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Output Regulation with Prescribed Performance Control of Switched Strict-feedback Systems

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Abstract: In this work, we establish a complete procedure for the output regulation problem with prescribed performance for a class of switched strict-feedback systems by employing barrier Lyapunov function and speed function method. Under the framework of output regulation control for general switched nonlinear systems, a switched regulator equation is constructed. Compared to the existing works which require the solvability of output regulation for each subsystem, the designed switched regulator equation based method need not each subsystem has a solution to the problem, which releases the conservatism. Moreover, the state feedback control scheme based on speed function technique and a new state dependent switching rule are constructed to achieve that the error output keeps in a prescribed performance bound in a given finite time and tracks a desired trajectory asymptotically. The effectiveness of the results is demonstrated by a simulation example.

Keywords: Finite time, output regulation, prescribed performance, switched nonlinear systems, switched regulator equation.

1. INTRODUCTION

Switched systems, composed of a family of subsystems and a switching law that determine the sequence of activated subsystem [1]. Because of its significant theoretical value in hybrid systems and control theory, as well as its widely applied in practical systems, switched systems have attracted much attention in the last several decades, and some elementary control issues about switched systems have been comprehensively researched, such as stability and stabilization, tracking control, filter design and dissipativity analysis, see [2,3] and the references therein. It should be noticed is that constructing specific switching rule for a class of dynamic systems can add accessional design freedom, for instance, when a certain performance of a switched system cannot be obtained by a single subsystem, one may achieve this goal by switching between a collection of subsystems or controllers.

Output regulation problem is a significant and essential control issue in control theory. The classical output regulation problem aims to achieve steady state performance including asymptotic tracking for desired reference inputs and/or asymptotic rejecting undesired disturbances generated by a system named as the exosystem which is usually assumed to be precisely known. A large majority of results on the classical output regulation problem for nonlinear system can be founded in [4-6]. More recently, the output regulation problem is extended to hybrid systems [7,8] and multi-agent systems [9-12]. As a basic problem in control theory, the output regulation problem is also addressed for switched systems. Besides the output regulation problem for switched linear system that constructed state feedback controllers and error feedback controllers via common Lyapunov function and average dwell time method [13-15], the output regulation problem for nonlinear switched systems has attracted much more attention since the work of [16]. The output regulation problem of positive switched systems and switched non-linear systems produce switched internal model and a state-input dependent switching rule in [17,18] respectively. Moreover, the output regulation problem for switched system is researched by incremental passivity [19,20]. It is necessary to note that virtually aforementioned results study the output regulation problem for switched systems under the condition that each subsystem is required to be solvable. It is still a challenging problem when consider the case in which the problem for subsystems is unsolvable. One major difficulty in dealing with this problem is that the switched regulator make the switching rule and control design further complicate.

Also, it is worth noting that the majority of the work on output regulation problem focus on the steady state performance of the switched system, i.e., the solvability conditions of the output regulation problem for switched systems. Noting that constraints are commonly occurring in practical applications, the transient performance is also an

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important issue to be considered in the output regulation problem. For example, when robot manipulators perform complicated tasks, perform complicated tasks, they are required to keep tracking error convergeing to a designed compact zone in a given finite time [21-23]. In [24], uncertain Euler-Lagrange systems with state constraints is designed to achieving tracking error converges to the designed compact zone in a given finite time by constructing a speed function and adopting error transformation technique. A result has been made in [25], which is designing a distributed controller and adaptive laws for the multi-agent systems to achieve prescribed performance control. Thus, the methods that guarantee prescribed transient performance are more preferred in many applications.

In this paper, we propose a prescribed performance control for a class of nonlinear switched systems in strict feedback form by constructing the state dependent switching rule and state feedback controllers. A general framework to tackle the output regulation with finite time prescribed performance for a class of nonlinear switched system is established by combining the coordinate transformation technique and barrier Lyapunov function technique. By constructing switched regulator equation and performing a state transformation, the output regulation problem with output error constraint is converted to a constrained stabilization problem of a transformed system still in strict feedback form. Then, barrier Lyapunov functions and speed function are introduced to handle the state constraints, based on which, a state feedback controller and a state dependent switching signal are designed to solve the constrained stabilization problem, which also solves the output regulation problem with prescribed performance. Compared to the existing results, the features of this paper are as follows:

1) Different from the traditional output regulation problem in which the steady state of the system is addressed [4-9], the proposed output regulation with prescribed performance control considers both the steady state performance and transient performance. The error output of the output regulation system not only is confined in the prescribed performance bound in a given finite time, but also tends to zero asymptotically. Moreover, compared with the existing transient performance control for nonlinear systems [26-28], the initial value of error output need not to be restricted in prescribed performance bound by our proposed method, which enlarge the degree of freedom in the design. To deal with this issue, a novel state feedback controller based on the barrier Lyapunov function and speed function is constructed.

2) The existing methods which require that every subsystem has its own regulator equation [16,17]. To come up with a less conservatism for this more complicated output regulation with prescribed performance problem, this paper constructs a switched regulator equation based a state dependent switching rule, by which the output regulation problem of each subsystem need not to be solved.

