International Journal of Control, Automation, and Systems 20(9) (2022) 2792-2804
http://dx.doi.org/10.1007/s12555-021-0097-x
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Fixed-time Disturbance Observer-based Sliding Mode Control for Mismatched Uncertain Systems

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Abstract: This paper concerns with fixed-time control for the mismatched uncertain system. We propose a fixed-time disturbance observer (DO). Then, a non-singular fixed-time sliding-mode surface and controller are designed based on the estimations of proposed DO. Existing DO-based sliding-mode control (DOSMC) and finite-time DOSMC schemes can eliminate the mismatched uncertain problem by using traditional sliding-mode control (TSMC). However, the convergence times of these DOSMC schemes are infinite or related to the initial system conditions. Unlike existing approaches, the proposed fixed-time DOSMC can guarantee the convergence time is uniformly bounded and the upper bound is independent on initial system conditions, which implies that the proposed scheme can provide similar convergence performance with the same control parameters under different initial conditions. Simulation result shows the excellent convergence performance of the proposed scheme.

Keywords: Finite-time convergence, fixed-time convergence, mismatched disturbance, observer, sliding-mode control.

1. INTRODUCTION

Sliding-mode control (SMC) is a effective scheme to handle the disturbances [1–4]. Due to the significance of suppressing disturbances, SMC schemes have been applied to many actual systems in recent years [5–10]. However, the robustness of the traditional SMC (TSMC) can be held only if disturbances and control inputs are in the same channel, this is known as the matching condition.

In fact, many practical systems are affected by various mismatched disturbances such as flight control system [11,12], permanent magnet synchronous motor system [13] and flexible joint manipulator system [14]. To apply the TSMC for the system with mismatched disturbances, various control schemes like Riccati-based SMC [15,16], LMI-based SMC [17–19], and adaptive SMC [20] have been developed. However, these modified SMC schemes in [15–20] require that the mismatched disturbances satisfy the H_2 norm-bounded assumption, which means that the mismatched uncertainties must belong to vanishing uncertainties. This assumption is unreasonable for most of practical systems [11–14]. In recent year, some results have adopted the disturbance observer (DO) to modify the SMC to suppress the mismatched and matched disturbances [21–25]. These disturbance observer-based SMC (DOSMC) schemes not only can relax the unreasonable H_2 norm-bounded assumption in [15–20], but also retain the nominal control performance of SMC [21–25]. Thus, the study on the DOSMC has increasingly became a research hotspot. The DOMSC in [21–25] assumed that the derivatives of the mismatched disturbances are to vanish when the time approaches to infinity. However, the assumption is still too restrictive. The DOMSC in [26] extended the results in [21–25]. The main feature of the method in [26] is that, it assumed that the disturbances and the derivatives of disturbances are bounded.

It is worthy of noting that, although DOSMC provides the high robust performance for mismatched uncertain system, the DOSMC schemes [21–26] are designed by using the asymptotic stability analysis. It is implied that these schemes in [21–26] only achieve the infinite convergence time. The fast convergence is critical desired performance in control systems. Thus, the finite-time SMC is necessary for these control systems. With the development of finite-time convergence theory, serval SMC schemes with finite convergence time have been developed, such as the TSMC [27], nonsingular TSMC [28,29], finite-time integral SMC [30]. However, most of existing finite-time SMC schemes [27–30] can only deal with the matched uncertain problem. Recently, for the mismatched uncertain

Manuscript received February 4, 2021; revised July 16, 2021 and October 5, 2021; accepted November 10, 2021. Recommended by Associate Editor Xiao-Heng Chang under the direction of Editor Hamid Reza Karimi. This work was supported by the Natural Science Basic Research Program of Shaanxi (2021JQ-880,2021JM-533,2020JM-646), Scientific Research Program Funded by Shaanxi Provincial Education Department (Program No.21JK0956, Program No.21JK0960) and the Scientific Research Fund for High-Level Talents of Xijing University (XJ21B01).

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system, a few DOSMC schemes with finite convergence time have been developed [31,32] based on the finite-time control and extending DOSMC schemes in [21–26]. In [31], a finite-time DOSMC scheme was proposed by using the finite-time DO and nonsingular terminal sliding-mode surface. In [32], a finite-time DOSMC was developed by combining the finite-time DO and finite-time integral sliding-mode surface. Under mismatched disturbances, the finite convergence can be guaranteed by the DOSMC schemes in [31,32]. However, the convergence times in [31,32] are mainly related to the initial system conditions. And, the convergence time may be very large if the initial system conditions increase greatly.

Recently, as a novel fast convergent control notion [33,34], the fixed-time convergence has been studied in some control problems. Fixed-time control guarantees that convergence time is bounded by a fixed constant under different initial system conditions. Then, the above problem of finite-time control can be avoided. In [35], for a power system, a fast fixed-time SMC was developed. In [36], the authors designed a fixed-time TSMC for a general nonlinear second-order system. In [37], based on fixed-time SMC and DO, a fixed-time formation tracking scheme was developed. However, these fixed-time SMC scheme in [35–37] are only suitable for the system which is subjected to matched disturbances.

According to the discussions above, even if the DOSMC is a good candidate to solve the mismatched uncertain problem in TSMC, existing DOSMC schemes still have limitations: 1) the convergence time of conventional DOSMC is infinite; 2) For different initial system conditions, the finite-time DOSMC cannot achieve similar fast convergence performance under same control parameters. In this paper, to avoid these limitations, we propose a novel fixed-time DOSMC. The main contribution of this paper is

1) In comparison with existing DOSMC schemes in [21–26,31,32], the proposed DOSMC is fixed-time stable, which can guarantee the convergence times of system states and estimation errors are uniformly bounded and the upper bounds are not related to the initial system condition. Thus, the limitations of convergence performance in the existing DOSMC schemes can be eliminated.
2) In comparison with the recent fixed-time SMC schemes in [35–37], the propose DOMSC scheme can extend the fixed-time SMC to suppress the mismatched disturbances.

The rest parts are given as follows: In Section 2, the preliminaries and motivation are given. The disturbance observer and controller are design in Section 3. The simulation results demonstrate the fast convergent performance of proposed controller in Section 4. The conclusion is given in Section 5.

Notations: t represents time (Initial time is zero). R^n represents the Euclidean space with n dimension. $\text{sign}(\cdot)$ denotes the symbolic function.

2. PRELIMINARIES AND MOTIVATION

2.1. System description

A system with mismatched and matched disturbances can be given as in [21–26,31,32]

$$\begin{cases} \dot{x}_1 = x_2 + d_1, \\ \dot{x}_2 = f(x_1, x_2) + u + d_2, \\ y = x_1, \end{cases} \tag{1}$$

where u represents the control input, x_1 and x_2 represent the system states, y is the system output and $f(x_1, x_2)$ denotes a nonlinear function. d_j $(j = 1,2)$ denote the time-varying mismatched and matched disturbances, respectively. $x_1(0)$ and $x_2(0)$ are the initial system conditions. The disturbances satisfy.

Assumption 1: The disturbances d_j $(j = 1, 2)$ are differentiable, $|d_j| \le D_{j\max}$ and $|\dot{d}_j| \le D^d_{j\max}$, where $D_{j\max}$ and $D^d_{j\max}$ are positive constants.

Assumption 1 means that the matched and mismatched disturbances are bounded by a positive constant, and the derivatives of these disturbances are also bounded. This assumption not only releases the H_2 norm-bounded assumption in [15–20], but also the assumption in [21–25] that the derivatives of the mismatched disturbances are to vanish when the time approaches to infinity.

Control objective: The objective of design of control input u is that u can drive the system output x_1 to zero despite the presence of matched and mismatched disturbances. And convergence time is always bounded by a constant for different initial system conditions.

2.2. Problem formulation

In order to explain the motivation of this paper, we now briefly introduce TSMC, DOSMC, and finite-time DOSMC.

