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Adaptive Event Triggered Optimal Control for Constrained Continuous-time Nonlinear Systems

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Abstract: This paper considers the event-triggered optimal control (ETOC) strategy for constrained continuous-time nonlinear systems via adaptive dynamic programming (ADP). First, a novel event-triggering condition is proposed, which can guarantee the stability of the closed-loop system. Meanwhile, the existence of a lower bound for the execution time is proved, which can guarantee that the designed event-trigger scheme avoids Zeno behavior. Then, to solve the partial differential Hamilton-Jacobi-Bellman (HJB) equation,the critic Neural Network (NN) is designed to approximate the cost function. So that the ADP-based ETOC scheme is designed. Moreover, through Lyapunov stability analysis, the stability of the closed-loop system can be ensured. Also, the uniform ultimate boundedness of the states and the weight estimation error can also be guaranteed. Last, a numerical example is given to illustrate the effectiveness and advantages of the proposed control scheme.

Keywords: ADP, constrained input, event-triggered, neural networks, optimal control.

1. INTRODUCTION

In the field of optimal control, from the view point of system analysis and design, the methods that consider both periodic communication and traversal [1–3] can be regarded as the most effective method. However, if we take the limited resources into account, then the periodic manner is no longer the best one. In recent years, due to the fact that the amount of web data is keeping increasing and the network resources are limited, some researchers have proposed an improved method called event-triggered control or event-driven control or event-based control [4–11]. Event-trigger (ET) scheme [12] is on the basis of intermittent principle. It means that the agent does not need to feedback in real time but executes the control action according to the demand based on the error of measured information. In other words, ET scheme can tremendously reduce the sampling frequency of the control system.

For the optimal control problem of nonlinear system, the difficulty lies in solving the HJB equation because it involves solving nonlinear partial differential equation [10,11]. Meanwhile, as is well known, constrained-input system is more reasonable because the control input is usually constrained by the physical constraints of components. So the optimal control problem of nonlinear system with input constraints is a much more challenging problem. In order to solve this problem, the idea of approximation is adopted to the solution of the HJB equation. A popular method is the Adaptive dynamic programming (ADP) based on Neural Network approximation. In fact, there are many literature concerned with this problem. For instance, Abu-Khalaf and Lewis [10] employed the NN-based HJB approach to address the near optimal control of nonlinear systems with saturating control inputs. NN-based near optimal control for nonlinear discrete-time systems with control constraints was studied in [13]. For partially unknown constrained-input systems was proposed in [14]. NN-based online optimal control for uncertain nonlinear systems with saturation constraints was discussed in [15]. But all these literature didn't combine the ET scheme with the ADP scheme, so that couldn't make better use of the network resources.

Many researchers have integrated ET scheme into the ADP method to propose the ADP-based ETOC scheme. Researchers [16,17] proposed ADP-based ETOC scheme for nonlinear continuous time systems. Researchers [18, 19] discussed about the uncertain nonlinear system.

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Vamvoudakis and Ferraz [20] presented a model-free ETOC for continuous-time linear systems. Zhao *et al*. [21] presented an event-triggered robust tracking control based ADP for unknown systems with disturbances. Yang and He [22] developed a novel event-triggered robust control strategy for systems with unknown dynamics. Though, it should be noted that all of the above literature are excellent which combined ADP method with ET scheme, yet none of them aimed at the constrained-input control systems. Although there are several documents related to constrained-input control system, such as [23,24], the Adaptive ETOC scheme for constrained-input nonlinear systems is still a challenging problem, so in this paper we will focus on the optimal control problem based on Adaptive dynamic programming and event-triggered scheme for nonlinear system with constrained input control.

Motivated by the above discussions, we focus on the adaptive ETOC of constrained-input nonlinear continuous-time system in this paper, the contributions include 1) As we know, the previous ET condition is established on the comparison of $\|x\|_2$ and $\|e\|_2$, in our paper a novel event-triggered condition which is based on the relationship between $\|x\|_2^2$ and $\|e\|_2$ is designed for constrained-input continuous-time nonlinear systems. It can reduce the number of triggered which was illustrated by simulation examples. 2) We present that our ET scheme can avoid the Zeno behavior by proving the existence of the lower bound for inter-execution time strictly. 3) An ADP-based ETOC scheme is designed which only employs a single critic network in order to reduce the computational load, moreover, the uniform ultimate boundedness (UUB) of the states and the weight approximation error are proved.

The remainder of this paper is organized as follows: Several notations used in this paper are presented in Section 2. The problem is formulated in Section 3. The event-triggering condition is designed and the lower bound of the inter-event intervals is determined in Section 4. Stability of ADP-based ETOC scheme and UUB of the states and the weight estimation error are analyzed in Section 5. Simulation results are shown in Section 6. In the end, some conclusions of this paper are shown in Section 7.

2. PRELIMINARIES

In this paper, $\mathbb{R}^m$ denotes the real m-dimensional Euclidean space with spectral norm $||\cdot||_2$, $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. I_m denotes the $m\times m$ identity matrix. For a matrix $A\in\mathbb{R}^{n\times n}$, A^T denotes the transposed of A. $\lambda_{\min}(A)$ is the minimum eigenvalue of matrix A.

3. PROBLEM FORMULATION

Consider a class of constrained continuous-time nonlinear systems described by

$$\dot{x}(t)=f(x(t))+g(x(t))u(x(t)),\ x(0)=x_0, \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $f(x(t))\in\mathbb{R}^n$ denotes the internal dynamics, $g(x(t))\in\mathbb{R}^{n\times m}$ is the input gain matrix; $u=(u_1,u_2,\ldots,u_m)^T\in\mathbb{R}^m$ is the control input with $|u_i|\le\lambda$, and λ is a positive constant.

Define the infinite horizontal cost function which requires to be optimized as

$$V(x,u)=\int_0^\infty (Q(x)+W(u))dt, \tag{2}$$

where $Q(x)$ and $W(u)$ are positive definite. For the function $W(u)$, a common selection to treat the control constraints is shown in [10,14], that is

$$\begin{aligned}W(u)&=2\lambda\int_0^u(\tanh^{-1}(v/\lambda))^Tdv\\&=2\lambda u^T\tanh^{-1}(u/\lambda)+\lambda^2\overline{R}\ln\left(\mathbf{1}-\frac{u^2}{\lambda^2}\right),\end{aligned} \tag{3}$$

where $v\in\mathbb{R}^m$, $\overline{R}=[1,\ldots,1]\in\mathbb{R}^{1\times m}$, $\mathbf{1}=[1,\ldots,1]^T\in\mathbb{R}^m$.

