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Maximum Likelihood Recursive Generalized Extended Least Squares Estimation Methods for a Bilinear-parameter Systems with ARMA Noise Based on the Over-parameterization Model

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Abstract: Maximum likelihood methods have wide applications in system modeling and parameter estimation. For the purpose of improving the precision of parameter estimation, this paper presents a maximum likelihood recursive generalized extended least squares (ML-RLS) algorithm for a bilinear-parameter system with autoregressive moving average noise based on the over-parameterization identification model. An over-parameterization-based recursive generalized extended least squares algorithm is presented to show the effectiveness of the proposed ML-RLS algorithm for comparison. The simulation test shows that the proposed algorithm has a higher estimation accuracy than the recursive least squares algorithm.

Keywords: Bilinear-parameter system, least squares, maximum likelihood, parameter estimation, recursive identification.

1. INTRODUCTION

System identification constitutes a crucial part in controller designs if the model parameters of a system are unknown [1–3]. Parameter estimation is the foundation of dynamic systems modeling [4–8], and has been widely used in the fields of science and engineering. For decades, the parameter estimation methods have received much attention [9–11]. Some parameter estimation methods such as the auxiliary model identification idea, the multi-innovation identification theory and the data filtering technique have been developed to identify linear and nonlinear systems [12–14]. Block-oriented nonlinear models consisting of linear dynamic blocks and static nonlinear parts are generally used for describing all sorts of nonlinear processes [15,16]. When the output of nonlinear block can be represented a linear combination of the unknown parameters and known basis functions, such a nonlinear system can be transformed into a bilinear-parameter system. These systems involve two product terms of the parameter vectors, and the outputs are linear with respect to any one of two parameter vectors [17]. For example, Ji *et al*. considered the parameter estimation problems of block-oriented nonlinear systems using the hierarchial identification principle and the multi-innovation identification theory for improving the parameter estimation accuracy [18].

The maximum likelihood principle is widely used in the field of stochastic system identification such as linear systems, bilinear systems [19–22], nonlinear systems [23–25] and multivariable systems [26–31], because it has good statistical properties. The maximum likelihood identification method needs to construct a likelihood function with respect to the observed data and the unknown parameters, and the parameter estimation can be obtained by maximizing the likelihood function [32]. The objective of maximum likelihood principle is to construct a likelihood function with respect to the observed data and the unmeasurable parameters, and to obtain the parameter estimation by maximizing the likelihood function. The main contributions of this paper are as follows:

- The identification model of the bilinear-parameter systems is obtained based on the over-parameterization method.
- Based on the over-parameterization model, a maximum likelihood recursive least squares (ML-RLS) algorithm is derived for identifying the parameters. Moreover, am over-parameterization-based recursive least squares (RLS) algorithm is provided as a comparison.
- The simulation example showed the effectiveness of the ML-RLS algorithm, which can give more accurate parameter estimates than the RLS algorithm.

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Briefly, the rest of this paper is organized as follows: Section 2 introduces the identification model of a bilinear-parameter system with autoregressive moving average noise based on the over-parameterization method. A ML-RLS algorithm is introduced in Section 3. In Section 4, a RLS algorithm is presented. Simulation examples are provided to verify the effectiveness in Section 5. Finally, some concluding remarks are given in Section 6.

2. SYSTEM DESCRIPTION AND IDENTIFICATION MODEL

Let us introduce some symbols used in this paper. A=:X stands for X is defined as A, or X:=A stands for A is defined as X, the symbol $\boldsymbol{I}_n$ denotes an identity matrix of appropriate size ($n\times n$); $\mathbf{1}_n$ stands for an n-dimensional column vector whose elements are 1, $\mathbf{1}_{m\times n}$ represents a matrix of size ($m\times n$) whose elements are 1; the superscript T stands for the vector/matrix transpose; the norm of a matrix $\boldsymbol{X}$ is defined by $\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$; $\mathrm{sgn}(x)$ denotes the sign of x.

Consider a bilinear-parameter system with autoregressive moving average noise [17,33],

$$\begin{aligned} y(t) &= \boldsymbol{a}^{\mathrm{T}}\boldsymbol{F}(t)\boldsymbol{\beta} + \frac{D(z)}{C(z)}v(t), \\ D(z) &:= 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{n_d} z^{-n_d}, \\ C(z) &:= 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{n_c} z^{-n_c}, \end{aligned} \tag{1}$$

where $y(t)$ is the output, and $v(t)$ is a Gaussian distributed white noise with zero mean and variance σ^2, $\boldsymbol{F}(t) := \boldsymbol{F}(u(t)) \in \mathbb{R}^{m\times n}$ consists of available input $u(t-j)$, $j=1$, 2, $\cdots$, n, $\boldsymbol{a}\in\mathbb{R}^m$ and $\boldsymbol{\beta}\in\mathbb{R}^n$ are the parameter vectors to be identified, z^{-1} is a unit backward shift operator, i.e., z^{-1} $[z^{-1}y(t) = y(t-1)]$. Let $\boldsymbol{a}$ and $\boldsymbol{\beta}$ and the information matrix $\boldsymbol{F}(t)$ as

$$\begin{aligned} \boldsymbol{a} &:= [a_1,\ a_2,\ \ldots,\ a_m]^{\mathrm{T}} \in \mathbb{R}^m, \\ \boldsymbol{\beta} &:= [\beta_1,\ \beta_2,\ \ldots,\ \beta_n]^{\mathrm{T}} \in \mathbb{R}^n, \\ \boldsymbol{F}(t) &:= \begin{bmatrix} f_1(u(t-1)) & \cdots & f_1(u(t-n)) \\ f_2(u(t-1)) & \cdots & f_2(u(t-n)) \\ \vdots & & \vdots \\ f_m(u(t-1)) & \cdots & f_m(u(t-n)) \end{bmatrix}, \end{aligned}$$

where $u(t)$ is the input of the system, $\boldsymbol{f}_i(u(t-i))$'s are the known nonlinear basis functions. Here, introduce the intermediate variable