2.2.1 TSMC

The traditional linear sliding-mode surface can be designed as

$$s_1 = x_2 + c_1 x_1, \tag{2}$$

where the constant $c_1 > 0$.
Calculating time derivative of s_1 gives

$$\begin{aligned} \dot{s}_1 &= \dot{x}_2 + c_1 \dot{x}_1 \\ &= f(x_1, x_2) + u + d_2 + c_1 (x_2 + d_1). \end{aligned} \tag{3}$$

Then TSMC can be designed as

$$u = -f(x_1, x_2) - c_1 x_2 - k_1 \text{sign}(s_1), \tag{4}$$

where k_1 is positive constant. According to (3) and (4), it can be known that $s_1\dot{s}_1 \le -(k_1-(d_2+c_1d_1))<0$ if we select $k_1 > D_{2\max}+c_1D_{1\max}$ ($D_{j\max}$ $(j=1,2)$ has been defined in Assumption 1). Then $s_1=0$ can be guaranteed in finite time by using controller (4). According to the condition $s_1=0$ in (2) and the system (1), the sliding motion can be given as

$$\dot{x}_1+c_1x_1=d_1. \tag{5}$$

From (5), we know that the matched disturbance d_2 can be directly compensated, but x_1 still be affected by the mismatched disturbance d_1. As stated in [21–26,33,34], the reason is that, for the mismatched disturbances, invariability of TSMC cannot be guaranteed, and the system states still be affected by the disturbances on the sliding mode surface.

2.2.2 DOSMC

According to [25,26], the DO-based linear sliding-mode surface can be designed as

$$s_2=x_2+\hat{d}_{DO}+c_2x_1. \tag{6}$$

The estimation of mismatched disturbance $\hat{d}_{DO}$ is given by following DO

$$\begin{cases}\dot{p}_{DO}=-l_{DO}p_{DO}-l_{DO}\left(l_{DO}x_1+x_2\right),\\ \hat{d}_{DO}=p_{DO}+l_{DO}x_1,\end{cases} \tag{7}$$

where the constant $l_{DO}>0$. Then calculating time derivative of s_2 and considering (1), we have

$$\begin{aligned}\dot{s}_2&=\dot{x}_2+\dot{\hat{d}}_{DO}+c_2\dot{x}_1\\ &=f(x_1,\,x_2)+u+d_2+\dot{p}_{DO}\\ &\quad+l_{DO}\dot{x}_1+c_2\left(x_2+d_1\right)\\ &=f(x_1,\,x_2)+u+d_2-l_{DO}p_{DO}\\ &\quad-l_{DO}\left(l_{DO}x_1+x_2\right)+l_{DO}\left(x_2+d_1\right)\\ &\quad+c_2\left(x_2+d_1\right).\end{aligned} \tag{8}$$

Then DOSMC can be designed as

$$\begin{aligned}u=&-f(x_1,\,x_2)+l_{DO}p_{DO}+l_{DO}\left(l_{DO}x_1+x_2\right)-l_{DO}x_2\\ &-c_2x_2-k_2\operatorname{sign}(s_2).\end{aligned} \tag{9}$$

According to (8) and (9), it is easy to know that $s_2\dot{s}_2\le 0$ if we select $k_2>D_{2\max}+l_{DO}D_{1\max}+c_2D_{1\max}$. Thus, $s_2=0$ is guaranteed in finite time by using DOSMC (9). Then the sliding motion can be given as

$$\dot{x}_1+c_2x_1=\hat{d}_{DO}-d_1. \tag{10}$$

From [25,26], we can know that DO (7) can guarantee $\left|\hat{d}_{DO}-d_1\right|\le\bar{d}_{\max}$, where $\bar{d}_{\max}$ is a small positive constant. Then, from (10), we can know that the mismatched disturbance d_1 can be suppressed by designing the DO-based linear sliding-mode surface (6). Thus, by using the estimation of DO to modified the traditional sliding-mode surface, these DOSMC can compensate the disturbance on the sliding-mode surface, thus the invariability can be guaranteed in these DOSMC methods. However, the linear characteristic of sliding-mode surface (6) can only guarantee the slow asymptotic stability. Recently, to suppress the mismatched disturbance and achieve the fast convergence performance, the DO-based finite-time SMC schemes have been developed in [31,32].

2.2.3 Finite-time DOSMC

According to [32], for system (1), the DO-based integral sliding-mode surface is given as

$$s_3=x_2+z_{11}+\int_0^t(f_{s3})d\tau, \tag{11}$$

where

$$\begin{aligned}f_{s3}=&k_{31}|x_1|^{\alpha_{31}}\operatorname{sign}(x_1)\\ &+k_{32}|x_2+z_{11}|^{\alpha_{32}}\operatorname{sign}(x_2+z_{11}).\end{aligned} \tag{12}$$

The value range of the positive constants k_{31}, k_{32}, α_{31} and α_{32} can be found in [32]. Then the finite-time DOSMC can be designed as

$$\begin{aligned}u=&-\left(f(x_1,\,x_2)+v_{11}+z_{12}\right)-f_{s3}\\ &-k_{33}|s_3|^{1/2}\operatorname{sign}(s_3)-\int_0^t k_{34}\operatorname{sign}(s_3)\,d\tau,\end{aligned} \tag{13}$$

where the estimations of disturbances z_{11} and z_{12} are given by following finite-time DO

$$\begin{cases}\dot{z}_{0j}=v_{0j}+x_{j+1},\\ \dot{z}_{1j}=v_{1j},\\ \dot{z}_{2j}=v_{2j},\\ v_{0j}=-\lambda_0L_j^{1/3}|z_{0j}-x_j|^{2/3}\operatorname{sign}(z_{0j}-x_j)\\ \qquad-\rho_0(z_{0j}-x_j)+z_{1j},\\ v_{1j}=-\lambda_1L_j^{1/2}|z_{1j}-v_{0j}|^{1/2}\operatorname{sign}(z_{1j}-v_{0j})\\ \qquad-\rho_1(z_{1j}-v_{0j})+z_{2j},\\ v_{2j}=-\lambda_2L_j\operatorname{sign}(z_{2j}-v_{1j})-\rho_2(z_{2j}-v_{1j}),\end{cases} \tag{14}$$

where $x_3=f(x_1,\,x_2)+u$ and $j=1,\,2$. λ_0, λ_1, ρ_0 and ρ_1 are observer parameters and can be found in [32]. z_{0j} is the estimation of x_j, and z_{1j} is estimation of disturbance d_j. According to [32], $s_3=0$ can be guaranteed in finite time. And, then $x_1=0$ can achieved in finite time

$$x_1=0,\text{if } t\ge T_f\left(x_1(0),\,x_2(0)\right), \tag{15}$$

where $T_f(x_1(0),x_2(0))$ is the convergence time. Compared with DOSMC, finite-time DOSMC (13) not only suppress the mismatched disturbance, but also achieve a faster convergence rate. But $T_f(x_1(0),\,x_2(0))$ is unbounded and related to the initial conditions $x_1(0)$ and $x_2(0)$. Especially,

$T_f(x_1(0), x_2(0))$ may be large if the initial conditions increase greatly. Thus, the fast convergence performance of finite-time control in [31,32] may not be guaranteed in this case.

2.2.4 Motivation

Motivation: From the previous discussion, the DOSMC is a good candidate to solve the mismatched uncertain problem of TSMC. Finite-time DOSMC can not only eliminate the mismatched uncertain problem, but also further improve the convergence rate. However, the convergence performance of finite-time DOSMC is still related to initial system conditions. In some initial condition cases, the convergence perfromance may be affected. This provies the motivation of this paper, that is, designing a new fixed-time DOSMC to suppress the mismatched and matched disturbances, and guarantee the convergence time is bounded by a fixed constant, and the constant is not related the initial system conditions.

