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Adaptive Containment Control for a Class of Uncertain Multi-agent Systems With Unknown Virtual Control Gain Functions

Meng-Yi Jiang , Yong-Hui Yang , Li-Bing Wu , and Qi Li*

Abstract: In this paper, we consider a containment control problem for a class of uncertain multi-agent systems (MASs). The systems contain unknown parameters and virtual control gain functions. By introducing lower bounds of virtual control gain functions into the Lyapunov functions, a novel controller design scheme is proposed based on an adaptive control design approach and bound estimation method. The designed controller is simpler in comparison with other controllers. The simulation results show that three followers converge into the convex hull rapidly following the sinusoidal leaders, and the effectiveness of the designed controller is verified.

Keywords: Adaptive backstepping control, containment control, multi-agent systems, uncertain nonlinear systems.

1. INTRODUCTION

Backstepping adaptive control [1-3] is one of the frontier topics of adaptive control theory and application [4]. In recent years, this method has shown great potential in improving the quality of transition process when dealing with nonlinear systems. Backstepping can be used to design control schemes to meet the matching conditions of triangular single input single output nonlinear systems. Therefore, backstepping design method has attracted great attention all over the world. In order to deal with unmodeled dynamics in nonlinear systems, a backstepping controller was proposed in [5], and the proposed control scheme does not require any dynamic dominating signal to guarantee the robustness property of Lagrange stability. In [6], the designed observer and controller are time-delay independent. Based on Lyapunov stability theory, and the asymptotically stable of the closed-loop system was achieved. In [7], a command filter and universal approximator was designed for uncertain nonlinear systems to avoid differential explosion problem.

Uncertain systems widely exist in nature and practical engineering applications. Objectively speaking, each actual system has different degrees of uncertainties. The internal uncertainties of the system usually refer to the structure and parameters of mathematical model, such as unmodeled dynamics, unknown system parameters and unknown control parameters, which can not be accurately known to the designer in advance. As an external environment, the impact of uncertainty on the system is often equivalent to unpredictable or random disturbances. In [8], an adaptive backstepping control scheme was designed for uncertain systems with unknown input time-delay. By considering input saturation and external disturbance, a novel adaptive control was realized in [9]. In [10], an adaptive stabilization was implemented for uncertain switched nonlinear systems. To deal with the problem of quantized input signal, a novel adaptive backstepping controller was designed in [11]. In [12], an adaptive fault-tolerant control scheme was given for a class of uncertain nonlinear systems by combining with command filtered technique. From [8-12], the diversity and necessity of studying uncertain systems were presented in details.

If the control direction of the closed-loop system is unknown, it will undoubtedly bring great difficulties for controller design. In fact, nonlinear systems with unknown control direction have a wide range of applications, hence the design of adaptive unknown control direction controller is a crucial concern and many results have been achieved. In [13], an adaptive controller was designed for nonlinear time-delay systems based on neural networks, and an adaptive fuzzy tracking control was realized for nonlinear time-delay systems in [14]. In [15], an adaptive robust control was proposed for uncertain dynamical systems. In [16], an adaptive asymptotic tracking was presented based on barrier Lyapunov function. In [17], an adaptive finite-time control was designed for stochastic nonlinear systems. From [13-17], it can be seen that Nussbaum gain technique, neural networks and fuzzy approximation are commonly used to deal with unknown control

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Meng-Yi Jiang, Yong-Hui Yang, and Qi Li are with the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, China (e-mails: {mengyimiao, yangyh2636688, liqi_as}@163.com). Li-Bing Wu is with the School of Science, University of Science and Technology Liaoning, Anshan, Liaoning, China (e-mail: beyondwlb@163.com).
* Corresponding author.

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direction for nonlinear systems.

A multi-agent system (MAS) is a set of multiple agents, and research of MAS involves the coordination and cooperation between agents. In recent years, multi-agent cooperative control has become a hot research field. Cooperative control problems of multi-agent systems include consistency control [18], containment control [19] and formation control [20]. The latter two can be regarded as the generalizations of consistency control. Containment control refers to a group of followers moving in the smallest geometric space (convex hull) surrounded by the guidance of multiple dynamic leaders. Containment control has a large number of potential applications in multiple agents jointly completing cooperative tasks such as hazardous material handling, enemy area search, fire rescue and cooperative transportation [21]. Therefore, containment control of MASs has been widely studied in recent years [22-27]. Generally, the agent may carry different devices during the cooperative control, which may cause the MASs with unknown control directions. To ensure the stability of the MASs, we need to design an effective controller to ensure that the follower enter and maintain in the convex hull surrounded by the leaders. From the existing achievements, there are few achievements in this area. Therefore, it is significant to study the problem of containment control with unknown control direction. As a result, we consider a containment control problem for a class of uncertain multi-agent systems. The agent contains unknown parameters and virtual control gain functions. Different from the related design schemes, the following contributions are summarized.

1) An adaptive controller with simpler structure is designed, and it is first attempt to the application in MASs. Compared with other related methods, the designed controller is effective in structure and calculation. The proposed controller ensures that all the signals of the closed-loop MASs are uniformly and ultimately bounded, and the followers will converge into the convex hull surrounded by the leaders.
2) In order to solve the problem of virtual control direction in MASs, different from [13-17], we propose a simple controller design method based on backstepping method. By introducing the bound of virtual control gain function into the Lyapunov function, the design procedure is based on adaptive control method and bound estimation method. The controller does not need to know the upper or lower bounds of unknown virtual control gain function, and the applications of upper or lower bounds are only used for stability analysis. The controller design is not restricted by the sign of gain function.

2. STATEMENTS AND CONTROL OBJECTIVE OF MULTI-AGENT SYSTEMS

Consider the nonlinear strict-feedback MASs with the form

$$\begin{cases} \dot{x}_{\beta,\gamma} = g_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right) x_{\beta,\gamma+1} + \theta_\beta^T f_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right), \\ \qquad 1 \le \gamma \le n-1, \\ \dot{x}_{\beta,n} = g_{\beta,n}\left(\bar{x}_{\beta,\gamma}\right) u_\beta + \theta_\beta^T f_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right), \\ y_\beta = x_{\beta,1}, \end{cases} \tag{1}$$

where $\bar{x}_{\beta,\gamma} = [x_{\beta,1}, x_{\beta,2}, \ldots, x_{\beta,\gamma}]^T \in R^\gamma$, $\beta = 1, 2, \ldots, N$, $\gamma = 1, 2, \ldots, n$ denote the states of the βth agent, $u_\beta \in R$ represents the control input of the βth agent, $y_\beta \in R$ denotes the output of the βth agent, $\theta_\beta \in R^l$ is an unknown system parameter vector, $g_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right) \in R$, $\gamma = 1, 2, \ldots, n$, are unknown control gain functions, and $f_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right) \in R^\gamma$, $\gamma = 1, 2, \ldots, n$, denote known smooth nonlinear functions. For the convenience of derivation, $g_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right)$ and $f_{\beta,\gamma}\left(\bar{x}_{\beta,\gamma}\right)$ are abbreviated as $g_{\beta,\gamma}$ and $f_{\beta,\gamma}$, respectively, and $\|\cdot\|$ denotes the L_2 norm of the vector.

