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Fixed-time Fuzzy Adaptive Decentralized Control for High-order Nonlinear Large-scale Systems

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Abstract: This paper studies a fuzzy adaptive fixed-time tracking control issue for nonlinear high-order large-scale systems. Fuzzy logic systems (FLSs) are utilized to identify unknown nonlinearities. Through using adaptive backstepping and adding a power integrator technique, the fixed-time decentralized control method is presented. It is proved that the tracking errors converge to a small neighborhood of a fixed time. A simulation example is presented to confirm the validity of the developed control method.

Keywords: Adaptive decentralized control, adding a power integrator technique, fixed-time control, fuzzy logic systems, high-order large-scale systems.

1. INTRODUCTION

Backstepping control decomposes the high-order nonlinear system into multiple subsystems to reduce the complexity of designing the system controller. The backstepping method starts from the first subsystem to design the virtual controller, and then step by step recursively until the actual controller of the whole system is designed. Therefore, the researchers frequently make use of backstepping method to handle the control issues for nonlinear systems. Ma *et al*. [1] presented a novel adaptive control strategy for nonlinear strict-feedback systems. Wang *et al*. [2] studied the control issue of n-order semi-strict nonlinear systems with uncertainty and constraint. Cai *et al*. [3] presented a robust adaptive control scheme of nonlinear systems. The authors in [4,5] studied robust output feedback control issues for a class of nonlinear systems. Hua *et al*. [6] devised a state-feedback controller of nonlinear time-delay systems, which indicated that this system is asymptotically stable. Cui and Xie [7] considered adaptive state-feedback stabilization control problem of stochastic high-order nonlinear systems with state-constrained. To solve synchronization problem of the systems, Peng *et al*. [8] developed a Nussbaum-type adaptive distributed controller for nonlinear high-order systems with uncertainties. Davila [9] designed an exponential exact tracking controller by using the backstepping method for nonlinear high-order systems with disturbances.

As we know, the research results mentioned above are only limited that the nonlinearities are known accurately or can be linearly parameterized for control systems. The design process has certain complexities and limitations. In order to solve this issue, based on the universal approximation property of FLSs or neural network (NN), the researchers have put forward many adaptive fuzzy and NN backstepping control technologies of nonlinear systems with completely unknown nonlinearities (see [10-14]). Among them, in [10,11], the authors proposed backstepping NN control schemes for nonlinear systems. Li *et al*. [12] proposed the adaptive fuzzy output tracking control scheme for nonlinear switched systems. Combining with backstepping technology, Tong *et al*. [13,14] designed a fuzzy adaptive output feedback controller of multi-input and multi-output (MIMO) nonlinear systems.

It is worth noting that the literatures [1-14] studied single-input and single-output (SISO) or MIMO nonlinear systems. Different from these types of systems, the nonlinear large-scale systems have the nature of multi-layered, high-dimensional, time-distributed and space-distributed, which lead to complex system construction, diverse objectives and limited information structure in the system. Moreover, many practical control systems can be regarded as the large-scale nonlinear systems, such as transportation systems, ecological systems, and digital communication networks, etc. Therefore, the decentralized control theory has been proposed, as an important branch of large-scale systems control theory (see [15-23]). Among them, the authors in [15-18] proposed state-feedback and output-feedback decentralized control algorithms for a class of large-scale systems. Li *et al*. [19] discussed the issue of

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adaptive fuzzy tracking fault-tolerant control for stochastic large-scale systems with output-constrained. Li *et al.* [20] designed an fixed-time fuzzy decentralized control algorithm for stochastic large-scale nonlinear systems. In [21-23], the authors studied adaptive NN and fuzzy decentralized state-feedback control issues for nonlinear large-scale systems. However, the existing literatures only studied the asymptotic stability for nonlinear large-scale systems. For systems like this one, it is still a challenge to study fixed-time fuzzy decentralized control issue.

The stability analysis has always been a hot issue in control field for nonlinear systems. According to the convergence time, the system stability is usually divided into infinite-time stability, finite-time stability and fixed-time stability. For a class of discrete-time systems, the authors discussed the issues of H^∞ dynamic output feedback control and peak-to-peak filtering in [24,25]. Wang and Zhu [26] designed a finite-time state-feedback controller for high-order nonlinear system. Fan and Li presented an adaptive finite-time optimal control scheme of switched nonlinear systems in [27]. The authors studied the issues of adaptive finite-time decentralized control for large-scale nonlinear systems in [28-30]. In [31], the authors studied a decentralised adaptive finite-time prescribed performance control issue for large-scale nonlinear interconnected systems. However, for finite-time control, the system convergence speed and performance depend on the initial conditions, and the convergence time will vary as the initial value of the state change. In order to deal with this problem, Polyakov [32] first presented the concept and basic theory of fixed-time control. Hong *et al.* [33] considered distributed fixed-time consensus protocol problem for multi-agent nonlinear systems. The authors in [34,35] studied the adaptive fixed-time tracking control issues for strict-feedback and nonstrict-feedback nonlinear systems, and certified that the convergence time is unconnected with the initial conditions. Zhou *et al.* focused on the decentralized adaptive fuzzy fixed-time control issue for a class of interconnected nonlinear systems in [36]. Zhang *et al.* [37] developed a fixed-time adaptive attitude tracking controller for spacecraft. It is worth pointing out that fixed-time control has not only faster convergence rate, stronger robustness and higher control accuracy than asymptotic stability, but also the convergence time is independent of the initial conditions, related to some design parameters. These advantages make the fixed-time control have a broad application prospect in spacecraft tracking control [38,39], multi-robot formation tracking control [40,41] and other fields.

Based on the above analysis, the literature [24-41] all studied the finite time or fixed time control for a class of nonlinear systems, there is no specific research for high-order nonlinear large-scale systems, and the fixed-time adaptive control for nonlinear systems still remains an open issue. Therefore, this paper takes this as an innovation point to discuss a fixed-time fuzzy adaptive tracking control problem for high-order nonlinear large-scale systems. In the control design, the unknown nonlinearities are recognized by utilizing FLSs. Under the fixed-time Lyapunov stability theory, a fixed-time fuzzy adaptive decentralized control scheme is proposed to guarantee the tracking performance is well within a fixed time.

2. SYSTEM DESCRIPTION AND PRELIMINARY KNOWLEDGE

2.1. System description and assumption

Consider the following strict-feedback high-order nonlinear systems as

$$\begin{cases} \dot{x}_{i,1} = x_{i,2}^{p_{i,1}} + f_{i,1}(x_{i,1}) + \Delta_{i,1}(\bar{y}), \\ \dot{x}_{i,2} = x_{i,3}^{p_{i,2}} + f_{i,2}(\bar{x}_{i,2}) + \Delta_{i,2}(\bar{y}), \\ \vdots \\ \dot{x}_{i,m_i} = u_i^{p_{i,m_i}} + f_{i,m_i}(\bar{x}_{i,m_i}) + \Delta_{i,m_i}(\bar{y}), \\ y_i = x_{i,1}, \end{cases} \tag{1}$$

where $\bar{x}_{i,j_i} = [x_{i,1}, x_{i,2}, \ldots, x_{i,j_i}]^T$, $(i = 1, \ldots, N;\ j_i = 1, \ldots, m_i)$ are the system state vectors, $u_i \in R$ are the i-th subsystem control input, $y_i \in R$ are control output. $f_{i,j_i}(\bar{x}_{i,j_i})$ are the unknown nonlinear functions, $\Delta_{i,j_i}(\bar{y})$ $(\bar{y} = [y_1, \ldots, y_N]^T)$ are interconnected functions and exist in every subsystem, $p_{i,j_i} \geq 1$ are the ratio of a positive even integer to odd integer.

Remark 1: Compared with the current researches for large-scale systems, the system (1) is an extension of this class of nonlinear large-scale systems. The fixed-time control has been studied for nonlinear system, with the index of virtual controller input $p_{i,j_i} = 1$ in [26-31]. However, this paper considered fixed-time tracking control issue for high-order nonlinear systems. In that case, the control methods developed by the previous researches are no longer feasible.

Assumption 1 [19]**:** The nonlinear interconnected terms $\Delta_{i,j_i}(\bar{y})$ $(i = 1, \ldots, N;\ j_i = 1, \ldots, m_i)$ satisfy

$$|\Delta_{i,j_i}(\bar{y})| \leq \sum_{r=1}^{n_{i,j_i}} \sum_{\iota=1}^{N} q_{i,j_i,\iota}^{r} |y_\iota|^r, \tag{2}$$

where $q_{i,j_i,\iota}^{r}$ are unknown constants.

2.2. Preliminaries

The following several lemmas and definitions on fixed-time control are given for high-order nonlinear large-scale systems.

Definition 1 [42]**:** For the i-th subsystem $\dot{x} = f(x,t)$, the large-scale nonlinear systems are semiglobal practical fixed-time stable (SGPFS). If for any $x_i(t_0) = x_0$, there is a positive constant ε and a settling time $T(\varepsilon, x_0) < \infty$, such that $\|x_i(t)\| < \varepsilon$ and $x(t) = 0$ for all $t \geq t_0 + T$.

Lemma 1 [35]**:** There is a function $V(X) \geq 0$, the constants β_1, $\beta_2 > 0$, $0 < \kappa_1 < 1$, $\kappa_2 > 1$, and $\lambda > 0$, such that

$$\dot{V}(X) \leq -\beta_1 V^{\kappa_1}(X) - \beta_2 V^{\kappa_2}(X) + \lambda, \tag{3}$$

the system (1) is practically fixed-time stable and the settling time T is bounded as

$$T \leq T_{\max} = \frac{1}{\beta_1 \delta (1-\kappa_1)} + \frac{1}{\beta_2 \delta (\kappa_2 - 1)}, \tag{4}$$

where the constant $0 < \delta < 1$.

Remark 2: Finite-time control strategies of nonlinear systems have been presented in [43-45], which only can ensure the system states converge to a region of equilibrium point. Thus, this paper provides a fuzzy adaptive fixed-time control scheme of high-order nonlinear large-scale systems (1). By applying Lemma 1, we can design a fuzzy adaptive fixed-time controller, which reaches the expected tracking precision in fixed time without relying on initial states.

