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Fixed-time Parameter Estimation and Control Design for Unknown Robot Manipulators with Asymmetric Motion Constraints

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Abstract: Most of the existing identification/control algorithm of uncertain robot manipulators have been proposed to achieve model identification and trajectory tracking with expected precision, but the convergence time and transient tracking performance have been rarely discussed. In this paper, an adaptive fixed-time estimation algorithm is proposed for an uncertain robot. A recursive update law combined with an auxiliary filtering technique has been exploited such that the measurement of acceleration signals could be avoided during the estimation process. Based on the results of parameter identification, we propose a fixed-time control scheme which can guarantee the specified motion performance and prescribed convergence time simultaneously. The tiny practical error of parameter identification can be effectively handled with the proposed control scheme. Finally, the simulation results based on an uncertain 2-DOF robot have demonstrated the effectiveness of the proposed identification/control algorithm.

Keywords: Asymmetric motion constraints, fixed-time control, parameter estimation, robot manipulators.

1. INTRODUCTION

Recently, effective control schemes [1,2] have been proposed in the robotics community to guarantee the desired control performance. A relatively comprehensive review involving these control schemes has been presented in [3]. It has been proved that model-based control schemes can guarantee good control performance when the robot model information is available.

However, for general robotic systems, the model information can not often be available precisely because of the unpredictability in dynamics and the unknown disturbance. In real applications, the imprecision of robot models may negatively affect the control performance, which could be one of the factors blocking the widespread application of the model-based control schemes.

In recent years, neural network and fuzzy logic algorithms have been widely used to compensate the uncertain systems [4–6]. An adaptive sliding mode neural network (NN) control method has been proposed for input delay tractor-trailer system with two degrees of freedom in [7]. In [8], a neural network-based robust anti-sway control scheme has been proposed for a crane system transporting an underwater object. It has been proved that the convergence performance can be improved when the estimation error is integrated into the adaptive law. However, long convergence time and weights training process have become the universal drawback for the approximation-based control schemes. Therefore, the efficient parameter identification algorithm for unknown robot has always been the topic that researchers pay attention to.

In the existing identification/control schemes, the convergence rate of the system states have been rarely investigated. In [9], an improved function augmented sliding mode control scheme has been proposed for uncertain nonlinear systems with preassigned settling time. In [10], a novel sliding mode control approach with finite-time convergence has been proposed. In [11], the global finite-time stabilization problem for a class of system with unknown virtual control coefficients has been studied. An adaptive finite-time algorithm has been proposed for wearable exoskeletons to realize trajectory tracking in [12]. However, the settling time in the universal finite-time control algorithm is always heavily related to the initial condition of the systems, which is hard to obtain in practice. In order to make the settling time pre-established by the designer and independent of the initial conditions, Polyakov firstly proposed the definition of fixed-time stability in [13]. An observer-based tracking control scheme in fixed-time was proposed for a second-order system in [14]. In

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[15], a fixed-time parameter estimation scheme has been proposed for a class of linearly parameterized polynomial system. The distributed fixed-time formation-containment problem of unknown multiple Euler-Lagrange systems has been studied under directed graphs in [16]. In [17], a fixed-time attitude control scheme has been proposed for a flexible spacecraft with unknown actuator faults, external disturbance and coupling effect of flexible modes.

In some special practical operations with robots, the transient performance and the output constraint requirement should be taken into account. For example, when the end-effector of the manipulator moves near the surface of the object, or moves in a specific narrow channel, the output of the system should be asymmetrically constrained and the error tolerance in different directions should be different. Besides, in some existing fixed-time control schemes [18–20], the system states converge to some convergence region in fixed-time with Lyapunov analysis. However, the unpredictability of the convergence region and the chattering problems are the universal disadvantages for these control methods. Some researchers have proposed effective schemes to deal with the problem of the system constraints. A kind of control algorithm has been proposed for non-affine multi-agent systems with output constraint in [21]. A new adaptive control strategy has been proposed in [22] for a class of uncertain MIMO nonlinear systems with guaranteed constraints.

The definition of barrier Lyapunov functions (BLFs) has been proposed in recent years, which were originally proposed to achieve the states constraints for nonlinear systems. The BLFs have been widely exploited in the community of nonlinear system control problems such that the system converges without violating the constraint requirements [23–25]. In [23], a full-state constrained control algorithm with adaptive neural network technique has been proposed for wheeled mobile robot. Neural control algorithm of bimanual robots with guaranteed stability and motion performance has been proposed in [24]. In [25], an adaptive control algorithm with fixed-time convergence has been proposed for a class of MIMO nonlinear systems with guaranteed constraints.

Motivated by the work mentioned above, a fixed-time identification scheme has been proposed for an unknown robot in our previous work [26]. This paper is extended from our previous conference paper [26]. Based on the results of parameter identification, we specify the time-varying constraints for the transient and steady state tracking errors. And a fixed-time control scheme has been proposed to guarantee the prescribed motion performance and preestablished convergence time simultaneously. The main contributions of our work in this paper can be listed as follows:

1) An auxiliary filtering technique has been integrated into the fixed-time identification scheme such that the measurement of the acceleration signals, which are sensitive to the interference signals, can be avoided. Moreover, the real-time inversion of square matrix in common identification algorithm can be avoided in the proposed identification scheme.

2) Superior to the universal finite-time control algorithm, the unknown parameters and the tracking errors of the dynamic system can all be convergent in fixed-time regardless of the initial conditions of the system.

3) The BLFs have been integrated into the proposed control scheme such that the prescribed motion performance can be achieved and the output constraints will not be violated.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem formulation

For a general n-DOF (degree of freedom) robot manipulator, the dynamic equation can be described as

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau, \tag{1}$$

where $q\in R^n$, $\dot{q}\in R^n$, and $\ddot{q}\in R^n$ denote respectively the joint position vector, velocity vector, and acceleration vector. The term τ represents the control torque applied to the joints by the controller. $M(q)\in R^{n\times n}$, $C(q,\dot{q})\in R^{n\times n}$, $G(q)\in R^n$ denote the inertia matrix, the Coriolis and centripetal torque matrix and the gravity vector, respectively.

For simplicity, we assume that the mass of the robot in (1) is concentrated at the end of each link. According to the results in [27], the terms $M(q)$, $C(q,\dot{q})$ and $G(q)$ can be expressed as the product of different regression matrix and the parameter vector Θ respectively, with Θ being an unknown parameter vector about the mass and length of the robot links. Then we can express (1) as a linear parameterized system in the following form

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau=\varphi(q,\dot{q},\ddot{q})\Theta, \tag{2}$$

where $\varphi(q,\dot{q},\ddot{q})$ is the dynamic regression matrix containing the terms q, $\dot{q}$, $\ddot{q}$.

The objective of our global control scheme is to identify the unknown parameter vector Θ in fixed-time. Based on the estimation results, the proposed control scheme is supposed to make the joint positions track the desired trajectory in fixed-time regardless of the initial conditions. Besides, the prescribed motion performance should also be guaranteed during the control operation.

2.2. Preliminaries

Property 1 [28]**:** The terms $M(q)$, $C(q,\dot{q})$ and $G(q)$ are bounded by some positive constants M_m, C_m, G_m.

Assumption 1: The first-order derivative of the desired trajectory q_d exists and continuous. Moreover, q_d and $\dot{q}_d$ are both bounded.

Assumption 2 [29,30]**:** All the joint position, velocity, and acceleration are bounded by $\|q\| \leq P_a$, $\|\dot{q}\| \leq P_b$, $\|\ddot{q}\| \leq P_c$ because of the mechanic limitations of the space manipulator where P_a, P_b, P_c are some unknown positive constants.

Definition 1 (Fixed-time stability) [13]**:** For the general system $\dot{x} = g(t,x)$ with initial values x_0, where $x \in R^n$ can be seen as the system variable and $g(t,x)$ can be continuous or discontinuous. The system $\dot{x} = g(t,x)$ is globally fixed-time stable if the convergence time $T \leq T_{max}$ with T_{max} being the upper bound of the convergence time independent of initial values x_0.

Definition 2 (PE condition) [31]**:** A vector or matrix φ_f is defined as "persistently excited" (PE) if there exist a bounded $T > 0, \beta > 0$ such that $\int_t^{t+T} \varphi_f(r)^T \varphi_f(r) dr \geq \beta I$, $\forall t \geq 0$ where I is the identity matrix with the corresponding dimension.

Lemma 1 [20,32]**:** For a positive definite Lyapunov function $V(x)$ which satisfies the inequality like this: $\dot{V}(x) \leq -\alpha_1 V^{a_1}(x) - \beta_1 V^{a_2}(x) + \rho_1$, where α_1, β_1 and ρ_1 are all positive constants and the parameters $a_1 \in (0,1)$ and $a_2 \in (1,\infty)$, the system $\dot{x}(t) = f(x)$ is practical fixed-time convergent. The final convergence domain of the system can be expressed as follows:

$$\Omega_1 = \Big\{x|V \leq \max\Big(\left[\rho_1/(\alpha_1(1-\theta))\right]^{1/a_1}, \left[\rho_1/(\beta_1(1-\theta))\right]^{1/a_2}\Big)\Big\},$$

where θ is a positive constant $\in (0,1)$. The settling time in such a system can be expressed as $T \leq T_{max} = \frac{1}{\alpha_1 \theta (1-a_1)} + \frac{1}{\beta_1 \theta (a_2 - 1)}$.