$$w(t) := \frac{D(z)}{C(z)}v(t), \tag{2}$$

and define the parameter vector $\boldsymbol{\rho}$ and the noise vector $\boldsymbol{\psi}(t)$ as

$$\begin{aligned} \boldsymbol{\rho} &:= [c_1,\ c_2,\ \ldots,\ c_{n_c},\ d_1,\ d_2,\ \ldots,\ d_{n_d}]^{\mathrm{T}} \in \mathbb{R}^{n_c+n_d}, \\ \boldsymbol{\psi}(t) &:= [-w(t-1),\ -w(t-2),\ \ldots,\ -w(t-n_c), \\ &\qquad v(t-1),\ v(t-2),\ \ldots,\ v(t-n_d)]^{\mathrm{T}} \in \mathbb{R}^{n_c+n_d}, \end{aligned}$$

where $w(t)$ is an autoregressive moving average process. Then, (2) can be rewritten as

$$\begin{aligned} w(t) &= [1-C(z)]w(t) + [D(z)-1]v(t) + v(t) \\ &= \boldsymbol{\psi}^{\mathrm{T}}(t)\boldsymbol{\rho} + v(t). \end{aligned} \tag{3}$$

Let $\boldsymbol{F}_i(t)$ be the ith row of the information matrix $\boldsymbol{F}(t)$. Define the parameter vectors $\boldsymbol{\theta}$ and $\boldsymbol{\vartheta}$ and the information vectors $\boldsymbol{\phi}(t)$ and $\boldsymbol{\Phi}(t)$ as

$$\begin{aligned} \boldsymbol{\theta} &:= [a_1\boldsymbol{\beta}^{\mathrm{T}},\ a_2\boldsymbol{\beta}^{\mathrm{T}},\ \ldots,\ a_m\boldsymbol{\beta}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{mn}, \\ \boldsymbol{\phi}(t) &:= [\boldsymbol{F}_1(t),\ \boldsymbol{F}_2(t),\ \ldots,\ \boldsymbol{F}_m(t)]^{\mathrm{T}} \in \mathbb{R}^{mn}, \\ \boldsymbol{\vartheta} &:= [\boldsymbol{\theta}^{\mathrm{T}},\ \boldsymbol{\rho}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{mn+n_c+n_d}, \\ \boldsymbol{\Phi}(t) &:= [\boldsymbol{\phi}^{\mathrm{T}}(t),\ \boldsymbol{\psi}^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{mn+n_c+n_d}. \end{aligned}$$

Then referring to the method in [17], equation (1) can be written as

$$\begin{aligned} y(t) &= \boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{\theta} + w(t) \\ &= \boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{\theta} + \boldsymbol{\psi}^{\mathrm{T}}(t)\boldsymbol{\rho} + v(t) \\ &= \boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{\vartheta} + v(t). \end{aligned} \tag{4}$$

For identifiability, we adopt the normalization constraint on $\boldsymbol{a}$ or $\boldsymbol{\beta}$. This paper uses the assumption: $\|\boldsymbol{\beta}\| = 1$, and the first nonzero element of the parameter vector $\boldsymbol{\beta}$ is positive, i.e., $\beta_1 > 0$.

3. THE MAXIMUM LIKELIHOOD RECURSIVE RECURSIVE GENERALIZED LEAST SQUARES ALGORITHM

According to the maximum likelihood principle, define

$$J(\boldsymbol{\vartheta}) := \frac{1}{2}\sum_{j=1}^{t} v^2(j), \tag{5}$$

$$v(t) = y(t) - \boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{\vartheta}. \tag{6}$$

The objective function can be written as a recursive form

$$J(\boldsymbol{\vartheta},t) := J(\boldsymbol{\vartheta},t-1) + \frac{1}{2}v^2(t). \tag{7}$$

Using the first-order Taylor expansion, $v(t)$ at $\boldsymbol{\vartheta} = \hat{\boldsymbol{\vartheta}}(t-1)$ can be approximately expressed as

$$v(t) \approx v(t)|_{\boldsymbol{\vartheta}(t-1)} + \left[\frac{\partial v(t)}{\partial \boldsymbol{\vartheta}}\right]^{\mathrm{T}}_{\hat{\boldsymbol{\vartheta}}(t-1)} [\boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}(t-1)].$$

Define the filtered information vectors

$$\begin{aligned} \hat{\boldsymbol{\Phi}}_f(t) &:= -\frac{\partial v(t)}{\partial \boldsymbol{\vartheta}}\bigg|_{\hat{\boldsymbol{\vartheta}}(t-1)} \\ &= \begin{bmatrix} \hat{\boldsymbol{\phi}}_f(t) \\ \hat{\boldsymbol{\psi}}_f(t) \end{bmatrix} \in \mathbb{R}^{mn+n_c+n_d}, \end{aligned}$$

$$\hat{\boldsymbol{\phi}}_f(t) := -\frac{\partial v(t)}{\partial \boldsymbol{\theta}}\bigg|_{\hat{\boldsymbol{\theta}}(t-1)} = -\begin{bmatrix} \dfrac{\partial v(t)}{a_1\boldsymbol{\beta}} \\ \dfrac{\partial v(t)}{a_2\boldsymbol{\beta}} \\ \vdots \\ \dfrac{\partial v(t)}{a_m\boldsymbol{\beta}} \end{bmatrix}_{\hat{\boldsymbol{\theta}}(t-1)}$$
$$= [\hat{f}_{1,f}(u(t-1)),\ \ldots,\ \hat{f}_{1,f}(u(t-n)),\ \ldots,\ \hat{f}_{m,f}(u(t-1)),\ \ldots,\ \hat{f}_{m,f}(u(t-n))]^{\mathrm{T}},$$
$$\hat{\boldsymbol{\psi}}_f(t) := -\frac{\partial v(t)}{\partial \boldsymbol{\rho}}\bigg|_{\hat{\boldsymbol{\rho}}(t-1)} = [-\hat{w}_f(t-1),\ -\hat{w}_f(t-2),\ \ldots,\ -\hat{w}_f(t-n_c),\ \hat{v}_f(t-1),\ \hat{v}_f(t-2),\ \ldots,\ \hat{v}_f(t-n_d)]^{\mathrm{T}},$$

