Adaptive Neural Network Control of a 2-DOF Helicopter System with Input Saturation

Jian Zhang, Yubao Yang, Zhijia Zhao* (D), and Keum-Shik Hong

Abstract: This paper investigates an adaptive neural network control strategy for a two-degree-of-freedom helicopter system with input saturation and unknown external disturbances. Firstly, the radial basis function neural network is used to compensate the uncertainty and input saturation error of the system. Furthermore, a disturbance observer is designed to deal with complex disturbances composed of unknown disturbances and neural network errors. By constructing and analyzing the Lyapunov function, the stability of the helicopter system is strictly guaranteed. Finally, the numerical simulations and experiments conducted on the Quanser laboratory platform reveal that the proposed control strategy is suitable and effective.

Keywords: Adaptive neural network control, external disturbances, input saturation, 2-DOF helicopter.

1. INTRODUCTION

Unmanned aerial vehicles (UAVs) have attracted wide attention and have thus undergone dramatical developments recently [1–4]. For example, in [1], for attitude and vibration control problems in flexible spacecraft systems, the authors proposed an adaptive control. In [2–4], the authors innovatively designed a bionic flapping-wing aerial robot with low energy consumption and high agility. Motivated by the above applications, the UAVs deserve further investigatation.

Helicopters represent a typical UAV with characteristics such as simple operation, low requirements for taking off and landing environment, and hovering flight, and have been widely used in transportation, emergency rescue, geological exploration, and other fields [5–7]. Motivated by a better application prospect, helicopters have thus attracted extensive attention from researchers. Over the past decade, researchers have proposed several types of control methods for helicopter systems including PID control, model-based control, and linear quadratic regulator control [8–11]. For instance, in [8], a linear-quadratic controller was designed and applied alone with a high performance adaptive enhancement method. Moreover, the controller implementation with the adaptive enhancement method was studied. The performance and robustness of the model due to the uncertainty in parameters and unmodeled dynamic process were also discussed. In [9], an LQR adaptive particle swarm optimization (APSO) control approach was developed for angle tracking of a 2degree of freedom (DOF) helicopter. In [10], a new model adaptive control based on a direct adaptive linear quadratic regulator (LQR) was used for tracking control of 2-DOF helicopters. In [11], a new Q-learning method for unknown discrete time LQR was presented, and the effectiveness of the method was demonstrated on a 2-DOF helicopter. However, the helicopter system was a greatly cross-coupled multi-input multi-output (MIMO) nonlinear system [12,13]. The above studies [8–11] linearized the nonlinear model of the helicopter, and neglected the nonlinear terms and uncertainties of the nonlinear helicopter system, which did not reflect the real behavior of the helicopter system. Therefore, further studies considering the nonlinear behavior of a helicopter are required.

In recent years, many control methods have been applied to nonlinear helicopter systems including sliding mode control, robust control, and fuzzy control [14–16]. For instance, in [14], a controller with two integral sliding

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surfaces was designed, which combined the continuous control technology with the super torsion control method, and the angle tracking control was implemented on a 2-DOF helicopter experimental platform. In [15], a sliding mode control strategy assisted by a generalized proportional integral observer was presented, and the effective-ness of the proposed approach was verified on a 2-DOF helicopter. In [16], a backward controller was designed to make the helicopter track the desired trajectory attitude. It should be pointed out that the above research was limited to the case with the known system model parameters. In practical situations, however, part of the model parameters are unknown and uncertain. Therefore, the development of control schemes considering uncertainties in the nonlinear system is necessary.

For the past few years, the neural network (NN) control has been a typically and generally used tool to handle uncertainties in the system model [17,18]. In [19], the authors proposed an adaptive neural fault-tolerant control approach to tackle the uncertainty, unknown external disturbances and actuator failures of a 3-DOF helicopter system. In [20], an adaptive NN control with a full state output feedback was designed to achieve the attitude and position control of a flying wing micro air vehicle (FW-MAV). In [21], The authors developed an adaptive NN rewind technique to compensate for the error effects of unmodeled dynamics and external disturbances in 3-DOF helicopter systems. In [22], based on the 3-DOF helicopter system, the authors designed an adaptive NN control strategy to solve the input and output constraint influence in the system.