Lemma 2 (Comparison lemma) [33]**:** Consider the continuous system $\dot{g}_{c1} = f(t, g_{c1})$ with initial value $g_{c1}(t_0)$, where $f(t, g_{c1})$ is locally Lipschitz with respect to g_{c1}. Consider another continuous function $g_{c2}(t)$, if the following inequality exists in the interval $[t_0, T)$ (T could be infinity)

$$D^+ g_{c2}(t) \leq f(t, g_{c2}(t)), g_{c2}(t_0) \leq g_{c1}(t_0) \tag{3}$$

where $D^+ g_{c2}(t)$ represents the upper right-hand derivative of $g_{c2}(t)$. Then, we can conclude that $g_{c2}(t) \leq g_{c1}(t)$ in the interval $[t_0, T)$ if $g_{c1}(t)$ has a solution over $[t_0, T)$.

Lemma 3 [34]**:** For any positive constant $\varepsilon > 0$ and any variable $x \in R$, the inequality exists as $0 \leq |x| - \frac{x^2}{\sqrt{x^2 + \varepsilon^2}} < \varepsilon$.

Lemma 4 [18]**:** For any constant satisfying $0 < m \leq 1$ or $m > 1$, the following inequalities can be respectively obtained

$$\sum_{i=1}^{n} |b_i|^m \geq \left(\sum_{i=1}^{n} |b_i|\right)^m, \quad 0 < m \leq 1, \tag{4}$$

$$\sum_{i=1}^{n} |b_i|^m \geq n^{1-m} \left(\sum_{i=1}^{n} |b_i|\right)^m, \quad m > 1. \tag{5}$$

Remark 1: Inspired by [35], a novel function can be defined for a vector $\mu \in R^n$ as follows:

$$Sig^p(\mu) = [sig^p(\mu_1), \ldots, sig^p(\mu_n)]^T, \tag{6}$$

where μ_i represents the ith term of μ, p can be chosen as a positive constant. The term $sig^p(\mu_i)$ in (6) can be defined as [35]

$$sig^p(\mu_i) = |\mu_i|^p sign(\mu_i), \quad i = 1, .., n, \tag{7}$$

where $sign(\cdot)$ represents the universal signum function.

3. ADAPTIVE PARAMETER ESTIMATION

Follow our previous work in [26], for the parameter identification algorithm proposed below, the estimation strategy is demonstrated in Fig. 1.

As we can see from (2), the matrix φ contains the joint acceleration term which is often unavailable in practice. Inspired by the work in [36], (2) can be rewritten as

$$\dot{J}(q,\dot{q}) + B(q,\dot{q}) = \tau, \tag{8}$$

where $J(q,\dot{q})$ and $B(q,\dot{q})$ can be defined respectively as

$$\begin{cases} J(q,\dot{q}) = M(q)\dot{q}, \\ B(q,\dot{q}) = -\dot{M}(q)\dot{q} + C(q,\dot{q})\dot{q} + G(q), \end{cases} \tag{9}$$

where $\dot{J}(q,\dot{q}) = d[M(q)\dot{q}]/dt$ represents the derivative of $J(q,\dot{q})$ with respect to time. Then the term $B(q,\dot{q})$ in (9) can be divided into two parts as follows:

$$B(q,\dot{q}) = B_1(q,\dot{q}) + B_2(q,\dot{q}), \tag{10}$$

where $B_1(q,\dot{q}) = -\dot{M}(q)\dot{q}$ and $B_2(q,\dot{q}) = C(q,\dot{q})\dot{q} + G(q)$. According to the results in [27], We can express the terms $J(q,\dot{q})$, $B_1(q,\dot{q})$, $B_2(q,\dot{q})$ in linear parametric form with respect to Θ as follows:

$$J(q,\dot{q}) = M(q)\dot{q} = \phi_1(q,\dot{q})\Theta, \tag{11}$$

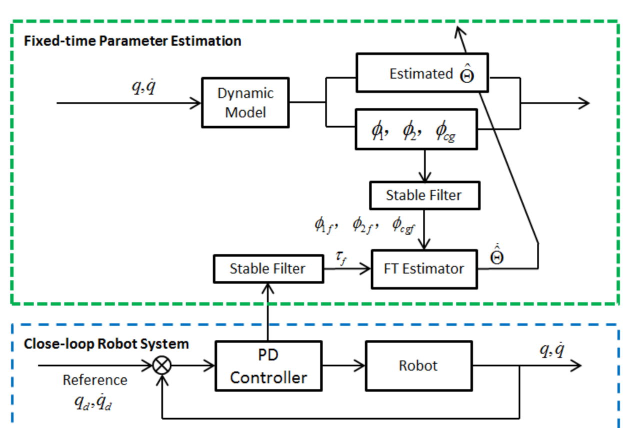

Fig. 1. Fixed-time dynamic estimation strategy of a robot manipulator.

$$B_1(q,\dot{q}) = -\dot{M}(q)\dot{q} = \phi_2(q,\dot{q})\Theta, \tag{12}$$
$$B_2(q,\dot{q}) = C(q,\dot{q})\dot{q} + G(q) = \phi_{cg}(q,\dot{q})\Theta. \tag{13}$$

In order to eliminate the joint acceleration term in (2), we use a stable, regular filter $(\cdot)f = \frac{1}{\eta_1 s+1}$ to filter both sides of (8) with η_1 being a positive filter parameter which can be completed set by the operator. Then we can obtain [37]

$$\begin{aligned} &\eta_1\dot{\phi}_{1f}(q,\dot{q}) + \phi_{1f}(q,\dot{q}) = \phi_1(q,\dot{q}),\ \phi_{1f}|_{t=0} = 0,\\ &\eta_1\dot{\phi}_{2f}(q,\dot{q}) + \phi_{2f}(q,\dot{q}) = \phi_2(q,\dot{q}),\ \phi_{2f}|_{t=0} = 0,\\ &\eta_1\dot{\phi}_{cgf}(q,\dot{q}) + \phi_{cgf}(q,\dot{q}) = \phi_{cg}(q,\dot{q}),\ \phi_{cgf}|_{t=0} = 0, \end{aligned} \tag{14}$$

where ϕ_{1f}, ϕ_{2f}, ϕ_{cgf} are the filtered items corresponding to ϕ_1, ϕ_2, ϕ_{cg} respectively. Then the filtered system can be rewritten as

$$\tau_f = \left[\frac{\phi_1(q,\dot{q}) - \phi_{1f}(q,\dot{q})}{\eta_1} + \phi_{2f}(q,\dot{q}) + \phi_{cgf}(q,\dot{q})\right]\Theta, \tag{15}$$

i.e.,

$$\tau_f = \varphi_f(q,\dot{q})\Theta, \tag{16}$$

where $\varphi_f(q,\dot{q}) \in R^{n\times k}$ can be seen as the filtered regression matrix. $\Theta \in R^k$ is the unknown parameter vector about the physical properties of the links where k is the number of the unknown terms in Θ. So the linear parameterized system of the unknown n-DOF robot without the acceleration term has been shown as (16).

Follow the work in [38], we design the auxiliary filtered terms as follows:

$$\begin{cases} \dot{P} = -\vartheta P + \varphi_f(q,\dot{q})^T\varphi_f(q,\dot{q}), & P(0) = 0,\\ \dot{Q} = -\vartheta Q + \varphi_f(q,\dot{q})^T\tau_f, & Q(0) = 0,\\ D = P\hat{\Theta} - Q, \end{cases} \tag{17}$$

where ϑ is an appropriate positive constant which can act as a forgetting factor. We can see that ϑ can not only guarantee the boundedness of $P \in R^{k\times k}$ and $Q \in R^k$, but also balance the robustness and convergence rate of the system. That $\hat{\Theta}$ is the estimation value of the term Θ. We can see that all the terms in (17) are bounded combined with Assumption 2.

Then we can obtain the expressions of P and Q by integrating two sides of (17) with respect to time:

$$\begin{cases} P(t) = \int_0^t e^{-\vartheta(t-r)}\varphi_f(r)^T\varphi_f(r)dr,\\ Q(t) = \int_0^t e^{-\vartheta(t-r)}\varphi_f(r)^T\tau_f dr. \end{cases} \tag{18}$$

Remark 2: According to Definition 2, we can know that if $\varphi_f(r)$ satisfies the PE condition, the following inequality holds when $t > T > 0$: $\int_{t-T}^t \varphi_f(r)^T\varphi_f(r)dr \geq \beta I$.

Considering the integration interval $r \in [t-T, t]$, we can obtain the following inequality according to the mathematical property of exponential functions

$$e^{-\vartheta(t-r)} \geq e^{-\vartheta T} > 0,\ (t-r \leq T). \tag{19}$$

Further, we can obtain the following results

$$\begin{aligned} &\int_{t-T}^t e^{-\vartheta(t-r)}\varphi_f(r)^T\varphi_f(r)dr\\ &\geq \int_{t-T}^t e^{-\vartheta T}\varphi_f(r)^T\varphi_f(r)dr\\ &\geq e^{-\vartheta T}\beta I. \end{aligned} \tag{20}$$

Combined with the inequality which holds when $t > T > 0$

$$\begin{aligned} &\int_0^t e^{-\vartheta(t-r)}\varphi_f(r)^T\varphi_f(r)dr\\ &> \int_{t-T}^t e^{-\vartheta(t-r)}\varphi_f(r)^T\varphi_f(r)dr, \end{aligned} \tag{21}$$

the following inequality for the matrix P can be obtained

$$\begin{aligned} P &= \int_0^t e^{-\vartheta(t-r)}\varphi_f(r)^T\varphi_f(r)dr\\ &> e^{-\vartheta T}\int_{t-T}^t \varphi_f(r)^T\varphi_f(r)dr\\ &\geq e^{-\vartheta T}\beta I. \end{aligned} \tag{22}$$

From (22) we can obtain the conclusion that when $t > T$, matrix P is full-rank and symmetric positive definite whose minimum eigenvalue satisfies that $\lambda_{min}(P) > \rho$ with $\rho = e^{-\vartheta T}\beta$.