Lemma 2 [35]**:** For any constants $d_1, d_2, \phi > 0$, and $0 < d_3 < 1$, real variables A and B, the following inequalities hold

$$|A|^{d_1} |B|^{d_2} \leq \frac{d_1}{d_1+d_2} \phi |A|^{d_1+d_2} + \frac{d_2}{d_1+d_2} \phi^{-\frac{d_1}{d_2}} |B|^{d_1+d_2}, \tag{5}$$

$$\left| A^{d_3} - B^{d_3} \right| \leq 2^{1-d_3} |A-B|^{d_3}. \tag{6}$$

Lemma 3 [18]**:** The unknown nonlinearity $f(X)$ is continuous on a compact set Ω, for any constant $\varepsilon > 0$, there exists a FLS $\theta^{*T}\varphi(X)$ such as

$$\sup_{X \in \Omega} \left| f(X) - \theta^{*T} \varphi(X) \right| \leq \varepsilon, \tag{7}$$

where ε is the fuzzy minimum approximation error, $\theta^{*T} = [\theta_1^*, \theta_2^*, \ldots, \theta_l^*]^T$ is the weight vector, the number of fuzzy rules $l > 1$. $\varphi(x) = [\varphi_1(x), \varphi_2(x), \ldots, \varphi_l(x)]^T$, $\varphi_i(x)$ is the fuzzy basis function. The fuzzy basis functions and fuzzy membership functions are designed Gaussian functions.

Remark 3: It should be mentioned that since neural networks and Takagi-Sugeno fuzzy systems also have the property of Lemma 3, the FLSs used in this paper can be replaced by neural networks [10,11] or Takagi-Sugeno fuzzy systems [46-48].

3. FUZZY ADAPTIVE FIXED-TIME CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, combining adaptive backstepping design technique and adding a power integrator method, a fuzzy adaptive fixed-time stabilization controller will be developed. Further, the stability of the controlled system (1) will be proved by constructing the form of Lemma 1.

Before the adaptive backstepping design, for $i = 1, \ldots, N$; $j_i = 1, \ldots, m_i$, we define $\Theta^*_{i,j_i} = \left\| \theta^*_{i,j_i} \right\|^{1+\nu}$, $\tilde{\Theta}_{i,j_i} = \Theta^*_{i,j_i} - \hat{\Theta}_{i,j_i}$, and $\hat{\Theta}_{i,j_i}$ is the estimation of Θ^*_{i,j_i}. Then, introduce coordinate transformation as follows:

$$\begin{cases} e_{i,1} = x_{i,1} - y_{i,d}, \\ e_{i,j_i} = x_{i,j_i}^{\sigma_{i,j_i}} - \alpha_{i,j_i}^{\sigma_{i,j_i}}, \end{cases} \tag{8}$$

where e_{i,j_i} is the virtual error, $y_{i,d}$ is the reference signal, α_{i,j_i} is a virtual controller, it is designed later. For convenience, we can define $1 + \frac{p_{i,j_i}}{\sigma_{i,j_i+1}} = \frac{1}{\sigma_{i,j_i}} + \nu$, the design parameter $\sigma_{i,1} = 1$ and $\sigma_{i,j_i} > 1 (j_i = 2, \ldots, m_i)$, the constant ν satisfies $\nu \in (0, 1)$.

Step *i*,1: Choose the Lyapunov function as

$$V_1 = \sum_{i=1}^{N} \left(W_{i,1} + \frac{1}{2} \tilde{\Theta}_{i,1}^2 + \frac{1}{2} \tilde{\Psi}_{i,1}^2 \right), \tag{9}$$

where $\tilde{\Psi}_{i,1} = \Psi^*_{i,1} - \hat{\Psi}_{i,1}$, $\hat{\Psi}_{i,1}$ is the estimation of $\Psi^*_{i,1}$. We can define the positive-definite function $W_{i,1} = \int_{\alpha_{i,1}}^{x_{i,1}} (s^{\sigma_{i,1}} - \alpha_{i,1}^{\sigma_{i,1}})^{2-1/\sigma_{i,1}} ds$.

The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{N} \left[e_{i,1} \dot{e}_{i,1} - \tilde{\Theta}_{i,1} \dot{\hat{\Theta}}_{i,1} - \tilde{\Psi}_{i,1} \dot{\hat{\Psi}}_{i,1} \right] \\ &= \sum_{i=1}^{N} \left[e_{i,1} (x_{i,2}^{p_{i,1}} + f_{i,1}(x_{i,1}) + \Delta_{i,1}(\bar{y}) \right. \\ &\quad \left. - \dot{y}_{i,d}) - \tilde{\Theta}_{i,1} \dot{\hat{\Theta}}_{i,1} - \tilde{\Psi}_{i,1} \dot{\hat{\Psi}}_{i,1} \right]. \end{aligned} \tag{10}$$

Since $F_{i,1}(\chi_{i,1}) = f_{i,1}(x_{i,1}) - \dot{y}_{i,d}$ is completely unknown nonlinear function, a FLS is used to recognize $F_{i,1}(\chi_{i,1})$. One can get

$$F_{i,1}(\chi_{i,1}) = \theta_{i,1}^{*T} \varphi_{i,1}(\chi_{i,1}) + \varepsilon_{i,1}(\chi_{i,1}), \tag{11}$$

where $|\varepsilon_{i,1}(\chi_{i,1})| \leq \varepsilon^*_{i,1}$, $\varepsilon^*_{i,1}$ is a positive constant, $\chi_{i,1} = [x_{i,1}, y_{i,d}, \dot{y}_{i,d}]^T$.

By using Young's inequality, we can obtain

$$\begin{aligned} &\sum_{i=1}^{N} e_{i,1} F_{i,1}(\chi_{i,1}) \\ &= \sum_{i=1}^{N} e_{i,1} (\theta_{i,1}^{*T} \varphi_{i,1}(\chi_{i,1}) + \varepsilon_{i,1}(\chi_{i,1})) \\ &\leq \sum_{i=1}^{N} \left[\frac{1}{1+\nu} a_{i,1}^{1+\nu} e_{i,1}^{1+\nu} \left\| \theta^*_{i,1} \right\|^{1+\nu} \left\| \varphi_{i,1}(\chi_{i,1}) \right\|^{1+\nu} \right. \\ &\quad + \frac{\nu}{1+\nu} a_{i,1}^{-(1+\nu)/\nu} + \frac{1}{1+\nu} b_{i,1}^{1+\nu} e_{i,1}^{1+\nu} \\ &\quad \left. + \frac{\nu}{1+\nu} b_{i,1}^{-(1+\nu)/\nu} (\varepsilon^*_{i,1})^{(1+\nu)/\nu} \right] \\ &\leq \sum_{i=1}^{N} \left[e_{i,1}^{1+\nu} (a_{i,1}^{1+\nu} \Theta^*_{i,1} \| \varphi_{i,1}(\chi_{i,1}) \|^{1+\nu} + b_{i,1}^{1+\nu}) + \omega_{i,1} \right], \end{aligned} \tag{12}$$

where $a_{i,1}$, $b_{i,1}$ are positive design parameters, $\Theta^*_{i,1} = \left\|\theta^*_{i,1}\right\|^{1+\nu}$ and $\omega_{i,1} = a_{i,1}^{-(1+\nu)/\nu} + b_{i,1}^{-(1+\nu)/\nu}(\varepsilon^*_{i,1})^{(1+\nu)/\nu}$.

Furthermore, invoking Assumption 1, Young's inequality and Cauchy-Schwartz inequality yields

$$\begin{aligned}
&\sum_{i=1}^{N} e_{i,1}\Delta_{i,1}(\bar{y}) \\
&\le \sum_{i=1}^{N}\left\{\frac{1}{2d_{i,1}} + \frac{d_{i,1}}{2}(e_{i,1}\Delta_{i,1}(\bar{y}))^2\right\} \\
&\le \sum_{i=1}^{N}\left\{\frac{1}{2d_{i,1}} + \frac{d_{i,1}}{2}e_{i,1}^2\left(\sum_{r=1}^{k}\sum_{\iota=1}^{N} q^r_{i,1,\iota}|y_\iota|^r\right)^2\right\} \\
&\le \sum_{i=1}^{N}\frac{1}{2d_{i,1}} + \sum_{i=1}^{N}\sum_{r=1}^{k}\sum_{\iota=1}^{N}\frac{d_{i,1}}{2}e_{i,1}^2(q^r_{i,1,\iota}|y_\iota|^r)^2 kN,
\end{aligned} \tag{13}$$

where $d_{i,1}$ is the design parameter. $k = \max\{n_{i,j_i} | 1 \le i \le N, 1 \le j_i \le m_i\}$ is known.

From the inequality $(\sum_{i=1}^{2} x_i)^{2r} \le 2^{2r}\sum_{i=1}^{2} x_i^{2r}$, the above formula can be transformed into

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{r=1}^{k}\sum_{\iota=1}^{N}\frac{d_{i,1}}{2}e_{i,1}^2(q^r_{i,1,\iota}|y_\iota|^r)^2 kN \\
&\le kN\sum_{i=1}^{N}\sum_{r=1}^{k}\sum_{\iota=1}^{N}\frac{d_{\iota,1}}{2}e_{i,1}^2 q^{2r}_{\iota,1,i}(e_{i,1}^{2r} + y_{i,d}^{2r}) \times 2^{2r} \\
&\le \sum_{i=1}^{N}\sum_{r=1}^{k} e_{i,1}^2(e_{i,1}^{2r} + y_{i,d}^{2r})q_{1,i,r},
\end{aligned} \tag{14}$$

where $q_{1,i,r} = 2^{2r-1} \times kNd_{\iota,1}\sum_{\iota=1}^{N} q^{2r}_{\iota,1,i}$.

