\mathsf{T}}(s)R_i\tilde{z}(s)ds\\&\quad+\int_{t-\hat{\tau}}^{t}e^{\varrho(s-t)}\tilde{z}^{\mathsf{T}}(s)T_i\tilde{z}(s)ds.\end{aligned}\qquad(19)$$

Performing the time derivative of $V_1(t)$ becomes

$$\begin{aligned}\dot{V}_1(i,t)&=2z^{\mathsf{T}}(t)Z_i\dot{z}(t)+2\tilde{z}^{\mathsf{T}}Z_i\dot{\tilde{z}}(t)\\&=2z^{\mathsf{T}}(t)Z_i(A_i-B_iK_i)z(t)\\&\quad+2z^{\mathsf{T}}(t)Z_iA_{di}z(t-d(t))\\&\quad+2z^{\mathsf{T}}(t)Z_i[\Delta A_i(t)z(t)\\&\quad+\Delta A_{di}(t)z(t-d(t))]\\&\quad+2[z^{\mathsf{T}}(t)+\tilde{z}^{\mathsf{T}}(t)]Z_iB_i[u(t)+\tilde{g}_i(t,z)]\\&\quad+2\tilde{z}^{\mathsf{T}}(t)Z_iA_{di}z(t-d(t))\\&\quad+2\tilde{z}^{\mathsf{T}}(t)Z_i(A_i-L_iC_i)\tilde{z}(t)\\&\quad+2\tilde{z}^{\mathsf{T}}(t)Z_i[\Delta A_i(t)z(t)+\Delta A_{di}(t)z(t-d(t))]\\&\quad-2\tilde{z}^{\mathsf{T}}(t)Z_iB_iK_iz(t)\\&\quad-2\tilde{z}^{\mathsf{T}}(t)Z_iA_{di}\hat{z}(t-\hat{\tau}).\end{aligned}\qquad(20)$$

In addition, the following hold

$$\begin{aligned}&2z^{\mathsf{T}}(t)Z_i[\Delta A_i(t)z(t)+\Delta A_{di}(t)z(t-d(t))]\\&\leq\varepsilon_1^{-1}z^{\mathsf{T}}(t)Z_iM_iM^{\mathsf{T}}{}_iZ_iz(t)+\varepsilon_1[N_iz(t)\\&\quad+N_{di}z(t-d(t))]^{\mathsf{T}}[N_iz(t)+N_{di}z(t-d(t))],\end{aligned}\qquad(21)$$

$$\begin{aligned}&2\tilde{z}^{\mathsf{T}}(t)Z_i[\Delta A_i(t)z(t)+\Delta A_{di}(t)z(t-d(t))]\\&\leq\varepsilon_2^{-1}z^{\mathsf{T}}(t)Z_iM_iM^{\mathsf{T}}{}_iZ_iz(t)+\varepsilon_2[N_iz(t)\\&\quad+N_{di}z(t-d(t))]^{\mathsf{T}}[N_iz(t)+N_{di}z(t-d(t))].\end{aligned}\qquad(22)$$

Since the STDS arrives and stays at the SM that has been pre-programmed, $\sigma(t)$ then $=Y_i[2y(t)-C_i\hat{z}(t)]=B_i^{\mathsf{T}}Z_i[z(t)+\tilde{z}(t)]=0$ in view of (15), it further gives

$\sigma^\top(t) = [z^\top(t) + \tilde{z}^\top(t)]Z_iB_i$ equals to a zero vector. As a result, the preceding formulation can also be condensed as follows:

$$\begin{aligned}\dot{V}_1(i,t) \le\ & z^\top(t)[Z_i(A_i - B_iK_i) + (A_i - B_iK_i)^\top Z_i \\ & + \varepsilon_1^{-1} Z_iM_iM_i^\top Z_i + (\varepsilon_1 + \varepsilon_2)N_i^\top N_i]z(t) \\ & + 2z^\top(t)[Z_iA_{di} + (\varepsilon_1 + \varepsilon_2)N_i^\top N_{di}]z(t - d(t)) \\ & + \tilde{z}^\top(t)[Z_i(A_i - L_iC_i) \\ & + (A_i - L_iC_i)^\top Z_i + \varepsilon_2^{-1} Z_iM_iM_i^\top Z_i]\tilde{z}(t) \\ & + 2\tilde{z}^\top(t)Z_iA_{di}z(t - d(t)) - 2\tilde{z}^\top(t)Z_iB_iK_iz(t) \\ & - 2\tilde{z}^\top(t)Z_iA_{di}\hat{z}(t - \hat{\tau}) \\ & + z^\top(t - d(t))[(\varepsilon_1 + \varepsilon_2)N_{di}^\top N_{di}]z(t - d(t)).\end{aligned} \tag{23}$$