Inspired by [15], in order to identify the unknown parameters in vector Θ in fixed-time, an adaptive updating law for $\hat{\Theta}$ can be proposed as follows:

$$\dot{\hat{\Theta}} = -\Xi P[Sig^{a_1}(P\tilde{\Theta}) + Sig^{a_2}(P\tilde{\Theta})], \tag{23}$$

where Ξ is the appropriately chosen symmetric positive definite matrix. The exponents a_1 and a_2 satisfy that $a_1 \in [0,1)$ and $a_2 > 1$ respectively.

Remark 3: From (18), we can get the result that $Q = P\Theta$. Combined with (17), we can get the expression of the vector D as $D = P\hat{\Theta} - P\Theta = P\tilde{\Theta}$ where $\tilde{\Theta}$ expresses the estimated error vector. Then the term $P\tilde{\Theta}$ in (23) can be expressed by the auxiliary filtered terms in (17) instead of the real values of the parameters.

Based on Definition 1 and updating law in (23), we have the theorem as follows:

Theorem 1: Consider the linear parameterized filtered system (16) with the updating law in (23), if the regression matrix $\varphi_f(q,\dot{q})$ in (16) meets the PE condition, i.e., there exists positive constants T and ρ such that $P \geq \rho I$ where $\rho = e^{-\vartheta T}\beta$ when $t \geq T$, the unknown parameter vector Θ

can be fixed-time convergent whose convergence time can be expressed as

$$t \geq 2T + \frac{2^{\frac{1-a_1}{2}} k^{a_1} \lambda_M^{\frac{1+a_1}{2}}(\Xi^{-1})}{(1-a_1)\rho^{(1+a_1)}} + \frac{2^{\frac{1-a_2}{2}} k^{a_2} \lambda_M^{\frac{1+a_2}{2}}(\Xi^{-1})}{(a_2-1)\rho^{(1+a_2)}}, \tag{24}$$

where $\lambda_M(\Xi^{-1})$ represents the largest eigenvalue of Ξ^{-1}. From (24) we can see that the convergence time will not change despite of the initial conditions of the system, then the system can be regarded as global uniformly fixed-time convergent.

It should be noted that in (23), the terms involved in a_1 and a_2 contribute to the finite time convergence quality and the uniformity involving the initial conditions respectively. Moreover, when $a_1 = a_2 = 1$, the updating law in (23) degenerates into the classical linear gradient descent algorithm. So the updating law we have proposed can be seen as an improvement on the classical identification algorithm by introducing fractional order terms.

Proof: As mentioned before, the terms in Θ are all the constants about the physical properties of the links. So we can derive that

$$\dot{\tilde{\Theta}} = \dot{\hat{\Theta}} = -\Xi P\left[Sig^{a_1}(P\tilde{\Theta}) + Sig^{a_2}(P\tilde{\Theta})\right]. \tag{25}$$

A following candidate Lyapunov function can be defined

$$L(t) = \frac{1}{2}\tilde{\Theta}^T \Xi^{-1}\tilde{\Theta}. \tag{26}$$

Then we take the derivative of L with respect to time, the results can be obtained as follows:

$$\begin{aligned}\dot{L} =& \tilde{\Theta}^T \Xi^{-1}\dot{\tilde{\Theta}}\\ =& \tilde{\Theta}^T \Xi^{-1}\left\{-\Xi P\left[Sig^{a_1}(P\tilde{\Theta}) + Sig^{a_2}(P\tilde{\Theta})\right]\right\}\\ =& -\tilde{\Theta}^T P\left[Sig^{a_1}(P\tilde{\Theta}) + Sig^{a_2}(P\tilde{\Theta})\right].\end{aligned} \tag{27}$$

We assume that $P\tilde{\Theta} = [h_1, h_2, ..., h_k]^T \in R^k$, then we have $\tilde{\Theta}^T P = [h_1, h_2, ..., h_k]$ where the symmetry of the matrix P has been used. Thus we have

$$\begin{aligned}\dot{L} =& -[h_1, h_2, ..., h_k]\Big\{[sig^{a_1}(h_1), ..., sig^{a_1}(h_k)]^T\\ &+ [sig^{a_2}(h_1), ..., sig^{a_2}(h_k)]^T\Big\}.\end{aligned} \tag{28}$$

Combined with (7), we can derive that

$$\dot{L} = -\left[\sum_{i=1}^{k}|h_i|^{1+a_1} + \sum_{i=1}^{k}|h_i|^{1+a_2}\right]. \tag{29}$$

Then the following inequality can be obtained according to Jensen inequality

$$\frac{1}{k}\sum_{i=1}^{k}|h_i|^{1+a_i} \geq \left(\frac{1}{k}\sum_{i=1}^{k}|h_i|\right)^{1+a_i}, \tag{30}$$

i.e.,

$$\sum_{i=1}^{k}|h_i|^{1+a_i} \geq k\frac{1}{k^{1+a_i}}\left(\sum_{i=1}^{k}|h_i|\right)^{1+a_i}. \tag{31}$$

Then, substituting (31) into (29) obtains

$$\dot{L} \leq -\frac{1}{k^{a_1}}\left(\sum_{i=1}^{k}|h_i|\right)^{1+a_1} - \frac{1}{k^{a_2}}\left(\sum_{i=1}^{k}|h_i|\right)^{1+a_2}. \tag{32}$$

From the negative definiteness of $\dot{L}$, we can obtain the global asymptotic stability of $\tilde{\Theta}$. Before we derive the convergence time required for the system, several results are listed as follows:

$$L(t) = \frac{1}{2}\tilde{\Theta}^T \Xi^{-1}\tilde{\Theta} \leq \frac{1}{2}\lambda_M(\Xi^{-1}), \tag{33}$$

$$\dot{L} \leq -\frac{1}{k^{a_i}}\left\|P\tilde{\Theta}\right\|_1^{1+a_i}, \tag{34}$$

$$P \geq \rho I. \tag{35}$$

Moreover, the result that $\rho = e^{-\vartheta T}\beta I$ mentioned above should also be noted. In the derivation of (33) and (34), the knowledge of matrix theory and the conclusion that $\dot{L}$ is less than each term including $a_i, i = 1, 2$ in (32) have been considered.

According to (33), we have

$$L^{\frac{1+a_i}{2}} \leq 2^{-\frac{1+a_i}{2}}\lambda_M^{\frac{1+a_i}{2}}(\Xi^{-1})\left\|\tilde{\Theta}\right\|_2^{1+a_i}. \tag{36}$$

According to (35), we have

$$\left\|P\tilde{\Theta}\right\|_1^{1+a_i} \geq \rho^{1+a_i}\left\|\tilde{\Theta}\right\|_1^{1+a_i} \geq \rho^{1+a_i}\left\|\tilde{\Theta}\right\|_2^{1+a_i}. \tag{37}$$

Then substituting (37) into (34) obtains

$$\dot{L} \leq -\frac{\rho^{1+a_i}}{k^{a_i}}\left\|\tilde{\Theta}\right\|_2^{1+a_i}. \tag{38}$$

Then substituting (36) into (38) obtains

$$\dot{L} \leq -2^{\frac{1+a_i}{2}}\frac{\rho^{1+a_i}}{k^{a_i}\lambda_M^{\frac{1+a_i}{2}}(\Xi^{-1})}L^{\frac{1+a_i}{2}}. \tag{39}$$

When $i = 1$ in (39), we have $\frac{1+a_i}{2} \in [0, 1)$. According to [39], the finite-time uniform stability of the system can be obtained.

From (39) we can obtain

$$\begin{aligned}\dot{L} \leq& -2^{\frac{1+a_i}{2}}\frac{2^{\frac{1-a_i}{2}}\rho^{1+a_i}}{2^{\frac{1-a_i}{2}}k^{a_i}\lambda_M^{\frac{1+a_i}{2}}(\Xi^{-1})}L^{\frac{1+a_i}{2}}\\ \leq& -2\frac{\rho^{1+a_i}}{\sigma_i}L^{\frac{1+a_i}{2}},\end{aligned} \tag{40}$$

where $\sigma_i = 2^{\frac{1-a_i}{2}}k^{a_i}\lambda_M^{\frac{1+a_i}{2}}(\Xi^{-1})$.

Take the upper bounded condition in (40), and applying the integration over $[0,t]$ combined with Lemma 2, we can obtain the inequality as follows:

$$L(t) \leq \left[L^{\frac{1-a_i}{2}}(0) - \frac{1-a_i}{\sigma_i}\rho^{1+a_i}t\right]^{\frac{2}{1-a_i}}. \tag{41}$$

Take $i=1$, we have $\frac{2}{1-a_i} > 2$. Then we have the inequality as follows:

$$\begin{aligned} &L^{\frac{1-a_i}{2}}(0) - \frac{1-a_i}{\sigma_i}\rho^{1+a_i}(t-T) \\ &> L^{\frac{1-a_i}{2}}(0) - \frac{1-a_i}{\sigma_i}\rho^{1+a_i}t. \end{aligned} \tag{42}$$

Then we have

$$\begin{aligned} &\left[L^{\frac{1-a_i}{2}}(0) - \frac{1-a_i}{\sigma_i}\rho^{1+a_i}(t-T)\right]^{\frac{2}{1-a_i}} \\ &> \left[L^{\frac{1-a_i}{2}}(0) - \frac{1-a_i}{\sigma_i}\rho^{1+a_i}t\right]^{\frac{2}{1-a_i}}. \end{aligned} \tag{43}$$

Combined with (41) and (43), we can get the inequality as follows:

$$L(t) \leq \left[L^{\frac{1-a_i}{2}}(0) - \frac{1-a_i}{\sigma_i}\rho^{1+a_i}(t-T)\right]^{\frac{2}{1-a_i}}. \tag{44}$$

Take $i=2$, with the derivation similar to (42) and (43), we can obtain the same inequality as (44).

Now we're going to compute T_{max} in Definition 1 based on (44).

