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Output Feedback Stabilization of Stochastic Nonlinear Time-varying Delay Systems with Unknown Output Function

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Abstract: This paper addresses the problem of output feedback stabilization for stochastic nonlinear time-varying delay systems with an unknown output function. A remarkable feature of the system to be considered is the simultaneous presence of a continuous unknown sensitivity function and the stochastic disturbances, which have not been treated together before. A new observer is designed by using a dynamic gain without using the information on unknown time-varying delay and nonlinearities. With the aid of the observer, an output feedback controller is constructed by the stochastic double-domination method, where two gains are used to handle the unknown output function and nonlinearities. The performance of the system is analyzed in detail via two integral Lyapunov functions. Finally, the efficiency of the control strategy is illustrated by a simulation example.

Keywords: Double-domination method, output feedback, stochastic nonlinear systems, time-varying delay.

1. INTRODUCTION

In practical engineering systems, the existence of time-delay is a pervasive phenomenon. In general, time-delay of controlled system deteriorates the performance of the system and even affects the stability of the system. Therefore, the problems of time-delay have quite practical significance. In recent years, Lyapunov-Krasovskii methodology plays an important role in dealing with time-delay systems. Based on this method, Pepe and Jiang [1] studied the input state and integral input state stability of time-delay nonlinear systems. Zhang *et al*. designed a controller with dynamic gain for nonlinear time-delay feedforward system to achieve system stability in [2]. However, some problems of uncertain high-order time-delay nonlinear systems cannot be solved because it is difficult to find a suitable Lyapunov functional. To get over this difficulty, a new method should be put forward. Fortunately, Song *et al*. studied the stabilization of high-order feedforward delay nonlinear systems by means of the saturation function technique, homogeneous control and Lyapunov method in [3]. Yang *et al*. [5] investigated the state feedback stabilization for uncertain time-delay nonlinear systems based on the homogeneous control idea and the Lyapunov-Krasovskii functional. There are also other research results on time-delay, such as [6–9] and the references therein.

On the other hand, output feedback stabilization which is the primary task of considering other control problems, such as tracking problem and disturbance rejection problem, has always been a hot topic in the field of control theory. In addition, the output feedback stabilization has more practical application value because of the unmeasurable state existing in most physical systems. Fortunately, many interesting results have emerged to manipulate the issues of output feedback stabilization, for the details, please see [11–16]. It is worth noting that some results also have been extended to output feedback stabilization of nonlinear time-delay systems. Recently, Sun *et al*. [23] studied the global output feedback stabilization for nonlinear time-varying delays systems. He *et al*. studied the global sampled output feedback stabilization for non-strict feedback stochastic time-varying delay nonlinear systems in [24]. Wang *et al*. [25] addressed the output feedback control for generalized dissipative asynchronous repeated scalar time-varying delay nonlinear systems with Markov jump. There are other literatures on the time-delay systems with output feedback, such as [26–28] and so on.

What worse is, the actual systems in engineering technology, environmental ecology, social economy and other fields are generally accompanied with random disturbance. It makes control problems more complex to handle, so the study of stochastic systems is necessary for practical application. In recent years, some results have been obtained by means of Lyapunov-Krasovskii approach and homogeneous domination idea, such as Ai *et al*. [30], Xie

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and Jiang [31], to name just a few. Inspired by the large amount of literatures above, this paper addresses the output feedback stabilization for stochastic time-varying delay nonlinear system. The difficulties are elaborated from the following aspects: it is not clear how to ingeniously design an observer for the controlled system in the presence of an unknown sensitivity function, and how to achieve system stabilization by output feedback under the stochastic setting. In view of the designing of integral Lyapunov functions, an output feedback controller is adopted, which averts the differentiability of unknown output sensitivity function and reduces the complexity of the computation. In this essay, the contributions of are described as follows:

1) The stochastic double-domination approach is delicately used to nonlinear time-varying delay stochastic systems with unknown sensitivity function in this paper.
2) An output feedback controller is designed by the transformation technique, the reasonable combination of several systems and the double-domination approach. The designing of integral Lyapunov functions avoid the differentiability of output function and can compensate the time-delay terms effectively. Finally, we analyze the performance of the controlled system.

Notations: For a real vector $x(t) = [x_1(t), \ldots, x_n(t)]^T$, denote the norm of $x(t)$ as $\|x\| = (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$. For a given real matrix $A = (a_{ij})_{n\times m}$, A^T is the transpose of A; $\|A\|_F = (\sum_{i=1}^n \sum_{j=1}^m a_{ij}^2)^{\frac{1}{2}}$ denotes Frobenius norm; and $\|A\|_\infty = \max_{1\leq i\leq n} \sum_{j=1}^m |a_{ij}|$ is referred to as the norm of row sum. $Tr(A) = \sum_{i=1}^n a_{ii}$ represents the trace of A, when $m = n$.

2. PROBLEM STATEMENT AND PRELIMINARIES

2.1. Problem statement

Consider the stochastic time-varying delay nonlinear system as follows:

$$\begin{cases} dx_i(t) = x_{i+1}(t)dt + f_i(t,x(t),x(t-\tau(t)))dt \\ \qquad + g_i(t,x(t),x(t-\tau(t)))d\omega(t), \\ \qquad i = 1, \ \ldots, \ n-1, \\ dx_n(t) = u(t)dt + f_n(t,x(t), \ x(t-\tau(t)))dt \\ \qquad + g_n(t, \ x(t), \ x(t-\tau(t)))d\omega(t), \\ y(t) = \phi(t)x_1(t), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), \ \ldots, \ x_n(t)]^T \in \mathbb{R}^n$ is the state, $x(t-\tau(t)) = [x_1(t-\tau(t)), \ldots, x_n(t-\tau(t))]^T$, $u(t) \triangleq x_{n+1}(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the time-delayed state, the input and the output, respectively. The unknown continuous functions $f_i(\cdot) : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^n$ and $g_i(\cdot) : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^n$ represent nonlinearities of system (1). $\omega(t)$ is an m-dimensional standard Brownian motion, which is defined on a probability space $(\Omega, \mathscr{F}, P)$. The unknown continuous function $\phi(t) : \mathbb{R} \to \mathbb{R}$ is a sensitivity error. The time-varying delay $\tau(t)$ is unknown and satisfies $\tau(t) \in [0, \tilde{\tau}]$, $\dot{\tau} \leq \bar{\tau} < 1$, where the known constants $\tilde{\tau} \geq 0$ and $\bar{\tau} \geq 0$. $x(\Xi) = x_0(\Xi)$ is the initial value for $-\tilde{\tau} \leq \Xi \leq 0$ and $x_0(\cdot)$ is a known continuous function.

This paper aims to construct a controller $u(t)$ ingeniously such that state $x(t)$ of system (1) approaches to zero in probability. Assumptions in the following are necessary.