This paper is organized as follows: In Section 2, an output regulation problem with prescribed performance control is formulated for a class of strict-feedback systems with state constraints. Section 3 establishes the main result on the solvability of the proposed output regulation problem. In Section 4, a numerical example is presented to illustrate the validity of our proposed method. Section 5 ends the paper with some concluding remarks.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of switched strict-feedback nonlinear systems described as follows:

$$\begin{aligned} \dot{x}_1 &= f_1^{\sigma(t)}(x_1,v) + g_1^{\sigma(t)}(x_1,v)x_2, \\ \dot{x}_i &= f_i^{\sigma(t)}(x_1,\dots,x_i,v) + g_i^{\sigma(t)}(x_1,\dots,x_i,v)x_{i+1}, \\ \dot{x}_n &= f_n^{\sigma(t)}(x_1,\dots,x_n,v) + g_n^{\sigma(t)}(x_1,\dots,x_n,v)u, \end{aligned} \tag{1}$$

where $2 \le i \le n-1$, $x_i \in \mathbb{R}$, $i = 1, 2, \dots, n$,are the system states. $\sigma(t)$ is a switching signal denoting the mth subsystem with $\sigma(t) = m$, which takes its values in a finite set $\mathcal{M} = \{1, \dots, M\}$ and $M > 1$ is the number of subsystems. Moreover, for a switching sequence $0 < t_1 < \dots < t_i < \dots$, symbol t_i denotes the moment of the ith switching. $u \in \mathbb{R}$ is the control input, $v \in \mathbb{R}^q$ is the exogenous signal representing the disturbance and tracking signal, which is given by

$$\dot{v} = Sv \tag{2}$$

with $v(0) \in \mathcal{V}_0$ where $\mathcal{V}_0$ is a compact set. It is assumed that, for $\forall i = 1, \dots, n$, the functions $f_i^{\sigma(t)}(x_1,\dots,x_i,v)$ and $g_i^{\sigma(t)}(x_1,\dots,x_i,v)$ are smooth with $f_i^{\sigma(t)}(0,\dots,0,0) = 0$. The control objective is to design a switching feedback control law

$$u = k_{\sigma(t)}(x,v), \tag{3}$$

and a state dependent switching rule $\sigma(t)$ such that the output of the closed-loop system $y = x_1$ will asymptotically track a reference trajectory $y_d(v)$.

Defining the error output of the system as

$$e = y(t) - y_d(v) = x_1 - y_d(v), \tag{4}$$

the above described control problem can be formulated as an output regulation problem with prescribed performance control as follows.

Output regulation problem with prescribed performance control: Consider switched strict-feedback nonlinear systems (1), the exosystem (2), and the error output

(4), design a state feedback control law in the form of (3) and a switching rule $\sigma(t)$ such that the closed-loop system consisting of (1), (2) and (3) has the following three properties:

Property 1: all the signals in the closed-loop system are bounded for all $t \geq 0$;

Property 2: the system output $y = x_1$ tracks a reference trajectory $y_d(v)$ asymptotically, i.e.,

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (y(t) - y_d(v)) = 0;$$

Property 3: within a given finite time T, the tracking error $e(t)$ satisfies a prescribed performance bound,

$$-cK < e(t) < cK, \quad \forall t \geq T,$$

where K is positive constant and $0 < c \ll 1$ is a design parameter.

Remark 1: The output regulation problem with prescribed performance is an extension of the traditional output regulation problem by addressing the transient performance of the closed-loop error output $e(t)$. In fact, without Property 3, it is exactly a traditional output regulation problem which is extensively investigated in the literature. Property 3 defines a performance boundary for the transient performance of the closed-loop error output $e(t)$. The error output $e(t)$ are confined to a given region which is predefined by the performance bound function cK in a designed finite time. Comparing with traditional output regulation problem, we not only guarantee that the steady states converge to constructed invariant manifold, but also achieve that the transient states without violation of the preferred boundary. Moreover, switching between each subsystems raise the difficulty of constructing switched invariant manifold.

The following assumptions and definition are necessary to derive the main results in this paper.

Assumption 1: The exosystem (2) is neutrally stable, i.e., all the eigenvalues of S are simple and have zero real parts.

Assumption 2: $g_i^m(x_1,\ldots,x_i,v)$, $i = 1,\ldots,n$, are known, and there exists some unknowns constants b_i such that $|g_i^m(x_1,\ldots,x_i,v)| \geq b_i > 0$. Without loss of generality, we assumed that $g_i^m(x_1,\ldots,x_i,v) \geq b_i > 0$.

Remark 2: Assumption 1 is quite common in the literature on nonlinear output regulation problems. To simplify the stability analysis of the closed-system, Assumption 2 is made to guarantee that the control direction is not changed during the system is running, i.e., $g_i^m \geq b_i > 0$ or $g_i^m \leq -b_i < 0$ with $b_i > 0$. However, if $g_i^m \leq -b_i < 0$, let $\tilde{g}_i^m = -g_i^m$, we have $\tilde{g}_i^m \geq b_i > 0$. Thus, without loss of generality, we assumed that $g_i^m \geq b_i > 0$.

