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An Event-triggered Output Feedback Robust MPC Scheme for Time-varying System with Packet Loss and Bounded Disturbance

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Abstract: This paper is concerned with the event-triggered output feedback robust model predictive control (RMPC) for time-varying discrete-time systems via networks with data quantization, packet loss and bounded disturbance. An observer-based event-triggered scheme is introduced according to the error between the estimated state at the current time and the latest event-triggered state. The overall designed controller consists of two components, a state observer which is offline designed by using the notion of robust positively invariant (RPI) set, and an online RMPC optimization problem which minimizes the upper bound of the expect value of the infinite horizon performance cost based on the obtained estimated state. Applying the S-procedure and the sufficient conditions of RPI sets, a constraint tightening method of estimated error bound is utilized to ensure the recursive feasibility of RMPC optimization problem. An example is performed to illustrate the availability of the developed technique.

Keywords: Event-triggered, output feedback, packet loss, robust model predictive control (RMPC), robust positively invariant (RPI).

1. INTRODUCTION

Model predictive control (MPC), as an effective control method to solve optimal control problems with physical constraints, has attracted the academic and industrial interests in the past decades [1,2]. The studies on MPC have been experienced from the classical algorithms to the robust MPC (RMPC) which is mainly regarded as a class of MPC with unique ability to tackle the non-ideal system with model uncertainties and/or bounded disturbances. There have been many works focusing on RMPC for linear systems [3–5] or nonlinear systems [6,7]. Kothare *et al*. [3] proposed a typical synthesis approach of RMPC and adopted the robust positively invariant (RPI) set theory which provided the sufficient conditions to guarantee the recursive feasibility and closed-loop systems stability. Since then, great advancements on RMPC have been pushed: the one is for the improvement of the control performance [8,9], the other mainly focuses on the issue of decreasing the burden of calculation [10,11]. However, the afore-mentioned works supposed that the system states are exactly known, and ignored they are not always measurable in practical applications, hence output feedback RMPC (OFRMPC) is needed to be further considered.

Meanwhile, networked control systems (NCSs) have received increasing attention due to its enormous advantages over the traditional control systems [12–14]. However, some challenges are also brought by inserting the communication networks into the control loop. In recent years, a large number of results have been developed on the problems of controller design [15,16] and stability analysis [17,18] for NCSs with packet loss and/or data quantization. Moreover, some works have started to extend RMPC to deal with the existing problems in NCSs [19–21]. Although these papers obtained some remarkable results and made a great progress in this field, few of them considered the synthesis approach of RMPC, i.e., RMPC with guaranteed feasibility and stability. Recently, Zou *et al*. [22] investigated the synthesis approach of RMPC by minimizing an upper bound on the expected value of an infinite horizon cost function with the satisfaction of physical constraints for NCSs involving packet loss and data quantization. Similar work can be found in [23]. However, the results obtained in [22,23] were based on the measurable system states and the external disturbance was not taken into consideration. This motivates our study here.

Another problem, i.e., most of the above approaches adopt the time-triggered scheme where data are transmitted within a fixed time interval, would lead to unnecessary data transmissions and control calculations. Hence, to solve such problem, the event-triggered scheme where the control task is executed after the satisfaction of an event, that is, the data is transmitted only when the corresponding error passes or at least attain the given thresh-

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old, has been adopted [24]. In the past few years, scholars have conducted in-depth research on the event-triggered scheme [25–30]. Based on the stability problem of the delayed neural network in [25], researchers [26,27] studied the stability of a delayed memristive neural network with a novel threshold function event-triggered mechanism. In [28], the input-based trigger method was used to study the distributed average consensus of multiagent systems under denial-of-service attacks. Some advanced works exploiting the event-triggered RMPC approach have been reported. Researchers [31,32] investigated an event-triggered RMPC method for the systems with bounded disturbance and obtained higher computational efficiency. Relevantly, Wu *et al.* [33] has successfully extended the event-triggered predictive control to more complex NCSs and achieved the desired closed-loop stability while consuming less communication resources. However, none of them considered the synthesis approach of OFRMPC with event-triggered scheme, which is another motivation of this paper.

This paper addresses a synthesis approach of event-triggered networked OFRMPC scheme for time-varying discrete-time systems with packet loss and bounded disturbance. The main contributions of this paper are concluded as follows: 1) An event-triggered scheme is exploited to decide whether the estimated state at the current time can be transmitted to controller or not, so that to control the event-triggered frequency to reduce the transmission burden under guaranteeing of the closed-loop stability. 2) A synthesis approach of OFRMPC involving packet loss and external disturbance without sacrificing the property of the recursive feasibility and closed-loop stability is provided by solving an infinite horizon expected cost function with input constraint. 3) By resorting the conditions of RPI set in [35], the state observer is offline designed and the estimated error bound is online refreshed at each time which ensures the system states converge to near the origin point at each time, and the ellipsoidal bound of the estimated error will become tighter as time goes by and tend to be stable eventually.

Notations: $P > 0$ means that P is a positive definite matrix. $\|x\|_P := \sqrt{x^{\mathrm{T}}Px}$, where $P > 0$. $*$ denotes a symmetric term in a symmetric block matrix. $\mathrm{Co}\{\cdots\}$ denotes the boundaries of a convex hull, i.e., $\mathbb{F} = \mathrm{Co}\{A_1, A_2, \cdots, A_L\} = \{\Sigma_{i=1}^{L} a_i A_i | \Sigma_{i=1}^{L} a_i = 1,\ a_i \geq 0\}$, A_i, $i = 1, \ldots, L$ denote matrices and represent the vertices of the convex hull. $\mathbb{E}_x$ denotes the expectation operator with respect to x.

2. PROBLEM FORMULATION

2.1. System description

Consider the following time-varying discrete-time system:

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k + D_k w_k, \\ y_k &= C_k x_k + E_k w_k, \end{aligned} \tag{1}$$

where $x_k \in R^{n_x}$, $u_k \in R^{n_u}$, $y_k \in R^{n_y}$ represent the system state, control input, regulated output, respectively. $w_k \in R^{n_w}$ is bounded disturbance and satisfies $\|w_k\|_{P^w}^2 \leq 1$. The following input constraint should be considered:

$$-\tilde{u} \leq u_k \leq \tilde{u}, \tag{2}$$

where $\tilde{u} = [\tilde{u}_1,\ \tilde{u}_2,\ \cdots,\ \tilde{u}_{n_u}]^{\mathrm{T}}$, $\tilde{u}_t > 0$, $t \in \{1,\ \ldots,\ n_u\}$. Assume that

$$\begin{aligned} &[A_k|B_k|C_k|D_k|E_k] \in \mathcal{E} := \mathrm{Co}\{[A_1|B_1|C_1|D_1|E_1], \\ &[A_2|B_2|C_2|D_2|E_2], \cdots, [A_L|B_L|C_L|D_L|E_L]\}, \end{aligned}$$

where $[A_l|B_l|\ldots.|E_l]$, $l = 1, \ldots, L$ represents the vertices of the convex hull $\mathcal{E}$. It implies that there exist unknown nonnegative coefficients $\varphi_{k,l}$, $l = 1, \ldots, L$ such that $\sum_{l=1}^{L} \varphi_{k,l} = 1$, and $[A_k|B_k|C_k|D_k|E_k] = \sum_{l=1}^{L} \varphi_{k,l} [A_l|B_l|C_l|D_l|E_l]$.

Assumption 1: The system (1) is both controllable and observable. Besides, the output vector y_k can be obtained and the system state x_k and the external disturbance w_k are not measurable at each sampling time instant.

2.2. Event-triggered scheme

The overall structure of the time-varying discrete-time system is shown in Fig. 1. An event-trigger is introduced to reduce calculation burden and the waste of communication resources:

$$\|\eta_k\|_{P_1^x}^2 \geq \tau \|x_{k_s}\|_{P_1^x}^2, \tag{3}$$

where $k_s (s \in \{0, 1, 2, 3, \ldots\})$ is event-triggered time, x_{k_s} is the latest event-triggered state, $\eta_k = \hat{x}_k - x_{k_s}$ is the error between the latest event-triggered state and the current estimated state, $\tau \in (0, 1)$ is pre-specified and $P_1^x > 0$ will be

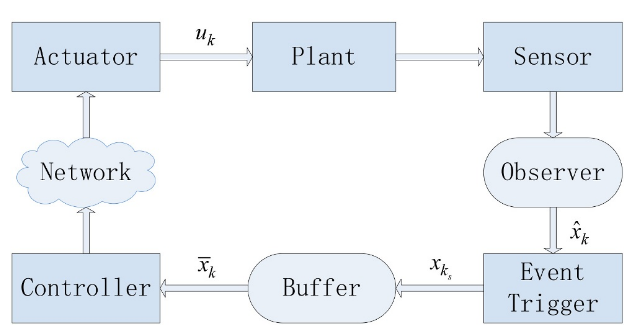

Fig. 1. Diagram of the time-varying system with event-triggered scheme.

designed. It determines whether the newly estimated state should be released to controller. If the current estimated state $\hat{x}_k$ satisfies (3), the latest event-triggered state will be updated, i.e., $x_{k_{s+1}} = \hat{x}_k$ and $k_{s+1} = k$; else, the current estimated state would not be transmitted into the controller. Then, we can get the received data $\bar{x}_k$ of the controller as

$$\bar{x}_k = \begin{cases} x_{k_{s+1}} = \hat{x}_k, & \text{when (3) is satisfied,} \\ x_{k_s}, & \text{when (3) is not satisfied.} \end{cases} \quad (4)$$

Based on (4), when $k \in [k_s, k_{s+1})$, since $x_{k_s} = \hat{x}_k - \eta_k$ holds, we can obtain $\bar{x}_k = \hat{x}_k - \eta_k$.