Assumption 1: There exists a known parameter $\bar{\phi} > 0$ such that $|1-\phi(t)| \leq \bar{\phi} < 1$.

Assumption 2: For given constants $c \geq 0$, $p > 0$, there holds

$$\begin{aligned} &\|f_i\| + \|g_i\| \\ &\leq c(1+|y|^p)\sum_{j=1}^i |x_j| \\ &\quad + c(1+|y(t-\tau(t))|^p)\sum_{j=1}^i |x_j(t-\tau(t))|, \\ &\quad i = 1, \ \ldots, \ n. \end{aligned} \tag{2}$$

2.2. Preliminaries

In order to facilitate the controller design, the following definitions and lemmas are needed. Consider a stochastic system of the form

$$\begin{aligned} dx(t) = {}& f(t,x(t),x(t-\tau(t)))dt \\ &+ g(t,x(t),x(t-\tau(t)))d\omega(t), \ t \geq t_0. \end{aligned} \tag{3}$$

Definition 1: For a given function V is $\mathcal{C}^2$ with respect to the system (3), then $\mathcal{L}V$ is said to be the infinitesimal generator of V defined by

$$\mathcal{L}V = \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\left\{ g^T \frac{\partial^2 V}{\partial x^2} g \right\}.$$

Lemma 1 [23]**:** For a given constant $q \geq 1$, and $x_i \in \mathbb{R}$, $i = 1, \ldots, n$, there holds

$$(\sum_{i=1}^n |x_i|)^q \leq n^{q-1} \sum_{i=1}^n |x_i|^q. \tag{4}$$

Lemma 2 [23]**:** For given constants $a > 0, b > 0$ and positive smooth function $r(x,y) > 0$, following inequality holds

$$\begin{aligned} |x^a y^b| \leq {}& \frac{a}{a+b} r(x,y)|x|^{a+b} \\ &+ \frac{b}{a+b} r^{-\frac{a}{b}}(x,y)|y|^{a+b}, \ x \in \mathbb{R}, \ y \in \mathbb{R}. \end{aligned} \tag{5}$$

Lemma 3 [19]**:** For a given real matrix $A = (a_{ij})_{m\times n}$, there is $\|A\|_\infty \leq \sqrt{m}\|A\|_F$. If $m = n$, then $Tr(A) \leq n\|A\|_\infty$.

3. DESIGN PROCEDURES

Since the states $x_2(t)$, ..., $x_n(t)$ of system (1) are unmeasurable, we will design an observer for the system at first, and construct a controller to stabilize the system.

3.1. Observer design

To begin with, the linear observer is designed as follows:

$$\begin{cases}\dot{\hat{x}}_i(t) = \hat{x}_{i+1}(t) - k_i\gamma^i(t)\hat{x}_1(t),\ i = 1,\ \cdots,\ n-1,\\ \dot{\hat{x}}_n(t) = u(t) - k_n\gamma^n(t)\hat{x}_1(t),\end{cases} \tag{6}$$

where $\gamma(t)$ is a dynamic gain and

$$\dot{\gamma}(t) = \frac{1}{\eta}\max\{0, -\gamma(t)(l\gamma(t) - \mathcal{X}(y(t)))\}, \tag{7}$$

$\gamma(t) \equiv 1$ for $-\tilde{\tau} \le t \le 0$, the positive constants l, η, k_1, $\cdots$, k_n can be specified later, $\mathcal{X}(y(t))$ is an undetermined continuous function.

Remark 1: The dynamic gain $\gamma(t)$ is used to dominate the unknown nonlinearities that are bounded by states multiplied by output polynomial. The intrinsic feature of $\gamma(t)$ and the appropriate choices of k_i guarantee the stability of the observer in the case of $u(t) = 0$. On the other hand, because $\gamma(t)$ can be increased to a sufficient large value, it can compensate the polynomial growth rate $c(1+|y|^p)$ in Assumption 2.

The estimation error is defined as follows:

$$\varepsilon_i = \frac{x_i - \hat{x}_i}{\gamma^{i-1+\vartheta}(t)},\ i = 1,\ \ldots,\ n, \tag{8}$$

where $\vartheta > 0$, and it is not hard to obtain

$$\begin{aligned} d\varepsilon_i &= d\frac{x_i - \hat{x}_i}{\gamma^{i-1+\vartheta}(t)}\\ &= \frac{x_{i+1}dt + f_i(\cdot)dt + g_i^T(\cdot)d\omega - \hat{x}_{i+1}dt + k_i\gamma^i(t)\hat{x}_1 dt}{\gamma^{i-1+\vartheta}(t)}\\ &\quad - (i-1+\vartheta)\frac{(x_i - \hat{x}_i)\dot{\gamma}(t)dt}{\gamma^{i+\vartheta}(t)}\\ &= \gamma(t)\varepsilon_{i+1}dt - \gamma(t)k_i\varepsilon_1 dt - (i-1+\vartheta)\frac{\dot{\gamma}(t)}{\gamma(t)}\varepsilon_i dt\\ &\quad + \frac{\gamma(t)}{\gamma^{\vartheta}(t)}k_i x_1 dt + \frac{f_i(\cdot)dt}{\gamma^{i-1+\vartheta}(t)} + \frac{g_i^T(\cdot)d\omega}{\gamma^{i-1+\vartheta}(t)}, \end{aligned} \tag{9}$$

where $\varepsilon_{n+1} \triangleq 0$. Rewrite (9) into the following form

$$\begin{aligned} d\varepsilon(t) &= (\gamma(t)A\varepsilon + \frac{\gamma(t)}{\gamma^{\vartheta}(t)}Mx_1 - \frac{\dot{\gamma}(t)}{\gamma(t)}(\vartheta I + D)\varepsilon)dt\\ &\quad + f(\cdot)dt + g^T(\cdot)d\omega, \end{aligned} \tag{10}$$

where

$$A = \begin{pmatrix} -k_1 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ -k_{n-1} & 0 & \cdots & 1\\ -k_n & 0 & \cdots & 0 \end{pmatrix},\ \varepsilon = \begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \vdots\\ \varepsilon_n\end{pmatrix},\ M = \begin{pmatrix}k_1\\ k_2\\ \vdots\\ k_n\end{pmatrix},$$

$$f(\cdot) = \begin{pmatrix}\frac{f_1(\cdot)}{\gamma^{\vartheta}(t)}\\ \frac{f_2(\cdot)}{\gamma^{1+\vartheta}(t)}\\ \vdots\\ \frac{f_n(\cdot)}{\gamma^{n-1+\vartheta}(t)}\end{pmatrix},\ g^T(\cdot) = \begin{pmatrix}\frac{g_1^T(\cdot)}{\gamma^{\vartheta}(t)}\\ \frac{g_2^T(\cdot)}{\gamma^{1+\vartheta}(t)}\\ \vdots\\ \frac{g_n^T(\cdot)}{\gamma^{n-1+\vartheta}(t)}\end{pmatrix},$$

$$D = \begin{pmatrix}0 & & & \\ & 1 & & \\ & & \ddots & \\ & & & n-1\end{pmatrix}.$$