The goal of optimal control is to find an optimal constrained input $u^*=u^*(x)$ to minimize the cost function, meanwhile the system is driven to asymptotic stability. The Lyapunov equation (LE) for nonlinear systems is proposed in [10], which is equivalent to the HJB equation with the substitution of the optimal value V^* as follows [25]:

$$V_x^{*T}(f+gu^*)+Q(x)+W(u^*)=0,\ \ V(0)=0, \tag{4}$$

where $V_x^*=\partial V^*(x)/\partial x\in\mathbb{R}^n$. For the constrained input, the HJB equation is as follows:

$$\begin{aligned}H(x,u^*(x),V_x^*)&=\min_{u(x)\in\Omega}H(x,u(x),V_x^*)\\&=V_x^{*T}(f+gu^*)+Q(x)\\&\quad+2\lambda\int_0^{u^*}(\tanh^{-1}(v/\lambda))^Tdv\\&=0.\end{aligned} \tag{5}$$

Then, through calculations, the optimal control u^* can be obtained as

$$u^*=-\lambda\tanh\left(\frac{1}{2\lambda}g^T(x)V_x^*\right). \tag{6}$$

We can obtain the equivalent form as follows:

$$\begin{aligned}W(u^*)&=2\lambda u^{*T}\tanh^{-1}(u^*/\lambda)+\lambda^2\overline{R}\ln\left(\mathbf{1}-\frac{u^{*2}}{\lambda^2}\right)\\&=-2\lambda u^{*T}D^*+\lambda^2\overline{R}\ln\left(\mathbf{1}-\tanh^2(D^*)\right),\end{aligned} \tag{7}$$

where D^* is defined as

$$D^*=D^*(x)=\frac{1}{2\lambda}g^T(x)V_x^*. \tag{8}$$

4. EVENT-TRIGGERED OPTIMAL CONTROL

In this section, a qualified ETOC scheme is proposed to guarantee the stability of the closed-loop system. Moreover, it will be shown that there exists a low bound for the execution time, or in other words, the minimum inter-event interval is positive.

4.1. The ETOC scheme

In ETOC scheme, the system state is sampled at the triggering instants $\{t_k\}_{k\in N}$, and t_k is the k-th consecutive release time at which the triggering condition is violated, where $t_0 = 0$. $\{\hat{x}_k\}_{k\in N}$ denotes the sampled state, where $\hat{x}_k = x(t_k)$. Then, the ETOC is obtained by mapping the sampled state as:

$$u^*(\hat{x}_k) = -\lambda \tanh\left(\frac{1}{2\lambda} g^T(\hat{x}_k) V_x^*(\hat{x}_k)\right), \ \ t \in [t_k, t_{k+1}), \tag{9}$$

which is transformed into a continuous signal by employing the zero-order holder without delay.

The error between the current state and the last released state is defined as

$$e_k = e_k(t) = \hat{x}_k - x(t), \ \ t \in [t_k, t_{k+1}). \tag{10}$$

Accordingly, by using (10), the closed-loop system can be written as

$$\dot{x} = f(x) + g(x)u^*(\hat{x}_k) = f(x) + g(x)u^*(x + e_k), \tag{11}$$

and the LE under the ET scheme is as follows:

$$\begin{aligned} H(x, u^*(\hat{x}_k), V_x^*) = {} & V_x^{*T}(f + gu^*(\hat{x}_k)) \\ & + Q(x) + W(u^*(\hat{x}_k)), \end{aligned} \tag{12}$$

in which

$$\begin{aligned} W(u^*(\hat{x}_k)) = {} & 2\lambda u^{*T}(\hat{x}_k) \tanh^{-1}\left(\frac{u^*(\hat{x}_k)}{\lambda}\right) \\ & + \lambda^2 \overline{R} \ln\left(\mathbf{1} - \frac{u^*(\hat{x}_k)^2}{\lambda^2}\right). \end{aligned} \tag{13}$$

Next, the novel triggering condition and the stability of the closed-loop system under the novel triggering condition will be given in Theorem 1. Before that, we need to introduce the following assumptions.

Assumption 1 [4,9,17,26]**:** Suppose that system (1) is Lipschitz continuous with respect to x and e on a set $\Omega \in \mathbb{R}^n$ and $f(0) = 0$, that is, there exists a positive constant L such that

$$\|f(x) + g(x)u(x + e)\|_2 \le L\|x\|_2 + L\|e\|_2, \ \forall x, e \in \mathbb{R}^n.$$

Assumption 2 [9,17,24]**:** Suppose that u^* is Lipschitz continuous on a compact set, that is, there exists a constant $L_u > 0$ such that

$$\|u^*(x) - u^*(\hat{x}_k)\|_2 \le L_u\|x - \hat{x}_k\|_2.$$

Assumption 3 [23]**:** Suppose that D^* defined in (8) is Lipschitz continuous on a compact set, that is, there exists a constant $L_D > 0$ such that

$$\|D^*(x) - D^*(\hat{x}_k)\|_2 \le L_D\|x - \hat{x}_k\|_2.$$

From (4), we can get

$$V_x^{*T}(f + gu^*) = -Q(x) - W(u^*). \tag{14}$$

Since $Q(x)$ and $W(u)$ are positive definite, the negative definiteness of the time-derivative of V^* can be guaranteed, that is, V is decreasing. Furthermore, we can guarantee that the derivative of V^* satisfies

$$\dot{V}^* \le -\gamma[Q(x) + W(u^*)], \tag{15}$$

where $\gamma \in (0, 1)$ is a tuning parameter. Note that (15) is important in this paper by which we can derive the new ET condition.

Theorem 1: Suppose Assumptions 1-3 hold. Then, closed-loop system (11) under the ETOC scheme (9) is asymptotically stable, if the following triggering condition is satisfied:

$$\begin{aligned} TC = {} & (2L_u\lambda \overline{M}_{tanh^{-1}} + 2\lambda^2 m L_D)\|e_k\|_2 \\ & + (\gamma - 1)\lambda_{min}(Q)\|x\|_2^2 \\ \le {} & 0, \end{aligned} \tag{16}$$

that is to say, the triggering instants are given by

$$t_{k+1} = \inf\{t | TC > 0, \ t > t_k\}. \tag{17}$$

Proof: In order to prove the theorem, subtracting (5) from (12) yields

$$\begin{aligned} & H(x, u^*(\hat{x}_k), V_x^*) - H(x, u^*(x), V_x^*) \\ & = V_x^{*T}(f + gu^*(\hat{x}_k)) + W(u^*(\hat{x}_k)) \\ & \quad - V_x^{*T}(f + gu^*) - W(u^*). \end{aligned} \tag{18}$$

Combined $H(x, u^*(x), V_x^*) = 0$ with (7) and (8), we can get

$$\begin{aligned} H(x, u^*(\hat{x}_k), V_x^*) = {} & \lambda^2 \overline{R} \ln[\mathbf{1} - \tanh^2(D^*(\hat{x}_k))] \\ & - \lambda^2 \overline{R} \ln[\mathbf{1} - \tanh^2(D^*(x))]. \end{aligned} \tag{19}$$