Let $x_{i+1} = a_1 x_{i+1,1} + a_2 x_{i+1,2} + \ldots + a_M x_{i+1,M}$, with $i = 0$, ..., n, $a_j = \begin{cases} 1, & \delta(t) - j = 0, \\ 0, & \text{else}, \end{cases}$ $j \in \mathcal{M} = \{1, \ldots, M\}$ and $\mathbf{x}_1(v) = y_d(v)$, we can define

$$\mathbf{x}_{(i+1)}(v) = \frac{\frac{\partial \mathbf{x}_i(v)}{\partial v} Sv - f_i^{\sigma(t)}(\mathbf{x}_1(v),\ldots,\mathbf{x}_i(v),v)}{g_i^{\sigma(t)}(\mathbf{x}_1(v),\ldots,\mathbf{x}_i(v),v)},$$
$$i = 1, \ldots, n. \tag{5}$$

It is clear that $\mathbf{x}(v) = [\mathbf{x}_1(v) \ \cdots \ \mathbf{x}_n(v)]^T$ and $\mathbf{u}(v) = \mathbf{x}_{n+1}(v)$ satisfy the regulator equations of the output regulation problem for the system (1), (2) and (4).

Remark 3: Equation (5) constructs a switched regulator equation based a state dependent $\sigma(t)$, which is made to guarantee that the solvable of the output regulation problem of switched system. The existing results require that each subsystem states converge to invariant manifold, which is $x_{im} = \mathbf{x}_i(v)$. Comparing with existing results, the output regulation problem of each subsystem is not required to be solvable, which release the conservatism.

Definition 1 [24]**:** A rate function is given as follows:

$$\bar{\chi}(t) = \begin{cases} (\frac{T}{T-t})^4 \chi(t), & 0 \leq t < T, \\ \infty, & t \geq T, \end{cases} \tag{6}$$

where $0 < T < \infty$ is a designed finite time, and $\chi(t)$ represents nondecreasing and smooth function meeting $\chi(0) = 1$ and $\dot{\chi} \geq 0$, which should be noticed is that when $t \geq T$, $\bar{\chi}(t) = \infty$.

Assumption 3: The finite time T is designed to satisfy $T > T_c$. T_c is a small time interval which is necessary for signal computing and transmission.

According to the introduced speed function and Assumption 2, the speed function $\beta(t)$ is given as follows:

$$\beta(t) = \frac{1}{(1-c)\bar{\chi}(t)^{-1} + c}, \tag{7}$$

where c is a given parameter meeting $0 < c \ll 1$. Based on the expression of $\bar{\chi}(t)$ in (6), we obtain

$$\beta(t) = \begin{cases} \frac{T^4 \chi(t)}{(1-c)(T-t)^4 + cT^4 \chi(t)}, & 0 \leq t < T, \\ \frac{1}{c}, & t \geq T. \end{cases} \tag{8}$$

The properties of the speed function $\beta(t)$ are listed in [24].

3. MAIN RESULTS

In this section, we will give sufficient conditions for the solvability of the output regulation problem with prescribed performance of the system (1), (2), and (4). To this end, we first define

$$\bar{x}_i = a_1 x_{i,1} + a_2 x_{i,2} + \ldots + a_M x_{i,M} - \mathbf{x}_i(v),$$
$$\bar{u} = a_1 u_1 + a_2 u_2 + \ldots + a_M u_M - \mathbf{u}(v), \tag{9}$$

where $a_j = \begin{cases} 1, & \delta(t) - j = 0, \\ 0, & \text{else}, \end{cases}$ $j \in \mathcal{M} = \{1, ..., M\}$.
Then, the system (1), (2) and (4) can be rewritten as

$$\begin{aligned}
\dot{\bar{x}}_1 &= f_1^{\sigma(t)}(\bar{x}_1 + \mathbf{x}_1(v), v) + g_1^{\sigma(t)}(\bar{x}_1 + \mathbf{x}_1(v), v)(\bar{x}_2 + \mathbf{x}_2) \\
&\quad - \frac{\partial \mathbf{x}_1}{\partial v} Sv = h_1^{\sigma(t)}(\bar{x}_1, \bar{x}_2, v), \\
\dot{\bar{x}}_i &= f_i^{\sigma(t)}(\bar{x}_1 + \mathbf{x}_1(v), ..., \bar{x}_i + \mathbf{x}_i(v), v) \\
&\quad + g_i^{\sigma(t)}(\bar{x}_1 + \mathbf{x}_1(v), ..., \bar{x}_i + \mathbf{x}_i(v), v)(\bar{x}_{i+1} + \mathbf{x}_{i+1}) \\
&\quad - \frac{\partial \mathbf{x}_i}{\partial v} Sv \\
&= h_i^{\sigma(t)}(\bar{x}_1, ..., \bar{x}_i, v), \ i = 1, \ ..., \ n-1 \\
\dot{\bar{x}}_n &= f_n^{\sigma(t)}(\bar{x}_1 + \mathbf{x}_1(v), ..., \bar{x}_n + \mathbf{x}_n(v), v) \\
&\quad + g_n^{\sigma(t)}(\bar{x}_1 + \mathbf{x}_1(v), ..., \bar{x}_n + \mathbf{x}_n(v), v)(\bar{u} + \mathbf{u}) \\
&\quad - \frac{\partial \mathbf{x}_n}{\partial v} Sv) \\
&= h_n^{\sigma(t)}(\bar{x}_1, ..., \bar{x}_n, \bar{u}, v), \\
\dot{v} &= Sv, \\
e &= x_1 - y_d(v) = \bar{x}_1.
\end{aligned} \tag{10}$$