On the other hand, in practice, the input of the actuator is affected by a variety of constraints, including input saturation and input dead zones [23-26]. Among them, input saturation is an important nonlinear factor affecting the stability of the closed-loop system. Ignoring the effects of input saturation in the design of the control system can degrade the performance of the closed-loop control and even cause system instability. During the past few years, many advanced control strategies have been proposed to address the effects of input saturation. For example, in [27], the authors designed a distributed collaborative control strategy, and solved the effect of input saturation in the system by using robust adaptive control method. In [28], in order to eliminate the input saturation effect in unmanned helicopter system, the authors designed a robust adaptive control strategy with compensation control input. In [29], an inner-outer loop control method combining conventional PID control and saturated thrust was designed to solve the position control of a quadrotor with state and input constraints. In [30], in order to make an UAV with input saturation have a hovering flight function, the authors proposed a strategy based on backstepping control. Although considerable progress has been made in the study of input saturation for UAV systems, there is little research reported on the input saturation for unknown nonlinear 2-DOF helicopter systems up to now.

Moreover, external disturbances in real-world environments can reduce the control performance of the actuator and even cause damage to the actuator [31,32]. In order to reduce the impact of external disturbances on the system, many types of disturbance observer (DO) have been developed. In [33], the authors designed a disturbance observer that ensures that the flexible Timoshenko manipulator converges in effective time when subjected to external disturbances. In [34], for the unknown approximation error and disturbance in flexible string system, a composite disturbance observer was developed. In [35], the authors designed a DO, which successfully estimated the upper bound of unknown disturbance and used it as a switch gain for a universal robust controller. In [36], in the study of the control of the force and position of the collaborative robot arm, the authors proposed a NN control based on DO to eliminates the negative effects of external disturbance. Although the above studies of DO have made great progress, there is no research of the DO for the 2-DOF helicopter system with unknown disturbances.

Motivated by the above researches, we propose an adaptive neural network control strategy for the 2-DOF helicopter system with input saturation and unknown disturbance. Compared to the previous studies, the contributions of this study are as follows:

- (i) Unlike [14–16], this paper considers the uncertainty problem in a nonlinear 2-DOF helicopter system in practice and employs a NN to estimate the uncertainty term in the system. In addition, compared with [28], this paper directly uses an adaptive NN to deal with the saturation errors in the system, which improves the robustness of the system.
- (ii) Different from [34,35], this study considers a composite disturbance that combines NN errors and external perturbations, and designs a DO to make the system more stable.
- (iii) The proposed control algorithm is successfully applied to a Quanser's 2-DOF helicopter experimental platform, and the experimental consequences verify the effectiveness of the controller design.

The remainder of this paper is displayed as follows. Dynamical model of the 2-DOF helicopter system and related preliminaries are arranged in Section 2. The adaptive control scheme based on DO and the radial basis function (RBF) NN are proposed in Section 3. In Section 4, the simulation results show the effectiveness of the presented control. Section 5 details the experiment performed. Section 6 summarizes and concludes the study.

2. PROBLEM STATEMENT

Fig. 1 shows a model sketch of the 2-DOF helicopter. It can be considered as a simple model of a dual-rotor helicopter that can be controlled using two DC motors. One motor locates at the back that generates the thrust forces (F_y) for controlling the yaw (ϕ) motion, and another motor locates at the front that generates the thrust forces (F_p) for controlling the pitch (θ) motion. These two motors are controlled via two finite voltage inputs V_p and V_y , respectively.

According to the Lagrangian formulation [37], the nonlinear equations of the 2-DOF helicopter system can be written as follows:

$$(J_p + M_a l_a^2) \ddot{\theta} = K_{pp} V_p + K_{py} V_y - M_a g l_a \cos(\theta) - D_p \dot{\theta} - M_a l_a^2 \dot{\phi}^2 \sin(\theta) \cos(\theta), \quad (1) (J_y + M_a l_a^2 \cos^2(\theta)) \ddot{\phi} = K_{yp} V_p + K_{yy} V_y - D_y \dot{\phi} + 2M_a l_a^2 \dot{\phi} \dot{\theta} \sin(\theta) \cos(\theta), \quad (2)$$

where θ and ϕ are the pitch and yaw angles, respectively. D_p and D_y denote the viscous friction coefficients. J_p and J_y denote the moments of inertia with respect to the pitch and yaw axes. g represents the acceleration of gravity. l_a is the distance from the origin of the fixed frame of the body to the center of mass. M_a is the total weight of the body. K_{pp} , K_{py} , K_{yp} , and K_{yy} are the thrust torque constants generated by the helicopter system [38].