Without loss of generality, the initial value of the Lyapunov function are assumed to be greater than 1. Then T_{max} can be derived in two stages, i.e., $L(t)$ from the initial value to $L(t)=1$ and $L(t)$ from $L(t)=1$ to $L(t)=0$.

Take $i=2$ in (44), we can obtain the time needed in the first stage as follows:

$$T_1 \geq T + \left[1 - L^{\frac{1-a_2}{2}}(0)\right]\frac{\sigma_2}{(a_2-1)\rho^{1+a_2}}. \tag{45}$$

When the initial value $L(0) \to \infty$, the limit of T_1 can be obtained

$$T_1 = T + \frac{\sigma_2}{(a_2-1)\rho^{1+a_2}}. \tag{46}$$

Similarly, take $i=1$ in (44), the time needed in the second stage can be obtained

$$T_2 = T + \frac{\sigma_1}{(1-a_1)\rho^{1+a_1}}. \tag{47}$$

So all the unknown terms in vector Θ can achieve convergence after the two stages in fixed-time for

$$T_c = T_1 + T_2 = 2T + \frac{\sigma_2}{(a_2-1)\rho^{1+a_2}} + \frac{\sigma_1}{(1-a_1)\rho^{1+a_1}}. \tag{48}$$

So far, the result in (48) has proved the correctness of Theorem 1.

Remark 4: In the parameter identification algorithm proposed above, the PD controller with gravity compensation has been used to construct the updating law in (23).

$$\tau = K_P e + K_D \dot{e} + \hat{G}(q), \tag{49}$$

where $e = q_d - q$ and $\dot{e} = \dot{q}_d - \dot{q}$ are the position and velocity tracking error vector respectively. The terms q_d and $\dot{q}_d$ are the reference position and velocity vector respectively. And $\hat{G}(q)$ is the gravity compensation term provided by the estimation results above.

4. CONTROL DESIGN

For the n-DOF robot dynamic model, we reconstruct the dynamic equation in (1) as follows:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2}, \\ \dot{x}_{i,2} = \dfrac{1}{m_{ii}}\tau_i + f_i, \quad i = 1,2\ldots,n, \end{cases} \tag{50}$$

where m_{ii} is the ith element in the main diagonal of $M(q)$ in (1). Moreover, $x_{i,1} = q_i$ is the position of the ith joint, $x_{i,2} = \dot{q}_i$ is the velocity of the ith joint and τ_i is the control torque applied on the ith joint. The values of f_i can be obtained by the expansion equation of (1).

In practice, there are occasionally the parameter estimation errors caused by operator's reading in the identification algorithm above. Then there will be a missing term, i.e., $f_{\Delta i} = f(\ddot{q}_i, \dot{q}_i, q_i, \Delta\Theta)$ compared to the original expression of f_i in (50), where $\Delta\Theta$ is the unknown reading error vector of the parameter vector Θ. Define $\bar{\Theta} = \Theta - \Delta\Theta$ where $\bar{\Theta}$ is the estimated parameter vector obtained by the operator, then the dynamic (50) can be rewritten as follows:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2}, \\ \dot{x}_{i,2} = \dfrac{1}{\bar{m}_{ii}}\tau_i + F_i, \quad i = 1,2,\ldots,n, \end{cases} \tag{51}$$

where $\bar{m}_{ii}$ can be regarded as known because of the obtained parameter vector $\bar{\Theta}$ and F_i is unknown because of the unpredictability of $f_{\Delta i}$. And we can know that the structure of the dynamic equation has not changed from (50) and (51).

Remark 5: By the boundedness of the reading error vector $\Delta\Theta$, Property 1 and Assumption 2, we can obtain the boundedness of F_i, i.e., $|F_i| \leq \bar{F}_i$ where $\bar{F}_i$ can be unknown.

4.1. Requirement for transient and steady state tracking performance

As described in Section 1, in some special operating environment, the allowable transient and steady state tracking performance at different directions could be different, and the required convergence rate in different directions may be different too. In this case, we propose boundary function $\xi_{i,1}$ and $\xi_{i,2}$ to constrain the tracking errors of ith joint position in the opposite directions as follows:

$$\xi_{i,1} = (\rho_{i,01} - \rho_{i,\infty 1})e^{-a_{i,1}t} + \rho_{i,\infty 1}, \tag{52}$$

$$\xi_{i,2} = (\rho_{i,02} - \rho_{i,\infty 2})e^{-a_{i,2}t} + \rho_{i,\infty 2}, \tag{53}$$

where $\rho_{i,0j}$, $\rho_{i,\infty j}$, $a_{i,j}$ $(i = 1, 2, ..., n,\ j = 1, 2)$ are all properly positive constants.

Remark 6: We define the output error for $x_{i,1}$ as $e_{i,1} = x_{i,1} - x_{i,d}$, where $x_{i,d} = q_{di}$ is the desired trajectory of the ith joint. The constraint requirement on the system output is formulated as $-\xi_{i,2} < e_{i,1} < \xi_{i,1}$ where $\xi_{i,2} > 0$ and $\xi_{i,1} > 0$. When $\xi_{i,2} = \xi_{i,1}$, we say that the motion constraint requirement is symmetric. When $\xi_{i,2} \neq \xi_{i,1}$, we say that the motion constraint requirement is asymmetric. For the proposed boundary function in (52) and (53), the terms $a_{i,1}$ and $a_{i,2}$ are different to satisfy different convergence rates in opposite directions for the tracking errors. From the monotonicity of the exponential function, we can know that when $t = 0$ and $t \to \infty$, we can specify the overshoot and the steady-state errors by the appropriate values of $\rho_{i,0j}$, $\rho_{i,\infty j}$. So we can specify the transient and steady state tracking performance by properly choosing parameters $\rho_{i,0j}$, $\rho_{i,\infty j}$, $a_{i,j}$ $(i = 1, 2, ..., n,\ j = 1, 2)$.

4.2. Controller design involving BLF

For the dynamic system in (51), we propose a fixed-time controller to achieve the prescribed tracking performance and convergence time simultaneously. The control strategy is demonstrated in Fig. 2.

To make the tracking errors satisfy the output constraint: $-\xi_{i,2} < e_{i,1} < \xi_{i,1}$, inspired by [25], an asymmetric barrier function is exploited for the ith joint position $x_{i,1}$ as follows:

$$\delta_i = \frac{\xi_{i,1}\xi_{i,2}e_{i,1}}{(\xi_{i,1} - e_{i,1})(\xi_{i,2} + e_{i,1})}, \tag{54}$$

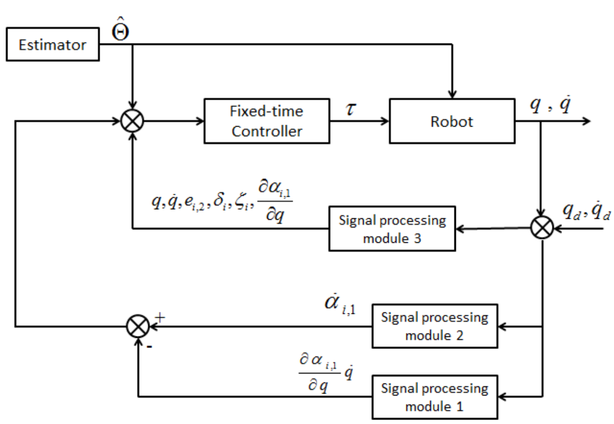

Fig. 2. Fixed-time control strategy of a robot manipulator with guaranteed motion constraints.

where the initial condition of $e_{i,1}$ satisfies that $-\xi_{i,2}(0) < e_{i,1}(0) < \xi_{i,1}(0)$.

We define the candidate Lyapunov function as follows:

$$V_1 = \sum_{i=1}^{n} V_{1,i} = \sum_{i=1}^{n} \frac{1}{2}\delta_i^2. \tag{55}$$

Take the derivative of (54), we can get

$$\dot{\delta}_i = \gamma_i\dot{\xi}_{i,1} + \varepsilon_i\dot{\xi}_{i,2} + \zeta_i\dot{e}_{i,1}, \tag{56}$$

where

$$\gamma_i = \frac{\partial\delta_i}{\partial\xi_{i,1}} = -\frac{\xi_{i,2}e_{i,1}^2}{(\xi_{i,1} - e_{i,1})^2(\xi_{i,2} + e_{i,1})}, \tag{57}$$

$$\varepsilon_i = \frac{\partial\delta_i}{\partial\xi_{i,2}} = \frac{\xi_{i,1}e_{i,1}^2}{(\xi_{i,1} - e_{i,1})(\xi_{i,2} + e_{i,1})^2}, \tag{58}$$

$$\zeta_i = \frac{\partial\delta_i}{\partial e_{i,1}} = \frac{\xi_{i,1}\xi_{i,2}(e_{i,1}^2 + \xi_{i,1}\xi_{i,2})}{(\xi_{i,1} - e_{i,1})^2(\xi_{i,2} + e_{i,1})^2}. \tag{59}$$

We define the variable δ_{Pi} as $\delta_{Pi} = \gamma_i\dot{\xi}_{i,1} + \varepsilon_i\dot{\xi}_{i,2}$, then we have

$$\dot{\delta}_i = \delta_{Pi} + \zeta_i\dot{e}_{i,1}. \tag{60}$$

Take the derivative of $V_{1,i}$, we can get

$$\dot{V}_{1,i} = \dot{\delta}_i\delta_i = \delta_i\delta_{Pi} + \delta_i\zeta_i\dot{e}_{i,1}. \tag{61}$$

Define $e_{i,2} = x_{i,2} - \alpha_{i,1}$ where $\alpha_{i,1}$ is the virtual controller to be designed. Combined with (51), we have

$$\dot{e}_{i,1} = e_{i,2} + \alpha_{i,1} - \dot{x}_{i,d}. \tag{62}$$

Substituting (62) into (61), we have

$$\dot{V}_{1,i} = \delta_i\delta_{Pi} + \delta_i\zeta_i e_{i,2} + \delta_i\zeta_i\alpha_{i,1} - \delta_i\zeta_i\dot{x}_{i,d}. \tag{63}$$

