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Dynamic Event-triggered Fuzzy Filtering for Semi-linear Parabolic PDE Systems: A Reduced-order Approach

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Abstract: This paper investigates dynamic event-triggered fuzzy reduced-order filtering for a class of nonlinear semi-linear parabolic partial differential equation (PDE) systems. First, the considered systems are reconstructed by a Takagi-Sugeno (T-S) fuzzy model based on the sector nonlinearity approach. Furthermore, a dynamic event-triggered mechanism is developed to improve network resource utilization. Based on the non-parallel distribution compensation principle, several theorems that guarantee the augmented system's asymptotic stability with L_2-L_∞ performance are provided. Finally, two examples are introduced to illustrate the effectiveness of the proposed method.

Keywords: Dynamic event-triggered mechanism, PDE systems, reduced-order filtering, T-S fuzzy model.

1. INTRODUCTION

Numerous practical systems like chemical reaction processes, community dynamics systems, and fluid heat exchange systems [1-4] tend to have distributional parameter properties, i.e., their behavior is not only time-dependent but also spatially location-dependent and often described by partial differential equations (PDE) [5-11]. These PDE systems can be classified into three types according to the properties of spatial differential operators: hyperbolic [5,6], parabolic [7-9], and elliptic [10,11]. Since these PDE systems have infinite dimensions, it is not easy to employ the control theory of lumped parametric systems (LPS) directly for controller/filter design. Therefore, researchers have developed many new methods to solve the controller/filter design problem for PDE systems. For example, Wang and Wu [12] investigated output feedback control for nonlinear parabolic PDE systems; control design for parabolic PDE systems was discussed in [13]. These works employ the Takagi-Sugeno (T-S) fuzzy model [14-16] to linearize the nonlinear part of the PDE systems and construct the fuzzy controller for fuzzy systems by the parallel distribution compensation (PDC) method (fuzzy control rules share premise variables and fuzzy sets with fuzzy PDE models). However, due to complex factors such as time delay and packet loss in the network, the premise variables and fuzzy sets are often difficult to match perfectly. Therefore, applying the non-parallel distribution compensation (non-PDC) technique is meaningful for the stability analysis of PDE systems [17-19]. In [17], the problem of fuzzy filter design based on the event-triggered mechanism (ETM) was addressed by using the non-PDC technique; event-triggered fuzzy control based on the non-PDC method was addressed in [18].

As described in [17,18], the introduction of ETM in controller/filter design can effectively reduce the network communication burden and improve network resource utilization compared with traditional time-triggered mechanisms (TTM) (periodic sampling). However, the traditional ETM [20,21] cannot reflect the time-varying nature of event-triggered and systems updates. Therefore, it is essential to introduce variables that adapt to dynamic environments in ETM, also referred to as dynamic event-triggered mechanisms (DETM) [22-25]. Based on the DETM, the authors in [22] designed a controller for networked control systems; the dynamic event-triggered control for linear stochastic systems was considered in [23]. Currently, DETM has been applied to some nonlinear systems, and the results show that DETM can effectively reduce the number of signal transmissions compared to ETM. However, to the authors' knowledge, few

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articles apply DETM for PDE systems, allowing us to carry out the current study.

Nonlinear dynamic systems in engineering are commonly multiple-input multiple-output systems [26-33], which may lead to difficulty and complexity in evaluating systems performance and analyzing systems stability. More seriously, due to the requirement of system control/filtering, the designed controller/filter needs to be in the same order as the original system, further increasing the difficulty of the controller and filter design. Therefore, it is necessary to simplify the mathematical model by adopting a suitable reduced-order (RO) criterion, which is a hot topic of current research [34-36]. To this end, several techniques have been proposed to design a lower-order filter than the original system according to specific criteria [37,38]. In [37], a RO filter design method was proposed for fuzzy systems with ETM. Based on ETM, the filter design for networked control systems with time-varying delays was investigated in [38]. These results indicate that the RO filter is more flexible and simple. It is worth mentioning that the existing works on fuzzy PDE system filtering are limited, and to the authors' knowledge, there are few precedents of fuzzy RO filtering applied to PDE systems, which stimulates the current research.

Based on the above research motivation, we aim to design a RO filter with L_2-L_∞ performance for fuzzy PDE systems. The main contributions of our research results are as follows:

1) In [38], a RO filter design method is proposed. However, this method failed to take into account the systems' spatial characteristics. In addition, combined with the existing works [17], it is the first attempt to design a fuzzy RO filter for semi-linear parabolic fuzzy PDE systems.
2) Different from [37], the fuzzy filter shares the same premise variables and fuzzy sets with the original systems, which will make the filter design unreasonable and inflexible. Therefore, employing the non-PDC method to design filters with L_2-L_∞ performance is more general.
3) In contrast to [20], the traditional ETM can not reflect the systems' dynamic characteristics effectively. Therefore, a suitable DETM is designed for PDE systems with spatiotemporal characteristics, which can effectively improve the efficiency of network resource transmission.

The remainder of the paper is organized as follows: Section 2 introduces the fuzzy PDE systems with the filter design, the DETM, some definitions and citations. The main results on RO filter design and non-PDC techniques are presented in Section 3. In Section 4, two simulation examples are developed to indicate the effectiveness of the adopted approaches. Finally, Section 5 concludes this article and gives future research guidelines.

Notations: $\mathbb{R}^n$ stands for n-dimensional Euclidean space. $\mathbb{H}^n$ is the infinite-dimensional Hilbert space. I, diag$\{\cdot\}$ and col$[\cdot]$ are represents identity matrix, block-diagonal matrix and column vectors, respectively. $P>0$ means that P is a positive definite matrix. The symbol $*$ denotes the transpose of the symmetric matrix. $\mathcal{L}_2$ represents the space of the square-integrable vector functions and $\|\cdot\|$ denotes the usual $\mathcal{L}_2[0,\infty)$ norm. For simplicity, define $y=y(x,t)$, $\bar{y}=\bar{y}(x,t)$, $z_o=z_o(x,t)$, $\hat{z}_o=\hat{z}_o(x,t)$, $\tilde{z}_o=\tilde{z}_o(x,t)$, $\tilde{z}_o=z_o-\hat{z}_o$, $z_m=z_m(x,t)$, $z_m(t_k)=z_m(x,t_k)$, $z_m(t_k+n\mathcal{T})=z_m(x,t_k+n\mathcal{T})$, $z_{mh}=z_m(x,t-h(t))$, $e_{mh}=e_m(x,t-h(t))$, $\omega=\omega(x,t)$, $\omega(s)=\omega(x,s)$, $\eta=\eta(x,t)$, $\xi=\text{col}[y\ \hat{y}]$, $\xi_h=\xi(x,t-h)$, $\xi_{ht}=\xi(x,t-h(t))$, $\xi(s)=\xi(x,s)$, $y_t=\frac{\partial y(x,t)}{\partial t}$, $y_{xx}=\frac{\partial^2 y(x,t)}{\partial x^2}$, $\eta_t=\frac{\partial \eta(x,t)}{\partial t}$, $\xi_x=\frac{\partial \xi(x,t)}{\partial x}$, $\xi_s(s)=\frac{\partial \xi(s)}{\partial s}$, $\theta=\theta(x,t)$, $\vartheta=\vartheta(x,t)$, $h_i=h_i(\theta)$, $g_j=g_j(\vartheta)$. Matrices not explicitly stated in the text are assumed to have the appropriate dimensionality.