Define $x_1 = [\theta, \phi]^T$ and $x_2 = [\dot{\theta}, \dot{\phi}]^T$, where $x_{11} = \theta$, $x_{12} = \phi$, $x_{21} = \dot{\theta}$, and $x_{22} = \dot{\phi}$. Considering the system uncertainty and unknown external disturbance, we transform the 2-DOF helicopter system model into a general MIMO nonlinear system

$$\dot{x}_1 = x_2, \tag{3}$$

$$\dot{x}_2 = Q(x_1, x_2) + \Delta Q(x_1, x_2)$$

+
$$(P(x_1, x_2) + \Delta P(x_1, x_2))$$
sat $(u) + d(t)$, (4)

$$y = x_1, \tag{5}$$

where $\Delta Q(x_1, x_2)$ and $\Delta P(x_1, x_2)$ are the system uncertainties, sat $(u) = [V_p, V_y]^T$ represents the control input, *u* represents the saturated nonlinear input, and d(t) is external interference to the system, $Q(x_1, x_2)$ and $P(x_1, x_2)$ are given as follows:

$$Q(x_1, x_2) = \begin{bmatrix} \frac{-M_a g l_a \cos(x_{11}) - D_p x_{21} - M_a l_a^2 x_{22}^2 \sin(x_{11}) \cos(x_{11})}{J_p + M_a l_a^2} \\ \frac{-D_y x_{22} + 2M_a l_a^2 x_{22} x_{21} \sin(x_{11}) \cos(x_{11})}{J_y + M_a l_a^2 \cos^2(x_{11})} \end{bmatrix},$$
(6)

$$P(x_1, x_2) = \begin{bmatrix} \frac{K_{pp}}{J_p + M_a l_a^2} & \frac{K_{py}}{J_p + M_a l_a^2} \\ \frac{K_{yp}}{J_y + M_a l_a^2 \cos^2(x_{11})} & \frac{K_{yy}}{J_y + M_a l_a^2 \cos^2(x_{11})} \end{bmatrix}.$$
 (7)

In addition, due to physical limitations, there will be saturation phenomenon inside the actuator, so the corresponding output is described as follows:



Fig. 1. The 2-DOF helicopter model sketch.

$$\operatorname{sat}(u_i) = \begin{cases} S_{\max}\operatorname{sgn}(u_i) & \text{if } |u_i| \ge S_{\max}, \\ u_i & \text{if } |u_i| < S_{\max}, \ i = 1, \ 2, \end{cases}$$
(8)

where S_{max} is a known bound of u_i , and by defining saturation error $\Delta u_i = \text{sat}(u_i) - u_i$.

Since the inverse matrix of the gain matrix $P(x_1, x_2)$ in the system may not exist, in order to develop an adaptive NN control scheme, we propose $u = P^T(x_1, x_2)v$, and vis a desired control input signal, which will be analyzed later. Therefore, \dot{x}_2 in the system can be rewritten as

$$\begin{split} \dot{x}_2 &= Q(x_1, x_2) + \Delta Q(x_1, x_2) + P(x_1, x_2)u \\ &+ P(x_1, x_2)\Delta u + \Delta P(x_1, x_2)(u + \Delta u) + d(t) \\ &= Q(x_1, x_2) + \Delta Q(x_1, x_2) + P(x_1, x_2)P^T(x_1, x_2)\upsilon \\ &+ P(x_1, x_2)\Delta u + \Delta P(x_1, x_2)(u + \Delta u) + d(t) \\ &= Q(x_1, x_2) + \Delta Q(x_1, x_2) - \gamma I_{2 \times 2}\upsilon \\ &+ (P(x_1, x_2)P^T(x_1, x_2) + \gamma I_{2 \times 2})\upsilon \\ &+ P(x_1, x_2)\Delta u + \Delta P(x_1, x_2)(u + \Delta u) + d(t), \end{split}$$
(9)

where $\gamma > 0$ is a design parameter, and define

$$J(x,u) = F\Delta J(x,u), \tag{10}$$

where $F = F^T$ is a design parameter, and $\Delta J(x, u) = \Delta Q(x_1, x_2) - \gamma I_{2 \times 2} \upsilon + P(x_1, x_2) \Delta u + \Delta P(x_1, x_2)(u + \Delta u)$. Substituting (10) into (9), we obtain

$$\dot{x}_2 = Q(x_1, x_2) + (P(x_1, x_2)P^T(x_1, x_2) + \gamma I_{2\times 2})\upsilon + F^{-1}J(x, u) + d(t).$$
(11)

Assumption 1: Assume that the 2-DOF helicopter system is smooth and bounded by external interference, and there is an unknown normal number δ_0 such that $|\dot{d}(t)| < \delta_0$.

Assumption 2: In this paper, the trajectory x_d is expected to be continuously bounded and derivable.

Assumption 3: For a continuously differentiable function $\Theta(l)$, if $\Theta(l)$ satisfies $\Theta \leq \kappa$, $\forall l \in [0, \infty)$, and κ is a positive constant, then $\dot{\Theta}(l)$ is bounded $\forall l \in [0, \infty)$.