where $\hat{f}_{i,f}(u(t))$, $\hat{w}_f(t)$ and $\hat{v}_f(t)$ are the filtered values of $f_i(u(t))$, $w(t)$ and $v(t)$, respectively, and defined as

$$\begin{aligned}\hat{f}_{i,f}(u(t)) &:= f_{i,f}(u(t)) + \hat{c}_1(t-1)f_{i,f}(u(t-1)) + \cdots + \hat{c}_{n_c}(t-1)f_{i,f}(u(t-n_c)) \\ &\quad - \hat{d}_1(t-1)\hat{f}_{i,f}(u(t-1)) - \cdots - \hat{d}_{n_d}(t-1)\hat{f}_{i,f}(u(t-n_d)),\\ \hat{w}_f(t) &:= \hat{w}(t) - \hat{d}_1(t-1)\hat{w}_f(t-1) - \cdots - \hat{d}_{n_d}(t-1)\hat{w}_f(t-n_d)),\\ \hat{v}_f(t) &:= \hat{v}(t) - \hat{d}_1(t-1)\hat{v}_f(t-1) - \cdots - \hat{d}_{n_d}(t-1)\hat{v}_f(t-n_d)).\end{aligned}$$

Define the estimated information vector

$$\hat{\boldsymbol{\Phi}}(t) = [\boldsymbol{\phi}^{\mathrm{T}}(t),\ -\hat{w}(t-1),\ -\hat{w}(t-2),\ \ldots,\ -\hat{w}(t-n_c),\ \hat{v}(t-1),\ \hat{v}(t-2),\ \ldots,\ \hat{v}(t-n_d)]^{\mathrm{T}}.$$

The the estimate $\hat{w}(t)$ and $\hat{v}(t)$ can be computed by

$$\hat{w}(t) = y(t) - \boldsymbol{\phi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1),$$
$$\hat{v}(t) = y(t) - \hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1).$$

Applying the Taylor series expansion to $J(\boldsymbol{\vartheta},t-1)$ gives

$$\begin{aligned}J(\boldsymbol{\vartheta},t-1) \approx& \frac{\partial J(\hat{\boldsymbol{\vartheta}}(t-1),t-1)}{\partial \boldsymbol{\vartheta}}[\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] \\ &+ \frac{1}{2}[\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)]^{\mathrm{T}}\frac{\partial^2 J(\hat{\boldsymbol{\vartheta}}(t-1),t-1)}{\partial \boldsymbol{\vartheta}\partial \boldsymbol{\vartheta}^{\mathrm{T}}} \\ &\times [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] + \frac{1}{2}\eta(t),\end{aligned}$$

where the variable $\eta(t)$ is the residual of the Taylor expansion of $J(\boldsymbol{\vartheta},t-1)$. Since the first-order derivative of $J(\boldsymbol{\vartheta},t-1)$ at $\boldsymbol{\vartheta} = \hat{\boldsymbol{\vartheta}}(t-1)$ approximately equals zero, and

$$\boldsymbol{P}_f^{-1}(t) := \frac{\partial^2 J(\boldsymbol{\vartheta},t-1)}{\partial \boldsymbol{\vartheta}\partial \boldsymbol{\vartheta}^{\mathrm{T}}}\bigg|_{\hat{\boldsymbol{\vartheta}}(t-1)}$$

is a positive-definite matrix, equation (7) can be written as

$$J(\boldsymbol{\vartheta},t) \approx \frac{1}{2}[\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)]^{\mathrm{T}}\boldsymbol{P}_f^{-1}(t)[\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] + \frac{1}{2}\eta(t) + \frac{1}{2}v^2(t). \tag{8}$$

From (8) and (7), we have

$$\begin{aligned}2J(\boldsymbol{\vartheta},t) \approx& [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)]^{\mathrm{T}}\boldsymbol{P}_f^{-1}(t)[\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] + \eta(t) + v^2(t)\\ =& [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)]^{\mathrm{T}}\boldsymbol{P}_f^{-1}(t)[\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] \\ &+ \eta(t)\left[v(t)|_{\hat{\boldsymbol{\vartheta}}(t-1)} + \left[\frac{\partial v(t)}{\partial \boldsymbol{\vartheta}}\right]^{\mathrm{T}}_{\hat{\boldsymbol{\vartheta}}(t-1)} \times [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)]\right]^2\\ =& [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)]^{\mathrm{T}}[\boldsymbol{P}_f^{-1}(t) + \hat{\boldsymbol{\Phi}}_f(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)] \\ &\times [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] - 2\hat{v}(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t) \\ &\times [\boldsymbol{\vartheta}-\hat{\boldsymbol{\vartheta}}(t-1)] + v^2(t) + \eta(t).\end{aligned}$$

The second-order derivative of $J(\boldsymbol{\vartheta},t)$ in (7) with respect to $\boldsymbol{\vartheta}$ at $\boldsymbol{\vartheta} = \hat{\boldsymbol{\vartheta}}(t)$ is

$$\boldsymbol{P}_f^{-1}(t) = \frac{\partial^2 J(\boldsymbol{\vartheta},t)}{\partial \boldsymbol{\vartheta}\partial \boldsymbol{\vartheta}^{\mathrm{T}}} = \frac{\partial^2 J(\boldsymbol{\vartheta},t-1)}{\partial \boldsymbol{\vartheta}\partial \boldsymbol{\vartheta}^{\mathrm{T}}} + \hat{v}(t)\frac{\partial^2 \hat{v}(t)}{\partial \boldsymbol{\vartheta}\partial \boldsymbol{\vartheta}^{\mathrm{T}}} + \frac{\partial \hat{v}(t)}{\partial \boldsymbol{\vartheta}}\left[\frac{\partial \hat{v}(t)}{\partial \boldsymbol{\vartheta}}\right]^{\mathrm{T}}. \tag{9}$$