Choose positive constants $k_1, \cdots, k_n$ such that the matrix A is Hurwitz. There is a positive-definite and symmetric matrix Q_1 satisfies

$$A^TQ_1 + Q_1A \le -I, \tag{11}$$

$$\vartheta Q_1 \le DQ_1 + Q_1D + 2\vartheta Q_1. \tag{12}$$

Then, choose the candidate function as

$$\begin{aligned} V_1 = \varepsilon^TQ_1\varepsilon + \sum_{i=1}^{n}\frac{2cL_1 i(n-i+1)}{1-\bar{\tau}}\\ \times\int_{t-\tau(t)}^{t}\frac{(1+|y|^p)^2x_i^2(s)}{\gamma^{2i-2+2\vartheta}(s)}ds, \end{aligned} \tag{13}$$

where $L_1 = 1 + 2n\sqrt{n}\|Q_1\|_F$.

With the help of (11) and (12), one can figure out that

$$\begin{aligned} \mathcal{L}V_1 &= \frac{\partial V_1}{\partial\varepsilon}(\gamma(t)A\varepsilon + \frac{\gamma(t)}{\gamma^{\vartheta}(t)}Mx_1 + f(\cdot)\\ &\quad - \frac{\dot{\gamma}(t)}{\gamma(t)}(\vartheta I + D)\varepsilon) + \frac{1}{2}Tr\left\{g\frac{\partial^2V_1}{\partial\varepsilon^2}g^T\right\}\\ &\quad + \sum_{i=1}^{n}\frac{2cL_1 i(n-i+1)}{1-\bar{\tau}}\frac{x_i^2(1+|y|^p)^2}{\gamma^{2i-2+2\vartheta}(t)}\\ &\quad - \sum_{i=1}^{n}\frac{2cL_1 i(n-i+1)}{1-\bar{\tau}}\\ &\quad \times\frac{x_i^2(t-\tau(t))(1+|y(t-\tau(t))|^p)^2}{\gamma^{2i-2+2\vartheta}(t-\tau(t))}(1-\dot{\tau})\\ &\le 2\gamma(t)\varepsilon^TQ_1A\varepsilon + \frac{2\gamma(t)}{\gamma^{\vartheta}(t)}\varepsilon^TQ_1Mx_1 + 2\varepsilon^TQ_1f(\cdot)\\ &\quad - \vartheta\lambda_1\frac{\dot{\gamma}(t)}{\gamma(t)}\|\varepsilon\|^2 + Tr\{gQ_1g^T\}\\ &\quad + \sum_{i=1}^{n}\frac{2cL_1 i(n-i+1)}{1-\bar{\tau}}\frac{x_i^2(1+|y|^p)^2}{\gamma^{2i-2+2\vartheta}(t)}\\ &\quad - \sum_{i=1}^{n}\frac{2cL_1 i(n-i+1)}{\gamma^{2i-2+2\vartheta}(t-\tau(t))}\\ &\quad \times(1+|y(t-\tau(t))|^p)^2x_i^2(t-\tau(t)), \end{aligned} \tag{14}$$

where $1-\dot{\tau} \ge 1-\bar{\tau}$ and $\lambda_1 > 0$ is the minimum eigenvalue of the matrix Q_1. In the following, we need to handle the indefinite item in the right side of (14). To begin with, there holds

$$2\gamma\varepsilon^TQ_1A\varepsilon \le -\gamma(t)\|\varepsilon\|^2. \tag{15}$$

In view of Lemma 2, it follows that

$$\begin{aligned}&\frac{2\gamma(t)}{\gamma^{\vartheta}(t)}\varepsilon^T Q_1 M x_1\\ &\le \frac{\gamma(t)}{2}\|\varepsilon\|^2+\frac{2\gamma(t)}{\gamma^{2\vartheta}(t)}\|Q_1\|^2\|M\|^2x_1^2. \end{aligned}\tag{16}$$

By Assumption 1, it can be obtained that

$$\begin{aligned}&2\varepsilon^T Q_1 f(\cdot)\\ &\le 2c\|\varepsilon\|\|Q_1\|\sum_{i=1}^{n}\frac{(n-i+1)}{\gamma^{i-1+\vartheta}(t)}|x_i|(1+|y|^p)\\ &\quad+2c\|\varepsilon\|\|Q_1\|\sum_{i=1}^{n}\frac{(n-i+1)}{\gamma^{i-1+\vartheta}(t-\tau(t))}\\ &\quad\times|x_i(t-\tau(t))|(1+|y(t-\tau(t))|^p)\\ &\le cn(n+1)\|Q_1\|(\frac{2n+1}{6}\|Q_1\|+1)\|\varepsilon\|^2\\ &\quad+c\|Q_1\|\sum_{i=1}^{n}\frac{(n-i+1)}{\gamma^{2i-2+2\vartheta}(t)}|x_i|(1+|y|^p)\\ &\quad+c\sum_{i=1}^{n}\frac{(n-i+1)}{\gamma^{2i-2+2\vartheta}(t-\tau(t))}\\ &\quad\times|x_i(t-\tau(t))|(1+|y(t-\tau(t))|^p). \end{aligned}\tag{17}$$

In light of Assumption 1 and Lemma 1, it is not difficult to deduce that

$$\begin{aligned}&\|g\|_F^2\\ &=\|[\frac{g_1}{\gamma^k},\frac{g_2}{\gamma^{\vartheta+1}},\cdots,\frac{g_n}{\gamma^{n-1+\vartheta}}]\|_F^2\\ &\le\sum_{i=1}^{n}\frac{2i(n-i+1)c^2(1+|y|^p)^2|x_i|^2}{\gamma^{2i-2+2\vartheta}(t-\tau(t))}\\ &\quad+\frac{2i(n-i+1)c^2(1+|y(t-\tau(t))|^p)^2|x_i(t-\tau(t))|^2}{\gamma^{2i-2+2\vartheta}(t-\tau(t))}, \end{aligned}$$

and with Lemma 3 in mind, there holds

$$\begin{aligned}Tr\{gQ_1g^T\}&=n\|gQ_1g^T\|_\infty\le n\sqrt{n}\|gQ_1g^T\|_F\\ &\le n\sqrt{n}\|Q_1\|_F\|g\|_F^2\le n\sqrt{n}\|Q_1\|_F\\ &\quad\times\sum_{i=1}^{n}2ic^2(n-i+1)(\frac{|x_i|^2(1+|y|^p)^2}{\gamma^{2i-2+2\vartheta}(t)}\\ &\quad+\frac{|x_i(t-\tau(t))|^2(1+|y(t-\tau(t))|^p)^2}{\gamma^{2i-2+2\vartheta}(t-\tau(t))}). \end{aligned}\tag{18}$$