Lemma 1 [39]: For a continuous function V(x,t), if it satisfies $\omega_1(||x||) \le V(x,t) \le \omega_2(||x||)$ and its derivative satisfies

$$\dot{V}(x,t) \le -\mu V + D_1,\tag{12}$$

where $\mu > 0$ and $D_1 > 0$.

Lemma 2 [40,41]: It is well known that radial basis neural nets have good approximation and learning capabilities, and any unknown smooth function $f(X), R^m \to R$ can be estimated by the following RBFNN

$$f(X) = W^T D(X), \tag{13}$$

where $X \in \Omega_X \subset \mathbb{R}^m$ is the RBFNN input vector, Ω_X is a compact set, $W = [W_1, W_2, \ldots, W_q]^T \in \mathbb{R}^{q \times n}$ is the weight vector and $D(X) = [D_1(X), D_2(X), \ldots, D_q(X)]^T$ is Gaussian basis function vector, with $D_j(X) = \exp\left(\frac{-(X-\vartheta_j)^T(X-\vartheta_j)}{b_j^2}\right)$, $j = 1, 2, \ldots, q$, where b_j is the width vector of Gaussian function, and $\vartheta_j = [\vartheta_{j1}, \vartheta_{j2}, \ldots, \vartheta_{jm}]^T$ is the center vector of the *j*th hidden layer neuron.

Since RBFNN can approximate an unknown continuous smooth function to an arbitrary desired accuracy by increasing the amount of neurons while choosing appropriate design parameters. For the smooth function f(X), we have

$$f(X) = W^{*T}D(X) + v(X),$$
 (14)

where W^{*T} is the ideal weight vector, v(X) is the approximation error vector that is bounded, and satisfies $||v|| \le v^*$ with v^* being a small positive constant.

Remark 1: In order to avoid the singularity of gain matrix $P(x_1, x_2)$, the saturated nonlinear input *u* is designed as $u = P^T(x_1, x_2)v$, and a design parameter γ is introduced to make $(P(x_1, x_2)P^T(x_1, x_2) + \gamma I_{2\times 2})$ non-singular.

3. CONTROL DESIGN

According to (11), we can see in the actual situation, $\Delta Q(x_1, x_2)$, $\Delta P(x_1, x_2)$, and Δu are unknown, so the J(x, u) term is uncertain. The RBFNN can be used to estimate the unknown parameters. Therefore, we have

$$J(x,u) = W^{*T}H(X) + v(X),$$
(15)

where W^* is the ideal weight, H(X) contains the activation function, $X = [x_1^T, x_2^T, x_d^T, \dot{x}_d^T, u^T]^T$ is the input vector of the NN, and $v \in \mathbb{R}^m$ is a bounded approximation error vector and satisfying $||v|| \le v^*$, and v^* is a very small positive number. Moreover, the estimated weight is defined as \hat{W} with the weight error $\tilde{W} = \hat{W} - W^*$. Substituting (15) into (11), we get

$$\dot{x}_{2} = Q + (PP^{T} + \gamma I_{2 \times 2})\upsilon + F^{-1}W^{*T}H + F^{-1}\nu + d(t).$$
(16)

To handle the neural network approximation error v, we define $\Xi(t) = F^{-1}v + d(t)$. Then, (16) can be written as

$$\dot{x}_{2} = Q + (PP^{T} + \gamma I_{2 \times 2})\upsilon + F^{-1}W^{*T}H + \Xi(t).$$
(17)

3.1. Design of nonlinear disturbance observer

The compound disturbance $\Xi(t)$ including unknown external disturbances and neural network errors are unknown and cannot be used in subsequent control laws. So, we propose a nonlinear disturbance observer to estimate $\Xi(t)$.

By the fact that $||v|| \le v^*$ and v^* is a very small positive constant, we have $||\Xi(t)|| \le ||F^{-1}||v^* + d(t)$. Thus, invoking Assumptions 1 and 3 yields

 $\|\dot{\Xi}(t)\| \leq \delta,$

where δ is an unknown positive constant.