In order to connect the information between agents, graph theory is adopted in this paper. Define information exchange graph of the agents as $G = \left(s, \varepsilon, \bar{C}\right)$, where $s = \{s_1, \ldots, s_N, s_{N+1}, \ldots, s_{N+M}\}$ is a set of agents, and the followers are marked as $\beta = 1, 2, \ldots, N$. In other words, in set s, the first N agents are followers and the last M agents are leaders. For $j = 1, \ldots, N+M$, $\varepsilon = \left\{\left(s_\beta, s_j\right)\right\} \subseteq s \times s$ denotes a edge set, where $\times$ represents the Cartesian product, and $\bar{C} = \left[c_{\beta,j}\right] \in R^{(N+M)\times(N+M)}$ is called an adjacency matrix. $\left(s_\beta, s_j\right) \in \varepsilon$ represents the follower β can receive the information from its neighbour agent j. If $\left(s_\beta, s_j\right) \notin \varepsilon$, $c_{\beta,j} = 0$, otherwise, $c_{\beta,j} = 1$. Noticing that $c_{j,j} = 0$ (no self-edges) and $c_{i,j} = 0$ (there is no neighbour for leaders) for $i = N+1, \ldots, N+M$. Define a Laplacian matrix as $L = \left[L_{\beta,j}\right] \in R^{(N+M)\times(N+M)} = D - \bar{C}$, and $D = diag(d_1, \ldots, d_j, \ldots, d_{N+M})$ is a degree matrix of agent j, and $d_\beta = \sum_{j=1}^{N+M} c_{\beta,j}$. Suppose that there is at least one neighbour for each follower. Under this condition, the Laplacian matrix of the graph can be partitioned as

$$L = \begin{bmatrix} \hat{L}_1 & \hat{L}_2 \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix}, \tag{2}$$

where $\hat{L}_1 \in R^{N\times N}$ represents the communication between N followers and $\hat{L}_2 \in R^{N\times M}$ denotes the communication between N followers and M leaders.

Definition 1 [28]**:** For a convex set $\Theta \subseteq R^l$, there exists arbitrary $x_1, x_2 \subseteq \Theta$ and $\varrho \in [0, 1]$ such that the point $\varrho x_1 + (1-\rho)x_2$ is also in set Θ. For a set of points $\Lambda = \{x_1, \ldots, x_n\}$, convex hull $Co(\Lambda)$ is a minimal convex set and defined as $Co(\Lambda) = \left\{\sum_{i=1}^{l} \varrho_i x_i \mid x_i \in \Lambda,\ \varrho_i > 0,\ \sum_{i=1}^{l} \varrho_i = 1\right\}$.

In this paper, control objective is to design a feasible adaptive controller for MAS (1) such that the outputs y_β

($\beta = 1, 2, \ldots, N$) of the followers converge to the convex hull following the leaders $y_{d,j}$, $j = N+1, \ldots, N+M$, i.e., $\inf_{w\in Y} |y_\beta - w| < \epsilon$, $Y = Co\{y_{N+1}, \ldots, y_{N+M}\}$, and $\epsilon > 0$ is a sufficiently small constant. In addition, all the signals of the closed-loop MAS are uniformly and ultimately bounded.

To achieve the control objective, the following Assumptions are needed.

Assumption 1: The dynamic leaders $y_{d,j} \in R$, $j = N+1, \ldots, N+M$ are bounded and differentiable, and their nth-order derivatives $y_{d,j}^{(n)}$ are continuous and bounded.

Assumption 2: The sign of $g_{\beta,\gamma}$, $\gamma = 1, 2, \ldots, n$ is known. In addition, if $g_{\beta,\gamma} > 0$, the lower bound $\grave{g}_{\beta,\gamma}$ of $g_{\beta,\gamma}$ is unknown such that $0 < \grave{g}_{\beta,\gamma} \le g_{\beta,\gamma}$ and $g_{\beta,\gamma}/\grave{g}_{\beta,\gamma} \ge 1$; if $g_{\beta,\gamma} < 0$, the upper bound $\acute{g}_{\beta,\gamma}$ of $g_{\beta,\gamma}$ is unknown such that $g_{\beta,\gamma} \le \acute{g}_{\beta,\gamma} < 0$ and $g_{\beta,\gamma}/\acute{g}_{\beta,\gamma} \ge 1$. Without loss of generality, $g_{\beta,\gamma} > 0$ is used in this paper.

Assumption 3: Each follower agent has at least one directed path communicated from the dynamic leader.

Lemma 1 [29]**:** Based on Assumption 3, $\hat{L}_1$ is a nonsingular M-matrix, which means it is invertible. In addition, the sum of each row of $-\hat{L}_1^{-1}\hat{L}_2$ is equal to 1, and all the entries of $-\hat{L}_1^{-1}\hat{L}_2$ are nonnegative.

Define $\check{y} = [y_{d,N+1}, \ldots, y_{d,N+M}]^T$ and $y_f = [y_{f,1}, \ldots, y_{f,N}]^T = -\hat{L}_1^{-1}\hat{L}_2\check{y}$, where $y_{f,p} \in R$ with $p = 1, 2, \ldots, N$. From Lemma 1, it is easy to know that $\inf_{w\in Y} |y_{f,p} - w| < \epsilon$ with $p = 1, 2, \ldots, N$ for all $t \ge 0$. Then, containment control design can be transformed into tracking control design for each follower such that $|y_\beta - y_{f,p}| < \check{\epsilon}$ with $p = 1, 2, \ldots, N$ and $\check{\epsilon} > 0$ is a constant that can be sufficiently small.