Substituting (15)-(18) into (14) yields that

$$\begin{aligned}\mathcal{L}V_1\le&-\frac{\gamma(t)}{2}\|\varepsilon\|^2-\vartheta\lambda_1\frac{\dot\gamma(t)}{\gamma(t)}\|\varepsilon\|^2+\frac{\gamma(t)\varrho_1}{\gamma^{2\vartheta}(t)}x_1^2\\ &+\varrho_2\|\varepsilon\|^2+\sum_{i=1}^{n}\frac{\varrho_{i+2}(y)x_i^2}{\gamma^{2i-2+2\vartheta}(t)}, \end{aligned}\tag{19}$$

where positive constants ϱ_1, ϱ_2 and functions $\varrho_{i+2}(y)$ are defined by

$$\begin{cases}\varrho_1=2\|Q_1\|^2\|M\|^2,\\ \varrho_2=cn(n+1)\|Q_1\|(\frac{2n+1}{6}\|Q_1\|+1),\\ \varrho_{i+2}(y)=\frac{c(n-i+1)}{2}(1+|y|^p)^2\|Q_1\|\\ \qquad+\frac{2cL_1i(n-i+1)}{1-\bar\tau}(1+|y|^p)^2\\ \qquad+2c^2i(n-1+i)n\sqrt{n}\|Q_1\|_F(1+|y|^p)^2,\\ \qquad i=1,\ 2,\ \cdots,\ n.\end{cases}\tag{20}$$

3.2. Controller design

In view of (1), (6) and (8), it follows that

$$\begin{cases}dx_1(t)=x_2(t)dt+f_1(t,x(t),x(t-\tau(t)))dt\\ \qquad+g_1(t,x(t),x(t-\tau(t)))d\omega(t),\\ \dot{\hat x}_i(t)=\hat x_{i+1}(t)+\gamma^i(t)k_i(\gamma^{\vartheta}(t)\varepsilon_1-x_1),\\ \qquad i=2,\ \ldots,\ n-1,\\ \dot{\hat x}_n(t)=u(t)+\gamma^n(t)k_n(\gamma^{\vartheta}(t)\varepsilon_1-x_1).\end{cases}\tag{21}$$

The coordinate transformations are introduced for dealing with the nonlinear function f_1 and g_1 as follows:

$$\begin{aligned}&\zeta_1(t)=\frac{x_1(t)}{\gamma^{\vartheta}(t)},\cdots,\zeta_i(t)=\frac{\hat x_i(t)}{\gamma^{i-1+\vartheta}(t)q^{i-1}},\\ &\upsilon(t)=\frac{u(t)}{\gamma^{n+\vartheta}(t)q^n},\ i=2,\ \ldots,\ n,\end{aligned}\tag{22}$$

where constant $q\ge1$ can be specified later. Combining (8), (21) and (22) yields

$$\begin{cases}d\zeta_1(t)=(\gamma(t)q\zeta_2(t)-\vartheta\frac{\dot\gamma(t)}{\gamma(t)}\zeta_1(t)+\gamma(t)\varepsilon_2)dt\\ \qquad+\frac{f_1(\cdot)}{\gamma^{\vartheta}}dt+\frac{g_1^T(\cdot)}{\gamma^{\vartheta}(t)}d\omega,\\ d\zeta_i(t)=\gamma(t)q\zeta_{i+1}(t)dt-(i-1+\vartheta)\frac{\dot\gamma(t)}{\gamma(t)}\zeta_i(t)dt\\ \qquad+\frac{\gamma(t)k_i(\varepsilon_1-\zeta_1(t))}{q^{i-1}}dt,\ i=2,\ \cdots,\ n-1,\\ d\zeta_n(t)=\gamma(t)q\upsilon(t)dt-(n-1+\vartheta)\frac{\dot\gamma(t)}{\gamma(t)}\zeta_n(t)dt\\ \qquad+\frac{\gamma(t)k_n(\varepsilon_1-\zeta_1(t))}{q^{n-1}}dt.\end{cases}\tag{23}$$

Then, the following actual control law is designed:

$$\upsilon(t)=-\frac{b_1}{\gamma^{\vartheta}(t)}y(t)-b_2\zeta_2(t)-\cdots-b_n\zeta_n(t),\tag{24}$$

where $b_1, \cdots, b_n$ are positive constants to be determined later. Substituting (24) into (23) to render

$$\begin{aligned} d\zeta(t) =& (\gamma(t)qB\zeta + \tilde{f}(\cdot) - \frac{\dot{\gamma}(t)}{\gamma(t)}(\vartheta I + D)\varepsilon \\ &+ \frac{\gamma(t)}{q}H_3(\varepsilon_1 - \zeta_1) + \gamma(t)H_2\varepsilon_2 \\ &+ \gamma(t)qH_1b_1(1-\phi(t))\zeta_1)dt + \tilde{g}^T(\cdot)d\omega, \end{aligned} \tag{25}$$

where

$$B = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_1 & -b_2 & \cdots & -b_n \end{pmatrix}, \ \zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_n \end{pmatrix},$$

$$\tilde{f}(\cdot) = \begin{pmatrix} \frac{f_1(\cdot)}{\gamma^{\vartheta}(t)} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ \tilde{g}^T(\cdot) = \begin{pmatrix} \frac{g_1^T(\cdot)}{\gamma^{\vartheta}(t)} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ H_1 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix},$$

$$H_2 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ H_3 = \begin{pmatrix} 0 \\ k_2 \\ \frac{k_3}{q} \\ \vdots \\ \frac{k_n}{q^{n-2}} \end{pmatrix}.$$

Choose positive constants $b_1, \cdots, b_n$ such that B is a Hurwitz matrix. Similarly to the matrix Q_1, there exists a matrix Q_2 satisfying

$$B^T Q_2 + Q_2 B \le -I, \tag{26}$$
$$\vartheta Q_2 \le DQ_2 + Q_2 D + 2\vartheta Q_2. \tag{27}$$

Then, choose the function as

$$V_2 = \zeta^T Q_2 \zeta + \frac{cL_2}{1-\bar{\tau}}\int_{t-\tau(t)}^{t} \frac{(1+|y|^p)^2 x_1^2(s)}{\gamma^{2\vartheta}(s)}ds, \tag{28}$$

where $L_2 = 1 + 2n\sqrt{n}\|Q_2\|_F$.

