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The Filtering Based Maximum Likelihood Recursive Least Squares Parameter Estimation Algorithms for a Class of Nonlinear Stochastic Systems with Colored Noise

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Abstract: This paper focuses on the maximum likelihood estimation for bilinear systems in the presence of colored noise. The state variables in the model is eliminated and an input-output expression is provided. The input-output data of the system is filtered by an estimated noise transfer function, and the system is transformed into two subsystems. A filtering based maximum likelihood recursive least squares algorithm is proposed to strengthen the identification accuracy and improve computational efficiency. The superior performance of the developed methods are demonstrated by numerical simulations.

Keywords: Bilinear system, data filtering, least squares, maximum likelihood.

1. INTRODUCTION

Parameter identification is crucial for controller design, signal processing and system analysis [1-5]. Plenty of parameter estimation approaches are presented for modeling linear and nonlinear systems [6-8]. Zhang and Yang considered the identification of bilinear systems under unknown states, where a state estimator based recursive algorithm was presented by integrating the parameter estimation and state estimation [9]. Fan and Liu presented a gradient-based iterative method for input variable-gain nonlinear systems using auxiliary model [10]. Pan *et al.* studied the estimation of autoregressive moving average (ARMA) systems, where a filtering based extended stochastic gradient algorithm was proposed [11-14].

The least squares algorithm is a popular approach in the field of parameter estimation, and can be divided into two categories: iteration [15-19] and recursion [20-22]. In [23], a decomposed least squares based iterative (LSI) method was proposed for dual-rate stochastic systems. However, the LSI methods can only be applied to off-line identification and have the high computational efforts since the whole data is used in each iteration. Compared with the LSI methods, the recursive least squares (RLS) algorithms have less computational cost and they are suitable for online identification [24-27].

Parameter estimation accuracy is important indicator to evaluating the estimation algorithms, and different methods have been proposed to enhance the estimation accuracy. Among these methods, the data filtering based estimation methods are effective to improve the estimation quality for the systems in the presence of colored noise, and have been utilized extensively in parameter estimation [28-30]. In [31], a RLS algorithm was derived for bilinear systems using data filtering. Another useful approach for enhancing the estimation precision is adopting the maximum likelihood (ML) principle. The maximum likelihood estimation approaches have wide applications in parameter estimation and system modeling due to their good statistical characteristics [32, 33]. For instance, Li and Liu presented a maximum likelihood LSI method for bilinear systems using data filtering [34]. However, their presented method can be only used for off-line identification.

Motivated by aforementioned analysis, this paper develops a filtering based maximum likelihood recursive least squares (F-ML-RLS) method to depress the computational burden and strengthen the estimation precision. The main contributions of this paper may be summarized as follows:

- Because of the special model structure of bilinear state-space systems, providing an input-output expression by eradicating the state variables is the top priority to identify such systems.

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- By integrating the ML principle and the data filtering technique, a F-ML-RLS algorithm is developed to strengthen the estimation precision and improve the computational efficiency.

The rest parts are provided in the following. The problem formulation are presented in Section 2. Section 3 derives a RGELS method. Section 4 proposes a F-RELS method. Section 5 presents a F-ML-RLS method by means of data filtering and ML principle, respectively. Comparative simulations are carried out in Section 6. Finally, general conclusions are summarized in Section 7.

2. PROBLEM FORMULATION

Considering a bilinear state-space systems as follows:

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t) + B\boldsymbol{x}(t)u(t) + \boldsymbol{h}u(t), \tag{1}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t), \tag{2}$$

where $y(t) \in \mathbb{R}$, $u(t) \in \mathbb{R}$ and $\boldsymbol{x}(t) \in \mathbb{R}^n$ denote the system output, input and state variables, respectively. The system matrices A, B and the system parameter vectors $\boldsymbol{h}$ and $\boldsymbol{c}$ are given by

$$A := \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$\boldsymbol{h} := \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{n-1} \\ h_n \end{bmatrix} \in \mathbb{R}^{n\times 1}, \ B := \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{b} \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$\boldsymbol{b} := [-b_n, -b_{n-1}, -b_{n-2}, \cdots, -b_1] \in \mathbb{R}^{1\times n},$$

$$\boldsymbol{c} := [1, 0, 0, \cdots, 0] \in \mathbb{R}^{1\times n}.$$

Referring to the work in [35], from (1) and (2), the input-output form can be derived by eliminating the state variables

$$[A(z) + u(t-n)B(z)]y(t) = [C(z)u(t) + D(z)u(t-n)]u(t), \tag{3}$$

$$A(z) := 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n},$$

$$B(z) := b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n},$$

$$C(z) := c_1 z^{-1} + c_2 z^{-2} + \cdots + c_n z^{-n},$$

$$D(z) := d_2 z^{-2} + d_3 z^{-3} + \cdots + d_n z^{-n}.$$

The relations between parameters a_i, b_i, h_i and coefficients c_i, d_i are

$$[c_n, \cdots, c_2, c_1] := [h_n + a_{n-1}h_1 + a_{n-2}h_2 + \cdots + a_1 h_{n-1}, \cdots, h_2 + a_1 h_1, h_1] \in \mathbb{R}^{1\times n},$$

$$[d_n, \cdots, d_3, d_2] := [b_{n-1}h_1 + b_{n-2}h_2 + \cdots + b_1 h_{n-1}, \cdots, b_1 h_1] \in \mathbb{R}^{1\times(n-1)}.$$