The nonlinear disturbance observer is proposed as

$$\hat{\Xi} = F(x_2 - z_3),\tag{18}$$

$$\dot{z}_3 = Q + (PP^T + \gamma I_{2 \times 2})\upsilon + F^{-1}\hat{W}^T H(X) + \hat{\Xi},$$
 (19)

where $F = F^T$ is a design parameter. According to (18) and (19), we obtain

$$\dot{\hat{\Xi}} = F(\dot{x}_2 - \dot{z}_3)
= F(\Xi - \hat{\Xi}) + (W^{*T}H(X) - \hat{W}^TH(X)),$$
(20)

where $\tilde{\Xi} = \hat{\Xi} - \Xi$ and $\tilde{W} = \hat{W} - W^*$. Considering (20), we get

$$\dot{\tilde{\Xi}} = \dot{\tilde{\Xi}} - \dot{\Xi} = -F\tilde{\Xi} - \tilde{W}H(X) - \dot{\Xi}.$$
(21)

Invoking (21), we have

$$\tilde{\Xi}^T \dot{\Xi} = -\tilde{\Xi}^T F \tilde{\Xi} - \tilde{\Xi}^T \tilde{W} H(X) - \tilde{\Xi}^T \dot{\Xi}.$$
(22)

Consider the following two inequalities

$$-\tilde{\Xi}^{T}\tilde{W}H(X) \leq \|\tilde{\Xi}\| \|\tilde{W}\| \|H(X)\| \\ \leq k_{3}\zeta^{2} \|\tilde{\Xi}\|^{2} + \frac{1}{4k_{3}} \|\tilde{W}\|^{2},$$
(23)

$$-\tilde{\Xi}^{T}\dot{\Xi} \le \|\tilde{\Xi}\|^{2} + \frac{1}{4}\|\dot{\Xi}\|^{2} \le \|\tilde{\Xi}\|^{2} + \frac{1}{4}\delta^{2},$$
(24)

where $||H(X)|| \le \zeta$ and k_3 is a positive constant. Substituting (23) and (24) into (22), we get

$$\widetilde{\Xi}^{T}\dot{\Xi} \leq -\widetilde{\Xi}^{T}F\widetilde{\Xi} + k_{3}\zeta^{2}\|\widetilde{\Xi}\|^{2} + \frac{1}{4k_{3}}\|\widetilde{W}\|^{2} + \|\widetilde{\Xi}\|^{2} + \frac{1}{4}\delta^{2} \\
\leq -\widetilde{\Xi}^{T}\left(F - \left(1 + k_{3}\zeta^{2}\right)I_{2\times2}\right)\widetilde{\Xi} + \frac{1}{4}\delta^{2} \\
+ \frac{1}{4k_{3}}\|\widetilde{W}\|^{2}.$$
(25)

3.2. Design of adaptive neural network control

The error variable z_1 is defined as $z_1 = x_1 - x_d$, where $x_d = [\theta_d, \phi_d]^T$ is the desired trajectory, and we have $\dot{z}_1 = \dot{x}_1 - \dot{x}_d$. Subsequently, we define the second error variable as $z_2 = x_2 - \alpha$, with α being the virtual control variable. The time derivative of z_2 is as follows:

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}.\tag{26}$$

Consider the following candidate Lyapunov function:

$$V_1 = \frac{1}{2} z_1^T z_1.$$
 (27)

The time derivative of V_1 yields

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T (z_2 + \alpha - \dot{x}_d).$$
 (28)

The virtual control law α is proposed as

$$\alpha = -k_1 z_1 + \dot{x}_d,\tag{29}$$

where k_1 is a positive constant matrix.

Substituting (29) into (28), we have

$$\dot{V}_1 = z_1^T z_2 - z_1^T k_1 z_1. \tag{30}$$

Consider the Lyapunov function candidate as below

$$V_2 = V_1 + \frac{1}{2} z_2^T z_2.$$
(31)

The time derivative of V_2 leads to

$$\dot{V}_2 = \dot{V}_1 + z_2^T \dot{z}_2. \tag{32}$$

Substituting (17), (26), and (30) into (32), we have

$$\dot{V}_{2} = z_{1}^{T} z_{2} - z_{1}^{T} k_{1} z_{1} + z_{2}^{T} (Q + (PP^{T} + \gamma I_{2 \times 2}) \upsilon + F^{-1} W^{*T} H + \Xi - \dot{\alpha}).$$
(33)

Then the control law is selected as

$$\upsilon = (PP^{T} + \gamma I_{2\times 2})^{-1} (-Q - F^{-1} \hat{W}^{T} H(X) + \dot{\alpha} -z_{1} - k_{2} z_{2} - \hat{\Xi}),$$
(34)

where k_2 is a positive constant matrix.