According to (12), we have

$$\begin{aligned} \dot{V}^* = {} & \lambda^2 \overline{R} \ln[\mathbf{1} - \tanh^2(D^*(\hat{x}_k))] \\ & - \lambda^2 \overline{R} \ln[\mathbf{1} - \tanh^2(D^*(x))] \\ & - Q(x) - W(u^*(\hat{x}_k)), \end{aligned} \tag{20}$$

$$\begin{aligned} & \dot{V}^* + \gamma[Q(x) + W(u^*)] \\ & = (\gamma - 1)Q(x) + \{\gamma W(u^*(x)) - W(u^*(\hat{x}_k))\} \\ & \quad + \{\lambda^2 \overline{R} \ln[\mathbf{1} - \tanh^2(D^*(\hat{x}_k))] \\ & \quad - \lambda^2 \overline{R} \ln[\mathbf{1} - \tanh^2(D^*(x))]\}. \end{aligned} \tag{21}$$

Next, we will rewrite the right-hand side of (21) in a simplified equivalent form. Based on the mean value theorem and combined with $\gamma < 1$ and (3), the second term in the right-hand side of (21) can be rewritten as follows:

$$\begin{aligned}&\gamma W(u^*(x)) - W(u^*(\hat{x}_k))\\&\leq 2\lambda\int_0^{u^*(x)}(\tanh^{-1}(v/\lambda))^T dv\\&\quad -2\lambda\int_0^{u^*(\hat{x}_k)}(\tanh^{-1}(v/\lambda))^T dv\\&= 2\lambda(\tanh^{-1}(\xi/\lambda))^T(u^*(x)-u^*(\hat{x}_k)),\end{aligned}$$

where $\xi = u^*(\hat{x}_k)+\theta(u^*(x)-u^*(\hat{x}_k)) \in \mathbb{R}^m$, $\theta \in (0,1)$. Since the monotonicity of $\tanh^{-1}(\cdot)$, let $\overline{M}_{\tanh^{-1}} = \max\left\{\|\tanh^{-1}\left(\frac{u^*(x)}{\lambda}\right)\|_2, \|\tanh^{-1}\left(\frac{u^*(\hat{x}_k)}{\lambda}\right)\|_2\right\}$, by using the Lipschitz condition in Assumption 2, the second term can be estimated as follows:

$$\gamma W(u^*(x)) - W(u^*(\hat{x}_k)) \leq 2L_u\lambda\overline{M}_{\tanh^{-1}}\|e_k\|_2. \tag{22}$$

Based on the mean value theorem and Assumption 3, the third term of the right-hand side of (21) is

$$\begin{aligned}&\lambda^2\overline{R}\ln[\mathbf{1}-\tanh^2(D^*(\hat{x}_k))] - \lambda^2\overline{R}\ln[\mathbf{1}-\tanh^2(D^*(x))]\\&= -2\lambda^2\tanh^T\eta(D^*(\hat{x}_k))-D^*(x))\\&\leq 2\lambda^2 mL_D\|e_k\|_2,\end{aligned}\tag{23}$$

where $\eta = (\eta_1,\ldots,\eta_m)^T$, $\eta_i = D_i^*(x)+\theta_i[D_i^*(\hat{x}_k)-D_i^*(x)]$, $\theta_i \in (0,1), i = 1,\ldots,m$.
Combining (22), (23) and $Q(x) \geq \lambda_{\min}(Q)\|x\|_2^2$, ultimately, we can rewrite (20) as follows:

$$\begin{aligned}&\dot{V}^* + \gamma[Q(x)+W(u^*)]\\&\leq \left[2L_u\lambda\overline{M}_{\tanh^{-1}} + 2\lambda^2 mL_D\right]\|e_k\|_2\\&\quad +(\gamma-1)\lambda_{\min}(Q)\|x\|_2^2.\end{aligned}$$

Under the condition (16), in inter-event intervals, we have

$$\dot{V}^* \leq -\gamma[Q(x)+W(u^*)] < 0.$$

This result corresponds to (15) and therefore guarantees the stability of the closed-loop system. Meanwhile, the next triggering instant can be determined as

$$t_{k+1} = \inf\{t|TC>0,\ t>t_k\}.$$

This completes the proof. □

4.2. The lower bound for the execution time

Define the execution time as $\tau_k = t_{k+1}-t_k$, $k = 0, 1, \ldots$. In what follows, we will discuss the bound of the inter-event interval of the proposed ET scheme. Firstly, define the execution time as

$$\tau_k = t_{k+1}-t_k,\ k = 0,1,\ldots$$

Assumption 4: Suppose that the following conditions hold.

1) $f(x)$ and $g(x)$ are bounded on the compact set Ω, i.e., $\|f(x)\|_2 \leq f_b$, $\|g(x)\|_2 \leq g_b$;
2) $c_1\|x\|_2 \leq \|u^*(x)\|_2 \leq c_2\|x\|_2$.

Theorem 2: Let Assumptions 1 and 4 hold. Consider the event-triggered system (11) with the triggering condition (16), then there exists a non-zero lower bound for the execution time τ_k, $k = 0, 1, \ldots$.

Proof: By using Assumptions 1 and 4, we can get

$$\frac{d\|x(t)\|_2}{dt} \leq \|\dot{x}(t)\|_2 = \|f+gu\|_2 \leq f_b + g_b c_2\|x(t)\|_2,$$

that is

$$\|x(t)\|_2 \leq \left[\|x(t_k)\|_2 + \frac{f_b}{g_b c_2}\right]e^{g_b c_2(t-t_k)} - \frac{f_b}{g_b c_2},\ \ t \geq t_k. \tag{24}$$

From (10), we have $e_k(t) = x(t_k)-x(t)$, $t \in [t_k,\ t_{k+1})$, which can be written as follows by using Assumptions 1 and 4:

$$\frac{d\|e_k(t)\|_2}{dt} \leq \|\dot{e}_k(t)\|_2 \leq f_b + g_b c_2\|x(t)\|_2,$$

by using (24) we can get

$$\begin{cases}\dfrac{d\|e_k(t)\|_2}{dt} \leq \alpha e^{g_b c_2(t-t_k)},\ \forall t \geq t_k,\\ e_k(t_k) = 0,\end{cases}$$

where $\alpha = [g_b c_2\|x(t_k)\|_2 + f_b]$.
So we have

$$\|e_k(t)\|_2 \leq \frac{\alpha}{g_b c_2}[e^{g_b c_2(t-t_k)}-1],$$

$$\begin{aligned}\|e_k(t_{k+1})\|_2 &\leq \frac{\alpha}{g_b c_2}[e^{g_b c_2\tau_k}-1],\\ &\leq \frac{\alpha}{g_b c_2}[e^{g_b c_2\tau_k}-\frac{1}{\delta}],\ \ (\delta>1).\end{aligned}\tag{25}$$

From (16) we can get $\|e_k(t_{k+1})\|_2 \geq \beta\|x(t_{k+1})\|_2^2$, where $\beta = \frac{(1-\gamma)\lambda_{\min}(Q)}{2L_u\lambda\overline{M}_{\tanh^{-1}}+2\lambda^2 mL_D}$, combined with (25), so we have

$$\beta\|x(t_{k+1})\|_2^2 \leq \|e_k(t_{k+1})\|_2 \leq \frac{\alpha}{g_b c_2}[e^{g_b c_2\tau_k}-\frac{1}{\delta}],$$

$$\tau_k \geq \frac{1}{g_b c_2}\ln\left[\frac{1}{\delta}+\frac{g_b c_2\beta\|x(t_{k+1})\|_2^2}{\alpha}\right].$$

It is obvious that if δ takes the appropriate value, we can all guarantee

$$\ln\left[\frac{1}{\delta}+\frac{g_b c_2\beta\|x(t_{k+1})\|_2^2}{\alpha}\right] > 0,$$

that is

$$\tau_k \geq \frac{1}{g_b c_2}\ln\left[\frac{1}{\delta}+\frac{g_b c_2\beta\|x(t_{k+1})\|_2^2}{\alpha}\right] > 0,\ k = 0,1,\ldots$$

Hence, there exists a non-zero lower bound for the inter-execution time, which can prevent the proposed ET scheme from Zeno behavior. □

5. ADP-BASED ETOC

Since the HJB is a nonlinear equation, it is hard to be solved, we employ Neural Network to approximate the solution of the HJB equation.