2. PROBLEM FORMULATION

2.1. Fuzzy system model

In this paper, a class of nonlinear distributed parameter systems described by PDE is considered:

$$\begin{cases} y_t=\Theta y_{xx}+f(y)+c(y)\omega, \\ z_o=d(y), \\ z_m=e(y), \end{cases} \tag{1}$$

where $y\in\mathbb{H}^n$ indicates the state with $x\in[0,\ l]\subset\mathbb{R}$ and $t>0$; z_o stands for the signal to be estimated; z_m represents the measured output; $\omega\in\mathcal{L}_2[0,\infty)$ means the external disturbance. Θ is a constant; $f(y)$, $c(y)$, $d(y)$ and $e(y)$ are adequately smooth nonlinear functions satisfying $f(0)=c(0)=d(0)=e(0)=0$, which is reasonable and feasible in combination with the sector nonlinear method [12,17]. The above systems with the following boundary conditions

$$y_x(0,t)=y_x(l,t)=y(l,t)=0, \tag{2}$$

and the initial condition

$$y(0,t)=y_0(x). \tag{3}$$

Next, the following T-S fuzzy rule is employed to reconstruct the nonlinear systems to deal with the unknown nonlinear function:

Plant Rule $\aleph^i$: IF θ_1 is F_1^i, ..., and θ_z is F_z^i, THEN

$$\begin{cases} y_t=\Theta y_{xx}+A_i y+C_i\omega, \\ z_o=D_i y, \\ z_m=E_i y, \end{cases} \tag{4}$$

where $\theta_q=[\theta_1,\cdots,\theta_z]$ stand for the premise variable vectors; F_q^i represent the fuzzy sets of rule i corresponding to

θ_q with $i \in \{1, 2, \cdots, r\}$ and $q \in \{1, 2, \cdots, z\}$. A_i, C_i, D_i and E_i indicate known matrices with appropriate dimensions. Then, (4) can be represented as

$$\begin{cases} y_t = \Theta y_{xx} + \sum_{i=1}^{r} h_i [A_i y + C_i \omega], \\ z_o = \sum_{i=1}^{r} h_i D_i y, \\ z_m = \sum_{i=1}^{r} h_i E_i y, \end{cases} \tag{5}$$

where $h_i = \frac{\prod_{q=1}^{z} F_q^i(\theta_q)}{\sum_{i=1}^{r} \prod_{q=1}^{z} F_q^i(\theta_q)}$ is the normalized membership grade; $F_q^i(\theta_q)$ is the membership function corresponding to the fuzzy F_q^i and satisfies $h_i \geq 0$, $\sum_{i=1}^{r} h_i = 1$.

Remark 1 (The well-posedness analysis)**:** Inspired by [39], the PDE system (1) based on the boundary conditions (2) and the initial condition (3) is formulated as the following abstract differential equation

$$\dot{y}(t) = \mathcal{A} y(t) + f(y(t)) + \omega(t),\ y(0) = y_0(\cdot), \tag{6}$$

where $y(t) = y(\cdot, t) \triangleq \{y(x,t), x \in [0, l]\}$, $\omega(t) = \omega(\cdot, t) \triangleq \{\omega(x,t), x \in [0, l]\}$ and the operator $\mathcal{A}$ is defined as

$$\mathcal{A}\bar{y}(x) \triangleq \frac{d}{dx}\left(\frac{d\bar{y}(x)}{dx}\right),$$

whose domain is $\mathcal{D}(\mathcal{A}) \triangleq \{\bar{y} \in \mathbb{H}^n(0,l) : d\bar{y}(x)/dx|_{x=0} = \bar{y}(l) = 0\}$ and the nonlinear term $f(y(t))$ is chosen as $f(y(t)) \triangleq f(y(\cdot, t))$. By Exercise 2.10 (see [40]), it is easily checked that the operator $\mathcal{A}$ with $\mathcal{D}(\mathcal{A})$ generates a C_0 semigroup $\exp(\mathcal{A}t)$ on $\mathcal{L}_2([0,l])$. As $f(y)$ is of class C^1 in y, by Theorem 3.1.3 in Chapter 3 on page 103 of [40], Theorem 1.5 in Chapter 6 on page 187 of [41], one can easily conclude that the abstract evolution equation (6) has a unique classical solution.

Assumption 1: The systems (1) are quadratically stable.

This assumption ensures that the estimation error is bounded since the asymptotic stability of the error dynamics also depends on the systems' state.

2.2. Dynamic event-trigger mechanism

As shown in Fig. 1, for the purpose of saving the limited communication resources, a novel DETM (see [25]) is adopted with a sampler and zero-order holder (ZOH). Define $z_m(t_k\mathcal{T} + n\mathcal{T})$, $z_m(t_k\mathcal{T})$ as the current sample signal and latest transmitted signal, respectively. Then, the DETM is established as follows:

$$\eta + \theta[\delta z_m^T(t_k\mathcal{T})\Omega z_m(t_k\mathcal{T}) - e_m^T(s_k\mathcal{T})\Omega e_m^T(s_k\mathcal{T})] \geq 0, \tag{7}$$

where $\delta \in [0, 1)$, $\theta > 0$, Ω is a positive definite symmetric matrix; $e_m(s_k\mathcal{T}) = z_m(t_k\mathcal{T} + n\mathcal{T}) - z_m(t_k\mathcal{T})$ is expressed as

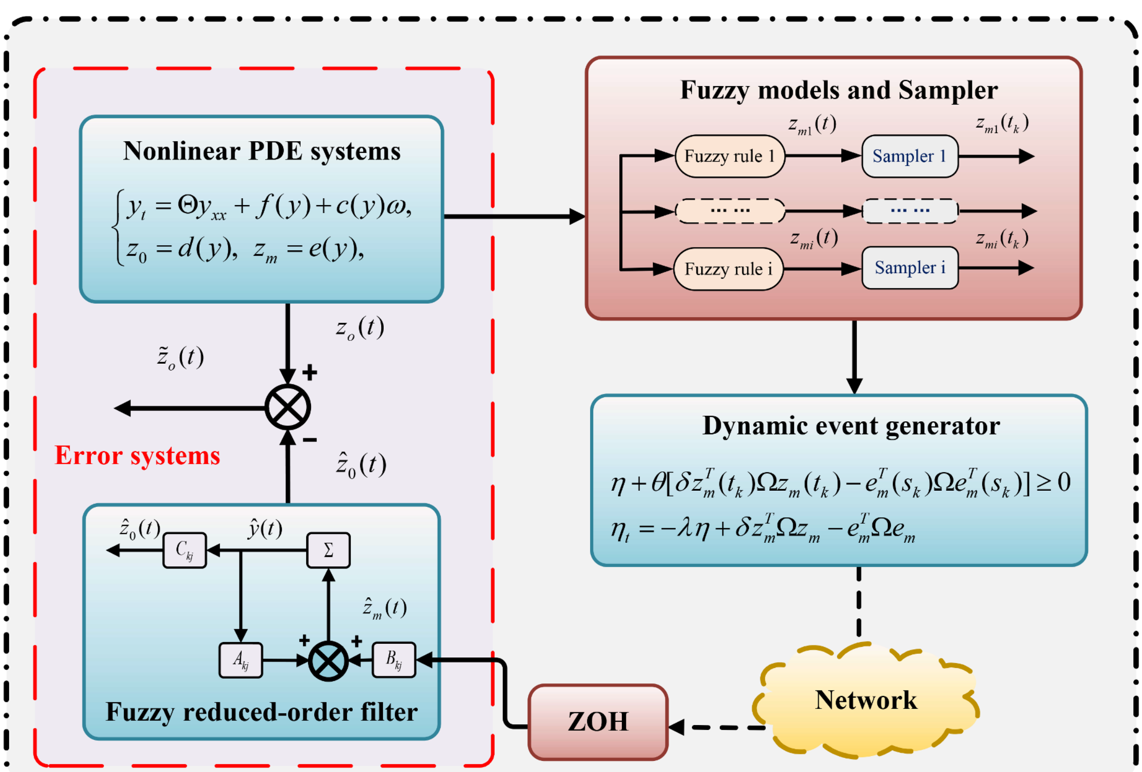

Fig. 1. The frame of networked filtering system.

the transmission error; $s_k\mathcal{T} = t_k\mathcal{T} + n\mathcal{T}$ stands for the sampling time between $t_k\mathcal{T}$ (current instant) and $t_{k+1}\mathcal{T}$ (approaching instant). η is a dynamic variable and satisfies the following equation

$$\eta_t = -\lambda\eta + \delta z_m^T\Omega z_m - e_m^T\Omega e_m, \tag{8}$$

where $\lambda > 0$, $\eta_0 > 0$ is the initial value of η. Assuming that α is a non-zero natural number satisfying $t_{k+1} = t_k + \alpha + 1$. Then, consider the time interval of ZOH

$$[t_k\mathcal{T} + \chi_{t_k}, t_{k+1}\mathcal{T} + \chi_{t_{k+1}}) = \bigcup_{\mathfrak{n}=0}^{\alpha} \Gamma_{\mathfrak{n},k},$$

where

$$\Gamma_{\mathfrak{n},k} \triangleq [t_k\mathcal{T} + \mathfrak{n}\mathcal{T} + \chi_{t_k+\mathfrak{n}}, t_k\mathcal{T} + (\mathfrak{n}+1)\mathcal{T} + \chi_{t_k+\mathfrak{n}+1}),$$
$$\mathfrak{n} = 0,\ 1,\ \dots,\ \alpha.$$

Define the network transmission delay as follows:

$$h(t) = t - (t_k\mathcal{T} + n\mathcal{T}) = t - s_k\mathcal{T},$$
$$t \in \Gamma_{\mathfrak{n},k},\ 0 \le h(t) \le h,$$

and the input of the RO filter under the behavior of the ZOH can be expressed as follows:

$$\begin{aligned}\hat{z}_m &= z_m(t_k\mathcal{T}) = z_m(s_k\mathcal{T}) - e_m(s_k\mathcal{T}) \\ &= z_{mh} - e_{mh},\ t \in [t_k + \chi_{t_k}, t_{k+1} + \chi_{t_{k+1}}).\end{aligned} \tag{9}$$

Remark 2: When the dynamic variable $\eta = 0$, (7) can be regarded as $\delta z_m^T(t_k\mathcal{T})\Omega z_m(t_k\mathcal{T}) - e_m^T(s_k\mathcal{T})\Omega e_m(s_k\mathcal{T})$. At this time, the DETM (7) is converted to a traditional static ETM as follows:

$$\delta z_m^T(t_k\mathcal{T})\Omega z_m(t_k\mathcal{T}) - e_m^T(s_k\mathcal{T})\Omega e_m(s_k\mathcal{T}) \ge 0, \tag{10}$$

which means that the ETM can be used as a special situation of DETM (7) as $\theta \to +\infty$. The introduction of dynamic variables is more helpful to widen the interval between two consecutive triggered times.