The time derivative along the trajectories of (25) can be calculated as follows:

$$\begin{aligned} \mathcal{L}V_2 =& 2\zeta^T Q_2(\gamma(t)qB\zeta - \frac{\dot{\gamma}(t)}{\gamma(t)}(\vartheta I + D)\zeta + \gamma(t)H_2\varepsilon_2 \\ &+ \frac{\gamma(t)}{q}H_3(\varepsilon_1 - \zeta_1) \\ &+ \gamma(t)qH_1b_1(1-\phi(t))\zeta_1 + \tilde{f}(\cdot)) \\ &+ Tr\{\tilde{g}Q_2\tilde{g}^T\} + \frac{cL_2}{1-\bar{\tau}}(1+|y|^p)^2\|\zeta\|^2 \\ &- \frac{cL_2(1+|y(t-\tau(t))|^p)^2 x_1^2(t-\tau(t))}{(1-\bar{\tau})\gamma^{2\vartheta}(t-\tau(t))}(1-\dot{\tau}) \\ \le& 2\gamma(t)q\zeta^T Q_2 B\zeta - \vartheta\lambda_2\frac{\dot{\gamma}(t)}{\gamma(t)}\|\zeta\|^2 \\ &+ \frac{cL_2}{1-\bar{\tau}}(1+|y|^p)^2\|\zeta\|^2 + 2\gamma(t)\zeta^T Q_2 H_2\varepsilon_2 \\ &- \frac{cL_2 x_1^2(t-\tau(t))(1+|y(t-\tau(t))|^p)^2}{\gamma^{2\vartheta}(t-\tau(t))} \\ &+ 2\gamma(t)qb_1\zeta^T Q_2 H_1(1-\phi(t))\zeta + 2\zeta^T Q_2\tilde{f}(\cdot) \\ &+ 2\zeta^T Q_2\frac{\gamma(t)}{q}H_3(\varepsilon_1 - \zeta_1) + Tr\{\tilde{g}Q_2\tilde{g}^T\}, \end{aligned} \tag{29}$$

where λ_2 is the minimum eigenvalue of the matrix Q_2, $|\zeta_1| \le \|\zeta\|$ and $\frac{\dot{\gamma}(t)}{\gamma(t)} \ge 0$.

In what follows, it is necessary to manipulate indefinite terms in the right hand side of (29). By means of (26), one immediately has

$$2\gamma(t)q\zeta^T Q_2 B\zeta \le -\gamma(t)q\|\zeta\|^2. \tag{30}$$

It is not difficult to deduce that

$$\begin{aligned} &2\gamma(t)qb_1\zeta^T Q_2 H_1(1-\phi(t))\zeta \\ &\le 2\gamma(t)qb_1\|Q_2H_1\|(1-\phi(t))\|\zeta\|^2. \end{aligned} \tag{31}$$

With the aid of $\|H_2\| = 1$ and $\|H_3\| \le (\sum_{i=2}^n k_i^2)^{\frac{1}{2}} \triangleq \delta$, it is not hard to arrive at

$$\begin{aligned} &2\gamma(t)\zeta^T Q_2 H_2\varepsilon_2 + 2\zeta^T Q_2\frac{\gamma(t)}{q}H_3(\varepsilon_1 - \zeta_1) \\ &\le \frac{\gamma(t)}{4}\|\varepsilon\|^2 + \frac{\gamma(t)}{8}\|Q_2\|^2\|\zeta\|^2 \\ &\quad + \frac{8\gamma(t)\delta^2}{q^2}\|Q_2\|^2\|\zeta\|^2 + 2\delta\frac{\gamma(t)}{q}\|Q_2\|\|\zeta\|^2. \end{aligned} \tag{32}$$

Then, by Assumption 1 and Lemma 2, one can obtain that

$$\begin{aligned} &2\zeta^T Q_2\tilde{f}(\cdot) \\ &\le 2\|\zeta\|\|Q_2\|\|\tilde{f}(\cdot)\| \\ &\le \frac{2\|\zeta\|\|Q_2\|c(1+|y|^p)|x_1|}{\gamma^{\vartheta}(t)} \\ &\quad + \frac{2\|\zeta\|\|Q_2\|c(1+|y(t-\tau(t))|^p)|x_1(t-\tau(t))|}{\gamma^{\vartheta}(t)} \\ &\le 2c\|Q_2\|(1+|y|^p)^2\|\zeta\|^2 + c\|\zeta\|^2\|Q_2\|^2 \\ &\quad + \frac{c}{\gamma^{2\vartheta}(t-\tau(t))}(1+|y(t-\tau(t))|^p)^2 \\ &\quad \times |x_1(t-\tau(t))|^2. \end{aligned} \tag{33}$$

By Assumption 1 and Lemma 3, there holds

$$\begin{aligned} Tr\{\tilde{g}Q_2\tilde{g}^T\} &\le n\sqrt{n}\|Q_2\|_F\|\tilde{g}\|_F^2 \\ &\le 2c^2n\sqrt{n}\|Q_2\|_F(1+|y|^p)^2\|\zeta\|^2 \\ &\quad + 2c^2n\sqrt{n}\|Q_2\|_F \\ &\quad \times \frac{(1+|y(t-\tau(t))|^p)^2|x_1(t-\tau(t))|^2)}{\gamma^{2\vartheta}(t-\tau(t))}. \end{aligned} \tag{34}$$

Substituting (30)-(34) into (29), there exist $\gamma(t) \geq 1$ and $q > 1$ such that

$$\begin{aligned}\mathcal{L}V_2 \leq &-\gamma(t)q(1-2b_1(1-\phi(t))\|Q_2\|)\|\zeta\|^2\\&-\vartheta\lambda_2\frac{\dot{\gamma}(t)}{\gamma(t)}\|\zeta\|^2+\frac{\gamma(t)}{4}\|\varepsilon\|^2\\&+\gamma(t)q(\frac{2\|Q_2\|[\delta(1+4\delta\|Q_2\|)+4\|Q_2\|]}{q}\\&+\frac{c(2(1+|y|^p)+\|Q_2\|)\|Q_2\|}{\gamma(t)}+\frac{cL_2(1+|y|^p)^2}{(1-\bar{\tau})}\\&+\frac{2c^2n\sqrt{n}\|Q_2\|_F(1+|y|^p)^2}{\gamma(t)})\|\zeta\|^2\\=&-\gamma(t)q(1-2b_1(1-\phi(t))\|Q_2\|)\|\zeta\|^2\\&+\frac{\gamma(t)}{4}\|\varepsilon\|^2-\vartheta\lambda_2\frac{\dot{\gamma}(t)}{\gamma(t)}\|\zeta\|^2\\&+\gamma(t)q(\frac{\bar{\varrho}_1}{q}+\frac{\bar{\varrho}_2}{\gamma(t)})\|\zeta\|^2,\end{aligned} \tag{35}$$

where the positive constant $\bar{\varrho}_1$ and function $\bar{\varrho}_2$ are defined as follows:

$$\begin{cases}\bar{\varrho}_1 = 2\|Q_2\|[\delta(1+4\delta\|Q_2\|)+4\|Q_2\|],\\ \bar{\varrho}_2 = c(2(1+|y|^p)+\|Q_2\|)\|Q_2\|\\ \quad +2c^2n\sqrt{n}\|Q_2\|_F(1+|y|^p)^2\\ \quad +\dfrac{cL_2}{(1-\bar{\tau})}(1+|y|^p)^2.\end{cases} \tag{36}$$

4. MAIN RESULTS

Combining (22) and (24), one immediately has

$$\begin{aligned}u(t) = &-\gamma^n(t)q^nb_1y(t)-\gamma^{n-1}(t)q^{n-1}b_2\hat{x}_2(t)\\&-\cdots-\gamma^2(t)q^2b_{n-1}\hat{x}_{n-1}(t)-\gamma(t)qb_n\hat{x}_n(t).\end{aligned} \tag{37}$$

Next, we will formulate the main results of this paper.