In practice, the bilinear system are usually subject to various disturbances, such as stochastic noise. Therefore, introducing $w(t) \in \mathbb{R}$ to (3) yields

$$[A(z) + u(t-n)B(z)]y(t) = [C(z)u(t) + D(z)u(t-n)]u(t) + w(t). \tag{4}$$

In general, the colored noise is more difficult to deal with. Therefore, an ARMA noise is taken as the stochastic noise in this paper, which can be expressed as

$$E(z)w(t) = F(z)v(t), \tag{5}$$

where $v(t) \in \mathbb{R}$ denotes the white noise sequence, $E(z)$ and $F(z)$ are given by

$$E(z) := 1 + e_1 z^{-1} + \cdots + e_{n_e} z^{-n_e}, \ e_i \in \mathbb{R},$$

$$F(z) := 1 + f_1 z^{-1} + \cdots + f_{n_f} z^{-n_f}, \ f_i \in \mathbb{R}.$$

To derive the identification model, the parameter vector $\boldsymbol{\theta}$ is defined as

$$\boldsymbol{\theta} := \begin{bmatrix} \boldsymbol{\theta}_s \\ \boldsymbol{\theta}_n \end{bmatrix} \in \mathbb{R}^{n_0}, \quad n_0 := 4n + n_e + n_f - 1,$$

$$\boldsymbol{\theta}_s := [a_1, \cdots, a_n, b_1, \cdots, b_n, c_1, \cdots, c_n, d_2, \cdots, d_n]^T \in \mathbb{R}^{n_1},$$

$$\boldsymbol{\theta}_n := [e_1, \cdots, e_{n_e}, f_1, \cdots, f_{n_f}]^T \in \mathbb{R}^{n_2},$$

$$n_1 := 4n - 1, \quad n_2 := n_e + n_f,$$

and the information vector $\boldsymbol{\varphi}(t)$ is defined as

$$\boldsymbol{\varphi}(t) := \begin{bmatrix} \boldsymbol{\varphi}_s(t) \\ \boldsymbol{\varphi}_n(t) \end{bmatrix} \in \mathbb{R}^{n_0},$$

$$\boldsymbol{\varphi}_s(t) := [-y(t-1), \cdots, -y(t-n), -u(t-n)y(t-1), \cdots, -u(t-n)y(t-n), u(t-1), \cdots, u(t-n), u(t-n)u(t-2), \cdots, u(t-n)u(t-n)]^T \in \mathbb{R}^{n_1},$$

$$\boldsymbol{\varphi}_n(t) := [-w(t-1), \cdots, -w(t-n_e), v(t-1), \cdots, v(t-n_f)]^T \in \mathbb{R}^{n_2}.$$

Thus, the identification model of the bilinear system in (4) is rewritten as

$$y(t) = \boldsymbol{\varphi}^T(t)\boldsymbol{\theta} + v(t). \tag{6}$$

In this paper, the proposed algorithms are found on this identification model. Various estimation approaches are proposed on the basis of the identification models [36-42] and these methods can be employed to identify other systems [43-47] and can be used in other areas [48-52] like ocean engineering systems. This paper is aiming at developing new methods to estimate $\boldsymbol{\theta}$ based the obtained input-output data.

3. THE RGELS ALGORITHM

To show the superiority of the developed F-ML-RLS method in Section 5, this section provides a RGELS algorithm for comparison.

From (6), define a performance index as follows:

$$J_1(\boldsymbol{\theta}) := \sum_{i=1}^{t} \|y(i) - \boldsymbol{\varphi}^T(t)\boldsymbol{\theta}\|^2. \tag{7}$$

Minimizing $J_1(\boldsymbol{\theta})$ and let its partial derivative be zero, then the RGELS method for identifying the parameter vector $\hat{\boldsymbol{\theta}}(t)$ can be summarized as