3. MAIN RESULTS

Theorem 1: Consider the uncertain MASs (1), based on Assumptions 1-3, for arbitrary initial conditions, the following state feedback control laws and adaptive law guarantee that the outputs y_β ($\beta = 1, 2, \ldots, N$) of the followers can track the desired signals $y_{d,j}$, $j = N+1, \ldots, N+M$, of the leaders, and all the signals of the closed-loop MASs are uniformly and ultimately bounded. Additionally, containment errors between leaders and followers converge to a tuned neighborhood of the origin.

$$\begin{cases} \alpha_{\beta,1} = -\dfrac{1}{d_\beta}\left(b_{\beta,1}z_{\beta,1}\bar{\alpha}_{\beta,1}^2 + a_{\beta,1}z_{\beta,1}\right), \\ \alpha_{\beta,\gamma} = -\left(b_{\beta,\gamma}z_{\beta,\gamma}\bar{\alpha}_{\beta,\gamma}^2 + a_{\beta,\gamma}z_{\beta,\gamma}\right), \\ \qquad \gamma = 2, 3, \ldots, n, \ u_\beta = \alpha_{\beta,n}, \\ \dot{\hat{\mu}}_\beta = \sum_{k=1}^{n} r_\beta\left(|z_{\beta,k}|\,\|W_{\beta,k}\|\right) - \sigma_\beta\hat{\mu}_\beta, \end{cases} \tag{3}$$

where $d_\beta = \sum_{j=1}^{N+M} c_{\beta,j}$, $a_{\beta,\gamma} > 0$, $b_{\beta,\gamma} > 0$, $\gamma = 1, \ldots, n$, $r_\beta > 0$ and $\sigma_\beta > 0$ are positive design parameters. $\alpha_{\beta,\gamma-1}$, $\gamma = 2, \ldots, n$ denotes the virtual control laws for the γth-subsystem of the βth agent. The transformation errors $z_{\beta,\gamma}$ are given by

$$\begin{cases} z_{\beta,1} = \sum_{j=1}^{N} c_{\beta,j}\left(y_\beta - y_j\right) + \sum_{j=N+1}^{N+M} c_{\beta,j}\left(y_\beta - y_{d,j}\right) \\ \qquad = d_\beta y_\beta - \sum_{j=1}^{N} c_{\beta,j}y_j - \sum_{j=N+1}^{N+M} c_{\beta,j}y_{d,j}, \\ z_{\beta,\gamma} = x_{\beta,\gamma} - \alpha_{\beta,\gamma-1}, \ \gamma = 2, \ldots, n. \end{cases} \tag{4}$$

Suppose that μ_β is the upper bound of the unknown vectors $\upsilon_{\beta,\gamma}$ and which is defined by

$$\mu_\beta = \max\left\{\left\|\upsilon_{\beta,\gamma}\right\|, \gamma = 1, \ldots, n\right\}, \tag{5}$$

where

$$\upsilon_{\beta,1} = \left[\frac{\theta_\beta^T}{\grave{g}_{\beta,1}}, \ \frac{1}{\grave{g}_{\beta,1}}\sum_{j=1}^{N} c_{\beta,j}(g_{j,1}x_{j,2} - \theta_j^T f_{j,1}), \ \frac{1}{\grave{g}_{\beta,1}}\right]^T, \tag{6}$$

and

$$\upsilon_{\beta,\gamma} = \left[\upsilon_{\beta,\gamma,1}^T, \ \upsilon_{\beta,\gamma,2}^T, \ \upsilon_{\beta,\gamma,3}^T, \ \upsilon_{\beta,\gamma,4}^T, \ \upsilon_{\beta,\gamma,5}^T\right]^T, \tag{7}$$
$$\gamma = 2, 3, \ldots, n,$$

with

$$\upsilon_{\beta,\gamma,1} = \left[\frac{g_{\beta,\gamma-1}}{\grave{g}_{\beta,\gamma-1}}, \ \frac{\theta_\beta^T}{\grave{g}_{\beta,\gamma}}\right]^T,$$

$$\upsilon_{\beta,\gamma,2} = \left[\frac{g_{\beta,1}}{\grave{g}_{\beta,\gamma}}, \ \frac{g_{\beta,2}}{\grave{g}_{\beta,\gamma}}, \ \ldots, \ \frac{g_{\beta,\gamma-1}}{\grave{g}_{\beta,\gamma}}\right]^T,$$

$$\upsilon_{\beta,\gamma,3} = \left[\frac{1}{\grave{g}_{\beta,\gamma}}, \ \frac{\dot{\hat{\mu}}_\beta}{\grave{g}_{\beta,\gamma}}\right]^T,$$

$$\upsilon_{\beta,\gamma,4} = \begin{bmatrix} \frac{g_{1,1}}{\grave{g}_{\beta,\gamma}}c_{\beta,1}, \ \ldots, \ \frac{g_{1,\gamma-1}}{\grave{g}_{\beta,\gamma}}c_{\beta,1}, \ \ldots, \ \frac{g_{j,1}}{\grave{g}_{\beta,\gamma}}c_{\beta,j}, \ \ldots, \\ \frac{g_{j,\gamma-1}}{\grave{g}_{\beta,\gamma}}c_{\beta,j}, \ \ldots, \ \frac{g_{N,1}}{\grave{g}_{\beta,\gamma}}c_{\beta,N}, \ \ldots, \ \frac{g_{N,\gamma-1}}{\grave{g}_{\beta,\gamma}}c_{\beta,N} \end{bmatrix}^T,$$

$$\upsilon_{\beta,\gamma,5} = \left[\frac{\theta_1^T}{\grave{g}_{\beta,\gamma}}c_{\beta,1}, \ldots, \frac{\theta_j^T}{\grave{g}_{\beta,\gamma}}c_{\beta,j}, \ldots, \frac{\theta_N^T}{\grave{g}_{\beta,\gamma}}c_{\beta,N}\right]^T,$$

with $j = 1, 2, \ldots, N$ and $j \ne \beta$.

$\hat{\mu}_\beta$ is the estimate of μ_β, then the estimation error is degined as $\tilde{\mu}_\beta = \mu_\beta - \hat{\mu}_\beta$. The vectors $W_{\beta,k}$ in (3) are designed as follows:

$$W_{\beta,1} = \left[d_\beta f_{\beta,1}^T, -1, -\sum_{j=N+1}^{N+M} c_{\beta,j}\dot{y}_{d,j}\right]^T, \tag{8}$$

and

$$W_{\beta,\gamma} = \left[W_{\beta,\gamma,1}^T, \ W_{\beta,\gamma,2}^T, \ W_{\beta,\gamma,3}^T, \ W_{\beta,\gamma,4}^T, \ W_{\beta,\gamma,5}^T\right]^T, \tag{9}$$
$$\gamma = 2, 3, \ldots, n,$$

with

$$W_{\beta,\gamma,1}=\left[z_{\beta,\gamma-1},f_{\beta,\gamma}^T-\sum_{k=1}^{\gamma-1}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{\beta,k}}f_{\beta,k}^T\right]^T,$$

$$W_{\beta,\gamma,2}=\left[-\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{\beta,1}}x_{\beta,2},-\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{\beta,2}}x_{\beta,3},\ldots,-\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{\beta,\gamma-1}}x_\gamma\right]^T,$$

$$W_{\beta,\gamma,3}=\left[-\sum_{j=N+1}^{N+M}\sum_{k=0}^{\gamma-1}c_{\beta,j}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial y_{d,j}^{(k)}}y_{d,j}^{(k+1)},-\frac{\partial\alpha_{\beta,\gamma-1}}{\partial\hat{\mu}_\beta}\right]^T,$$

$$W_{\beta,\gamma,4}=\begin{bmatrix}-c_{\beta,1}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{1,1}}x_{1,2},\ldots,-c_{\beta,1}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{1,\gamma-1}}x_{1,\gamma},\ldots,\\-c_{\beta,j}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{j,1}}x_{j,2},\ldots,-c_{\beta,j}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{j,\gamma-1}}x_{j,\gamma},\ldots,\\-c_{\beta,N}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{N,1}}x_{N,2},\ldots,-c_{\beta,N}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{N,\gamma-1}}x_{N,\gamma}\end{bmatrix}^T,$$

$$W_{\beta,\gamma,5}=\begin{bmatrix}-\sum_{k=1}^{\gamma-1}c_{\beta,1}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{1,k}}f_{1,k},\ldots,\\-\sum_{k=1}^{\gamma-1}c_{\beta,j}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{2,k}}f_{j,k},\ldots,-\sum_{k=1}^{\gamma-1}c_{\beta,N}\frac{\partial\alpha_{\beta,\gamma-1}}{\partial x_{N,k}}f_{N,k}\end{bmatrix}^T,$$

where $j=1,2,\ldots,N$ and $j\neq\beta$.