According to (5), $\mathbf{x}(v)$ and $\mathbf{u}(v)$ satisfy the regulator equations, which implies that

$$h_i^{\sigma(t)}(0, ..., 0, v) = 0,$$

for all $v \in \mathbb{R}^q$. Thus, it is not difficult to show that the output regulation problem with prescribed performance for the system (1), (2), and (4) is equivalent to a constraint stabilization problem of (10). In fact, if there exists a controller

$$\bar{u} = \bar{k}_{\sigma(t)}(\bar{x}_1, ... \bar{x}_n, v) \tag{11}$$

solves the stabilization problem of (10), then $\lim_{t\to\infty} \bar{x}_i(t) = 0$, i.e., $\lim_{t\to\infty} x_i(t) = \mathbf{x}_i(v)$ and $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \bar{x}_1(t) = 0$, which guarantees Properties 1 and 2.

Moreover, Property 3 can be satisfied if $\bar{x}_1$ is controlled in

$$\bar{x}_1 \in \mathcal{E}_i := \{-cK < \bar{x}_1(t) < cK, \ \forall t > T\}. \tag{12}$$

Next, we will construct a control law by the following steps to solve the stabilization problem of (10) with constraint (12), which equivalently solves the output regulation problem with prescribed performance for the system (1), (2) and (4).

Step 1: Define $\bar{x}_1 = z_1$, error transformation $\eta = \beta z_1$, and $z_2 = \bar{x}_2 - \alpha_1(\eta)$. Consider the following common Lyapunov function candidate

$$V_{1,\sigma(t)}(\eta) = \frac{1}{2} \ln(\frac{K^2}{K^2 - \eta_{\sigma(t)}^2}), \ \forall \sigma(t) \in \mathcal{M}. \tag{13}$$

The derivative of $V_{1,\sigma(t)}(z_1)$ along the transformed system (10) is given by

$$\begin{aligned}
&\dot{V}_{1,\sigma(t)}(\eta_{\sigma(t)}) \\
&= \frac{\eta}{K^2 - \eta_{\sigma(t)}^2} \Big[f_1^{\sigma(t)}\Big(\frac{\eta}{\beta} + \mathbf{x}_1(v)\Big) \\
&\quad + g_1^{\sigma(t)}\Big(\frac{\eta}{\beta} + \mathbf{x}_1(v)\Big)(z_2 + \mathbf{x}_2(v) + \alpha_1(\eta)) - \dot{\mathbf{x}}_1(v) \Big].
\end{aligned} \tag{14}$$

Design the state dependent switching law

$$\sigma(t) = \arg \min_{m\in\mathcal{M}} \{|x_{1m} - \mathbf{x}_1(v)|\}, \tag{15}$$

when $\sigma(t) = m$, the mth subsystem is active, one obtains

$$\begin{aligned}
\dot{V}_{1,m}(\eta_m) = &\frac{\eta_m}{K^2 - \eta_m^2} [f_1^m(\frac{\eta}{\beta} + \mathbf{x}_1(v)) + g_1^m(\frac{\eta}{\beta} + \\
&\mathbf{x}_1(v))(z_2 + \mathbf{x}_2(v) + \alpha_1(\eta)) - \dot{\mathbf{x}}_1(v) + \frac{\dot{\beta}}{\beta}\eta].
\end{aligned} \tag{16}$$

Design the stabilizing function $\alpha_1(z_1)$ as

$$\begin{aligned}
\alpha_1(\eta) = &\frac{1}{g_1^m}\Big(-f_1^m - k_{1,m}(K^2 - \eta^2)\eta + \dot{\mathbf{x}}_1(v) - \frac{\dot{\beta}}{\beta}\eta \Big) \\
&- \mathbf{x}_2(v),
\end{aligned} \tag{17}$$

where $k_{1,m} > 0$ are positive constants. Combining (14) and (17), we can obtain

$$\dot{V}_{1,m}(z_1) = -k_{1,m}\eta^2 + \frac{1}{K^2 - \eta^2} g_1^m \eta z_2, \tag{18}$$

where the coupling term $\frac{1}{K^2-\eta^2} g_1^m \eta z_2$ is canceled in the subsequent step.