Since the second-order derivative of $v(t)$ with respect to $\boldsymbol{\vartheta}$ at $\boldsymbol{\vartheta} = \hat{\boldsymbol{\vartheta}}(t-1)$ is zero, equation (9) can be written as

$$\boldsymbol{P}_f^{-1}(t) = \boldsymbol{P}_f^{-1}(t-1) + \hat{\boldsymbol{\Phi}}_f(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t). \tag{10}$$

Furthermore, applying the matrix inversion

$$(\boldsymbol{A}+\boldsymbol{BC})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I}+\boldsymbol{CA}^{-1}\boldsymbol{B})^{-1}\boldsymbol{CA}^{-1},$$

to (10) gives

$$\boldsymbol{P}_f(t) = \boldsymbol{P}_f(t-1) - \frac{\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)}{1+\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}. \tag{11}$$

Define the filtered gain vector $\boldsymbol{L}_f(t) := \boldsymbol{P}_f(t)\hat{\boldsymbol{\Phi}}_f(t)$. Then

$$\begin{aligned}\boldsymbol{L}_f(t) =& \boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t) - \frac{\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}{1+\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}\\ =& \frac{\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}{1+\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}.\end{aligned}$$

Bring $\boldsymbol{L}_f(t)$ into (11), we have

$$\boldsymbol{P}_f(t) = \boldsymbol{P}_f(t-1) - \boldsymbol{L}(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)$$

$$= [\boldsymbol{I} - \boldsymbol{L}_f(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)]\boldsymbol{P}_f(t-1).$$

The objective function $J(\boldsymbol{\vartheta}, t)$ can be written as

$$\begin{aligned}2J(\boldsymbol{\vartheta}, t) \approx & [\boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}(t-1) - \boldsymbol{L}(t)\hat{v}(t)]^{\mathrm{T}}\boldsymbol{P}_f^{-1}(t) \\ & \times [\boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}(t-1) - \boldsymbol{L}(t)\hat{v}(t)] \\ & + [\boldsymbol{L}(t)\hat{v}(t)]^{\mathrm{T}}\boldsymbol{P}_f^{-1}(t)[\boldsymbol{L}(t)\hat{v}(t)] + \hat{v}^2(t) \\ & + \eta(t).\end{aligned}$$

Minimizing $2J(\boldsymbol{\vartheta}, t)$ gives the estimate $\hat{\boldsymbol{\vartheta}}(t)$ of $\boldsymbol{\vartheta}$ as

$$\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}(t)\hat{v}(t).$$

Then, we can summarize the maximum likelihood recursive generalized extended least squares (ML-RLS) identification algorithm based on the over-parameterization model as

$$\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}_f(t)\hat{v}(t), \tag{12}$$

$$\boldsymbol{L}_f(t) = \frac{\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}{1 + \hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_f(t-1)\hat{\boldsymbol{\Phi}}_f(t)}, \tag{13}$$

$$\boldsymbol{P}_f(t) = [\boldsymbol{I} - \boldsymbol{L}_f(t)\hat{\boldsymbol{\Phi}}_f^{\mathrm{T}}(t)]\boldsymbol{P}_f(t-1), \tag{14}$$

$$\hat{v}(t) = y(t) - \hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1), \tag{15}$$

$$\hat{\boldsymbol{\Phi}}_f(t) = \begin{bmatrix}\hat{\boldsymbol{\phi}}_f(t) \\ \hat{\boldsymbol{\psi}}_f(t)\end{bmatrix}, \tag{16}$$

$$\hat{\boldsymbol{\Phi}}(t) = \begin{bmatrix}\boldsymbol{\phi}(t) \\ \hat{\boldsymbol{\psi}}(t)\end{bmatrix}, \tag{17}$$

$$\begin{aligned}\hat{\boldsymbol{\phi}}_f(t) = & [\hat{f}_{1,f}(u(t-1)),\ \ldots,\ \hat{f}_{1,f}(u(t-n)),\ \ldots, \\ & \hat{f}_{m,f}(u(t-1)),\ \ldots,\ \hat{f}_{m,f}(u(t-n))]^{\mathrm{T}},\end{aligned} \tag{18}$$

$$\begin{aligned}\hat{\boldsymbol{\psi}}_f(t) = & [-\hat{w}_f(t-1),\ \ldots,\ -\hat{w}_f(t-n_c), \\ & \hat{v}_f(t-1),\ \ldots,\ \hat{v}_f(t-n_d)]^{\mathrm{T}},\end{aligned} \tag{19}$$

$$\boldsymbol{\phi}(t) = [\boldsymbol{F}_1(t),\ \boldsymbol{F}_2(t),\ \ldots,\ \boldsymbol{F}_m(t)]^{\mathrm{T}}, \tag{20}$$

$$\begin{aligned}\hat{\boldsymbol{\psi}}(t) = & [-\hat{w}(t-1),\ \ldots,\ -\hat{w}(t-n_c), \\ & \hat{v}(t-1),\ \ldots,\ \hat{v}(t-n_d)]^{\mathrm{T}},\end{aligned} \tag{21}$$

$$\begin{aligned}\hat{f}_{i,f}(u(t)) = & f_{i,f}(u(t)) + \hat{c}_1(t-1)f_{i,f}(u(t-1)) + \cdots \\ & + \hat{c}_{n_c}(t-1)f_{i,f}(u(t-n_c)) \\ & - \hat{d}_1(t-1)\hat{f}_{i,f}(u(t-1)) - \cdots \\ & - \hat{d}_{n_d}(t-1)\hat{f}_{i,f}(u(t-n_d)),\end{aligned} \tag{22}$$