Remark 3: The triggered threshold $\delta \in [0, 1)$ is a given constant. When $\delta = 0$, the DETM will be changed to the traditional TTM. As δ gradually increases from 0 to 1, the network transmission data will gradually decrease to save the network transmission resources.

2.3. Structure of reduced-order fuzzy filter

Next, using the non-PDC method to design a fuzzy RO filter, which is expressed as follows:

Filter Pule $\mathfrak{I}^j$: IF ϑ_1 is N_1^j, ..., and ϑ_p is N_p^j, THEN

$$\begin{cases}\hat{y}_t = \Theta_k\hat{y}_{xx} + A_{kj}\hat{y} + B_{kj}\hat{z}_m, \\ \hat{z}_o = C_{kj}\hat{y},\end{cases} \tag{11}$$

where $\hat{y} \in \mathbb{R}^l$ represents the filter's state and $l < n$; $\hat{z}_m$ stands for the filter's practical input signal; $\hat{z}_o$ expresses the estimate signal of z_o; $\vartheta_v = [\vartheta_1, \dots, \vartheta_p]$ stand for the filter premise variable vectors; N_v^j denote the fuzzy sets of rule j corresponding to ϑ_v with $j \in \{1, 2, \cdots, r\}$ and $v \in \{1, 2, \cdots, p\}$. A_{kj}, B_{kj} and C_{kj} are the filter coefficient matrices to be determined. Then, the following fuzzy filter can be obtained

$$\begin{cases}\hat{y}_t = \Theta_k\hat{y}_{xx} + \sum_{j=1}^{r} g_j[A_{kj}\hat{y} + B_{kj}\hat{z}_m], \\ \hat{z}_o = \sum_{j=1}^{r} g_j C_{kj}\hat{y},\end{cases} \tag{12}$$

where $g_j = \frac{\prod_{v=1}^{p} N_v^j(\vartheta_v)}{\sum_{j=1}^{r}\prod_{v=1}^{p} N_v^j(\vartheta_v)}$ is the normalized membership grade; $N_v^j(\vartheta_v)$ is the membership function corresponding to the fuzzy N_v^j and satisfies $g_j \ge 0$, $\sum_{j=1}^{r} g_j = 1$.

Although the PDC method has made significant progress, it is difficult to match the premise variables and fuzzy sets of the filter with the original systems due to many complex factors, such as network delay and packet loss. Therefore, this paper adopts a non-PDC method to design the proposed fuzzy RO filter.

2.4. Problem formulation

Combining (5) and (12), the filtering error system is constructed as follows:

$$\begin{aligned}\xi_t &= \bar{\Theta}\xi_{xx} + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i g_j[\bar{A}_{ij}\xi + \bar{C}_i\omega + \bar{B}_{ij}\xi_{ht} - \bar{B}_j e_{mh}], \\ \tilde{z}_o &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i g_j \bar{D}_{ij}\xi,\end{aligned} \tag{13}$$

where

$$\bar{\Theta} = \begin{bmatrix}\Theta & 0 \\ 0 & \Theta_k\end{bmatrix},\ \bar{A}_{ij} = \begin{bmatrix}A_i & 0 \\ 0 & A_{kj}\end{bmatrix},\ \bar{B}_{ij} = \begin{bmatrix}0 & 0 \\ B_{kj}E_i & 0\end{bmatrix},$$
$$\bar{B}_j = \begin{bmatrix}0 \\ B_{kj}\mathcal{H}^T\end{bmatrix},\ \bar{C}_i = \begin{bmatrix}C_i \\ 0\end{bmatrix},\ \bar{E}_i = \begin{bmatrix}E_i \\ 0\end{bmatrix},$$
$$\bar{D}_{ij} = \begin{bmatrix}D_i^T \\ -C_{kj}^T\end{bmatrix}^T.$$

The aim of robust RO filtering is to obtain an estimate $\hat{z}_o$ of the signal z_o such that a guaranteed performance criterion is minimized in an estimation error sense. Next, the problem of robust L_2-L_∞ fuzzy filter can be defined as follows:

Definition 1: Given noise attenuation level $\gamma > 0$, determine the filter parameters so that for any time $h(t)$ satisfying

i) The system (13) is asymptotically stable;
ii) Under zero initial conditions, the system (13) satisfies

$$\gamma\|\omega\|_2 > \|\tilde{z}_o\|_\infty, \tag{14}$$

for any non-zero $\omega \in \mathcal{L}_2[0,\infty)$, where $\|\tilde{z}_o\|_\infty^2 = \sup_t\{\tilde{z}_o^T\tilde{z}_o\}$.

Lemma 1 [42]**:** Assume that $\varsigma(t)$, $\varrho(t)$ are sequential functions. The function $\pounds(t,\star)$ is the difference of continuity for all $t \geq 0$. $\varrho(t) \leq \varsigma(t)$ is established for all $t \geq 0$ by satisfying the following conditions

$$\dot{\varsigma}(t) = \pounds(t,\varsigma(t)),\ \varsigma(t_0) = \varsigma_0,$$
$$\dot{\varrho}(t) \leq \pounds(t,\varrho(t)),\ \varrho(t_0) \leq \varsigma_0.$$

Lemma 2 (Jensen's inequality) [43]**:** Assume $\zeta \in \mathbb{H}^n$ be a vector function. Then, for any matrix $M > 0$, the following inequality holds

$$\int_a^b \zeta^T(x)M\zeta(x)dx \geq \frac{1}{b-a}\left(\int_a^b \zeta^T(x)dx\right)M\left(\int_a^b \zeta(x)dx\right),$$

Lemma 3 [44]**:** Let $\psi_1,\ \psi_2,\ \cdots,\ \psi_N : \mathbb{R}^m \mapsto \mathbb{R}$ have non-negative values in an open subset $\mathbb{C}$ of $\mathbb{R}$. Then, the reciprocally convex combination of ψ_i to $\mathbb{C}$ satisfies

$$\min_{\{\beta_i|\beta_i>0,\sum_i\beta_i=1\}} \sum_i \frac{1}{\beta_i}\psi_i(t) = \sum_i \psi_i(t) + \max_{\varphi_{i,j}(t)} \sum_{i\neq j} \varphi_{i.j}(t),$$

subject to

$$\left\{\varphi_{i,j}(t) :\ \mathbb{R}^m \mapsto \mathbb{R}, \varphi_{j,i}(t) \triangleq \varphi_{i,j}(t), \begin{bmatrix}\psi_i(t) & \varphi_{i,j}(t)\\ \varphi_{i,j}(t) & \psi_j(t)\end{bmatrix} \geq 0\right\}.$$

3. MAIN RESULT

In this section, the filter design and the stability analysis with L_2-L_∞ performance will be presented in Theorem 1. On this basis, the non-PDC problem is solved in Theorem 2. Finally, the solution of the RO filter parameters is given in Theorem 3.

