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Adaptive Decentralized Event-triggered Tracking Control for Large-scale Strongly Interconnected Nonlinear System with Global Performance

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Abstract: In this paper, an adaptive decentralized event-triggered global performance control of a class of large-scale strongly interconnected nonlinear systems with external disturbances is investigated. Firstly, the original performance constrained large-scale nonlinear system is transformed into an equivalent unconstrained nonlinear large-scale system by barrier function transformation. Secondly, the additional assumptions of interconnect terms such as upper bound function and matching conditions are eliminated by using the inherent properties of Gaussian function. In addition, an event-triggered mechanism is designed to reduce unnecessary transfers between the controller and the actuator for better resource efficiency. It is shown that the proposed control schemes guarantee that all signals of the closed-loop system are bounded, and the output tracking error is always kept within the given boundary. Finally, a numerical system and a mass-spring damping system are taken as examples to verify the effectiveness of the proposed control method.

Keywords: Decentralized control, event-triggered control, large-scale nonlinear systems, prescribed performance.

1. INTRODUCTION

In the past few decades, the decentralized control of large-scale nonlinear systems has attracted handsome attention. Compared with centralized control, decentralized control is a control mechanism that only needs local signals to construct controller, which can simplify the control process and reduce the amount of calculation. Many important results have been achieved in the decentralized control of large-scale nonlinear systems, such as [1-5]. However, the above control methods have certain limitations in that the uncertain nonlinearities in the system are either certain linear functions with unknown parameters or scoped by known nonlinear functions. To remove the constraint, adaptive decentralized control methods using neural networks [6-8] or fuzzy logic systems [9-11] to identify unknown nonlinear functions have been widely studied. In [6], the decentralized output feedback control mechanism of adaptive neural network was proposed for uncertain large-scale interconnected nonlinear system with nonconstant control gain. In [10], an improved fault-tolerant controller was constructed by using fuzzy control for a class of nonstrict-feedback nonlinear systems with actuator faults, which ensured that all signals in the closed-loop system are semi-globally finite-time stable.

On the other hand, adaptive event-triggered control for nonlinear system has attracted extensive attention. Different from traditional time-triggered control, event-triggered control is a control mechanism that applies the controller output to the system only when the system needs it. It can effectively save communication resources and reduce communication burden. Lately, some adaptive event-triggered control schemes have been proposed by combining fuzzy and neural network approximation methods with backstepping techniques for nonlinear systems with different triggering mechanisms, see [12-19]. A fuzzy adaptive finite-time event-triggered control strategy on the basis of variable threshold scheme was came up with in [12] for a class of stochastic nonlinear systems with unmodeled dynamics. In [16], a new adaptive event-triggered mechanism was contrived for a class of uncertain strict-feedback nonlinear systems such that the controller and parameter estimators were triggered simultaneously. However, the above control scheme cannot be employed to control the transient behavior and steady-state performance of the system. In many practical applications, in addition to tracking stability, we often need the tracking performance of the system to meet our predetermined constraints, and require the system to achieve the desired steady-state tracking accuracy.

Recently, predefined performance control (PPC) technology has attracted wide attention once it was proposed

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in [20]. This method deals with predefined performance constraint by drawing on prescribed performance functions (PPF) and constructing error variations, see [21-28]. In [23], an adaptive predetermined performance control manner was proposed for a sort of uncertain nonlinear systems with unknown actuator faults and gap nonlinearity simultaneously by using command filtering theory. In [26], an adaptive prescribed performance controller was designed by utilizing backstepping technique for a category of strict-feedback nonlinear system with zero dynamics, which can ensure that the tracking error always evolves within the performance constraints and converges asymptotically to zero. However, the control methods described above depend on initial conditions and are semi-global results. To remove the constraints on the initial condition, Zhao *et al*. [29] proposed an improved predefined performance tracking control method, which make the tracking error not only evolve in a predefined funnel, but also achieve global asymptotic stability. However, as far as the authors know, there are few research results on event-triggered tracking control for nonlinear interconnected systems with global performance constraints. This is what drives our research.

Based on the above discussion, this paper will study the problem of adaptive event-triggered control for a category of large-scale nonlinear strongly interconnected systems with global performance and external disturbance. The main contributions of this article are as follows:

1) Compared with the based-PPF predefined performance control in [24-28], where the requirements on initial conditions are relaxed in this paper such that the initial value of the proposed prescribed function is infinite instead of a bounded constant
2) Compared with existing control of large-scale interconnected systems [6-11] in which the interconnection term is constrained by a known or partially known function, the proposed decentralized control scheme in this paper removes such conditions, and thus successfully handles the completely unknown strongly interconnection term.
3) Compared with the results of [20-23], an event-triggered controller is designed by using neural network system, event-triggered mechanism and backstepping technique, and the proposed controller can effectively reduce the transmission burden between the controller and the actuator, and greatly save communication resources.

The rest of the work is arranged as below: In Section 2, the preliminary knowledge and system statement are shown. The dynamic event-triggered scheme is designed in Sections 3-5. The effectiveness of the proposed method is demonstrated by the simulation results in Section 6. Finally, concludes this study in Section 7.

2. PROBLEM FORMULATION

In this paper, we consider a class of large-scale nonlinear systems with nonstrict-feedback structure, which consists of N interconnected subsystems, the ith subsystem ($i = 1, \dots, N$) can be modeled as follows:

$$\begin{cases} \dot{x}_{ij} = x_{ij+1} + f_{ij}(x_i) + g_{ij}(x_1, \cdots, x_N) + d_{ij}(t), \\ \dot{x}_{in_i} = u_i + f_{in_i}(x_i) + g_{in_i}(x_1, \cdots, x_N) + d_{in_i}(t), \\ y_i = x_{i1}, \end{cases} \tag{1}$$

where $j = 1, \cdots, n_i - 1$, $x_i = [x_{i1}, \dots, x_{in_i}]^T \in R^{n_i}$, $u_i \in R$, $y_i \in R$, $i = 1, \dots, N$ represent the system state vector, input and output respectively. $f_{ij}(x_i) : R^{n_i} \longrightarrow R$ is an unknown smooth function. $g_{ij}(x_1, \cdots, x_N)$ is an unknown smooth function representing the interconnection between the ith subsystem and other subsystems. $d_{ij}(t)$ is an external disturbance.

Below we will give some theorems and assumptions to facilitate the subsequent research:

Assumption 1: The desired trajectory y_{d_i} ($i = 1, \cdots, N$), and its n_i-th derivatives are known, bounded, and piecewise continuous.

Assumption 2: External disturbances $d_{ij}(t)$ satisfying

$$|d_{ij}(t)| \le \overline{d}_{ij}. \tag{2}$$

Assumption 3: There exist ideal constant weights θ^*, such that $|\sigma| \le \overline{\sigma}$ with unknown constant $\overline{\sigma}$ for all $X \in \Lambda_X$.

Next, we introduce the basic work of NNs. Radial basis NNs is a linear parameterized neural networks, which can be expressed as

$$f_{nn}(\theta, X) = \theta^T W(X), \tag{3}$$

where $X = [X_1, \cdots, X_{n_i}] \subset R^{n_i}$ is the input vector, n_i is the input dimension of the NNs; $\theta = [\theta_1, \cdots, \theta_l]^T \subset R^l$ is the weight vector of NNs, $l > 1$ is the number of nodes of NNs; $W(X) = [W_1(X), \cdots, W_l(X)]^T$ is the radial basis function vector, $W_i(X)$ is the basis function, which usually selected as a Gaussian function $W_i(X) = \exp\left[-\frac{(X-\Phi_i)^T(X-\Phi_i)}{\varrho_i^2}\right]$, where $i = 1, \cdots, l$, $\Phi_i = [\Phi_{i1}, \cdots, \Phi_{in_i}]$ is the center of the basis function and ϱ_i is the height of the Gaussian function.

Lemma 1 [6]**:** $f(X)$ be a uncertain continuous and smooth function defined on a compact set Λ_X, for any given constant σ, there exists a neural network system such that

$$f(X) = \theta^{*T} W(X) + \sigma(X), \tag{4}$$

where $\theta^* = \arg\min_{\theta \subset R^l} \left\{ \sup_{X \subset \Lambda_X} |f(X) - \theta^T W(X)| \right\}$ is the optimal weight. σ represent the minimum approximation error.

Lemma 2 [30]**:** For $\bar{x}_q = [x_1, \cdots, x_q]^T$, the basis function vector of a radial basis function neural network is defined as $W(\bar{x}_q) = [W_1(\bar{x}_q), \cdots, W_l(\bar{x}_q)]^T$. When any positive integer $k \leq q$, the following inequality holds

$$W^T(\bar{x}_q)W(\bar{x}_q) \leq W^T(\bar{x}_k)W(\bar{x}_k). \tag{5}$$

Based on the above assumptions, the control objective in this paper is to design a event-triggered controller u_i for system (1) such that

1) All signals of the closed-loop system are bounded.
2) The tracking error always evolves in a predetermined performance region.