Step 2: Define $z_3 = \bar{x}_3 - \alpha_2(\eta, z_2)$, and consider the following common Lyapunov function candidate

$$V_{2,\sigma(t)}(\eta, z_2) = V_{1,\sigma(t)}(\eta) + \frac{1}{2} z_2^2, \ \forall \sigma(t) \in \mathcal{M}. \tag{19}$$

The derivative of $V_{2,\sigma(t)}(\eta, z_2)$ is given by

$$\begin{aligned}
&\dot{V}_{2,\sigma(t)}(\eta, z_2) \\
&= -k_{1,\sigma(t)}\eta^2 + \frac{1}{K^2 - \eta^2} g_2^{\sigma(t)} \eta z_2 \\
&\quad + z_2[f_2^{\sigma(t)} + g_1^{\sigma(t)}(z_3 + \mathbf{x}_3(v) + \alpha_2(\eta, z_2)) \\
&\quad - \dot{\mathbf{x}}_2(v) - \dot{\alpha}_1].
\end{aligned} \tag{20}$$

Design the state dependent switching law

$$\sigma(t) = \arg \min_{m\in\mathcal{M}} \{|x_{2m} - \mathbf{x}_2(v)|\}, \tag{21}$$

when $\sigma(t) = m$, the mth subsystem is active, one obtains

$$\begin{aligned}
\dot{V}_{2,m}(\eta, z_2) = &- k_{1,m}\eta^2 + \frac{1}{K^2 - \eta^2} g_1^m \eta z_2 \\
&+ z_2[f_2^m + g_2^m(z_3 + \mathbf{x}_3(v) + \alpha_2(\eta, z_2)) \\
&- \dot{\mathbf{x}}_2(v) - \dot{\alpha}_1].
\end{aligned} \tag{22}$$

Design the stabilizing function $\alpha_2(\eta, z_2)$ as

$$\alpha_2(\eta, z_2) = \frac{1}{g_2^m}(-f_2^m - k_{2,m}z_2 - \frac{1}{K^2-\eta^2}g_1^m\eta + \dot{\mathbf{x}}_2(v) + \dot{\alpha}_1) - \mathbf{x}_3(v), \tag{23}$$

where $k_{2,m} > 0$ are positive constants. Substituting (23) into (20), we have

$$\dot{V}_{2,m}(\eta, z_2) = -k_{1,m}\eta^2 - k_{2,m}z_2^2 + g_2^m z_2 z_3, \tag{24}$$

where the coupling term $g_2^m z_2 z_3$ is canceled in the subsequent step.

Step *i*: Define $z_i = \bar{x}_i - \alpha_{i-1}(\eta, ..., z_{i-1})$, and consider the following common Lyapunov function candidate

$$V_{i,\sigma(t)}(\eta, ..., z_i) = V_{i-1,\sigma(t)}(z_{i-1}) + \frac{1}{2}z_i^2,\ \forall \sigma(t) \in \mathcal{M}. \tag{25}$$

The derivative of $V_{i,\sigma(t)}(z_i)$ is given by

$$\begin{aligned}\dot{V}_{i,\sigma(t)}(\eta, ..., z_i) = &- k_{1,\sigma(t)}\eta^2 - \sum_{j=2}^{i-1} k_{j,\sigma(t)}z_j^2 \\ &+ g_{i-1}^{\sigma}(t)z_{i-1}z_i \\ &+ z_i[f_i^{\sigma(t)} + g_i^{\sigma(t)}(z_{i+1} + \alpha_i(z_i) \\ &+ \mathbf{x}_{i+1}^{\sigma(t)}(v)) - \dot{\mathbf{x}}_i^{\sigma(t)}(v) - \dot{\alpha}_{i-1}].\end{aligned} \tag{26}$$

Design the state dependent switching law

$$\sigma(t) = \arg\min_{m\in\mathcal{M}}\{|x_{im} - \mathbf{x}_i(v)|\}, \tag{27}$$

when $\sigma(t) = m$, the mth subsystem is active, one obtains

$$\begin{aligned}\dot{V}_{i,m}(\eta, ..., z_i) = &- k_{1,m}\eta^2 - \sum_{j=2}^{i-1} k_{j,m}z_j^2 \\ &+ g_{i-1}^m z_{i-1}z_i \\ &+ z_i[f_i^m + g_i^m(z_{i+1} + \alpha_i(z_i) + \mathbf{x}_{i+1}^m(v)) \\ &- \dot{\mathbf{x}}_i^m(v) - \dot{\alpha}_{i-1}].\end{aligned} \tag{28}$$

Design the stabilizing function $\alpha_i(z_i)$ as

$$\alpha_i(\eta, ..., z_i) = \frac{1}{g_i^m}(-f_i^m - k_{i,m}z_i - g_{i-1}^m z_{i-1} + \dot{\mathbf{x}}_i^m(v) + \dot{\alpha}_{i-1}) - \mathbf{x}_{i+1}^m(v), \tag{29}$$

where $k_{i,m} > 0$ are positive constants. Substituting (29) into (26), we have

$$\dot{V}_{i,m}(z_1, ..., z_i) = -k_{1,m}\eta^2 - \sum_{j=2}^{i-1} k_{j,m}z_j^2 + g_i^m z_i z_{i+1}, \tag{30}$$

where the coupling term $g_i^m z_i z_{i+1}$ is canceled in the subsequent step.