The virtual controller $\alpha_{i,1}$ is designed as follows:

$$\alpha_{i,1} = -\frac{\delta_i\zeta_i|\bar{\alpha}_{i,1}|^2}{\sqrt{\delta_i^2\zeta_i^2|\bar{\alpha}_{i,1}|^2 + \varepsilon_1^2}}, \tag{64}$$

where

$$\bar{\alpha}_{i,1} = \frac{1}{\zeta_i}\breve{\alpha}_{i,1}. \tag{65}$$

We design the intermediate variable as follows:

$$\breve{\alpha}_{i,1} = \delta_{Pi} - \zeta_i\dot{x}_{i,d} + \left(\frac{1}{2}\right)^{\frac{3}{4}} K_{1,1}\frac{\delta_i}{\delta_i^2}U_{\delta_i} + \left(\frac{1}{2}\right)^2 K_{1,2}\delta_i^3, \tag{66}$$

$$U_{\delta_i} = \begin{cases} (\delta_i^2)^{\frac{3}{4}}, & |\delta_i| \geq \varepsilon_{10}, \\ \frac{5}{4}\delta_i^2(\varepsilon_{10}^2)^{-\frac{1}{4}} - \frac{1}{4}\delta_i^4(\varepsilon_{10}^2)^{-\frac{5}{4}}, & |\delta_i| < \varepsilon_{10}, \end{cases} \tag{67}$$

where ε_{10}, $K_{1,1}$, $K_{1,2}$ are the properly chosen positive constants.

Remark 7: The setting of (67) for U_{δ_i} is to make the term including $\frac{\delta_i}{\delta_i^2}U_{\delta_i}$ avoid the singularity in (66) when $e_{i,1}=0$, i.e., $\delta_i=0$. In the controller design below, we need to take the derivative of the term including $\frac{\delta_i}{\delta_i^2}U_{\delta_i}$. We can calculate that the derivative of the term $\frac{\delta_i}{\delta_i^2}U_{\delta_i}$ is continuous along the boundary $|\delta_i|=\varepsilon_{10}$. So we can solve the problem of the singularity and avoid the chattering problem simultaneously.

By (64) we can get that

$$\delta_i\zeta_i\alpha_{i,1}=-\frac{\delta_i^2\zeta_i^2\left|\bar{\alpha}_{i,1}\right|^2}{\sqrt{\delta_i^2\zeta_i^2\left|\bar{\alpha}_{i,1}\right|^2+\varepsilon_1^2}}. \tag{68}$$

By (65) combined with Lemma 3, we can obtain the following inequality

$$\delta_i\zeta_i\alpha_{i,1}<\varepsilon_1-\delta_i\zeta_i\bar{\alpha}_{i,1}<\varepsilon_1-\delta_i\breve{\alpha}_{i,1}. \tag{69}$$

Substituting (69) into (63), we can get that

$$\dot{V}_{1,i}<\delta_i\delta_{Pi}+\delta_i\zeta_ie_{i,2}-\delta_i\breve{\alpha}_{i,1}-\delta_i\zeta_i\dot{x}_{i,d}+\varepsilon_1. \tag{70}$$

When $|\delta_i|\geq\varepsilon_{10}$, we can get that

$$\begin{aligned}\delta_i\breve{\alpha}_{i,1}=&\delta_i\delta_{Pi}-\delta_i\zeta_i\dot{x}_{i,d}+\left(\frac{1}{2}\right)^{\frac{3}{4}}K_{1,1}\left(\delta_i^2\right)^{\frac{3}{4}}\\&+K_{1,2}\left(\frac{1}{2}\delta_i^2\right)^2.\end{aligned} \tag{71}$$

Substituting (71) into (70), we can get that

$$\dot{V}_{1,i}<\delta_i\zeta_ie_{i,2}-K_{1,1}\left(\frac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-K_{1,2}\left(\frac{1}{2}\delta_i^2\right)^2+\varepsilon_1. \tag{72}$$

Remark 8: When $|\delta_i|<\varepsilon_{10}$, there will be an extra term in (72) as $\Delta_{s1}=K_{1,1}\left(\frac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{3}{4}}K_{1,1}\left[\frac{5}{4}\delta_i^2\left(\varepsilon_{10}^2\right)^{-\frac{1}{4}}-\frac{1}{4}\delta_i^4\left(\varepsilon_{10}^2\right)^{-\frac{5}{4}}\right]$. We can calculate that the term Δ_{s1} is bounded by some positive constant $\Delta_{s1}\leq\varepsilon_{11}$ when $|\delta_i|<\varepsilon_{10}$. So we can conclude that the structure of (72) would not change as follows:

$$\dot{V}_{1,i}<\delta_i\zeta_ie_{i,2}-K_{1,1}\left(\frac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-K_{1,2}\left(\frac{1}{2}\delta_i^2\right)^2+C_1, \tag{73}$$

where $C_1=\varepsilon_1$ and $C_1=\varepsilon_1+\varepsilon_{11}$ for both the conditions in (67) respectively. Then the boundness of C_1 can be easily deduced.

By the definition of $e_{i,2}$ and (51), we can get that

$$\dot{e}_{i,2}=\frac{1}{\bar{m}_{ii}}\tau_i+F_i-\dot{\alpha}_{i,1}. \tag{74}$$

By the definition of $\alpha_{i,1}$ in (64), (65) and (66), we can get that

$$\begin{aligned}\dot{\alpha}_{i,1}=&\sum_{j=1}^{2}\frac{\partial\alpha_{i,1}}{\partial x_{i,d}^{(j-1)}}x_{i,d}^{(j)}+\sum_{j=1}^{2}\frac{\partial\alpha_{i,1}}{\partial\xi_{i,2}^{(j-1)}}\xi_{i,2}^{(j)}\\&+\sum_{j=1}^{2}\frac{\partial\alpha_{i,1}}{\partial\xi_{i,1}^{(j-1)}}\xi_{i,1}^{(j)}+\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2}.\end{aligned} \tag{75}$$

Let local Lyapunov function $V_{2,i}=\frac{1}{2}e_{i,2}^2$, then from (74) and (75) we have

$$\begin{aligned}\dot{V}_{2,i}=&\dot{e}_{i,2}e_{i,2}\\=&e_{i,2}\left[\frac{1}{\bar{m}_{ii}}\tau_i+F_i+\psi_{i,2}-\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2}\right],\end{aligned} \tag{76}$$

where

$$\begin{aligned}\psi_{i,2}=&-\sum_{j=1}^{2}\frac{\partial\alpha_{i,1}}{\partial x_{i,d}^{(j-1)}}x_{i,d}^{(j)}-\sum_{j=1}^{2}\frac{\partial\alpha_{i,1}}{\partial\xi_{i,2}^{(j-1)}}\xi_{i,2}^{(j)}\\&-\sum_{j=1}^{2}\frac{\partial\alpha_{i,1}}{\partial\xi_{i,1}^{(j-1)}}\xi_{i,1}^{(j)}.\end{aligned}$$

Then the global Lyapunov function at this step can be designed as follows:

$$V=V_1+V_2=\sum_{i=1}^{n}(V_{1,i}+V_{2,i})=\sum_{i=1}^{n}\left(\frac{1}{2}\delta_i^2+\frac{1}{2}e_{i,2}^2\right). \tag{77}$$

From (73) and (76), we have

$$\begin{aligned}\dot{V}<&\sum_{i=1}^{n}\left[\delta_i\zeta_ie_{i,2}-K_{1,1}\left(\frac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-K_{1,2}\left(\frac{1}{2}\delta_i^2\right)^2+C_1\right.\\&\left.+e_{i,2}\frac{1}{\bar{m}_{ii}}\tau_i+e_{i,2}F_i+e_{i,2}\psi_{i,2}-e_{i,2}\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2}\right].\end{aligned} \tag{78}$$

For the items in (78), we have the following inequality

$$\begin{aligned}&\delta_i\zeta_ie_{i,2}-e_{i,2}\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2}\\&le\left|\delta_i\zeta_i\right|\left|e_{i,2}\right|+\left|e_{i,2}\right|\left|\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right|\left|x_{i,2}\right|.\end{aligned} \tag{79}$$

Combined with Lemma 3, we can get that

$$\begin{aligned}&\delta_i\zeta_ie_{i,2}-e_{i,2}\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2}\\&<2\varepsilon_2+\frac{\left|\delta_i\zeta_i\right|^2\left|e_{i,2}\right|^2}{\sqrt{\left|\delta_i\zeta_i\right|^2\left|e_{i,2}\right|^2+\varepsilon_2^2}}\\&\quad+\frac{\left|e_{i,2}\right|^2\left|\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right|^2\left|x_{i,2}\right|^2}{\sqrt{\left|e_{i,2}\right|^2\left|\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right|^2\left|x_{i,2}\right|^2+\varepsilon_2^2}}.\end{aligned} \tag{80}$$

Similarly with Remark 5, we have

$$e_{i,2}F_i \le |e_{i,2}|\,|F_i| < \bar{F}_i\left(\varepsilon_2 + \frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}\right). \tag{81}$$

Substituting (80) and (81) into (78), we have

$$\begin{aligned}\dot{V} <\sum_{i=1}^{n}\Big[&-K_{1,1}\left(\tfrac{1}{2}\delta_i^2\right)^{\frac{3}{4}} - K_{1,2}\left(\tfrac{1}{2}\delta_i^2\right)^2 - K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}\\ &-K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2 + K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}} + K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2\\ &+2\varepsilon_2 + \frac{(\delta_i\zeta_i)^2e_{i,2}^2}{\sqrt{(\delta_i\zeta_i)^2e_{i,2}^2+\varepsilon_2^2}} + \frac{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2}{\sqrt{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2+\varepsilon_2^2}}\\ &+\bar{F}_i\varepsilon_2 + \bar{F}_i\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}} + e_{i,2}\psi_{i,2} + e_{i,2}\frac{1}{\bar{m}_{ii}}\tau_i + C_1\Big],\end{aligned} \tag{82}$$

where $K_{2,1}$, $K_{2,2}$ are the properly chosen positive constants.