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + L(t)[y(t) - \hat{\boldsymbol{\varphi}}^T(t)\hat{\boldsymbol{\theta}}(t-1)], \tag{8}$$

$$L(t) = \frac{P(t-1)\hat{\boldsymbol{\varphi}}(t)}{1+\hat{\boldsymbol{\varphi}}^T(t)P(t-1)\hat{\boldsymbol{\varphi}}(t)}, \tag{9}$$

$$P(t) = [\boldsymbol{I}_{n_0} - L(t)\hat{\boldsymbol{\varphi}}^T(t)]P(t-1), \tag{10}$$

$$\hat{\boldsymbol{\theta}}(t) := \begin{bmatrix}\hat{\boldsymbol{\theta}}_s(t)\\ \hat{\boldsymbol{\theta}}_n(t)\end{bmatrix},\ \hat{\boldsymbol{\varphi}}(t) := \begin{bmatrix}\boldsymbol{\varphi}_s(t)\\ \hat{\boldsymbol{\varphi}}_n(t)\end{bmatrix}, \tag{11}$$

$$\hat{w}(t) = y(t) - \boldsymbol{\varphi}_s^T(t)\hat{\boldsymbol{\theta}}_s(t), \tag{12}$$

$$\hat{v}(t) = y(t) - \hat{\boldsymbol{\varphi}}^T(t)\hat{\boldsymbol{\theta}}(t). \tag{13}$$

4. THE F-RELS ALGORITHM

The RGELS method is often adopted to estimate the model parameters. However, the estimation precision is reduced because of the stochastic noise. The data filter technique is an effective way to handle the stochastic noise through filtering the input-output data. Based on the data filtering method, this section derives a F-RELS algorithm to improve the identification accuracy.

Before giving the F-RELS algorithm, some variables are firstly defined as

$$y_1(t) := E(z)y(t), \quad u_1(t) := E(z)u(t),$$

$$\boldsymbol{\theta}_1 := \left[\boldsymbol{\theta}_s^T, \boldsymbol{\theta}_{n_f}^T\right]^T \in \mathbb{R}^{n_3}, \quad n_3 := 4n + n_f - 1,$$

$$\boldsymbol{\varphi}_1(t) := \left[\boldsymbol{\varphi}_{s1}^T(t), \boldsymbol{\varphi}_{n_f}^T(t)\right]^T \in \mathbb{R}^{n_3},$$

$$\begin{aligned}\boldsymbol{\varphi}_{s1}(t) := [&-y_1(t-1), \cdots, -y_1(t-n),\\ &-u(t-n)y_1(t-1), -u(t-n)y_1(t-2),\\ &\cdots, -u(t-n)y_1(t-n), u_1(t-1),\\ &\cdots, u_1(t-n), u(t-n)u_1(t-2),\\ &u(t-n)u_1(t-3), \cdots,\\ &u(t-n)u_1(t-n)]^T \in \mathbb{R}^{n_1},\end{aligned}$$

$$\boldsymbol{\varphi}_{n_f}(t) := [v(t-1), \cdots, v(t-n_f)]^T \in \mathbb{R}^{n_f},$$

$$\boldsymbol{\varphi}_{n_e}(t) := [-w(t-1), \cdots, -w(t-n_e)]^T \in \mathbb{R}^{n_e},$$

$$\boldsymbol{\theta}_{n_e} := [e_1, \cdots, e_{n_e}]^T \in \mathbb{R}^{n_e},$$

$$\boldsymbol{\theta}_{n_f} := [f_1, \cdots, f_{n_f}]^T \in \mathbb{R}^{n_f}.$$

Multiplying both sides of (3) by $E(z)$ yields

$$\begin{aligned}&[A(z) + u(t-n)B(z)]y_1(t)\\ &= [C(z) + u(t-n)D(z)]u_1(t) + F(z)v(t).\end{aligned} \tag{14}$$

Then we have the following identification model

$$\begin{aligned}y_1(t) = &-\sum_{i=1}^{n} a_i y_1(t-i) - u(t-n)\sum_{i=1}^{n} b_i y_1(t-i)\\ &+\sum_{i=1}^{n} c_i u_1(t-i) + u(t-n)\sum_{i=2}^{n} d_i u_1(t-i)\\ &+\sum_{i=1}^{n_f} f_i v(t-i) + v(t)\\ = &\ \boldsymbol{\varphi}_1^T(t)\boldsymbol{\theta}_1 + v(t).\end{aligned} \tag{15}$$

According to (5), we have

$$w(t) = \boldsymbol{\varphi}_{n_e}^T(t)\boldsymbol{\theta}_{n_e} + \boldsymbol{\varphi}_{n_f}^T(t)\boldsymbol{\theta}_{n_f} + v(t) \tag{16}$$

$$= y(t) - \boldsymbol{\varphi}_s^T(t)\boldsymbol{\theta}_s. \tag{17}$$

For the identification model in (15) and (16), we can obtain the F-RELS algorithm

$$\begin{aligned}\hat{\boldsymbol{\theta}}_1(t) = &\ \hat{\boldsymbol{\theta}}_1(t-1)\\ &+ L_1(t)[\hat{y}_1(t) - \hat{\boldsymbol{\varphi}}_1^T(t)\hat{\boldsymbol{\theta}}_1(t-1)],\end{aligned} \tag{18}$$

$$L_1(t) = \frac{P_1(t-1)\hat{\boldsymbol{\varphi}}_1(t)}{1+\hat{\boldsymbol{\varphi}}_1^T(t)P_1(t-1)\hat{\boldsymbol{\varphi}}_1(t)}, \tag{19}$$