The adaptive law is proposed as

$$\hat{W} = \Gamma_w (HF^{-1}z_2^T - \sigma_1 \hat{W}), \qquad (35)$$

where $\sigma_1 > 0$ is a design parameter and $\Gamma_w = \Gamma_w^T \in R^{q \times q}$. Consider the following candidate Lyapunov function

$$V_3 = V_2 + \frac{1}{2} \operatorname{tr} \left\{ \tilde{W}^T \Gamma_w^{-1} \tilde{W} \right\} + \frac{1}{2} \tilde{\Xi}^T \tilde{\Xi}.$$
 (36)

Invoking (33), and the time derivative of V_3 is given as

$$\dot{V}_3 = z_1^T \dot{z}_1 + z_2^T \dot{z}_2 + \operatorname{tr}\left\{\tilde{W}^T \Gamma_w^{-1} \dot{W}\right\} + \tilde{\Xi}^T \dot{\Xi}$$

$$= z_1^T z_2 - z_1^T k_1 z_1 + z_2^T (Q + (PP^T + \gamma I_{2\times 2})\upsilon)$$

+ $F^{-1}W^{*T}H + \Xi - \dot{\alpha}$ + tr $\left\{ \tilde{W}^T \Gamma_w^{-1} \dot{\hat{W}} \right\}$
+ $\tilde{\Xi}^T \dot{\Xi}$. (37)

Substituting (25), (34), and (35) into (37), we obtain

$$\begin{split} \dot{V}_{3} &= z_{1}^{T} z_{2} - z_{1}^{T} k_{1} z_{1} + z_{2}^{T} (-F^{-1} \tilde{W} H - \tilde{\Xi} - k_{2} z_{2} - z_{1}) \\ &+ \operatorname{tr} \left\{ \tilde{W}^{T} \Gamma_{w}^{-1} \dot{W} \right\} + \tilde{\Xi}^{T} \dot{\Xi} \\ &\leq - z_{1}^{T} k_{1} z_{1} - z_{2}^{T} \tilde{\Xi} - z_{2}^{T} k_{2} z_{2} - \sigma_{1} \operatorname{tr} \left\{ \tilde{W}^{T} \hat{W} \right\} \\ &- \tilde{\Xi}^{T} (F - (1 + k_{3} \zeta^{2}) I_{2 \times 2}) \tilde{\Xi} + \frac{1}{4} \delta^{2} + \frac{1}{4 k_{3}} \| \tilde{W} \|^{2}. \end{split}$$

$$(38)$$

Considering the following fact, we have

$$-\sigma_{1} \operatorname{tr}\left\{\tilde{W}^{T}\hat{W}\right\} = -\frac{\sigma_{1}}{2} \|\tilde{W}\|_{F}^{2} - \frac{\sigma_{1}}{2} \|\hat{W}\|_{F}^{2} + \frac{\sigma_{1}}{2} \|W^{*}\|_{F}^{2}$$
$$\leq -\frac{\sigma_{1}}{2} \|\tilde{W}\|_{F}^{2} + \frac{\sigma_{1}}{2} \|W^{*}\|_{F}^{2}, \qquad (39)$$

and

$$-z_2^T \tilde{\Xi} \le \frac{1}{2} \|z_2\|^2 + \frac{1}{2} \|\tilde{\Xi}\|^2.$$
(40)

Substituting (39) and (40) into (38), we derive

$$\dot{V}_{3} \leq -z_{1}^{T}k_{1}z_{1} - z_{2}^{T}(k_{2} - \frac{1}{2}I_{2\times2})z_{2} - (\frac{\sigma_{1}}{2} - \frac{1}{4k_{3}})\|\tilde{W}\|_{F}^{2} + \frac{\sigma_{1}}{2}\|W^{*}\|_{F}^{2} - \tilde{\Xi}^{T}\left(F - (1.5 + k_{3}\zeta^{2})I_{2\times2}\right)\tilde{\Xi} + \frac{1}{4}\delta^{2}.$$
(41)

Then, we obtain

$$\dot{V}_3 \le -\mu V_3 + B,\tag{42}$$

where

$$\mu = \min\left(2\lambda_{\min}(k_{1}), 2\lambda_{\min}(k_{2} - \frac{1}{2}I_{2\times 2}), \\ \frac{2(\frac{\sigma_{1}}{2} - \frac{1}{4k_{3}})}{\lambda_{\max}(\Gamma^{-1})}, 2\lambda_{\min}\left(F - (1.5 + k_{3}\zeta^{2})I_{2\times 2}\right)\right),$$
(43)

$$B = \frac{1}{2}\sigma_1 \|W^*\|_F^2 + \frac{1}{4}\delta^2.$$
(44)

To ensure $\mu > 0$, k_1 , k_2 , and k_3 can be chosen as

$$\begin{aligned} \lambda_{\min}(k_1) &> 0, \ \lambda_{\min}(k_2 - I) > 0, \ \frac{\sigma_1}{2} - \frac{1}{4k_3} > 0, \\ \lambda_{\min}\left(F - \left(1.5 + k_3\zeta^2\right)I_{2\times 2}\right) > 0. \end{aligned}$$
(45)