In like manner, it follows that

$$\begin{aligned}\dot{V}_2(i,t) \le & -(1-\theta)e^{-\varrho\hat{\tau}}z^\top(t - d(t))P_iz(t - d(t)) \\ & -(1-\theta)e^{-\varrho\hat{\tau}}\tilde{z}^\top(t - d(t))R_i\tilde{z}(t - d(t)) \\ & - e^{-\varrho\hat{\tau}}z^\top(t - \hat{\tau})Q_iz(t - \hat{\tau}) \\ & - e^{-\varrho\hat{\tau}}\tilde{z}^\top(t - \hat{\tau})T_i\tilde{z}(t - \hat{\tau}) \\ & - \varrho\int_{t-d(t)}^{t} e^{\varrho(s-t)}z^\top(s)P_iz(s)ds + z^\top(t)P_iz(t) \\ & - \varrho\int_{t-\hat{\tau}}^{t} e^{\varrho(s-t)}z^\top(s)Q_iz(s)ds + z^\top(t)Q_iz(t) \\ & - \varrho\int_{t-d(t)}^{t} e^{\varrho(s-t)}\tilde{z}^\top(s)R_i\tilde{z}(s)ds + \tilde{z}^\top(t)R_i\tilde{z}(t) \\ & - \varrho\int_{t-\hat{\tau}}^{t} e^{\varrho(s-t)}\tilde{z}^\top(s)T_i\tilde{z}(s)ds + \tilde{z}^\top(t)T_i\tilde{z}(t).\end{aligned} \tag{24}$$

Combining (19)-(24), the following holds

$$\dot{V}(i,t) + \varrho V(i,t) \le \xi^\top(t)\Phi_i\xi(t), \tag{25}$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \Phi_{14} & 0 & 0 \\ * & \Phi_{22} & 0 & (Z_iA_{di})^\top & 0 & 0 \\ * & * & \Phi_{33} & -(Z_iA_{di})^\top & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & Z_iA_{di} \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix}$$

with $\Phi_{11} = Z_i(A_i - B_iK_i) + (A_i - B_iK_i)^\top Z_i + P_i + Q_i + \varrho Z_i + \varepsilon_1^{-1}Z_iM_iM_i^\top Z_i + (\varepsilon_1 + \varepsilon_2)N_i^\top N_i$, $\Phi_{12} = Z_iA_{di} + (\varepsilon_1 + \varepsilon_2)N_i^\top N_{di}$, $\Phi_{14} = -(Z_iB_iK_i)^\top$, $\Phi_{22} = -(1-\theta)e^{-\varrho\hat{\tau}}P_i + (\varepsilon_1 + \varepsilon_2)N_{di}^\top N_{di}$, $\Phi_{33} = -e^{-\varrho\hat{\tau}}Q_i$, $\Phi_{44} = Z_i(A_i - L_iC_i) + (A_i - L_iC_i)^\top Z_i + R_i + T_i + \varrho Z_i + \varepsilon_2^{-1}Z_iM_iM_i^\top Z_i$, $\Phi_{55} = -(1-\theta)e^{-\varrho\hat{\tau}}R_i$, $\Phi_{66} = -e^{-\varrho\hat{\tau}}T_i$.
According to the Shur complement and (14), (25) gives

$$\dot{V}(i,t) + \varrho V(i,t) \le 0. \tag{26}$$

Define $\text{ß}(t_p^-) = j$ and $\text{ß}(t_p^+) = i$ for certain switching moment t_p, $p \in \{1, 2, ..., N_{\text{ß}}\}$. Integrating (26) from t_p to t, we have

$$V(i,t) \le e^{-\varrho(t-t_p)}V(i,t_p). \tag{27}$$

It is obtained by combining (16) and (27)