Two useful definitions are given as follows:

Definition 1 [38]**:** The system (1) is finite-time output stable if $x_1 = 0$ is asymptotically stable and $x_1 = 0$ can be guaranteed at finite time time, i.e., $\forall t \geq T_f(x_1(0), x_2(0))$: $x_1 = 0$, where $T_f(x_1(0), x_2(0)) : R \to R_+ \cup \{0\}$ is finite.

Definition 2 [39]**:** The system (1) is fixed-time output stable if $x_1 = 0$ is finite-time stable and the convergence time $T_f(x_1(0), x_2(0))$ is bounded by a fixed value $T_{\max}$, i.e., $\exists T_{\max} \geq 0 : \forall x_1(0) \in R$ and $\forall x_2(0) \in R$, $T_f(x_1(0), x_2(0)) \leq T_{\max}$.

3. MAIN RESULT

A fixed-time DO will be proposed in this section to estimate the mismatched and matched disturbances of system (1) by using the control algorithm of [40]. Then, a fixed-time DOSMC will be designed by using the output of the proposed fixed-time DO. Unlike the DOSMC (9) and the finite-time DOSMC (13), the proposed fixed-time DOSMC can achieve fixed-time stability.

3.1. Fixed-time disturbance observer

Before giving the fixed-time DO and DOSMC, two useful lemmas are given as follows:

Lemma 1 [41, Theorem 2]**:** Consider a dynamic system with disturbance $d(t)$:

$$\begin{cases} \dot{\sigma}_0 = -\bar{k}_1 f_{\sigma 0} + \sigma_1, \\ \dot{\sigma}_1 = -\bar{k}_2 f_{\sigma 1} - d(t), \end{cases} \tag{16}$$

$$\begin{cases} f_{\sigma 0} = |\sigma_0|^{1/2}\mathrm{sign}(\sigma_0) + \theta|\sigma_0|^{3/2}\mathrm{sign}(\sigma_0), \\ f_{\sigma 1} = \dfrac{1}{2}\mathrm{sign}(\sigma_0) + 2\theta\sigma_0 + \dfrac{3}{2}\theta^2|\sigma_0|^2\mathrm{sign}(\sigma_0). \end{cases} \tag{17}$$

If the parameter $\theta > 0$, the disturbance $d(t)$ is bounded as $|d(t)| \leq d_{\max}$, and the parameters satisfy

$$\kappa = \Big\{ (\bar{k}_1, \bar{k}_2) \in R^2 \mid 0 < \bar{k}_1 \leq 2\sqrt{d_{\max}}, \bar{k}_2 > \frac{\bar{k}_1^2}{4} + \frac{4d_{\max}^2}{\bar{k}_1^2} \Big\} \cup \Big\{ (\bar{k}_1, \bar{k}_2) \in R^2 \mid \bar{k}_1 > 2\sqrt{d_{\max}}, \bar{k}_2 > 2d_{\max} \Big\}. \tag{18}$$

Then $\sigma_0 = 0$ and $\sigma_1 = 0$ can be achieved in time t_f, where $t_f \leq T_{f\max}$ and $T_{f\max}$ is a constant which is not related to the initial conditions $\sigma_0(0)$ and $\sigma_1(0)$.

The detailed proof process of fixed-time convergence of Lemma 1 can be found in the Section 3 of [41].

Lemma 2 [40, Theorem 3.1]**:** Consider a dynamic system as follows:

$$\ddot{e}_1 = -K_p|e_1|^{\alpha_1}\mathrm{sign}(e_1) - K_d|\dot{e}_1|^{\alpha_2}\mathrm{sign}(\dot{e}_1) - L_p|e_1|^{\beta_1}\mathrm{sign}(e_1) - L_d|\dot{e}_1|^{\beta_2}\mathrm{sign}(\dot{e}_1), \tag{19}$$

where e_1 and $\dot{e}_1$ are the system states. If the constants $K_p > 0$, $K_d > 0$, $L_p > 0$, $L_d > 0$, $0 < \alpha_1 < 0.5$, $\alpha_2 = 2\alpha_1/(\alpha_1+1)$, $\beta_1 = 2\alpha_1 + 1$ and $\beta_2 = (2\alpha_1+1)/(\alpha_1+1)$, then the states e_1 and e_2 satisfy

$$e_1 = 0,\ \dot{e}_1 = 0, \text{if } t \geq t_e, \tag{20}$$

and

$$t_e \leq T_e, \tag{21}$$

where T_e is a constant and not related to the initial system conditions.

Lemma 3 [38]**:** Provided that $M(t)$ is differentiable and nonnegative, and satisfies $\dot{M}(t) \leq -\alpha M^{\psi}(t)$, where α and ψ are positive constants, and $\alpha > 1$, $0 < \psi < 1$. Then, we have

$$M(t) = 0, \text{if } t \geq t_M, \tag{22}$$

where the finite time $t_M \leq M^{1-\psi}(0)/(\alpha(1-\psi))$.

The fixed-time DO and the stability proof are given as follows:

Theorem 1: For the mismatched uncertain system (1), the fixed-time DO is constructed as

$$\begin{cases} \dot{h}_{11} = -\mu_{11} f_{h11} + h_{12} + x_2, \\ \dot{h}_{12} = -\mu_{13} f_{h12}, \\ \bar{h}_1 = h_{11} - x_1, \\ \dot{h}_{21} = -\mu_{21} f_{h21} + h_{22} + f(x_1, x_2) + u, \\ \dot{h}_{22} = -\mu_{23} f_{h22}, \\ \bar{h}_2 = h_{21} - x_2, \end{cases} \tag{23}$$

where

$$\begin{cases} f_{h11} = \left|\bar{h}_1\right|^{1/2}\text{sign}\left(\bar{h}_1\right) + \mu_{12}\left|\bar{h}_1\right|^{3/2}\text{sign}\left(\bar{h}_1\right), \\ f_{h12} = \frac{1}{2}\text{sign}\left(\bar{h}_1\right) + 2\mu_{12}\bar{h}_1 + \frac{3}{2}\mu_{12}^2\left|\bar{h}_1\right|^2\text{sign}\left(\bar{h}_1\right), \\ f_{h21} = \left|\bar{h}_2\right|^{1/2}\text{sign}\left(\bar{h}_2\right) + \mu_{22}\left|\bar{h}_2\right|^{3/2}\text{sign}\left(\bar{h}_2\right), \\ f_{h22} = \frac{1}{2}\text{sign}\left(\bar{h}_2\right) + 2\mu_{22}\bar{h}_2 + \frac{3}{2}\mu_{22}^2\left|\bar{h}_2\right|^2\text{sign}\left(\bar{h}_2\right), \end{cases} \tag{24}$$

where h_{11} and h_{21} are the states of observer. h_{12} is the estimation of mismatched disturbance d_1. h_{22} is the estimation of matched disturbance d_2. Assumption 1 is valid. For $j = 1, 2$, $\mu_{j2} > 0$, μ_{j1} and μ_{j3} are in the following set:

$$\Omega = \left\{(\mu_{j1}, \mu_{j3}) \in R^2 \left| \begin{array}{l} 0 < \mu_{j1} \le 2\sqrt{D_{j\max}^d}, \\ \mu_{j3} > \frac{(\mu_{j1})^2}{4} + \frac{4\left(D_{j\max}^d\right)^2}{(\mu_{j1})^2} \end{array} \right.\right\} \cup \left\{(\mu_{j1}, \mu_{j3}) \in R^2 \left| \begin{array}{l} \mu_{j1} > 2\sqrt{D_{j\max}^d}, \\ \mu_{j3} > 2D_{j\max}^d \end{array} \right.\right\}, \tag{25}$$

where $D_{j\max}^d$ $(j = 1, 2)$ is defined in Assumption 1. $\exists T_{DO} > 0 : x_1(0) \in R$ and $x_2(0) \in R$, the estimation errors will converge to zero

$$d_1 - h_{12} = 0 \text{ and } d_2 - h_{22} = 0, \text{if } t > T_{DO}, \tag{26}$$

where T_{DO} is a constant and not related to the initial system conditions.

