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Nonsingular Terminal Sliding Mode Control-based Prescribed Performance Guidance Law with Impact Angle Constraints

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Abstract: Conventional guidance law designs can only guarantee steady-state performance. However, transient performance is also the key performance index in practical guidance applications. In this paper, a novel terminal guidance law is presented for missile intercepting maneuvering target with impact angle constraints, which can strictly guarantee the prescribed steady-state and transient performances of interception. By utilizing the prescribed performance control technique, the prescribed performance tracking control problem is transformed into an equivalent unconstrained form such that the tracking error can be limit to the prescribed performance bound. Then, on the basis of transferred the tracking error, a novel nonsingular terminal sliding mode control-based guidance law is proposed with impact angle constraint, and the extended state observer is incorporated to online estimate the external disturbances and unknown target maneuver. The closed-loop system stability and the convergence characteristic are rigorously proved. Finally, extensive contrast simulations are conducted to demonstrate the efficiency and superiority of the proposed guidance law for different engagement scenarios.

Keywords: Extended state observer, impact angle, nonsingular terminal sliding mode control, prescribed performance control, terminal guidance law.

1. INTRODUCTION

With the rapid development of technology, the targets have become more and more intelligent and mobile. It is a challenge for the design of the guidance law whether it can ensure the missiles to strike the target accurately or not. As is well known, the significant objective of guided missiles is to intercept targets with a minimum miss distance and predetermined impact angle [1]. For the current tactical requirement and the high maneuverable target, it is essential to further research the guidance law with impact angle constrained for missiles to improve the attacking efficiency.

Over the past decades, extensive efforts have been expended to the design of terminal homing guidance laws. It is widely known that the proportional navigation guidance (PNG) law has been extensively adopted to intercept the weakly maneuverable targets as to its convenient implementation and high efficiency [2–4]. However, the PNG law is more applicable for the task of intercepting a non-maneuvering target or a weakly maneuvering target. In practice, target acceleration can change rapidly. The PNG is inappropriate to intercept a highly maneuverable target under the required tactics index and a significant miss distance may be resulted. With the development of the homing guidance technique in recent years, more and more mordent terminal angle constraint guidance laws have been explored to obtain a small miss distance which are based on optimal control [5], trajectory shaping theory [6], feedback linearization control [7], nonlinear $H\infty$ control [8], $L2$ gain control [9], adaptive control [10], sliding mode control [11–13] and references therein.

It is worthwhile to mention that sliding model control (SMC) is well known for its good robustness to external disturbances and parametric uncertainties [14–16], which has been widely applied into the terminal homing guidance law design with terminal impact angle constraint due to its robustness to system uncertainties and external disturbances of nonlinear system. A novel sliding mode-based impact angle guidance law for intercepting a maneuvering target is proposed in [12], and this guidance concept is further investigated in [13], which is insensitive to uncertainties and disturbances. In order to achieve the finite time convergence and fast response, the terminal sliding mode control (TSMC), whose sliding mode manifold is a nonlinear function, is introduced into the guidance law design for missiles in [17–19]. However, the proposed TSMC guidance laws have an obvious draw-

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back of the singularity problem owing to the existing of the negative exponential term. To avoid the singularity problem, the nonsingular terminal sliding mode control (NTSMC)-based guidance law is developed in [20–23] to intercept the target at desired impact angle without exhibiting any singularity. But unfortunately, the convergent rate of NTSM controllers will be extremely slow when the system state is far away from the equilibrium. Thus, it is urgent to investigate the NTSMC based-guidance law design with the favorable transient performance.

Fortunately, Bechlioulis [24,25] originally proposed a remarkable prescribed performance control (PPC) methodology, in which both the transient and steady-state performance can be quantitatively examined and analytically studied. The main characteristic of this approach is that the tracking error can be converge to zero with converge rate no less than a prescribed value and maximum overshoot less than a sufficiently small constant. Inspired by this idea, the PPC was extended to robot systems [26,27], turntable servo mechanisms [28], near space vehicles [29], vehicle suspensions system [30,31] and air-breathing supersonic missiles [32], etc. However, the research that the design of guidance law with the PPC technique is insufficient.

Note that there also remains an unavoidable chattering problem in the implementation of the sliding mode control process. In order to resolve this problem, an efficient way is to estimate the disturbance by using an observer, the guidance law in [33] was presented combing the NTSMC and extended state observer (ESO). Based on the effective estimation of disturbance, the proposed guidance law requires no priori information on disturbances including the target maneuver. It is worth noting that the ESO was originally developed in the research of the active disturbance rejection control by Han [34], and successfully applied in many important control fields [35–37] and the references therein. Although the guidance law in [33] achieved a good guidance effect. But it can only guarantee that the steady-state performance, but not the transient performance which is particularly important for the safe operation, because the control system with aggressive transient response (e.g., large overshoot) may be broken before they reach a stable steady-state. Therefore, it is vitally essential for the missile guidance system design that can guarantee the system for both the steady-state performance and transient performance.