Step *n*: By repeating the above steps, let $z_n = \bar{x}_n - \alpha_{n-1}(z_1, ..., z_{n-1})$ and $V_{n,\sigma(t)}(\eta, ..., z_n) = V_{n-1,\sigma(t)}(z_{n-1}) + \frac{1}{2}z_n^2$. It is straightforward to obtain

$$\begin{aligned}&\dot{V}_{n,\sigma(t)}(\eta, ..., z_n) \\ &= -k_{1,\sigma(t)}\eta^2 - \sum_{j=2}^{n-1} k_{j,\sigma(t)}z_j^2 \\ &\quad + g_{n-1}^{\sigma}(t)z_{n-1}z_n + z_n[f_n^{\sigma(t)} + g_n^{\sigma(t)}(z_{n+1} + \alpha_n(z_n) \\ &\quad + \mathbf{x}_{n+1}^{\sigma(t)}(v)) - \dot{\mathbf{x}}_n^{\sigma(t)}(v) - \dot{\alpha}_{n-1}].\end{aligned} \tag{31}$$

Design the state dependent switching law

$$\sigma(t) = \arg\min_{m\in\mathcal{M}}\{|x_{nm} - \mathbf{x}_n(v)|\}, \tag{32}$$

when $\sigma(t) = m$, the mth subsystem is active, one obtains

$$\begin{aligned}&\dot{V}_{n,m}(\eta, ..., z_n) \\ &= -k_{1,m}\eta^2 - \sum_{j=2}^{n-1} k_{j,m}z_j^2 \\ &\quad + g_{n-1}^m z_{n-1}z_n + z_n[f_n^m + g_n^m(z_{n+1} + \alpha_n(z_n) \\ &\quad + \mathbf{x}_{n+1}^m(v)) - \dot{\mathbf{x}}_n^m(v) - \dot{\alpha}_{n-1}].\end{aligned} \tag{33}$$

Then, the control law is designed as

$$\begin{aligned}\bar{u} &= \alpha_n(\eta, ..., z_n) \\ &= \frac{1}{g_n^m}(-f_n^m - k_{n,m}z_i - g_{n-1}^m z_{n-1} + \dot{\mathbf{x}}_n^m(v) + \dot{\alpha}_{n-1}) \\ &\quad - \mathbf{x}_{n+1}^m(v),\end{aligned} \tag{34}$$

where $k_{n,m} > 0$ are positive constants. Substituting (34) into (31), we have

$$\dot{V}_{n,m}(\eta, ..., z_n) = -k_{1,m}\eta^2 - \sum_{j=2}^{n} k_{j,m}z_j^2. \tag{35}$$

Consequently, we can conclude the main result which is declared by the following theorem.

Theorem 1: For the switched system described by (1) and (2) that satisfies (5) with the state feedback controller in the form of (34) and state dependent switching rule $\sigma(t)$ satisfies the condition (15), (27), and (32), the output regulation problem with prescribed performance is achieved.

Proof: To show Theorem 1, we just need to verify that Properties 1-3 are satisfied:

(i) We first show that the trajectories of the closed-loop system are bounded for all $t \geq 0$. According to (35), we get

$$\dot{V}_{n,m} < 0 \Rightarrow V_{n,m}(t) \leq V_{n,m}(0),\ \ t \geq 0. \tag{36}$$

This implies that

$$\begin{aligned}&\ln\left(\frac{1}{1-\eta^2(t)}\right) + \sum_{i=2}^{n} z_i^2(t) \\ &\leq \ln\left(\frac{1}{1-\eta^2(t_0)}\right) + \sum_{i=2}^{n} z_i^2(t_0).\end{aligned} \tag{37}$$

Boundedness of the left-hand side of (37) implies that η, z_i, $i = 2, ..., n$, are bounded. Apparently, α_i are bounded by constructed. After coordinate transformation $z_i = \bar{x}_i - \alpha_{i-1}(\eta, ..., z_{i-1})$, we can obtain $\bar{x}_i$, $i = 1, ..., n$ are bounded. Furthermore, based on $\bar{x}_i = x_i - x_i(v)$, $i = 1, ..., n$, it is easy to get all x_i are bounded, i.e., Property 1 is satisfied.

(ii) (35) implies that η convergence to zero, as $t \to \infty$, thus

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \frac{\eta(t)}{\beta} = 0,$$

which is Property 2.