2.3. Symmetric quantizer

Using the following quantizer, the control input is of the following form:

$$\begin{aligned} v_k &= F_k \bar{x}_k, \\ \bar{u}_k &= f(v_k), \end{aligned} \quad (5)$$

where F_k is the state-feedback gain, and $f(\cdot)$ is a symmetric quantizer.

From [36], the quantized level set is constructed as $\Theta = \{\pm\sigma_m : \sigma_m = \varepsilon^m \sigma_0, m = \pm 1, \pm 2, \ldots\} \cup \{\pm\sigma_0\} \cup \{0\}$, $0 < \varepsilon < 1$, $\sigma_0 > 0$. Then, the logarithmic static quantizer $f(\cdot)$ is defined as follows:

$$f(v_k) = \begin{cases} \sigma_m, \ \text{if } \dfrac{1}{1+\flat}\sigma_m < v_k \leq \dfrac{1}{1-\flat}\sigma_m, \ v_k > 0, \\ 0, \ \text{if } v_k = 0, \\ -f(-v_k), \ \text{if } v_k < 0, \end{cases}$$

where $\flat = \frac{1-\varepsilon}{1+\varepsilon}$, and ε is the density of quantization. $f(v_k) = (1+\varsigma)v_k$, where $\varsigma \in [-\flat, \flat]$. For a multi-input system with $u_k \in R^{n_u}$, the quantized control inputs with channel t $(t = 1, \ldots, n_u)$ will become $\bar{u}_k = \Lambda_k v_k$, where $\Lambda_k = \text{diag}\{1+\varsigma_{k,1}, 1+\varsigma_{k,2}, \ldots, 1+\varsigma_{k,n_u}\} \in \mathbb{A} = \text{Co}\{\Lambda^{(1)}, \Lambda^{(2)}, \ldots, \Lambda^{(2^{n_u})}\}$, $\varsigma_{k,t} \in [-\flat_t, \flat_t]$, $\Lambda^{(l)}$ is a diagonal matrix with elements $1+\flat_t$ or $1-\flat_t$.

2.4. Process of Markov packet loss

In this paper, the packet loss phenomenon is modeled as the time-homogenous Markov process and a stochastic variable ϑ_k ($\vartheta_k = 1$ means no data loss, $\vartheta_k = 0$ means data loss) is used to describe whether or not a data packet is lost at time k. The corresponding transition probability matrix is defined as $\mathscr{F} = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}$, where $p = \text{Prob}(\vartheta_{k+1} = 0 | \vartheta_k = 1) \in [0,1]$, $q = \text{Prob}(\vartheta_{k+1} = 1 | \vartheta_k = 0) \in [0,1]$ denote the failure and recovery probability, respectively. At time k, define δ as the forgetting factor, the control signal can be described as

$$u_k = \vartheta_k \bar{u}_k + (1-\vartheta_k)\delta u_{k-1}. \quad (6)$$

Based on (1), (5), (6), the closed-loop system is derived as

$$x_{k+1} = A_k x_k + \vartheta_k B_k \Lambda_k v_k + (1-\vartheta_k)\delta B_k u_{k-1} + D_k w_k. \quad (7)$$

Definition 1 [34]**:** If $\mathbb{E}_{x_0}\left\{\sum_{k=0}^{\infty} x_k^{\mathrm{T}} x_k\right\} < \infty$ would be satisfied for any initial condition, then, the closed-loop system (7) is said to be stochastically stable.

Definition 2 [35]**:** Assume the set Ω is an RPI set. If $\forall x \in \Omega$, it holds $Ax + Dw \in \Omega$ for any admissible w_k.

Lemma 1 [35]**:** For a system with bounded disturbance $\|w\|^2_{P^w} \leq 1$, the following two conditions are equivalent:

(i) The ellipsoidal set $\Omega_p := \{x | x^{\mathrm{T}} P x \leq \xi\}$ with $P > 0$ is an RPI set;

(ii) The function $x^{\mathrm{T}} P x$ is not decreasing which implies $(Ax+Dw)^{\mathrm{T}} P (Ax+Dw) \leq x^{\mathrm{T}} P x$, if the disturbance is small enough such that $w^{\mathrm{T}} P^w w \leq \frac{1}{\xi} x^{\mathrm{T}} P x$ will be guaranteed.

3. STATE ESTIMATION

3.1. Minimal RPI set of observer design

The following observer is used to estimate the system state:

$$\hat{x}_{k+1} = A_k \hat{x}_k + B_k u_k + L_p(y_k - C_k \hat{x}_k), \quad (8)$$

where L_p is the determined observer gain, and $\hat{x}_k \in R^{n_x}$ is the estimated state. Let the estimated error $e_k = x_k - \hat{x}_k$, then the error dynamics is derived as

$$e_{k+1} = (A_k - L_p C_k) e_k + (D_k - L_p E_k) w_k. \quad (9)$$

Using Lemma 1, it is seen that $\Omega_{Q^e} := \{e_k | e_k^{\mathrm{T}} Q^e e_k \leq 1\}$ is an RPI set if and only if

$$e_{k+1}^{\mathrm{T}} Q^e e_{k+1} \leq e_k^{\mathrm{T}} Q^e e_k \quad (10)$$

holds under the condition

$$w_k^{\mathrm{T}} P^w w_k \leq e_k^{\mathrm{T}} Q^e e_k. \quad (11)$$

Applying the S-procedure, Ω_{Q^e} is an RPI set if there exists a positive scalar λ_1 satisfying

$$e_k^{\mathrm{T}} Q^e e_k - e_{k+1}^{\mathrm{T}} Q^e e_{k+1} - \lambda_1 (e_k^{\mathrm{T}} Q^e e_k - w_k^{\mathrm{T}} P^w w_k) \geq 0. \quad (12)$$

(12) is affine in $[A_k | B_k | C_k | D_k | E_k]$ and holds for all $\varphi_{k,l}$, $l = 1, \ldots, L$ satisfying $\sum_{l=1}^{L} \varphi_{k,l} = 1$. Substituting (9) into (12), then it is guaranteed by

$$\begin{aligned} &\begin{bmatrix} (1-\lambda_1) Q^e & 0 \\ 0 & \lambda_1 P^w \end{bmatrix} - \begin{bmatrix} (A_l - L_p C_l)^{\mathrm{T}} \\ (D_l - L_p E_l)^{\mathrm{T}} \end{bmatrix} \\ &\quad \times Q^e \left[\ (A_l - L_p C_l) \ \ (D_l - L_p E_l) \ \right] \geq 0, \\ &l = \{1, 2, \ldots, L\}. \end{aligned}$$

Applying the Schur complement, and substituting $Y = Q^e L_p$, we can obtain the following form:

$$\begin{bmatrix} (1-\lambda_1)Q^e & * & * \\ 0 & \lambda_1 P^w & * \\ Q^e A_l - YC_l & Q^e D_l - YE_l & Q^e \end{bmatrix} \geq 0,$$
$$l = \{1,2,\dots,L\}.$$

3.2. Update the estimated error bound

At time k, we assume e_k satisfies $e_k^{\mathrm{T}} Q^e e_k \leq \phi_k$, where ϕ_k is an appropriate constant to be determined. Then, ϕ_{k+1} should satisfy $e_{k+1}^{\mathrm{T}} Q^e e_{k+1} \leq \phi_{k+1}$. Thus, the problem of updating the estimated error bound is formulated as

$$\begin{aligned} &\min \ \phi_{k+1} \\ &\text{s.t. } e_{k+1}^{\mathrm{T}} Q^e e_{k+1} \leq \phi_{k+1}, \\ &e_k^{\mathrm{T}} Q^e e_k \leq \phi_k, \\ &w_k^{\mathrm{T}} P^w w_k \leq 1. \end{aligned} \tag{13}$$