Proof: The controller design process is consisting of the following n steps based on Lyapunov second method and backstepping method.

Step 1: For the 1st subsystem of the βth agent, select the Lyapunov function candidate $V_{\beta,1}$ as follows:

$$V_{\beta,1}=\frac{1}{2\grave{g}_{\beta,1}}z_{\beta,1}^2+\frac{1}{2r_\beta}\tilde{\mu}_\beta^2. \tag{10}$$

The time derivative of $V_{\beta,1}$ is obtained by

$$\dot{V}_{\beta,1}=\frac{1}{\grave{g}_{\beta,1}}z_{\beta,1}\dot{z}_{\beta,1}+\frac{1}{r_\beta}\tilde{\mu}_\beta\dot{\tilde{\mu}}_\beta. \tag{11}$$

From (1), (4), (11) and $\tilde{\mu}_\beta=\mu_\beta-\hat{\mu}_\beta$, the following results hold.

$$\begin{aligned}\dot{V}_{\beta,1}=&\frac{1}{\grave{g}_{\beta,1}}z_{\beta,1}d_\beta\left(g_{\beta,1}x_{\beta,2}+\theta_\beta^Tf_{\beta,1}\right)\\&-\frac{1}{\grave{g}_{\beta,1}}z_{\beta,1}\sum_{j=1}^{N}c_{\beta,j}\left(g_{j,1}x_{j,2}-\theta_j^Tf_{j,1}\right)\\&-\frac{1}{\grave{g}_{\beta,1}}z_{\beta,1}\sum_{j=N+1}^{N+M}c_{\beta,j}\dot{y}_{d,j}-\frac{1}{r_\beta}\tilde{\mu}_\beta\dot{\hat{\mu}}_\beta\\=&\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}z_{\beta,2}+\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}\alpha_{\beta,1}-\frac{1}{r_\beta}\tilde{\mu}_\beta\dot{\hat{\mu}}_\beta\\&+\frac{1}{\grave{g}_{\beta,1}}z_{\beta,1}\left(d_\beta\theta_\beta^Tf_{\beta,1}-\sum_{j=N+1}^{N+M}c_{\beta,j}\dot{y}_{d,j}\right)\\&-\frac{1}{\grave{g}_{\beta,1}}z_{\beta,1}\sum_{j=1}^{N}c_{\beta,j}\left(g_{j,1}x_{j,2}-\theta_j^Tf_{j,1}\right)\\=&\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}z_{\beta,2}+\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}\alpha_{\beta,1}\\&-\frac{1}{r_\beta}\tilde{\mu}_\beta\dot{\hat{\mu}}_\beta+z_{\beta,1}\upsilon_{\beta,1}^TW_{\beta,1}.\end{aligned} \tag{12}$$

From (5), we have $z_{\beta,1}\upsilon_{\beta,1}^TW_{\beta,1}\leq|z_{\beta,1}|\left\|\upsilon_{\beta,1}^T\right\|\left\|W_{\beta,1}\right\|\leq|z_{\beta,1}|\mu_\beta\left\|W_{\beta,1}\right\|$. By applying $\tilde{\mu}_\beta=\mu_\beta-\hat{\mu}_\beta$ and (12), it gives

$$\begin{aligned}\dot{V}_{\beta,1}\leq&\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}z_{\beta,2}+\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}\alpha_{\beta,1}\\&-\frac{1}{r_\beta}\tilde{\mu}_\beta\dot{\hat{\mu}}_\beta+|z_{\beta,1}|\mu_\beta\left\|W_{\beta,1}\right\| \qquad (13)\\\leq&\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}z_{\beta,2}+\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}\alpha_{\beta,1}+|z_{\beta,1}|\bar{\alpha}_{\beta,1}\\&+\frac{1}{r_\beta}\tilde{\mu}_\beta\left(r_\beta|z_{\beta,1}|\left\|W_{\beta,1}\right\|-\dot{\hat{\mu}}_\beta\right), \qquad (14)\end{aligned}$$

where $\bar{\alpha}_{\beta,1}=\hat{\mu}_\beta\left\|W_{\beta,1}\right\|$. According to (3), the following inequality holds.

$$\begin{aligned}&\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}\alpha_{\beta,1}+|z_{\beta,1}|\bar{\alpha}_{\beta,1}\\&\leq d_\beta z_{\beta,1}\alpha_{\beta,1}+|z_{\beta,1}|\bar{\alpha}_{\beta,1}\\&=-\left(b_{\beta,1}z_{\beta,1}^2\bar{\alpha}_{\beta,1}^2+a_{\beta,1}z_{\beta,1}^2\right)+|z_{\beta,1}|\bar{\alpha}_{\beta,1}\\&=-b_{\beta,1}\left(z_{\beta,1}^2\bar{\alpha}_{\beta,1}^2-\frac{1}{b_{\beta,1}}|z_{\beta,1}|\bar{\alpha}_{\beta,1}+\frac{1}{4b_{\beta,1}^2}\right)\\&\quad+\frac{1}{4b_{\beta,1}}-a_{\beta,1}z_{\beta,1}^2\\&=-b_{\beta,1}\left(z_{\beta,1}\bar{\alpha}_{\beta,1}-\frac{1}{2b_{\beta,1}}\right)^2+\frac{1}{4b_{\beta,1}}-a_{\beta,1}z_{\beta,1}^2\\&\leq\frac{1}{4b_{\beta,1}}-a_{\beta,1}z_{\beta,1}^2.\end{aligned} \tag{15}$$

Substituting (15) into (14) yields that

$$\begin{aligned}\dot{V}_{\beta,1}\leq&-a_{\beta,1}z_{\beta,1}^2+\frac{1}{4b_{\beta,1}}+\frac{g_{\beta,1}}{\grave{g}_{\beta,1}}d_\beta z_{\beta,1}\alpha_{\beta,1}\\&+\frac{1}{r_\beta}\tilde{\mu}_\beta\left(r_\beta|z_{\beta,1}|\left\|W_{\beta,1}\right\|-\dot{\hat{\mu}}_\beta\right).\end{aligned} \tag{16}$$