$$P_1(t) = [\boldsymbol{I}_{n_3} - L_1(t)\hat{\boldsymbol{\varphi}}_1^T(t)]P(t-1), \tag{20}$$

$$\hat{y}_1(t) = y(t) + \sum_{i=1}^{n_e} \hat{e}_i(t-1)y(t-i), \tag{21}$$

$$\hat{u}_1(t) = u(t) + \sum_{i=1}^{n_e} \hat{e}_i(t-1)u(t-i), \tag{22}$$

$$\hat{v}(t) = \hat{y}_1(t) - \hat{\boldsymbol{\varphi}}_1^T(t)\hat{\boldsymbol{\theta}}_1(t), \tag{23}$$

$$\begin{aligned}\hat{\boldsymbol{\theta}}_{n_e}(t) = &\ \hat{\boldsymbol{\theta}}_{n_e}(t-1) + L_e(t)[\hat{\omega}(t)\\ &- \hat{\boldsymbol{\varphi}}_{n_f}^T(t)\hat{\boldsymbol{\theta}}_{n_f}(t) - \hat{\boldsymbol{\varphi}}_{n_e}^T(t)\hat{\boldsymbol{\theta}}_{n_e}(t-1)],\end{aligned} \tag{24}$$

$$L_e(t) = \frac{P_e(t-1)\hat{\boldsymbol{\varphi}}_{n_e}(t)}{1+\hat{\boldsymbol{\varphi}}_{n_e}^T(t)P_e(t-1)\hat{\boldsymbol{\varphi}}_{n_e}(t)}, \tag{25}$$

$$P_e(t) = [\boldsymbol{I}_{n_e} - L_e(t)\hat{\boldsymbol{\varphi}}_{n_e}^T(t)]P_e(t-1), \tag{26}$$

$$\hat{w}(t) = y(t) - \boldsymbol{\varphi}_s^T(t)\hat{\boldsymbol{\theta}}_s(t). \tag{27}$$

In order to derive the F-RELS algorithm, a new identification model in (15) is obtained. From (21) and (22), the input and output data is filtered. Thus, the identification accuracy is enhanced. Meanwhile, the original system (8) is decomposed into two subsystems (18) and (24) to identify, which the computational efficiency is improved.

5. THE F-ML-RLS ALGORITHM

In this section, a F-ML-RLS algorithm is developed to further strengthen the parameter estimation precision.

For identification model (15), the ML identification for $\boldsymbol{\theta}_1$ can be achieved through maximizing the likelihood function, or the probability distribution function of the observation $y_N := \{y_1(1),\ y_2(2),\ \cdots,\ y_1(N)\}$ and $u_N := \{u_1(1), u_2(2), \cdots, u_1(N)\}$, that is

$$\hat{\boldsymbol{\theta}}_1 = \arg\max_{\boldsymbol{\theta}_1} L(y_N|u_{N-1}, \boldsymbol{\theta}_1). \tag{28}$$

Because $v(t)$ is uncorrelated with y_N and u_N, the ML function is obtained as

$$L(y_N|u_{N-1}, \boldsymbol{\theta}_1) = (2\pi\sigma^2)^{-\frac{N}{2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{t=1}^{N} v^2(t)\right) + k, \tag{29}$$

where k denote constant term which is determined by the previous data.

Taking the natural logarithm of both sides of (29) and maximizing the logarithm likelihood function, we can obtain the following equivalent performance index

$$J_2(\boldsymbol{\theta}_1) := \frac{1}{2}\sum_{t=1}^{N} v^2(t)\Big|_{\hat{\boldsymbol{\theta}}_1} = \min, \tag{30}$$

$$v(t) = \frac{1}{F(z)}\{[A(z) + u(t-n)B(z)]y_1(t) - [C(z) + u(t-n)D(z)]u_1(t)\}. \tag{31}$$

Using $\hat{\boldsymbol{\theta}}_{n_f}(t)$ and $\hat{\boldsymbol{\theta}}_{n_e}(t)$ to establish the estimates of $F(z)$ and $E(z)$ at time t as follows:

$$\hat{F}(t,z) := 1 + \hat{f}_1(t)z^{-1} + \cdots + \hat{f}_{n_f}(t)z^{-n_f}, \tag{32}$$

$$\hat{E}(t,z) := 1 + \hat{e}_1(t)z^{-1} + \cdots + \hat{e}_{n_e}(t)z^{-n_e}. \tag{33}$$