$$\begin{aligned}\hat{w}_f(t) = & \hat{w}(t) - \hat{d}_1(t-1)\hat{w}_f(t-1) \\ & - \cdots - \hat{d}_{n_d}(t-1)\hat{w}_f(t-n_d)),\end{aligned} \tag{23}$$

$$\begin{aligned}\hat{v}_f(t) = & \hat{v}(t) - \hat{d}_1(t-1)\hat{v}_f(t-1) \\ & - \cdots - \hat{d}_{n_d}(t-1)\hat{v}_f(t-n_d)),\end{aligned} \tag{24}$$

$$\hat{w}(t) = y(t) - \boldsymbol{\phi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1), \tag{25}$$

$$\hat{\boldsymbol{\vartheta}}(t) = [\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t), \hat{\boldsymbol{\rho}}^{\mathrm{T}}(t)]^{\mathrm{T}}, \tag{26}$$

$$\hat{\boldsymbol{\theta}}(t) = [\widehat{a_1\boldsymbol{\beta}^{\mathrm{T}}(t)},\ \widehat{a_2\boldsymbol{\beta}^{\mathrm{T}}(t)},\ \ldots,\ \widehat{a_m\boldsymbol{\beta}^{\mathrm{T}}(t)}]^{\mathrm{T}}, \tag{27}$$

$$\hat{\boldsymbol{\rho}}(t) = [\hat{c}_1(t),\ \ldots,\ \hat{c}_{n_c}(t),\ \hat{d}_1,\ \ldots,\ \hat{d}_{n_d}(t)]^{\mathrm{T}}. \tag{28}$$

The parameter vector $\hat{\boldsymbol{\theta}}(t)$ contains the products of a_i and β_l ($i = 1, 2, \ldots, m$, $l = 1, 2, \ldots, n$). Then we have

$$\begin{aligned}\sum_{l=1}^{m}\widehat{a_i\beta_l(t)}^2 &= \widehat{a_i\beta_1(t)}^2 + \widehat{a_i\beta_2(t)}^2 + \cdots + \widehat{a_i\beta_n(t)}^2 \\ &= (\hat{a}_i(t)\hat{\beta}_1(t))^2 + \cdots + (\hat{a}_m(t)\hat{\beta}_n(t))^2 \\ &= \hat{a}_i^2(t)\|\hat{\boldsymbol{\beta}}(t)\|^2.\end{aligned}$$

From the normalization hypothesis $\|\boldsymbol{\beta}\| = 1$, $\beta_1 > 0$. The estimate $\hat{a}_i(t)$ can be computed by

$$\hat{a}_i(t) = \mathrm{sgn}[\widehat{a_i\beta_1(t)}]\sqrt{\sum_{l=1}^{m}\widehat{a_i\beta_l(t)}}. \tag{29}$$

Then we can obtain the estimates $\hat{\beta}_{1,i}(t), \hat{\beta}_{2,i}(t), \ldots, \hat{\beta}_{n,i}(t)$ of $\beta_1, \beta_2, \ldots, \beta_n$ as

$$\hat{\beta}_{1,i}(t) = \frac{\widehat{a_i\beta_1(t)}}{\hat{a}_i(t)},\ \ldots,\ \hat{\beta}_{n,i}(t) = \frac{\widehat{a_i\beta_n(t)}}{\hat{a}_i(t)}. \tag{30}$$

In order to improve the estimation accuracy, take their average as the estimates of $\beta_1, \beta_2, \ldots, \beta_n$, that is

$$\hat{\beta}_1(t) = \frac{1}{m}\sum_{i=1}^{m}\hat{\beta}_{1,i}(t),\ \ldots,\ \hat{\beta}_n(t) = \frac{1}{m}\sum_{i=1}^{m}\hat{\beta}_{n,i}(t). \tag{31}$$

The steps for implementing the ML-RLS algorithm in (12)-(31) are as follows:

1) To initialize: set the parameter estimation accuracy ε. Set the initial values $\hat{\boldsymbol{\vartheta}}(0) = \mathbf{1}_{(mn+n_c+n_d)\times 1}/p_0$, $\hat{f}_{i,f}(u(t-j)) = 1/p_0$, $\hat{w}_f(t-j) = 1/p_0$, $\hat{w}(t-j) = 1/p_0$, $\hat{v}_f(t-j) = 1/p_0$, $\hat{v}(t-j) = 1/p_0$ and $\boldsymbol{P}_f(0) = p_0\boldsymbol{I}_{(mn+n_c+n_d)}$, $j = 1, 2, \ldots, \max[n_c, n_d]$, $i = 1, 2, \ldots, m$, $p_0 = 10^6$.
2) Collect the observation data $u(t)$ and $y(t)$, form the information vector $\hat{\boldsymbol{\Phi}}(t)$ by (17) and form the filtered information vector $\hat{\boldsymbol{\Phi}}_f(t)$ by (16).
3) Compute the gain vector $\boldsymbol{L}_f(t)$ by (13) and the covariance matrix $\boldsymbol{P}_f(t)$ by (14).
4) Update the parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t)$ by (12).
5) Compute the residual $\hat{v}(t)$ and $\hat{w}(t)$ by (15), (25). Compute the filtered variables $\hat{f}_{i,f}(u(t))$, $\hat{w}_f(t)$ and $\hat{v}_f(t)$ by (22)-(24). Compute $\hat{a}_i(t)$, $\hat{\beta}_l(t)$ in (29)-(31).
6) If $\|\hat{\boldsymbol{\vartheta}}(t) - \hat{\boldsymbol{\vartheta}}(t-1)\| > \varepsilon$, increase t by 1 go to Step 4; Otherwise, terminate the recursive calculation procedure and obtain the parameter estimate $\hat{\boldsymbol{\vartheta}}(t)$.

4. THE RECURSIVE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM

In order to illustrate the advantage of the ML-RLS identification algorithm proposed in Section 3, we derive an over-parameterization-based recursive generalized extended least squares identification algorithm as a comparison.