Step γ ($\gamma=2,\ldots,n-1$): For the γth subsystem of the βth agent, select the Lyapunov function candidate $V_{\beta,\gamma}$ as follows:

$$V_{\beta,\gamma}=V_{\beta,\gamma-1}+\frac{1}{2\grave{g}_{\beta,\gamma}}z_{\beta,\gamma}^2. \tag{17}$$

The time derivative of $V_{\beta,\gamma}$ is derived as

$$\begin{aligned}\dot{V}_{\beta,\gamma}=&\dot{V}_{\beta,\gamma-1}+\frac{1}{\grave{g}_{\beta,\gamma}}z_{\beta,\gamma}\dot{z}_{\beta,\gamma}\\=&\dot{V}_{\beta,\gamma-1}+\frac{1}{\grave{g}_{\beta,\gamma}}z_{\beta,\gamma}\left(\dot{x}_{\beta,\gamma}-\dot{\alpha}_{\beta,\gamma-1}\right).\end{aligned} \tag{18}$$

From (1) and (3), it gives that

$$\begin{aligned}\dot{V}_{\beta,\gamma} =& \dot{V}_{\beta,\gamma-1} + \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} x_{\beta,\gamma+1} + \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \theta_\beta^T f_{\beta,\gamma} \\ &- \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \sum_{k=1}^{\gamma-1} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{\beta,k}} \left(g_{\beta,k} x_{\beta,k+1} + \theta_\beta^T f_{\beta,k} \right) \\ &- \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \sum_{j=1}^{N} \sum_{k=1}^{\gamma-1} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{j,k}} \big(g_{j,k} x_{j,k+1} \\ &+ \theta_j^T f_{j,k} \big) - \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \sum_{j=N+1}^{N+M} \sum_{k=0}^{\gamma-1} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial y_{d,j}^{(k)}} y_{d,j}^{(k+1)} \\ &- \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial \hat{\mu}_\beta} \dot{\hat{\mu}}_\beta. \end{aligned} \tag{19}$$

Then, the following result holds.

$$\begin{aligned}\dot{V}_{\beta,\gamma} =& \dot{V}_{\beta,\gamma-1} + \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} x_{\beta,\gamma+1} \\ &+ \frac{\theta_\beta^T}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \left(f_{\beta,\gamma} - \sum_{k=1}^{\gamma-1} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{\beta,k}} f_{\beta,k} \right) \\ &- z_{\beta,\gamma} \sum_{k=1}^{\gamma-1} \frac{g_{\beta,k}}{\grave{g}_{\beta,\gamma}} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{\beta,k}} x_{\beta,k+1} \\ &- z_{\beta,\gamma} \sum_{j=1}^{N} \sum_{k=1}^{\gamma-1} \frac{g_{j,k}}{\grave{g}_{\beta,\gamma}} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{j,k}} x_{j,k+1} \\ &- z_{\beta,\gamma} \sum_{j=1}^{N} \sum_{k=1}^{\gamma-1} \frac{\theta_j^T}{\grave{g}_{\beta,\gamma}} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{j,k}} f_{j,k} \\ &- \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \sum_{j=N+1}^{N+M} \sum_{k=0}^{\gamma-1} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial y_{d,j}^{(k)}} y_{d,j}^{(k+1)} \\ &- \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial \hat{\mu}_\beta} \dot{\hat{\mu}}_\beta. \end{aligned} \tag{20}$$

From $x_{\beta,\gamma+1} = z_{\beta,\gamma+1} + \alpha_{\beta,\gamma}$ and (16), (20) can be further derived as follows:

$$\begin{aligned}\dot{V}_{\beta,\gamma} \le& -\sum_{k=1}^{\gamma-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{\gamma-1} \frac{1}{4b_{\beta,k}} \\ &+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{\gamma-1} r_\beta |z_{\beta,k}| \|W_{\beta,k}\| - \dot{\hat{\mu}}_\beta \right) \\ &+ \frac{g_{\beta,\gamma-1}}{\grave{g}_{\beta,\gamma-1}} z_{\beta,\gamma-1} z_{\beta,\gamma} + \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} z_{\beta,\gamma+1} \\ &+ \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} \alpha_{\beta,\gamma} \\ &+ \frac{\theta_\beta^T}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \left(f_{\beta,\gamma} - \sum_{k=1}^{\gamma-1} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{\beta,k}} f_{\beta,k} \right) \\ &- z_{\beta,\gamma} \sum_{k=1}^{\gamma-1} \frac{g_{\beta,k}}{\grave{g}_{\beta,\gamma}} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{\beta,k}} x_{\beta,k+1} \\ &- z_{\beta,\gamma} \sum_{j=1}^{N} \sum_{k=1}^{\gamma-1} \frac{g_{j,k}}{\grave{g}_{\beta,\gamma}} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{j,k}} x_{j,k+1} \\ &- z_{\beta,\gamma} \sum_{j=1}^{N} \frac{\theta_j^T}{\grave{g}_{\beta,\gamma}} \sum_{k=1}^{\gamma-1} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial x_{j,k}} f_{j,k} \\ &- \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \sum_{j=N+1}^{N+M} \sum_{k=0}^{\gamma-1} c_{\beta,j} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial y_{d,j}^{(k)}} y_{d,j}^{(k+1)} \\ &- \frac{\dot{\hat{\mu}}_\beta}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \frac{\partial \alpha_{\beta,\gamma-1}}{\partial \hat{\mu}_\beta}. \end{aligned} \tag{21}$$

From (7), it follows from (21) that

$$\begin{aligned}\dot{V}_{\beta,\gamma} \le& -\sum_{k=1}^{\gamma-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{\gamma-1} \frac{1}{4b_{\beta,k}} \\ &+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{\gamma-1} r_\beta |z_{\beta,k}| \|W_{\beta,k}\| - \dot{\hat{\mu}}_\beta \right) \\ &+ \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} z_{\beta,\gamma+1} + \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} \alpha_{\beta,\gamma} \\ &+ z_{\beta,\gamma} \upsilon_{\beta,\gamma}^T W_{\beta,\gamma} \\ \le& -\sum_{k=1}^{\gamma-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{\gamma-1} \frac{1}{4b_{\beta,k}} \\ &+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{\gamma-1} r_\beta |z_{\beta,k}| \|W_{\beta,k}\| - \dot{\hat{\mu}}_\beta \right) \\ &+ \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} z_{\beta,\gamma+1} \\ &+ \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} \alpha_{\beta,\gamma} + |z_{\beta,\gamma}| \bar{\alpha}_{\beta,\gamma}, \end{aligned} \tag{22}$$

with $\bar{\alpha}_{\beta,\gamma} = \hat{\mu}_\beta \|W_{\beta,\gamma}\|$.