$$V(i,t_p) \le \kappa V(j,t_p{}^-). \tag{28}$$

Combining (27), (28) and $N_{\text{ß}}(t_0,t) \le N_0 + \dfrac{t - t_0}{T_a}$, it leads to

$$\begin{aligned} V(i,t) \le\ & e^{-\varrho(t-t_p)}\kappa V(j,t_p{}^-) \\ & \vdots \\ \le\ & e^{-\varrho(t-t_0)}\kappa^{N_{\text{ß}}(t_0,t)}V(\text{ß}(t_0),t_0) \\ \le\ & e^{-(\varrho-\frac{\ln\kappa}{T_a})(t-t_0)}V(\text{ß}(t_0),t_0).\end{aligned} \tag{29}$$

Furthermore, it is deduced from (18) that

$$p\|\xi(t)\|^2 \le V(i,t), V(\text{ß}(t_0),t_0) \le q\|\xi(t_0)\|_{c^1}^2. \tag{30}$$

Together with (29), we have

$$\|\xi(t)\|^2 \le \frac{1}{p}V(i,t) \le \frac{q}{p}e^{-(\varrho-\frac{\ln\kappa}{T_a})(t-t_0)}\|\xi(t_0)\|_{c^1}^2, \tag{31}$$

which implies (17). The dynamics of the STDS (2) are exponentially stable. □

4. ILLUSTRATIVE EXAMPLES

4.1. Example 1

The system parameters are provided as follows:
Subsystem 1:

$$A_1 = \begin{bmatrix} -5 & 0.5 \\ -4 & -3 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}^\top, M_1 = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}^\top, N_1 = \begin{bmatrix} 0.2 \\ -0.3 \end{bmatrix}^\top,$$

$$F_1(t) = \sin(t),\ N_{d1} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}^\top,\ g_1(t,z) = 0.5e^{-t}.$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -4 & -0.5 \\ -2 & -3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}^\top, M_2 = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}^\top, N_2 = \begin{bmatrix} 0.1 \\ -0.3 \end{bmatrix}^\top,$$

$$F_2(t) = \cos(t),\ N_{d1} = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}^\top,\ g_2(t,z) = 0.5e^{-t}.$$

K_i is selected as $\begin{bmatrix} -1 & -1 \end{bmatrix}$ to meet that $A_i - B_iK_i$ is Hurwitz, $i = 1, 2$. The time-vary delay is set by $d(t) =$

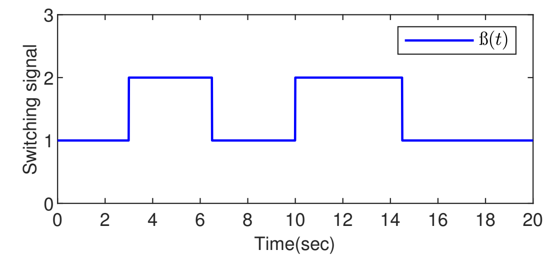

Fig. 1. Switching signal.

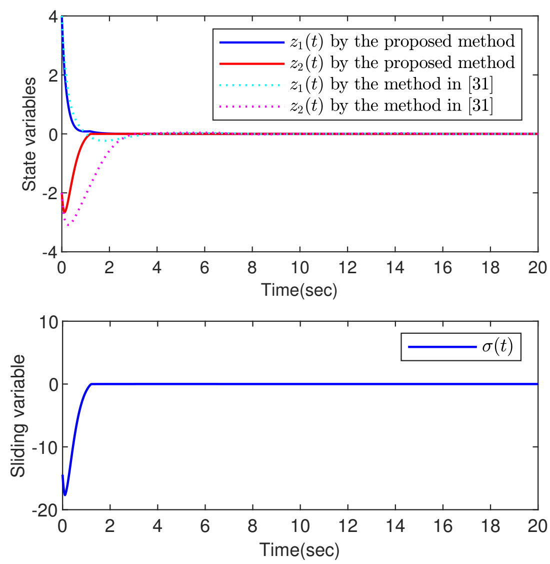

Fig. 2. Responses of system states and sliding variable.