3. PRESCRIBED PERFORMANCE FUNCTION

To achieve the control objective, we construct the following performance function

$$I_i(\gamma_i(t)) = \frac{\sqrt{\lambda_i}\gamma_i(t)}{\sqrt{1-\gamma_i^2(t)}}, i = 1, \cdots, N, \tag{6}$$

where λ_i is a designed constant, $\gamma_i(t) = \frac{1}{\varphi_i(t)}$. $\varphi_i(t)$ is a monotonically increasing function, and its derivative $\varphi_i^{(k)}(t)$, $k = 0, 1, \cdots, n_i$ is bounded and piecewise continue. The initial of $\varphi_i(t)$ is defined as $\varphi_i(0) = 1$ and $\lim_{t\to\infty} \varphi_i(t) = \frac{1}{\overline{\varphi}_i}$, $0 < \overline{\varphi}_i < 1$ is a constant. Thus, $\gamma_i(t)$ is a strictly monotonically decreasing function, $\gamma_i(0) = 1$ and $\lim_{t\to\infty} \gamma_i(t) = \overline{\varphi}_i$. According to (6), $I_i(\gamma_i(t))$ is a monotonically decreasing function with respect to time, i.e.,

$$\begin{cases} I_i(\gamma_i(0)) = I_i(1) = \infty, \\ \lim_{t\to\infty} I_i(\gamma_i(t)) = I_i(\overline{\varphi}_i) = \frac{\sqrt{\lambda_i}\overline{\varphi}_i}{\sqrt{1-\overline{\varphi}_i^2}}. \end{cases}$$

If λ_i is chosen as $\lambda_i = 1 - \overline{\varphi}_i^2$, then $\lim_{t\to\infty} I_i(\gamma_i(t)) = \overline{\varphi}_i$.

Remark 1: Compared with PPF in [28], the initial value of the prescribed performance function $I_i(\gamma_i(t))$ proposed in this paper is infinite instead of a bounded constant, which loosens the requirements on initial conditions.

The prescribed region is defined as the set

$$F_{\gamma_i} := \{(t, e_i) \in R^+ \times R^N \mid I_i(-\gamma_i(t)) < e_i < I_i(\gamma_i(t))\}, \tag{7}$$

where $i = 1, \cdots, N$, $e_i = y_i - y_{d_i}$ is the tracking error. The trajectory of tracking errors under predetermined performance boundary is shown in Fig. 1. From Fig. 1 we can see that the performance function $I_i(\gamma_i(t))$ converges from infinity to a bounded constant. The second control objective can be achieved if we can design an controller so that the tracking error evolves within the performance region, i.e., $(t, e_i) \in F_{\gamma_i}$, $\forall t > 0$.

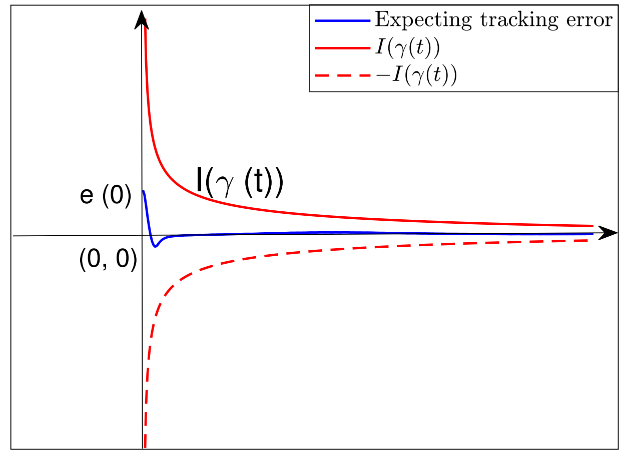

Fig. 1. A diagram of the prescribed tracking behavior.

To achieve the control objective, we define the following error transformation

$$\varpi_i(e_i) = \frac{e_i}{\sqrt{\lambda_i + e_i^2}}. \tag{8}$$

Remark 2: We know from (8) that $\varpi_i(e_i)$ is a strictly monotonically increasing function, and $\varpi_i(e_i) \in (-1, 1)$, for $e_i \in R$. Thus, if $\varpi_i(e_i)$ is bounded, then $e_i = \frac{\varpi_i\sqrt{\lambda_i}}{\sqrt{1-\varpi_i^2}}$ is bounded.

4. EVENT-TRIGGERED CONTROLLER

In this section, an event-triggered controller is designed for system. Using the definitions of $\varphi_i(t)$ and $\varpi_i(e_i)$ given in Section 3, we define the following function

$$\vartheta_i(t) = \varphi_i(t)\varpi_i(e_i). \tag{9}$$

To eliminate the effect of performance constraints, the following barrier function is adopted

$$z_{i1}(t) = \frac{\vartheta_i}{1 - \vartheta_i^2}. \tag{10}$$

From the expression of $z_{i1}(t)$, we can see that as long as initial condition $|\vartheta_i(0)| < 1$, the properties $z_{i1}(t) \to +\infty \Leftrightarrow \vartheta_i \to +1$, $z_{i1}(t) \to -\infty \Leftrightarrow \vartheta_i \to -1$ are obtained. Thus, in the initial condition $|\vartheta_i(0)| < 1$, $z_{i1}(t)$ must be bounded as long as ϑ_i is bounded.

Remark 3: If there is a constant $\bar{\varepsilon}_i$ such that $|\vartheta_i(t)| \leq \bar{\varepsilon}_i < 1$, according to (9) and the function $\varpi_i(e_i)$, it can be inferred that

$$-\gamma_i(t) = -\frac{1}{\varphi_i(t)} < -\frac{\bar{\varepsilon}_i}{\varphi_i(t)} \leq \varpi_i \leq \frac{\bar{\varepsilon}_i}{\varphi_i(t)} \leq \frac{1}{\varphi_i(t)} = \gamma_i(t).$$

Thus, from (8) and the properties of the function $I(\gamma_i)$, we have

$$I(-\gamma_i(t)) < I(\varpi_i) = e_i < I(\gamma_i(t)).$$

Based on the above analysis and Remark 3, we use the barrier function to transform the system with performance constraint into an equivalent unconstraint system. Hence, as long as we can assure that $\vartheta_i(t)$ is bounded, the tracking error will evolve within a predetermined performance range under the initial condition $|\vartheta_i(0)| < 1$.

Now, we give the recursive design by the following steps.

Step *i*, 1: Define error transformation as follows:

$$\begin{cases} z_{i1} = \frac{\vartheta_i}{1-\vartheta_i^2}, \\ z_{ij} = x_{ij} - \alpha_{ij-1},\ j = 2,\ \cdots,\ n_i, \end{cases} \tag{11}$$

where α_{ij-1} is the virtual controller. To develop a backstepping-based design procedure, we define a constant as follows:

$$\zeta_i^* = \max\{\parallel \theta_{i1}^* \parallel^2, \cdots, \parallel \theta_{in_i}^* \parallel^2\}. \tag{12}$$

From (6), (9) and the definition of z_{i1}, we obtain the derivative of z_{i1}

$$\begin{aligned} \dot{z}_{i1}(t) &= \eta_{i1}(\dot{\varphi}_i\varpi_i + \varphi_i\dot{\varpi}_i) \\ &= \eta_{i1}\dot{\varphi}_i\varpi_i + \gamma_i\eta_{i1}\eta_{i2}\dot{e}_i \\ &= \mu_{i1} + \mu_{i2}(\alpha_{i1} + z_{i2} - \dot{y}_{d_i} + f_{i1}(x_i) \\ &\quad + d_{i1}(t) + g_{i1}(x_1, \cdots, x_N)), \end{aligned} \tag{13}$$

where

$$\begin{aligned} \eta_{i1} &= \frac{1+\vartheta_i^2}{(1-\vartheta_i^2)^2}, \\ \eta_{i2} &= \frac{\lambda_i}{\sqrt{\lambda_i + e_i^2}(\lambda_i + e_i^2)}, \\ \mu_{i1} &= \eta_{i1}\dot{\varphi}_i\varpi_i, \\ \mu_{i2} &= \varphi_i\eta_{i1}\eta_{i2}. \end{aligned}$$

Select the following Lyapunov function candidates

$$V_{i1} = \frac{1}{2}z_{i1}^2 + \frac{1}{2r_i}\tilde{\zeta}_i^2, \tag{14}$$

where $\tilde{\zeta}_i = \zeta_i^* - \hat{\zeta}_i$, $\hat{\zeta}_i$ is the estimate of ζ_i^*, and r_i is a positive constant.