Applying the S-procedure, (13) can be cast into

$$\begin{aligned} &(e_{k+1}^{\mathrm{T}} Q^e e_{k+1} - \phi_{k+1}) - \lambda_2 (e_k^{\mathrm{T}} Q^e e_k - \phi_k) \\ &- \lambda_3 (w_k^{\mathrm{T}} P^w w_k - 1) \leq 0, \end{aligned} \tag{14}$$

where $\lambda_2, \lambda_3 \in (0,1)$. Using the Schur complement, it yields

$$\begin{aligned} &\min_{\lambda_2,\lambda_3,\phi_{k+1}} \ \phi_{k+1} \\ &\text{s.t. } \begin{bmatrix} -\lambda_2 Q^e & * & * \\ 0 & -\lambda_3 P^w & * \\ 0 & 0 & -\phi_{k+1} + \lambda_2 \phi_k + \lambda_3 \end{bmatrix} + \Gamma \\ &\leq 0, \ l = \{1,2,\dots,L\}, \end{aligned} \tag{15}$$

where

$$\Gamma = \begin{bmatrix} \|(A_l - L_p C_l)\|_{Q^e}^2 & * & * \\ (D_l - L_p E_l)^{\mathrm{T}} Q^e (A_l - L_p C_l) & \|(D_l - L_p E_l)\|_{Q^e}^2 & * \\ 0 & 0 & 0 \end{bmatrix}.$$

Remark 1: It should be noted that ϕ_k keeps decreasing as time goes by until $\Omega_{Q^e} := \{e_k | {e_k}^{\mathrm{T}} Q^e e_k \leq \phi_k\}$ becomes the minimal ellipsoidal RPI set, that is ϕ_k becomes the minimum. There is a high correlation between this refreshing method and the recursive feasibility of the presented synthesis approach of OFRMPC which will be discussed in the next section.

4. THE ONLINE SYNTHESIS APPROACH OF EVENT- TRIGGERED NETWORKED OFRMPC

In this section, an online synthesis approach of event-triggered networked OFRMPC is investigated. The RMPC method in [3] is properly extended to the networked environment with packet loss and bounded disturbance when the system states are unmeasurable, and both the recursive feasibility and closed-loop stability of this synthesis approach are discussed.

4.1. Event-triggered networked OFRMPC

According to (4), (6) can be described as

$$u_k = \vartheta_k \Lambda_k F_k (\hat{x}_k - \eta_k) + (1-\vartheta_k)\delta u_{k-1}. \tag{16}$$

Let $z_k = [\hat{x}_k^{\mathrm{T}} \ u_{k-1}^{\mathrm{T}} \ e_k^{\mathrm{T}}]^{\mathrm{T}}$, the augmented model of (8), (9) and (16) is

$$z_{k+1} = \Upsilon_k^1 z_k + \Upsilon_k^2 w_k + \Upsilon_k^3 \eta_k, \tag{17}$$

where

$$\Upsilon_k^1 = \begin{bmatrix} A_k + \vartheta_k B_k \Lambda_k F_k & (1-\vartheta_k)\delta B_k & L_p C_k \\ \vartheta_k \Lambda_k F_k & (1-\vartheta_k)\delta I & 0 \\ 0 & 0 & A_k - L_p C_k \end{bmatrix},$$

$$\Upsilon_k^2 = \begin{bmatrix} L_p E_k \\ 0 \\ D_k - L_p E_k \end{bmatrix}, \Upsilon_k^3 = \begin{bmatrix} -\vartheta_k B_k \Lambda_k F_k \\ -\vartheta_k \Lambda_k F_k \\ 0 \end{bmatrix}.$$

Choose the following expected cost function: $J_\infty = \sum_{i=0}^{\infty} \mathbb{E}_{z_k} \{\|z_{k+i}\|_{\mathfrak{L}}^2 + \|u_{k+i}\|_{\mathfrak{R}}^2\}$, $\mathfrak{L} = \mathrm{diag}\{\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3\}$ and $\mathfrak{R}$ are suitable weighting matrices. The quadratic Lyapunov function is $V(z_{k+i}) = z_{k+i}^{\mathrm{T}} \Psi_{\vartheta_{k+i}} z_{k+i}$, where $\Psi_{\vartheta_{k+i}} = \mathrm{diag}\{P_{\vartheta_{k+i}}^x, P_{\vartheta_{k+i}}^u, P_{\vartheta_{k+i}}^e\}$. Suppose that the following stability constraint is satisfied at each time k:

$$\begin{aligned} &\mathbb{E}_{z_k}\{V(z_{k+i})\} \geq 1 \Longrightarrow \mathbb{E}_{z_k}\{V(z_{k+i}) - V(z_{k+i+1})\} \\ &\geq \mathbb{E}_{z_k}\left\{\|z_{k+i}\|_{\mathfrak{L}}^2 + \|u_{k+i}\|_{\mathfrak{R}}^2\right\}. \end{aligned} \tag{18}$$

For robust stability, by summing (18) from $i = 0 \to \infty$, we can get

$$J_\infty \leq \mathbb{E}_{z_k}\{V(z_k)\} \leq \gamma_k, \tag{19}$$

where γ_k is the upper bound of J_∞. Based on (18) and (19), the whole event-triggered networked OFRMPC optimization problem can be developed as

$$\min \gamma_k \ \text{ s.t. } (2), \ (18) \text{ and } (19).$$

In the following analysis, the optimization problem will be changed into LMI constraint conditions.

4.2. Solving optimization problem via LMI technique

In order to facilitate the analysis of packet loss phenomenon, we introduce the following Lemma 2.

Lemma 2: The Markov packet loss phenomenon would be simplified into two cases if the following LMIs are satisfied:

$$\begin{bmatrix} \Xi_1 & * \\ \Xi_2 & \Xi_3 \end{bmatrix} \geq 0, \tag{20}$$

$$\begin{bmatrix} \Xi_1 & * \\ \Xi_4 & \Xi_3 \end{bmatrix} \geq 0, \tag{21}$$

where

$$\Xi_1 = \mathrm{diag}\{\tilde{Q}^x, \tilde{Q}^u, \gamma Q^{e-1}\},$$

$$\Xi_2 = \begin{bmatrix} p^{\frac{1}{2}}\tilde{Q}^x & 0 & 0 \\ (1-p)^{\frac{1}{2}}\tilde{Q}^x & 0 & 0 \\ 0 & p^{\frac{1}{2}}\tilde{Q}^u & 0 \\ 0 & (1-p)^{\frac{1}{2}}\tilde{Q}^u & 0 \\ 0 & 0 & p^{\frac{1}{2}}\gamma Q^{e-1} \\ 0 & 0 & (1-p)^{\frac{1}{2}}\gamma Q^{e-1} \end{bmatrix},$$

$$\Xi_3 = \mathrm{diag}\{\tilde{P}_0^x, \tilde{P}_1^x, \tilde{P}_0^u, \tilde{P}_1^u, \tilde{P}_0^e, \tilde{P}_1^e\},$$

$$\Xi_4 = \begin{bmatrix} (1-q)^{\frac{1}{2}}\tilde{Q}^x & 0 & 0 \\ q^{\frac{1}{2}}\tilde{Q}^x & 0 & 0 \\ 0 & (1-q)^{\frac{1}{2}}\tilde{Q}^u & 0 \\ 0 & q^{\frac{1}{2}}\tilde{Q}^u & 0 \\ 0 & 0 & (1-q)^{\frac{1}{2}}\gamma Q^{e-1} \\ 0 & 0 & q^{\frac{1}{2}}\gamma Q^{e-1} \end{bmatrix}.$$

Proof: Consider the transition probability matrix of Markov chain which involves four cases of packet loss phenomenon: $1 \longrightarrow 1$, $1 \longrightarrow 0$, $0 \longrightarrow 1$, $0 \longrightarrow 0$, we introduce a matrix $\Phi = \mathrm{diag}\{Q^x, Q^u, Q^e\}$ to analyze two cases of packet loss synthetically: $1 \longrightarrow 1/0$ and $0 \longrightarrow 1/0$, then we have

$$\mathbb{E}_{\vartheta_{k+i}}\{\Psi_{\vartheta_{k+i+1}}\} = p\Psi_0 + (1-p)\Psi_1 \leq \Phi, \tag{22}$$

$$\mathbb{E}_{\vartheta_{k+i}}\{\Psi_{\vartheta_{k+i+1}}\} = (1-q)\Psi_0 + q\Psi_1 \leq \Phi. \tag{23}$$

Pre and post-multiplying (22) and (23) by Φ^{-1}, and using the Schur complement with $\gamma P_1^{x-1} = \tilde{P}_1^x$, $\gamma P_1^{u-1} = \tilde{P}_1^u$, $\gamma P_1^{e-1} = \tilde{P}_1^e$, $\gamma P_0^{x-1} = \tilde{P}_0^x$, $\gamma P_0^{u-1} = \tilde{P}_0^u$, $\gamma P_0^{e-1} = \tilde{P}_0^e$, $\gamma Q^{x-1} = \tilde{Q}^x$, $\gamma Q^{u-1} = \tilde{Q}^u$, we can get (20) and (21). □