5.1. NN-based algorithm

By the approximation theorem of neural networks, it is reasonable to assume that the cost function $V^*(x)$ is approximated by

$$V^*(x) = \delta_1^T \Phi_1(x) + \varepsilon_1(x), \tag{26}$$

where $\delta_1 \in \mathbb{R}^N$ is the ideal weight, $\Phi_1 : \mathbb{R}^n \to \mathbb{R}^N$ is the activation function, N is the number of hidden neurons and $\varepsilon_1(x)$ is the Neural Network approximation error. Thus, Neural Network Lyapunov equation can be rewritten as

$$\begin{aligned} H(x,u,\delta_1) &= \delta_1^T \nabla\Phi_1(f+gu) + Q(x) + W(u) \\ &= \varepsilon_H, \end{aligned} \tag{27}$$

where $\varepsilon_H = -\nabla\varepsilon_1^T(x)(f+gu)$ is the residual error.

Since the weight δ_1 which can achieve the optimal cost function is unknown, the actual output is given by

$$\hat{V}^*(x) = \hat{\delta}_1^T \Phi_1(x), \tag{28}$$

where $\hat{\delta}_1$ is the estimated value of weight δ_1, meanwhile the weight estimation error can be defined as $\widetilde{\delta}_1 = \delta_1 - \hat{\delta}_1$. By using (28), the approximate optimal control law can be described as

$$\begin{aligned} \hat{u}(\hat{x}_k) = &-\lambda \tanh\left(\frac{1}{2\lambda} g^T(\hat{x}_k)\nabla\Phi_1^T(\hat{x}_k)\hat{\delta}_1\right), \\ &t \in [t_k, t_{k+1}). \end{aligned} \tag{29}$$

Furthermore, the approximate Neural Network Lyapunov equation can be rewritten as

$$\begin{aligned} H(x,u,\hat{\delta}_1) &= \hat{\delta}_1^T \nabla\Phi_1(f+gu) + Q(x) + W(u) \\ &= \hat{\varepsilon}_H, \end{aligned} \tag{30}$$

where

$$\hat{\varepsilon}_H = \varepsilon_H - \widetilde{\delta}_1^T \nabla\Phi_1(f+gu). \tag{31}$$

In order to achieve $\hat{\delta}_1 \to \delta_1$, $\hat{\varepsilon}_H \to \varepsilon_H$, we aim at minimizing $E_1 = \frac{1}{2}\hat{\varepsilon}_H^T\hat{\varepsilon}_H$ by refining the parameter $\hat{\delta}_1$. The normalized gradient descent algorithm is used to tune the critic weight $\hat{\delta}_1$, so that

$$\begin{aligned} \dot{\hat{\delta}}_1 &= -\frac{l_1}{(\rho^T\rho+1)^2}\frac{\partial E_1}{\partial \hat{\delta}_1} \\ &= -l_1\frac{\rho}{(\rho^T\rho+1)^2}\left[\rho^T\hat{\delta}_1 + Q(x) + W(u)\right], \end{aligned} \tag{32}$$

can be achieved, where $\rho = \nabla\Phi_1(f+gu)$. So, the time derivative of the estimation error is

$$\dot{\widetilde{\delta}}_1 = -\dot{\hat{\delta}}_1 = l_1\frac{\rho}{(\rho^T\rho+1)^2}\left[\rho^T\hat{\delta}_1 + Q(x) + W(u)\right]. \tag{33}$$

From (27), we have

$$Q(x) + W(u) = \varepsilon_H - \delta_1^T\nabla\Phi_1(f+gu) = \varepsilon_H - \rho^T\delta_1. \tag{34}$$

Combining (33) with (34), the derivative of the estimation error can be rewritten as

$$\dot{\widetilde{\delta}}_1 = -l_1 G\widetilde{\delta}_1 + l_1\frac{\vartheta}{\rho^T\rho+1}\varepsilon_H, \tag{35}$$

in which $G = \frac{\rho^T\rho}{(\rho^T\rho+1)^2}$, $\vartheta = \frac{\rho}{\rho^T\rho+1}$. It is obvious that $G > 0$, $\|\vartheta\|_2 \le \frac{1}{2}$, $\|\frac{\vartheta}{\rho^T\rho+1}\|_2 \le \frac{1}{2}$ by using the existing conclusions.

5.2. Uniform ultimate boundedness

In this subsection, Theorem 3 is given to guarantee not only the stability of the closed-loop ET system, but the uniform ultimate boundedness (UUB) of $x(t)$, $\hat{x}(t)$ and parameter $\widetilde{\delta}_1$, which is inspired by [5,17,18,27].

Assumption 5: Suppose that the following conditions hold

1) $\Phi_1(x)$ and $\nabla\Phi_1(x)$ are bounded on the compact set Ω, i.e., $\|\Phi_1(x)\|_2 \le \Phi_{1b}$, $\|\nabla\Phi_1(x)\|_2 \le \nabla\Phi_{1b}$;
2) The residual error is bounded, i.e., $\|\varepsilon_H\|_2 \le \varepsilon$;
3) δ_1 is bounded, i.e., $\|\delta_1\|_2 \le \delta_{1b}$;
4) The NN approximation error is bounded over the compact set Ω, i.e., $\|\varepsilon_1(x)\|_2 \le \varepsilon_{1b}$.

Theorem 3: Consider the constrained nonlinear system (1) with the triggering condition given by Theorem 1, the ETOC input given by (29) and the critic NN given by (28). Let Assumptions 1-5 hold, then $x(t)$, $\hat{x}(t)$ and parameter $\widetilde{\delta}_1$ are uniformly ultimately bounded.

Proof: First, define the candidate Lyapunov function as

$$L = V^*(x) + V^*(\hat{x}) + V_c,$$

where $V^*(x)$ is the optimal cost function and $V_c = \frac{1}{2}tr\left(\widetilde{\delta}_1^T\widetilde{\delta}_1\right)$.

The proof of Theorem 3 will be divided into two cases, that is, the flow dynamics and the jump dynamics.

Case 1: The flow dynamics, i.e., the events are not triggered, i.e., $t \in [t_k, t_{k+1})$.