The derivative of V_{i1} is

$$\begin{aligned} \dot{V}_{i1} = z_{i1}[\mu_{i1} + \mu_{i2}(z_{i2} + \alpha_{i1} + f_{i1} + d_{i1}(t) \\ + g_{i1}(x_1, x_2, \cdots x_N) - \dot{y}_{d_i})] - \frac{1}{r_i}\tilde{\zeta}_i\dot{\hat{\zeta}}_i. \end{aligned} \tag{15}$$

Using Young's inequality and Assumption 2, we have

$$z_{i1}\mu_{i2}d_{i1}(t) \leq \frac{1}{2}\mu_{i2}^2 z_{i1}^2 + \frac{1}{2}\bar{d}_{i1}^2. \tag{16}$$

where $\bar{d}_{i1}$ is a positive constant. Substituting (16) into (15) yields

$$\begin{aligned} \dot{V}_{i1} \leq\ & z_{i1}[\mu_{i1} + \mu_{i2}(z_{i2} + \frac{1}{2}\mu_{i2}z_{i1} + \alpha_{i1} + h_{i1}(S_{i1}))] \\ & - \frac{1}{r_i}\tilde{\zeta}_i\dot{\hat{\zeta}}_i + \frac{1}{2}\bar{d}_{i1}^2, \end{aligned} \tag{17}$$

where $h_{i1}(S_{i1}) = f_{i1}(x_i) + g_{i1}(x_1, x_2, \cdots x_N) - \dot{y}_{d_i}$, $S_{i1} = [x_1^T, x_2^T, \cdots, x_N^T, y_{d_i}, \dot{y}_{d_i}]^T$. Since $h_{i1}(S_{i1})$ is an unknown function, we use a neural network to model the unknown function $h_{i1}(S_{i1})$, and for $\sigma_{i1} > 0$, we can obtain the following

$$h_{i1}(S_{i1}) = \theta_{i1}^{*T} W_{i1}(S_{i1}) + \sigma_{i1}(S_{i1}), \tag{18}$$

where $|\sigma_{i1}| \leq \bar{\sigma}_{i1}$ and $\bar{\sigma}_{i1} > 0$ is positive constant. By utilizing Lemma 2 and Young's inequality, one can obtain that

$$\begin{aligned} z_{i1}\mu_{i2}h_{i1}(S_{i1}) &= z_{i1}\mu_{i2}[\theta^{*T}{}_{i1}W_{i1}(S_{i1}) + \sigma_{i1}(S_{i1})] \\ &\leq \frac{1}{2c_{i1}^2}\mu_{i2}^2 z_{i1}^2 \zeta_i^* W_{i1}^T W_{i1} + \frac{1}{2}c_{i1}^2 \\ &\quad + \frac{1}{2}\mu_{i2}^2 z_{i1}^2 + \frac{1}{2}\bar{\sigma}_{i1}^2, \end{aligned} \tag{19}$$

where $W_{i1} = W_{i1}(Z_{i1})$, $Z_{i1} = [x_{i1}, y_{d_i}, \dot{y}_{d_i}]^T$, and c_{i1} is a positive constant. From (19), (17) can be rewritten

$$\begin{aligned} \dot{V}_{i1} \leq\ & z_{i1}[\mu_{i1} + \mu_{i2}(z_{i2} + \alpha_{i1} + \frac{1}{2c_{i1}^2}\mu_{i2}z_{i1}\hat{\zeta}_i W_{i1}^T W_{i1} \\ & + \mu_{i2}z_{i1})] + \frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2 z_{i1}^2 W_{i1}^T W_{i1} - \dot{\hat{\zeta}}_i) \\ & + \frac{1}{2}\bar{\sigma}_{i1}^2 + \frac{1}{2}c_{i1}^2 + \frac{1}{2}\bar{d}_{i1}^2. \end{aligned} \tag{20}$$

Define the virtual controller α_{i1} as follows:

$$\begin{aligned} \alpha_{i1} =\ & -\frac{1}{2c_{i1}^2}\mu_{i2}z_{i1}\hat{\zeta}_i W_{i1}^T W_{i1} - l_{i1}\mu_{i2}^{-1}z_{i1} \\ & - \mu_{i2}z_{i1} - \mu_{i2}^{-1}\mu_{i1}, \end{aligned} \tag{21}$$

where l_{i1} is designed positive constant.

Substituting (21) into (20), one has

$$\begin{aligned} \dot{V}_{i1} \leq\ & -l_{i1}z_{i1}^2 + \mu_{i2}z_{i1}z_{i2} + \frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2 z_{i1}^2 W_{i1}^T W_{i1} \\ & - \dot{\hat{\zeta}}_i) + \frac{1}{2}\bar{\sigma}_{i1}^2 + \frac{1}{2}c_{i1}^2 + \frac{1}{2}\bar{d}_{i1}^2. \end{aligned} \tag{22}$$

Step *i*, 2: Taking the derivative of z_{i2}, we have

$$\begin{aligned} \dot{z}_{i2} =\ & z_{i3} + \alpha_{i2} + d_{i2}(t) + f_{i2}(x_i) - \dot{\alpha}_{i1} \\ & + g_{i2}(x_1, x_2, \cdots x_N), \end{aligned} \tag{23}$$

where

$$\dot{\alpha}_{i1} = \frac{\partial \alpha_{i1}}{\partial x_{i1}}\dot{x}_{i1} + \sum_{k=0}^{1}\frac{\partial \alpha_{i1}}{\partial y_{d_i}^{(k)}}y_{d_i}^{(k+1)} + \frac{\partial \alpha_{i1}}{\partial \hat{\zeta}_i}\dot{\hat{\zeta}}_i$$

$$+\sum_{k=0}^{1}\frac{\partial\alpha_{i1}}{\partial\varphi_i^{(k)}}\varphi_i^{(k+1)}.$$

The following Lyapunov function candidate V_{i2} is defined

$$V_{i2}=V_{i1}+\frac{1}{2}z_{i2}^2. \tag{24}$$

Derivative of V_{i2}, one has

$$\dot{V}_{i2}=\dot{V}_{i1}+z_{i2}(\alpha_{i2}+d_{i2}(t)+g_{i2}(x_1,x_2,\cdots x_N)+z_{i3}-\dot{\alpha}_{i1}+f_{i2}(x_i)). \tag{25}$$

As the same of (16), we have

$$z_{i2}d_{i2}(t)\le\frac{1}{2}z_{i2}^2+\frac{1}{2}\overline{d}_{i2}^2. \tag{26}$$

Substituting (26) into (25), it can be obtained

$$\dot{V}_{i2}\le\dot{V}_{i1}+z_{i2}(\alpha_{i2}+\frac{1}{2}z_{i2}+z_{i3}+h_{i2}(S_{i2}))+\frac{1}{2}\overline{d}_{i2}^2, \tag{27}$$

where $h_{i2}(S_{i2})=f_{i2}(x_i)+g_{i2}(x_1,x_2,\cdots x_N)-\dot{\alpha}_{i1}$, $S_{i2}=[x_1^T, x_2^T, \cdots, x_N^T, y_{d_i}, \dot{y}_{d_i}]^T$. Since $h_{i2}(S_{i2})$ is an unknown function, we use a neural network to model the unknown function $h_{i2}(S_{i2})$, and for $\sigma_{i2}>0$, we can obtain

$$h_{i2}(S_{i2})=\theta_{i2}^{*T}W_{i2}(S_{i2})+\sigma_{i2}(S_{i2}), \tag{28}$$

where $|\sigma_{i2}|\le\overline{\sigma}_{i2}$ and $\overline{\sigma}_{i2}>0$ is positive constant. By utilizing Lemma 2 and Young's inequality, one can obtain that

$$z_{i2}h_{i2}(S_{i2})\le\frac{1}{2c_{i2}^2}z_{i2}^2\zeta_i^*W_{i2}^TW_{i2}+\frac{1}{2}c_{i2}^2+\frac{1}{2}z_{i2}^2+\frac{1}{2}\overline{\sigma}_{i2}^2, \tag{29}$$

where $W_{i2}=W_{i2}(Z_{i2})$, $Z_{i2}=[x_{i1}, x_{i2}, y_{d_i}, \dot{y}_{d_i}]^T$, and c_{i2} is a positive constant. From (29), (27) can be rewritten

$$\begin{aligned}\dot{V}_{i2}\le&-l_{i1}z_{i1}^2+\sum_{m=1}^{2}\frac{1}{2}c_{im}^2+\frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2z_{i1}^2W_{i1}^TW_{i1}\\&+\frac{r_i}{2c_{i2}^2}z_{i2}^2W_{i2}^TW_{i2}-\dot{\hat{\zeta}}_i)+z_{i2}(\mu_{i2}z_{i1}+z_{i3}\\&+\alpha_{i2}+\frac{1}{2c_{i2}^2}z_{i2}\hat{\zeta}_iW_{i2}^TW_{i2}+z_{i2})\\&+\sum_{m=1}^{2}\frac{1}{2}\overline{\sigma}_{im}^2+\sum_{m=1}^{2}\frac{1}{2}\overline{d}_{im}^2.\end{aligned} \tag{30}$$

Define the virtual controller α_{i2} as follows:

$$\alpha_{i2}=-\mu_{i2}z_{i1}-l_{i2}z_{i2}-\frac{1}{2c_{i2}^2}z_{i2}\hat{\zeta}_iW_{i2}^TW_{i2}-z_{i2}, \tag{31}$$

where l_{i2} is designed positive constant.