Theorem 1: If system (1) satisfies Assumptions 1 and 2, the states of the closed-loop systems (1), (6), (7) and (37), which are defined on $[-\tilde{\tau}, +\infty)$ are uniformly bounded, and the closed-loop systems are stochastic stable in probability.

Proof: The aforementioned theorem is proved from the following aspects.

1) Calculation of time derivative of V_1. By (8) and (22), it is not hard to obtain

$$\begin{aligned}&x_1 = \gamma^{\vartheta}(t)\zeta_1,\\&x_i = \gamma^{i-1+\vartheta}(t)\varepsilon_i+\gamma^{i-1+\vartheta}(t)q^{i-1}\zeta_i,\ i=2,\ \ldots,\ n.\end{aligned} \tag{38}$$

Then, applying Lemma 1 and the fact $|\varepsilon_i| \leq \|\varepsilon\|$, $|\zeta_i| \leq \|\zeta\|$ for $i = 2, \ldots, n$, one can obtain

$$\frac{1}{\gamma^{2\vartheta+2i-2}(t)}x_i^2 \leq 2(\|\varepsilon\|^2+q^{2i-2}\|\zeta\|^2). \tag{39}$$

Substituting (38),(39) into (19), one can get

$$\begin{aligned}\mathcal{L}V_1 \leq &-\frac{\gamma(t)}{2}\|\varepsilon\|^2-\vartheta\lambda_1\frac{\dot{\gamma}(t)}{\gamma(t)}\|\varepsilon\|^2\\&+(\varrho_1+2\sum_{i=2}^{n}\varrho_{i+2})\|\varepsilon\|^2\\&+(\gamma(t)\varrho_2+q(\varrho_3+2\sum_{i=2}^{n}\varrho_{i+2}q^{2i-3}))\|\zeta\|^2,\end{aligned} \tag{40}$$

where defining functions $\tilde{\varrho}_1$ and $\tilde{\varrho}_2$ as

$$\begin{cases}\tilde{\varrho}_1(y) = \varrho_1+\sum_{i=2}^{n}2\varrho_{i+2}(y) \geq 0,\\ \tilde{\varrho}_2(y) = \varrho_3(y)+\sum_{i=2}^{n}2q^{2i-3}\varrho_{i+2}(y) \geq 0.\end{cases} \tag{41}$$

It is not hard to verify that (19) can be further simplified to

$$\begin{aligned}\mathcal{L}V_1 \leq &-\frac{\gamma(t)}{2}\|\varepsilon\|^2-\vartheta\lambda_1\frac{\dot{\gamma}(t)}{\gamma(t)}\|\varepsilon\|^2\\&+\tilde{\varrho}_1\|\varepsilon\|^2+(\gamma(t)\varrho_2+q\tilde{\varrho}_2(y))\|\zeta\|^2.\end{aligned} \tag{42}$$

2) Specification of design parameters. Firstly, by the inequality $\bar{\phi} < \min\{1, \frac{1}{2b_1\|Q_2\|}\}$, the sensitivity error $\bar{\phi}$ is specified, one gets

$$1-2b_1|1-\phi|\cdot\|Q_2\| \geq 1-2b_1\bar{\phi}\|Q_2\| \triangleq \rho, \tag{43}$$

$$\begin{aligned}V = &\varepsilon^TQ_1\varepsilon+\sum_{i=1}^{n}\frac{2cL_1i(n-i+1)}{1-\bar{\tau}}\\&\times\int_{t-\tau(t)}^{t}\frac{(1+|y|^p)^2x_i^2(s)}{\gamma^{2i-2+2\vartheta}(s)}ds+\zeta^TQ_2\zeta\\&+\frac{cL_2}{1-\bar{\tau}}\int_{t-\tau(t)}^{t}\frac{(1+|y|^p)^2x_1^2(s)}{\gamma^{2\vartheta}(s)}ds,\end{aligned} \tag{44}$$

then

$$\begin{aligned}\mathcal{L}V \leq &-\gamma(t)(\frac{1}{4}+\vartheta\lambda_1\frac{\dot{\gamma}(t)}{\gamma^2(t)}-\frac{\tilde{\varrho}_1}{\gamma(t)})\|\varepsilon\|^2\\&-\gamma(t)q(\rho+\vartheta\lambda_2\frac{\dot{\gamma}(t)}{\gamma^2(t)q}-\frac{\varrho_2+\bar{\varrho}_1}{q}\\&-\frac{\tilde{\varrho}_2+\bar{\varrho}_2}{\gamma(t)})\|\zeta\|^2,\end{aligned} \tag{45}$$

$$\rho-\frac{\varrho_2+\bar{\varrho}_1}{q} \geq \frac{1}{4}\rho, \tag{46}$$

where $q \geq \max\left\{1, \frac{4(\varrho_2+\bar{\varrho}_1)}{3\rho}\right\}$. Let $\eta = \min\{\vartheta\lambda_1, \vartheta\lambda_2\}$, $l = \frac{q\rho}{8}$, $\dot{\gamma}(t) \geq -\frac{q\rho\gamma^2(t)}{8\eta}+\frac{\gamma(t)\mathcal{X}(y)}{\eta}$, one has

$$\eta\frac{\dot{\gamma}(t)}{\gamma^2(t)q} \geq -\frac{\rho}{8}+\frac{\mathcal{X}(y)}{\gamma(t)q}. \tag{47}$$