Proof of Theorem 1: Define the estimation errors as $\varphi_j = d_j - h_{j2}$ $(j = 1, 2)$, Differentiating φ_j $(j = 1, 2)$ gives

$$\begin{cases} \dot{\varphi}_1 = \dot{d}_1 - \dot{h}_{12}, \\ \dot{\varphi}_2 = \dot{d}_2 - \dot{h}_{22}. \end{cases} \tag{27}$$

Substituting the expressions of $\dot{h}_{12}$ and $\dot{h}_{22}$ in (23) into (27), we have

$$\begin{cases} \dot{\varphi}_1 = \dot{d}_1 + \mu_{13} f_{h12}, \\ \dot{\varphi}_2 = \dot{d}_2 + \mu_{23} f_{h22}, \end{cases} \tag{28}$$

where f_{h12} and f_{h22} are defined in (24). Differentiating $\bar{h}_j$ $(j = 1, 2)$ gives

$$\begin{cases} \dot{\bar{h}}_1 = \dot{h}_{11} - \dot{x}_1, \\ \dot{\bar{h}}_2 = \dot{h}_{21} - \dot{x}_2. \end{cases} \tag{29}$$

Substituting the expressions of $\dot{h}_{11}$ and $\dot{h}_{21}$ in (23), $\dot{x}_1$ and $\dot{x}_2$ in (1) into (29), we have

$$\begin{cases} \dot{\bar{h}}_1 = -\mu_{11} f_{h11} + h_{12} - d_1, \\ \dot{\bar{h}}_2 = -\mu_{21} f_{h21} + h_{22} - d_2. \end{cases} \tag{30}$$

Considering the definition $\varphi_j = d_j - h_{j2}$ $(j = 1, 2)$, then we have

$$\begin{cases} \dot{\bar{h}}_1 = -\mu_{11} f_{h11} - \varphi_1, \\ \dot{\bar{h}}_2 = -\mu_{21} f_{h21} - \varphi_2. \end{cases} \tag{31}$$

Combining (28) and (31), we have

$$\begin{cases} \dot{\bar{h}}_1 = -\mu_{11} f_{h11} - \varphi_1, \\ \dot{\varphi}_1 = \mu_{13} f_{h12} + \dot{d}_1, \end{cases} \tag{32}$$

$$\begin{cases} \dot{\bar{h}}_2 = -\mu_{21} f_{h21} - \varphi_2, \\ \dot{\varphi}_2 = \mu_{23} f_{h22} + \dot{d}_2. \end{cases} \tag{33}$$

According to Lemma 1, for $j = 1, 2$, if $\mu_{j2} > 0$, μ_{j1} and μ_{j3} are chosen in the set (25), then the estimation errors φ_j $(j = 1, 2)$ are bounded from the initial time and fixed-time stable

$$\varphi_1 = d_1 - h_{12} = 0 \text{ and } \varphi_2 = d_2 - h_{22} = 0, \quad \text{if } t > T_{DO}, \tag{34}$$

where the constant $T_{DO} > 0$ is not related to the initial system conditions.

The proof is finished. □

Remark 1: For the fixed-time DO (23), the fixed convergence time T_{DO} is determined by the parameters μ_{ij} $(i = 1, 2; j = 1, 2, 3)$. The estimation of T_{DO} can be found in the Section IV of [41].

3.2. Fixed-time sliding mode control

By using the estimations of fixed-time DO (23), a novel DO-based integral sliding-mode surface is proposed as

$$s = x_{2g} + \int_0^t f_s d\tau, \tag{35}$$

where $f_s = K_1|x_1|^{\omega_1}\text{sign}(x_1) + K_2|x_{2g}|^{\omega_2}\text{sign}(x_{2g}) + L_3|x_1|^{\gamma_1}\text{sign}(x_1) + L_4|x_{2g}|^{\gamma_2}\text{sign}(x_{2g})$, $x_{2g} = x_2 + h_{12}$ and h_{12} is given by the fixed-time DO (23). The constants $K_j > 0$, $L_j > 0$ $(j = 1, 2)$, $0 < \omega_1 < 0.5$, $\omega_2 = 2\omega_1/(\omega_1 + 1)$, $\gamma_1 = 2\omega_1 + 1$ and $\gamma_2 = (2\omega_1 + 1)/(\omega_1 + 1)$. Then the fixed-time DOSMC is designed as

$$u = -\phi\left(|s|^q + |s|^{2-q}\right)\text{sign}(s) - f(x_1, x_2) + \mu_{13} f_{h12} - f_s - h_{22}, \tag{36}$$

where ϕ and q are control parameters and are positive constants. q satisfies the inequation $0 < q < 1$.

Then, stability proof of proposed fixed-time DOSMC (36) are given as

Theorem 2: The system (1) adopts fixed-time DOSMC (36). If Assumption 1 can be satisfied, then $s = 0$ is achieved in fixed time. And the system (1) is fixed-time output stable, i.e., the system output $x_1 = 0$ can be satisfied in finite time t_{all}

$$x_1 = 0, \text{if } t \ge t_{all}, \tag{37}$$

and

$$t_{all} \le T_m, \tag{38}$$

where T_m is a constant and not related to the initial system conditions.

Proof of Theorem 2: Calculating the derivative of s, and considering (1) and (23), we have

$$\dot{s} = u + f(x_1, x_2) + d_2 - \mu_{13} f_{h12} + f_s. \tag{39}$$

Substituting the proposed fixed-time DOSMC (36) into (39), we have

$$\dot{s} = -\phi \left(s^q + s^{2-q}\right) \operatorname{sign}(s) + (d_2 - h_{22}). \tag{40}$$

Construct Lyapunov function V_1 as

$$V_1 = \frac{1}{2}s^2. \tag{41}$$

Then calculating derivative of V_1 and considering (40), we get

$$\begin{aligned}\dot{V}_1 &= s\dot{s}\\ &= -\phi\left(|s|^{1+q} + |s|^{3-q}\right) + (d_2 - h_{22})s\\ &\le -\phi\left(2^{\frac{(1+q)}{2}} V_1^{\frac{(1+q)}{2}} + 2^{\frac{(3-q)}{2}} V_1^{\frac{(3-q)}{2}}\right)\\ &\quad + \sqrt{2}|\varphi_2| V_1^{1/2},\end{aligned} \tag{42}$$

where $\varphi_2 = d_2 - h_{22}$ is the estimation error and has been defined in Theorem 1. From (42), it is clear that V_1 is affected by the estimation error φ_2. Although Theorem 1 has proved that the estimation error φ_2 is fixed-time stable, the convergence stability of sliding-mode surface s and state x_1 may not be guaranteed during the convergence period of φ_2.

In order to consider the affection of φ_2, we consider four proof steps: First, it will be proved that sliding-mode surface s is bounded before the estimation error φ_2 converges to zero. Second, we will prove that $s = 0$ will be achieved in fixed time after $\varphi_2 = 0$. Third, we will prove that the states x_1 and x_{2g} are bounded before the estimation error φ_2 converges to zero. Fourth, we will prove that the output state $x_1 = 0$ will be achieved in fixed time after $\varphi_2 = 0$, and $x_1 = 0$ is fixed-time output stable.