In this case, the derivative of the Lyapunov function is given by

$$\dot{L} = \dot{V}^*(x) + \dot{V}^*(\hat{x}) + \dot{V}_c. \tag{36}$$

For the first term of (36), we have

$$\dot{V}^*(x) = V_x^{*T}(f + g\hat{u}(\hat{x}_k)). \tag{37}$$

According to the triggering condition in Theorem 1, we have

$$\dot{V}^*(x) = V_x^{*T}(f + gu^*) < -\gamma[Q(x) + W(u^*)].$$

Thus, for the first term of (36), we have

$$\dot{V}^*(x) < -\gamma[Q(x) + W(\hat{u}(\hat{x}_k))]. \tag{38}$$

Next, we design an equation about $-\gamma W(\hat{u}(\hat{x}_k))$, which is of benefit to the proof, as follows:

$$-\gamma W(\hat{u}(\hat{x}_k)) = \|\hat{u}(\hat{x}_k)\|_2^2 - \|\hat{u}(\hat{x}_k)\|_2^2 - \gamma W(\hat{u}(\hat{x}_k)).$$

Furthermore, combing (6), (8) and $W(u) > 0$, we can rewrite the above equation as

$$\begin{aligned} -\gamma W(\hat{u}(\hat{x}_k)) =& \|\lambda \tanh(\hat{D}(\hat{x}_k))\|_2^2 - \|\hat{u}(\hat{x}_k)\|_2^2 \\ &- \gamma W(\hat{u}(\hat{x}_k)), \\ <& \|\lambda \tanh(\hat{D}(\hat{x}_k))\|_2^2 - \|\hat{u}(\hat{x}_k)\|_2^2. \end{aligned} \tag{39}$$

According to Assumption 4 and $|\tanh(\cdot)| < 1$, (39) can be rewritten as

$$-\gamma W(\hat{u}(\hat{x}_k)) < \lambda^2 m - c_1\|\hat{x}_k\|_2^2. \tag{40}$$

Hence, for the first term of (36), we can get

$$\dot{V}^*(x) < -\gamma\lambda_{\min}(Q)\|x\|_2^2 - c_1\|\hat{x}_k\|_2^2 + \lambda^2 m. \tag{41}$$

For the second term of (36), we have $\dot{V}^*(\hat{x}) = 0$.

For the third term of (36), by employing (35) and Assumption 5, we have

$$\begin{aligned} \dot{V}_c &= \widetilde{\delta}_1^T \dot{\widetilde{\delta}}_1, \\ &= \widetilde{\delta}_1^T \left(-l_1 G\widetilde{\delta}_1 + l_1 \frac{\vartheta}{\rho^T\rho + 1}\varepsilon_H \right) \\ &= -l_1 \widetilde{\delta}_1^T G\widetilde{\delta}_1 + l_1 \widetilde{\delta}_1^T \frac{\vartheta}{\rho^T\rho + 1}\varepsilon_H, \\ &\leq -l_1\lambda_{\min}(G)\|\widetilde{\delta}_1\|_2^2 + \frac{1}{2} l_1 \varepsilon \|\widetilde{\delta}_1\|_2. \end{aligned} \tag{42}$$

Thus, by combining (41) with (42), we can obtain

$$\begin{aligned} \dot{L} &= \dot{V}^*(x) + \dot{V}_c, \\ &\leq -\gamma\lambda_{\min}(Q)\|x\|_2^2 - l_1\lambda_{\min}(G)\|\widetilde{\delta}_1\|_2^2 - c_1\|\hat{x}_k\|_2^2 \\ &\quad + \frac{1}{2} l_1 \varepsilon \|\widetilde{\delta}_1\|_2 + \lambda^2 m. \end{aligned}$$

Let $\eta = [\|x\|_2,\ \|\widetilde{\delta}_1\|_2,\ \|\hat{x}\|_2]^T$, we have

$$\dot{L} \leq -\eta^T A\eta + B\eta + C, \tag{43}$$

where

$$A = \begin{bmatrix} \gamma\lambda_{\min}(Q) & 0 & 0 \\ 0 & l_1\lambda_{\min}(G) & 0 \\ 0 & 0 & c_1 \end{bmatrix} > 0,$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{2} l_1 \varepsilon \\ 0 \end{bmatrix},$$

$$C = \lambda^2 m.$$

We can rewrite (43) as $\dot{L} \leq -\lambda_{\min}(A)\|\eta\|_2^2 + \|B\|_2\|\eta\|_2 + C$, then $\dot{L}$ is negative if

$$\|\eta\|_2 \geq \frac{\|B\|_2 + \sqrt{\|B\|_2^2 + 4\lambda_{\min}(A)C}}{2\lambda_{\min}(A)} = \underline{\eta},$$

which can ensure the boundedness of $x(t)$, $\hat{x}(t)$ and parameter $\widetilde{\delta}_1$, i.e., they are uniformly ultimately bounded.

Case 2: The jump dynamics, i.e., events are triggered, i.e., $t = t_k$.

In this case, we only need to consider the deviation of the candidate Lyapunov function between these two cases. According to the result in Case 1, we have

$$\begin{aligned} \Delta L =& V^*(x^+(t_k)) - V^*(x(t_k)) + V^*(\hat{x}^+(t_k)) \\ &- V^*(\hat{x}(t_k)) + V_c(x^+(t_k)) - V_c(x(t_k)). \end{aligned}$$

where $x^+(t_k)$, $\hat{x}^+(t_k)$ denote the right limit at t_k.

Due to the conclusion in Case 1, that is, the flow dynamics is asymptotically stable, we have

$$V^*(x^+(t_k)) < V^*(x(t_k)),\ \ V_c(x^+(t_k)) < V_c(x(t_k)).$$

Moreover, in view of the triggering condition, that is

$$\dot{V} \leq -\gamma[Q(x) + W(u)] < 0,$$

we have

$$\dot{V}^*(\hat{x}(t_k)) < 0, i.e., V^*(\hat{x}^+(t_k)) < V^*(\hat{x}(t_k)).$$

To sum up, when $\|\eta\|_2 \geq \underline{\eta}$, $\Delta L < 0$ hold at the sampling instants $t = t_k$. So, we can obtain that $x(t)$, $\hat{x}(t)$ and parameter $\widetilde{\delta}_1$ are uniformly ultimately bounded at the sampling instants. □

The relationship between NN estimation error and the bounds for performance index and inter-execution time will be discussed in the following theorem and Remark 1.

Theorem 4: Consider the system (1). The input control is given by (29). Let Assumptions 1-5 hold, then there is a upper bound for the performance index for the proposed ADP-based ETOC.