Substituting (31) into (30), one has

$$\begin{aligned}\dot{V}_{i2}\le&-\sum_{m=1}^{2}l_{im}z_{im}^2+\frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2z_{i1}^2W_{i1}^TW_{i1}\\&+\frac{r_i}{2c_{i2}^2}z_{i2}^2W_{i2}^TW_{i2}-\dot{\hat{\zeta}}_i)+\sum_{m=1}^{2}\frac{1}{2}\overline{\sigma}_{im}^2\\&+\sum_{m=1}^{2}\frac{1}{2}c_{im}^2+\sum_{m=1}^{2}\frac{1}{2}\overline{d}_{im}^2.\end{aligned} \tag{32}$$

Step *i*, *j* ($j=3,\ldots,n_i-1$): Taking the derivative of z_{ij}, we have

$$\begin{aligned}\dot{z}_{ij}=&z_{ij+1}+\alpha_{ij}+d_{ij}(t)+f_{ij}(x_i)-\dot{\alpha}_{ij}\\&+g_{ij}(x_1,\cdots,x_N),\end{aligned} \tag{33}$$

where

$$\begin{aligned}\dot{\alpha}_{ij-1}=&\sum_{k=1}^{j-1}\frac{\partial\alpha_{ij-1}}{\partial x_{ik}}\dot{x}_{ik}+\sum_{k=0}^{j-1}\frac{\partial\alpha_{ij-1}}{\partial y_{d_i^{(k)}}}y_{d_i}^{(k+1)}\\&+\sum_{k=0}^{j-1}\frac{\partial\alpha_{ij-1}}{\partial\varphi_i^{(k)}}\varphi_i^{(k+1)}+\frac{\partial\alpha_{i1}}{\partial\hat{\zeta}_i}\dot{\hat{\zeta}}_i.\end{aligned}$$

We choose the following Lyapunov function candidate as follows:

$$V_{ij}=V_{ij-1}+\frac{1}{2}z_{ij}^2. \tag{34}$$

Derivative of V_{ij}, one has

$$\begin{aligned}\dot{V}_{ij}=&\dot{V}_{ij-1}+z_{ij}(z_{ij+1}+f_{ij}(x_i)+g_{ij}(x_1,\cdots,x_N)\\&+\alpha_{ij}+d_{ij}(t)-\dot{\alpha}_{ij}).\end{aligned} \tag{35}$$

As the same of (26), we have

$$z_{ij}d_{ij}(t)\le\frac{1}{2}z_{ij}^2+\frac{1}{2}\overline{d}_{ij}^2. \tag{36}$$

Substituting (36) into (35), we get

$$\dot{V}_{ij}\le\dot{V}_{ij-1}+z_{ij}(z_{ij+1}+\alpha_{ij}+h_{ij}(S_{ij})+\frac{1}{2}z_{ij})+\frac{1}{2}\overline{d}_{ij}^2, \tag{37}$$

where $h_{ij}(S_{ij})=f_{ij}(x_i)-\dot{\alpha}_{ij}+g_{ij}(x_1,\cdots,x_N)$, $S_{ij}=[x_1^T, x_2^T, \cdots, x_N^T, y_{d_i}, \dot{y}_{d_i}]^T$. Since $h_{ij}(S_{ij})$ is an unknown function, we use a neural network to model the unknown function $h_{ij}(S_{ij})$, and for $\sigma_{ij}>0$, we can obtain

$$h_{ij}(S_{ij})=\theta_{ij}^{*T}W_{ij}(S_{ij})+\sigma_{ij}(S_{ij}), \tag{38}$$

where $|\sigma_{ij}|\le\overline{\sigma}_{ij}$ and $\overline{\sigma}_{ij}>0$ is positive constant. By utilizing Lemma 2 and Young's inequality, one can obtain that

$$z_{ij}h_{ij}(S_{ij})\le\frac{1}{2c_{ij}^2}z_{ij}^2\zeta_i^*W_{ij}^TW_{ij}+\frac{1}{2}c_{ij}^2+\frac{1}{2}z_{ij}^2+\frac{1}{2}\overline{\sigma}_{ij}^2, \tag{39}$$

where $W_{ij}=W_{ij}(Z_{ij})$, $Z_{ij}=[x_{i1}, x_{i2}, \cdots, x_{ij}, y_{d_i}, \dot{y}_{d_i}]^T$, and c_{ij} is a positive constant. From (39), (37) can be rewritten

$$\dot{V}_{ij}\le-\sum_{m=1}^{j-1}l_{im}z_{im}^2+\sum_{m=1}^{j}\frac{1}{2}\overline{\sigma}_{im}^2+z_{ij}(z_{ij+1}+\alpha_{ij}$$

$$+\frac{1}{2c_{ij}^2}z_{ij}\hat{\zeta}_i W_{ij}^T W_{ij}+z_{ij}+z_{ij-1})+\sum_{m=1}^{j}\frac{1}{2}c_{im}^2 + \sum_{m=1}^{j}\frac{1}{2}\overline{d}_{im}^2+\frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2 z_{i1}^2 W_{i1}^T W_{i1} + \sum_{m=2}^{j}\frac{r_i}{2c_{im}^2}z_{im}^2 W_{im}^T W_{im}-\dot{\hat{\zeta}}_i). \quad (40)$$

Define the virtual controller α_{ij} as follows:

$$\alpha_{ij}=-z_{ij-1}-l_{ij}z_{ij}-\frac{1}{2c_{ij}^2}z_{ij}\hat{\zeta}_i W_{ij}^T W_{ij}-z_{ij}, \quad (41)$$

where l_{ij} is designed positive constant.

Substituting (41) into (40), one has

$$\dot{V}_{ij}\le -\sum_{m=1}^{j}l_{im}z_{im}^2+\sum_{m=1}^{j}\frac{1}{2}c_{im}^2+z_{ij}z_{ij+1}+\sum_{m=1}^{j}\frac{1}{2}\overline{\sigma}_{im}^2 + \sum_{m=1}^{j}\frac{1}{2}\overline{d}_{im}^2+\frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2 z_{i1}^2 W_{i1}^T W_{i1} + \sum_{m=2}^{j}\frac{r_i}{2c_{im}^2}z_{im}^2 W_{im}^T W_{im}-\dot{\hat{\zeta}}_i). \quad (42)$$

Step i, n_i: The event triggering controller is designed as

$$\omega_i(t)=-(1+k_i)(\alpha_{in_i}\tanh\frac{z_{in_i}\alpha_{in_i}}{\epsilon_i}+\overline{\kappa}_i\tanh\frac{z_{in_i}\overline{\kappa}_i}{\epsilon_i}), \quad (43)$$

$$u_i(t)=\omega_i(t_k), \forall t\in[t_k,\ t_{k+1}), \quad (44)$$

which the event triggering mechanism is defined as

$$t_{k+1}=\inf\{t>t_k \mid |\eta_i|\ge k_i|u_i|+q_i\}, \quad (45)$$

where $\eta_i(t)=\omega_i(t)-u_i(t)$, $q_i>0$, $0<k_i<1$ and $\bar{\kappa}_i>\frac{q_i}{1-k_i}$ are positive parameters that need to be designed. From (45), we can obtain

$$\omega_i(t)=(1+k_i\lambda_{i_1}(t))u_i(t)+\lambda_{i_2}(t)q_i, \quad (46)$$

where $t_k\le t<t_{k+1}$, $|\lambda_{i_1}(t)|\le 1$ and $|\lambda_{i_2}(t)|\le 1$. Thus, (46) can be rewritten as

$$u_i(t)=\frac{\omega_i(t)}{1+k_i\lambda_{i_1}(t)}-\frac{\lambda_{i_2}(t)q_i}{1+k_i\lambda_{i_1}(t)}. \quad (47)$$

Taking the derivative of z_{in_i}, we have

$$\dot{z}_{in_i}=f_{in_i}(x_i)+u_i-\dot{\alpha}_{in_i-1} + g_{in_i}(x_1,x_2,\cdots x_N)+d_{in_i}(t), \quad (48)$$

where

$$\dot{\alpha}_{in_i-1}=\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{in_i-1}}{\partial x_{ik}}\dot{x}_{ik}+\sum_{k=0}^{n_i-1}\frac{\partial\alpha_{in_i-1}}{\partial y_{d_i}^{(k)}}y_{d_i}^{(k+1)} + \sum_{k=0}^{n_i-1}\frac{\partial\alpha_{in_i-1}}{\partial\varphi_i^{(k)}}\varphi_i^{(k+1)}+\frac{\partial\alpha_{i1}}{\partial\hat{\zeta}_i}\dot{\hat{\zeta}}_i.$$

Choose the following Lyapunov function candidate:

$$V_{in_i}=V_{in_i-1}+\frac{1}{2}z_{in_i}^2. \quad (49)$$

The derivative of V_{in_i} is

$$\dot{V}_{in_i}=\dot{V}_{in_i-1}+z_{in_i}(\frac{\omega_i(t)}{1+k_i\lambda_{i_1}(t)}-\frac{\lambda_{i_2}(t)q_i}{1+k_i\lambda_{i_1}(t)}+d_{in_i}(t) + f_{in_i}(x_i)+g_{in_i}(x_1,x_2,\cdots x_N)-\dot{\alpha}_{in_i-1}). \quad (50)$$

Due to $|\lambda_{i_1}(t)|\le 1$, $|\lambda_{i_2}(t)|\le 1$, one gets

$$z_{in_i}\frac{\omega_i(t)}{1+k_i\lambda_{i_1}(t)}\le z_{i,n_i}\frac{\omega_i(t)}{1+k_i}, \quad (51)$$

$$|z_{in_i}\frac{\lambda_{i_2}(t)q_i}{1+k_i\lambda_{i_1}(t)}|\le|z_{in_i}\frac{q_i}{1-k_i}|. \quad (52)$$