Calculating the partial derivative of $v(t)$, $\left(\frac{\partial v(t)}{\partial \boldsymbol{\theta}_1}\right)$, in (31) yields

$$\frac{\partial v(t)}{\partial a_j}\Big|_{\hat{\boldsymbol{\theta}}_1(t-1)} = \frac{z^{-j}\hat{y}_1(t)}{\hat{F}(t-1,z)} =: z^{-j}\hat{y}_f(t), \tag{34}$$

$$\frac{\partial v(t)}{\partial b_j}\Big|_{\hat{\boldsymbol{\theta}}_1(t-1)} = u(t-n)\frac{z^{-j}\hat{y}_1(t)}{\hat{F}(t-1,z)} =: u(t-n)z^{-j}\hat{y}_f(t), \tag{35}$$

$$\frac{\partial v(t)}{\partial c_j}\Big|_{\hat{\boldsymbol{\theta}}_1(t-1)} = -\frac{z^{-j}\hat{u}_1(t)}{\hat{F}(t-1,z)} =: -z^{-j}\hat{u}_f(t), \tag{36}$$

$$\frac{\partial v(t)}{\partial d_j}\Big|_{\hat{\boldsymbol{\theta}}_1(t-1)} = -u(t-n)\frac{z^{-j}\hat{u}_1(t)}{\hat{F}(t-1,z)} =: -u(t-1)z^{-j}\hat{u}_f(t), \tag{37}$$

$$\frac{\partial v(t)}{\partial f_j}\Big|_{\hat{\boldsymbol{\theta}}_1(t-1)} = -\frac{z^{-j}\hat{v}(t)}{\hat{F}(t-1,z)} =: -z^{-j}\hat{v}_f(t), \tag{38}$$

where the filtered data $\hat{y}_f(t), \hat{u}_f(t)$ and $v_f(t)$ are determined by

$$\hat{y}_f(t) := \frac{1}{\hat{F}(t-1,z)}\hat{y}_1(t) = \hat{y}_1(t) - \sum_{i=1}^{n_f}\hat{f}_i(t-1)\hat{y}_f(t-i), \tag{39}$$

$$\hat{u}_f(t) := \frac{1}{\hat{F}(t-1,z)}\hat{u}_1(t) = \hat{u}_1(t) - \sum_{i=1}^{n_f}\hat{f}_i(t-1)\hat{u}_f(t-i), \tag{40}$$

$$\hat{v}_f(t) := \frac{1}{\hat{F}(t-1,z)}\hat{v}_1(t) = \hat{v}(t) - \sum_{i=1}^{n_f}\hat{f}_i(t-1)\hat{v}_f(t-i). \tag{41}$$

Filtering $u(t)$ and $y(t)$ using $\hat{E}(t-1,z)$, then we can acquire the estimates of $u_1(t)$ and $y_1(t)$

$$\hat{u}_1(t) := \hat{E}(t-1,z)u(t) = u(t) + \sum_{i=1}^{n_e}\hat{e}_i(t-1)u(t-i), \tag{42}$$

$$\hat{y}_1(t) := \hat{E}(t-1,z)y(t) = y(t) + \sum_{i=1}^{n_e}\hat{e}_i(t-1)y(t-i). \tag{43}$$

Let the filtered information vector be defined as

$$\boldsymbol{\varphi}_{1f}(t) := -\frac{\partial v(t)}{\partial \boldsymbol{\theta}_1}\Big|_{\hat{\boldsymbol{\theta}}_1(t-1)} = -\left[\frac{\partial v(t)}{\partial a_1}, \cdots, \frac{\partial v(t)}{\partial a_n}, \frac{\partial v(t)}{\partial b_1}, \cdots, \frac{\partial v(t)}{\partial b_n}, \frac{\partial v(t)}{\partial c_n}, \frac{\partial v(t)}{\partial d_2}, \frac{\partial v(t)}{\partial c_1}, \cdots, \frac{\partial v(t)}{\partial d_3}, \cdots, \frac{\partial v(t)}{\partial d_n}, \frac{\partial v(t)}{\partial f_1}, \frac{\partial v(t)}{\partial f_2}, \cdots, \frac{\partial v(t)}{\partial f_{n_f}}\right]^T_{\hat{\boldsymbol{\theta}}_1(t-1)}. \tag{44}$$

Then, the F-ML-RLS algorithm for estimating $\boldsymbol{\theta}_1$ can be summarized as

$$\hat{\boldsymbol{\theta}}_1(t) = \hat{\boldsymbol{\theta}}_1(t-1) + L_1(t)[\hat{y}_1(t) - \hat{\boldsymbol{\varphi}}_1^T(t)\hat{\boldsymbol{\theta}}_1(t-1)], \tag{45}$$

$$L_1(t) = \frac{P_1(t-1)\hat{\boldsymbol{\varphi}}_{1f}(t)}{1 + \hat{\boldsymbol{\varphi}}_{1f}^T(t)P_1(t-1)\hat{\boldsymbol{\varphi}}_{1f}(t)}, \tag{46}$$

$$P_1(t) = [\boldsymbol{I}_{n_3} - L_1(t)\hat{\boldsymbol{\varphi}}_{1f}^T(t)]P_1(t-1), \tag{47}$$

$$\hat{y}_f(t) = \hat{y}_1(t) - \sum_{i=1}^{n_f}\hat{f}_i(t-1)\hat{y}_f(t-i), \tag{48}$$

$$\hat{u}_f(t) = \hat{u}_1(t) - \sum_{i=1}^{n_f}\hat{f}_i(t-1)\hat{u}_f(t-i), \tag{49}$$

$$\hat{v}_f(t) = \hat{v}(t) - \sum_{i=1}^{n_f}\hat{f}_i(t-1)\hat{v}_f(t-i), \tag{50}$$

$$\hat{v}(t) = \hat{y}_1(t) - \hat{\varphi}_1^T(t)\hat{\theta}_1(t), \tag{51}$$

Table 1. Computational efficiency of the algorithms.