(iii) According to (35), we can know that $V_{1m}(t) \leq V_{1m}(0)$, $\forall t > 0$. Based on the expression of $V_{1m}(t)$, we obtain

$$\log \frac{K^2}{K^2 - \eta_m^2(t)} \leq V_1(0), \tag{38}$$

which implies that $K^2 \leq \eta_m^{V_1(0)}(K^2 - \eta_m^2(t))$. Therefore

$$\|\eta\| \leq K\sqrt{1 - \eta^{-V_1(0)}} \leq K, \quad \forall t > 0. \tag{39}$$

Note that $\eta = \beta^{-1}\bar{x}_1$, thus, we can obtain

$$\|\bar{x}_1\| \leq \beta^{-1}K. \tag{40}$$

Then, according to (8), we obtain

$$\| \bar{x}_1\| \leq (1-c)\frac{T-t}{T}^4 \chi^{-1}K + cK,\ 0 \leq t < T, \tag{41}$$

$$\|\bar{x}_1\| \leq cK,\ t \geq T. \tag{42}$$

Owing to $e = \bar{x}_1$, which means that the output tracking error converges to the designed constrained set in a finite time T. Thus, Property 3 is achieved. This completes the proof. □

Remark 4: Under the proposed finite time prescribed performance control scheme defined by (17), (23), (29), and (34), the output error of the system satisfies $|e(t)| \leq cK$ after a given finite time T, which means that the speed function $\beta(t)$ restricts the decay rate no less than $\frac{T-t}{T}^4\chi^{-1}$ in $[0, T)$ and K defines a prescribed performance bound for the output error. As a special case, let $\beta(t) = 1$ and $K = \infty$, the control scheme defined by (17), (23), (29), and (34) will be reduced to a traditional control scheme

$$\begin{aligned}
\alpha_1(z_1) &= \frac{1}{g_1^m}(-f_1^m - k_{1,m}z_1 + \dot{\mathbf{x}}_1(v)) - \mathbf{x}_2(v),\\
\alpha_2(z_1, z_2) &= \frac{1}{g_2^m}(-f_2^m - k_{2,m}z_2 - g_1^m z_1 + \dot{\mathbf{x}}_2(v) + \dot{\alpha}_1)\\
&\quad - \mathbf{x}_3(v),\\
\alpha_i(z_1, ..., z_i) &= \frac{1}{g_i^m}(-f_i^m - k_{i,m}z_i - g_{i-1}^m z_{i-1} + \dot{\mathbf{x}}_i^m(v)\\
&\quad + \dot{\alpha}_{i-1}) - \mathbf{x}_{i+1}^m(v),\\
\bar{u} = \alpha_n(z_1, ..., z_n) &= \frac{1}{g_n^m}(-f_n^m - k_{n,m}z_i - g_{n-1}^m z_{n-1}\\
&\quad + \dot{\mathbf{x}}_n^m(v) + \dot{\alpha}_{n-1}) - \mathbf{x}_{n+1}^m(v).
\end{aligned} \tag{43}$$

The traditional control scheme also solves the output regulation problem which is Properties 1 and 2, but the requirement on transient performance for the error output $e(t)$ is removed which is Property 3.

Remark 5: Compared with the previous output regulation problem with transient performance result [26,27], which only considers that the output be restricted in the given bound with limited initial output value, this paper considers that the error output is confined in the prescribed performance bound with any initial value in a designed finite time. In addition, this article consider switched systems require constructing specific switching rule and this result can be applied to non-switched systems.

4. SIMULATION RESULTS

In this section, a numerical example is given to illustrate the effectiveness of the proposed control scheme. Consider the following switched strict-feedback nonlinear system

$$\begin{aligned}
\dot{x}_1 &= f_1^{\sigma(t)}(\bar{x}_1, v) + g_1^{\sigma(t)}(\bar{x}_1, v)x_2,\\
\dot{x}_2 &= f_2^{\sigma(t)}(\bar{x}_2, v) + g_2^{\sigma(t)}(\bar{x}_2, v)u,\\
\dot{v} &= Sv,\\
e &= x_1 - y_d(v),
\end{aligned} \tag{44}$$

where $\sigma(t) \in \mathcal{M} = \{1,\ 2\}$, $f_1^1(x_1, v) = 0.5x_1 + v_2$, $g_1^1(x_1, v) = 1$, $f_2^1(x_1, x_2, v) = -x_1 - 2e^{-x_2} + 2$, $g_2^1(x_1, x_2, v) = 1$, $f_1^2(x_1, v) = x_1 + 2v_2$, $g_1^2(x_1, v) = 1$, $f_2^2(x_1, x_2, v) = x_1 + e^{-x_2} - 1$, $g_2^2(x_1, x_2, v) = 1$. The exosystem is selected as: $\dot{v}_1 = v_2$, $\dot{v}_2 = -v_1$. For the output tracking error $e = x_1 - v_1$, the required constraints is considered in $-0.2 < e(t) < 0.2$. We choose the speed function $\beta(t)$ is as in (8) with $c = 0.1$ the rate function $\chi = e^t$ and the finite time $T = 2$ s.

Based on the state dependent switching rule (32), the solution of the switched regulator equations can be given as: $\mathbf{x}_1(v) = v_1$, $\mathbf{x}_2(v) = -v_2 - v_1$. Then, according to the proposed finite time prescribed performance control scheme defined by (17), (23), (29), and (34), the design parameters of controllers are chosen as $k_{11} = 0.3$, $k_{12} = 0.5$, $k_{21} = 0.4$, $k_{22} = 0.8$. We select the initial values as $x_1(0) = 0.5$, $x_2(0) = -0.5$, and the other initial conditions are chosen as zero.