Recalling (13) and (14) leads to

$$\sum_{i=1}^{N} e_{i,1}\Delta_{i,1}(\bar{y}) \le \sum_{i=1}^{N}\frac{1}{2d_{i,1}} + \sum_{i=1}^{N}\sum_{r=1}^{k} e_{i,1}^2(e_{i,1}^{2r} + y_{i,d}^{2r})q_{1,i,r}. \tag{15}$$

According to (12) and (15), (10) can be rewritten as

$$\begin{aligned}
\dot{V}_1 \le \sum_{i=1}^{N}\Bigg[& e_{i,1}(x_{i,2}^{p_{i,1}} - \alpha_{i,2}^{p_{i,1}}) \\
&+ e_{i,1}^{1+\nu}(a_{i,1}^{1+\nu}\Theta^*_{i,1}\|\varphi_{i,1}(\chi_{i,1})\|^{1+\nu} + b_{i,1}^{1+\nu}) \\
&+ e_{i,1}\alpha_{i,2}^{p_{i,1}} + \sum_{r=1}^{k} e_{i,1}^2(e_{i,1}^{2r} + y_{i,d}^{2r})\Psi^*_{i,1} + \frac{1}{2d_{i,1}} \\
&+ \omega_{i,1} - \tilde{\Theta}_{i,1}\dot{\hat{\Theta}}_{i,1} - \tilde{\Psi}_{i,1}\dot{\hat{\Psi}}_{i,1}\Bigg],
\end{aligned} \tag{16}$$

where $\Psi^*_{i,1} = \max_{1\le r\le k} q_{1,i,r}$.

Design the virtual controller $\alpha_{i,2}$, the adaptive laws $\dot{\hat{\Theta}}_{i,1}$ and $\dot{\hat{\Psi}}_{i,1}$ as

$$\begin{aligned}
\alpha_{i,2} = &-e_{i,1}^{1/\sigma_{i,2}}(c_{i,1} + c'_{i,1}e_{i,1}^{2+\nu} + a_{i,1}^{1+\nu}\hat{\Theta}_{i,1} \\
&\times \|\varphi_{i,1}(h_{i,1})\|^{1+\nu} + b_{i,1}^{1+\nu} + D_{i,1})^{1/p_{i,1}},
\end{aligned} \tag{17}$$

$$\dot{\hat{\Theta}}_{i,1} = e_{i,1}^{1+\nu}a_{i,1}^{1+\nu}\|\varphi_{i,1}(h_{i,1})\|^{1+\nu} - \gamma_{i,1}\hat{\Theta}_{i,1}, \tag{18}$$

$$\dot{\hat{\Psi}}_{i,1} = \sum_{r=1}^{k} e_{i,1}^2(e_{i,1}^{2r} + y_{i,d}^{2r}) - \gamma'_{i,1}\hat{\Psi}_{i,1}, \tag{19}$$

where $D_{i,1} = e_{i,1}^{1-p_{i,1}/\sigma_{i,2}}\hat{\Psi}_{i.1}\sum_{r=1}^{k}(e_{i,1}^{2r} + y_{i,d}^{2r})$, the design parameters $c_{i,1}$, $c'_{i,1}$, $\gamma_{i,1}$ and $\gamma'_{i,1}$ are positive constants.

Substituting (17)-(19) into (16) yields

$$\begin{aligned}
\dot{V}_1 \le \sum_{i=1}^{N}[&-c_{i,1}e_{i,1}^{1+\nu} - c'_{i,1}e_{i,1}^{3+2\nu} + e_{i,1}(x_{i,2}^{p_{i,1}} - \alpha_{i,2}^{p_{i,1}}) \\
&+ \gamma_{i,1}\tilde{\Theta}_{i,1}\hat{\Theta}_{i,1} + \gamma'_{i,1}\tilde{\Psi}_{i,1}\hat{\Psi}_{i,1} + \bar{\omega}_{i,1}],
\end{aligned} \tag{20}$$

where $\bar{\omega}_{i,1} = \omega_{i,1} + \frac{1}{2d_{i,1}}$.

Step i, j_i $(j_i = 2, \ldots, m_i - 1)$**:** Choose the Lyapunov function as

$$V_{j_i} = V_{j_i-1} + \sum_{i=1}^{N}(W_{i,j_i} + \frac{1}{2}\tilde{\Theta}^2_{i,j_i} + \frac{1}{2}\tilde{\Psi}^2_{i,j_i}), \tag{21}$$

where $\tilde{\Psi}_{i,j_i} = \Psi^*_{i,j_i} - \hat{\Psi}_{i,j_i}$, $\hat{\Psi}_{i,j_i}$ is the estimation of Ψ^*_{i,j_i}, we can define $W_{i,j_i} = \int_{\alpha_{i,j_i}}^{x_{i,j_i}}(s^{\sigma_{i,j_i}} - \alpha_{i,j_i}^{\sigma_{i,j_i}})^{2-1/\sigma_{i,j_i}}ds$.

The time derivative of V_{j_i} is

$$\begin{aligned}
\dot{V}_{j_i} = \dot{V}_{j_i-1} + \sum_{i=1}^{N}\Bigg[&\frac{\partial W_{i,j_i}}{\partial x_{i,j_i}}\dot{x}_{i,j_i} \\
&+ \sum_{l=1}^{j_i-1}\left(\frac{\partial W_{i,j_i}}{\partial x_{i,l}}\dot{x}_{i,l} + \frac{\partial W_{i,j_i}}{\partial \hat{\Theta}_{i,l}}\dot{\hat{\Theta}}_{i,l} + \frac{\partial W_{i,j_i}}{\partial \hat{\Psi}_{i,l}}\dot{\hat{\Psi}}_{i,l}\right) \\
&- \tilde{\Theta}_{i,j_i}\dot{\hat{\Theta}}_{i,j_i} - \tilde{\Psi}_{i,j_i}\dot{\hat{\Psi}}_{i,j_i}\Bigg].
\end{aligned} \tag{22}$$

According to $W_{i,j_i} = \int_{\alpha_{i.j_i}}^{x_{i.j_i}}(s^{\sigma_{i.j_i}} - \alpha_{i,j_i}^{\sigma_{i.j_i}})^{2-1/\sigma_{i.j_i}}ds$ and Lemma 2, one has

$$\begin{aligned}
&|\partial W_{i,j_i}/\partial x_{i,l}\dot{x}_{i,l}| \\
&\le (2 - 1/\sigma_{i.j_i})|(x_{i.j_i}^{\sigma_{i.j_i}})^{1/\sigma_{i.j_i}} - (\alpha_{i.j_i}^{\sigma_{i.j_i}})^{1/\sigma_{i.j_i}}| \\
&\quad \times |(x_{i.j_i}^{\sigma_{i.j_i}} - \alpha_{i.j_i}^{\sigma_{i.j_i}})|^{1-1/\sigma_{i.j_i}}|\partial\alpha_{i.j_i}^{\sigma_{i.j_i}}/\partial x_{i,l}\dot{x}_{i,l}| \\
&\le (2 - 1/\sigma_{i.j_i})2^{1-1/\sigma_{i.j_i}} \\
&\quad \times \left|e_{i.j_i}^{1/\sigma_{i.j_i}}\right|\left|e_{i.j_i}^{1-1/\sigma_{i.j_i}}\right|\left|\partial\alpha_{i.j_i}^{\sigma_{i.j_i}}/\partial x_{i,l}\dot{x}_{i,l}\right| \\
&\le \psi_{i.j_i}|e_{i.j_i}||\partial\alpha_{i.j_i}^{\sigma_{i.j_i}}/\partial x_{i,l}\dot{x}_{i,l}|,
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\partial W_{i,j_i}/\partial x_{i,j_i}\dot{x}_{i,j_i} \\
&= (x_{i.j_i}^{\sigma_{i.j_i}} - \alpha_{i.j_i}^{\sigma_{i.j_i}})^{2-1/\sigma_{i.j_i}}\dot{x}_{i,j_i} \\
&= e_{i.j_i}^{2-1/\sigma_{i.j_i}}[x_{i,j_i+1}^{p_{i,j_i}} + f_{i,j_i}(\hat{x}_{i,j_i}) + \Delta_{i,j_i}(\bar{y})],
\end{aligned} \tag{24}$$

where $\psi_{i.j_i} \ge 2^{1-1/\sigma_{i.j_i}}(2 - 1/\sigma_{i.j_i})$. Similarly to the previous step (23), one gets

$$|\partial W_{i,j_i}/\partial\hat{h}_{i,l}\dot{\hat{h}}_{i,l}| \le \psi_{i.j_i}|e_{i.j_i}|\left|\partial\alpha_{i.j_i}^{\sigma_{i.j_i}}/\partial\hat{h}_{i,l}\dot{\hat{h}}_{i,l}\right|, \tag{25}$$

where $\hat{h}_{i,l} = [\hat{\Theta}_{i,l}, \hat{\Psi}_{i,l}]^T$ and $\dot{\hat{h}}_{i,l} = [\dot{\hat{\Theta}}_{i,l}, \dot{\hat{\Psi}}_{i,l}]^T$.

Substituting (23)-(25) into (22) yields

$$\begin{aligned}\dot{V}_{j_i} \leq & \dot{V}_{j_i-1} + \sum_{i=1}^{N} [\psi_{i.j_i} |e_{i.j_i}| \sum_{l=1}^{j_i-1} (\left|\partial \alpha_{i.j_i}^{\sigma_{i.j_i}} / \partial x_{i,l} \dot{x}_{i,l}\right| \\ & + \left|\partial \alpha_{i.j_i}^{\sigma_{i.j_i}} / \partial \hat{\Theta}_{i,l} \dot{\hat{\Theta}}_{i,l}\right| + \left|\partial \alpha_{i.j_i}^{\sigma_{i.j_i}} / \partial \hat{\Psi}_{i,l} \dot{\hat{\Psi}}_{i,l}\right|) \\ & + e_{i.j_i}^{2-1/\sigma_{i.j_i}} (x_{i,j_i+1}^{p_{i,j_i}} - \alpha_{i,j_i+1}^{p_{i,j_i}} + f_{i,j_i}(\hat{x}_{i,j_i}) + \Delta_{i,j_i}(\bar{y})) \\ & + e_{i.j_i}^{2-1/\sigma_{i.j_i}} \alpha_{i,j_i+1}^{p_{i,j_i}} - \tilde{\Theta}_{i,j_i} \dot{\hat{\Theta}}_{i,j_i} - \tilde{\Psi}_{i,j_i} \dot{\hat{\Psi}}_{i,j_i}]. \quad (26)\end{aligned}$$

Utilizing the inequailty (5), it follows that

$$\begin{aligned}& e_{i.j_i-1}^{2-1/\sigma_{i.j_i}} (x_{i,j_i}^{p_{i,j_i-1}} - \alpha_{i,j_i}^{p_{i,j_i-1}}) \\ & = e_{i.j_i-1}^{2-1/\sigma_{i.j_i}} [(x_{i.j_i}^{\sigma_{i.j_i}})^{p_{i,j_i-1}/\sigma_{i.j_i}} - (\alpha_{i.j_i}^{\sigma_{i.j_i}})^{p_{i,j_i-1}/\sigma_{i.j_i}}] \\ & \leq e_{i.j_i-1}^{1+\nu} + \varsigma_{i,j_i} e_{i.j_i}^{1+\nu}, \quad (27)\end{aligned}$$

where $\varsigma_{i,j_i} = \nu/(1+\nu) \times (2^{1-p_{i,j_i-1}/\sigma_{i.j_i}})^{1+\nu/\nu}$.