According to [12], it yields

$$0\le|x_i|-x_i\tanh\left(\frac{x_i}{\varepsilon_i}\right)\le 0.2785\varepsilon_i, \quad (53)$$

where ε_i and $x_i\in R$. Thus, according to the definition of event triggering controller (43)-(44), and inequality (51)-(52), (50) can be written

$$\begin{aligned}\dot{V}_{in_i}\le&\ \dot{V}_{in_i-1}+z_{in_i}(\frac{\omega_i(t)}{1+k_i}-\dot{\alpha}_{in_i-1}+f_{in_i}(x_i)\\&+g_{in_i}(x_1,x_2,\cdots x_N)+d_{in_i}(t))+|\frac{z_{in_i}q_i}{1-k_i}|\\ \le&\ \dot{V}_{in_i-1}+z_{in_i}\big(f_{in_i}(x_i)+g_{in_i}(x_1,x_2,\cdots x_N)\\&+d_{in_i}(t)-\dot{\alpha}_{in_i-1}\big)+z_{in_i}\alpha_{in_i}+|z_{in_i}\alpha_{in_i}|\\&-z_{in_i}\alpha_{in_i}\tanh\frac{z_{in_i}\alpha_{in_i}}{\epsilon_i}-z_{in_i}\overline{\kappa}_i\tanh\frac{z_{in_i}\overline{\kappa}_i}{\epsilon_i}\\&-|z_{in_i}\alpha_{in_i}|-z_{in_i}\alpha_{in_i}+|z_{in_i}\overline{\kappa}_i|-|z_{in_i}\overline{\kappa}_i|+|\frac{z_{in_i}q_i}{1-k_i}|\\ \le&\ \dot{V}_{in_i-1}+z_{in_i}\big(\alpha_{in_i}+f_{in_i}(x_i)+g_{in_i}(x_1,x_2,\cdots x_N)\\&+d_{in_i}(t)-\dot{\alpha}_{in_i-1}\big)+0.557\epsilon_i.\end{aligned} \quad (54)$$

Using Young's inequality and Assumption 2, we have

$$z_{in_i}d_{in_i}(t)\le\frac{1}{2}z_{in_i}^2+\frac{1}{2}\overline{d}_{in_i}^2. \quad (55)$$

Substituting (55) into (54), we have

$$\dot{V}_{in_i}\le\dot{V}_{in_i-1}+z_{in_i}(\alpha_{in_i}+h_{in_i}(S_{in_i})+\frac{1}{2}z_{in_i}) + 0.557\epsilon_i+\frac{1}{2}\overline{d}_{in_i}^2, \quad (56)$$

where $h_{in_i}(S_{in_i})=f_{in_i}(x_i)+g_{in_i}(x_1,x_2,\cdots x_N)-\dot{\alpha}_{in_i-1}$, $S_{in_i}=[x_1^T,\ x_2^T,\ \cdots,\ x_N^T,\ y_{d_i},\ \dot{y}_{d_i}]^T$. Since $h_{in_i}(S_{in_i})$ is an unknown function, we use a neural network to model the unknown function $h_{in_i}(S_{in_i})$, and for $\sigma_{in_i}>0$, we can obtain

$$h_{in_i}(S_{in_i})=\theta_{in_i}^{*T}W_{in_i}(S_{in_i})+\sigma_{in_i}(S_{in_i}), \quad (57)$$

where $|\sigma_{in_i}| \leq \overline{\sigma}_{in_i}$. By utilizing Lemma 2 and Young's inequality, one can obtain that

$$z_{in_i}h_{in_i}(S_{in_i}) \leq \frac{1}{2c_{in_i}^2}z_{in_i}^2\zeta_i^*W_{in_i}^TW_{in_i} + \frac{1}{2}c_{in_i}^2 + \frac{1}{2}z_{in_i}^2 + \frac{1}{2}\overline{\sigma}_{in_i}^2, \tag{58}$$

where $W_{in_i} = W_{in_i}(Z_{in_i})$, $Z_{in_i} = [x_{i1}, \cdots, x_{in_i}, y_{d_i}, \dot{y}_{d_i}]^T$, and c_{in_i} is a positive constant. From (58), (56) can be rewritten

$$\begin{aligned}\dot{V}_{in_i} \leq & -\sum_{m=1}^{n_i-1} l_{im}z_{im}^2 + z_{in_i}(\alpha_{in_i} + \frac{1}{2c_{in_i}^2}z_{in_i}\hat{\zeta}_iW_{in_i}^TW_{in_i} \\ & + z_{in_i-1} + z_{in_i}) + \sum_{m=1}^{n_i}\frac{1}{2}\overline{\sigma}_{im}^2 + \sum_{m=1}^{n_i}\frac{1}{2}c_{im}^2 \\ & + \frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2z_{i1}^2W_{i1}^TW_{i1} + \sum_{m=2}^{n_i}\frac{r_i}{2c_{im}^2}z_{im}^2W_{im}^TW_{im} \\ & - \dot{\hat{\zeta}}_i) + 0.557\epsilon_i + \sum_{m=1}^{n_i}\frac{1}{2}\overline{d}_{im}^2. \end{aligned} \tag{59}$$

The virtual controller α_{in_i} is designed as

$$\alpha_{in_i} = -z_{in_i-1} - l_{in_i}z_{in_i} - \frac{1}{2c_{in_i}^2}z_{in_i}\hat{\zeta}_iW_{in_i}^TW_{in_i} - z_{in_i}, \tag{60}$$

where l_{in_i} is designed positive constant.

Substituting (60) into (59), we have

$$\begin{aligned}\dot{V}_{in_i} \leq & -\sum_{m=1}^{n_i} l_{im}z_{im}^2 + \sum_{m=1}^{n_i}\frac{1}{2}\overline{\sigma}_{im}^2 + 0.557\epsilon_i \\ & + \frac{\tilde{\zeta}_i}{r_i}(\frac{r_i}{2c_{i1}^2}\mu_{i2}^2z_{i1}^2W_{i1}^TW_{i1} + \sum_{m=2}^{n_i}\frac{r_i}{2c_{im}^2}z_{im}^2W_{im}^TW_{im} \\ & - \dot{\hat{\zeta}}_i) + \sum_{m=1}^{n_i}\frac{1}{2}\overline{d}_{im}^2 + \sum_{m=1}^{n_i}\frac{1}{2}c_{im}^2. \end{aligned} \tag{61}$$

The adaptive law is designed as

$$\dot{\hat{\zeta}}_i = \frac{r_i}{2c_{i1}^2}\mu_{i2}^2z_{i1}^2W_{i1}^TW_{i1} + \sum_{m=2}^{n_i}\frac{r_i}{2c_{im}^2}z_{im}^2W_{im}^TW_{im} - \tau_i\hat{\zeta}_i. \tag{62}$$

Thus, (61) can be written as

$$\begin{aligned}\dot{V}_{in_i} \leq & -\sum_{m=1}^{n_i} l_{im}z_{im}^2 + \sum_{m=1}^{n_i}\frac{1}{2}\overline{\sigma}_{im}^2 + \sum_{m=1}^{n_i}\frac{1}{2}c_{im}^2 \\ & + \frac{\tau_i}{r_i}\tilde{\zeta}_i\hat{\zeta}_i + \sum_{m=1}^{n_i}\frac{1}{2}\overline{d}_{im}^2 + 0.557\epsilon_i. \end{aligned} \tag{63}$$

5. STABILITY ANALYSIS

Theorem 1: Based on Assumptions 1-3, consider the closed-loop system containing of the uncertain system (1), the event-triggered based adaptive predefined performance controller design in (43)-(44), (60), and adaptive laws (62). Then, the following statements hold

1) All signals in closed-loop system are semi-globally uniformly bounded (SGUB).
2) The trajectory of the tracking error is always within a defined performance constraint.

Proof: We can get the result of the theorem by proving it in the following steps:

The following Lyapunov function candidate is constructed for the whole system

$$V = \sum_{i=1}^{N} V_{in_i}. \tag{64}$$

Using Young's inequality, one obtains

$$\frac{\tau_i}{r_i}\tilde{\zeta}_i\hat{\zeta}_i \leq -\frac{\tau_i}{2r_i}\tilde{\zeta}_i^2 + \frac{\tau_i}{2r_i}\zeta_i^{*2}. \tag{65}$$

Substituting (65) into (64), one gets

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N}\sum_{m=1}^{n_i} l_{im}z_{im}^2 + \sum_{i=1}^{N}\sum_{m=1}^{n_i}\frac{1}{2}\overline{d}_{im}^2 - \sum_{i=1}^{N}\frac{\tau_i}{2r_i}\tilde{\zeta}_i^2 \\ & + \sum_{i=1}^{N}\frac{\tau_i}{2r_i}\zeta_i^{*2} + \sum_{i=1}^{N}\sum_{m=1}^{n_i}\frac{1}{2}\overline{\sigma}_{i,m}^2 \\ & + \sum_{i=1}^{N}0.557\epsilon_i + \sum_{i=1}^{N}\sum_{m=1}^{n_i}\frac{1}{2}c_{im}^2 \\ \leq & -a_0V + b_0, \end{aligned} \tag{66}$$

where $a_0 = \min\{2l_{im_i}, \tau_i, m = 1, \cdots, n_i, i = 1, \cdots, N\}$, $b_0 = \sum_{i=1}^{N}\sum_{m=1}^{n_i}\frac{1}{2}\overline{d}_{im}^2 + \sum_{i=1}^{N}\frac{\tau_i}{2r_i}\zeta_i^{*2} + \sum_{i=1}^{N}0.557\epsilon_i + \sum_{i=1}^{N}\sum_{m=1}^{n_i}(\frac{1}{2}c_{im}^2 + \frac{1}{2}\bar{\sigma}_{im}^2)$.