Proof: By employing (15), we can obtain that there is a upper bound for cost function in the ET scheme as follows:

$$\begin{aligned}
&\because Q(x)+W(u^*(\hat{x})) \le -\frac{1}{\gamma}\dot{V}^*,\\
&\therefore V(x_0,u) = \int_0^\infty [Q(x)+W(u^*(\hat{x}))]dt,\\
&\qquad \le -\frac{1}{\gamma}\int_0^\infty \dot{V}^* dt,\\
&\qquad = \frac{1}{\gamma}V^*(x_0).
\end{aligned}$$

While the cost function $V(x)$ is approximated by the critic Neural Network in (26), we can prove that the performance index for the proposed ADP-based ETOC still has the upper bound which can be presented as follows:

$$\begin{aligned}
&\because \dot{V}^* = \dot{\hat{V}}^* + \dot{\varepsilon}_1(x),\\
&= \hat{V}_x^{*T}[f+g\hat{u}(\hat{x})] + \dot{\varepsilon}_1(x),\\
&\le -\gamma[Q(x)+W(\hat{u}(\hat{x}))] + \dot{\varepsilon}_1(x),\\
&\therefore Q(x)+W(\hat{u}(\hat{x})) \le \frac{1}{\gamma}\left[\dot{\varepsilon}_1(x) - \dot{V}^*\right],\\
&\therefore \hat{V}^*(x_0) = \int_0^\infty [Q(x)+W(\hat{u}(\hat{x}))]dt,\\
&\qquad \le \frac{1}{\gamma}\int_0^\infty [\dot{\varepsilon}_1(x) - \dot{V}^*]dt,\\
&\qquad = \frac{1}{\gamma}[V^*(x_0)+2\varepsilon_{1b}].
\end{aligned}$$

As shown in [30], as long as neurons $N \to \infty$, the approximation error $\varepsilon_1(x) \to 0$, in this case, the performance index will approach to the expected value $\frac{1}{\gamma}V^*(x_0)$. So it is feasible to realize a satisfying result in the ADP scheme in our paper. □

Remark 1: Combined (26) with (23), we can get

$$\dot{V}^* \le \varepsilon_1^{'T}(x)(f+gu^*) - \gamma[Q(x)+W(u^*)] + TC.$$

As shown in [30], as long as neurons $N \to \infty$, the approximation error $\varepsilon_1^{'}(x) \to 0$. Furthermore, based on the proof of Theorem 2, we can consider that the lower bound for the inter-execution time is positive for the proposed ADP-based ETOC.

6. NUMERICAL EXAMPLES

In this section, we illustrate the feasibility and effectiveness of the proposed ETOC scheme by employing two examples.

Example 1: The system dynamics is given as follows [28]:

$$\begin{cases}
\dot{x}_1 = -x_1 + x_2,\\
\dot{x}_2 = -\frac{1}{2}(x_1+x_2) + \frac{1}{2}x_2\sin^2(x_1) + \sin(x_1)u.
\end{cases}$$

Assume that $x_0 = [1, -1]^T$, the control constraint is $\|u\|_2 \le 1.2$.

As shown in (2) and (3), the cost function is defined as

$$V(x) = \int_0^\infty [x^T Q x + 2\lambda \int_0^u (\tanh^{-1}(v/\lambda))^T dv]dt,$$

where Q is the identity matrix of appropriate dimension, and $\lambda = 1.2$. We select the sample interval as 0.01 s, the simulation time as 10s, parameter l_1 in (32) as 0.4, the parameter γ in (15) as 0.1. The basis function of the cost function is chosen as $\phi(x) = [x_1^2; x_1x_2; x_2^2]$, which was described in [29]. The NN weights are initialized at random from a uniform distribution in the interval $(-1, 1)$.

The simulation results are presented in Figs. 1-6. Fig. 1 shows the convergence of system states $x_i(t)$ and $x_i(t_k)$, $i = 1, 2$. Fig. 2 presents the nearly optimal control input u. These figures imply that the closed-loop system is stable in the sense of uniform ultimate boundedness.

Fig. 3 shows the evolution of the trigger threshold and the event-trigger error in the proposed ET scheme in Theorem 1. We can observe that the trend of the event-trigger error $\|e_k\|_2$, that is, in each event-trigger interval (t_k, t_{k+1}), $\|e_k\|_2$ is below the trigger threshold until it reaches the threshold. Meanwhile, the triggering instants t_{k+1} is generated when $\|e_{k+1}\|_2$ is reset to zero. The event threshold converges to zero ultimately.

By using our ET scheme, the total number of triggering times is only 323. It is far below 1000, which denotes the sampling times obtained by using the time-triggered controller. Fig. 4 presents the sampling period, that is, the execution time between two adjacent triggering instants. We can see that the execution time has a lower bound, which is greater than zero, so our method can avoid the Zeno behavior.

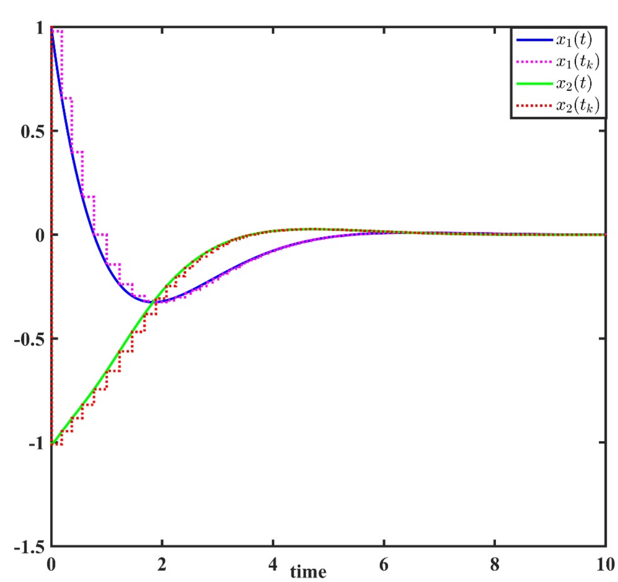

Fig. 1. The trajectories of $x_i(t)$ and $x_i(t_k)$.

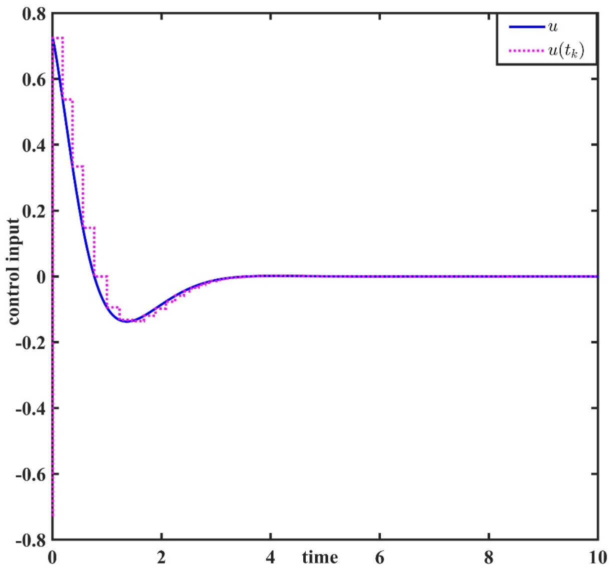

Fig. 2. The optimal control input.