Theorem 1: According to the application of the presented NN control law (34), the nonlinear disturbance observer (18) and adaptive updating law (35) in the 2-DOF helicopter system, we conclude that z_1 , z_2 , $\tilde{\Xi}$, and \tilde{W} are semi-globally bounded. Furthermore, the tracking errors z_1 , z_2 , and approximation errors \tilde{W} , $\tilde{\Xi}$ converge to their respective compact sets Ω_{z_1} , Ω_{z_2} , $\Omega_{\tilde{W}}$, and, $\Omega_{\tilde{\Xi}}$ respectively, defined as follows:

$$\Omega_{z_1} = \left\{ z_1 \in \mathbb{R}^2 | \| z_1 \| \le \sqrt{\Lambda} \right\},\tag{46}$$

$$\Omega_{z_2} = \left\{ z_2 \in \mathbb{R}^2 | \| z_2 \| \le \sqrt{\Lambda} \right\},\tag{47}$$

$$\Omega_{\tilde{W}} = \left\{ \tilde{W} \in \mathbb{R}^{q \times 2} | \|\tilde{W}\| \le \sqrt{\frac{\Lambda}{\lambda_{max}(\Gamma_w^{-1})}} \right\},$$
(48)

$$\Omega_{\Xi} = \left\{ \tilde{\Xi} \in R^2 | \| \tilde{\Xi} \| \le \sqrt{\Lambda} \right\},\tag{49}$$

where $\Lambda = 2(V_3(0) + \frac{B}{\mu})$, and *B* and μ are defined in (44) and (43), respectively.

Proof: Multiplying (42) by $e^{\mu t}$ yields

$$\frac{\mathrm{d}}{\mathrm{d}t}(V_3 e^{\mu t}) \le B e^{\mu t}.$$
(50)

Integrating (50), we obtain

$$0 \le V_3 \le (V_3(0) - \frac{B}{\mu})e^{-\mu t} + \frac{B}{\mu}.$$
(51)

Thus, for z_1, z_2, \tilde{W} , and $\tilde{\Xi}$ we further have

$$\|z_1\| \le \sqrt{\Lambda},\tag{52}$$

$$\|z_2\| \le \sqrt{\Lambda},\tag{53}$$

$$\|\tilde{W}\| \le \sqrt{\frac{\Lambda}{\lambda_{min}(\Gamma_w^{-1})}},\tag{54}$$

$$\|\tilde{\Xi}\| \le \sqrt{\Lambda}.\tag{55}$$

Remark 2: With references such as [28], we find that when the helicopter system is subjected to input saturation and external disturbance, the methods used in most papers are to design auxiliary systems to eliminate the saturation error and, at the same time, to use adaptive control methods to cope with the externally generated perturbations. In addition, all these methods have only been studied theoretically and their effectiveness has not been experimentally verified. Therefore, this paper proposes an adaptive NN control strategy in which the NN both estimates the system uncertainty and eliminates the input saturation error, while the designed DO also eliminates the helicopter system from external disturbances, and is experimentally validated on a Quanser's 2-DOF helicopter platform.

4. NUMERICAL SIMULATION

In this section, the presented algorithms are verified through some simulation examples of a Quanser's 2-DOF helicopter system. The parameters of the Quanser laboratory platform used for simulations is shown in Table 1.

Tabl	e I	. S	ystem	paramet	ers.
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Symbol	Value	Unit	Symbol	Value	Unit
J_p	0.0215	kg·m ²	l_a	0.002	m
J_y	0.0237	kg·m ²	K_{pp}	0.0011	N·m/V
M_a	1.0750	kg	K_{py}	0.0021	N·m/V
D_p	0.0071	N/V	Kyp	-0.0027	N·m/V
D_y	0.0220	N/V	K _{yy}	0.0022	N·m/V

The initial conditions are selected as $x_1(0) = [0, 0]^T$. Moreover, we consider the following desired tracking trajectory $x_d = \left[\frac{\pi}{18}\sin(t), \frac{5\pi}{36}\sin(t)\right]^T$. The simulation study in this section is intended to verify the tracking performance of the control algorithm proposed in this article with input saturation and unknown external disturbance.

The input saturation values are chosen as $S_{\text{max}} = 24$, the external disturbance is $d(t) = [-0.2 \sin(t), -0.1(\sin(0.5t) + \sin(0.2t))]^T$. The design parameters are chosen as $k_1 = \text{diag}[15, 15], k_2 = \text{diag}[20, 20], k_3 = 6$, $\Gamma_w = 15I, \sigma_1 = 0.1, \gamma = 0.1$, and F = diag[15, 15].