According to $\alpha_{\beta,\gamma}$ in (3) and (15), then the following inequality holds.

$$\frac{g_{\beta,\gamma}}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} \alpha_{\beta,\gamma} + |z_{\beta,\gamma}| \bar{\alpha}_{\beta,\gamma} \le \frac{1}{4b_{\beta,\gamma}} - a_{\beta,\gamma} z_{\beta,\gamma}^2. \tag{23}$$

Substituting (23) into (22) gives that

$$\begin{aligned}\dot{V}_{\beta,\gamma} \le& -\sum_{k=1}^{\gamma} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{\gamma} \frac{1}{4b_{\beta,\gamma k}} \\ &+ \frac{1}{\grave{g}_{\beta,\gamma}} z_{\beta,\gamma} g_{\beta,\gamma} z_{\beta,\gamma+1} \\ &+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{\gamma} r_\beta |z_{\beta,k}| \|W_{\beta,k}\| - \dot{\hat{\mu}}_\beta \right). \end{aligned} \tag{24}$$

Step n: For the nth subsystem of the βth agent, choose the Lyapunov function candidate $V_{\beta,n}$ as follows:

$$V_{\beta,n} = V_{\beta,n-1} + \frac{1}{2\grave{g}_{\beta,n}} z_{\beta,n}^2. \tag{25}$$

Then, the time derivative of $V_{\beta,n}$ is obtained by

$$\dot{V}_{\beta,n} = \dot{V}_{\beta,n-1} + \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \dot{z}_{\beta,n}$$

$$
\begin{aligned}
&= \dot{V}_{\beta,n-1} + \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \left(\dot{x}_{\beta,n} - \dot{\alpha}_{\beta,n-1}\right) \\
&= \dot{V}_{\beta,n-1} + \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \left(g_{\beta,n} u_\beta + \theta_\beta^T f_{\beta,n} - \dot{\alpha}_{\beta,n-1}\right).
\end{aligned} \tag{26}
$$

From the results in Step γ, it follows that

$$
\begin{aligned}
\dot{V}_n = &\dot{V}_{n-1} + \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} g_{\beta,n} u_\beta + \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \theta_\beta^T f_{\beta,n} \\
&- \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \sum_{k=1}^{n-1} \frac{\partial \alpha_{\beta,n-1}}{\partial x_{\beta,k}} \left(g_{\beta,k} x_{\beta,k+1} + \theta_\beta^T f_{\beta,k}\right) \\
&- \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \sum_{j=N+1}^{N+M} \sum_{k=0}^{n-1} c_{\beta,j} \frac{\partial \alpha_{\beta,n-1}}{\partial y_{d,j}^{(k)}} y_{d,j}^{(k+1)} \\
&- \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \frac{\partial \alpha_{\beta,n-1}}{\partial \hat{\mu}_\beta} \dot{\hat{\mu}}_\beta.
\end{aligned} \tag{27}
$$

Then, the following inequality holds.

$$
\begin{aligned}
\dot{V}_n \le &-\sum_{k=1}^{n-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n-1} \frac{1}{4b_{\beta,k}} + z_{\beta,n} \frac{g_{\beta,n-1}}{\grave{g}_{\beta,n-1}} z_{\beta,n-1} \\
&+ \frac{g_{\beta,n}}{\grave{g}_{\beta,n}} z_{\beta,n} u_\beta + \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{n-1} r_\beta |z_{\beta,k}| \|W_{\beta,k}\| - \dot{\hat{\mu}}_\beta\right) \\
&- \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \frac{\partial \alpha_{\beta,n-1}}{\partial \hat{\mu}_\beta} \dot{\hat{\mu}}_\beta \\
&+ z_{\beta,n} \frac{\theta_\beta^T}{\grave{g}_{\beta,n}} \left(f_{\beta,n} - \sum_{k=1}^{n-1} \frac{\partial \alpha_{\beta,n-1}}{\partial x_{\beta,k}} f_{\beta,k}\right) \\
&- z_{\beta,n} \sum_{k=1}^{n-1} \frac{g_{\beta,k}}{\grave{g}_{\beta,n}} \frac{\partial \alpha_{\beta,n-1}}{\partial x_{\beta,k}} x_{\beta,k+1} \\
&- \frac{1}{\grave{g}_{\beta,n}} z_{\beta,n} \sum_{j=N+1}^{N+M} \sum_{k=0}^{n-1} c_{\beta,j} \frac{\partial \alpha_{\beta,n-1}}{\partial y_{d,j}^{(k)}} y_{d,j}^{(k+1)}.
\end{aligned} \tag{28}
$$

Then, it follows from (28) that

$$
\begin{aligned}
\dot{V}_{\beta,n} \le &-\sum_{k=1}^{n-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n-1} \frac{1}{4b_{\beta,k}} \\
&+ \frac{g_{\beta,n}}{\grave{g}_{\beta,n}} z_{\beta,n} u_\beta + z_{\beta,n} \upsilon_{\beta,n}^T W_{\beta,n} \\
&+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{n-1} r\beta \left|z_{\beta,k}\right| \left\|W_{\beta,k}\right\| - \dot{\hat{\mu}}_\beta\right) \\
\le &-\sum_{k=1}^{n-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n-1} \frac{1}{4b_{\beta,k}} \\
&+ \frac{g_{\beta,n}}{\grave{g}_{\beta,n}} z_{\beta,n} u_\beta + \left|z_{\beta,n}\right| \bar{\alpha}_{\beta,n} \\
&+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{n-1} r_\beta \left|z_{\beta,k}\right| \left\|W_{\beta,k}\right\| - \dot{\hat{\mu}}_\beta\right),
\end{aligned} \tag{29}
$$

where $\bar{\alpha}_{\beta,n} = \hat{\mu}_\beta \left\|W_{\beta,n}\right\|$. According to (23), substituting u_β in (3) into (29) yields that

$$
\begin{aligned}
\dot{V}_{\beta,n} \le &-\sum_{k=1}^{n-1} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n-1} \frac{1}{4b_{\beta,k}} \\
&+ \frac{1}{4b_{\beta,n}} - a_{\beta,n} z_{\beta,n}^2 \\
&+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{n-1} r_\beta \left|z_{\beta,k}\right| \left\|W_{\beta,k}\right\| - \dot{\hat{\mu}}_\beta\right) \\
= &-\sum_{k=1}^{n} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n} \frac{1}{4b_{\beta,k}} \\
&+ \frac{1}{r_\beta} \tilde{\mu}_\beta \left(\sum_{k=1}^{n-1} r_\beta \left|z_{\beta,k}\right| \left\|W_{\beta,k}\right\| - \dot{\hat{\mu}}_\beta\right).
\end{aligned} \tag{30}
$$

Then, it gives the following result from substituting the adaptive law in (3) into (30).