4.2.1 Transform the constraint (19) into LMI

According to Lemma 2, we can obtain $\mathbb{E}_{z_k}\{V(z_k)\} \leq z_k^{\mathrm{T}}\Phi z_k \leq \gamma_k$, which is equivalent to $\|\hat{x}_k\|_{Q^x}^2 + \|u_{k-1}\|_{Q^u}^2 + \|e_k\|_{Q^e}^2 \leq \gamma_k$. Since $e_k^{\mathrm{T}} Q^e e_k \leq \phi_k$, then $\|\hat{x}_k\|_{Q^x}^2 + \|u_{k-1}\|_{Q^u}^2 \leq \gamma_k - \phi_k$. Applying the Schur complement, it yields

$$\begin{bmatrix} \gamma_k - \phi_k & * & * \\ \hat{x}_k & \tilde{Q}^x & * \\ u_{k-1} & 0 & \tilde{Q}^u \end{bmatrix} \geq 0. \tag{24}$$

4.2.2 Transform stability constraint condition (18) into LMIs

Since w_k satisfies $\|w_k\|_{P^w}^2 \leq 1$, $\mathbb{E}_{z_k}\{V(z_{k+i})\} \geq 1$ can be expressed as $\mathbb{E}_{z_k}\{V(z_{k+i})\} \geq \|w_{k+i}\|_{P^w}^2$, then (18) is guaranteed by

$$\mathbb{E}_{z_k}\{V(z_{k+i})\} \geq \|w_{k+i}\|_{P^w}^2 \Longrightarrow$$
$$\mathbb{E}_{z_k}\{V(z_{k+i}) - V(z_{k+i+1})\} \geq \mathbb{E}_{z_k}\left\{\|z_{k+i}\|_{\mathfrak{L}}^2 + \|u_{k+i}\|_{\mathfrak{R}}^2\right\}. \tag{25}$$

When $k \in [k_s, k_{s+1})$, according to (4), $\tau\|\hat{x}_k - \eta_k\|_{P_1^x}^2 \geq \|\eta_k\|_{P_1^x}^2$ is obtained. Hence, (25) is guaranteed by

$$\begin{aligned} &\mathbb{E}_{z_k}\{V(z_{k+i})\} \geq \|w_{k+i}\|_{P^w}^2 \Longrightarrow \\ &\mathbb{E}_{z_k}\{V(z_{k+i}) - V(z_{k+i+1})\} \geq \mathbb{E}_{z_k}\left\{\|z_{k+i}\|_{\mathfrak{L}}^2 + \|u_{k+i}\|_{\mathfrak{R}}^2\right\} \\ &\qquad + \tau\|\hat{x}_k - \eta_k\|_{P_1^x}^2 - \|\eta_k\|_{P_1^x}^2. \end{aligned} \tag{26}$$

Applying the S-procedure, if there exists $\lambda_4 > 0$, we can get

$$\begin{aligned} &\mathbb{E}_{z_k}\{V(z_{k+i}) - V(z_{k+i+1})\} - \mathbb{E}_{z_k}\left\{\|z_{k+i}\|_{\mathfrak{L}}^2 + \|u_{k+i}\|_{\mathfrak{R}}^2\right\} \\ &- \tau\|\hat{x}_k - \eta_k\|_{P_1^x}^2 + \|\eta_k\|_{P_1^x}^2 - \lambda_4 \mathbb{E}_{z_k}\{V(z_{k+i})\} \\ &+ \lambda_4\|w_{k+i}\|_{P^w}^2 \geq 0. \end{aligned} \tag{27}$$

Consider $\vartheta_{k+i} = 1$, combining (20), (27) can be guaranteed if the following condition is satisfied:

$$\begin{aligned} &\mathrm{diag}\{\mathcal{M}_1, (1-\lambda_4)P_1^u - \mathfrak{L}_2, (1-\lambda_4)P_1^e - \mathfrak{L}_3, \lambda_4 P^w, P_1^x\} \\ &- \begin{bmatrix} A_l + B_l\Lambda F & 0 & L_pC_l & L_pE_l & -B_l\Lambda F \\ \Lambda F & 0 & 0 & 0 & -\Lambda F \\ 0 & 0 & A_l - L_pC_l & D_l - L_pE_l & 0 \end{bmatrix}^{\mathrm{T}} \\ &\times \mathrm{diag}\{Q^x, Q^u, Q^e\} \\ &\times \begin{bmatrix} A_l + B_l\Lambda F & 0 & L_pC_l & L_pE_l & -B_l\Lambda F \\ \Lambda F & 0 & 0 & 0 & -\Lambda F \\ 0 & 0 & A_l - L_pC_l & D_l - L_pE_l & 0 \end{bmatrix} \\ &- [\Lambda F\ 0\ 0\ 0\ -\Lambda F]^{\mathrm{T}}\mathfrak{R}[\Lambda F\ 0\ 0\ 0\ -\Lambda F] \\ &- [\tau I\ 0\ 0\ 0\ -\tau I]^{\mathrm{T}} P_1^x [\tau I\ 0\ 0\ 0\ -\tau I] \\ &\geq 0,\ l = \{1, 2, ..., L\}, \end{aligned} \tag{28}$$

where $\mathcal{M}_1 = (1-\lambda_4)P_1^x - \mathfrak{L}_1$. Pre and post-multiplying (28) by $\mathrm{diag}\{\gamma^{\frac{1}{2}}P_1^{x-1}, \gamma^{\frac{1}{2}}P_1^{u-1}, \gamma^{\frac{1}{2}}P_1^{e-1}, \gamma^{\frac{1}{2}}, \gamma^{\frac{1}{2}}P_1^{x-1}\}$ and its transpose, and using the Schur complement, we can get the following LMIs:

$$\begin{bmatrix} \Xi_5 & * \\ \Xi_6 & \Xi_7 \end{bmatrix} \geq 0,\ l = \{1, 2, ..., L\}, \tag{29}$$

where

$$\Xi_5 = \mathrm{diag}\{(1-\lambda_4)\tilde{P}_1^x, (1-\lambda_4)\tilde{P}_1^u, (1-\lambda_4)\tilde{P}_1^e, \gamma\lambda_4 P^w, \tilde{P}_1^x\},$$

$$\Xi_6 = \begin{bmatrix} A_l\tilde{P}_1^x + B_l\Lambda K & 0 & L_pC_l\tilde{P}_1^e & \gamma L_pE_l & -B_l\Lambda K \\ \Lambda K & 0 & 0 & 0 & -\Lambda K \\ 0 & 0 & \Delta_1 & \gamma(D_l - L_pE_l) & 0 \\ \tilde{P}_1^x & 0 & 0 & 0 & 0 \\ 0 & \tilde{P}_1^u & 0 & 0 & 0 \\ 0 & 0 & \tilde{P}_1^e & 0 & 0 \\ \Lambda K & 0 & 0 & 0 & -\Lambda K \\ \tau\tilde{P}_1^x & 0 & 0 & 0 & -\tau\tilde{P}_1^x \end{bmatrix},$$

$$\Delta_1 = A_l\tilde{P}_1^e - L_pC_l\tilde{P}_1^e,$$

$$\Xi_7 = \text{diag}\{\tilde{Q}^x, \tilde{Q}^u, \gamma Q^{e-1}, \gamma\mathfrak{L}_1^{-1}, \gamma\mathfrak{L}_2^{-1}, \gamma\mathfrak{L}_3^{-1}, \gamma\mathfrak{R}^{-1}, \tilde{P}_1^x\}.$$

Similarly, consider $\vartheta_{k+i} = 0$, (27) is guaranteed by the following LMIs:

$$\begin{bmatrix} \Xi_8 & * \\ \Xi_9 & \Xi_7 \end{bmatrix} \geq 0,\ l = \{1,2,...,L\}, \tag{30}$$

where

$$\Xi_8 = \text{diag}\{(1-\lambda_4)\tilde{P}_0^x, (1-\lambda_4)\tilde{P}_0^u, (1-\lambda_4)\tilde{P}_0^e, \gamma\lambda_4 P^w, \tilde{P}_1^x\},$$

$$\Xi_9 = \begin{bmatrix} A_l\tilde{P}_0^x & \delta B_l\tilde{P}_0^u & L_pC_l\tilde{P}_0^e & \gamma L_pE_l & 0 \\ 0 & \delta\tilde{P}_0^u & 0 & 0 & 0 \\ 0 & 0 & A_l\tilde{P}_0^e - L_pC_l\tilde{P}_0^e & \gamma(D_l - L_pE_l) & 0 \\ \tilde{P}_0^x & 0 & 0 & 0 & 0 \\ 0 & \tilde{P}_0^u & 0 & 0 & 0 \\ 0 & 0 & \tilde{P}_0^e & 0 & 0 \\ 0 & \delta\tilde{P}_0^u & 0 & 0 & 0 \\ \tau\tilde{P}_1^x & 0 & 0 & 0 & -\tau\tilde{P}_1^x \end{bmatrix}.$$

Moreover, the control gain is given as $F = K\tilde{P}_1^{x-1}$.