From (27), it follows that

$$\begin{aligned}\dot{V}_{j_i} \leq & \sum_{i=1}^{N} \{-\sum_{l=1}^{j_i-1} (c_{i,l} - j_i + 1) e_{i,l}^{1+\nu} - \sum_{l=1}^{j_i-1} c'_{i,l} e_{i,l}^{3+2\nu} \\ & + e_{i.j_i}^{2-1/\sigma_{i.j_i}} \alpha_{i,j_i+1}^{p_{i,j_i}} + e_{i.j_i}^{2-1/\sigma_{i.j_i}} (x_{i,j_i+1}^{p_{i,j_i}} - \alpha_{i,j_i+1}^{p_{i,j_i}}) \\ & + |e_{i.j_i}| \Bigg[\varsigma_{i,j_i} e_{i.j_i}^{\nu} + e_{i.j_i}^{2-1/\sigma_{i.j_i}} f_{i,j_i}(\hat{x}_{i,j_i}) \\ & + \psi_{i.j_i} \sum_{l=1}^{j_i-1} \left(\left| \frac{\partial \alpha_{i.j_i}^{\sigma_{i.j_i}}}{\partial x_{i,l}} \dot{x}_{i,l} \right| + \left| \frac{\partial \alpha_{i.j_i}^{\sigma_{i.j_i}}}{\partial \hat{\Theta}_{i,l}} \dot{\hat{\Theta}}_{i,l} \right| \right. \\ & \left. + \left| \frac{\partial \alpha_{i.j_i}^{\sigma_{i.j_i}}}{\partial \hat{\Psi}_{i,l}} \dot{\hat{\Psi}}_{i,l} \right| \right) \Bigg] + e_{i.j_i}^{2-1/\sigma_{i.j_i}} \Delta_{i,j_i}(\bar{y}) \\ & + \sum_{l=1}^{j_i-1} (\gamma_{i,l} \tilde{\Theta}_{i,l} \hat{\Theta}_{i,l} + \gamma'_{i,l} \tilde{\Psi}_{i,l} \hat{\Psi}_{i,l} + \bar{\omega}_{i,l}) \\ & - \tilde{\Theta}_{i,j_i} \dot{\hat{\Theta}}_{i,j_i} - \tilde{\Psi}_{i,j_i} \dot{\hat{\Psi}}_{i,j_i} \}. \quad (28)\end{aligned}$$

Define the unknown nonlinear function $F_{i,j_i}(\chi_{i,j_i}) = \varsigma_{i,j_i} e_{i.j_i}^{\nu} + e_{i.j_i}^{2-1/\sigma_{i.j_i}} f_{i,j_i}(\hat{x}_{i,j_i}) + \psi_{i.j_i} \sum_{l=1}^{j_i-1} \left(\left| \frac{\partial \alpha_{i.j_i}^{\sigma_{i.j_i}}}{\partial x_{i,l}} \dot{x}_{i,l} \right| + \left| \frac{\partial \alpha_{i.j_i}^{\sigma_{i.j_i}}}{\partial \hat{h}_{i,l}} \dot{\hat{h}}_{i,l} \right| \right)$, a FLS can be utilized to identify the function $F_{i,j_i}(\chi_{i,j_i})$ and assumes that

$$F_{i,j_i}(\chi_{i,j_i}) = \theta_{i,j_i}^{*T} \varphi_{i,j_i}(\chi_{i,j_i}) + \varepsilon_{i,j_i}(\chi_{i,j_i}), \quad (29)$$

where $|\varepsilon_{i,j_i}(\chi_{i,j_i})| \leq \varepsilon_{i,j_i}^*$ with ε_{i,j_i}^* is a positive constant, $\chi_{i,j_i} = [\bar{x}_{i,j_i}, \bar{\hat{\Theta}}_{i,j_i-1}, \bar{\hat{\Psi}}_{i,j_i-1}]^T$ with $\bar{\hat{\Theta}}_{i,j_i-1} = [\hat{\Theta}_{i,1}, \ldots, \hat{\Theta}_{i,j_i-1}]^T$, $\bar{\hat{\Psi}}_{i,j_i-1} = [\hat{\Psi}_{i,1}, \ldots, \hat{\Psi}_{i,j_i-1}]^T$.

By using Young's inequality, we can get

$$\begin{aligned}& \sum_{i=1}^{N} |e_{i,j_i}| F_{i,j_i}(\chi_{i,j_i}) \\ & = \sum_{i=1}^{N} |e_{i,j_i}| (\theta_{i,j_i}^{*T} \varphi_{i,j_i}(\chi_{i,j_i}) + \varepsilon_{i,j_i}(\chi_{i,j_i})) \\ & \leq \sum_{i=1}^{N} \Bigg[\frac{1}{1+\nu} a_{i,j_i}^{1+\nu} e_{i,j_i}^{1+\nu} \left\|\theta_{i,j_i}^*\right\|^{1+\nu} \|\varphi_{i,j_i}(\chi_{i,j_i})\|^{1+\nu} \\ & + \frac{\nu}{1+\nu} a_{i,j_i}^{-(1+\nu)/\nu} + \frac{1}{1+\nu} b_{i,j_i}^{1+\nu} e_{i,j_i}^{1+\nu} \\ & + \frac{\nu}{1+\nu} b_{i,j_i}^{-(1+\nu)/\nu} (\varepsilon_{i,j_i}^*)^{(1+\nu)/\nu} \Bigg] \\ & \leq \sum_{i=1}^{N} [e_{i,j_i}^{1+\nu} (a_{i,j_i}^{1+\nu} \Theta_{i,j_i}^* \|\varphi_{i,j_i}(\chi_{i,j_i})\|^{1+\nu} + b_{i,j_i}^{1+\nu}) \\ & + \omega_{i,j_i}], \quad (30)\end{aligned}$$

where a_{i,j_i} and b_{i,j_i} are positive design parameters, $\Theta_{i,j_i}^* = \left\|\theta_{i,j_i}^*\right\|^{1+\nu}$, and $\omega_{i,j_i} = a_{i,j_i}^{-(1+\nu)/\nu} + b_{i,j_i}^{-(1+\nu)/\nu} (\varepsilon_{i,j_i}^*)^{(1+\nu)/\nu}$.

Similar to (15)

$$\begin{aligned}& \sum_{i=1}^{N} e_{i.j_i}^{2-1/\sigma_{i.j_i}} \Delta_{i,j_i}(\bar{y}) \\ & \leq \sum_{i=1}^{N} \frac{1}{2d_{i,j_i}} + \sum_{i=1}^{N} \sum_{r=1}^{k} e_{i,j_i}^{4-2/\sigma_{i.j_i}} (e_{i,j_i}^{2r} + y_{i,d}^{2r}) q_{j_i,i,r}, \quad (31)\end{aligned}$$

where $q_{j_i,i,r} = 2^{2r-1} \times kNd_{\iota,1} \sum_{\iota=1}^{N} q_{\iota,j_i,i}^{2r}$.

According to (30) and (31), (28) can be rewritten as

$$\begin{aligned}\dot{V}_{j_i} \leq & \sum_{i=1}^{N} \Bigg[-\sum_{l=1}^{j_i-1} (c_{i,l} - j_i + 1) e_{i,l}^{1+\nu} - \sum_{l=1}^{j_i-1} c'_{i,l} e_{i,l}^{3+2\nu} \\ & + e_{i.j_i}^{2-1/\sigma_{i.j_i}} (x_{i,j_i+1}^{p_{i,j_i}} - \alpha_{i,j_i+1}^{p_{i,j_i}}) + e_{i.j_i}^{2-1/\sigma_{i.j_i}} \alpha_{i,j_i+1}^{p_{i,j_i}} \\ & + e_{i,j_i}^{1+\nu} (a_{i,j_i}^{1+\nu} \Theta_{i,j_i}^* \|\varphi_{i,j_i}(\chi_{i,j_i})\|^{1+\nu} + b_{i,j_i}^{1+\nu}) \\ & + \sum_{r=1}^{k} e_{i,j_i}^{4-2/\sigma_{i.j_i}} \Psi_{i,j_i}^* (e_{i,j_i}^{2r} + y_{i,d}^{2r}) + \omega_{i,j_i} + \frac{1}{2d_{i,j_i}} \\ & + \sum_{l=1}^{j_i-1} (\gamma_{i,l} \tilde{\Theta}_{i,l} \hat{\Theta}_{i,l} + \gamma'_{i,l} \tilde{\Psi}_{i,l} \hat{\Psi}_{i,l} + \bar{\omega}_{i,l}) \\ & - \tilde{\Theta}_{i,j_i} \dot{\hat{\Theta}}_{i,j_i} - \tilde{\Psi}_{i,j_i} \dot{\hat{\Psi}}_{i,j_i} \Bigg], \quad (32)\end{aligned}$$

where $\Psi_{i,j_i}^* = \max_{1 \leq r \leq k} q_{j_i,i,r}$.