We design the global controller as

$$\tau_i = -\bar{m}_{ii}\frac{e_{i,2}\bar{\alpha}_{i,2}^2}{\sqrt{e_{i,2}^2\bar{\alpha}_{i,2}^2+\varepsilon_2^2}}, \tag{83}$$

where

$$\begin{aligned}\bar{\alpha}_{i,2} = &\left(\frac{1}{2}\right)^{\frac{3}{4}}K_{2,1}\frac{e_{i,2}}{e_{i,2}^2}U_{e_{i,2}} + \left(\frac{1}{2}\right)^2K_{2,2}e_{i,2}^3\\ &+\psi_{i,2} + \frac{\chi_ie_{i,2}\|\omega_{i,2}\|^2}{\sqrt{e_{i,2}^2\|\omega_{i,2}\|^2+\varepsilon_2^2}}.\end{aligned} \tag{84}$$

Design the function

$$\begin{aligned}\omega_{i,2} = \Bigg[&\frac{(\delta_i\zeta_i)^2e_{i,2}}{\sqrt{(\delta_i\zeta_i)^2e_{i,2}^2+\varepsilon_2^2}} + \frac{e_{i,2}\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2}{\sqrt{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2+\varepsilon_2^2}},\\ &\frac{e_{i,2}}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}\Bigg]^T,\end{aligned} \tag{85}$$

and

$$\chi_i = \|\Phi_i\| = \left\|[1,|F_{ei}|]^T\right\|, \tag{86}$$

where F_{ei} is the arbitrary designed bounded function with $|F_{ei}| \le \bar{F}_{ei}$. So we can conclude the boundness of χ_i as $|\chi_i| \le \bar{\chi}_i$.

With the similar design to (67), we design function $U_{e_{i,2}}$ as

$$U_{e_{i,2}} = \begin{cases}\left(e_{i,2}^2\right)^{\frac{3}{4}}, & |e_{i,2}| \ge \varepsilon_{20},\\ \dfrac{5}{4}e_{i,2}^2\left(\varepsilon_{20}^2\right)^{-\frac{1}{4}} - \dfrac{1}{4}e_{i,2}^4\left(\varepsilon_{20}^2\right)^{-\frac{5}{4}}, & |e_{i,2}| < \varepsilon_{20},\end{cases} \tag{87}$$

where ε_{20} is a properly chosen small positive constant.

Remark 9: Similar to (67), the setting of (87) for $U_{e_{i,2}}$ is to make the term including $\frac{e_{i,2}}{e_{i,2}^2}U_{e_{i,2}}$ avoid the singularity in (84) when $e_{i,2} = 0$. We can calculate that the derivative of the term $\frac{e_{i,2}}{e_{i,2}^2}U_{e_{i,2}}$ is continuous along the boundary $|e_{i,2}| = \varepsilon_{20}$. So we can solve the problem of the singularity and avoid the chattering problem simultaneously.

Substituting (83) into (82), combined with Lemma 3, for the term $e_{i,2}\frac{1}{\bar{m}_{ii}}\tau_i$ in (82) we have

$$\begin{aligned}e_{i,2}\frac{1}{\bar{m}_{ii}}\tau_i &= -\frac{e_{i,2}^2\bar{\alpha}_{i,2}^2}{\sqrt{e_{i,2}^2\bar{\alpha}_{i,2}^2+\varepsilon_2^2}}\\ &< \varepsilon_2 - e_{i,2}\bar{\alpha}_{i,2}.\end{aligned} \tag{88}$$

Substituting (84) into (88), we have

$$\begin{aligned}e_{i,2}\frac{1}{\bar{m}_{ii}}\tau_i <&\varepsilon_2 - \left(\frac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}} - \left(\frac{1}{2}\right)^2K_{2,2}e_{i,2}^4\\ &- e_{i,2}\psi_{i,2} - \frac{\chi_ie_{i,2}^2\|\omega_{i,2}\|^2}{\sqrt{e_{i,2}^2\|\omega_{i,2}\|^2+\varepsilon_2^2}}.\end{aligned} \tag{89}$$

By Lemma 3, we can get

$$-\frac{\chi_ie_{i,2}^2\|\omega_{i,2}\|^2}{\sqrt{e_{i,2}^2\|\omega_{i,2}\|^2+\varepsilon_2^2}} < \chi_i\left(\varepsilon_2 - |e_{i,2}|\,\|\omega_{i,2}\|\right). \tag{90}$$

Substituting (89) and (90) into (82), we have

$$\begin{aligned}\dot{V} <\sum_{i=1}^{n}\Big[&-K_{1,1}\left(\tfrac{1}{2}\delta_i^2\right)^{\frac{3}{4}} - K_{1,2}\left(\tfrac{1}{2}\delta_i^2\right)^2 - K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}\\ &-K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2 + 3\varepsilon_2 + \bar{F}_i\varepsilon_2 + C_1 + \chi_i\varepsilon_2\\ &+K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}} + K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2 + \frac{(\delta_i\zeta_i)^2e_{i,2}^2}{\sqrt{(\delta_i\zeta_i)^2e_{i,2}^2+\varepsilon_2^2}}\\ &+\frac{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2}{\sqrt{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2+\varepsilon_2^2}} + \bar{F}_i\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}} + e_{i,2}\psi_{i,2}\\ &-\left(\tfrac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}} - \left(\tfrac{1}{2}\right)^2K_{2,2}e_{i,2}^4 - e_{i,2}\psi_{i,2}\\ &-\chi_i|e_{i,2}|\,\|\omega_{i,2}\|\Big].\end{aligned} \tag{91}$$

By the definition in (86), we have

$$-\chi_i|e_{i,2}|\,\|\omega_{i,2}\| \le -\left|e_{i,2}\Phi_i^T\omega_{i,2}\right|. \tag{92}$$

Substituting (92) into (91), we have

$$\begin{aligned}\dot{V} <\sum_{i=1}^{n}\Big[&-K_{1,1}\left(\tfrac{1}{2}\delta_i^2\right)^{\frac{3}{4}} - K_{1,2}\left(\tfrac{1}{2}\delta_i^2\right)^2 - K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}\\ &-K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2 + 3\varepsilon_2 + \bar{F}_i\varepsilon_2 + C_1 + \chi_i\varepsilon_2\\ &+K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}} + \frac{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2}{\sqrt{e_{i,2}^2\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2x_{i,2}^2+\varepsilon_2^2}}\end{aligned}$$

$$+\frac{(\delta_i\zeta_i)^2 e_{i,2}^2}{\sqrt{(\delta_i\zeta_i)^2 e_{i,2}^2+\varepsilon_2^2}}+\bar{F}_i\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}-\left(\tfrac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}} - \left|e_{i,2}\Phi_i^T\omega_{i,2}\right|\Big]. \quad (93)$$

Substituting (85) and (86) into (91) and omit the absolute value sign of the term $\left|e_{i,2}\Phi_i^T\omega_{i,2}\right|$, we have

$$\begin{aligned}\dot{V} <\sum_{i=1}^{n}\Big[&-K_{1,1}\left(\tfrac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-K_{1,2}\left(\tfrac{1}{2}\delta_i^2\right)^2-K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}\\ &-K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2+3\varepsilon_2+\bar{F}_i\varepsilon_2+C_1+\chi_i\varepsilon_2\\ &+K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}+\bar{F}_i\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}-\left(\tfrac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}}\\ &-\bar{F}_{ei}\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}\Big]. \end{aligned}\quad (94)$$

From (87), when $|e_{i,2}|\geq\varepsilon_{20}$, we can conclude in (94) that

$$K_{2,1}\left(\frac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}}=0. \quad (95)$$

With the similar derivation to Remark 8, when $|e_{i,2}|<\varepsilon_{20}$, we can conclude in (94) that

$$K_{2,1}\left(\frac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}}=\Delta_{s2}\leq\varepsilon_{21}, \quad (96)$$

where ε_{21} is a positive constant.

Then we can rewrite (94) as follows:

$$\begin{aligned}\dot{V} <\sum_{i=1}^{n}\Big[&-K_{1,1}\left(\tfrac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-K_{1,2}\left(\tfrac{1}{2}\delta_i^2\right)^2-K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}\\ &-K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2+3\varepsilon_2+\bar{F}_i\varepsilon_2+C_1+\bar{\chi}_i\varepsilon_2\\ &+K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}+\left(\bar{F}_i-\bar{F}_{ei}\right)\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}\\ &-\left(\tfrac{1}{2}\right)^{\frac{3}{4}}K_{2,1}U_{e_{i,2}}\Big]. \end{aligned}\quad (97)$$

Remark 10: For the term $\left(\bar{F}_i-\bar{F}_{ei}\right)\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}$ in (97), we can easily deduce the boundness of $e_{i,2}$ when $e_{i,1}$ satisfies the condition $-\xi_{i,2}<e_{i,1}<\xi_{i,1}$. The detailed derivation will be discussed later. Combined with the boundness of $\bar{F}_i$, $\bar{F}_{ei}$ and ε_2, we can conclude the boundness of the term $\left(\bar{F}_i-\bar{F}_{ei}\right)\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}$, i.e., $\left(\bar{F}_i-\bar{F}_{ei}\right)\frac{e_{i,2}^2}{\sqrt{e_{i,2}^2+\varepsilon_2^2}}\leq\varkappa_i$ where $\varkappa_i$ is a positive constant.