Step 1: According to Young's inequality [42], (42) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\le -\phi\left(2^{\frac{(1+q)}{2}} V_1^{\frac{(1+q)}{2}} + 2^{\frac{(3-q)}{2}} V_1^{\frac{(3-q)}{2}}\right) + \sqrt{2}|\varphi_2| V_1^{1/2}\\ &\le -\phi\left(2^{\frac{(1+q)}{2}} V_1^{\frac{(1+q)}{2}} + 2^{\frac{(3-q)}{2}} V_1^{\frac{(3-q)}{2}}\right)\\ &\quad + \left(\varphi_2^2 + V_1\right)/\sqrt{2}\\ &\le \left(1/\sqrt{2}\right) V_1 + R(t),\end{aligned} \tag{43}$$

where $R(t) = \varphi_2^2/\sqrt{2}$. It can be known from Theorem 1 that the estimation error φ_2 is bounded from the initial time. Thus $R(t)$ is bounded. Define the upper bound of $R(t)$ as $|R(t)| \le R_{\max}$, where $R_{\max}$ is a positive constant. Then, it can be obtained from (43) that

$$\dot{V}_1 \le \left(1/\sqrt{2}\right) V_1 + R_{\max}, \text{if } t \ge 0. \tag{44}$$

From (44), we have

$$\begin{aligned}V_1 &\le V_1(0) e^{\left(1/\sqrt{2}\right)t}\\ &\quad + \sqrt{2}R_{\max}\left(1 - e^{\left(1/\sqrt{2}\right)t}\right), \text{if } t \ge 0.\end{aligned} \tag{45}$$

From (45), we know that V_1 is bounded in arbitrary finite time. Thus it can be known that s is bounded in arbitrary finite time

$$s \le s_{\max},\ if\ t \le t_s, \tag{46}$$

where $s_{\max}$ is a unknow positive constant and t_s is an arbitrary finite time.

Step 2: Substituting (34) into (42), we have

$$\dot{V}_1 \le -\phi\left(2^{\frac{(1+q)}{2}} V_1^{\frac{(1+q)}{2}} + 2^{\frac{(3-q)}{2}} V_1^{\frac{(3-q)}{2}}\right), \text{if } t \ge T_{DO}. \tag{47}$$

Let $m = 1 - (1+q)/2$ and $V_2 = 2V_1$, we get

$$\dot{V}_2 \le -2\phi\left(V_2^{1-m} + V_2^{1+m}\right), \text{if } t \ge T_{DO}, \tag{48}$$

$$\dot{V}_2 \le -2\phi V_2^{1-m}, \text{if } t \ge T_{DO}. \tag{49}$$

Since $1 - m < 1$ and $\phi > 0$, according to Lemma 3, it can be obtained from (49) that

$$V_2 = 0, \text{if } t \ge t_{V_2}, \tag{50}$$

where the constant t_{V_2} is a finite convergence time. Integrating (48) from $t = T_{DO}$ to $t = t_{V_2}$ gives

$$\int_{V_2(T_{DO})}^{V_2\left(t_{V_2}\right)} \frac{1}{V_2^{1-m} + V_2^{1+m}} d\left(V_2\right) \le -\int_{T_{DO}}^{t_{V_2}} 2\phi\, dt. \tag{51}$$

Then, we have

$$\begin{aligned}t_{V_2} &\le \left(\int_{V_2(T_{DO})}^{V_2\left(t_{V_2}\right)} \frac{1}{V_2^{1-m} + V_2^{1+m}} dV_2\right)/(-2\phi)\\ &= \left(\int_{V_2\left(t_{V_2}\right)}^{V_2(T_{DO})} \frac{1}{V_2^{1-m} + V_2^{1+m}} dV_2\right)/(2\phi)\\ &= \operatorname{atan}\left(V_2^m\left(T_{DO}\right)\right)/(2m\phi)\\ &\quad - \operatorname{atan}\left(V_2^m\left(t_{V_2}\right)\right)/(2m\phi).\end{aligned} \tag{52}$$

Considering (50) and (46) (s is bounded in arbitrary finite time), it can be known that $\operatorname{atan}\left(V_2^m\left(t_{V_2}\right)\right)$ and $\operatorname{atan}\left(V_2^m\left(T_{DO}\right)\right)$ is bounded. Then we have

$$t_{V_2} \le \operatorname{atan}\left(V_2^m\left(T_{DO}\right)\right)/(2m\phi)$$

$$\leq \pi/(4m\phi). \tag{53}$$

Thus, it can be known from (50) and (53) that the sliding-mode surface is fixed-time stable

$$s = \dot{s} = 0, \text{if } t \geq \pi/(4m\phi). \tag{54}$$

Step 3: Consider $x_{2g} = x_2 + h_{12}$ and the estimation errors $\varphi_j = d_j - h_{j2}$ $(j = 1,2)$. Then, substituting the control input (36) into (1) gives

$$\begin{cases} \dot{x}_1 = x_{2g} + \varphi_1, \\ \dot{x}_{2g} = -f_s - \phi\left(s^q + s^{2-q}\right)\text{sign}(s) + \varphi_2. \end{cases} \tag{55}$$

Construct Lyapunov function V_3 as

$$V_3 = \frac{1}{2}\left(x_1^2 + x_{2g}^2\right). \tag{56}$$

Then calculating $\dot{V}_3$ and considering (55) give

$$\begin{aligned} \dot{V}_3 &= x_1\dot{x}_1 + x_{2g}\dot{x}_{2g} \\ &= x_1\left(x_{2g} + \varphi_1\right) - x_{2g}f_s \\ &\quad - x_{2g}\phi\left(|s|^q + |s|^{2-q}\right)\text{sign}(s) + x_{2g}\varphi_2. \end{aligned} \tag{57}$$

Consider $f_s = K_1|x_1|^{\omega_1}\text{sign}(x_1) + K_2|x_{2g}|^{\omega_2}\text{sign}(x_{2g}) + L_3|x_1|^{\gamma_1}\text{sign}(x_1) + L_4|x_{2g}|^{\gamma_2}\text{sign}(x_{2g})$, we have

$$\begin{aligned} \dot{V}_3 &= x_1\left(x_{2g} + \varphi_1\right) - x_{2g}\phi\left(|s|^q + |s|^{2-q}\right)\text{sign}(s) \\ &\quad - x_{2g}K_1|x_1|^{\omega_1}\text{sign}(x_1) - K_2|x_{2g}|^{1+\omega_2} \\ &\quad - x_{2g}L_3|x_1|^{\gamma_1}\text{sign}(x_1) - L_4|x_{2g}|^{1+\gamma_2} + x_{2g}\varphi_2 \\ &\leq x_1\left(x_{2g} + \varphi_1\right) - x_{2g}\phi\left(|s|^q + |s|^{2-q}\right)\text{sign}(s) \\ &\quad - x_{2g}K_1|x_1|^{\omega_1}\text{sign}(x_1) - x_{2g}L_3|x_1|^{\gamma_1}\text{sign}(x_1) \\ &\quad + x_{2g}\varphi_2. \end{aligned} \tag{58}$$

Considering $-x_{2g} = \varphi_1 - (x_{2g} + \varphi_1)$, we have

$$\begin{aligned} \dot{V}_3 &\leq x_1\left(x_{2g} + \varphi_1\right) - x_{2g}\phi\left(|s|^q + |s|^{2-q}\right)\text{sign}(s) \\ &\quad + \varphi_1K_1|x_1|^{\omega_1}\text{sign}(x_1) + \varphi_1L_3|x_1|^{\gamma_1}\text{sign}(x_1) \\ &\quad - \left(x_{2g} + \varphi_1\right)L_3|x_1|^{\gamma_1}\text{sign}(x_1) \\ &\quad - \left(x_{2g} + \varphi_1\right)K_1|x_1|^{\omega_1}\text{sign}(x_1) + x_{2g}\varphi_2. \end{aligned} \tag{59}$$

Then, two cases can be given as

Case 1: If $x_1\left(x_{2g} + \varphi_1\right) \leq 0$, we have $x_1\dot{x}_1 \leq 0$. Then, we know that x_1 is bounded.