Design the virtual controller α_{i,j_i+1}, the adaptive laws $\dot{\hat{\Theta}}_{i,j_i}$ and $\dot{\hat{\Psi}}_{i,j_i}$ as

$$\begin{aligned}\alpha_{i,j_i+1} = & -e_{i,j_i}^{1/\sigma_{i,j_i+1}} [(c_{i,j_i} - j_i + 1) + c'_{i,j_i} e_{i,j_i}^{2+\nu} + b_{i,j_i}^{1+\nu} \\ & + a_{i,j_i}^{1+\nu} \hat{\Theta}_{i,j_i} \|\varphi_{i,j_i}(\chi_{i,j_i})\|^{1+\nu} + D_{i,j_i}]^{1/p_{i,j_i}}, \quad (33)\end{aligned}$$

$$\dot{\hat{\Theta}}_{i,j_i} = e_{i,j_i}^{1+\nu} a_{i,j_i}^{1+\nu} \|\varphi_{i,j_i}(\chi_{i,j_i})\|^{1+\nu} - \gamma_{i,j_i} \hat{\Theta}_{i,j_i}, \quad (34)$$

$$\dot{\hat{\Psi}}_{i,j_i} = \sum_{r=1}^{k} e_{i,j_i}^{4-2/\sigma_{i.j_i}} (e_{i,j_i}^{2r} + y_{i,d}^{2r}) - \gamma'_{i,j_i} \hat{\Psi}_{i,j_i}, \quad (35)$$

where $D_{i,j_i} = e_{i,j_i}^{\pi_{i,j_i}} \hat{\Psi}_{i,j_i} \sum_{r=1}^{k} (e_{i,j_i}^{2r} + y_{i,d}^{2r})$ with $\pi_{i,j_i} = 2 -$

$\left(\frac{\sigma_{i,j_i+1}+p_{i,j_i}\sigma_{i,j_i}}{\sigma_{i,j_i}\sigma_{i,j_i+1}}\right)$, the design parameters c_{i,j_i}, c'_{i,j_i}, γ_{i,j_i}, and γ'_{i,j_i} are positive constants.

Substituting (33)-(35) into (32) yields

$$\dot{V}_{j_i} \leq \sum_{i=1}^{N}\Bigg[-\sum_{l=1}^{j_i}(c_{i,l}-j_i+1)e_{i,l}^{1+\nu} - \sum_{l=1}^{j_i}c'_{i,l}e_{i,l}^{3+2\nu} + e_{i.j_i}^{2-1/\sigma_{i.j_i}}(x_{i,j_i+1}^{p_{i,j_i}} - \alpha_{i,j_i+1}^{p_{i,j_i}}) + \sum_{l=1}^{j_i}(\gamma_{i,l}\tilde{\Theta}_{i,l}\hat{\Theta}_{i,l} + \gamma'_{i,l}\tilde{\Psi}_{i,l}\hat{\Psi}_{i,l} + \bar{\omega}_{i,l})\Bigg], \tag{36}$$

where $\bar{\omega}_{i,l} = \omega_{i,l} + \frac{1}{2d_{i,l}}$.

Step i, m_i: Consider the Lyapunov function as

$$V_{m_i} = V_{m_i-1} + \sum_{i=1}^{N}(W_{i,m_i} + \frac{1}{2}\tilde{\Theta}_{i,m_i}^2 + \frac{1}{2}\tilde{\Psi}_{i,m_i}^2), \tag{37}$$

where $\tilde{\Psi}_{i,j_i} = \Psi_{i,j_i}^* - \hat{\Psi}_{i,j_i}$, $\hat{\Psi}_{i,j_i}$ is the estimation of Ψ_{i,j_i}^*, we can define $W_{i,m_i} = \int_{\alpha_{i,m_i}}^{x_{i,m_i}}(s^{\sigma_{i,m_i}} - \alpha_{i.m_i}^{\sigma_{i,m_i}})^{2-1/\sigma_{i,m_i}}ds$.

The time derivative of V_{m_i} is

$$\begin{aligned}\dot{V}_{j_i} \leq \sum_{i=1}^{N}\Bigg[&-\sum_{l=1}^{m_i-1}(c_{i,l}-m_i+1)e_{i,l}^{1+\nu} - \sum_{l=1}^{m_i-1}c'_{i,l}e_{i,l}^{3+2\nu} \\ &+ e_{i,m_i}^{1+\nu}(a_{i,m_i}^{1+\nu}\Theta_{i,m_i}^*\|\varphi_{i,m_i}(\chi_{i,m_i})\|^{1+\nu} + b_{i,m_i}^{1+\nu}) \\ &+ e_{i.m_i}^{2-1/\sigma_{i.m_i}}u_i^{p_{i,m_i}} + \sum_{r=1}^{k}e_{i,m_i}^{4-2/\sigma_{i,m_i}}\Psi_{i,m_i}^*(e_{i,m_i}^{2r} + y_{i,d}^{2r}) \\ &+ \omega_{i,m_i} + \frac{1}{2d_{i,m_i}} - \tilde{\Theta}_{i,m_i}\dot{\hat{\Theta}}_{i,m_i} - \tilde{\Psi}_{i,m_i}\dot{\hat{\Psi}}_{i,m_i}\Bigg] \\ &+ \sum_{l=1}^{m_i-1}(\gamma_{i,l}\tilde{\Theta}_{i,l}\hat{\Theta}_{i,l} + \gamma'_{i,l}\tilde{\Psi}_{i,l}\hat{\Psi}_{i,l} + \bar{\omega}_{i,l}),\end{aligned} \tag{38}$$

where

$$\begin{aligned}\Theta_{i,m_i}^* &= \left\|\theta_{i,m_i}^*\right\|^{1+\nu}, \\ \omega_{i,m_i} &= a_{i,m_i}^{-(1+\nu)/\nu} + b_{i,m_i}^{-(1+\nu)/\nu}(\varepsilon_{i,m_i}^*)^{(1+\nu)/\nu}, \\ \Psi_{i,m_i}^* &= \max_{1\leq r\leq k} q_{m_i,i,r}.\end{aligned}$$

Design the virtual controller u_i, the adaptive laws $\dot{\hat{\Theta}}_{i,m_i}$ and $\dot{\hat{\Psi}}_{i,m_i}$ as

$$\begin{aligned}u_i = &-e_{i,m_i}^{1/\sigma_{i,m_i+1}}[(c_{i,m_i} - m_i + 1) + c'_{i,m_i}e_{i,m_i}^{2+\nu} + b_{i,m_i}^{1+\nu} \\ &+ a_{i,m_i}^{1+\nu}\hat{\Theta}_{i,m_i}\|\varphi_{i,m_i}(h_{i,m_i})\|^{1+\nu} + D_{i,m_i}]^{1/p_{i,m_i}},\end{aligned} \tag{39}$$

$$\dot{\hat{\Theta}}_{i,m_i} = e_{i,m_i}^{1+\nu}a_{i,m_i}^{1+\nu}\|\varphi_{i,m_i}(h_{i,m_i})\|^{1+\nu} - \gamma_{i,m_i}\hat{\Theta}_{i,m_i}, \tag{40}$$

$$\dot{\hat{\Psi}}_{i,m_i} = \sum_{r=1}^{k}e_{i,m_i}^{4-2/\sigma_{i,m_i}}(e_{i,1}^{2r} + y_{i,d}^{2r}) - \gamma'_{i,m_i}\hat{\Psi}_{i,m_i}, \tag{41}$$

where $D_{i,m_i} = e_{i,m_i}^{\pi_{i,m_i}}\hat{\Psi}_{i,m_i}\sum_{r=1}^{k}(e_{i,m_i}^{2r} + y_{i,d}^{2r})$ with $\pi_{i,m_i} = 2 - \left(\frac{\sigma_{i,m_i+1}+p_{i,j_i}\sigma_{i,m_i}}{\sigma_{i,m_i}\sigma_{i,m_i+1}}\right)$, the design parameters c_{i,m_i}, c'_{i,m_i}, γ_{i,m_i}, and γ'_{i,m_i} are positive constants.

Substituting (39)-(41) into (38) yields

$$\dot{V}_{m_i} \leq \sum_{i=1}^{N}\Bigg[-\sum_{l=1}^{m_i}(c_{i,l}-m_i+1)e_{i,l}^{1+\nu} - \sum_{l=1}^{m_i}c'_{i,l}e_{i,l}^{3+2\nu} + \sum_{l=1}^{m_i}(\gamma_{i,l}\tilde{\Theta}_{i,l}\hat{\Theta}_{i,l} + \gamma'_{i,l}\tilde{\Psi}_{i,l}\hat{\Psi}_{i,l} + \bar{\omega}_{i,l})\Bigg], \tag{42}$$

where $\bar{\omega}_{i,l} = \omega_{i,l} + \frac{1}{2d_{i,l}}$.

Furthermore, we can recapitulate the controller designed above as the following theorem.

Theorem 1: For high-order nonlinear large-scale systems (1), under Assumption 1, the controller (39), intermediate control functions (17) and (33), adaptive laws (18), (19), (34), (35), (40), and (41), can ensure that the system (1) is SPFTS, and tracking errors convergence to a small neighborhood of the origin within fixed time.

Proof: It follows from the definitions of $\tilde{\Theta}_{i,l}$, $\tilde{\Psi}_{i,l}$ $(l = 1, \ldots, m_i)$ that

$$\gamma_{i,l}\tilde{\Theta}_{i,l}\hat{\Theta}_{i,l} \leq -\frac{\gamma_{i,l}}{2}\tilde{\Theta}_{i,l}^2 + \frac{\gamma_{i,l}}{2}(\Theta_{i,l}^*)^2, \tag{43}$$

$$\gamma'_{i,l}\tilde{\Psi}_{i,l}\hat{\Psi}_{i,l} \leq -\frac{\gamma'_{i,l}}{2}\tilde{\Psi}_{i,l}^2 + \frac{\gamma'_{i,l}}{2}(\Psi_{i,l}^*)^2. \tag{44}$$

In view of (43) and (44), one obtains

$$\begin{aligned}\dot{V}_{m_i} \leq \sum_{i=1}^{N}\Bigg[&-\sum_{l=1}^{m_i}(c_{i,l}-m_i+1)e_{i,l}^{1+\nu} - \sum_{l=1}^{m_i}c'_{i,l}e_{i,l}^{3+2\nu} \\ &- \sum_{l=1}^{m_i}(\frac{\gamma_{i,l}}{2}\tilde{\Theta}_{i,l}^2 + \frac{\gamma'_{i,l}}{2}\tilde{\Psi}_{i,l}^2) + \sum_{l=1}^{m_i}(\frac{\gamma_{i,l}}{2}(\Theta_{i,l}^*)^2 \\ &+ \frac{\gamma'_{i,l}}{2}(\Psi_{i,l}^*)^2 + \bar{\omega}_{i,l})\Bigg].\end{aligned} \tag{45}$$