From (66), and integrating it, we have

$$V(t) \leq (V(0) - \frac{b_0}{a_0})e^{-a_0t} + \frac{b_0}{a_0}. \tag{67}$$

This means that all signals of the closed loop system are bounded, and the bounds can be expressed as $t \to \infty$

$$\|z_{im}\| \leq \sqrt{2\frac{b_0}{a_0}},$$

$$\|\tilde{\zeta}_i\| \leq \sqrt{2\frac{b_0}{a_0}}.$$

Due to $\tilde{\zeta}_i = \zeta_i^* - \hat{\zeta}_i$, then the boundedness of $\hat{\zeta}_i$ is ensured. From the definition of z_{i1} and discussion in Section 3, it can be seen that there exists a constant $\epsilon_i < 1$, which makes $|\vartheta_i = r_i\varpi_i| \leq \epsilon_i < 1$ valid, and $|\varpi_i| \leq \frac{\epsilon_i}{|r_i|} < 1$ can be derived. Then, it follows from the previous discussion that the tracking error e_i is bounded and always evolves within predefined performance functions. So μ_{i1} and μ_{i2} are bounded. Due to $e_i = x_{i1} - y_{d_i}$, $y_{d_i} \in L_\infty$, $e_i \in L_\infty$, so x_{i1} is bounded. Since α_{i1} is a function of x_{i1}, y_{d_i}, μ_{i1}, μ_{i2}, then it insured that α_{i1} is bounded. It is clear that x_{i2} is bounded from $z_{i2} = x_{i2} - \alpha_{i1}$. Similarly, it can be proved that x_{ij} and

α_{ij} are bounded. From the definition in (44), u_i is a function of $x_{i1}, \cdots, x_{in_i}, y_{d_i}, \dot{y}_{d_i}, \cdots, {y_{d_i}}^{(k)}, \zeta_i$, and since $x_{i1}, \cdots, x_{in_i}, y_{d_i}, \dot{y}_{d_i}, \cdots, {y_{d_i}}^{k}, \zeta_i$ are bounded, the controller u_i is bounded. Therefore, all signals of a closed-loop system are bounded and the tracking error is preserved within the performance region.

Next, let us justify that the Zeno phenomenon does not occur, i.e., There exists a t_* such that $k \in z^+, t_{k+1} - t_k \geq t_*$. Therefore, from the $\eta_i(t) = \omega_i(t) - u_i(t)$, $\forall t \in [t_k,\, t_{k+1})$, we have

$$\frac{d}{dt}|\eta_i| = \mathrm{sgn}(\eta_i)\dot{\eta}_i \leq |\dot{\omega}_i|. \tag{68}$$

By means of (43), we get that ω_i is differentiable. Inequality $|\dot{\omega}_i| \leq \iota_i$ holds where ι_i is positive normal. Due to $\eta_i(t_k) = 0$ and $\lim_{t \to t_{k+1}} \eta_i(t) = k_i|u_i| + q_i$, therefore $t_* > \frac{k_i|u_i|+q_i}{\iota_i}$, the Zeno phenomenon was successfully eliminated. □

6. ILLUSTRATIVE EXAMPLE

In this section, we use two examples to prove the effectiveness of the proposed control method.

Example 1: The effectiveness of the raised control method is explained with an numerical example. We consider the large nonlinear system as below

$$\begin{cases} \dot{x}_{11} = x_{12} + f_{11} + g_{11}(x_1, x_2) + d_{11}(t), \\ \dot{x}_{12} = u_1 + f_{12} + g_{12}(x_1, x_2) + d_{12}(t), \\ y_1 = x_{11}, \\ \dot{x}_{21} = x_{22} + f_{21} + g_{21}(x_1, x_2) + d_{21}(t), \\ \dot{x}_{22} = u_2 + f_{22} + g_{22}(x_1, x_2) + d_{22}(t), \\ y_2 = x_{21}, \end{cases} \tag{69}$$

where $f_{11} = 0.2x_{11}\sin(x_{11}) + x_{12}$, $f_{12} = x_{12}\sin(x_{1,1})$, $f_{21} = x_{21}\cos(x_{21})$, $f_{22} = 0.2x_{22}\sin(x_{21})$, $g_{11}(x_1, x_2) = 0.1x_{11}\sin(x_{21}) + x_{22}$, $g_{12}(x_1, x_2) = 0.5x_{12}x_{21}$, $g_{21}(x_1, x_2) = 0.2x_{11}x_{22}$, $g_{22}(x_1, x_2) = 0.5(x_{11}^2 + x_{21}^2) + x_{12}$, $d_{11}(t) = d_{12}(t) = 0.001\sin(t)$, $d_{21}(t) = d_{22}(t) = 0.001\sin(2t)$. And select the reference signals as $y_{d_1} = \sin(2t)$, $y_{d_2} = \sin(1.5t)$.

In the simulation experiment, the virtual controller $\alpha_{i,1}$ is designed as

$$\begin{aligned} \alpha_{i1} = &-\frac{1}{2c_{i1}^2}\mu_{i2}z_{i1}\hat{\zeta}_i W_{i1}^T W_{i1} - l_{i1}\mu_{i2}^{-1}z_{i1} - \mu_{i2}z_{i1} \\ &- \mu_{i2}^{-1}\mu_{i1}, \end{aligned} \tag{70}$$

and the event-triggered controller u_i is

$$\omega_i(t) = -(1+k_i)(\alpha_{i2}\tanh\frac{z_{i2}\alpha_{i,2}}{\epsilon_i} + \overline{\kappa}_i\tanh\frac{z_{i2}\overline{\kappa}_i}{\epsilon_i}), \tag{71}$$

$$u_i(t) = \omega_i(t_k), \forall t \in [t_k,\, t_{k+1}), \tag{72}$$

where

$$\alpha_{i2} = -\mu_{i2}z_{i1} - l_{i2}z_{i2} - \frac{1}{2c_{i2}^2}z_{i2}\hat{\zeta}_i W_{i2}^T W_{i2} - z_{i2}, \tag{73}$$

and the adaptive laws $\hat{\zeta}_i$ is designed as

$$\dot{\hat{\zeta}}_i = \frac{r_i}{2c_{i1}^2}\mu_{i2}^2 z_{i1}^2 W_{i1}^T W_{i1} + \frac{r_i}{2c_{i2}^2}z_{i2}^2 W_{i2}^T W_{i2} - \tau_i\hat{\zeta}_i. \tag{74}$$

Initial values and other parameters that need to be designed are selected as $\hat{\zeta}_1(0) = \hat{\zeta}_2(0) = 1$, $[x_{11}(0), x_{12}(0), x_{21}(0), x_{22}(0)]^T = [0.2, 0.3, 0.3, 0.5]^T$, $[0.5, 0.3, 0.7, 0.5]^T$, $[-0.5, 0.3, -0.7, 0.5]^T$, $l_{11} = l_{12} = l_{21} = l_{22} = 10$, $c_{11} = c_{12} = c_{21} = c_{22} = 10$, $r_1 = r_2 = 1$, $\tau_1 = \tau_2 = 8$, $b_{f_1} = 0.09$, $b_{f_2} = 0.08$, $\lambda_1 = \lambda_2 = 0.3$, $k_1 = k_2 = 0.4$, $\overline{\kappa}_1 = \overline{\kappa}_2 = 60$, $\epsilon_1 = \epsilon_2 = 20$, the time-varying scaling function $\varphi_1 = \frac{1}{(1-b_{f_1})\exp(-0.5t)+b_{f_1}}$, $\varphi_2 = \frac{1}{(1-b_{f_2})\exp(-0.4t)+b_{f_2}}$. The simulation results of Example 1 are shown in Figs. 2-7. Fig. 2 shows that the outputs of the system are in good agreement with the reference signals. As can be seen from Figs. 3 and 4, the transient property of tracking error always evolves in a given region under different initial conditions, and the proposed control mechanism loosens the requirements on initial conditions. Fig. 5 shows the trajectories of the adaptive laws ζ_i $(i = 1, 2)$. Fig. 6 describes the trajectory of the event triggering controller u_i $(i = 1, 2)$, and Fig. 7 shows the triggering interval $t_{k+1} - t_k$. As you can see from Figs. 5 and 7, both computation and communication resources are significantly reduced.

Example 2: We consider a mass-spring-damping (MSD) system as shown in Fig. 8. The dynamic model of the above system is

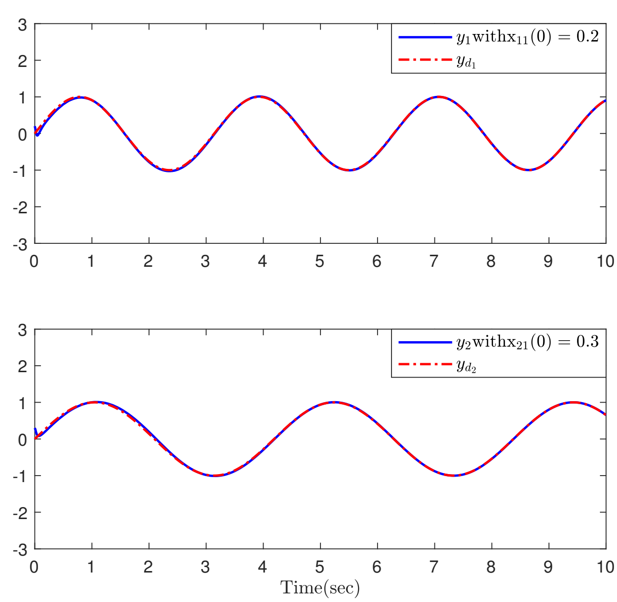

Fig. 2. The trajectories of x_{i1} and $y_{d_i}(t)$ $(i = 1, 2)$, for Example 1.