Algorithms	Multiplications	Additions	Flops
RGELS	$2n_0^2+5n_0+n_1$	$2n_0^2+3n_0+n_1$	$N_1:=4n_0^2+8n_0+2n_1$
F-RELS	$2n_3^2+2n_e^2+6n_0$	$2n_3^2+2n_e^2+4n_0+n_3$	$N_2:=4n_3^2+4n_e^2+10n_0+n_3$
F-ML-RLS	$2n_3^2+2n_e^2+3n_0+n_2+3n_3$	$2n_3^2+2n_e^2+n_0+3n_2+3n_3$	$N_3:=4n_3^2+4n_e^2+4n_0+4n_2+6n_3$

$$\hat{\boldsymbol{\theta}}_{n_e}(t)=\hat{\boldsymbol{\theta}}_{n_e}(t-1)+L_e(t)[\hat{w}(t)-\hat{\boldsymbol{\varphi}}_{n_f}^T(t)\hat{\boldsymbol{\theta}}_{n_f}(t)-\hat{\boldsymbol{\varphi}}_{n_e}^T(t)\hat{\boldsymbol{\theta}}_{n_e}(t-1)], \tag{52}$$

$$L_e(t)=\frac{P_e(t-1)\hat{\boldsymbol{\varphi}}_{n_e}(t)}{1+\hat{\boldsymbol{\varphi}}_{n_e}^T(t)P_e(t-1)\hat{\boldsymbol{\varphi}}_{n_e}(t)}, \tag{53}$$

$$P_e(t)=[\boldsymbol{I}_{n_e}-L_e(t)\hat{\boldsymbol{\varphi}}_{n_e}^T(t)]P_e(t-1), \tag{54}$$

$$\hat{w}(t)=y(t)-\boldsymbol{\varphi}_s^T(t)\hat{\boldsymbol{\theta}}_s(t). \tag{55}$$

The computational efficiency can be evaluated by the flop. Either an addition (including the subtraction) or a multiplication (including the division) is a flop. The flop amounts of these three methods are listed in Table 1, where $n_0=n_1+n_2$, $n_1=4n-1$, $n_2=n_e+n_f$, $n_3=4n+n_f-1$, and we have $N_1-N_3=3n_1+n_2+3n_3(n_e-1)+n_3n_e>0$. Therefore, the developed F-ML-RLS method can depress the computational burden. The proposed estimation algorithms in this paper can joint some adaptive estimation algorithms [53-56] to investigate new identification methods for various systems [57-60] and can be applied to control and schedule areas [61-65].

Remark 1: Different from the LSI methods, the proposed methods are recursive and suitable for online identification, which can be applied in practice easily.

Remark 2: By utilizing the data filtering method, the original complex model is transformed into two submodels and the dimension of identified parameter vector is reduced. Thus, the computational efficiency is improved.

Remark 3: By deriving the ML cost function (30), the F-ML-RLS method is developed for the identification model in (15) and (16). From (44), the information vector is obtained using the ML principle. Thus, the identification accuracy is strengthened.

6. SIMULATION STUDY

Considering a bilinear system such as in (4) and (5),

$$A(z)=1+a_1z^{-1}+a_2z^{-2}=1+0.91z^{-1}+0.63z^{-2},$$
$$B(z)=b_1z^{-1}+b_2z^{-2}=0.2z^{-1}-0.18z^{-2},$$
$$C(z)=c_1z^{-1}+c_2z^{-2}=0.8z^{-1}-2.3z^{-2},$$
$$D(z)=d_2z^{-2}=0.16z^{-2},$$
$$E(z)=1+e_1z^{-1}=1-0.17z^{-1},$$
$$F(z)=1+f_1z^{-1}=1+0.1z^{-1}.$$

In simulations, $\{u(t)\}$ is chosen as a persistent excitation signal, and $\{v(t)\}$ as a white noise signal with zero mean and variance $\sigma^2=2.0^2$.