According to the definition of W_{i,j_i} $(j_i = 1, \ldots, m_i)$ and the inequality (5), it holds

$$\begin{aligned}W_{i,j_i} &= \int_{\alpha_{i,j_i}}^{x_{i,j_i}}(s^{\sigma_{i,j_i}} - \alpha_{i,j_i}^{\sigma_{i,j_i}})^{2-1/\sigma_{i,j_i}}ds \\ &\leq |x_{i,j_i} - \alpha_{i,j_i}||e_{i,j_i}|^{2-1/\sigma_{i,j_i}} \\ &\leq 2|e_{i,j_i}|^2.\end{aligned} \tag{46}$$

Invoking (45) and (46) results in

$$\begin{aligned}\dot{V}_{m_i} \leq \sum_{i=1}^{N}\Bigg[&-\sum_{l=1}^{m_i}(c_{i,l}-m_i+1)(e_{i,l}^2)^{k_1} - \sum_{l=1}^{m_i}c'_{i,l}(e_{i,l}^2)^{k_2} \\ &- \sum_{l=1}^{m_i}\left\{\gamma_{i,l}\left(\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_1} + \gamma'_{i,l}\left(\frac{\tilde{\Psi}_{i,l}^2}{4}\right)^{k_1}\right\} \\ &- \sum_{l=1}^{m_i}\left\{\gamma_{i,l}\left(\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_2} + \gamma'_{i,l}\left(\frac{\tilde{\Psi}_{i,l}^2}{4}\right)^{k_2}\right\} \\ &+ \sum_{l=1}^{m_i}\left\{\gamma_{i,l}(\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_1} + \gamma'_{i,l}\left(\frac{\tilde{\Psi}_{i,l}^2}{4}\right)^{k_1}\right\} \\ &+ \sum_{l=1}^{m_i}\left\{\gamma_{i,l}\left(\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_2} + \gamma'_{i,l}\left(\frac{\tilde{\Psi}_{i,l}^2}{4}\right)^{k_2}\right\}\end{aligned}$$

$$-\sum_{l=1}^{m_i}\left(\frac{\gamma_{i,l}}{2}\tilde{\Theta}_{i,l}^2+\frac{\gamma'_{i,l}}{2}\tilde{\Psi}_{i,l}^2\right)$$
$$+\sum_{l=1}^{m_i}\left\{\frac{\gamma_{i,l}}{2}(\Theta_{i,l}^*)^2+\frac{\gamma'_{i,l}}{2}(\Psi_{i,l}^*)^2+\bar{\omega}_{i,l}\right\}\Bigg]. \quad (47)$$

After that

$$\dot{V}_{m_i}\le\sum_{i=1}^{N}\Bigg[-p\left(\sum_{l=1}^{m_i}\frac{e_{i,l}^2}{2}\right)^{k_1}-p\left(\sum_{l=1}^{m_i}\frac{e_{i,l}^2}{2}\right)^{k_2}$$
$$-p\left\{\sum_{l=1}^{m_i}\left(\frac{\tilde{\Theta}_{i,l}^2}{2}\right)^{k_1}+\left(\frac{\tilde{\Psi}_{i,l}^2}{2}\right)^{k_1}\right\}$$
$$-p\left\{\sum_{l=1}^{m_i}\left(\frac{\tilde{\Theta}_{i,l}^2}{2}\right)^{k_2}+\left(\frac{\tilde{\Psi}_{i,l}^2}{2}\right)^{k_2}\right\}$$
$$+\sum_{l=1}^{m_i}\left\{\gamma_{i,l}\left(\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_1}+\gamma'_{i,l}\left(\frac{\tilde{\Psi}_{i,l}^2}{4}\right)^{k_1}\right\}$$
$$+\sum_{l=1}^{m_i}\left\{\gamma_{i,l}\left(\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_2}+\gamma'_{i,l}\left(\frac{\tilde{\Psi}_{i,l}^2}{4}\right)^{k_2}\right\}$$
$$-\sum_{l=1}^{m_i}\left(\frac{\gamma_{i,l}}{2}\tilde{\Theta}_{i,l}^2+\frac{\gamma'_{i,l}}{2}\tilde{\Psi}_{i,l}^2\right)$$
$$+\sum_{l=1}^{m_i}\left\{\frac{\gamma_{i,l}}{2}(\Theta_{i,l}^*)^2+\frac{\gamma'_{i,l}}{2}(\Psi_{i,l}^*)^2+\bar{\omega}_{i,l}\right\}\Bigg], \quad (48)$$

where $k_1=(1+\nu)/2$, $k_2=(3+2\nu)/2$, and $p=\min\{2^{k_1}(c_{i,l}-m_i+1),2^{k_2}c'_{i,l},2\gamma_{i,l},2\gamma'_{i,l})$.
Due to Lemma 2, one has

$$\left(\sum_{l=1}^{m_i}\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_1}\le\sum_{l=1}^{m_i}\frac{\tilde{\Theta}_{i,l}^2}{4}+(1-k_1)k_1^{\tau_1}, \quad (49)$$
$$\left(\sum_{l=1}^{m_i}\frac{\tilde{\Theta}_{i,l}^2}{4}\right)^{k_2}\le\sum_{l=1}^{m_i}\frac{\tilde{\Theta}_{i,l}^2}{4}+(1-k_2)k_2^{\tau_2}, \quad (50)$$

where $\tau_1=\frac{k_1}{1-k_1}$, $\tau_2=\frac{k_2}{1-k_2}$ are the design parameters, and $\gamma'_{i,l}(\frac{\tilde{\Psi}_{i,l}^2}{4})^{k_1}$, $\gamma'_{i,l}(\frac{\tilde{\Psi}_{i,l}^2}{4})^{k_2}$ are similar to (48) and (49).
Substituting (49) and (50) into (48) yields

$$\dot{V}_{m_i}\le-\beta_1V_{m_i}^{k_1}-\beta_2V_{m_i}^{k_2}+\lambda, \quad (51)$$

where $\beta_1=p$, $\beta_2=p/(m_i+1)^{k_2}$ and

$$\lambda=\sum_{i=1}^{N}\sum_{l=1}^{m_i}\{\bar{\omega}_{i,l}+(\gamma_{i,l}/2)\Theta_{i,l}^{*2}+(\gamma'_{i,l}/2)\Psi_{i,l}^{*2})\}+(1-k_1)k_1^{\tau_1}+(1-k_2)k_2^{\tau_2}.$$

Then, the settling time is

$$T\le T_{\max}=\frac{1}{\beta_1\delta(1-k_1)}+\frac{1}{\beta_2\delta(k_2-1)}. \quad (52)$$

Moreover, from the definition of V_{m_i}, one has

$$|y_i-y_{i,d}|\le\sqrt{2}(\frac{\lambda}{(1-\delta)\beta_2})^{1/2k_2}. \quad (53)$$

Thus, the tracking errors can converge to a small neighborhood of zero within a fixed time by tuning the parameters appropriately.

Remark 4: From the proof of Theorem 1, the convergence rate of the tracking error $e_{i,1}$ depends on the design parameters β_2 and λ. We can increase the design parameters c_{i,j_i} and c'_{i,j_i}, or decrease the design parameters ν, γ_{i,j_i}, γ'_{i,j_i}, a_{i,j_i}, and b_{i,j_i} to better achieve the system performance from (51). However, increasing the design parameters c_{i,j_i} and c'_{i,j_i}, may make the control energy greater. Therefore, a tradeoff should be considered between the system tracking performance and the control effort.

4. SIMULATION RESULT

In this chapter, one simulation example confirms the feasibility of the fuzzy adaptive fixed-time decentralized control scheme for nonlinear high-order large-scale systems.

Example: Consider the nonlinear high-order large-scale systems as

$$\begin{cases}\dot{x}_{1,1}=x_{1,2}^{p_{1,1}}+f_{1,1}(x_{1,1})+\Delta_{1,1}(y_1,y_2),\\ \dot{x}_{1,2}=u_1^{p_{1,2}}+f_{1,2}(x_{1,1},x_{1,2})+\Delta_{1,2}(y_1,y_2),\\ y_1=x_{1,1},\end{cases} \quad (54)$$
$$\begin{cases}\dot{x}_{2,1}=x_{2,2}^{p_{2,1}}+f_{2,1}(x_{2,1})+\Delta_{2,1}(y_1,y_2),\\ \dot{x}_{2,2}=u_2^{p_{2,2}}+f_{2,2}(x_{2,1},x_{2,2})+\Delta_{2,2}(y_1,y_2),\\ y_2=x_{2,1},\end{cases} \quad (55)$$

where $f_{1,1}=0.2x_{1,1}e^{-0.5x_{1,1}^2}$, $f_{1,2}=-0.1x_{1,2}\cos(1/1+x_{1,1}^2)$, $f_{2,1}=0.07x_{2,1}$, $f_{2,2}=-2x_{2,1}^2x_{2,2}^2$, $\Delta_{1,1}=-0.12\sin(x_{1,1}x_{2,1})$, $\Delta_{1,2}=-0.1(x_{1,1}^2-x_{2,1}^2)$, $\Delta_{2,1}=0.2-0.2\cos(x_{1,1}x_{2,1})$, $\Delta_{2,2}=-0.1(x_{1,1}+x_{2,1})$, $y_{1,d}=\sin t$, and $y_{2,d}=\sin t$.