4.2.3 Transform input constraint (2) into LMIs

The following condition must be imposed based on the concept of RPI set: $z_{k+i} \in \Omega_{\tilde{\Psi}_1^{-1}} = \{z | z^{\mathrm{T}}\tilde{\Psi}_1^{-1}z \leq 1\}$ which implies $V\{(z_{k+i})\} \leq \gamma_k$, where $\tilde{\Psi}_1 = \text{diag}\{\tilde{P}_1^x, \tilde{P}_1^u, \tilde{P}_1^e\}$.

Lemma 3: The above condition for NCSs with multiple missing data is guaranteed if the additional LMIs are satisfied:

$$\begin{bmatrix} \bar{\Xi}_1 & * \\ \Xi & \bar{\Xi}_2 \end{bmatrix} \geq 0,\ (l = 1,2,\dots,L), \tag{31}$$

$$\begin{bmatrix} \bar{\Xi}_1 & * \\ \Xi & \bar{\Xi}_{0.1} \end{bmatrix} \geq 0,\ (l = 1,2,\dots,L), \tag{32}$$

$$\begin{bmatrix} \bar{\Xi}_{0.s} & * \\ \Xi_{0.s} & \bar{\Xi}_2 \end{bmatrix} \geq 0, \tag{33}$$

$$\begin{bmatrix} \bar{\Xi}_{0.s} & * \\ \Xi_{0.s} & \bar{\Xi}_{0.s+1} \end{bmatrix} \geq 0, \tag{34}$$

where $s = 1,\dots,\chi_{\max}$, $l = 1,2,\dots,L$.

$$\bar{\Xi}_1 = \text{diag}\{(1-\mu_1)\tilde{P}_1^x, (1-\mu_1)\tilde{P}_1^u, (1-\mu_1)\tilde{P}_1^e, \gamma\mu_1 P^w, \mu_2\tilde{P}_1^x\},$$

$$\Xi = \begin{bmatrix} A_l\tilde{P}_1^x + B_l\Lambda K & 0 & L_pC_l\tilde{P}_1^e & \gamma L_pE_l & -B_l\Lambda K \\ \Lambda K & 0 & 0 & 0 & -\Lambda K \\ 0 & 0 & \Delta_1 & \gamma(D_l - L_pE_l) & 0 \\ \tau\tilde{P}_1^x & 0 & 0 & 0 & -\tau\tilde{P}_1^x \end{bmatrix},$$

$$\bar{\Xi}_2 = \text{diag}\{\tilde{P}_1^x, \tilde{P}_1^u, \tilde{P}_1^e, \frac{1}{\mu_2}\tilde{P}_1^x\},$$

$$\bar{\Xi}_{0.1} = \text{diag}\{\tilde{P}_{0.1}^x, \tilde{P}_{0.1}^u, \tilde{P}_{0.1}^e, \frac{1}{\mu_2}\tilde{P}_1^x\},$$

$$\bar{\Xi}_{0.s} = \text{diag}\{(1-\mu_1)\tilde{P}_{0.s}^x, (1-\mu_1)\tilde{P}_{0.s}^u, (1-\mu_1)\tilde{P}_{0.s}^e, \gamma\mu_1 P^w, \mu_2\tilde{P}_1^x\},$$

$$\bar{\Xi}_{0.s+1} = \text{diag}\{\tilde{P}_{0.s+1}^x, \tilde{P}_{0.s+1}^u, \tilde{P}_{0.s+1}^e, \frac{1}{\mu_2}\tilde{P}_1^x\},$$

$$\Xi_{0.s} = \begin{bmatrix} A_l\tilde{P}_{0.s}^x & \delta B_l\tilde{P}_{0.s}^u & L_pC_l\tilde{P}_{0.s}^e & \gamma L_pE_l & 0 \\ 0 & \delta\tilde{P}_{0.s}^u & 0 & 0 & 0 \\ 0 & 0 & \Delta_{0.s} & \gamma(D_l - L_pE_l) & 0 \\ \tau\tilde{P}_1^x & 0 & 0 & 0 & -\tau\tilde{P}_1^x \end{bmatrix},$$

$$\Delta_{0.s} = A_l\tilde{P}_{0.s}^e - L_pC_l\tilde{P}_{0.s}^e.$$

Proof: At time $k+1$, since $z_{k+1} \in \Omega_{\tilde{\Psi}_1^{-1}}$ holds for $1 \to 0$, i.e., $\vartheta_k = 1$, $\vartheta_{k+1} = 0$, we only consider $1 \to 1$. Using Lemma 1 and considering the event-triggered scheme (3) with $k \in [k_s, k_{s+1})$, $z_{k+1} \in \Omega_{\tilde{\Psi}_1^{-1}}$ holds if and only if

$$V(z_{k+1}) \leq V(z_k) \tag{35}$$

holds under the condition

$$\|w_k\|_{P^w} \leq V(z_k) \ \text{ and } \ \|\eta_k\|_{P_1^x}^2 \leq \tau\|\hat{x}_k - \eta_k\|_{P_1^x}^2. \tag{36}$$

Applying the S-procedure, $z_{k+1} \in \Omega_{\tilde{\Psi}_1^{-1}}$ holds if there exist scalars $\mu_1, \mu_2 \in (0,1)$ satisfying

$$V(z_k) - V(z_{k+1}) - \mu_1(V(z_k) - \|w_k\|_{P^w}) - \mu_2(\tau\|\hat{x}_k - \eta_k\|_{P_1^x}^2 - \|\eta_k\|_{P_1^x}^2) \geq 0. \tag{37}$$

Equation (37) is described as

$$\begin{aligned} &\text{diag}\{(1-\mu_1)P_1^x, (1-\mu_1)P_1^u, (1-\mu_1)P_1^e, \mu_1 P^w, \mu_2 P_1^x\} \\ &- \begin{bmatrix} A_l + B_l\Lambda F & 0 & L_pC_l & L_pE_l & B_l\Lambda F \\ \Lambda F & 0 & 0 & 0 & \Lambda F \\ 0 & 0 & A_l - L_pC_l & D_l - L_pE_l & 0 \end{bmatrix}^{\mathrm{T}} \\ &\times \text{diag}\{P_1^x, P_1^u, P_1^e\} \\ &\times \begin{bmatrix} A_l + B_l\Lambda F & 0 & L_pC_l & L_pE_l & B_l\Lambda F \\ \Lambda F & 0 & 0 & 0 & \Lambda F \\ 0 & 0 & A_l - L_pC_l & D_l - L_pE_l & 0 \end{bmatrix} \\ &- \mu_2[\tau I\ 0\ 0\ 0\ -\tau I]^{\mathrm{T}} P_1^x [\tau I\ 0\ 0\ 0\ -\tau I] \geq 0. \end{aligned} \tag{38}$$

Pre and post-multiplying (38) by $\text{diag}\{\gamma^{\frac{1}{2}}P_1^{x-1}, \gamma^{\frac{1}{2}}P_1^{u-1}, \gamma^{\frac{1}{2}}P_1^{e-1}, \gamma^{\frac{1}{2}}, \gamma^{\frac{1}{2}}P_1^{x-1}\}$ and its transpose, and using the Schur complement, we can obtain (31). □

At time $k+2$, we need to analyze two cases for $\vartheta_{k+2} = 1$. (i) $1 \to 1 \to 1$, using (31), we have $z_{k+2} \in \Omega_{\tilde{\Psi}_1^{-1}}$. (ii) $1 \to 0 \to 1$, assume $z_{k+1} \in \Omega_{\tilde{\Psi}_{0.1}^{-1}} = \{z | z^{\mathrm{T}}\tilde{\Psi}_{0.1}^{-1}z \leq 1\}$, where $\tilde{\Psi}_{0.1} = \text{diag}\{\tilde{P}_{0.1}^x, \tilde{P}_{0.1}^u, \tilde{P}_{0.1}^e\}$. Like (37), $z_{k+1} \in$

$\Omega_{\tilde{\Psi}_{0,1}^{-1}}$ can be guaranteed by (32). Similarly, $z_{k+2} \in \Omega_{\tilde{\Psi}_1^{-1}}$ holds if the following condition is satisfied:

$$V(z_{k+1}) - V(z_{k+2}) - \mu_1(V(z_{k+1}) - \|w_{k+1}\|_{P^w}) - \mu_2(\tau\|\hat{x}_{k+1} - \eta_{k+1}\|^2_{\tilde{P}_1^x} - \|\eta_{k+1}\|^2_{\tilde{P}_1^x}) \geq 0. \tag{39}$$

Then, using the Schur complement, we can obtain (33).