Define a quadratic criterion function

$$J(\boldsymbol{\vartheta}) = \frac{1}{2}\sum_{j=1}^{t}[y(t) - \boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{\vartheta}(t-1)]^2.$$

Minimizing $J(\boldsymbol{\vartheta})$ and letting the derivatives of $J(\boldsymbol{\vartheta})$ with respect to $\boldsymbol{\vartheta}$ be zero, the estimate of the parameter vector $\boldsymbol{\vartheta}$ at time t is given by

$$\hat{\boldsymbol{\vartheta}}(t) = \left[\sum_{j=1}^{t}\boldsymbol{\Phi}(t)\boldsymbol{\Phi}^{\mathrm{T}}(t)\right]^{-1}\sum_{j=1}^{t}\boldsymbol{\Phi}(t)y(t). \tag{32}$$

The parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t)$ in (32) can be written as a recursive least squares algorithm

$$\begin{aligned}
\hat{\boldsymbol{\vartheta}}(t) &= \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}(t)[y(t) - \boldsymbol{\Phi}^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1)],\\
\boldsymbol{L}(t) &= \frac{\boldsymbol{P}(t-1)\hat{\boldsymbol{\Phi}}(t)}{1+\hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\Phi}}(t)},\\
\boldsymbol{P}(t) &= [\boldsymbol{I} - \boldsymbol{L}(t)\hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)]\boldsymbol{P}(t-1).
\end{aligned}$$

Similarly, replacing the unknown information $\boldsymbol{\psi}(t)$ in $\boldsymbol{\Phi}(t)$ with its estimate $\hat{\boldsymbol{\psi}}(t)$, we can obtain the recursive generalized extended least squares (RLS) parameter estimation algorithm based on the over-parameterization model as follows:

$$\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}(t)[y(t) - \hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1)], \tag{33}$$

$$\boldsymbol{L}(t) = \frac{\boldsymbol{P}(t-1)\hat{\boldsymbol{\Phi}}(t)}{1+\hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\Phi}}(t)}, \tag{34}$$

$$\boldsymbol{P}(t) = [\boldsymbol{I} - \boldsymbol{L}(t)\hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)]\boldsymbol{P}(t-1), \tag{35}$$

$$\hat{\boldsymbol{\Phi}}(t) = \begin{bmatrix}\boldsymbol{\phi}(t)\\ \hat{\boldsymbol{\psi}}(t)\end{bmatrix}, \tag{36}$$

$$\boldsymbol{\phi}(t) = [\boldsymbol{F}_1(t), \boldsymbol{F}_2(t), \ldots, \boldsymbol{F}_m(t)]^{\mathrm{T}}, \tag{37}$$

$$\begin{aligned}\hat{\boldsymbol{\psi}}(t) = [&-\hat{w}(t-1),\ \ldots,\ -\hat{w}(t-n_c),\\ &\hat{v}(t-1), \ldots, \hat{v}(t-n_d)]^{\mathrm{T}},\end{aligned} \tag{38}$$

$$\hat{w}(t) = y(t) - \boldsymbol{\phi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1), \tag{39}$$

$$\hat{v}(t) = y(t) - \hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1), \tag{40}$$

$$\hat{\boldsymbol{\vartheta}}(t) = [\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t), \hat{\boldsymbol{\rho}}^{\mathrm{T}}(t)]^{\mathrm{T}}, \tag{41}$$

$$\hat{\boldsymbol{\theta}}(t) = [\widehat{a_1\boldsymbol{\beta}^{\mathrm{T}}(t)},\ \widehat{a_2\boldsymbol{\beta}^{\mathrm{T}}(t)},\ \ldots,\ \widehat{a_m\boldsymbol{\beta}^{\mathrm{T}}(t)}]^{\mathrm{T}}, \tag{42}$$

$$\hat{\boldsymbol{\rho}}(t) = [\hat{c}_1(t),\ \ldots,\ \hat{c}_{n_c}(t),\ \hat{d}_1(t),\ \ldots,\ \hat{d}_{n_d}(t)]^{\mathrm{T}}. \tag{43}$$

The steps for implementing the RLS algorithm in (33)-(43) are as follows:

1) To initialize: Set the parameter estimation accuracy ε and the initial values $\hat{w}(t-j) = 1/p_0$, $\hat{v}(t-j) = 1/p_0$ and $\boldsymbol{P}(0) = p_0\boldsymbol{I}_{(mn+n_c+n_d)}$, $j = 1, 2, \ldots, \max[n_c, n_d]$, $p_0 = 10^6$.
2) Collect the observation data $u(t)$ and $y(t)$, form the information vector $\hat{\boldsymbol{\Phi}}(t)$ by (36).
3) Compute the gain vector $\boldsymbol{L}(t)$ by (34) and the covariance matrix $\boldsymbol{P}(t)$ by (35).
4) Update the parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t)$ by (33).
5) Compute the residual $\hat{v}(t)$ and $\hat{w}(t)$ by (39) and (40). Compute $\hat{a}_i(t)$ and $\hat{\beta}_l(t)$ using (29)-(31).
6) Compare $\hat{\boldsymbol{\vartheta}}(t)$ with $\hat{\boldsymbol{\vartheta}}(t-1)$: if $\|\hat{\boldsymbol{\vartheta}}(t) - \hat{\boldsymbol{\vartheta}}(t-1)\| > \varepsilon$, increase t by 1 go to Step 3; Otherwise, terminate the recursive calculation procedure and obtain the parameter estimate $\hat{\boldsymbol{\vartheta}}(t)$.

The proposed algorithms in this paper can combine other methods [34–38] to study parameter identification of different systems [39–41] and can be applied to other fields [42–47] such as chemical engineering systems. In practice, the design of many fault detection and control algorithms assume that the parameters of the considered system models are known [48–51]. If the case is not so, then some identification methods can be used first for obtaining the parameters of the systems from observation information.