Choose the fuzzy membership functions as ($i=1$, 2; $l=1,\ldots,5$)

$$\mu_{F_{i,1}^l}(x_{i,1})=\exp\left[-\frac{(x_{i,1}-3+l)^2}{2}\right],$$
$$\mu_{F_{i,2}^l}(\hat{x}_{i,2})=\exp\left[-\frac{(x_{i,1}-3+l)^2}{2}\right]\times\exp\left[-\frac{(x_{i,2}-3+l)^2}{2}\right].$$

Choose the design parameters as $\nu=0.2$; $p_{1,1}=2$, $p_{1,2}=1.2$, $p_{2,1}=1.2$, $p_{2,2}=1.2$; $\sigma_{1,1}=1$, $\sigma_{1,2}=10/9$, $\sigma_{1,3}=10/9$, $\sigma_{2,1}=1$, $\sigma_{2,2}=10/9$, $\sigma_{2,3}=10/9$; $c_{1,1}=8$, $c_{1,2}=16$, $c_{2,1}=10$, $c_{2,2}=16$; $c'_{1,1}=0.3$, $c'_{1,2}=1.2$, $c'_{2,1}=1.5$, $c'_{2,2}=1.5$; $a_{1,1}=0.3$, $a_{1,2}=0.3$, $a_{2,1}=0.5$, $a_{2,2}=0.3$; $b_{1,1}=0.3$, $b_{1,2}=0.1$, $b_{2,1}=0.3$, $b_{2,2}=0.1$; $\gamma_{1,1}=0.3$, $\gamma_{1,2}=0.6$, $\gamma_{2,1}=0.3$, $\gamma_{2,2}=0.6$; $\gamma'_{1,1}=0.2$, $\gamma'_{1,2}=0.3$, $\gamma'_{2,1}=0.3$, $\gamma'_{2,2}=0.3$.

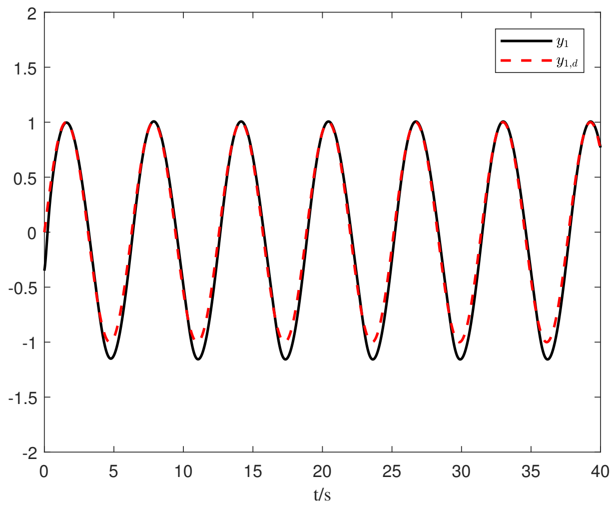

Fig. 1. The trajectories of y_1 and $y_{1,d}$.

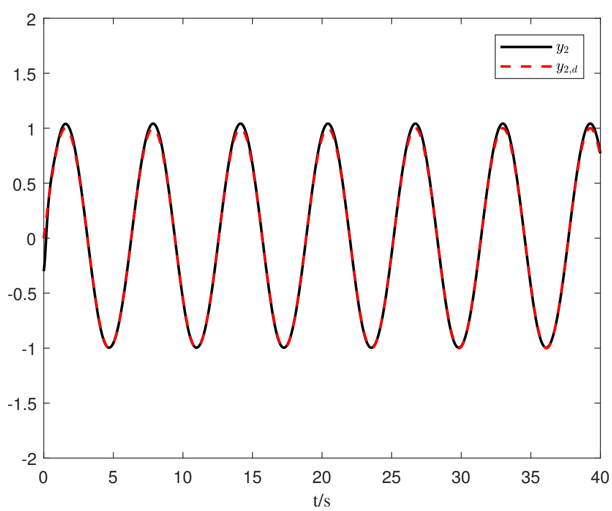

Fig. 2. The trajectories of y_2 and $y_{2,d}$.

Let the initial conditions be $x_{1,1}(0) = x_{1,2}(0) = -0.35$, $x_{2,1}(0) = x_{2,2}(0) = -0.3$, $\hat{\Theta}_{i,j_i} = 0$, $\hat{\Psi}_{i,j_i} = 0$ $(i = 1, 2;\ j = 1, 2)$.

Thus, the simulation results are shown in Figs. 1-7, where Figs. 1 and 2 reflect the trajectories of outputs y_i and their tracking signals $y_{i,d}$, respectively, it can be seen that the outputs y_i have excellent tracking performance; Figs. 3 and 4 indicate the trajectories of u_1 and u_2, respectively, which shows that u_1 and u_2 are saturated during the initialization transient phase; Figs. 5 and 6 show the trajectories of the adaptive parameters $\hat{\Theta}_{i,j_i}$ and $\hat{\Psi}_{i,j_i}$, $(i = 1, 2; j = 1, 2)$; Fig. 7 gets the trajectories of states $x_{1,2}$ and $x_{2,2}$.

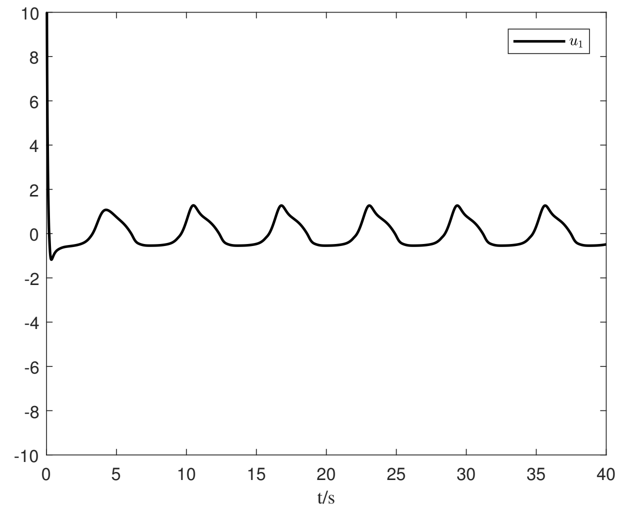

Fig. 3. The trajectory of u_1.

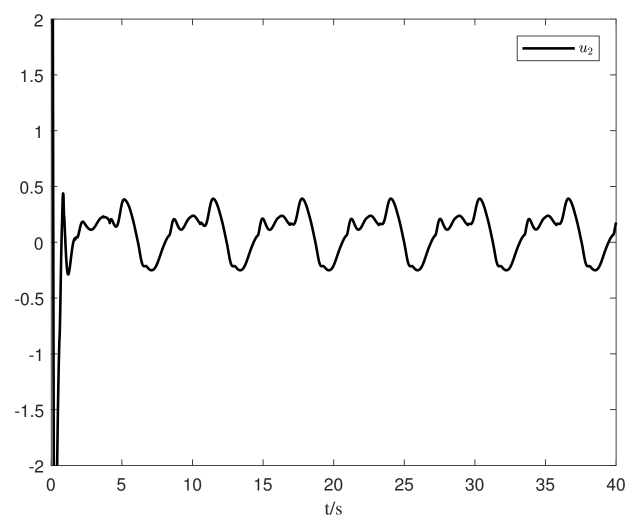

Fig. 4. The trajectory of u_2.

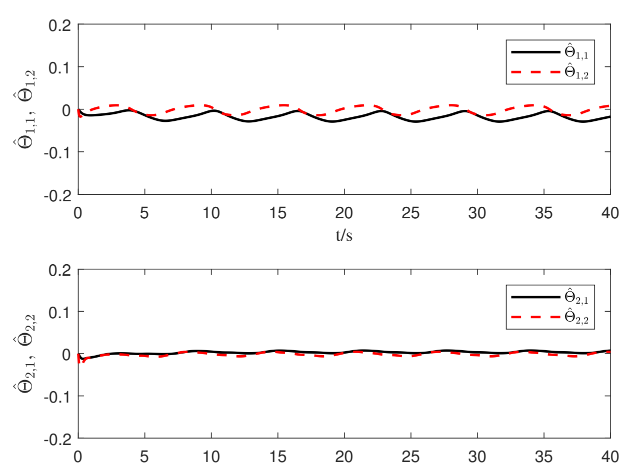

Fig. 5. The trajectories of $\hat{\Theta}_{i,j_i}$.

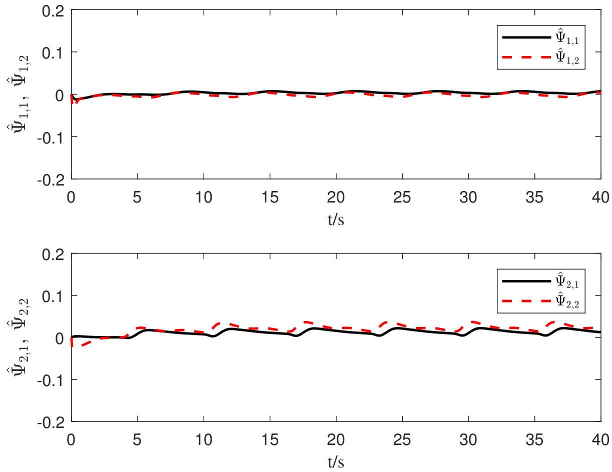

Fig. 6. The trajectories of $\hat{\Psi}_{i,j_i}$.

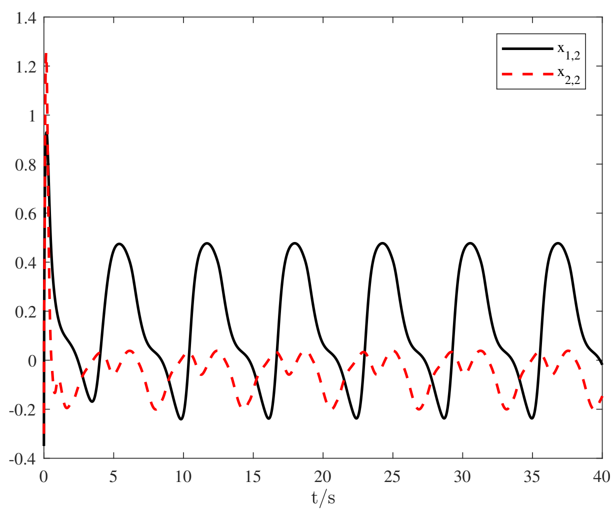

Fig. 7. The trajectories of $x_{1,2}$ and $x_{2,2}$.

5. CONCLUSION

This paper researches the problem of the fuzzy adaptive fixed-time tracking control for nonlinear high-order large-scale systems. The fuzzy adaptive fixed-time decentralized control scheme is proposed by using backstepping design technology and adding a power integrator method. The developed control algorithm can indicate that both the tracking performance and the closed-loop stability are preserved within a fixed time.The simulation results show the feasibility of the advanced control approach.

REFERENCES

[1] H. Ma, H. J. Liang, H. J. Ma, and Q. Zhou, "Nussbaum gain adaptive backstepping control of nonlinear strict-feedback systems with unmodeled dynamics and unknown dead zone," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 17, pp. 5326-5343, 2018.