$$
\begin{aligned}
\dot{V}_{\beta,n} \le &-\sum_{k=1}^{n} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n} \frac{1}{4b_{\beta,k}} + \frac{\sigma_\beta}{r_\beta} \tilde{\mu}_\beta \hat{\mu}_\beta \\
\le &-\sum_{k=1}^{n} a_{\beta,k} z_{\beta,k}^2 + \sum_{k=1}^{n} \frac{1}{4b_{\beta,k}} - \frac{\sigma_\beta \tilde{\mu}_\beta^2}{2r_\beta} + \frac{\sigma_\beta \mu_\beta^2}{2r_\beta} \\
= &-\sum_{k=1}^{n} a_{\beta,k} z_{\beta,k}^2 - \frac{\sigma_\beta \tilde{\mu}_\beta^2}{2r_\beta} + \sum_{k=1}^{n} \frac{1}{4b_{\beta,k}} + \frac{\sigma_\beta \mu_\beta^2}{2r_\beta} \\
\le &-\vartheta_\beta V_{\beta,n} + \varsigma_\beta,
\end{aligned} \tag{31}
$$

where $\vartheta_\beta = \min\{2a_{\beta,k},\ \sigma_\beta,\ k = 1,\ \ldots,\ n\}$ and $\varsigma_\beta = \sum_{k=1}^{n} \bar{b}_{\beta,k} + \sigma_\beta \mu_\beta^2 / \left(2r_\beta\right)$ with $\bar{b}_{\beta,k} = 1/\left(4b_{\beta,k}\right)$. Choose the following Lyapunov function candidate for all followers: $V = \sum_{k=1}^{N} V_{k,n}$, from the results of (31), we have $\dot{V} \le -\sum_{k=1}^{N} \vartheta_k V_{k,n} + \sum_{k=1}^{N} \varsigma_k$, it gives that $V \le \sum_{k=1}^{N} V_{k,n}(0) e^{-\vartheta_k t} + \sum_{k=1}^{N} \varsigma_k / \vartheta_k$. Accordingly, all the signals of the closed-loop MASs are uniformly and ultimately bounded. Additionally, it is implied that $(1/2)\,||z_1||^2 \le \sum_{k=1}^{N} V_{k,n}(0) e^{-\vartheta_k t} + \sum_{k=1}^{N} \varsigma_k / \vartheta_k$, with $z_1 = [z_{1,1},\ \ldots,\ z_{N,1}]^T$, it gives that $||z_1||^2 \le \sum_{k=1}^{N} 2V_{k,n}(0)$ $e^{-\vartheta_k t} + \sum_{k=1}^{N} 2\varsigma_k / \vartheta_k$. Therefore, the error surface vector $||z_1||$ converges into the bound of $\sum_{k=1}^{N} \sqrt{2\varsigma_k / \vartheta_k}$ with the increase of time. By increasing ϑ_k, $\sum_{k=1}^{N} \sqrt{2\varsigma_k / \vartheta_k}$ can be tuned to an arbitrarily small value. Then, define a vector $y_0 = [y_1,\ \ldots,\ y_N]^T$, from $z_1 = \hat{L}_1 y_0 + \hat{L}_2 \check{y}$, it can be seen that the output y_β of the followers converge into the convex hull following the leaders $y_{d,j}$, i.e., $\left|y_\beta - y_{f,p}\right| < \check{\epsilon}$ with $\beta = 1,\ 2,\ \ldots,\ N$ and $p = 1,\ 2,\ \ldots,\ N$. The proof of Theorem 1 is completed. □

4. SIMULATION

In order to show the performance of the designed control scheme, the following MAS is given:

$$
\begin{cases}
\dot{x}_{\beta,1} = \left(1 + 0.1\sin\left(x_{\beta,1}\right)\right) x_{\beta,2} + \sin\left(x_{\beta,1}\right), \\
\dot{x}_{\beta,2} = 2u_\beta + 0.2\sin\left(x_{\beta,1}\right) + 2\sin\left(x_{\beta,2}\right), \\
y_\beta = x_{\beta,1},\ \beta = 1,\ 2,\ 3,
\end{cases} \tag{32}
$$

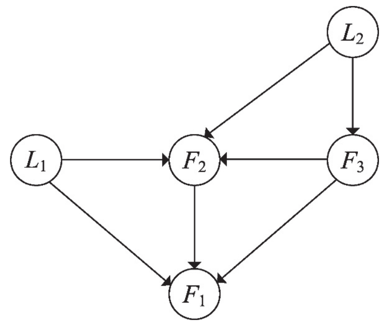

Fig. 1. Directed communication graph.

where $x_{\beta,1}$ and $x_{\beta,2}$ represents the state variables, y_β dentoes the system output, and u_β is taken as the actual control law. The dynamic leaders 1 and 2 are $y_{d,1} = \cos(t)+1$ and $y_{d,1} = \cos(t) - 0.7$, respectively. Define $f_{\beta,1} = 0.5\sin\left(x_{\beta,1}\right)$ and $f_{\beta,2} = 0.1\sin\left(x_{\beta,1}\right) + \sin\left(x_{\beta,2}\right)$. Then, the unknown parameter of the MAS is $\theta_\beta = 2$. According to the directed communication graph in Fig. 1, the relative adjacency matrix and the Laplacian matrix are determined as follows:

$$\bar{C} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{33}$$

$$L = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ 0 & 3 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{34}$$

From Theorem 1, the virtual control law $\alpha_{\beta,1}$ can be selected as

$$\alpha_{\beta,1} = -\frac{1}{d_\beta}\left(b_{\beta,1} z_{\beta,1} \bar{\alpha}_{\beta,1}^2 + a_{\beta,1} z_{\beta,1}\right), \tag{35}$$

where $a_{\beta,1}$ and $b_{\beta,1}$ are appropriately selected positive parameters, $z_{1,1} = 3x_{1,1} - x_{2,1} - x_{3,1} - y_{d,1}$, $z_{2,1} = 3x_{2,1} - x_{3,1} - y_{d,1} - y_{d,2}$, $z_{3,1} = 2x_{3,1} - x_{1,1} - y_{d,2}$ and $\bar{\alpha}_{\beta,1} = \hat{\mu}_\beta \left\|W_{\beta,1}\right\|$. The actual control law u_β and adaptive law $\dot{\hat{\mu}}_\beta$ are given as follows:

$$u_\beta = -\left(b_{\beta,2} z_{\beta,2} \bar{\alpha}_{\beta,2}^2 + a_{\beta,2} z_{\beta,2}\right), \tag{36}$$

$$\dot{\hat{\mu}}_\beta = \sum_{k=1}^{2} r_\beta \left(\left|z_{\beta,k}\right| \left\|W_{\beta,k}\right\|\right) - \sigma_\beta \hat{\mu}_\beta, \tag{37}$$

where $a_{\beta,2}$, $b_{\beta,2}$, r_β, and σ_β are positive designed parameters, $z_{\beta,2} = x_{\beta,2} - \alpha_{\beta,1}$ and $\bar{\alpha}_{\beta,2} = \hat{\mu}_\beta \left\|W_{\beta,2}\right\|$.