Theorem 1: For given scalars $0 < \delta < 1$ and $h > 0$, the system (13) is asymptotically stable if there exist matrices $P > 0$, $Q > 0$, $R > 0$, S satisfying $S\bar{\Theta} \geq 0$ and G that the following inequalities hold for all $i \in \{1, 2, \cdots, r\}$, $j \in \{1, 2, \cdots, r\}$

$$\Psi_{1ij} = \begin{bmatrix}\Psi_{1ij}^1 & \Psi_{1ij}^2\\ * & \Psi_1^3\end{bmatrix} < 0, \tag{15}$$

$$\Psi_{2ij} = \begin{bmatrix}P & \bar{D}_{ij}^T\\ * & \gamma^2 I\end{bmatrix} > 0, \tag{16}$$

where $\begin{bmatrix}R & G\\ G & R\end{bmatrix} \geq 0$, $\Psi_{1ij}^1 = \begin{bmatrix}\Psi_{1ij}^{11} & 0\\ * & \Psi_{1ij}^{12}\end{bmatrix}$, $\Psi_{1ij}^2 = \begin{bmatrix}\Psi_{1ij}^{21} & \Psi_{1ij}^{22}\\ 0 & 0\end{bmatrix}$, $\Psi_1^3 = \begin{bmatrix}\Psi_1^{31} & 0\\ 0 & \Psi_1^{32}\end{bmatrix}$, $\Psi_{1ij}^{11} = \begin{bmatrix}\Psi_{1ij}^{111} & \Psi_{1ij}^{112}\\ * & \Psi_{1ij}^{122}\end{bmatrix}$, $\Psi_{1ij}^{21} = \begin{bmatrix}\Psi_{1ij}^{211} & G^T\\ S\bar{B}_{ij} & 0\end{bmatrix}$, $\Psi_{1ij}^{22} = \begin{bmatrix}-S\bar{B}_j & S\bar{C}_i\\ -S\bar{B}_j & S\bar{C}_i\end{bmatrix}$, $\Psi_1^{31} = \begin{bmatrix}\Psi_1^{311} & \Psi_1^{312}\\ * & \Psi_1^{313}\end{bmatrix}$, $\Psi_{1ij}^{12} = \Psi_{1ij}^{133}$, $\Psi_1^{32} = \mathrm{diag}\{-\Omega,\ -I\}$, $\Psi_{1ij}^{111} = Q - R + S\bar{A}_{ij} + \bar{A}_{ij}^T S^T$, $\Psi_{1ij}^{112} = P + \bar{A}_{ij}^T S^T - S$, $\Psi_1^{122} = h^2R - S - S^T$. $\Psi_{1ij}^{133} = -S\bar{\Theta} - \bar{\Theta}S^T$, $\Psi_{1ij}^{211} = R - G^T + S\bar{B}_{ij}$, $\Psi_1^{311} = -2R + G + G^T + \delta\bar{E}^T\Omega\bar{E}$, $\Psi_1^{312} = R - G^T$, $\Psi_1^{313} = -Q - R$.

Proof: The dynamic variable η is positive according to (8), which is proved as follows:

$$\eta_t + \lambda\eta = \delta z_m^T\Omega z_m - e_m^T\Omega e_m \geq -\frac{\eta}{\theta}, \tag{17}$$

with $\eta_0 > 0$ and Lemma 1, we can get

$$\eta \geq \eta_0 e^{-(\lambda+\frac{1}{\theta})t},\ \forall t > 0. \tag{18}$$

The following Lyapunov-like functions are adopted

$$V(t) = \sum_{n=1}^5 V_n(t),$$

where

$$V_1(t) = \int_0^l \xi^T P\xi dx,\ V_2(t) = \int_0^l \xi_x^T S\bar{\Theta}\xi_x dx,$$
$$V_3(t) = h\int_0^l\int_{-h}^0\int_{t+\sigma}^t \xi_s^T(s)R\xi_s(s)dsd\sigma dx,$$
$$V_4(t) = \int_0^l\int_{t-h}^t \xi^T(s)Q\xi(s)dsdx,\ V_5(t) = \int_0^l \eta dx.$$

Calculating the derivative of $V(t)$, one obtains

$$\begin{aligned}\dot{V}_1(t) &= \int_0^l 2\xi^T P\xi_t dx,\ \dot{V}_2(t) = -\int_0^l 2\xi_t^T S\bar{\Theta}\xi_{xx}dx,\\ \dot{V}_3(t) &= \int_0^l [h^2\xi_t^T R\xi_t - h\int_{t-h}^t \xi_s^T(s)R\xi_s(s)ds]dx,\\ \dot{V}_4(t) &= \int_0^l [\xi^T Q\xi - \xi_h^T Q\xi_h]dx,\ \dot{V}_5(t) = \int_0^l \eta_t dx.\end{aligned} \tag{19}$$

Based on Lemma 2, the following inequality can be obtained

$$\begin{aligned}&-h\int_0^l\int_{t-h}^t \xi_s^T(s)R\xi_s(s)dsdx\\ &\leq -\int_0^l\{\frac{h}{h-h(t)}[\int_{t-h}^{t-h(t)} \xi_s^T(s)ds]R[\int_{t-h}^{t-h(t)} \xi_s(s)ds]\\ &\quad + \frac{h}{h(t)}[\int_{t-h(t)}^t \xi_s{}^T(s)ds]R[\int_{t-h(t)}^t \xi_s(s)ds]\}dx.\end{aligned} \tag{20}$$

Combing (20) and Lemma 3, it is obvious that the following inequality holds

$$-h\int_0^l\int_{t-h}^t \xi_s^T(s)R\xi_s(s)dsdx$$

$$\leq -\int_0^l \{[\int_{t-h}^{t-h(t)} \xi_s^T(s)ds]R[\int_{t-h}^{t-h(t)} \xi_s(s)ds]$$
$$+[\int_{t-h(t)}^{t} \xi_s^T(s)ds]R[\int_{t-h(t)}^{t} \xi_s(s)ds]$$
$$+2[\int_{t-h}^{t-h(t)} \xi_s^T(s)ds]G[\int_{t-h(t)}^{t} \xi_s(s)ds]\}dx. \quad (21)$$

From (13), we can get

$$0 = 2\int_0^l [\xi_t^T S + \xi^T S]\{-\xi_t + \bar{\Theta}\xi_{xx}$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r} h_i g_j [\bar{A}_{ij}\xi + \bar{B}_{ij}\xi_{ht} - \bar{B}_j e_{mh} + \bar{C}_i\omega]\}dx. \quad (22)$$

According to boundary conditions (2), the following equation can be obtained

$$2\int_0^l \xi^T S\bar{\Theta}\xi_{xx}dx = -\int_0^l \xi_x^T (S\bar{\Theta} + \bar{\Theta}S^T)\xi_x dx. \quad (23)$$

Define

$$\bar{y} \triangleq \mathrm{col}[\xi \quad \xi_t \quad \xi_x \quad \xi_{ht} \quad \xi_h \quad e_{mh} \quad \omega].$$

Inspired by [45], considering (19)-(23) with $\omega = 0$ and applying the Schur complement lemma to (15), it is easy to get

$$\dot{V}(t) \leq 0,$$

for all $t > 0$. Therefore, system (13) with $\omega = 0$ is asymptotically stable.

Next, the $L_2 - L_\infty$ performance of the filtering error system (13) is established under the zero initial condition. For all $\xi \neq 0$ and $\eta > 0$, the following inequalities can be obtained

$$\dot{V}(t) - \omega^T(t)\omega(t) \leq \int_0^l \sum_{i=1}^{r}\sum_{j=1}^{r} h_i g_j \bar{y}^T \Psi_{1ij}\bar{y}dx \leq 0, \quad (24)$$

which means

$$\int_0^l \xi^T P\xi dx \leq V(t) \leq \int_0^l \int_0^t \omega^T(s)\omega(s)dsdx,$$

and using the schur complement, (16) can guarantee

$$\sum_{i=1}^{r}\sum_{j=1}^{r} \bar{D}_{ij}^T \bar{D}_{ij} < \gamma^2 P.$$

Then, it is easily constructed the following inequality for $t > 0$,

$$\int_0^l \tilde{z}_o^T \tilde{z}_o dx = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i g_j \int_0^l \xi^T \bar{D}_{ij}^T \bar{D}_{ij}\xi dx$$
$$< \gamma^2 \int_0^l \xi^T P\xi dx \leq \gamma^2 \int_0^l \int_0^t \omega^T(s)\omega(s)dsdx$$
$$\leq \gamma^2 \int_0^l \int_0^\infty \omega^T(s)\omega(s)dsdx. \quad (25)$$

Therefore, $\gamma\|\omega\|_2 > \|\tilde{z}_o\|_\infty$, for any non-zero $\omega(t) \in \mathcal{L}_2[0,\infty)$, this completes the proof. □

Remark 4: If there exist matrices $P > 0$, $Q > 0$, $R > 0$ and $S\bar{\Theta} > 0$, combining with [12], we can get the Lyapunov function $\sum_{n=1}^{4} V_n(t) \geq 0$. Meanwhile, based on the (18) with $l > 0$, we can get $V_5 = \int_0^l \eta dx \geq 0$. Therefore, the Lyapunov-like functions satisfy the positive semidefinite.