At time $k+i$ $(i \geq 3)$, two cases should be considered for $\vartheta_{k+i} = 1$: (i) $1 \to 1 \to \cdots \to 1$, obviously, $z_{k+i} \in \Omega_{\tilde{\Psi}_1^{-1}}$ holds. (ii) $1 \to 0 \to \cdots \to 0 \to 1$, i.e., $\vartheta_{k+i-h-1} = 1$, $\vartheta_{k+i-s} = 0$, $\vartheta_{k+i} = 1$, where $h \in \{1, \dots, \chi_{\max}\}$ is the number of continuous data missing, $s = 1, \dots, h$. Assume $z_{k+i-(h+1-s)} \in \Omega_{\tilde{\Psi}_{0.s}^{-1}} = \{z | z^{\mathrm{T}}\tilde{\Psi}_{0.s}^{-1} z \leq 1\}$, where $\tilde{\Psi}_{0.s} = \mathrm{diag}\{\tilde{P}^x_{0.s}, \tilde{P}^u_{0.s}, \tilde{P}^e_{0.s}\}$. Then, together (32), (33) and (34), $z_{k+i} \in \Omega_{\tilde{\Psi}_1^{-1}}$ holds for $\vartheta_{k+i} = 1$ after h continuous times of data missing, and we can get $V(z_{k+i}) \leq \gamma_k$.

Since $z^{\mathrm{T}}\tilde{\Psi}_1^{-1}z \leq 1$ holds based on Lemma 3, and according to the event-triggered scheme (3) with $k \in [k_s, k_{s+1})$, $\|\eta_k\|^2_{\tilde{P}_1^x} \leq \tau$ holds, we have

$$\begin{aligned}
&\left|\left(\Lambda K\tilde{P}_1^{x^{-1}}(\hat{x}_{k+i} - \eta_{k+i})\right)_t\right|^2 \\
&\leq (1+\varsigma_t)^2 \left\|\left(\begin{bmatrix} K\tilde{P}_1^{x^{-\frac{1}{2}}} & -K\tilde{P}_1^{x^{-\frac{1}{2}}}\end{bmatrix}\right)_t\right\|^2 \\
&\quad \times \left\|\left(\begin{bmatrix} \tilde{P}_1^{x^{-\frac{1}{2}}} & 0 \\ 0 & \tilde{P}_1^{x^{-\frac{1}{2}}}\end{bmatrix}\begin{bmatrix}\hat{x}_{k+i} \\ \eta_{k+i}\end{bmatrix}\right)_t\right\|^2 \\
&\leq (1+\tau)(1+\flat_t)^2 \left\|\left(\begin{bmatrix} K\tilde{P}_1^{x^{-\frac{1}{2}}} & -K\tilde{P}_1^{x^{-\frac{1}{2}}}\end{bmatrix}\right)_t\right\|^2 < \bar{u}_t^2.
\end{aligned} \tag{40}$$

Then, (40) is guaranteed by the following LMI:

$$\begin{bmatrix} \frac{1}{(1+\tau)}\bar{U} & * & * \\ \emptyset K^{\mathrm{T}} & \tilde{P}_1^x & * \\ -\emptyset K^{\mathrm{T}} & 0 & \tilde{P}_1^x \end{bmatrix} \geq 0, \tag{41}$$

where $\emptyset = \mathrm{diag}\{1+\flat_1, 1+\flat_2, \dots, 1+\flat_{n_u}\}$, $\bar{U} = \mathrm{diag}\{\bar{u}_1^2, \bar{u}_2^2, \dots, \bar{u}_{n_u}^2\}$.

Theorem 1: For time-varying discrete-time system (1) subject to packet loss and bounded disturbance with given forgetting factor δ, quantization density ε and event-triggered parameter τ, the whole constrained event-triggered networked OFRMPC optimization problem can be formulated as follows:

$$\min_{\gamma, \tilde{P}_0^x, \tilde{P}_0^u, \tilde{P}_0^e, \tilde{P}_1^x, \tilde{P}_1^u, \tilde{P}_1^e, \tilde{P}_{0.s}^x, \tilde{P}_{0.s}^u, \tilde{P}_{0.s}^e, \tilde{Q}^x, \tilde{Q}^u, K} \gamma \tag{42}$$

s.t. (20)-(21), (24), (29), (30), (31)-(34) and (41).

The implementation steps of designing event-triggered OFRMPC is summarized in Algorithm 1.

Algorithm 1:

Off-line:

Step 1: Choose $\lambda_1 \in (0,1)$, solve

$$\begin{bmatrix} (1-\lambda_1)Q^e & * & * \\ 0 & \lambda_1 P^w & * \\ Q^eA_l - YC_l & Q^eD_l - YE_l & Q^e \end{bmatrix} \geq 0,\ l = \{1, 2, \dots, L\}$$

to obtain Q^e, Y and compute $L_p = (Q^e)^{-1}Y$.

Step 2: Set the value of ".......".

On-line:

Step 1: Solve the optimization problem (42), and obtain $v_k = F_k\bar{x}_k = K_k(\tilde{P}^x_{1k})^{-1}\bar{x}_k$.

Step 2: Compute the quantized control input $\bar{u}_k = f(v_k)$, and transmit it into the network. If the data is successfully arrived at the actuator, $u_k = \bar{u}_k$, else, $u_k = u_{k-h}$, $h \in \{1, 2, \dots, \chi_{\max}\}$.

Step 3: Compute x_{k+1}, y_k. According to (8) and (15), calculate estimate state $\hat{x}_{k+1}$ and ϕ_{k+1}, respectively.

Step 4: Update sampling instant to $k+1$ and go to Step 1.

4.3. Recursive feasibility and stochastic stability

Theorem 2: Consider time-varying discrete-time system (1) subject to packet loss and bounded disturbance. If the optimization problem (42) is feasible at time k, then (42) is also feasible for all the future time. Moreover, the closed-loop system is stochastically stable.

Proof: Recursive feasibility: At time k, it is assumed that the optimal solution of (42) is obtained. Note that when disturbance exists, recursive feasibility is no longer a natural feature of OFRMPC. It is seen that only the constraint (24) is time-dependent, where z_k is involved. Therefore, to prove recursive feasibility, we only need to prove $\Omega_{\tilde{\Phi}^{-1}} := \{z_k | z_k{}^{\mathrm{T}}\tilde{\Phi}^{-1}z_k \leq 1\}$ is an RPI set, where $\tilde{\Phi} = \mathrm{diag}\{\tilde{Q}^x, \tilde{Q}^u, \tilde{Q}^e\}$. From Lemma 1, it is known that the RPI set condition holds if and only if

$$\mathbb{E}_{z_k}\{\|z_{k+1}\|^2_{\tilde{\Phi}^{-1}}\} \leq \mathbb{E}_{z_k}\{\|z_k\|^2_{\tilde{\Phi}^{-1}}\}$$

holds under the condition

$$\frac{1}{\phi_k}e_k^{\mathrm{T}}Q^e e_k \leq \mathbb{E}_{z_k}\{\|z_k\|^2_{\tilde{\Phi}^{-1}}\}.$$

Applying the S-procedure, if there exists $\lambda_5 \in (0,1)$, we have

$$\mathbb{E}_{z_k}\{\|z_{k+1}\|^2_{\tilde{\Phi}^{-1}}\} - \mathbb{E}_{z_k}\{\|z_k\|^2_{\tilde{\Phi}^{-1}}\} - \lambda_5\left\{\frac{1}{\phi_k}e_k^{\mathrm{T}}Q^e e_k - \mathbb{E}_{z_k}\{\|z_k\|^2_{\tilde{\Phi}^{-1}}\}\right\} \leq 0. \tag{43}$$

Then, (43) can verify that $\Omega_{\tilde{\Phi}^{-1}}$ is an RPI set, and it is equivalent to

$$-\frac{\lambda_5}{\phi_k}e_k^{\mathrm{T}}Q^e e_k \leq -\mathbb{E}_{z_k}\{\|z_{k+1}\|^2_{\tilde{\Phi}^{-1}}\} + (1-\lambda_5)\mathbb{E}_{z_k}\{\|z_k\|^2_{\tilde{\Phi}^{-1}}\}. \tag{44}$$

As mentioned in Remark 1, ϕ_k decreases as time goes by, thus, $\phi_{k+1} < \phi_k$, and (44) holds by substituting ϕ_{k+1} for ϕ_k. Hence, the solution of (42) at time k is feasible at time $k+1$.

Stochastic stability: At time $k=0$, if (42) is satisfied, we can conclude $\sum_{i=0}^{\infty}\mathbb{E}_{z_k}\left\{\|z_{k+i}\|_{\mathfrak{L}}^2+\|u_{k+i}\|_{\mathfrak{R}}^2\right\}\leq\gamma_k$. Since the recursive feasibility is guaranteed, the upper bound γ_k keeps decreasing with time k. Let $k=0$, $\mathbb{E}_{z_0}\left\{\sum_{k=0}^{\infty}\|z_k\|_{\mathfrak{L}}^2\right\}<\infty$ holds. Assume $\partial=\lambda_{\min}\{\mathfrak{L}_1\}$, it yields $\mathbb{E}_{z_0}\left\{\sum_{k=0}^{\infty}x_k^{\mathrm{T}}x_k\right\}\leq\frac{1}{\partial}\mathbb{E}_{z_0}\left\{\sum_{k=0}^{\infty}x_k^{\mathrm{T}}\mathfrak{L}_1x_k\right\}<\infty$. According to Definition 1, the stochastic stability of the closed-loop system is guaranteed. □

5. SIMULATION EXAMPLE

Consider the continuous stirred tank reactor (CSTR) system which is regarded as a practical chemical industrial process and has been studied in [6], [37] and [38].