4.1. Case 1: Under the proposed control scheme

The simulation results are shown in Figs. 2(a)-2(d). Fig. 2(a) indicate that the output x_1 of the system can track the desired trajectory x_d accurately through input saturation. From Fig. 2(b), we can see that the tracking trace error eventually converges to a small value closer to zero. Figs. 2(c) and 2(d) show the saturated nonlinear input and control input. From the simulation results, it can be seen that the adaptive NN control proposed in this paper achieves



Fig. 2. Control performance under the proposed control scheme.



put *u*.

Fig. 3. Control performance under the proposed control scheme without disturbance observer.

robust tracking under the condition that the 2-DOF helicopter system has input saturation and external disturbance.

4.2. Case 2: Under the proposed control scheme without disturbance observer

In order to reflect the superiority of the control strategy designed in this paper, we design a control method without a DO. The design parameters to be used in this control method are the same as those in Case 1. The simulation results obtained are shown in Figs. 3(a)-3(d). Fig. 3(a) represents the response of the output variables of the system tracking the desired trajectory. Fig. 3(b) represents the saturated nonlinear input, and Fig. 3(d) represents the control input of the system, from which it can be seen that the control input of the system satisfies the input saturation.

4.3. Discussions on simulation

Comparing Case 1 with Case 2, it can be seen that the control method designed in Case 1 has a better tracking effect and produces a less tracking error. Therefore, we know that the control method with the DO improves the stability and robustness of the system when subjected to external disturbances.

5. EXPERIMENTAL RESULTS

In this section, we use the Quanser 2-DOF helicopter experimental platform to verify the effectiveness of the control strategy proposed in this paper. Fig. 4 shows the experimental equipment of the 2-DOF helicopter system



Fig. 4. Experiment setup.

produced by Quanser. In addition, we use a fan to generate the external disturbances required in the experiment.

This section is divided into two parts to discuss the experimental phenomena of 2-DOF helicopter system with disturbance observer and without disturbance observer after external disturbance. Moreover, in practice, the input voltage range of the two motors of the 2-DOF helicopter system is [-24V, +24V].

5.1. Case with disturbance observer

The results of the experiment are shown in Figs. 5(a)-5(d). Figs. 5(a) and 5(b) show the response of the helicopter system's pitch and yaw angles to track the desired trajectory. Fig. 5(c) represents the response of the tracking error. Fig. 5(d) indicates the input voltage of the motor in the helicopter system. The external disturbance in the experiment is caused by a fan with a speed of 8 rad/s. Based on the experimental results, we can find that the control strategy proposed in this study can obtain a good tracking performance while also maintaining a better input performance of the system.

5.2. Case without disturbance observer

In order to highlight the effectiveness of the adaptive NN control strategy with the DO, we will design an adaptive NN algorithm without the DO to see if the 2-DOF helicopter system can maintain robust tracking control when disturbed externally. The experimental results of this control strategy are shown in Figs. 6(a) - 6(d). Figs. 6(a) and 6(b) show the tracking response of the helicopter system's pitch and yaw angle. Fig. 6(c) represents the performance of the tracking error. Fig. 6(d) is the input to the system. The external disturbance in the experiment is caused by a fan with a speed of 8 rad/s.

When there is no DO, we can see from Figs. 6(a)-6(c) that the system is difficult to obtain better tracking performance in track tracking, therefore, the robust tracking control is not guaranteed. In addition, Fig. 6(d) shows that the motor input performance of the system is also poor.

5.3. Discussions on experiment

Based on the analysis of the above two experimental results, we can conclude that the adaptive NN control with



Fig. 5. Control performance with the disturbance observer.



Fig. 6. Case without disturbance observer.

the DO has a better tracking performance in tracking the desired trajectory when there exists external disturbances, the error obtained is small, and the motor input of the system has a good performance. The results from the numerical simulation experiments are also verified, proving that the method can enhance the stability and robustness of the system.

6. CONCLUSION

In this paper, we proposed an adaptive NN control scheme for a 2-DOF helicopter system with input saturation and unknown external disturbances. The RBFNN has been used to compensate for system uncertainty and saturation errors. In addition, the DO has been designed to handle a composite disturbance consisting of the unknown NN approximation error and the unknown external disturbances. Through the stability analysis of the Lyapunov function, it was illustrated that the closed-loop system is eventually bounded. Simulation and experiment results have proved the effectiveness and stability of the proposed control algorithm. In the future, we will extend this research to 4-DOF helicopters and will also work on the problem of actuator failures in helicopter systems.

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