The simulation results are given in Tables 2-4 and Figs. 1-2, where the estimation error is computed by $\delta(t):=\frac{\|\hat{\boldsymbol{\theta}}(t)-\boldsymbol{\theta}\|}{\|\boldsymbol{\theta}\|}\times 100\%$. From Tables 2-4 and Figs. 1 and 2, it can be seen that these three algorithms are effective for identifying the bilinear system subject to the colored noise, and the estimation errors becomes smaller as t increase. Compared with the RGELS algorithm, the F-RELS and F-ML-RLS methods can engender more precise estimates, which indicates that the data filtering technique can strengthen the identification precision. In addition, the F-ML-RLS method using the ML principle can also improve the estimation precision.

For model validation, the residual data from $t=4001$ to $t=4100$ and estimated models are used to calculate the predicted value $\hat{y}_i$. The real value y, predicted value $\hat{y}_i$ and error $\tilde{y}_i$ are plotted in Figs. 3-5 for these three algorithms. It can be observed that the estimated models are effective for predicting the system output and the developed F-ML-RLS method has smaller prediction error since more accurate model parameters contribute to the more accurate es-

Table 2. RGELS estimates and errors under $\sigma^2=2.0^2$.

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.87151	0.54553	0.21767	-0.19614	0.71698	-1.89078	0.16703	-0.07135	0.10725	16.28465
200	0.90381	0.64407	0.25461	-0.13887	0.83960	-2.01620	0.20104	-0.06286	0.16536	11.96890
500	0.91184	0.60867	0.22560	-0.16334	0.83212	-2.14372	0.19786	-0.07595	0.20724	8.16613
1000	0.89381	0.62183	0.22423	-0.17408	0.78631	-2.20656	0.18103	-0.06778	0.18966	6.28449
2000	0.89636	0.61329	0.20467	-0.18231	0.74278	-2.24673	0.15223	-0.07031	0.17741	5.56649
3000	0.90537	0.61349	0.20257	-0.18152	0.73460	-2.30385	0.15396	-0.06910	0.20526	5.95887
4000	0.90726	0.61859	0.20723	-0.17838	0.71565	-2.33390	0.15331	-0.07339	0.21256	6.46790
True values	0.91000	0.63000	0.20000	-0.18000	0.80000	-2.30000	0.16000	-0.17000	0.10000	

Table 3. F-RELS estimates and errors under $\sigma^2 = 2.0^2$.

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.89597	0.57609	0.24036	-0.00330	0.46419	-2.50611	0.22362	-0.12751	-0.02839	17.11281
200	0.93893	0.63711	0.24117	-0.08076	0.63614	-2.39795	0.21273	-0.25261	0.05444	9.11294
500	0.93828	0.65077	0.21492	-0.09479	0.67758	-2.34722	0.20893	-0.21519	0.05489	6.66603
1000	0.94561	0.64882	0.19589	-0.11023	0.64686	-2.27520	0.14423	-0.19965	0.06256	6.73774
2000	0.94323	0.64580	0.20328	-0.09981	0.74716	-2.26391	0.11919	-0.20760	0.05948	4.77273
3000	0.94089	0.64552	0.20363	-0.10310	0.78455	-2.23289	0.13597	-0.21174	0.06259	4.62309
4000	0.94337	0.65100	0.20956	-0.10425	0.79991	-2.22400	0.16645	-0.20697	0.05696	4.74549
True values	0.91000	0.63000	0.20000	-0.18000	0.80000	-2.30000	0.16000	-0.17000	0.10000	

Table 4. F-ML-RLS estimates and errors under $\sigma^2 = 2.0^2$

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.92979	0.68694	0.27398	-0.18973	0.49455	-2.58667	0.33366	-0.02131	0.08579	18.03398
200	0.88282	0.63258	0.23277	-0.17401	0.74672	-2.47652	0.20397	-0.05988	0.09446	8.27601
500	0.90729	0.62238	0.22722	-0.14681	0.96072	-2.32500	0.15883	-0.11993	0.12562	6.57557
1000	0.91906	0.63219	0.20950	-0.15689	0.84981	-2.22719	0.16954	-0.10944	0.12548	4.20633
2000	0.90990	0.62020	0.21186	-0.14832	0.83060	-2.30668	0.15881	-0.11851	0.11432	2.63932
3000	0.91334	0.62077	0.20893	-0.14561	0.82635	-2.31709	0.17570	-0.13396	0.12552	2.49570
4000	0.91614	0.62472	0.21321	-0.14080	0.83280	-2.27892	0.16519	-0.13918	0.13380	2.72540
True values	0.91000	0.63000	0.20000	-0.18000	0.80000	-2.30000	0.16000	-0.17000	0.10000	

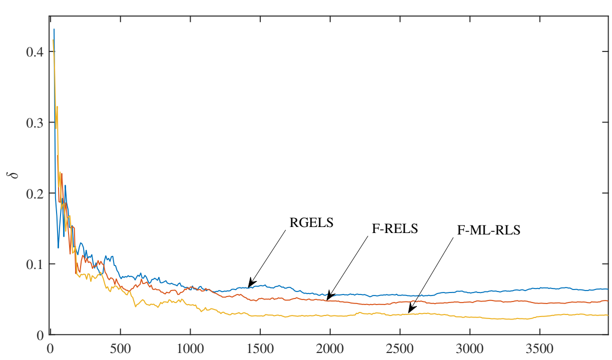

Fig. 1. Parameter estimation errors δ under $\sigma^2 = 2.0^2$.