Remark 5: The obtained conditions (15)-(16) in Theorem 1 guarantee that system (13) is asymptotically stable with L_2-L_∞ performance. However, adopting the same membership function for the RO filter as [37] will increase the complexity of the designed filter. Inspired by [46], we will solve this problem in the following Theorem 2.

Theorem 2: For given scalars $0 < \delta < 1$, $0 < \rho_j \leq 1$ and $h > 0$, the membership functions satisfying $g_j - \rho_j h_j \geq 0$, the systems (13) is asymptotically stable with L_2-L_∞ performance, if there exist matrices $P > 0$, $\Upsilon_{\iota i} = \Upsilon_{\iota i}^T$, $Q > 0$, $R > 0$, S satisfying $S\bar{\Theta} \geq 0$ and G, which make the following inequalities hold for all $i \in \{1, 2, \cdots, r\}$, $j \in \{1, 2, \cdots, r\}$, $\iota = \{1, 2\}$,

$$\begin{cases} \Psi_{\iota ij} - \Upsilon_{\iota i} < 0, \\ \rho_i \Psi_{\iota ij} - \rho_i \Upsilon_{\iota i} + \Upsilon_{\iota i} < 0, \ i < j, \\ \rho_j \Psi_{\iota ij} + \rho_i \Psi_{\iota ji} - \rho_j \Upsilon_{\iota i} - \rho_i \Upsilon_{\iota j} + \Upsilon_{\iota i} + \Upsilon_{\iota j} < 0, \end{cases} \quad (26)$$

where $\Psi_{\iota ij}$ are shown in Theorem 1.

Proof: Defining $\Upsilon_{\iota i} = \Upsilon_{\iota i}^T$, we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i [h_j - g_j]\Upsilon_{\iota i} = 0. \quad (27)$$

Inspired by [46], one can obtain

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i g_j \Psi_{\iota ij}$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\{[h_j - g_j + \rho_j g_j - \rho_j g_j]\Upsilon_{\iota i} + h_i g_j \Psi_{\iota ij}\}$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\{h_j[\rho_j \Psi_{\iota ij} - \rho_j \Upsilon_{\iota i} + \Upsilon_{\iota i}]$$
$$+ (g_j - \rho_j h_j)(\Psi_{\iota ij} - \Upsilon_{\iota i})\}$$
$$= \sum_{i=1}^{r} h_i^2(\rho_i \Psi_{\iota ii} - \rho_i \Upsilon_{\iota i} + \Upsilon_{\iota i})$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(g_j - \rho_j h_j)(\Psi_{\iota ij} - \Upsilon_{\iota i})$$
$$+ \sum_{i=1}^{r-1}\sum_{j=1}^{r} h_i h_j(\rho_j \Psi_{\iota ij} - \rho_j \Upsilon_{\iota i} + \Upsilon_{\iota i} + \rho_i \Psi_{\iota ji}$$

$$-\rho_i\Upsilon_{\iota j}+\Upsilon_{\iota j}). \tag{28}$$

Considering $g_j-\rho_j h_j \geq 0$ and (26), we can get $\sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_i g_j \Psi_{\iota ij}<0$. Thus, the system (13) is asymptotically stable with L_2-L_∞ performance, which completes the proof. □

The problem of the RO filter design will be solved by Theorem 3.

Theorem 3: Given scalars $0<\rho_j\leq 1$, $h>0$, $\gamma>0$ and $\varepsilon\in[0,1)$, if there exist matrices $\tilde{A}_{kj}$, $\tilde{B}_{kj}$, $\tilde{C}_{kj}$, $S_1>0$, $R>0$, $\Omega>0$, and $Q>0$ satisfying the following matrix inequations for $i\in\{1,2,\cdots,r\}$, $j\in\{1,2,\cdots,r\}$, $\iota=\{1,2\}$,

$$\begin{cases}\bar{\Psi}_{\iota ij}-\bar{\Upsilon}_{\iota i}<0,\\ \rho_i\bar{\Psi}_{\iota ij}-\rho_i\bar{\Upsilon}_{\iota i}+\bar{\Upsilon}_{\iota i}<0,\ i<j,\\ \rho_j\bar{\Psi}_{\iota ij}+\rho_i\bar{\Psi}_{\iota ji}-\rho_j\bar{\Upsilon}_{\iota i}-\rho_i\bar{\Upsilon}_{\iota j}+\bar{\Upsilon}_{\iota i}+\bar{\Upsilon}_{\iota j}<0,\end{cases} \tag{29}$$

where $\bar{\Psi}_{1ij}=\begin{bmatrix}\Phi^1_{1ij} & \Phi^2_{1ij}\\ * & \Phi^3_1\end{bmatrix}$, $\bar{\Psi}_{2ij}=\Psi_{2ij}$, $\Phi^1_{1ij}=\begin{bmatrix}\Phi^{11}_{1ij} & \Phi^{12}_{1ij} & 0\\ * & \Phi^{13}_{1ij} & 0\\ * & * & \Phi^{14}_{1ij}\end{bmatrix}$, $\Phi^2_{1ij}=\begin{bmatrix}\Phi^{21}_{1ij} & G & \Phi^{23}_{1ij}\\ \Phi^{22}_{1ij} & 0 & \Phi^{23}_{1ij}\\ 0 & 0 & 0\end{bmatrix}$, $\Phi^3_1=\Psi^3_1$, $\Phi^{11}_{1ij}=\begin{bmatrix}\Phi^{111}_{1ij} & \Phi^{112}_{1ij}\\ * & \Phi^{122}_{1ij}\end{bmatrix}$, $\Phi^{12}_{1ij}=\begin{bmatrix}\Phi^{113}_{1ij} & \Phi^{114}_{1ij}\\ \Phi^{123}_{1ij} & \Phi^{124}_{1ij}\end{bmatrix}$, $\Phi^{13}_{1ij}=\begin{bmatrix}\Phi^{133}_{1ij} & \Phi^{134}_{1ij}\\ * & \Phi^{144}_{1ij}\end{bmatrix}$, $\Phi^{14}_{1ij}=\begin{bmatrix}\Phi^{155}_{1ij} & \Phi^{156}_{1ij}\\ * & \Phi^{166}_{1ij}\end{bmatrix}$, $\Phi^{21}_{1ij}=\begin{bmatrix}\Phi^{211}_{1ij} & \Phi^{212}_{1ij}\\ \Phi^{221}_{1ij} & \Phi^{222}_{1ij}\end{bmatrix}$, $\Phi^{22}_{1ij}=\begin{bmatrix}\Phi^{231}_{1ij} & 0\\ \Phi^{241}_{1ij} & 0\end{bmatrix}$, $\Phi^{23}_{1ij}=\begin{bmatrix}\Phi^{215}_{1ij} & \Phi^{216}_{1ij}\\ \Phi^{225}_{1ij} & \Phi^{226}_{1ij}\end{bmatrix}$, $\Phi^{111}_{1ij}=Q_1-R_1+S_1A_i+A_i^TS_1^T$, $\Phi^{112}_{1ij}=Q_2-R_2+\mathcal{H}\tilde{A}_{kj}+A_i^T\mathcal{H}\mathcal{V}$, $\Phi^{113}_{1ij}=P_1+A_i^TS_1^T-S_1$, $\Phi^{114}_{1ij}=P_2+A_i^T\mathcal{H}\mathcal{V}-\mathcal{H}\mathcal{V}$, $\Phi^{122}_{1ij}=Q_3-R_3+\tilde{A}_{kj}+\tilde{A}_{kj}^T$, $\Phi^{123}_{1ij}=P_3+\tilde{A}_{kj}^T\mathcal{H}^T-\mathcal{V}^T\mathcal{H}^T$, $\Phi^{124}_{1ij}=P_4+\tilde{A}_{kj}^T-\mathcal{V}^T$, $\Phi^{133}_{1ij}=h^2R_1-S_1-S_1^T$, $\Phi^{134}_{1ij}=h^2R_2-2\mathcal{H}\mathcal{V}$, $\Phi^{144}_{1ij}=h^2R_3-\mathcal{V}^T-\mathcal{V}$, $\Phi^{155}_{1ij}=-S_1\Theta-\Theta S_1^T$, $\Phi^{156}_{1ij}=-\mathcal{H}S_2(\Theta+\Theta_k)$, $\Phi^{166}_{1ij}=-S_2\Theta_k-\Theta_kS_2^T$, $\Phi^{211}_{1ij}=R_1-G_1^T+\mathcal{H}\tilde{B}_{kj}E_i$, $\Phi^{212}_{1ij}=R_2-G_2^T$, $\Phi^{215}_{1ij}=-\mathcal{H}\tilde{B}_{kj}\mathcal{H}^T$, $\Phi^{216}_{1ij}=S_1C_i$, $\Phi^{221}_{1ij}=R_2^T-G_2+\tilde{B}_{kj}E_i$, $\Phi^{222}_{1ij}=R_3-G_3^T$, $\Phi^{225}_{1ij}=-\tilde{B}_{kj}\mathcal{H}^T$, $\Phi^{226}_{1ij}=\mathcal{V}^T\mathcal{H}^TC_i$, $\Phi^{231}_{1ij}=\mathcal{H}\tilde{B}_{kj}E_i$, $\Phi^{241}_{1ij}=\tilde{B}_{kj}E_i$, $\bar{\Upsilon}_{1i}=\text{diag}\{U^T, U^T, I, I, I, I, I\}\Upsilon_{1i}\{U, U, I, I, I, I, I\}$, $\bar{\Upsilon}_{2i}=\Upsilon_{2i}$, then the RO filter design problem is solvable. Furthermore, the desired RO filter parameters in (12) can be derived with the following transformation relation