5. EXAMPLE

Consider the following simulation system

$$\begin{aligned}
y(t) &= \boldsymbol{a}^{\mathrm{T}}\boldsymbol{F}(t)\boldsymbol{\beta} + \frac{D(z)}{C(z)}v(t),\\
\boldsymbol{F}(t) &= \begin{bmatrix}u(t-1) & u^2(t-1)\\ u(t-2) & u^2(t-2)\end{bmatrix},\\
\boldsymbol{a} &= [a_1,\ a_2]^{\mathrm{T}} = [-2.1,\ 1]^{\mathrm{T}},\\
\boldsymbol{\beta} &= [\beta_1,\ \beta_2]^{\mathrm{T}} = [0.912,\ -0.41]^{\mathrm{T}},\\
D(z) &= 1 + d_1z^{-1} = 1 + 0.1z^{-1},\\
C(z) &= 1 + c_1z^{-1} = 1 - 0.4z^{-1},\\
\boldsymbol{\Upsilon} &= [a_1,\ a_2,\ \beta_1,\ \beta_2,\ d_1,\ c_1]^{\mathrm{T}}\\
&= [-2.1,\ 1.0,\ 0.912,\ -0.41,\ -0.4,\ 0.1]^{\mathrm{T}}.
\end{aligned}$$

The inputs $\{u(t)\}$ is taken as an independent persistent signal sequence with zero mean and unit variance, $\{v(t)\}$ is an uncorrelated noise sequence with zero mean and variance $\sigma^2 = 0.50^2$, $\sigma^2 = 1.00^2$ and $\sigma^2 = 1.50^2$, respectively. Applying the ML-RLS algorithm and the RLS algorithm to estimate the parameters of this example system, the parameter estimates and their estimation errors $\delta := \|\hat{\boldsymbol{\Upsilon}}_t - \boldsymbol{\Upsilon}\|/\|\boldsymbol{\Upsilon}\|$ of the RLS algorithm and the ML-RLS algorithm are shown in Tables 1-3 and Figs. 1-4. The ML-RLS parameter estimates versus k with $\sigma^2 = 0.50^2$ are shown in Fig. 5.

From Tables 1-3 and Figs. 1-5, we can draw the following conclusions.

1) It can be seen that the estimation errors are becoming smaller as t increases, and then tends to be stationary. This shows that the ML-RLS algorithm and the RLS algorithm are effective.
2) The estimation errors are becoming smaller as the noise variance decreases in the ML-RLS algorithm and the RLS algorithm.

Table 1. The RLS and ML-RLS estimates and their errors with $\sigma^2 = 0.50^2$.

Algorithms	t	a_1	a_2	β_1	β_2	c_1	d_1	$\delta(\%)$
RLS	100	-1.97966	0.75807	0.94943	-0.28672	-0.12451	0.27310	17.23563
	200	-2.05863	0.91699	0.92993	-0.35685	-0.25117	0.21996	8.56679
	500	-2.03982	0.98246	0.91952	-0.38971	-0.33661	0.18206	4.79855
	1000	-2.05026	1.01979	0.92039	-0.39000	-0.37145	0.02228	3.93572
	2000	-2.07973	1.00311	0.91645	-0.40001	-0.39202	0.03507	2.70655
ML-RLS	100	-2.12771	0.99331	0.92520	-0.37917	-0.39068	0.23794	5.65630
	200	-2.09912	1.04162	0.91904	-0.39403	-0.40103	0.15360	2.73205
	500	-2.07329	0.99781	0.91751	-0.39737	-0.38119	0.15704	2.61963
	1000	-2.08792	1.00071	0.91734	-0.39795	-0.38776	0.04826	2.18695
	2000	-2.09465	1.00653	0.91438	-0.40486	-0.40718	0.08380	0.79657
True values		-2.10000	1.00000	0.91200	-0.41000	-0.40000	0.10000	

Table 2. The RLS and ML-RLS estimates and their errors with $\sigma^2 = 1.00^2$.

Algorithms	t	a_1	a_2	β_1	β_2	c_1	d_1	$\delta(\%)$
RLS	100	-1.94222	0.69348	0.95564	-0.22540	-0.02833	0.32741	22.88745
	200	-2.04614	0.89668	0.93394	-0.33637	-0.19965	0.25234	11.21966
	500	-2.01989	0.97964	0.92129	-0.38283	-0.31323	0.19420	6.04699
	1000	-2.03343	1.02786	0.92278	-0.38356	-0.35932	-0.02114	5.82843
	2000	-2.07277	1.00459	0.91789	-0.39662	-0.38620	0.00474	3.94526
ML-RLS	100	-2.22207	0.95829	0.94742	-0.31815	-0.33599	0.24764	8.90927
	200	-2.08815	1.12630	0.93050	-0.36568	-0.38978	0.19292	6.42219
	500	-2.01288	0.99549	0.92769	-0.37003	-0.32236	0.15786	5.34948
	1000	-2.06217	1.00590	0.92703	-0.37344	-0.34383	-0.01816	5.53326
	2000	-2.08319	1.02103	0.91891	-0.39446	-0.40130	0.01899	3.39379
True values		-2.10000	1.00000	0.91200	-0.41000	-0.40000	0.10000	

Table 3. The RLS and ML-RLS estimates and their errors with $\sigma^2 = 1.50^2$.