Then we can rewrite (97) as follows:

$$\begin{aligned}\dot{V} <\sum_{i=1}^{n}\Big[&-K_{1,1}\left(\tfrac{1}{2}\delta_i^2\right)^{\frac{3}{4}}-K_{1,2}\left(\tfrac{1}{2}\delta_i^2\right)^2-K_{2,1}\left(\tfrac{1}{2}e_{i,2}^2\right)^{\frac{3}{4}}\\ &-K_{2,2}\left(\tfrac{1}{2}e_{i,2}^2\right)^2+C_2\Big], \end{aligned}\quad (98)$$

where C_2 can be expressed as

$$C_2=3\varepsilon_2+\bar{F}_i\varepsilon_2+C_1+\bar{\chi}_i\varepsilon_2+\varkappa_i,\quad |e_{i,2}|\geq\varepsilon_{20}, \quad (99)$$

and

$$C_2=3\varepsilon_2+\bar{F}_i\varepsilon_2+C_1+\bar{\chi}_i\varepsilon_2+\varkappa_i+\varepsilon_{21},\quad |e_{i,2}|<\varepsilon_{20}. \quad (100)$$

We define the variable as follows:

$$\iota_1=\min\left(K_{1,1},K_{2,1}\right), \quad (101)$$

$$\iota_2=\min\left(K_{1,2},K_{2,2}\right). \quad (102)$$

By Lemma 4 and (77),we can rewrite (98) as follows:

$$\dot{V}<\sum_{i=1}^{n}\left[-\iota_1\bar{V}_i^{\frac{3}{4}}-\frac{\iota_2}{2}\bar{V}_i^2+C_2\right], \quad (103)$$

where $\bar{V}_i=\frac{1}{2}\delta_i^2+\frac{1}{2}e_{i,2}^2$.

In a similar way, by Lemma 4, we have

$$\dot{V}<-\iota_1V^{\frac{3}{4}}-\frac{\iota_2}{2n}V^2+nC_2. \quad (104)$$

Then we will prove that the BLF we exploited can guarantee the prescribed tracking performance and make the steady-state errors constrained completely by the designer, not by the convergence domain in Lemma 1.

With the similar process in parameter identification, we can prove the boundedness of $V(t)$ in two cases: $V(t)$ from the initial $V(0)$ to $V(t)=1$ and from $V(t)=1$ to $V(t)=0$.

Similarly, the initial value of the Lyapunov function $V(0)$ is assumed to be greater than 1. Then we can conclude the inequality existing from $V(0)$ to $V(t)=1$.

$$\dot{V}<-\iota_1V^{\frac{3}{4}}-\frac{\iota_2}{2n}V^2+nC_2<-\frac{\iota_2}{2n}V+nC_2. \quad (105)$$

Multiplying both sides by $e^{\frac{\iota_2}{2n}t}$ in (105), and then integrating both sides over $[0,t]$ in (105), we have

$$\begin{aligned}V(t)&<\left(V(0)-\frac{2n^2C_2}{\iota_2}\right)e^{-\frac{\iota_2}{2n}t}+\frac{2n^2C_2}{\iota_2}\\ &<V(0)+\frac{2n^2C_2}{\iota_2}.\end{aligned} \quad (106)$$

For the condition from $V(t)=1$ to $V(t)<1$, we have

$$\dot{V}<-\iota_1V^{\frac{3}{4}}-\frac{\iota_2}{2n}V^2+nC_2<-\iota_1V+nC_2. \quad (107)$$

With the similar process to (106), we have

$$V(t)<\left(V(0)-\frac{nC_2}{\iota_1}\right)e^{-\iota_1t}+\frac{nC_2}{\iota_1}<V(0)+\frac{nC_2}{\iota_1}. \quad (108)$$

By (106) and (108), we can obtain the boundness of the Lyapunov function $V(t)$. From the initial condition of $e_{i,1}(0)$ combined with Remark 9, the reasonability of the boundness derivation of $V(t)$ above can be guaranteed. From the definition in (77) and the boundness of $V(t)$, the boundedness of δ_i and $e_{i,2}$ can be concluded. So the tracking constraint of $-\xi_{i,2} < e_{i,1} < \xi_{i,1}$ will not be violated during operation according to the continuity of $e_{i,1}$. So the tracking performance can be prescribed by (52) and (53) instead of the uncertain convergence domain in Lemma 1.

Then from (57), (58) and (59) we can conclude the boundness of γ_i, ε_i, and ζ_i. Then the boundness of δ_{Pi} can be inferred. Then we can conclude that $\breve{\alpha}_{i,1}$, $\bar{\alpha}_{i,1}$, and $\alpha_{i,1}$ are all bounded by their expressions. By calculation, $\psi_{i,2}$ and $\frac{\partial \alpha_{i,1}}{\partial x_{i,1}}$ are continuous functions with bounded elements. Then we can easily conclude that $\bar{\alpha}_{i,2}$ is also bounded. Finally, the boundedness of the controller signal τ_i can also be implied.

From (104) we can know that the inequality we concluded conforms to the inequality structure in Lemma 1, according to which we can express the settling time in the system as follows:

$$T_g = \frac{4}{\iota_1 \varpi} + \frac{2n}{\iota_2 \varpi}, \tag{109}$$

where ϖ is a positive constant which satisfies $\varpi \in (0,1)$.

5. SIMULATION STUDY

For the sake of simplicity, the 2-DOF robot will be discussed to verify the effectiveness and superiority of the proposed identification/control scheme. In this section, the identification results for the unknown dynamic parameters of the 2-DOF robot in Fig. 3 will be shown, and the tracking performance with the guaranteed motion constraints under the proposed controller will be displayed.

5.1. Simulation setup

For the 2-DOF robot in Fig. 3, we can define the unknown parameters in Θ as follows:

$$\begin{aligned} \theta_1 &= m_2 l_2^2 + (m_1+m_2) l_l^2, \\ \theta_2 &= m_2 l_1 l_2, \\ \theta_3 &= m_2 l_2^2, \\ \theta_4 &= m_2 l_2, \\ \theta_5 &= (m_1+m_2) l_1. \end{aligned} \tag{110}$$

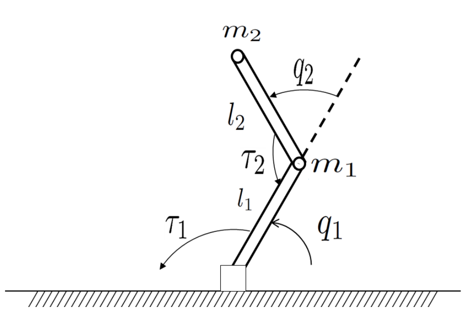

Fig. 3. 2-DOF robot system with the mass concentrated at the end of the links [26].

That $M(q)$, $C(q,\dot{q})$ and $G(q)$ in (1) can be defined respectively as follows:

$$\begin{aligned} M(q) &= \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} \theta_1 + 2c_2\theta_2 & \theta_3 + c_2\theta_2 \\ \theta_3 + c_2\theta_2 & \theta_3 \end{bmatrix}, \\ C(q,\dot{q}) &= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} -2\theta_2 s_2 \dot{q}_2 & -\theta_2 s_2 \dot{q}_2 \\ \theta_2 s_2 \dot{q}_1 & 0 \end{bmatrix}, \\ G(q) &= \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} g c_{s12}\theta_4 + g c_1 \theta_5 \\ g c_{s12} \theta_4 \end{bmatrix}, \end{aligned} \tag{111}$$

where m_i and l_i, $(i = 1, 2)$ are the mass and the length of the ith link respectively. The term $\Theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T \in R^{k\times 1}$, $k = 5$ is the unknown constant vector. That c_i and s_i mean $\cos(q_i)$ and $\sin(q_i)$, $(i = 1, 2)$, rspectively. And c_{s12} means $\cos(q_1+q_2)$.

The nominal values of m_i and l_i, $(i = 1, 2)$ in (110) are set to $m_1 = 4$ kg, $m_2 = 2$ kg, $l_1 = l_2 = 0.5$ m. Then all the unknown parameters values in Θ can be calculated as $\Theta = [2,\ 0.5,\ 0.5,\ 1,\ 3]^T$. The gain η_1 in (14) is set to $\eta_1 = 0.001$. We set the forgetting factor in (17) as $\vartheta = 1$. For the updating law in (23), we set the gain matrix as $\Xi = diag[5,\ 5,\ 11,\ 10,\ 5]$ and the fractional exponents are selected as $a_1 = 0.75$ and $a_2 = 1.2$. The reference trajectories to identify the unknown parameters are set as $q_{d1} = 2\sin(0.5t) + \cos(0.1t+0.3)$ and $q_{d2} = 3\sin(t+0.5)$. And all the initial values in Θ are set to zero. Moreover, the appropriate PD parameters in Remark 4 are chosen as $K_P = 3000$ and $K_D = 150$.

For the controller design, the desired joint position trajectories for the robot manipulator are set as $x_{1,d} = \sin(0.5t) + 0.5$ and $x_{2,d} = \cos(0.8t) + 1$.