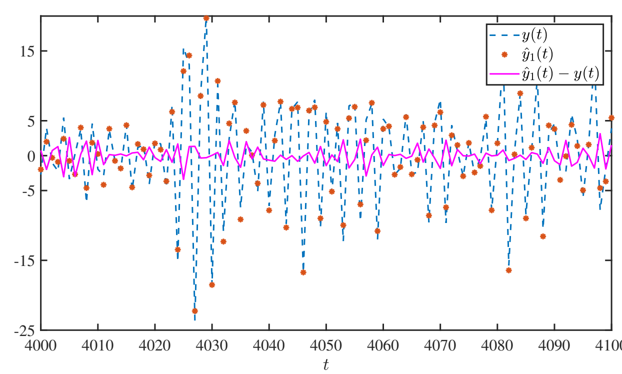

Fig. 3. Predicted value $\hat{y}_1$, real value y and error $\tilde{y}_1$ under RGELS estimates.

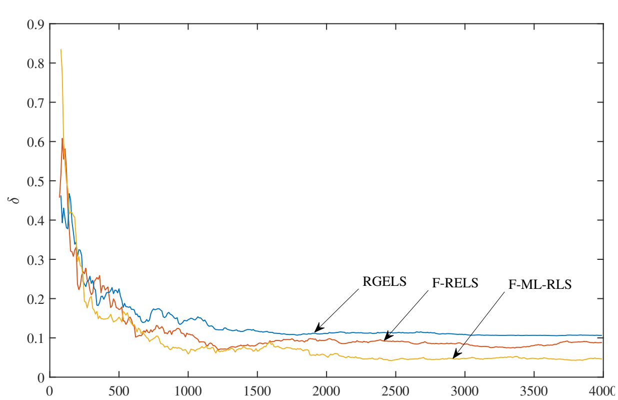

Fig. 2. Parameter estimation errors δ under $\sigma^2 = 5.0^2$.

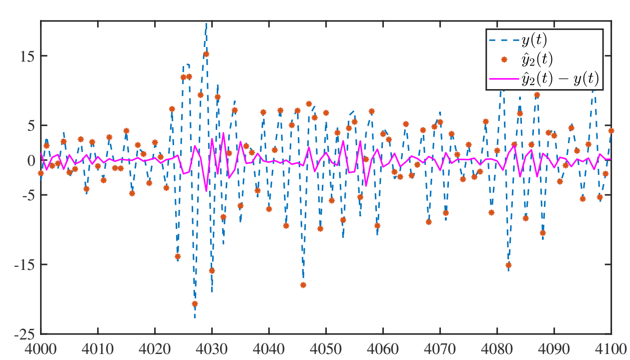

Fig. 4. Predicted value $\hat{y}_2$, real value y and error $\tilde{y}_2$ under F-RELS estimates.

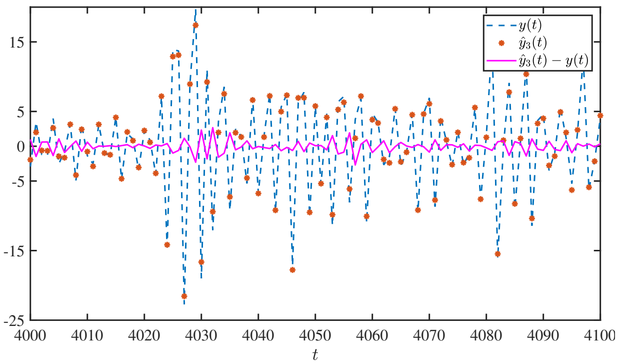

Fig. 5. Predicted value $\hat{y}_3$, real value y and error $\tilde{y}_3$ under F-ML-RLS estimates.

timation model. This also illustrates that the data filtering technique and ML principle can offer higher identification accuracy. From Tables 2-4 and Figs. 1-5, some conclusions are drawn as follows: Compared with the RGELS and F-RELS methods, the developed F-ML-RLS method can strengthen the identification accuracy - see Tables 2-4 and Figs. 1 and 2. The estimation errors of these three algorithms become smaller and smaller as t increases – see Tables 2-4. It is clear that the predicted outputs $\hat{y}_1(t)$, $\hat{y}_2(t)$ and $\hat{y}_3(t)$ are close to the actual output- see Figs. 3-5.

7. CONCLUSIONS

A F-ML-RLS parameter estimation algorithm is proposed to depress the computational cost and strengthen the identification accuracy by employing the data filtering technique and the ML principle, and a RGELS method and a F-RELS method are provided for comparison. The proposed algorithms in this article can integrate other identification algorithms [66-69] to study new estimation methods [70-77] and can be employed to other literature like signal processing and aerospace engineering.

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