$$\begin{bmatrix}A_{kj} & B_{kj}\\ C_{kj} & 0\end{bmatrix}\triangleq\begin{bmatrix}\mathcal{V}^{-1} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\tilde{A}_{kj} & \tilde{B}_{kj}\\ \tilde{C}_{kj} & 0\end{bmatrix}. \tag{30}$$

Proof: First, the non-singular matrix S is split into $S\triangleq\begin{bmatrix}S_1 & \mathcal{H}S_2\\ * & S_3\end{bmatrix}$ with $\mathcal{H}\triangleq[I_{r\times r}\ \ 0_{r\times(n-r)}]^T$, $S_1\in R^{n\times n}$, $S_2\in R^{r\times r}$, $S_3\in R^{r\times r}$.

Then the following matrix is defined

$$U\triangleq\begin{bmatrix}I & 0\\ 0 & S_3^{-1}S_2^T\end{bmatrix}, \mathcal{V}\triangleq S_2S_3^{-1}S_2^T, \tag{31}$$

$$\begin{bmatrix}\tilde{A}_{kj} & \tilde{B}_{kj}\\ \tilde{C}_{kj} & 0\end{bmatrix}\triangleq\begin{bmatrix}S_2 & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_{kj} & B_{kj}\\ C_{kj} & 0\end{bmatrix}\begin{bmatrix}S_3^{-1}S_2^T & 0\\ 0 & I\end{bmatrix}, \tag{32}$$

furthermore, we can get

$$\begin{aligned}&U^TS\bar{A}_{ij}U\triangleq\begin{bmatrix}S_1A_i & \mathcal{H}\tilde{A}_{kj}\\ \mathcal{V}^T\mathcal{H}^TA_i & \tilde{A}_{kj}\end{bmatrix},\\ &U^TS\bar{B}_j\triangleq\begin{bmatrix}\mathcal{H}\tilde{B}_{kj}\mathcal{H}^T\\ \tilde{B}_{kj}\mathcal{H}^T\end{bmatrix},\\ &U^T\bar{A}_{ij}^TS^TU\triangleq\begin{bmatrix}A_i^TS_1^T & A_i^T\mathcal{H}\mathcal{V}\\ \tilde{A}_{kj}^T\mathcal{H}^T & \tilde{A}_{kj}^T\end{bmatrix},\\ &U^TS\bar{C}_i\triangleq\begin{bmatrix}S_1C_i\\ \mathcal{V}^T\mathcal{H}^TC_i\end{bmatrix},\\ &U^TS^TU\triangleq\begin{bmatrix}S_1^T & \mathcal{H}\mathcal{V}\\ \mathcal{V}^T\mathcal{H}^T & \mathcal{V}^T\end{bmatrix},\\ &U^TSU\triangleq\begin{bmatrix}S_1 & \mathcal{H}\mathcal{V}\\ \mathcal{V}^T\mathcal{H}^T & \mathcal{V}\end{bmatrix},\\ &U^TS\bar{B}_{ij}\triangleq\begin{bmatrix}\mathcal{H}\tilde{B}_{kj}E_i & 0\\ \tilde{B}_{kj}E_i & 0\end{bmatrix}.\end{aligned} \tag{33}$$

Performing contract transformation on Ψ_{1ij} in (15) with diag$\{U\ \ U\ \ I\ \ I\ \ I\ \ I\ \ I\}$.

If the condition that (31)-(33) hold, inequality (29) holds. Therefore, the system (13) satisfies the asymptotic stability condition with L_2-L_∞ performance. Moreover, note that (32) is equivalent to

$$\begin{aligned}\begin{bmatrix}A_{kj} & B_{kj}\\ C_{kj} & 0\end{bmatrix}\triangleq&\begin{bmatrix}(S_2^{-T}S_3)^{-1}\mathcal{V}^{-1} & 0\\ 0 & I\end{bmatrix}\\ &\times\begin{bmatrix}\tilde{A}_{kj} & \tilde{B}_{kj}\\ \tilde{C}_{kj} & 0\end{bmatrix}\begin{bmatrix}S_2^{-T}S_3 & 0\\ 0 & I\end{bmatrix}.\end{aligned} \tag{34}$$

Consequently, the filter parameters (A_{kj}, B_{kj}, C_{kj}) are obtained from (34). In general, let $S_2^{-T}S_3=I$, (30) can be obtained, which can be adopted to construct the RO fuzzy filter in (12) and complete the proof. □

Based on the above analysis, the fuzzy RO filter design and DETM can be organized as in Algorithm 1.

Remark 6: The matrix $\mathcal{H}$ is called the RO factor and plays a crucial role in the RO filter design process. When $\mathcal{H}$ is a unit matrix, the filter designed at this time is a full-order (FO) filter. Thus, we can easily convert between the RO and FO filter by changing the order of the matrix $\mathcal{H}$.

Remark 7: The above non-PDC method only considers the partial mismatch problem between the fuzzy systems and the fuzzy filter (i.e., premise variables and fuzzy sets do not match) and does not consider the fuzzy rules mismatch case (complete mismatch). Therefore, inspired by [47], the completely mismatched fuzzy filter design

Algorithm 1: Fuzzy RO filter design with DETM.

Set appropriate DETM parameters θ, δ and λ;
for $t=0:3$ and $z=0:1$
  if Trigger condition are met; **then**
    Signal transmission;
  else
    Cancel transmission.
  end
end
The filter parameters A_{kj}, B_{kj} and C_{kj} are obtained by solving linear matrix inequalities (29);
for $i=1:2, j=1:2$
  if Satisfy tmin<0 in the feasp solver for (29);
    Get the filter parameters A_{kj}, B_{kj} and C_{kj};
  else
    Change the given systems parameters and DETM parameters to meet tmin< 0.
  end
end

method is introduced to further make the designed filter more flexible and reasonable, which is designed as follows:

The model of the fuzzy filter in (12) is given as follows:

$$\begin{cases} \hat{y}_t = \Theta_k \hat{y}_{xx} + \sum_{j=1}^{c} g_j [A_{kj}\hat{y} + B_{kj}\hat{z}_m], \\ \hat{z}_o = \sum_{j=1}^{c} g_j C_{kj}\hat{y}, \end{cases} \tag{35}$$

where $g_j = \frac{\prod_{v=1}^{p} N_v^j(\vartheta_v)}{\sum_{j=1}^{c}\prod_{v=1}^{p} N_v^j(\vartheta_v)}$ and satisfy $g_j \geq 0$, $\sum_{j=1}^{c} g_j = 1$ and $c \neq r$.

According to Theorem 1 in [47], the system (13) is asymptotically stable with L_2-L_∞ performance, if there exist ϖ_{ij} that $h_i g_j - \bar{h}_i \bar{g}_j - \varpi_{ij} > 0$ and matrices $M = M^T$, $W_{ij} = W_{ij}^T$, $W_{ij} \geq 0$, such that the following inequalities hold

$$\sum_{i=1}^{r}\sum_{j=1}^{c}((\bar{h}_i\bar{g}_j + \varpi_{ij})(\bar{\Psi}_{\iota ij} + W_{ij}) + \varpi_{ij}M) < 0, \tag{36}$$

where the $\bar{h}_i$ and $\bar{g}_j$ are the staircase membership functions that satisfy $\bar{h}_i \in [0,\ 1]$, $\bar{g}_j \in [0,\ 1]$, $\sum_{i=1}^{r}\bar{h}_i = 1$ and $\sum_{i=1}^{c}\bar{g}_j = 1$.