For the 2-DOF robot system in Fig. 3, the term f_i in (50) can be expressed as follows:

$$\begin{cases} f_1 = -(1/m_{11})(m_{12}\ddot{q}_2 + c_{11}\dot{q}_1 + c_{12}\dot{q}_2 + g_1), \\ f_2 = -(1/m_{22})(m_{21}\ddot{q}_1 + c_{21}\dot{q}_1 + c_{22}\dot{q}_2 + g_2). \end{cases} \tag{112}$$

Moreover, the term F_i in (51) can be expressed as follows:

$$\begin{cases} F_1 = -(1/\bar{m}_{11})(\bar{m}_{12}\ddot{q}_2 + \bar{c}_{11}\dot{q}_1 + \bar{c}_{12}\dot{q}_2 + \bar{g}_1 + f_{\Delta 1}), \\ F_2 = -(1/\bar{m}_{22})(\bar{m}_{21}\ddot{q}_1 + \bar{c}_{21}\dot{q}_1 + \bar{c}_{22}\dot{q}_2 + \bar{g}_2 + f_{\Delta 2}). \end{cases} \tag{113}$$

The term $\bar{m}_{ij}$, $\bar{c}_{ij}$, $\bar{g}_i$, $i,\ j = 1, 2$ can be all regarded as known because of the obtained parameter vector $\bar{\Theta}$. The unknown term $f_{\Delta i}$ in F_i can be expressed as

$$\begin{cases} f_{\Delta 1} = (\Delta\theta_1 + 2c_2\Delta\theta_2)\ddot{q}_1 + (\Delta\theta_3 + c_2\Delta\theta_2)\ddot{q}_2 \\ \qquad - 2s_2\dot{q}_1\dot{q}_2\Delta\theta_2 - s_2\dot{q}_2\dot{q}_2\Delta\theta_2 + g c_{s12}\Delta\theta_4 \\ \qquad + g c_1 \Delta\theta_5, \\ f_{\Delta 2} = (\Delta\theta_3 + c_2\Delta\theta_2)\ddot{q}_1 + \Delta\theta_3\ddot{q}_2 + s_2\dot{q}_1\dot{q}_1\Delta\theta_2 \\ \qquad + g c_{s12}\Delta\theta_4. \end{cases} \tag{114}$$

The constraint functions on the joint position tracking errors are set as

$$\xi_{1,2} = \xi_{2,2} = (0.5 - 0.05)e^{-0.5t} + 0.05, \tag{115}$$

$$\xi_{1,1} = \xi_{2,1} = (0.4 - 0.02)e^{-1.5t} + 0.02. \tag{116}$$

We can see that the tracking performance should be better in terms of convergence rate and steady-state error in the positive direction than that in the negative direction. The parameters in the controller design are set as $K_{1,1} = K_{1,2} = K_{2,1} = K_{2,2} = 70$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_{10} = \varepsilon_{20} = 0.01$. The initial conditions are set as$x_{1,1}(0) = 0.3$, $x_{2,1}(0) = 1.8$, $x_{1,2}(0) = x_{2,2}(0) = 0$. And the unknown reading error of the parameter vector $\Delta\Theta$ is set as $\Delta\Theta = [0.05, 0.05, 0.05, 0.05, 0.05]^T$ such that the estimated parameter vector from the identification process is set as $\bar{\Theta} = [1.95, 0.45, 0.45, 0.95, 2.95]^T$. The bounded function F_{ei} in (86) is designed as $F_{ei} = \sin(t)$.

To further verify the effectiveness and superiority of the proposed algorithm, comparison studies have been carried out based on the fixed-time backstepping controller in [20] and the fixed-time observer in [30], i.e., the controller in [20] with the observer in [30] has been applied to the same 2-DOF robot model in the paper. The unknown term F_i and the desired trajectory are respectively set the same as the corresponding term in our model.

Remark 11: As for the parameter identification, from (48) we can know that, when a_2 and Ξ are chosen relatively large while a_1 is chosen relatively small, shorter settling time can be obtained. So relatively large a_2, Ξ and relatively small a_1 should be chosen to guarantee better identification performance in the allowable conditions. From (104) we can know that when $K_{1,1}$, $K_{2,1}$, $K_{1,2}$, $K_{2,2}$ are chosen relatively large, the terms ι_1 and ι_2 will be made larger. And in terms of (99), (100) and Remark 8, if ε_1, ε_2, ε_{10}, ε_{20} are chosen relatively large, the term C_2 will be made larger. From (106) and (108) we can know that larger ι_1, ι_2 and smaller C_2 lead to faster convergence rate and smaller steady state errors. So relatively large $K_{1,1}$, $K_{2,1}$, $K_{1,2}$, $K_{2,2}$ and relatively small ε_1, ε_2, ε_{10}, ε_{20} should be chosen to guarantee better control performance in the allowable conditions.

5.2. Simulation results

The simulation results with the proposed schemes and the comparative schemes have been shown as Figs. 4-6. From Fig. 4 we can see that all parameters in Θ can achieve precise convergence very quickly with the proposed identification scheme. Compared with the finite-time identification algorithm in [38], where the required convergence time is positively related to the initial Lyapunov function value $V(0)$, the required convergence time for parameter identification will not change regardless of the initial conditions of the system in this paper. Fig. 5 represents the observation performance of the unknown F_i

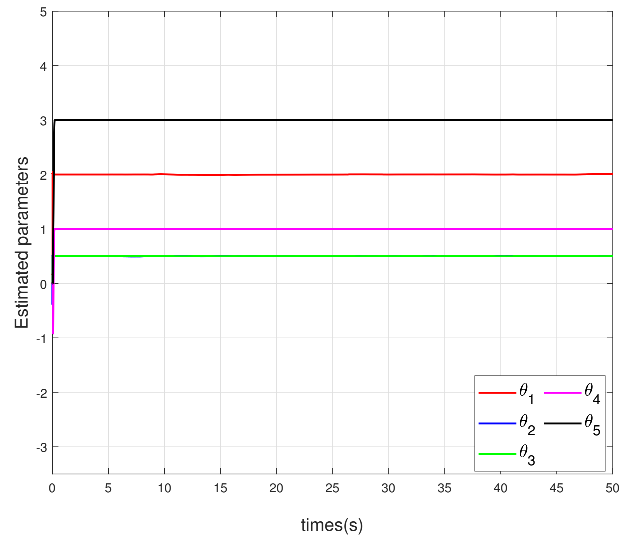

Fig. 4. Convergence of the estimated parameters in fixed-time with the proposed identification scheme.

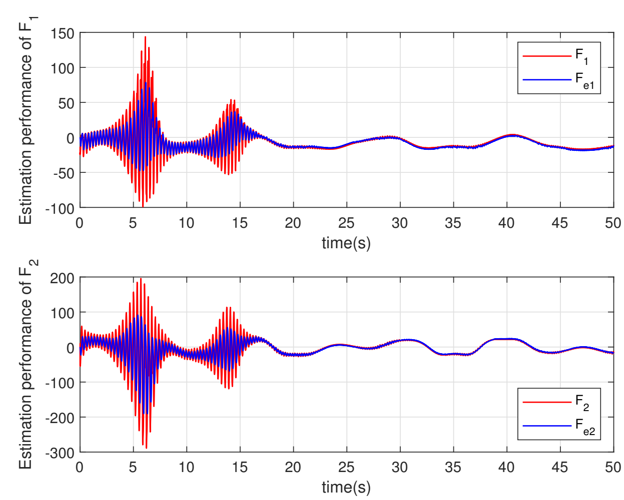

Fig. 5. Estimation performance of the unknown F_i with the fixed-time observer in [30], $i = 1, 2$.

with the fixed-time observer in [30]. And Fig. 6 has shown the tracking performance with the proposed controller and the fixed-time controller in [20] respectively.

It has been shown that the controller and the observer can both achieve fixed-time convergence in the comparative results. However, we can see from Figs. 5 and 6 that the estimation effect of the observer directly leads to the effect of the controller in [20]. So the output constraints in equations (115) and (116) may be violated because of the chattering problem of the observer. For the proposed control algorithm in the paper, the same asymmetric output constraints can be well handled because of the exploit of BLF, which can be seen in Fig. 6. Besides, in our control scheme, we design an arbitrary bounded function F_{ei} to handle the unknown term F_i in the system such that the ac-

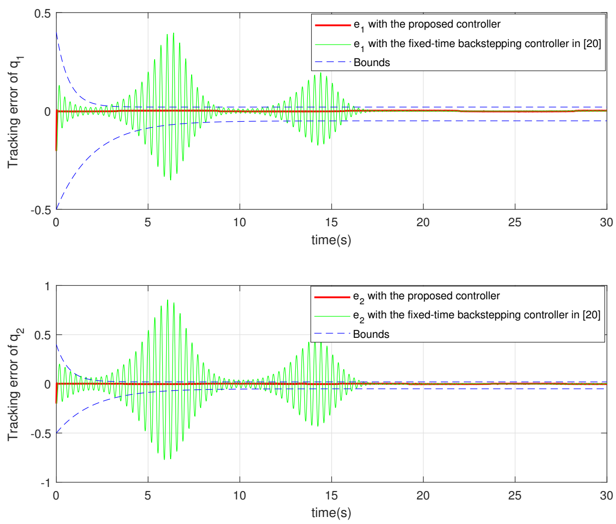

Fig. 6. Tracking error of the joint positions q_i with the proposed controller and the fixed-time backstepping controller in [20] respectively, $i = 1, 2$.

curate estimation requirement on unknown system terms can be relaxed. So the effectiveness and the superiority of the proposed identification/control algorithm can be seen from the simulation results above.

6. CONCLUSION

In this paper, a fixed-time parameter identification algorithm has been proposed for an uncertain robot. By introducing an auxiliary filtering technique, the measurement of joint acceleration can be avoided. Moreover, the computational burden is less than the traditional identification algorithm because the real-time inverse calculation of the square matrix has been omitted during the process. Based on the results of parameter identification, we propose a fixed-time control scheme which can guarantee the specified asymmetric motion performance and pre-established convergence time simultaneously. The problem of the uncertain convergence domain in a class of universal fixed-time control methods in [18–20] can be effectively solved by the exploit of BLF. The tiny practical errors of parameter identification can be effectively handled with the proposed control scheme. The simulation results has shown the effectiveness and the superiority of the proposed identification/control algorithm. Compared with the existing identification/control schemes, the schemes we proposed in the paper can guarantee that the convergence time will not change with different initial conditions of the system.

In the future work, neural control problem of unknown bimanual robots with fixed-time convergence will be discussed.

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