Case 2: If $x_1\left(x_{2g} + \varphi_1\right) > 0$, we have $\left(x_{2g} + \varphi_1\right)\text{sign}(x_1) > 0$. Then, it can be known from (59) that

$$\begin{aligned} \dot{V}_3 &\leq x_1\left(x_{2g} + \varphi_1\right) - x_{2g}\phi\left(|s|^q + |s|^{2-q}\right)\text{sign}(s) \\ &\quad + \varphi_1K_1|x_1|^{\omega_1}\text{sign}(x_1) + \varphi_1L_3|x_1|^{\gamma_1}\text{sign}(x_1) \\ &\quad + x_{2g}\varphi_2. \end{aligned} \tag{60}$$

Since $0 < \omega_1 < 0.5$ and $\gamma_1 = 2\omega_1 + 1$, we have

$$0 < \omega_1 < 0.5,\ 1 < \gamma_1 < 2. \tag{61}$$

Then, it can be known that

$$\begin{cases} |x_1|^{\omega_1} \leq 1 + |x_1|^2, \\ |x_1|^{\gamma_1} \leq 1 + |x_1|^2. \end{cases} \tag{62}$$

According to Young's inequality [42] and (62), then (60) can be rewritten as

$$\begin{aligned} \dot{V}_3 &\leq \left(1 + |\varphi_1|\left(K_1 + L_3\right)\right)x_1^2 + 3/2x_{2g}^2 \\ &\quad + \left(|\varphi_1|^2 + |\varphi_2|^2\right)/2 + |\phi|^2\left(|s|^q + |s|^{2-q}\right)^2/2 \\ &\quad + \left(K_1 + L_3\right)|\varphi_1|. \end{aligned} \tag{63}$$

It can be known from Theorem 1 that the estimation error $\varphi_j = d_j - h_{j2}$ $(j = 1,\ 2)$ is bounded from the initial time. And considering we have proved that s is bounded in arbitrary finite time (see (46)), we have

$$1 + |\varphi_1|\left(K_1 + L_3\right) \leq \bar{L}_{1\max}, \text{if } t \leq t_s, \tag{64}$$

and

$$\begin{aligned} &\left(|\varphi_1|^2 + |\varphi_2|^2\right)/2 + |\phi|^2\left(|s|^q + |s|^{2-q}\right)^2/2 \\ &\quad + \left(K_1 + L_3\right)|\varphi_1| \\ &\leq \bar{L}_{2\max}, \text{if } t \leq t_s, \end{aligned} \tag{65}$$

where $\bar{L}_{\mathrm{j}\max}$ $(j = 1,\ 2)$ are unknown constants and t_s is an arbitrary finite time. Then, combining (63), (64) and (65) yields

$$\begin{aligned} \dot{V}_3 &\leq 2K_{\max}\left(x_1^2 + x_{2g}^2\right)/2 + \bar{L}_{2\max} \\ &\leq 2K_{\max}V_3 + \bar{L}_{2\max}, \text{if } t \leq t_s, \end{aligned} \tag{66}$$

where $L_{\max}$ and $K_{\max}$ are unknown positive constants. And $K_{\max}$ is given as

$$K_{\max} = \max\left\{\bar{L}_{1\max},\ 3\right\}. \tag{67}$$

Thus, it can be known easily from (66) that V_3, x_1 and x_{2g} are bounded in arbitrary finite time t_s for Case 1. Combining the results of Cases 1 and 2, in arbitrary finite time, we know that V_3, x_1 and x_{2g} are bounded.

Step 4: Considering (54), the system dynamic (1) is reduced as follows in a fixed time

$$\dot{x}_1 = x_{2g} + \varphi_1 \text{ and } \dot{x}_{2g} = -f_s, \text{if } t \geq t_1. \tag{68}$$

Combining (34) and (68), the system dynamic (1) is reduced as follows in a fixed time $t_2 = \max\{\pi/(4m\phi),\ T_{DO}\}$

$$\begin{aligned} \ddot{x}_1 &= -f_s \\ &= -K_1|x_1|^{\omega_1}\text{sign}(x_1) - K_2|\dot{x}_1|^{\omega_2}\text{sign}(\dot{x}_1) \end{aligned}$$

$$-L_3|x_1|^{\gamma_1}\operatorname{sign}(x_1) - L_4|\dot{x}_1|^{\gamma_2}\operatorname{sign}(\dot{x}_1),\quad \text{if } t \geq t_2. \tag{69}$$

According to the Step 3 above, we know that $x_1(t_2)$ is bounded. Then, based on Lemma 2, we know that the system (1) is fixed-time output stable

$$x_1 = 0, \text{if } t \geq t_2 + t_x, \tag{70}$$

and

$$t_x < T_{x\max}, \tag{71}$$

where $T_{x\max}$ is a constant and not related to the initial system conditions. Since $t_2 = \max\{\pi/(4m\phi), T_{DO}\}$ is also a constant and not related to the initial system conditions, (37) and (38) can be guaranteed.

Remark 2: The stability proof in Theorem 2 considered the affect of estimation errors $\varphi_j = d_j - h_{j2}$ ($j = 1, 2$). Thus, this is a proof of fixed-time output stability for the whole hybrid system which contains the observer system and controller system. Based on this proof, it can be known that the fixed-time stability of sliding-mode surface s and output state x_1 will not be affected by the observer dynamic.

Remark 3: Based on the integral design of sliding-mode surface (35), we can know from (36) that the proposed controller does not contain any singular term.

Remark 4: Unlike the finite-time DOSMC (15), the proposed scheme (68) can achieve a convergence time which is bounded by a fixed constant. This implies that the proposed scheme provides a similar excellent convergence performance without adjusting the control parameters under different initial system conditions. To future illustrate this advantage, a simulation comparison for the finite-time and proposed fixed-time DOSMC with different initial system conditions will be given in Section 4.

4. SIMULATION RESULTS

The simulation is presented in this section. We consider the following mismatched uncertain system

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t), \\ \dot{x}_2 = -4x_1 - 10x_2 + 5\sin(x_1) + u + d_2(t), \\ y = x_1. \end{cases} \tag{72}$$

The disturbances are selected as

$$\begin{cases} d_1(t) = 2 + \sin(t) - 0.5\cos(t), \\ d_2(t) = \cos(t) + \cos(2t). \end{cases} \tag{73}$$

For the comparison, TSMC (4), DOSMC (9), finite-time DOSMC (13) and proposed fixed-time DOSMC (36) are considered in this section.
The parameters of TSMC are selected as

$$c_1 = 1,\ k_1 = 15. \tag{74}$$

The parameters of DOSMC are selected as

$$l_{DO} = 10,\ c_2 = 1,\ k_2 = 35. \tag{75}$$

The parameters of finite-time DOSMC are chosen as

$$\begin{aligned} &\lambda_0 = 5,\ \lambda_1 = 5,\ \lambda_2 = 5,\ \rho_0 = 2,\ \rho_1 = 2, \\ &\rho_2 = 2,\ L_1 = 3,\ L_2 = 3,\ \alpha_{31} = 0.3,\ \alpha_{32} = 0.6, \\ &k_{31} = 10,\ k_{32} = 7,\ k_{33} = 5,\ k_{34} = 5. \end{aligned} \tag{76}$$

The parameters of fixed-time DOSMC are chosen as

$$\begin{aligned} &q = 0.4,\ \phi = 4,\ \omega_1 = 0.3,\ K_1 = 8/3,\ K_2 = 5/3, \\ &L_3 = 8/3,\ L_4 = 5/3,\ L = 4,\ \mu_{11} = \mu_{21} = 3\sqrt{L}, \\ &\mu_{12} = \mu_{22} = 1,\ \mu_{13} = \mu_{23} = 2.2L. \end{aligned} \tag{77}$$

To remove the chattering, the symbolic function in fixed-time DOSMC (36), TSMC (4) and DOSMC (9) are modified by the following sigmf() as in [20]:

$$\operatorname{sigmf}(\upsilon) = \left(\frac{1}{1+e^{-\partial\upsilon}} - 1/2\right), \tag{78}$$

where ∂ is selected as 30.