The specific parameters are determined as follows: $a_{\beta,1} = 20$, $a_{\beta,2} = 20$, $b_{\beta,1} = 1$, $b_{\beta,2} = 1$, $r_\beta = 0.0001$ and $\sigma_\beta = 1$. The initial conditions of MAS are $x_{1,1}(0) = 4$, $x_{2,1}(0) = 3$, $x_{3,1}(0) = -2$, $x_{1,2}(0) = x_{2,2}(0) = x_{3,2}(0) = 0$, and $\hat{\mu}_1(0) = \hat{\mu}_2(0) = \hat{\mu}_3(0) = 0$.

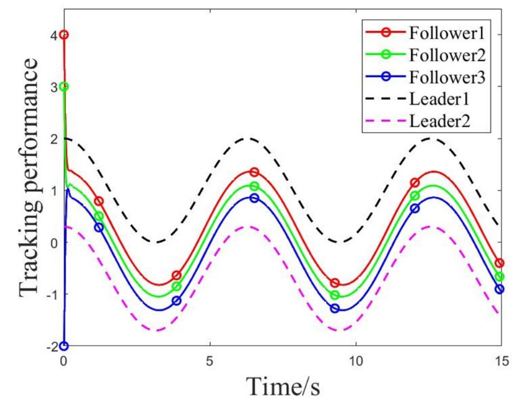

Fig. 2. Tracking performance of MAS.

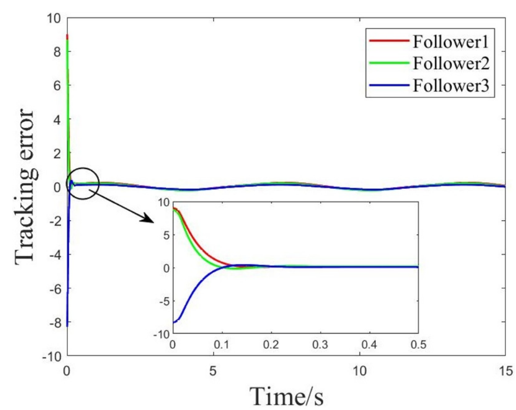

Fig. 3. Containment errors of MAS.

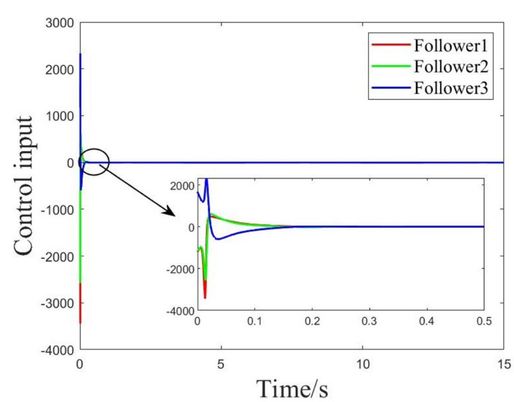

Fig. 4. Control inputs of MAS.

The simulation results are displayed in Figs. 2-5. The tracking performances of the followers are given in Fig. 2, it shows the position curves of three followers and two leaders outputs. The results imply that the followers F_1, F_2 and F_3 converge to the convex hull following the leaders L_1 and L_2. The containment errors $z_{\beta,1}$ are shown in Fig. 3, and the it shows the containment errors can converge to a tuned neighborhood of the origin. The control inputs for leaders and followers are displayed in Fig. 4. Fig. 5

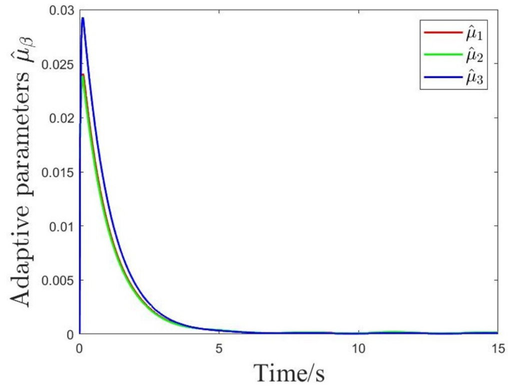

Fig. 5. Adaptive parameters $\hat{\mu}_\beta$.

shows the curves of adaptive parameter $\hat{\mu}_\beta$ for leaders and followers. Therefore, it illustrates that the stability of the MAS is achieved through the proposed adaptive backstepping containment controller, and the effectiveness of the designed controller is verified.

5. CONCLUSION

In this paper, a novel adaptive control scheme has been given for a class of MASs with parametric uncertainty, and the problem of containment control with unknown control direction has been solved. The proposed control scheme is based on backstepping method, adaptive control method and bound estimation method. The designed controller is effective in structure and calculation. It is proved that all the signals of the closed-loop MASs are uniformly and ultimately bounded. Additionally, containment errors will converge to a tuned neighborhood of the origin. The simulation results illustrate the effectiveness of the designed control scheme. For the future works, the combinations of fault-tolerant control and event-triggered mechanism are effective to make the MASs safe and save communication resources between agents.

CONFLICT OF INTERESTS

No potential competing interest was reported by the authors.

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Meng-Yi Jiang received her B.S. degree in mathematics and an M.S. degree in education from Liaoning Normal University, Dalian, China, in 2005 and 2011, respectively. She is currently a Ph.D. candidate in control science and engineering from University of Science and Technology Liaoning, Anshan, China. Her research interests include nonlinear system control and intelligent control.

Yong-Hui Yang received his B.Sc. degree in communication engineering, from Northeastern University, Shenyang, China, an M.Sc. degree in control engineering, and a Ph.D. degree in chemical process from University of Science and Technology Liaoning, Anshan, China, in 1995, 2010, and 2018, respectively. He is currently a Professor and Doctoral Supervisor at the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, China. His research interests include nonlinear control, intelligent process control, robot communication and control, and machine learning.

Li-Bing Wu was born in Chaoyang, Liaoning, China, in 1980, received his B.S. degree from the Department of Mathematics from Jinzhou Normal College, Jinzhou, China, an M.S. degree in basic mathematics from Northeastern University, Shenyang, China, and a Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2004, 2007, and 2016, respectively. He was a Postdoctoral Fellow with Yeungnam University, Kyongsan, Korea, from 2018 to 2019. He is currently an Associate Professor with the School of Science, University of Science and Technology Liaoning. His research interests include adaptive control, fault-tolerant control, nonlinear control, and fault estimation.

Qi Li received his B.S. degree in automation from Dalian University of Technology, Dalian, China, an M.S. degree in control theory and control engineering from University of Science and Technology Liaoning, Anshan, China, in 1997 and 2003, respectively. He is currently an Associate Professor with the School of Electronic and Information Engineering, University of Science and Technology Liaoning. His research interests include nonlinear system control, machine learning, and intelligent control.

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