4. NUMERICAL EXAMPLE

Example 1: In this section, the following FHN equation is considered to indicate the applicability of the proposed approach

$$\begin{cases} y_{1t} = y_{1xx} - y_1^3 - 1.2y_1 - y_2 + 2\omega, \\ y_{2t} = y_{2xx} - 0.1y_2 + 0.5\omega, \end{cases} \tag{37}$$

with the following boundary and initial conditions

$$y_x(0,t) = y_x(1,t) = 0,$$
$$y_1(s) = 0.5 + 0.3\cos(\pi x),$$
$$y_2(s) = 0.4\cos(\pi x).$$

Assume $y = \mathrm{col}[y_1\ \ y_2]$, and the external disturbance ω is given as follows:

$$\omega = \cos(0.05\pi x)\sin(4\pi t),\ t \in [0,\ 2).$$

Then, the systems (37) can be described as follows:

$$y_t = y_{xx} + Ay + C\omega,$$

with $A = \begin{bmatrix} -y_1^2 - 1.2 & -1 \\ 0 & -0.1 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$.

If $y_1 \in [-1.5\ \ 1.5]$, then $y_1^2 \in [0\ \ 2.25]$, y_1^2 can be expressed as follows:

$$y_1^2 = 2.25 \cdot h_1(y_1) + 0 \cdot h_2(y_1),$$

where $h_1(y_1) \in [0,\ 1]$, $h_2(y_1) \in [0,\ 1]$, $h_1(y_1) + h_2(y_1) = 1$. It can be noticeably obtained

$$h_1(y_1) = \frac{1}{2.25}y_1^2,\ h_2(y_1) = 1 - h_1(y_1).$$

Similar to the procedure in [17], the T-S fuzzy model can be generated as follows:

$$y_t = y_{xx} + \sum_{i=1}^{2} h_i(y_1)[A_i y + C_i\omega].$$

We can also obtain the following estimated output and measured output signals

$$z_o = \sum_{i=1}^{2} h_i(y_1)D_i y,\ z_m = \sum_{i=1}^{2} h_i(y_1)E_i y,$$

where

$$A_1 = \begin{bmatrix} -3.45 & -1 \\ 0 & -0.1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.2 & -1 \\ 0 & -0.1 \end{bmatrix},$$
$$C_1 = C_2 = \begin{bmatrix} 2 & 0.5 \end{bmatrix}^T,\ D_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0 & 1 \end{bmatrix},\ E_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

and the rest of the parameters are shown in Table 1.

Assume $g_2(\hat{y}_1) = \cos(0.8\hat{y}_1)$, $g_1(\hat{y}_1) = 1 - g_2(\hat{y}_1)$, and the filter parameters are obtained by Theorem 3

$$A_{k1} = -21.3842,\ B_{k1} = -3.6025,\ C_{k1} = -5.1507,$$

Table 1. Simulation parameters.

$\delta = 0.05$	$h = 1$ ms	$\Theta = 1$	$\Theta_k = 0.5$	$\lambda = 0.2$
$\rho_1 = 0.9$	$\rho_2 = 0.8$	$\theta = 20$	$\eta_0 = 1$	

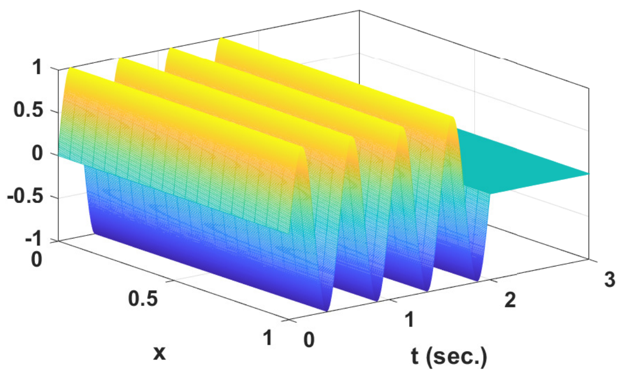

Fig. 2. The trajectory of $\omega(x,t)$.

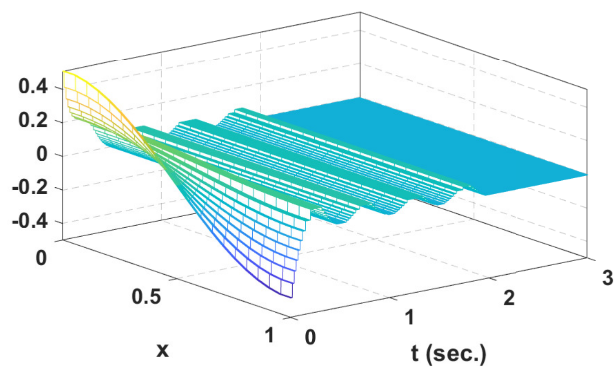

Fig. 3. The trajectory of $\hat{z}_m(x,t)$.

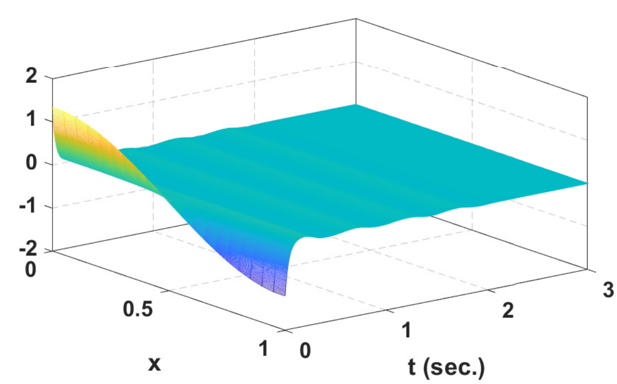

Fig. 4. The trajectory of $\tilde{z}(x,t)$.

$$A_{k2} = -19.9713,\ B_{k2} = -3.9856,\ C_{k2} = -4.1226,$$
$$\Omega = 32.6733,\ \gamma = 0.5596.$$

Under the zero initial condition, one gets $\frac{\|\tilde{z}_o\|_\infty}{\|\omega\|_2} = 0.0833 < 0.5596$.

Simulation results: The trajectory of the external disturbance is shown in Fig. 2. The actual input of the RO filter based on the DETM is given in Fig. 3, which shows that the DETM reduces the network data transmission effectively.

The trajectory of the filtering error is shown in Fig. 4, which can be seen that the filtering system satisfies asymptotically stable with $L_2 - L_\infty$ performance. Fig. 5 presents

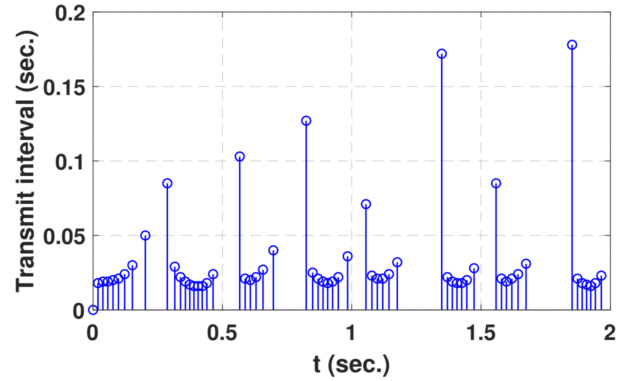

Fig. 5. Release instants and release interval.

Table 2. The NTs by different ETM.

δ	The dynamic event	The static event
$\delta = 0.01$	362	390
$\delta = 0.05$	185	208
$\delta = 0.1$	141	156
$\delta = 0.5$	61	67

Table 3. The NTs by different θ with δ.

θ	$\delta = 0.01$	$\delta = 0.05$	$\delta = 0.1$	$\delta = 0.5$
$\theta = 0.2$	362	185	141	61
$\theta = 20$	367	204	153	66
ETM	390	208	156	67

the release instants and release intervals under the DETM (7) to reduce network transmission data and improve the utilization of network resources. The following two tables further demonstrate the adopted approach's effectiveness. For different δ, the number of transmissions (NTs) with $\theta = 80$, $\lambda = 8$ are chosen in Table 2. In Table 3, varying the δ yields the NTs with different θ. As shown in the table above, the DETM is more effective in mitigating communication redundancy and improving resource utilization efficiency than static ETM, and with increasing θ and δ, the NTs decrease, further demonstrating the effectiveness of the adopted DETM.