[2] F. Wang, Q. Zou, and Q. Zong, "Robust adaptive backstepping control for an uncertain nonlinear system with input constraint based on Lyapunov redesign," *International Journal of Control, Automation, and Systems*, vol. 15, pp. 212-225, 2017.

[3] J. P. Cai, C. Y. Wen, and H. Y. Su, "Adaptive backstepping control for a class of nonlinear systems with non-triangular structural uncertainties," *IEEE Transactions on Automatic Control*, vol. 62, no. 10, pp. 5220-5226, 2017.

[4] C. C. Hua, X. P. Guan, and P. Shi, "Robust backstepping control for a class of time delayed systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 6, pp. 894-899, 2005.

[5] M. L. Chiang and L. C. Fu, "Adaptive stabilization of a class of uncertain switched nonlinear systems with backstepping control," *Automatica*, vol. 50, no.8, pp. 2128-2135, 2014.

[6] C. C. Hua, P. X. Liu, and X. P. Guan, "Backstepping control for nonlinear systems with time delays and applications to chemical reactor systems," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3723-3732, 2009.

[7] R. H. Cui and X. J. Xie, "Adaptive state-feedback stabilization of state-constrained stochastic high-order nonlinear systems," *Science China Information Sciences*, vol. 64, Article number 200203, 2021.

[8] J. M. Peng, C. Y. Li, and X. D. Ye, "Cooperative control of high-order nonlinear systems with unknown control directions," *Systems and Control Letters*, vol. 113, pp. 101-108, 2018.

[9] J. Davila, "Exact tracking using backstepping control design and high-order sliding modes," *IEEE Transactions on Automatic Control*, vol. 58, no. 8, pp. 2077-2081, 2013.

[10] M. Chen, C. S. Jiang, and Q. X. Wu, "Backstepping control for a class of uncertain nonlinear systems with neural network," *International Journal of Nonlinear Science*, vol. 3, no. 2, pp. 137-143, 2007.

[11] Y. H. Li, S. Qiang, and X. Y. Zhuang, "Robust and adaptive backstepping control for nonlinear systems using RBF neural networks," *IEEE Transactions on Neural Networks*, vol. 15, no. 3, pp. 693-701, 2004.

[12] Y. M. Li, S. C. Tong, and T. S. Li, "Adaptive fuzzy backstepping control design for a class of pure-feedback switched nonlinear systems," *Nonlinear Analysis: Hybrid Systems*, vol. 16, pp. 72-80, 2015.

[13] S. C. Tong, Y. M. Li, G. Feng, and T. S. Li, "Observer-based adaptive fuzzy backstepping dynamic surface control for a class of MIMO nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 41, no. 4, pp. 1124-1135, 2011.

[14] S. C. Tong, S. Sui, and Y. M. Li, "Fuzzy adaptive output feedback control of MIMO nonlinear systems with partial tracking errors constrained," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 729-742, 2015.

[15] X. Zhang and Y. Lin, "Nonlinear decentralized control of large-scale systems with strong interconnections," *Automatica*, vol. 50, no. 9, pp. 2419-2423, 2014.

[16] Y. M. Li, K. K. Sun, and S. C. Tong, "Adaptive fuzzy robust fault-tolerant optimal control for nonlinear large-scale systems," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 5, pp. 2899-2914, 2018.

[17] S. C. Tong, Y. M. Li, and L. Y. Jun, "Adaptive fuzzy output feedback decentralized control of pure-feedback nonlinear large-scale systems," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 5, pp. 930-954, 2012.

[18] Y. M. Li and S. C. Tong, "Fuzzy adaptive control design strategy of nonlinear switched large-scale systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2209-2218, 2018.

[19] Y. M. Li, Z. Y. Ma, and S. C. Tong, "Adaptive fuzzy output-constrained fault-tolerant control of nonlinear stochastic large-scale systems with actuator faults," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2362-2376, 2017.

[20] K. W. Li, Y. M. Li, and G. D. Zong, "Adaptive fuzzy fixed-time decentralized control for stochastic nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 11, pp. 3428-3440, 2021.

[21] C. C. Hua, X. P. Guan, and P. Shi, "Adaptive fuzzy control for uncertain interconnected time-delay systems," *Fuzzy Sets and Systems*, vol. 153, no. 3, pp. 447-458, 2005.

[22] S. J. Yoo and J. B. Park, "Neural-network-based decentralized adaptive control for a class of large-scale nonlinear systems with unknown timevarying delays," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 39, no. 5, pp. 1316-1323, 2009.

[23] H. Q. Wang, B. Chen, and C. Lin, "Adaptive fuzzy decentralized control for a class of large-scale stochastic nonlinear systems," *Neurocomputing*, vol. 103, pp. 155-163, 2013.

[24] B. Wu, X. H. Chang, and X. D. Zhao, "Fuzzy H^∞ output feedback control for nonlinear NCSs with quantization and stochastic communication protocol," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 9, pp. 2623-2634, 2021.

[25] X. H. Chang, Q. Liu, Y. M. Wang, and J. Xiong, "Fuzzy peak-to-peak filtering for networked nonlinear systems with multipath data packet dropouts," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 3, pp. 436-446, 2019.

[26] H. Wang and Q. X. Zhu, "Finite-time stabilization of high-order stochastic nonlinear systems in strict-feedback form," *Automatica*, vol. 54, pp. 284-291, 2015.

[27] Y. L. Fan and Y. M. Li, "Adaptive fuzzy finite-time optimal control for switched nonlinear systems," *Optimal Control Applications and Methods*, vol. 41, no. 5, pp. 1616-1631, 2020.

[28] L. Chu, T. Gao, M. X. Wang, Y. Q. Han, and S. L. Zhu, "Adaptive decentralized control for large-scale nonlinear systems with finite-time output constraints by multi-dimensional Taylor network," *Asian Journal of Control*, vol. 24, no. 4, pp. 1769-1779, 2022.

[29] L. Y. Hu and X. H. Li, "Decentralised adaptive neural connectively finitetime control for a class of p-normal form largescale nonlinear systems," *International Journal of Systems Science*, vol. 50, no. 16, pp. 3003-3021, 2019.

[30] P. H. Du, Y. N. Pan, H. Y. Li, and H. K. Lam, "Nonsingular finite-time event-triggered fuzzy control for large-scale nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 8, pp. 2088-2099, 2021.

[31] S. J. Kang, X. P. Liu, and H. Q. Wang, "Command filter-based adaptive fuzzy decentralized control for large-scale nonlinear systems," *Nonlinear Dynamics*, vol. 105, no. 4, pp. 3239-3253, 2021.

[32] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Transactions on Automatic Control*, vol. 57, pp. 2106-2110, 2012.

[33] H. F. Hong, H. Wang, and Z. L. Wang, "Finite-time and fixed-time consensus problems for second-order multi-agent systems with reduced state information," *Science China Information Sciences*, vol. 62, Article number 212201, 2019.

[34] M. Chen, H. Q. Wang, and X. P. Liu, "Adaptive fuzzy practical fixed-time tracking control of nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 3, pp. 664-673, 2021.

[35] D. S. Ba, Y. X. Li, and S. C. Tong, "Fixed-time adaptive neural tracking control for a class of uncertain nonstrict nonlinear systems," *Neurocomputing*, vol. 363, pp. 273-280, 2019.

[36] Q. Zhou, P. H. Du, H. Y. Li, and R. Q. Lu, "Adaptive fixed-time control of error-constrained pure-feedback interconnected nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 10, pp. 6369-6380, 2021.

[37] L. J. Zhang, Y. Q. Xia, and G. H. Shen, "Fixed-time attitude tracking control for spacecraft based on a fixed-time extended state observer," *Science China Information Sciences*, vol. 64, Article number 212201, 2021.

[38] Q. Chen, S. Xie, M. Sun, and X. He, "Adaptive nonsingular fixed-time attitude stabilization of uncertain spacecraft," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 54, no. 6, pp. 2937-2950, 2018.

[39] M. Tao, Q. Chen, X. He, and M. Sun, "Adaptive fixed-time fault-tolerant control for rigid spacecraft using a double power reaching law," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 12, pp. 4022-4040, 2019.

[40] B. Ning, J. Jin, Z. Zuo, and J. Zheng, "Distributed fixed-time cooperative tracking control for multi-robot systems," *Proc. of IEEE International Conference on Robotics and Automation*, pp. 833-838, 2017. DOI: 10.1109/ICRA.2017.7989101

[41] C. Wang, H. Tnunay, Z. Zuo, B. Lennox, and Z. T. Ding, "Fixed-time formation control of multirobot systems: Design and experiments," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 8, pp. 6292-6301, 2018.

[42] Z. Zhu, Y. Q. Xia, and M. Y. Fu, "Attitude stabilization of rigid spacecraft with finite-time convergence," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 6, pp. 686-702, 2011.

[43] S. Y. Khoo, J. L. Yin, Z. H. Man, and X. H. Yu, "Finite-time stabilization of stochastic nonlinear systems in strict-feedback form," *Automatica*, vol. 49, no. 5, pp. 1403-1410, 2013.

[44] Y. Z. Wang and G. Feng, "On finite-time stability and stabilization of nonlinear port-controlled hamiltonian systems," *Science China Information Sciences*, vol. 51, pp. 1-14, 2013.

[45] X. S. Yang, J. D. Cao, C. Xu, and J. W. Feng, "Finite-time stabilization of switched dynamical networks with quantized couplings via quantized controller," *Science China Technological Sciences*, vol. 61, pp. 299-308, 2018.

[46] Z. X. Zhang and J. X. Dong, "A novel H_∞ control for T-S fuzzy systems with membership functions online optimization learning," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 4, pp. 1129-1138, 2022.

[47] Z. X. Zhang and J. X. Dong, "Fault-tolerant containment control for IT2 fuzzy networked multiagent systems against denial-of-service attacks and actuator faults," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 4, pp. 2213-2224, 2022.

[48] J. X. Dong, Q. H. Hou, and M. M. Ren, "Control synthesis for discrete-time T-S fuzzy systems based on membership function-dependent H_∞ performance," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 3360-3366, 2019.

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