The CSTR model where chemical B is formed by the chemical A is shown in Fig. 2 and described by the following equations:

$$\dot{C}_A=\frac{q}{V}(C_{Af}-C_A)-k_0e^{-\frac{E/R}{T}}C_A,$$
$$\dot{T}=\frac{q}{V}(T_f-T)-\frac{(-\Delta H)}{\rho C_p}k_0e^{-\frac{E/R}{T}}C_A+\frac{UA}{V\rho C_p}(T_c-T),$$

where T is the reactor temperature, C_A is reactant concentration, and the manipulated variable is coolant stream temperature T_c. The other relevant parameters are listed in Table 1.

Define the system state variables $x=[C_A-C_A^{eq}\ \ T-T^{eq}]^{\mathrm{T}}$ (C_A^{eq} is the non-zero equilibrium points of concentration), input variables $u=T_c-T_c^{eq}$ (T_c^{eq} is coolant temperature), and x_2 satisfies $\underline{x}_2\leq x_2\leq\bar{x}_2(\underline{x}_2=T^l-T^{eq},\bar{x}_2=T^u-T^{eq})$. Hence, the system matrices can be expressed as follows:

$$A_{1c}=\begin{bmatrix}-\frac{q}{V}-\psi_1^0-2g_1(\bar{x}_2) & -\psi_2^0\\ \frac{(-\Delta H)}{\rho C_p}\psi_1^0+2\frac{(-\Delta H)}{\rho C_p}g_1(\bar{x}_2) & -\frac{q}{V}-\frac{UA}{V\rho C_p}+\frac{(-\Delta H)}{\rho C_p}\psi_2^0\end{bmatrix},$$

$$A_{2c}=\begin{bmatrix}-\frac{q}{V}-\psi_1^0-2g_1(\underline{x}_2) & -\psi_2^0\\ \frac{(-\Delta H)}{\rho C_p}\psi_1^0+2\frac{(-\Delta H)}{\rho C_p}g_1(\underline{x}_2) & -\frac{q}{V}-\frac{UA}{V\rho C_p}+\frac{(-\Delta H)}{\rho C_p}\psi_2^0\end{bmatrix},$$

$$A_{3c}=\begin{bmatrix}-\frac{q}{V}-\psi_1^0 & -\psi_2^0-2g_2(\bar{x}_2)\\ \frac{(-\Delta H)}{\rho C_p}\psi_1^0 & -\frac{q}{V}-\frac{UA}{V\rho C_p}+\frac{(-\Delta H)}{\rho C_p}\psi_2^0+2\frac{(-\Delta H)}{\rho C_p}g_2(\bar{x}_2)\end{bmatrix},$$

$$A_{4c}=\begin{bmatrix}-\frac{q}{V}-\psi_1^0 & -\psi_2^0-2g_2(\underline{x}_2)\\ \frac{(-\Delta H)}{\rho C_p}\psi_1^0 & -\frac{q}{V}-\frac{UA}{V\rho C_p}+\frac{(-\Delta H)}{\rho C_p}\psi_2^0+2\frac{(-\Delta H)}{\rho C_p}g_2(\underline{x}_2)\end{bmatrix},$$

$$B_{1c}=B_{2c}=B_{3c}=B_{4c}=\begin{bmatrix}0\\ \frac{UA}{V\rho C_p}\end{bmatrix}.$$

Taking the sampling time $T_s=0.05$ min, we can get

$$A_1=\begin{bmatrix}0.8227 & -0.0017\\ 6.1233 & 0.9367\end{bmatrix},\ B_1=\begin{bmatrix}-0.0001\\ 0.1014\end{bmatrix},$$

$$A_2=\begin{bmatrix}0.9654 & -0.0018\\ -0.6759 & 0.9433\end{bmatrix},\ B_2=\begin{bmatrix}-0.0001\\ 0.1016\end{bmatrix},$$

$$A_3=\begin{bmatrix}0.8895 & -0.0029\\ 2.9447 & 0.9968\end{bmatrix},\ B_3=\begin{bmatrix}-0.0002\\ 0.1045\end{bmatrix},$$

$$A_4=\begin{bmatrix}0.8930 & -0.0006\\ 2.7738 & 0.8864\end{bmatrix},\ B_4=\begin{bmatrix}-0.000034\\ 0.0986\end{bmatrix},$$

and $C_1=C_2=C_3=C_4=[0\ \ 1]$, $D_1=D_2=D_3=D_4=[0.0022;0.0564]$, $E_1=E_2=E_3=E_4=0.04$. Moreover, define $\psi_1(x_2)=k_0e^{-\frac{E/R}{x_2+T^{eq}}}$, $\psi_2(x_2)=k_0[e^{-\frac{E/R}{x_2+T^{eq}}}-e^{-\frac{E/R}{T^{eq}}}]C_A^{eq}\frac{1}{x_2}$, $\psi_1^0=[\psi_1(\underline{x}_2)+\psi_1(\bar{x}_2)]/2$, $\psi_2^0=[\psi_2(\underline{x}_2)+\psi_2(\bar{x}_2)]/2$, $g_1(x_2)=\psi_1(x_2)-\psi_1^0$, $g_2(x_2)=\psi_2(x_2)-\psi_2^0$, and $\varphi_1(x_2)=\frac{1}{2}\frac{g_1(x_2)-g_1(\underline{x}_2)}{g_1(\bar{x}_2)-g_1(\underline{x}_2)}$, $\varphi_2(x_2)=\frac{1}{2}\frac{g_1(\bar{x}_2)-g_1(x_2)}{g_1(\bar{x}_2)-g_1(\underline{x}_2)}$, $\varphi_3(x_2)=\frac{1}{2}\frac{g_2(x_2)-g_2(\underline{x}_2)}{g_2(\bar{x}_2)-g_2(\underline{x}_2)}$, $\varphi_4(x_2)=\frac{1}{2}\frac{g_2(\bar{x}_2)-g_2(x_2)}{g_2(\bar{x}_2)-g_2(\underline{x}_2)}$.

Assume that $|u_k|\leq 10$ and $w_k\in[-1,1]$. Set $\mathfrak{L}_1=0.1I_2$, $\mathfrak{L}_2=0.1I_1$, $\mathfrak{L}_3=I_2$, $\mathfrak{R}=0.1$. Choose $\phi(0)=100$, $\lambda_1=0.01$, $\lambda_4=\mu_1=\mu_2=0.03$, $P_w=100$, $\varepsilon=0.9$, $\sigma_0=10$, $\rho=0.053$, $\chi_{\max}=4$. The transition probability matrix is $\mathscr{F}=[0.48\ 0.52;0.54\ 0.46]$. The initial conditions are

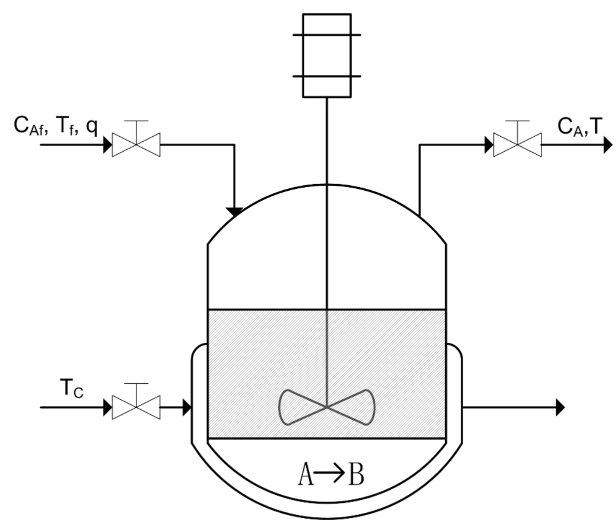

Fig. 2. CSTR schematic.

Table 1. The relevant parameters of CSTR.