The following three different kinds of initial system conditions which are chosen from the small value to the large value are considered

Case 1 (Small initial system states)**:** The initial system states are set as $x_1(0) = 4$, $x_2(0) = 0$. Figs. 1-4 give the simulation results of Case 1. From Fig. 1, the TSMC cannot guarantee the system output x_1 converge to zero. This is because TSMC cannot suppress the mismatched disturbance, as stated in Subsection 2.2. From Fig. 1, the three DOSMC methods (DOSMC, finite-time DOSMC and proposed fixed-time DOSMC) can suppress the mismatched disturbance. But the DOSMC cannot control the system output x_1 converge to zero, and the convergence rate of DOSMC is much slower than the finite-time and fixed time DOSMC. As stated in Subsection 2.2, the reason is that the DO of DOSMC cannot drive the estimation error of mismatched disturbance to zero (see Fig. 2), and the DOSMC is designed based on the asymptotic stability. Fig. 1 also show that both the finite-time and fixed-time DOSMC can effectively suppress the disturbances and drive the state x_1 to zero in 3 seconds. Then, we can know that the control parameters used in Case 1 for the finite-time DOSMC and fixed-time DOSMC guarantee a similar good convergence performance when the initial system states are small. Moreover, from Figs. 1-4, we can know that the state x_1, the estimation errors φ_j ($j = 1, 2$) and sliding-mode surface s of proposed fixed-time DOMSC are all bounded during the convergence period of system states and estimation errors. Thus, this simulation result illustrates the four steps of the proof of Theorem 2.

Case 2 (Larger initial system states)**:** To guarantee the fairness of comparison, we still chosen the same control

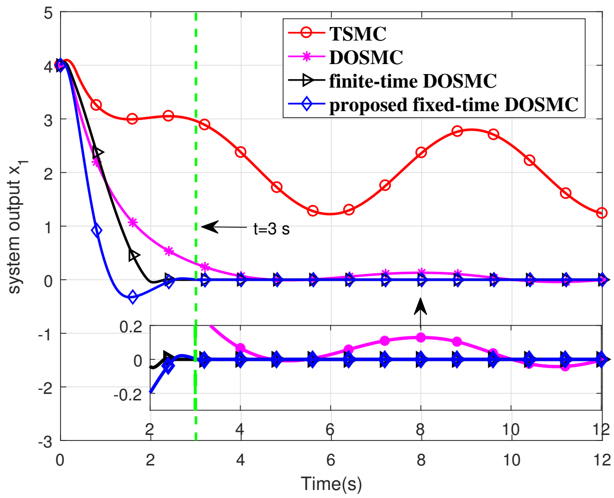

Fig. 1. The system output x_1 of Case 1 (small initial system states).

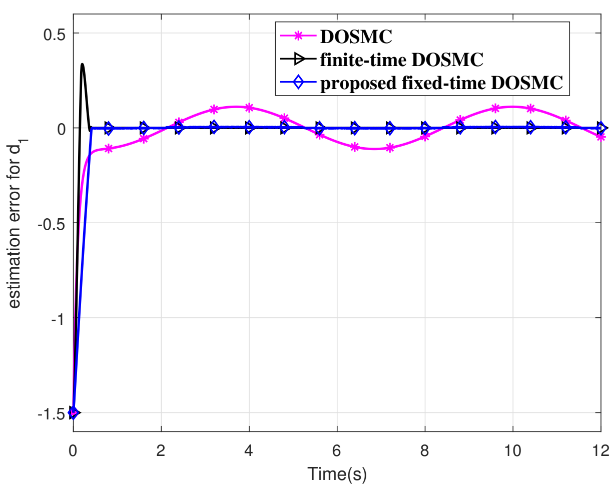

Fig. 2. The estimation error of d_1 of Case 1 (small initial system states).

parameters as in Case 1. We only increase initial states to $x_1(0) = 20$ and $x_2(0) = 0$ in this case. Figs. 5 and 6 give the simulation results of Case 2. From Fig. 5, unlike Case 1, we know that the finite-time DOSMC cannot guarantee the system output x_1 in 3 seconds. The convergence performance of the DOSMC and finite-time DOSMC both are affected. As stated in Subsection 2.2, this is because the convergence time of DOSMC and finite-time DOSMC are not independent on the initial system conditions. From Fig. 5, like the result in Case 1, the proposed fixed-time DOSMC still guarantees the system output x_1 is convergent to zero in 3 seconds and is not affected by the increase of initial system conditions.

Case 3 (Super large initial system states)**:** To guarantee the fairness of comparison, we still chosen the same control parameters as in Cases 1 and 2. Compared with Cases 1 and 2, we only increase the initial states to $x_1(0) = 50$ and $x_2(0) = 0$. It can be known that the initial system

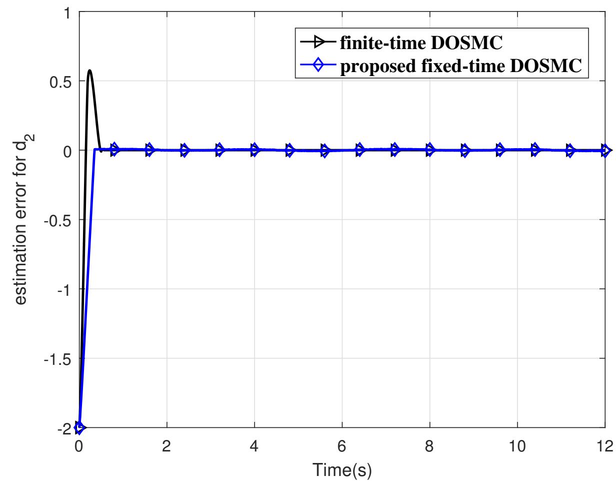

Fig. 3. The estimation error of d_2 of Case 1 (small initial system states).

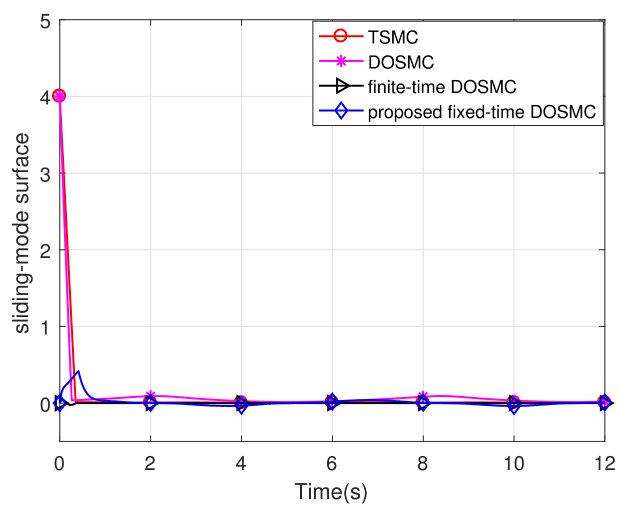

Fig. 4. The sliding-mode surface s of Case 1 (small initial system states).

states in Case 3 are much larger than that in Case 1 and 2. Figs. 7 and 8 give the simulation results of Case 3. From Fig. 7, the convergence performances of DOSMC and finite-time DOSMC are greatly affected. Like the results in Cases 1 and 2, Fig. 7 also shows the excellent convergence performance of fixed-time DOSMC still can be guaranteed.

For convenience, the convergence performances of finite-time DOSMC and proposed fixed-time DOSMC in the above three cases are plotted in the Figs. 9 and 10. From Figs. 9 and 10, the convergence performance of finite-time DOSMC are gradually affected with the increase of initial system states. The proposed fixed-time DOSMC still achieve similar excellent convergence performance under the three cases.

Then, the following conclusion of simulation can be given as follows:

1) The performance of TSMC is affected by the mis-

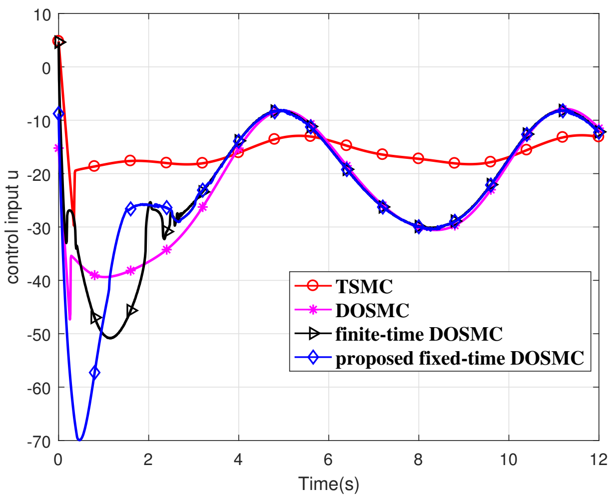

Fig. 5. The control input u of Case 1 (small initial system states).