The simulation results are shown in Figs. 1-3. Fig. 1 represents the unstable exosystem. Fig. 2 is the designed state dependent switching signal. Fig. 3 compares the trajectories of the error output between the finite time prescribed performance control scheme (34) with $\chi = e^t$, $T = 2$ s and $cK = 0.2$ and the traditional control scheme (43). The dashed line is the prescribed performance bound. It is clear that, under the finite time prescribed performance control scheme, the trajectory of the error output (the solid line) converges to the prescribed set in a given time

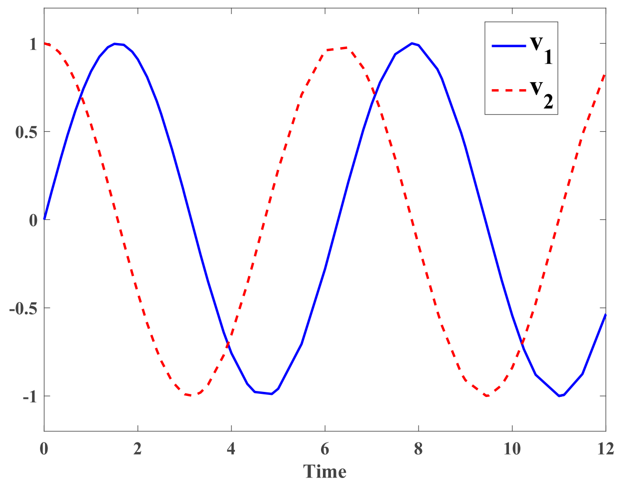

Fig. 1. The exosystem $v(t)$.

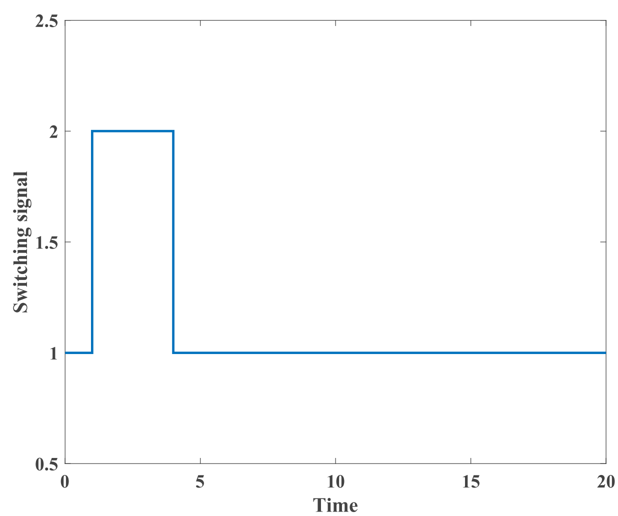

Fig. 2. Switching signal.

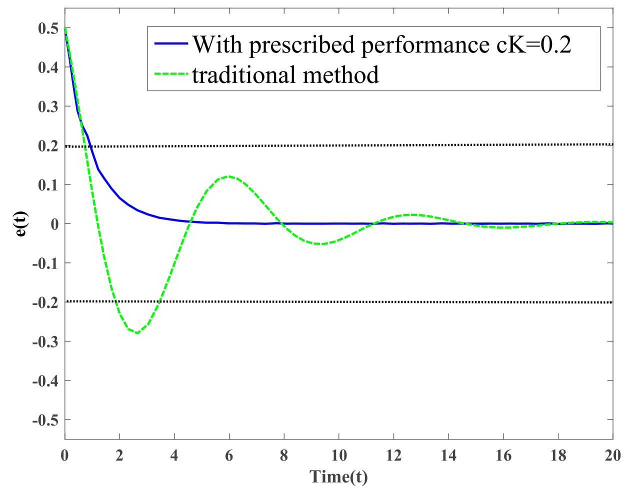

Fig. 3. Tracking error $e(t)$.

$T = 2$ s and decays to zero asymptotically. However, under the traditional control scheme, the prescribed performance bound is no longer kept after a give time $T = 2$ s, as shown by the dotted line in Fig. 3.

5. CONCLUSION

In this paper, a general framework is established to control switched strict-feedback nonlinear system to achieve output regulation with finite time prescribed performance. Firstly, we transform output regulation problem into a stabilization problem by constructing switched regulator equation and coordinate transformation. Then, speed function technique and barrier Lyapunov function technique are adopted to guarantee that the output error can be confined in a prescribed performance bound after a given finite time. Finally, by using the Lyapunov function and constructing a state dependent switching rule, the stability analysis is provided for the closed-loop systems. It should be noted that only the state feedback with feed-forward control is considered in this paper, and the control law (3) relies on the system state x and the exo-signal v. However, the states x and/or v may not be available for feedback control in practical applications. Thus, it is necessary but more difficult to design a dynamic output feedback control law. This will be one of interesting future works.

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