Algorithms	t	a_1	a_2	β_1	β_2	c_1	d_1	$\delta(\%)$
RLS	100	-2.28337	0.92833	0.96289	-0.26585	-0.28844	0.00378	11.28758
	200	-2.07573	1.20737	0.93803	-0.34525	-0.38274	0.00203	9.41775
	500	-1.96215	1.00186	0.93436	-0.34745	-0.27382	0.00445	8.58289
	1000	-2.04112	1.01266	0.93471	-0.35123	-0.30415	-0.03099	7.18253
	2000	-2.07471	1.03439	0.92286	-0.38512	-0.39596	-0.01823	5.01563
ML-RLS	100	-2.25222	0.92296	0.96124	-0.27244	-0.28908	0.01824	10.27323
	200	-2.06935	1.18723	0.93696	-0.34829	-0.38224	0.01431	8.54847
	500	-1.97019	0.99942	0.93333	-0.35149	-0.28283	0.01613	7.94086
	1000	-2.04466	1.01043	0.93295	-0.35650	-0.31297	0.00157	6.00622
	2000	-2.07584	1.03129	0.92195	-0.38731	-0.39718	0.00385	4.16785
True values		-2.10000	1.00000	0.91200	-0.41000	-0.40000	0.10000	

3) Under the same noise variances, compared with the RLS algorithm, the ML-RLS algorithm produces higher parameter estimation accuracy. Thus, the ML-RLS algorithm shows better performance.

6. CONCLUSIONS

This paper derives an ML-RLS algorithm and an RLS algorithm for identifying the bilinear-parameter systems with ARMA noise using the over-parameterization method and the maximum likelihood principle. The simulation results show that the ML-RLS algorithm has a higher estimation accuracy compared with the RLS algorithm. The proposed identification algorithm for bilinear-parameter stochastic systems with ARMA noises in this paper can joint some recirsive and iterative schemes [52–60] to explored new estimation algorithms for linear and nonlinear systems and can be applied to other control and

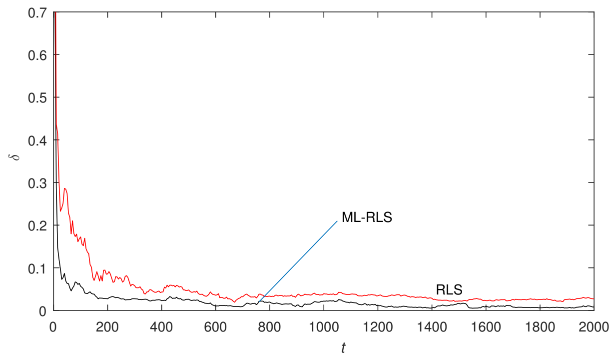

Fig. 1. The ML-RLS and RLS estimation errors versus t ($\sigma^2 = 0.50^2$).

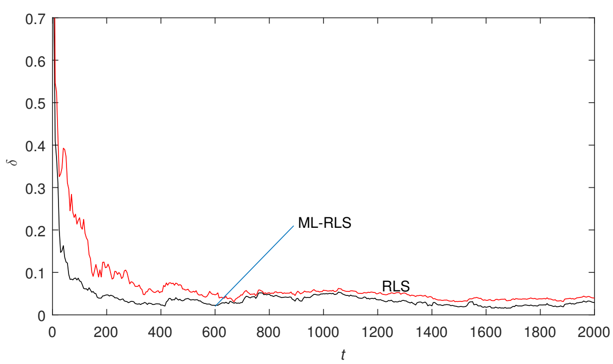

Fig. 2. The ML-RLS and RLS estimation errors versus t ($\sigma^2 = 1.00^2$).

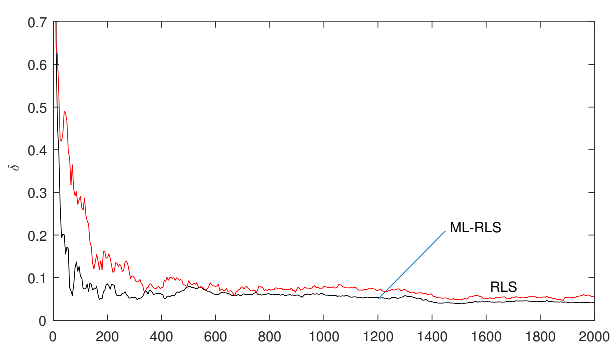

Fig. 3. The ML-RLS and RLS estimation errors versus t ($\sigma^2 = 1.50^2$).

schedule areas [61–68] such as the information processing and engineering systems by means of some mathematical tools [69–74].

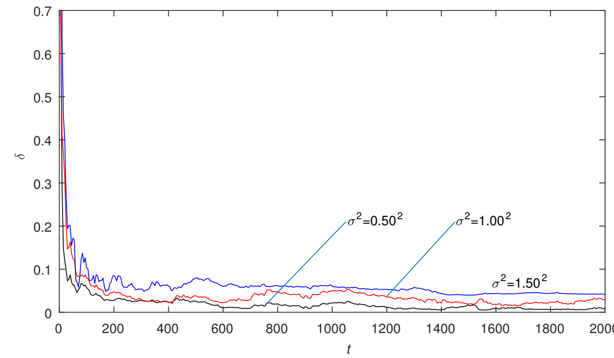

Fig. 4. The ML-RLS parameter estimation errors versus t with different σ^2.

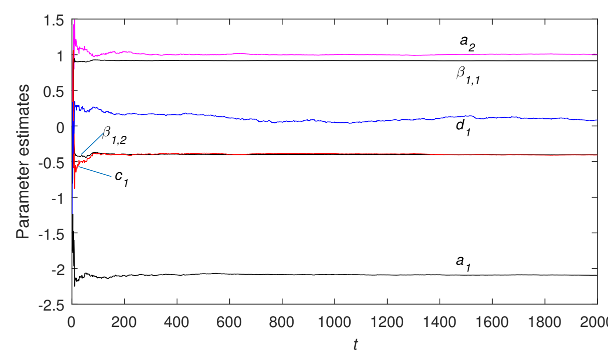

Fig. 5. The ML-RLS parameter estimates versus t with $\sigma^2 = 0.50^2$.

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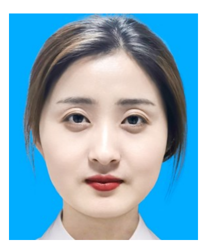

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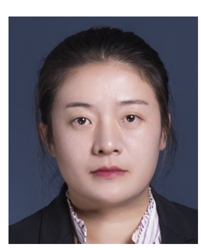

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