Example 2: Consider a class of nonlinear semi-linear parabolic PDE systems

$$\begin{cases} y_{1t} = y_{1xx} - \sin(y_1) - 1.2y_2 - y_2 + 2\omega, \\ y_{2t} = y_{2xx} - 0.1y_1 + y_2^3 + 0.5\omega, \end{cases} \tag{38}$$

and assuming the same boundary and initial conditions as Example 1. Based on the fuzzy method in the NUMERICAL EXAMPLES of [48] and assume

$$y_1 \in [-a,\ a],\ y_2 \in [-b,\ b],$$

where $a = 0.8$ and $b = 1.5$. The nonlinear PDE systems (38) can be represented by the following T-S fuzzy model

Plant rule 1: IF y_1 is F_{11} and y_2 is F_{21}, Then

$$y_t = y_{xx} + A_1 y + C_1 \omega,$$

Plant rule 2: IF y_1 is F_{11} and y_2 is F_{22}, Then

$$y_t = y_{xx} + A_2 y + C_2 \omega,$$

Plant rule 3: IF y_1 is F_{12} and y_2 is F_{21}, Then

$$y_t = y_{xx} + A_3 y + C_3 \omega,$$

Plant rule 4: IF y_1 is F_{12} and y_2 is F_{22}, Then

$$y_t = y_{xx} + A_4 y + C_4 \omega,$$

where

$$\begin{aligned}
&F_{11}(y_1) = \frac{y_1^2}{a^2},\ F_{12}(y_1) = 1 - F_{11}(y_1),\\
&F_{21}(y_2) = \begin{cases} \dfrac{b \sin y_2 - y_2 \sin b}{y_2(b - \sin b)}, & y_2 \neq 0,\\ 1, & y_2 = 0,\end{cases}\\
&F_{22}(y_1) = 1 - F_{21}(y_2),\\
&A_1 = \begin{bmatrix} 1 & -1.2\\ -0.1 & a^2 \end{bmatrix},\ A_2 = \begin{bmatrix} \frac{\sin(b)}{b} & -1.2\\ -0.1 & a^2 \end{bmatrix},\\
&A_3 = \begin{bmatrix} 1 & -1.2\\ -0.1 & 1 \end{bmatrix},\ A_4 = \begin{bmatrix} \frac{\sin(b)}{b} & -1.2\\ -0.1 & 1 \end{bmatrix},\\
&C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 2 & 0.5 \end{bmatrix}^T,
\end{aligned}$$

and the rest of the parameters are shown in Table 1. The external disturbance ω is given as follows:

$$\omega = e^{-t} \cos(0.5x) \sin(4\pi t),\ t \in [0,\ 2).$$

Assume $g_2(\hat{y}_1) = \cos(0.8\hat{y}_1)$, $g_1(\hat{y}_1) = 1 - g_2(\hat{y}_1)$. By using the method in [47], the filter parameters can be obtained

$$\begin{aligned}
&A_{k1} = -4.9312,\ B_{k1} = -2.7306,\ C_{k1} = -1.2654,\\
&A_{k2} = -4.8300,\ B_{k2} = -3.1016,\ C_{k2} = -1.5153,\\
&\Omega = 20.7299,\ \gamma = 0.6749.
\end{aligned}$$

Under the zero initial condition, one gets $\frac{\|\tilde{z}_o\|_\infty}{\|\omega\|_2} = 0.0027 < 0.6749$.

Simulation results: The trajectory of the external disturbance is shown in Fig. 6. Fig. 7 represents the actual input of the fuzzy RO filter. The trajectory of the filtering error is shown in Fig. 8. When $x = 0.5$, the original systems output z_o, the estimated output $\hat{z}_o$ and the filtering error $\tilde{z}_o$ are shown in Fig. 9. The release instants and intervals under the DETM are presented in Fig. 10. From these figures, it can be seen that the ideal estimation of z_o is achieved by the L_2-L_∞ RO filter based on the DETM, which can save communication bandwidth and computational resources to a certain extent.

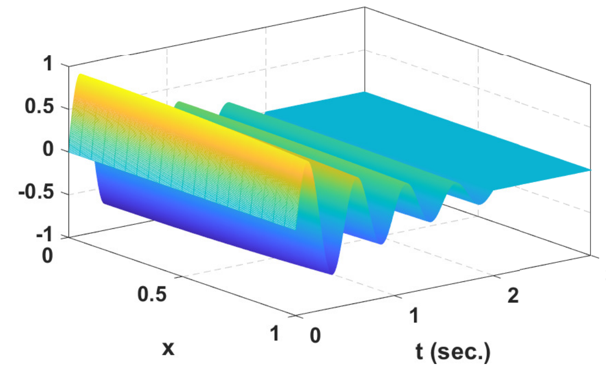

Fig. 6. The trajectory of $\omega(x,t)$.

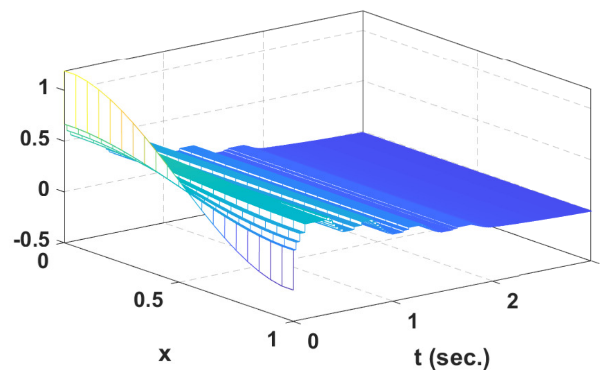

Fig. 7. The trajectory of $\hat{z}_m(x,t)$.

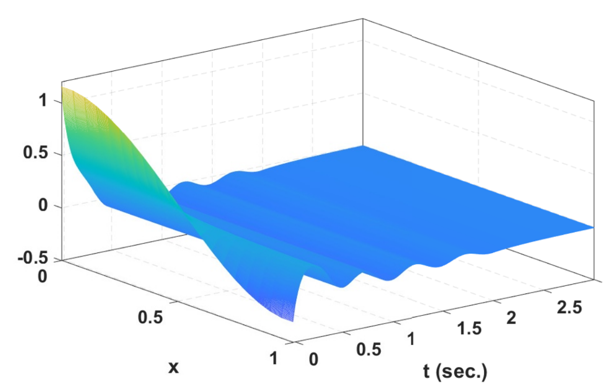

Fig. 8. The trajectory of $\tilde{z}(x,t)$.

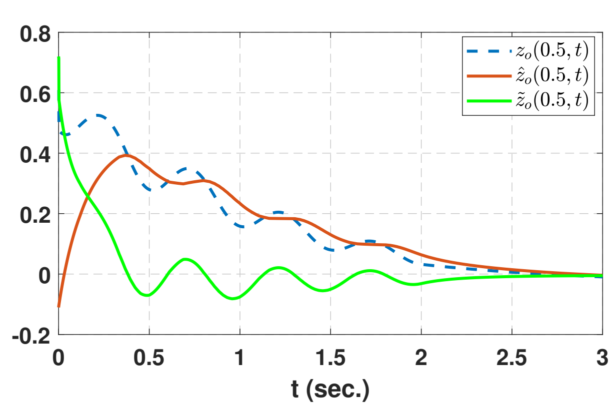

Fig. 9. Outputs z_o, estimated output $\hat{z}_o$, filtering error $\tilde{z}_o$.

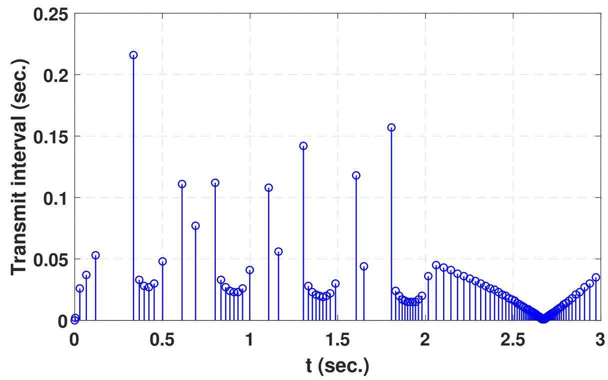

Fig. 10. Release instants and release interval.

5. CONCLUSION

In this work, a fuzzy dynamic event-triggered RO filter has been developed for fuzzy PDE systems. Initially, a T-S fuzzy model has been adopted to reconstruct the nonlinear systems. Then, a DETM has been adopted to improve resource utilization. Due to the DETM can lead to premise variables' incomplete match between the fuzzy filter and the original systems, the non-PDC method can effectively increase the flexibility of the filter design. In addition, a Lyapunov-like function has been constructed, and sufficient conditions for asymptotic stability of the augmented system have been obtained by the Lyapunov direct method. Furthermore, the relevant parameters of the fuzzy RO filter have been obtained by the linear matrix inequalities method. Eventually, two numerical examples illustrate that the proposed method is effective. Recently, a polynomial fuzzy model has been employed for PDE systems, which reduces conservatism effectively. Thus, polynomial fuzzy filtering is worth exploring.

CONFLICT OF INTERESTS

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

We are sure that we have obtained all necessary permissions for any material in our paper that has been previously published.

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