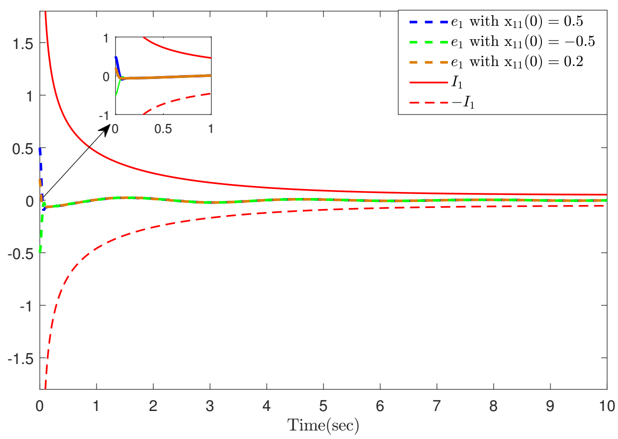

Fig. 3. The trajectories of e_1 under different initial conditions for Example 1.

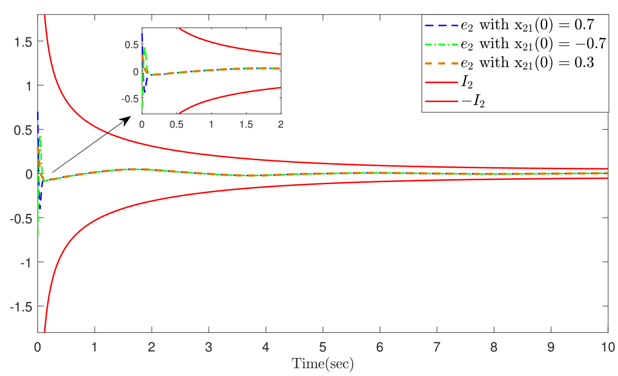

Fig. 4. The trajectories of e_2 under different initial conditions for Example 1.

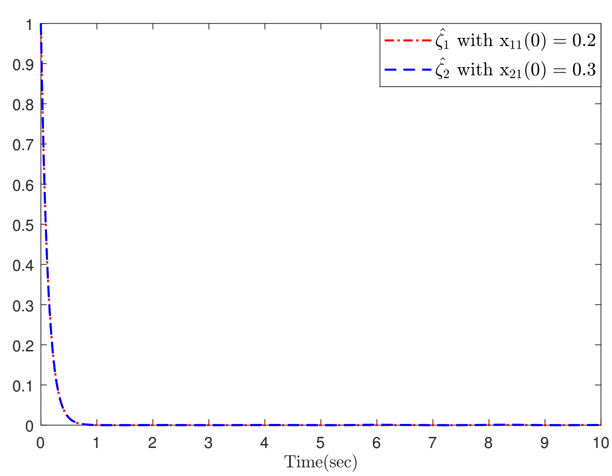

Fig. 5. The trajectories of $\hat{\zeta}_i$ $(i = 1, 2)$, for Example 1.

$$\begin{cases} M_1\ddot{y}_1 = u_1 - f_{t_1} - f_{g_1} + f_{t_2} + f_{g_2} - f_{a_1} + f_{a_2}, \\ M_2\ddot{y}_2 = u_2 - f_{t_2} - f_{g_1} - f_{a_2} + f_{a_2}, \end{cases} \tag{75}$$

where u_i, y_i, $(i = 1, 2)$ are the control inputs and out-

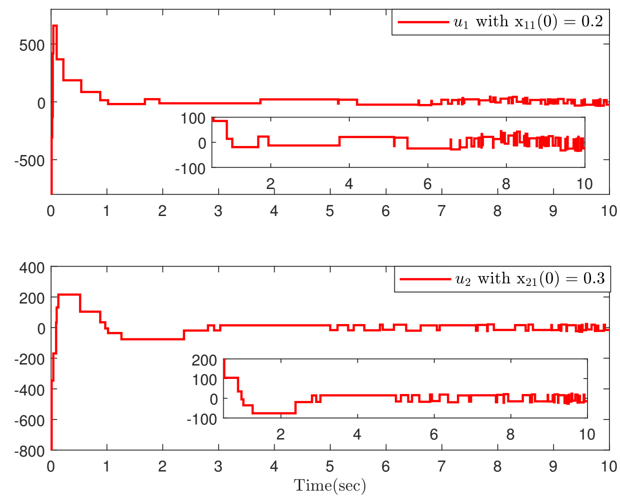

Fig. 6. The trajectories of u_i $(i = 1, 2)$, for Example 1.

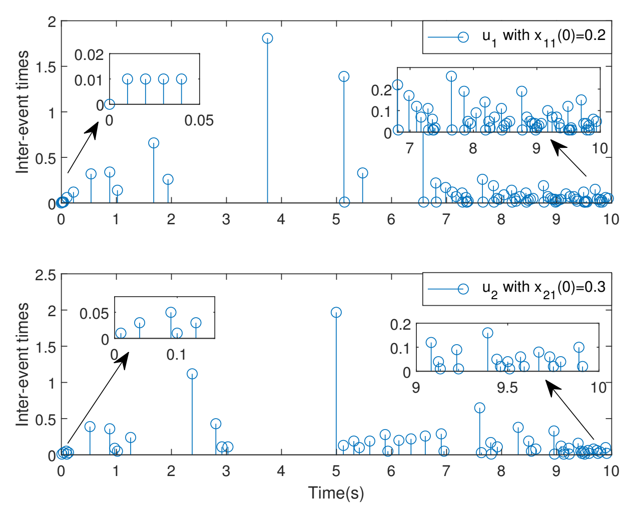

Fig. 7. The trigger time interval of u_i $(i = 1, 2)$, for Example 1.

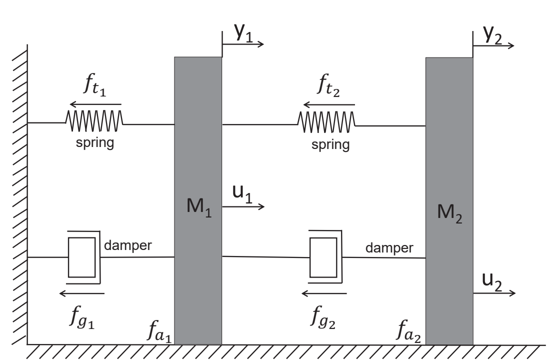

Fig. 8. Mass spring damping system.

puts, $f_{t_1} = y_1 + 0.1y_1^3$ and $f_{t_2} = 2(y_2 - y_1) + 0.12(y_2 - y_1)^3$ denote the force of the spring, $f_{g_1} = 2\dot{y}_1 + 0.2\dot{y}_1^2$ and $f_{g_2} = 2.2(\dot{y}_2 - \dot{y}_1) + 0.15(\dot{y}_2 - \dot{y}_1)^2$ stand for friction,

$f_{a_1} = 0.02\text{sign}(\dot{y}_1)$ and $f_{a_2} = 0.02\text{sign}(\dot{y}_2 - \dot{y}_1)$ stand for coulomb friction, $M_1 = 1$ kg, $M_2 = 1$ kg are mass of the MSD. Therefore, the expression of the state space can be written as

$$\begin{cases} \dot{x}_{11} = x_{12}, \\ \dot{x}_{12} = u_1 + f_{12} + g_{12}(x_1, x_2) + d_{12}(t), \\ y_1 = x_{11}, \\ \dot{x}_{21} = x_{22}, \\ \dot{x}_{22} = u_2 + f_{22} + g_{22}(x_1, x_2) + d_{22}(t), \\ y_2 = x_{21}, \end{cases} \tag{76}$$

where $f_{12} = -(x_{11} + 0.1x_{11}^3) - (2x_{12} + 0.2x_{12}^2)$, $g_{12}(x_1, x_2) = 2(x_{21} - x_{11}) + 0.12(x_{21} - x_{11})^3 + 2.2(x_{22} - x_{12}) + 0.15(x_{22} - x_{12})^2 - 0.2\text{sign}(x_{12}) + 0.02\text{sign}(x_{22} - x_{12})$, $f_{22} = -2x_{21}$, $g_{22} = 2x_{11} - 0.12(x_{21} - x_{11})^3 - 2x_{12} - 0.2x_{12}^2 - 0.02\text{sign}(x_{21} - x_{12})$, $d_{12} = 0.001\sin(2t)$, $d_{22} = 0.001\sin(t)$. Initial values and other parameters that need to be designed are selected as $\hat{\zeta}_1(0) = \hat{\zeta}_2(0) = 1$, $[x_{11}(0), x_{12}(0), x_{21}(0), x_{22}(0)]^T = [1.8, 0.3, 1.5, 0.3]^T$, $[0.8, 0.3, 0.5, 0.3]^T$, $[-0.8, 0.3, -0.5, 0.3]^T$, $l_{11} = l_{12} = l_{21} = l_{22} = 13$, $c_{11} = c_{12} = c_{21} = c_{22} = 8$, $r_1 = r_2 = 0.1$, $\tau_1 = \tau_2 = 10$, $b_{f_1} = 0.18$, $b_{f_2} = 0.16$, $\lambda_1 = \lambda_2 = 1$, $k_1 = k_2 = 0.5$, $\overline{\kappa}_1 = \overline{\kappa}_2 = 50$, $\epsilon_1 = \epsilon_2 = 20$, the time-varying function $\varphi_1 = \frac{1}{(1-b_{f_1})\exp(-0.5t)+b_{f_1}}$, $\varphi_2 = \frac{1}{(1-b_{f_2})\exp(-0.65t)+b_{f_2}}$. The simulation results of Example 2 are shown in Figs. 9-14. Fig. 9 shows that the outputs of the system are in good agreement with the reference signals. As can be seen from Figs. 10 and 11, the transient property of tracking error always evolves in a given region under different initial conditions, and the proposed control mechanism loosens the requirements on initial conditions. Fig. 12 shows the trajectories of the adaptive laws ζ_i $(i = 1, 2)$. Fig. 13 describes the trajectory of the event triggering controller