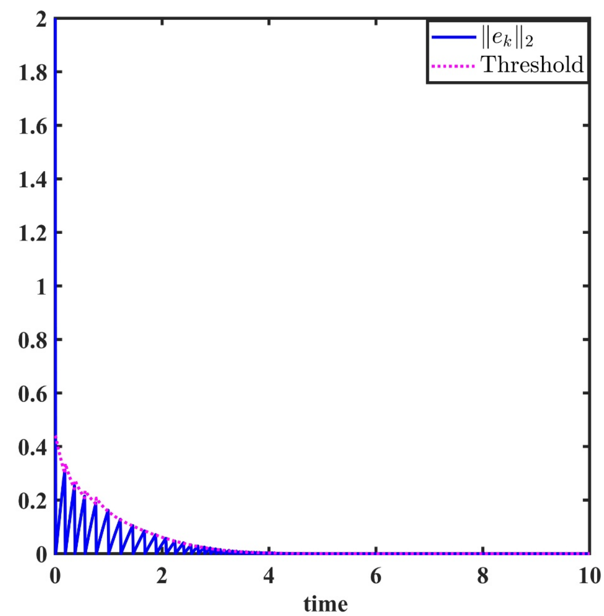

Fig. 3. The event-trigger error and the threshold.

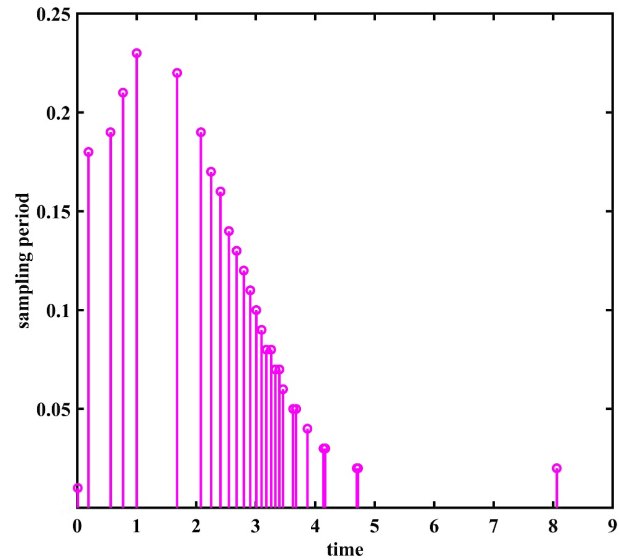

Fig. 4. Sampling period.

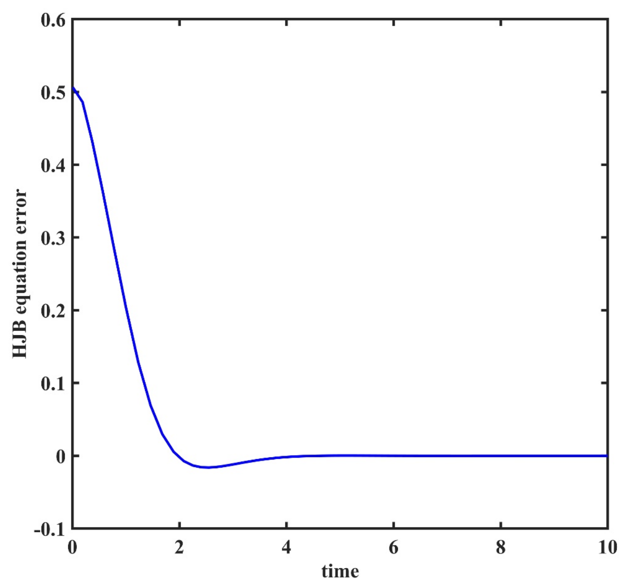

Fig. 5. HJB equation error.

Fig. 5 shows that the HJB equation error shown in (27) converges close to zero. It implies that the NN approximation of the value function converges to the neighborhood of the optimal value. The upper bound of the cost function $\frac{1}{\gamma}V^*(x_0)$ is shown in Fig. 6, in our paper we select γ as 0.1, so we can find the upper bound is ten times the value of $V^*(x_0)$.

Example 2: Consider the following nonlinear oscillator system [10]:

$$\begin{cases} \dot{x}_1 = x_1 + x_2 - x_1(x_1^2 + x_2^2), \\ \dot{x}_2 = -x_1 + x_2 - x_2(x_1^2 + x_2^2) + u. \end{cases}$$

Define the cost function as shown in (2) and (3), and Q, γ, $\phi(x)$, l_1 have the same selection as Example 1.

Assume that $x_0 = [0,\ 1]^T$, the control constraint is $\|u\|_2 \leq 1$. The NN weights are initialized at random from a uniform distribution in the interval $(-1,\ 1)$. We select the sample interval as 0.01 s, the simulation time as 15 s. The simulation results are presented in Figs. 7-12, which demonstrate the trajectories of the state, the control input, the event-triggered error, the sampling period, the HJB equation error, and the upper bound of the cost function, respectively.

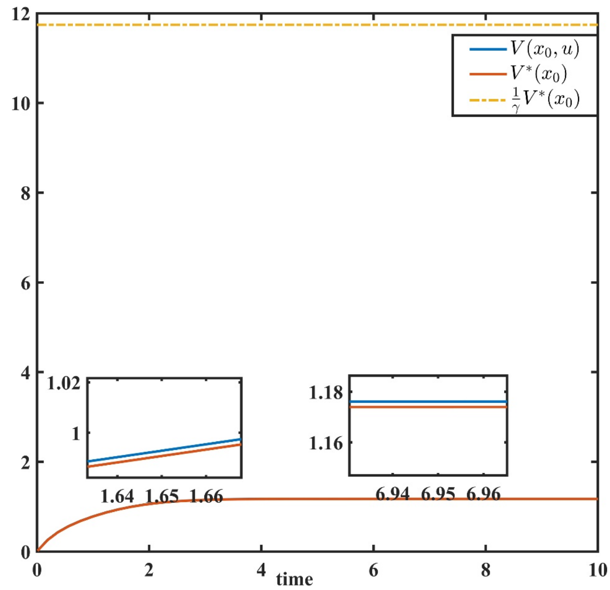

Fig. 6. The upper bound of the cost function.

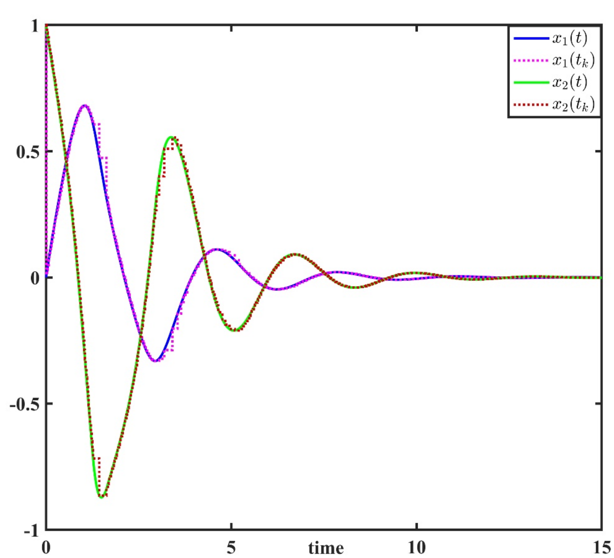

Fig. 7. The trajectories of $x_i(t)$ and $x_i(t_k)$.