Parameters	Descriptions	Values
ρ	Liquid density	1000 g/L
q	Process flow rate	100 L/min
V	Reactor volume	100 L
k_0	Reaction rate constant	7.2×10^{10} min^{-1}
T_f	Actual feed temperature	350 K
E/R	Activation energy	8750 K
C_{Af}	Feed concentration	1 mol/L
UA	Heat transfer coefficient	50000 J/min K
ΔH	Heat of reaction	−120000 J/mol
C_p	Heat capacity	0.239 J/g K

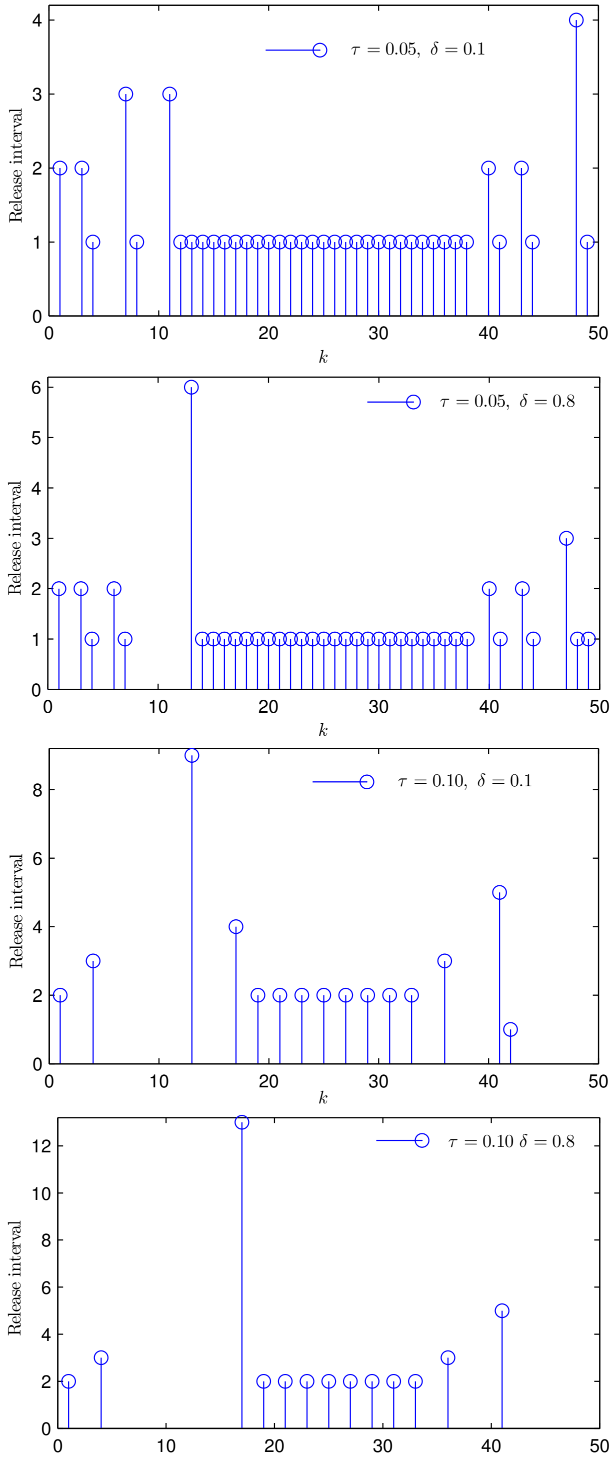

Fig. 3. The event-triggered release time and release interval.

$\hat{x}(0) = [0.2\ 0.5]^{\mathrm{T}}$, $e(0) = [0.2\ 1]^{\mathrm{T}}$ and $u(-1) = 5$.

In order to show how the forgetting factor δ and the event-triggered parameter τ affect the control performance, comparison experiments are performed by taking $\delta = 0.1$ (0.8) and $\tau = 0.05$ (0.10), and the results are shown in Figs. 3-7. The event-triggered release time and release interval are shown in Fig. 3. It is observed that the event-triggered frequency is reduced as event-triggered parameter τ is increased, and then it would naturally bring the reduction of computation resources. The control inputs are shown in Fig. 4, and the input constraint can be satisfied. The state responses of closed-loop with different δ and τ are depicted in Fig. 5. It is obvi-

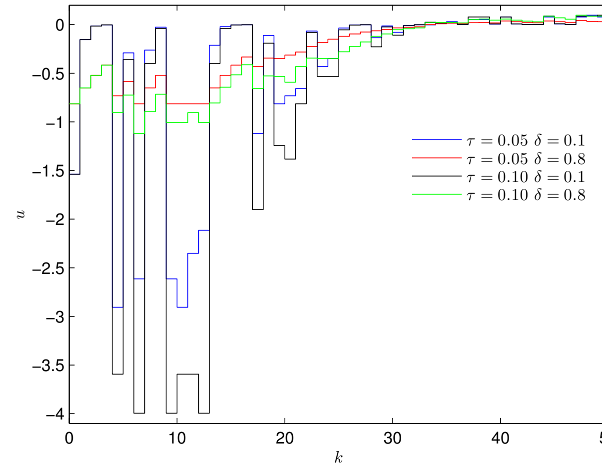

Fig. 4. Control input.

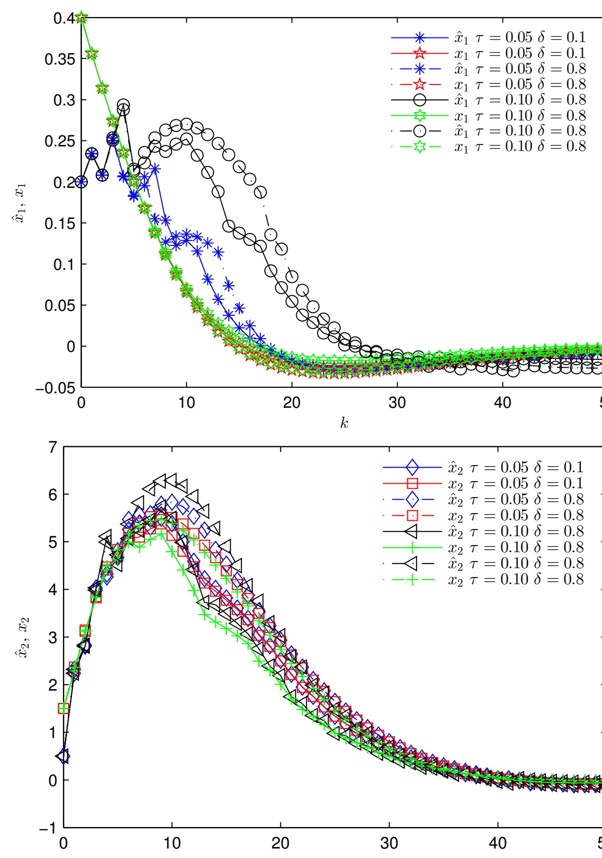

Fig. 5. State responses.

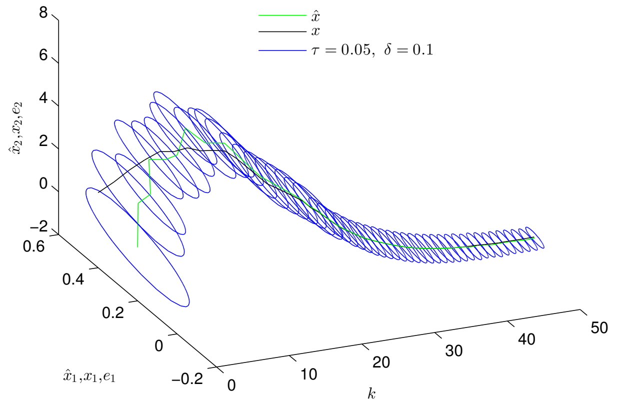

Fig. 6. State responses and error bounds.

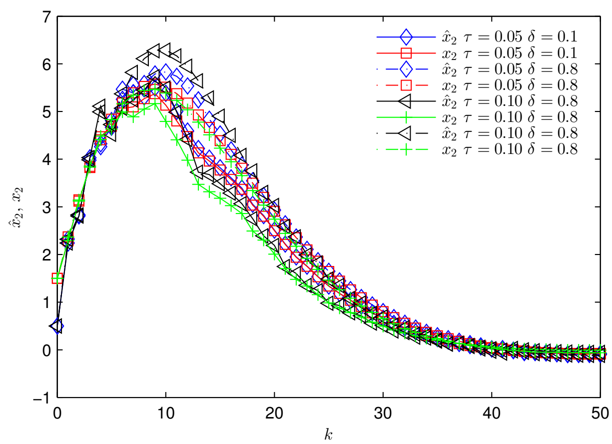

Fig. 7. Evolutions of γ.

ous that the system states will eventually converge to near $x = 0$ by using the proposed event-triggered networked OFRMPC strategy in spite of the occurrences of packet loss. The estimated error bounds are depicted in Fig. 6. It is seen that ϕ_k keeps decreasing as time k goes by until $\Omega_{Q^e} := \{e_k | e_k{}^{\mathrm{T}} Q^e e_k \le \phi_k\}$ becomes the minimal ellipsoidal RPI set. The upper bounds γ of quadratic cost function are depicted in Fig. 7, which is shown that the smaller event-triggered parameter τ is and the smaller forgetting factor δ is, the better control performance can be obtained.

6. CONCLUSION

In this paper, the problem of event-triggered networked OFRMPC for uncertain time-varying discrete-time systems with packet loss and bounded disturbance has been investigated. An event-triggered scheme is introduced to determine whether the current estimated state should be released to controller. Based on an offline designed observer, the output feedback predictive controller has been obtained by minimizing the upper bound of the expected cost function subject to input constraint and packet loss. A technique of refreshing the estimated error bound, which plays the key role of guaranteeing the recursive feasibility of optimization problem, has been provided based on the RPI set constraints.

CONFLICT OF INTERESTS

The author declares that there is no conflict of interest.

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