$0.1+0.1\sin(t)$, then $\hat{\tau}=0.2$, $a_1=a_2=1$ can be selected, θ can be chosen as 0.2 and $\varrho=0.5$. Then by solving the LMI (14), the solutions are obtained as

$$P_1=\begin{bmatrix}39.9718 & 1.0263\\ 1.0263 & 41.4087\end{bmatrix},$$

$$P_2=\begin{bmatrix}35.7126 & -1.1161\\ -1.1161 & 38.8274\end{bmatrix},$$

$$Q_1=\begin{bmatrix}35.9622 & -0.2490\\ -0.2490 & 37.1662\end{bmatrix},$$

$$Q_2=\begin{bmatrix}32.9687 & -1.7005\\ -1.7005 & 33.5379\end{bmatrix},$$

$$R_1=\begin{bmatrix}40.8548 & -0.0945\\ -0.0945 & 40.7130\end{bmatrix},$$

$$R_2=\begin{bmatrix}37.7819 & -0.1403\\ -0.1403 & 37.5714\end{bmatrix},$$

$$T_1=\begin{bmatrix}37.8111 & -0.0792\\ -0.0792 & 37.6922\end{bmatrix},$$

$$T_2=\begin{bmatrix}34.9578 & -0.1176\\ -0.1176 & 34.7814\end{bmatrix}.$$

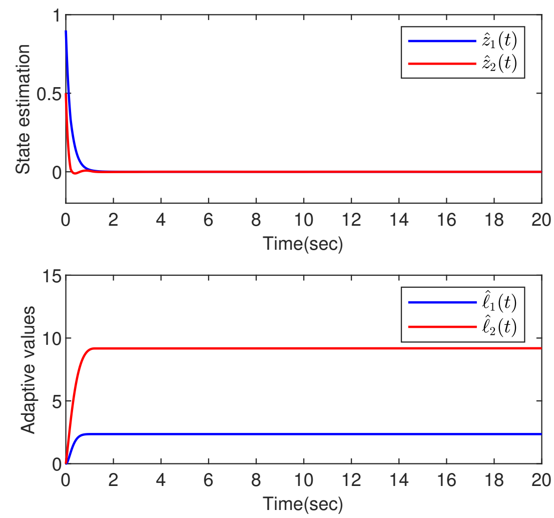

Fig. 3. Responses of observer and adaptive values.

Then, κ and T_a can be calculated as $\kappa=2.4802$ and $T_a=1.82$ s. For the method in [31], the parameters $\kappa=5.7021$, $T_a=3.48$ s are set under the proposed conditions. Fig. 1 shows the switching signal of the system. In contrast to [31], the proposed method does not require precise time delay information during the observer construction process. Fig. 2 shows that, for the same system parameters, the closed-loop system with the control strategy proposed in this paper has a faster convergence speed and a smaller overshoot, demonstrating the effectiveness of this method. In addition, Fig. 2 also shows the sliding variable. The evolutions of adaptive values and responses of the observer are provided in Fig. 3.

4.2. Example 2

Consider that both of the reactors are isothermal continuous stirred tanks reactors (CSTR) as shown in Fig. 4. The partition structure in the reactor automatically switches for a period of time to ensure the efficiency of the material re-

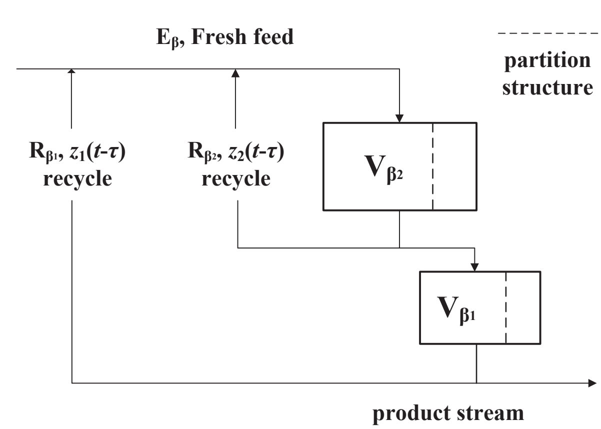

Fig. 4. CSTR.