Substituting (46) and (47) into (45), and let $0 < \rho < 1$, $q \geq 1$, it follows that

$$\mathcal{L}V \leq -\gamma(t)(\frac{1}{4}+\eta\frac{\dot{\gamma}(t)}{\gamma^2(t)q}-\frac{\tilde{\varrho}_1}{\gamma(t)})\|\varepsilon\|^2$$

$$
\begin{aligned}
&-\gamma(t)q(\frac{\rho}{4}+\eta\frac{\dot{\gamma}(t)}{\gamma^2(t)q}-\frac{\tilde{\varrho}_2+\bar{\varrho}_2}{\gamma(t)})\|\zeta\|^2\\
\le&-(\frac{\rho}{8}+\frac{\mathcal{X}(y)}{q}-\tilde{\varrho}_1)\|\varepsilon\|^2\\
&-(\frac{\rho}{8}+\mathcal{X}(y)-q\tilde{\varrho}_2-q\bar{\varrho}_2)\|\zeta\|^2. \qquad (48)
\end{aligned}
$$

Then, choosing

$$\mathcal{X}(y)=q\tilde{\varrho}_1+q\tilde{\varrho}_2+q\bar{\varrho}_2, \qquad (49)$$

yields

$$\mathcal{L}V\le-\frac{\rho}{8}\|\varepsilon\|^2-\frac{\rho}{8}\|\zeta\|^2. \qquad (50)$$

3) In light of the existence and the continuation properties of solutions, the state of the closed-loop systems composed of (1), (6), (7) and (22) is $X(t)\triangleq[\varepsilon(t),\,\zeta(t),\,\gamma(t)]^T$, which is defined on a time interval $[-\tilde{\tau},\,t_\kappa)$.

(i) $X(t)$ is bound on $[-\tilde{\tau},\,t_\kappa)$. Firstly, it sees that the function V is monotonically nonincreasing and nonnegative with respect to t from (44)and (50). Then, we have

$$
\begin{aligned}
&V(t)\\
&\le V(0)=\varepsilon(0)^TQ_1\varepsilon(0)\\
&\quad+\sum_{i=1}^{n}\frac{2cL_1i(n-i+1)}{1-\bar{\tau}}\int_{-\tau(0)}^{0}\frac{(1+|y|^p)^2x_i^2(s)}{\gamma^{2i-2+2\vartheta}(s)}ds\\
&\quad+\zeta(0)^TQ_2\zeta(0)+\frac{cL_2}{1-\bar{\tau}}\int_{-\tau(0)}^{0}\frac{(1+|y|^p)^2x_1^2(s)}{\gamma^{2\vartheta}(s)}ds\\
&\le\lambda_3\|\varepsilon(0)\|^2+\lambda_4\|\zeta(0)\|^2\\
&\quad+\sum_{i=1}^{n}\frac{3cL(n-i+1)}{1-\bar{\tau}}\int_{-\tau(0)}^{0}\frac{(1+|y|^p)^2x_i^2(s)}{\gamma^{2i-2+2\vartheta}(s)}ds,
\end{aligned}
\qquad (51)
$$

where the positive constants λ_3 and λ_4 are the maximum eigenvalue of the matrixes Q_1 and Q_2, respectively; The constant $L=\max\{L_1,L_2\}$. The term in the right hand side of (51) is bounded since $y(s)$ and $x_1(s),\cdots,x_n(s)$ are continuous functions with $s\in[-\tilde{\tau},\,0)$, and it is not hard to summarize from (51) that $V(t)$ is bounded for $-\tilde{\tau}\le t<t_\kappa$. Hence, $\varepsilon(t)$ and $\zeta(t)$ are bounded by (44).

Then, one needs to prove the boundness of $\gamma(t)$ on $[-\tilde{\tau},t_\kappa)$. There are the positive numbers M_1 and M_2 satisfying

$$\mathcal{X}(y)\le M_1(1+|y|^p)^2, \qquad (52)$$

and

$$|y|\le|\phi(t)|\cdot|x_1|\le(1+\bar{\phi})|\zeta|\gamma^{\vartheta}(t)\le M_2\gamma^{\vartheta}(t). \qquad (53)$$

It is not hard to deduce that ϑ can be fixed by the inequality $2p\vartheta<1$ by means of the constant p of Assumption 1. With the aid of Lemmas 1 and 2, substituting (53) into (52) yields

$$
\begin{aligned}
\mathcal{X}(y)&\le 2M_1(1+y^{2p})\\
&\le 2M_1(1+M_2^{2p}\gamma^{2p\vartheta}(t))\\
&=2M_1+2M_1M_2^{2p}\gamma^{2p\vartheta}(t)\\
&\le\frac{\rho q}{16}\gamma(t)+M_3,
\end{aligned}
\qquad (54)
$$

where the positive real number $M_3=2M_1+(1-2p\vartheta)(2M_1M_2^{2p}(\frac{32p\vartheta}{\rho q})^{2p\vartheta})^{\frac{1}{1-2p\vartheta}}$.

It is not hard to obtain

$$
\begin{aligned}
\dot{\gamma}(t)&=\frac{\mathcal{X}(y)}{\eta}\gamma(t)-\frac{q\rho}{8\eta}\gamma^2(t)\\
&\le\frac{M_3}{\eta}\gamma(t)-\frac{q\rho}{16\eta}\gamma^2(t)\\
&\triangleq-\varsigma_1\gamma^2(t)+\varsigma_2\gamma(t),
\end{aligned}
\qquad (55)
$$

where the positive real numbers $\varsigma_1=\frac{q\rho}{16\eta}$ and $\varsigma_2=\frac{M_3}{\eta}$, $\gamma(t)=\frac{\varsigma_2e^{\varsigma_2t}}{\varsigma_2-\varsigma_1+\varsigma_1e^{\varsigma_2t}}$ is the solution of the equation $\dot{\gamma}(t)=-\varsigma_1\gamma^2(t)+\varsigma_2\gamma(t)$, $\gamma(0)=1$. Then, we can see that

$$
\begin{aligned}
\gamma(t)&\le\frac{\varsigma_2e^{\varsigma_2t}}{\varsigma_2-\varsigma_1+\varsigma_1e^{\varsigma_2t}}\\
&\le\lim_{t\to+\infty}\frac{\varsigma_2e^{\varsigma_2t}}{\varsigma_2-\varsigma_1+\varsigma_1e^{\varsigma_2t}}=\frac{\varsigma_2}{\varsigma_1}.
\end{aligned}
\qquad (56)
$$

Thus, $\gamma(t)$ is bounded on $[-\tilde{\tau},\,t_\kappa)$.