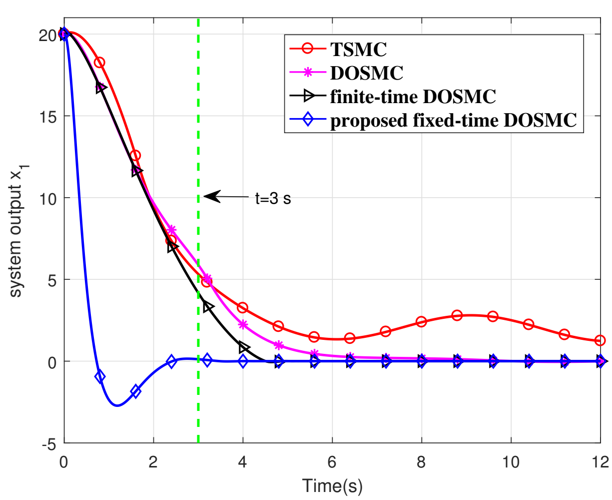

Fig. 6. The system output x_1 of Case 2 (larger initial system states).

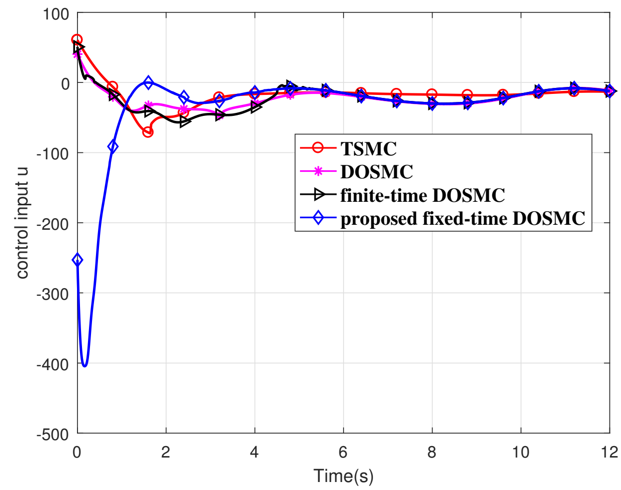

Fig. 7. The control input u of Case 2 (larger initial system states).

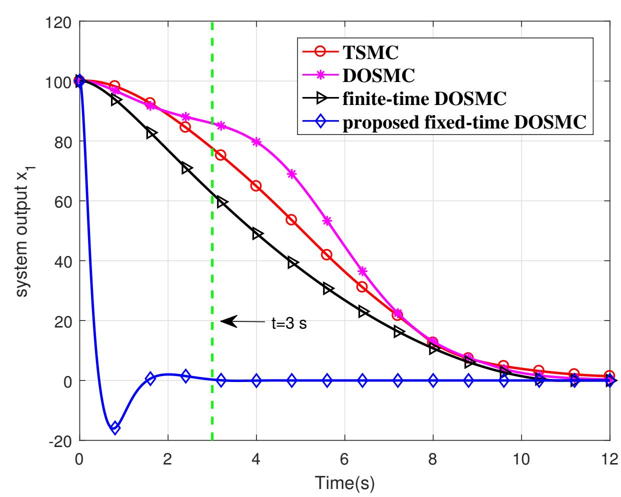

Fig. 8. The system output x_1 of Case 3 (super large initial system states).

matched disturbance (Figs. 1, 5, and 7). By using the estimation of mismatched disturbance to design the sliding-mode surface, the DOSMC effectively suppress the mismatched uncertainties. Meanwhile, with the help of nonlinear DO and nonlinear sliding-mode surface, the finite-time and fixed time DOSMC can provide faster convergence rate and higher precision than the conventional DOSMC (Figs. 1-3).

2) But, for the finite-time DOSMC, the convergence performance may be affected if the initial system conditions are change. In particular, the convergence performance of finite-time DOSMC may exhibit poor convergence performance if the initial conditions are large (Figs. 1, 5, 7, and 9). Unlike the existing finite-time DOSMC, the convergence rate of proposed fixed-time DOSMC are independent on the initial system conditions (Figs. 1, 5, 7, and 10).

5. CONCLUSION

A new fixed-time DOSMC was proposed for systems with mismatched and matched disturbances. The convergence rates of conventional DOSMC and finite-time DOSMC can be affected by the initial system conditions. The main contribution here is that the proposed fixed-time DOSMC scheme not only achieve a similar reject performance for mismatched disturbance like existing DOSMC schemes, but also can guarantee a excellent convergence performance which is not related to the initial system conditions. The proposed fixed-time DOSMC suppressed the mismatched uncertainties effectively, and achieved a better convergence performance than the existing DOSMC schemes when the initial system conditions

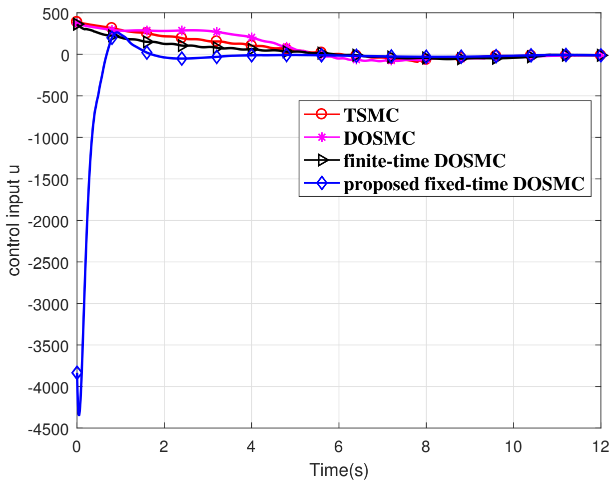

Fig. 9. The control input u of Case 3 (super large initial system states).

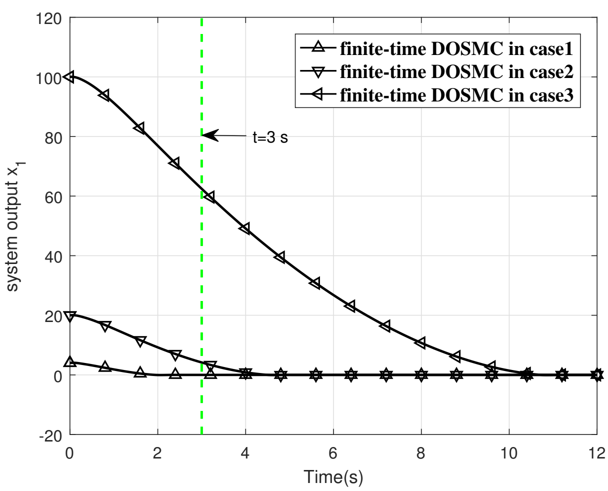

Fig. 10. The convergence rates of x_1 under finite-time DOSMC for the three cases.

change greatly. In the future, the proposed method will be improved from two aspect. First, many engineering systems not only are affected by mismatched uncertainties but also are high order, such as the flexible joint manipulator is a four-order mismatched system, and the integrated guidance and control system of missile is a three-order mismatched system. Second, although the convergence time of proposed scheme is bounded by a constant, the estimation of the upper bound is crude. Thus, we will focus on the improvement of estimation precision of convergence time in the future research.

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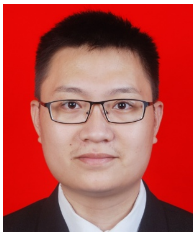

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