action. According to [39], the CSTR model can be transformed as (2), where

$$A(ß) = \begin{bmatrix} -\frac{1}{\theta_{ß1}} - k_{ß1} & \frac{1-\mathrm{R}_{ß2}}{\mathrm{V}_{ß1}} \\ 0 & -\frac{1}{\theta_{ß2}} - k_{ß2} \end{bmatrix},\ B(ß) = \begin{bmatrix} 0 \\ \frac{\mathrm{E}_ß}{\mathrm{V}_{ß2}} \end{bmatrix},$$

$$A_d(ß) = \begin{bmatrix} 0 & 0 \\ \frac{\mathrm{R}_{ß1}}{\mathrm{V}_{ß2}} & \frac{\mathrm{R}_{ß2}}{\mathrm{V}_{ß2}} \end{bmatrix},\ \Delta A(ß,t) = \begin{bmatrix} \Delta p_1(t) & \Delta p_2(t) \\ 0 & \Delta p_3(t) \end{bmatrix},$$

$$\Delta A_d(ß,t) = \begin{bmatrix} 0 & 0 \\ \Delta p_4(t) & \Delta p_5(t) \end{bmatrix},$$

where $\mathrm{R}_{ß1}$ and $\mathrm{R}_{ß1}$ are the recycle flow rates, $\theta_{ß1}$ and $\theta_{ß2}$ are the reactor residence times, k_i are the reaction constants, $\mathrm{E}_ß$ is the feed rate and $\mathrm{V}_{ß1}$ and $\mathrm{V}_{ß2}$ are the reactor volumes ($ß = 1, 2$). $\Delta p_j(t)$ ($j = 1, 2, 3, 4, 5$) are unknown variables. In this simulation, we choose $\theta_{11} = \theta_{12} = 1$, $k_{11} = k_{12} = 2$, $\mathrm{R}_{11} = \mathrm{R}_{12} = 2$, $\mathrm{V}_{11} = 1$, $\mathrm{V}_{12} = 2$, $\mathrm{E}_1 = 2$ and $\theta_{21} = \theta_{22} = 0.8$, $k_{21} = k_{22} = 1.55$, $\mathrm{R}_{21} = \mathrm{R}_{22} = 1.4$, $\mathrm{V}_{21} = 0.5$, $\mathrm{V}_{22} = 1.4$, $\mathrm{E}_2 = 1.4$. Then, one has

Subsystem 1:

$$A_1 = \begin{bmatrix} -3 & -1 \\ 0 & -3 \end{bmatrix},\ A_{d1} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^{\mathrm{T}},\ M_1 = I_{2\times 2},\ N_1 = \begin{bmatrix} -0.2 & -0.2 \\ 0 & 0.2 \end{bmatrix},$$

$$F_1(t) = \sin(t),\ N_{d1} = \begin{bmatrix} 0 & 0 \\ 0.7 & 0.7 \end{bmatrix},\ g_1(t,z) = \frac{1}{1+t^3}.$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -2.8 & -0.8 \\ 0 & -2.8 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^{\mathrm{T}},\ M_2 = I_{2\times 2},\ N_2 = \begin{bmatrix} -0.25 & -0.25 \\ 0 & 0.2 \end{bmatrix},$$

$$F_2(t) = \cos(t),\ N_{d1} = \begin{bmatrix} 0 & 0 \\ 0.7 & 0.7 \end{bmatrix},\ g_2(t,z) = \frac{1}{1+t^3}.$$

K_i is selected as $\begin{bmatrix} -1 & 0.8 \end{bmatrix}$ to meet that $A_i - B_iK_i$ is Hurwitz, $i = 1, 2$. The time-vary delay is set by $d(t) = 0.2 + 0.1\sin(t)$, then $\hat{\tau} = 0.3$, θ can be chosen as 0.2 and $\varrho = 0.5$. Then by solving the LMI (14), the solutions are obtained as

$$P_1 = \begin{bmatrix} 0.4621 & 0.4001 \\ 0.4001 & 0.4781 \end{bmatrix},\ P_2 = \begin{bmatrix} 0.4181 & 0.3803 \\ 0.3803 & 0.4743 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.1343 & 0.0692 \\ 0.0692 & 0.1433 \end{bmatrix},\ T_2 = \begin{bmatrix} 0.1181 & 0.0774 \\ 0.0774 & 0.1259 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.1076 & 0.0319 \\ 0.0319 & 0.0903 \end{bmatrix},\ Q_2 = \begin{bmatrix} 0.0778 & 0.0325 \\ 0.0325 & 0.0696 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.073 & -0.0018 \\ -0.0018 & 0.0847 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.0455 & -0.0019 \\ -0.0019 & 0.056 \end{bmatrix}.$$