(ii) $t_\kappa=+\infty$. If t_κ is finite, t_κ would be a finite escape time, which implies that at least one parameter of $X(t)$ would tend to ∞ when $t\to t_\kappa$. The boundedness of $X(t)$ at $t=t_\kappa$ can be guaranteed by the continuity of $\varepsilon(t)$, $\zeta(t)$, $\gamma(t)$ since $X(t)$ is bounded on $[-\tilde{\tau},t_\kappa)$. This contradicts the assumed result. Thus, $t_\kappa=+\infty$, and we've proved the theorem. □

5. SIMULATION EXAMPLE

Example 1: As an illustration of the proposed control method, this paper considers the following system:

$$
\begin{cases}
dx_1(t)\\
\quad=[x_2(t)+(1+y(t-\tau(t))x_1(t-\tau(t))\cos(10x_1))]dt\\
\qquad+(1+y(t-\tau(t))x_1(t-\tau(t))\cos(x_1))d\omega(t),\\
dx_2(t)\\
\quad=[u(t)+(1+y(t-\tau(t))x_2(t-\tau(t))\sin(2x_1))]dt\\
\qquad+(1+y(t-\tau(t))x_2(t-\tau(t))\sin(x_1))d\omega(t),\\
y(t)=\phi(t)x_1(t),
\end{cases}
$$

where $\phi(t)=0.1|\sin(100t^2)|+0.7$. Obviously, Assumption 1 satisfies $c=p=1$. By choosing $\vartheta=\frac{1}{3}$, $k_1=10$, $k_2=1$, $b_1=1$, $b_2=\frac{1}{2}$, $q=1$, one can construct the output feedback controller as follows:

$$
\begin{cases}
\dot{\hat{x}}_1(t)=\hat{x}_2(t)-10\gamma(t)\hat{x}_1(t),\\
\dot{\hat{x}}_2(t)=u(t)-\gamma^2(t)\hat{x}_1(t),\\
\dot{\gamma}(t)=\max\{0,-3\gamma^2(t)+(5+5(1+|y|)^2)\gamma(t)\},\\
u(t)=-0.5\gamma(t)\hat{x}_2(t)-y\gamma^2(t).
\end{cases}
$$

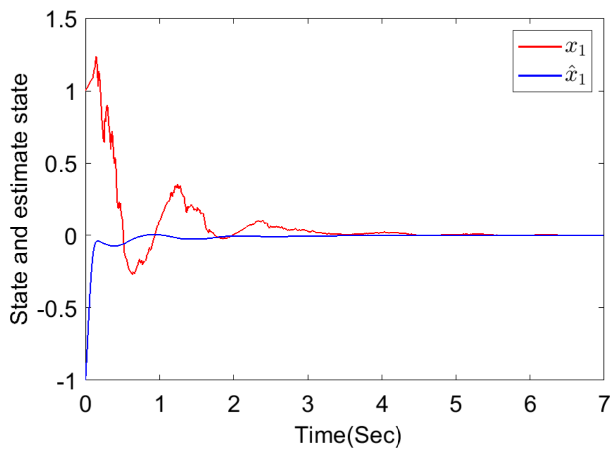

Fig. 1. The trajectories of $x_1(t)$ and $\hat{x}_1(t)$.

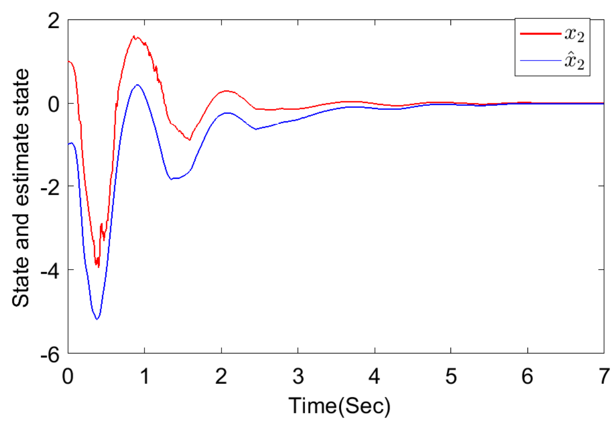

Fig. 2. The trajectories of $x_2(t)$ and $\hat{x}_2(t)$.

Next, we choose initial value as $x_1(\Xi) = 1$, $\hat{x}_1(\Xi) = -1$, $x_2(\Xi) = 1$, $\hat{x}_2(\Xi) = -1$, $\tau(t) = 0.01$ and $\gamma(\Xi) = 1$ for $-0.1 \leq \Xi \leq 0$. Figs. 1-4 exhibit the simulation results. One can be obtained from Fig. 1 that the states and the observed values converge to zero. That is to say, the aforementioned control strategy is effective.

6. CONCLUSION

This manuscript investigates the topic of output feedback stabilization for stochastic nonlinear systems with unknown time-varying delay and output function. According to the designing of integral Lyapunov functions and the double domination approach, the control method is achieved via the construction of the observer and controller. The above results indicate that the closed-loop system is stochastic stable in probability.

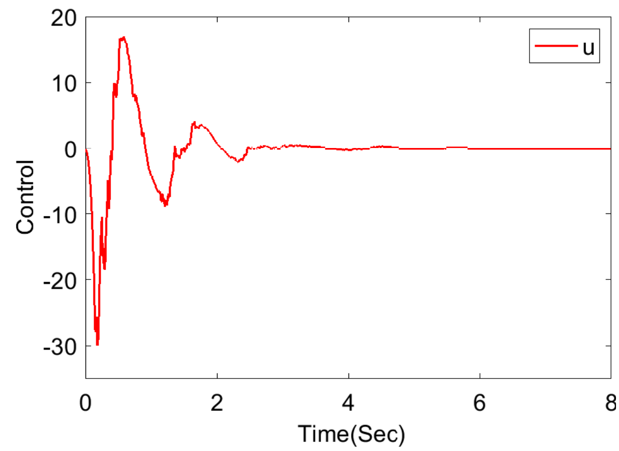

Fig. 3. The trajectories of u.

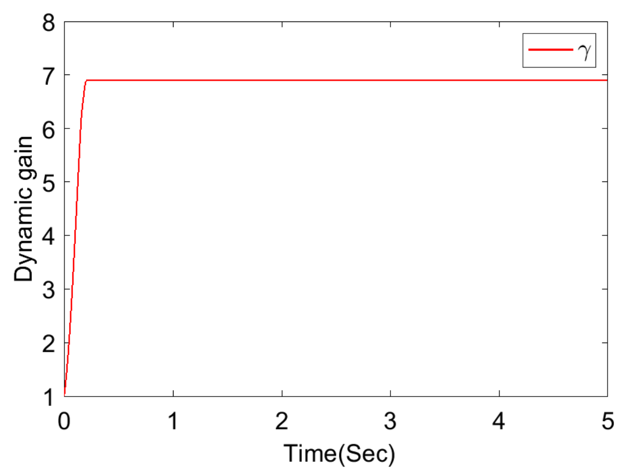

Fig. 4. The trajectories of γ.

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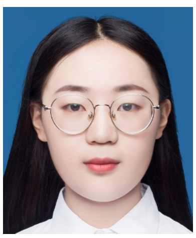

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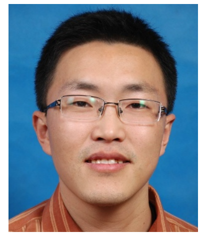

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