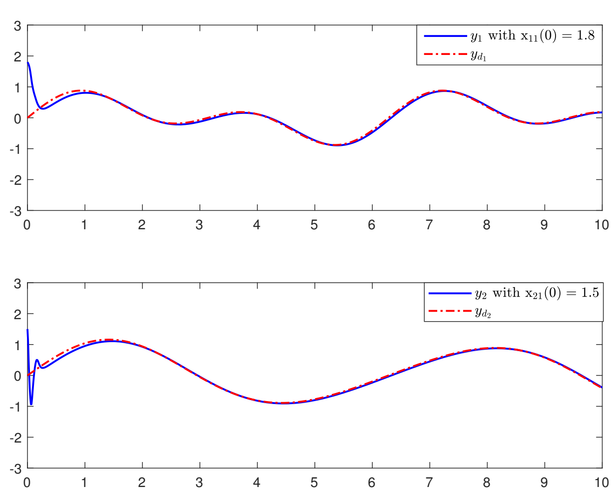

Fig. 9. The trajectories of x_{i1} and $y_{d_i}(t)$ $(i = 1, 2)$, for Example 2.

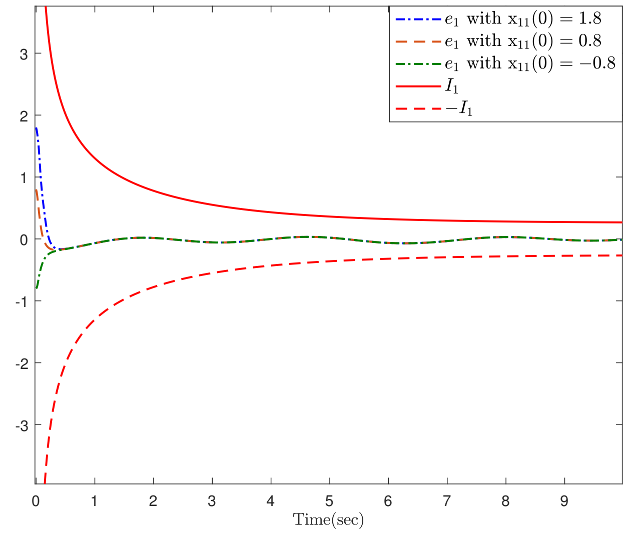

Fig. 10. The trajectories of e_1 under different initial conditions for Example 2.

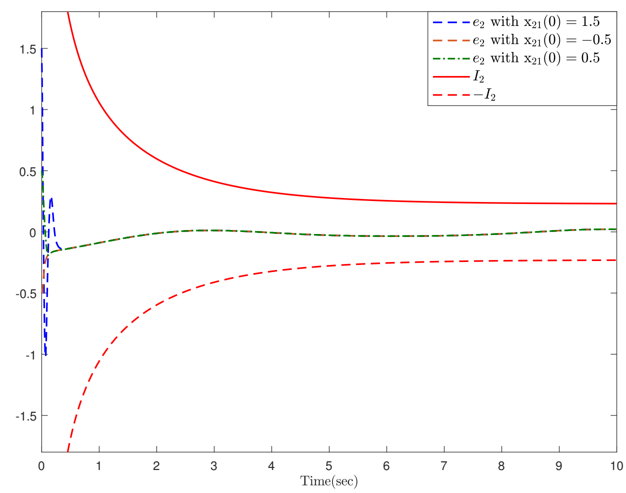

Fig. 11. The trajectories of e_2 under different initial conditions for Example 2.

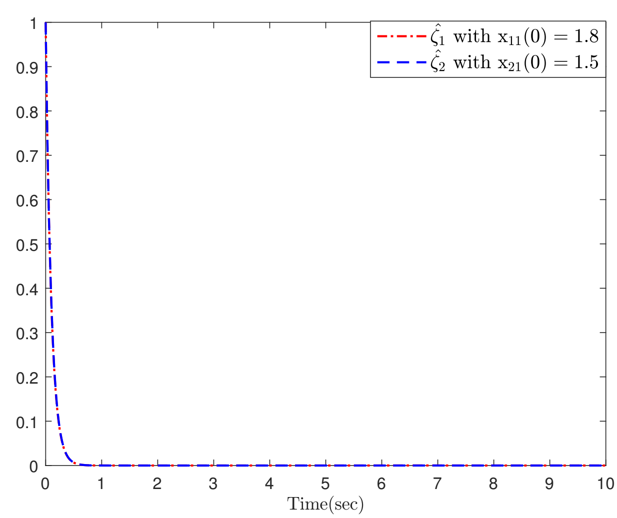

Fig. 12. The trajectories of $\hat{\zeta}_i$ $(i = 1, 2)$, for Example 2.

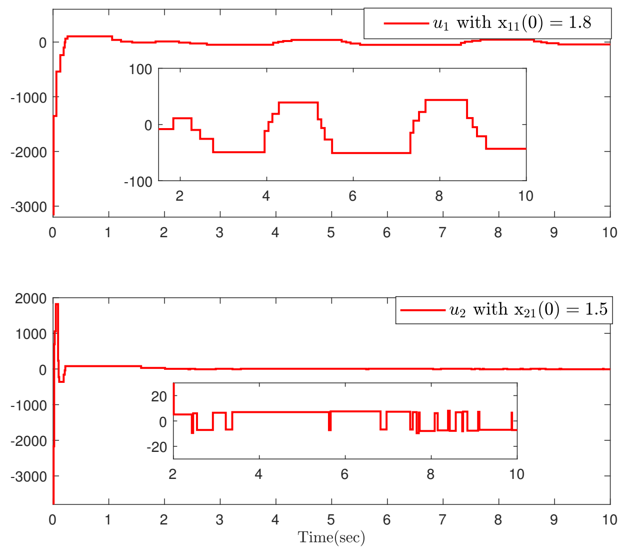

Fig. 13. The trajectories of u_i $(i = 1, 2)$, for Example 2.

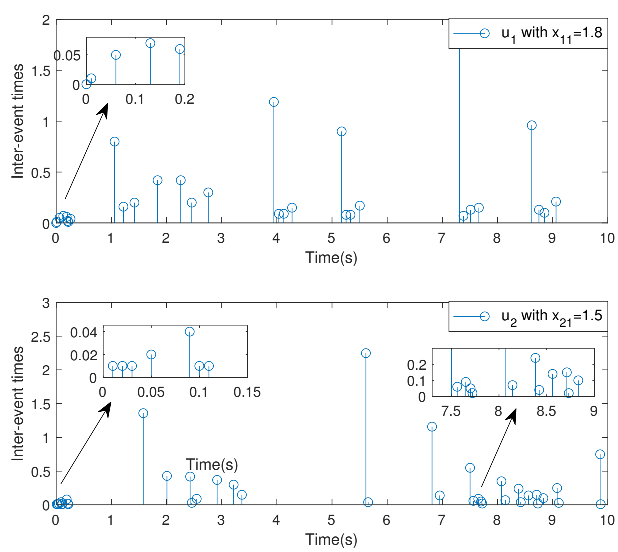

Fig. 14. The trigger time interval of u_i $(i = 1, 2)$, for Example 2.

u_i $(i = 1, 2)$, and Fig. 14 shows the triggering interval $t_{k+1} - t_k$. As you can see from Figs. 13 and 14, both computation and communication resources are significantly reduced.

7. CONCLUSIONS

In this paper, an adaptive event-triggered tracking control scheme is proposed for a class of strongly interconnected large-scale nonlinear systems with global performance and external disturbance. Predefined performance problems are solved by introducing barrier functions. Besides, all the extra assumptions about interconnect terms are eliminated by using the inherent properties of Gaussian function. The decentralized controller of each subsystem is constructed by combining backstepping technology, neural network system and event triggering mechanism. The designed controller could make sure that all signals of the closed-loop system are bounded and the output tracking error is kept within a given boundary. Finally, a numerical system and a mass spring damping system are taken as examples to verify the effectiveness of the proposed control method. In addition, the other topics include predefined performance time adaptive tracking control of nonlinear large-scale systems will be further studied in our future research.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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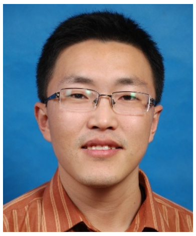

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