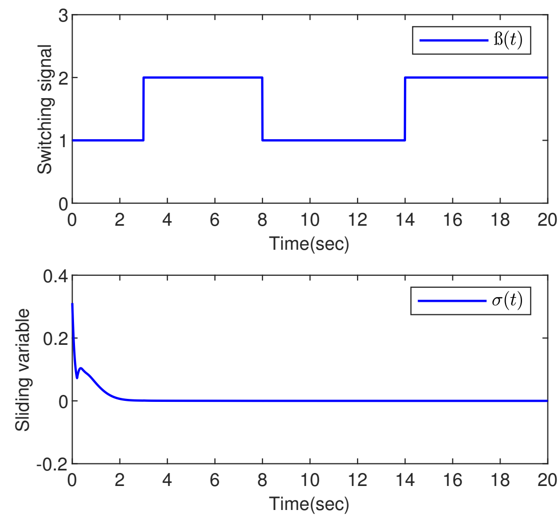

Fig. 5. Switching signal and sliding variable.

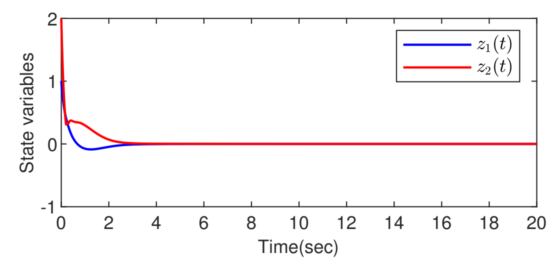

Fig. 6. Responses of system states.

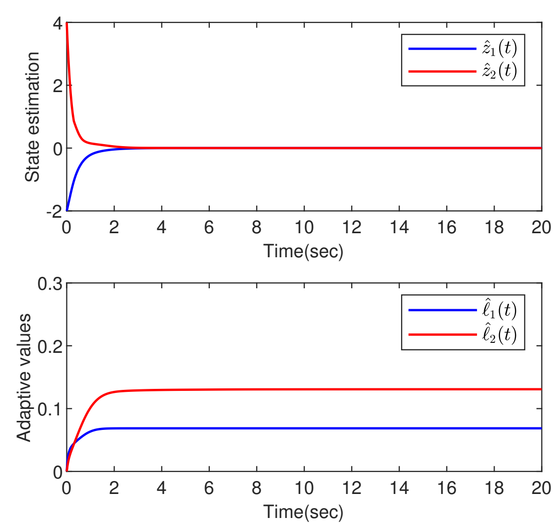

Fig. 7. Responses of observer and adaptive values.

Then, κ and T_a can be calculated as $\kappa = 13.4098$ and $T_a = 5.20$ s.

The simulation results are shown in Figs. 5-7. Fig. 5 represents the switching signal and the sliding variable. Fig. 6 shows the responses of system states. The evolutions of adaptive values and responses of the observer

Table 1. Range of residence time for different upper bounds $\hat{\tau}$ of time-delay.

$\hat{\tau}$	κ	T_a
0.3	13.4098	$T_a \geq 5.1920$
0.4	15.2705	$T_a \geq 5.4518$
0.5	17.1752	$T_a \geq 5.6869$
0.6	21.0557	$T_a \geq 6.0943$
0.7	25.9375	$T_a \geq 6.5114$

are provided in Fig. 7. Furthermore, some relationship between residence time and time-delay is probed, it can be found that the minimum value of residence time increases with the increment of the upper bound of time delay, as shown in Table 1. Therefore, the variation of time delay will affect the selection of residence time.

5. CONCLUSION

In this paper, a novel observer-based robust SMC issue of STDS subject to structural uncertainties, unknown state delay and unmeasured state variables has been addressed, including the simplified observer design, the ADT and associated adaptive SMC law to satisfy the control effect. The design of the observer no longer needs accurate time-delay information, which enhances the practicability of controller synthesis, and the SM establishment effectively reduces the derivation of the stability criterion. Finally, simulation examples have been performed to confirm the efficacy of the proposed control scheme.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no competing financial interests or personal relationships that could influence the work.

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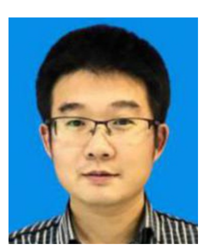

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