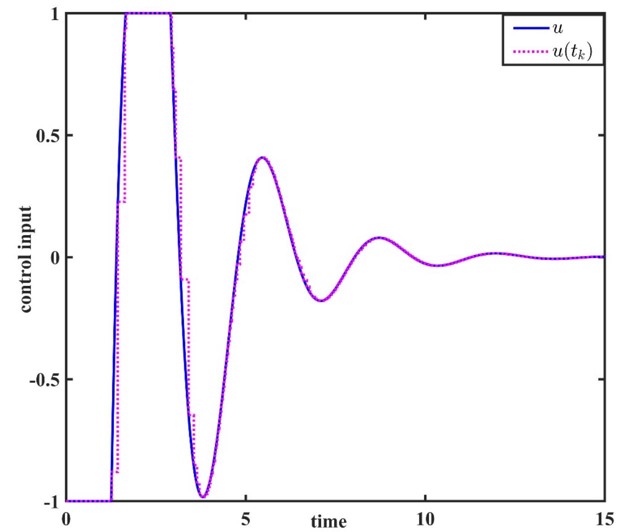

Fig. 8. The optimal control input.

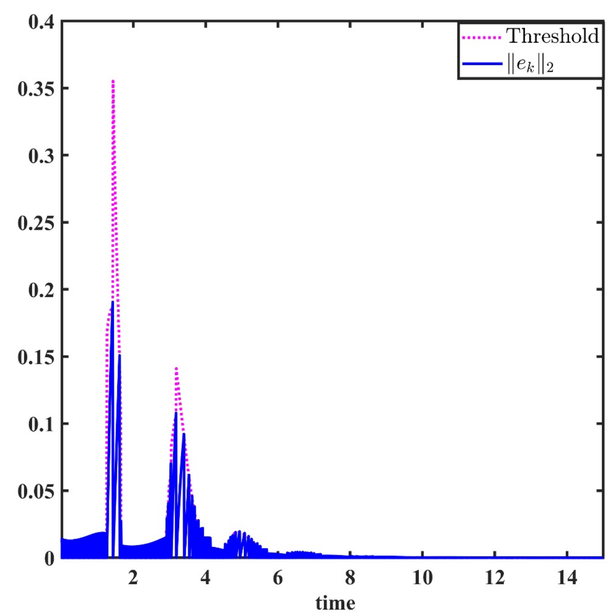

Fig. 9. The event-trigger error and the threshold.

7. CONCLUSION

In this paper, a novel ETOC scheme is designed for continuous-time nonlinear systems with constrained inputs and the critic NN is applied to approximate the cost function. The existence of a positive lower bound for the inter-execution time and the stability of the closed-loop system are proven through strict theoretical analysis. Meanwhile, all the states and the weight estimation error in the event-triggered system were guaranteed to be UUB. We also analyze the relationship between the bound for the cost function and the NN estimation error to illustrate the efficiency of the the NN approximation method. Numerical examples are offered to illustrate the effectiveness of our proposed ET scheme.

The proposed paper only uses a single critic structure to approximate the cost function to solve the HJB equation. Though the single critic network can reduce the computational load, yet the future work will extend to design more complex Neural Network to improve the accuracy of simulation. Additionally, the future work also will include adaptive design of more complex nonlinear systems.

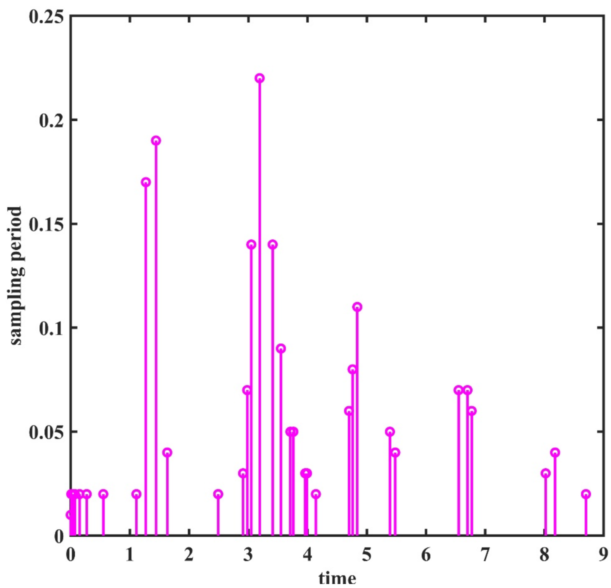

Fig. 10. Sampling period.

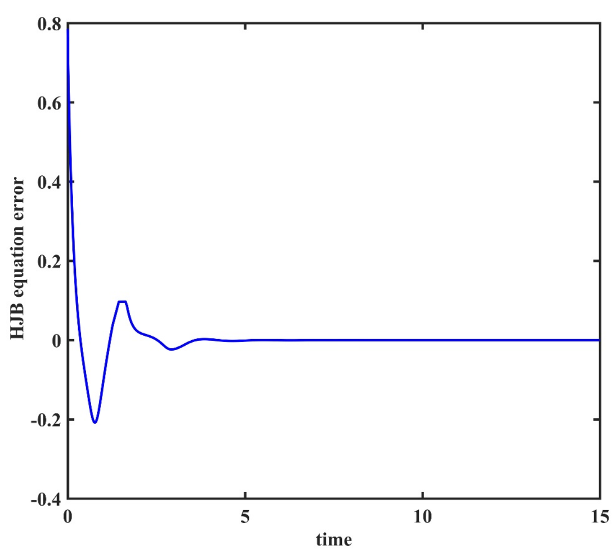

Fig. 11. HJB equation error.

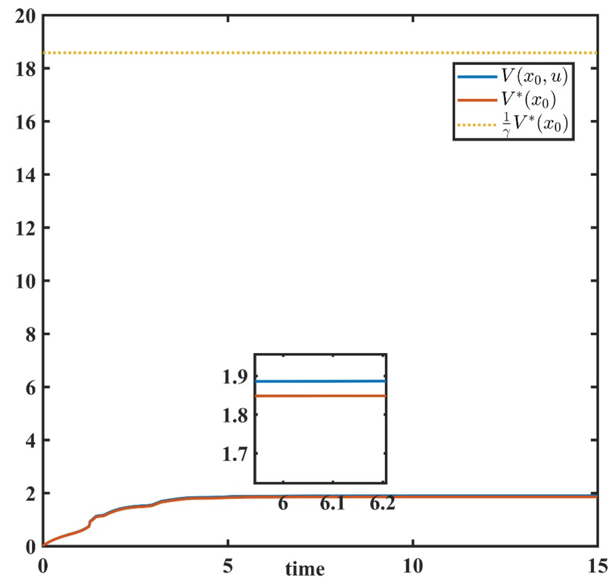

Fig. 12. The upper bound of the cost function.

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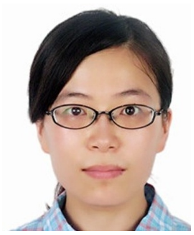

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