Motivated by the aforementioned discussions, this paper is to further study the guidance law design for missile guidance system subject to external disturbances. The main contributions of this paper are summarized as: 1) A novel NTSMC-based guidance law design is firstly proposed for missile with impact angle constraint by incorporating the PPC technique; 2) The proposed approach can improve the transient and steady-state performance of guidance system and the tracking error can be retained within a prescribed bound; 3) The proposed approach does not need the knowledge of the target movement information in advance; 4) The system stability and convergence characteristic are both proved strictly.

This paper is organized as follows: Section 2 presents a geometry of missile-target engagement and the problem formulation is given. In Section 3, a novel NTSMC guidance law design with terminal LOS angle constraint is proposed for missile, and the close-loop system stability is rigorously proved. Simulation results and analysis are presented in Section 4. Finally, some conclusions are summarized in Section 5.

2. PROBLEM FORMULATION

This section presents the equations of guidance system for missile intercepting the target. With design simplification and no loss of generality, we only consider the two-dimensional model here, and the engagement between a missile and a target is shown in Fig. 1, where the relative distance between the missile and the target are presented by r, and the line-of-sight (LOS) angle of the missile is defined by q, the magnitude velocity of missile and target v_m and v_t are assumed as fixed constants, their flight path angle are θ_m and θ_t, and their normal accelerations are denoted by a_m and a_t, respectively.

The equations of kinematic engagement are established as follows [19]:

$$\dot{r} = -v_m \cos(\theta_m - q) + v_t \cos(\theta_t - q), \tag{1}$$

$$r\dot{q} = v_m \sin(\theta_m - q) - v_t \sin(\theta_t - q), \tag{2}$$

$$\dot{\theta}_m = a_m / v_m, \tag{3}$$

$$\dot{\theta}_t = a_t / v_t. \tag{4}$$

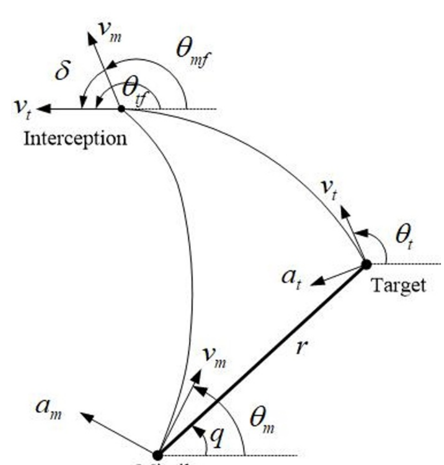

Fig. 1. The planar engagement between missile and target.

Differential (2) with respect to time, yields

$$\ddot{q} = \frac{\dot{r}}{r}\dot{q} - \frac{1}{r}a_m\cos(\theta_m - q) + \frac{1}{r}a_t\cos(\theta_t - q). \quad (5)$$

Achieving a desired impact angle is an additional objective along with the usual requirement of interception. The parameter δ is the desired impact angle, which is defined as the angle between the velocity vectors of the missile and the target at the time of interception, and can be formulated as

$$\delta = \theta_{tf} - \theta_{mf}, \quad (6)$$

where θ_{tf} and θ_{mf} represent the flight path angle of the target and the missile at the time of interception, respectively. For most engagement scenarios, a unique LOS angle exists for a particular impact angle. When the missile and target are on the collision course, there exists

$$v_m\sin(\theta_{mf} - q_d) - v_t\sin(\theta_{tf} - q_d) = 0, \quad (7)$$

where q_d is the desired terminal LOS angle. Under the assumption $\upsilon = v_t/v_m < 1$, and substituting (6) into (7), we can obtain that

$$q_d = \theta_{tf} - \arctan[\sin\delta/\cos\delta - \upsilon]. \quad (8)$$

From this relation, it can be seen that q_d and δ have the one-to-one correspondence to each other, which is beneficial for further research. Hence, the design of guidance with impact angle constrained can be transformed into the control problem of the terminal LOS angle, i.e., satisfying $q(t_f) = q_d$, where $t_f$is the guidance terminal time.

Define $x_1 = q - q_d$ and $x_2 = \dot{q}$ as the impact angle error and the relative velocity perpendicular to the LOS, system (5) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f + gu + d, \end{cases} \quad (9)$$

where

$$f = 2\frac{\dot{r}}{r}x_2, \quad g = -\frac{1}{r}\cos(\theta_m - q), \quad u = a_m,$$
$$d = \frac{1}{r}a_t\cos(\theta_t - q).$$

In (9), if the moving information of the target is known, the term d is the known dynamic of the system. Otherwise, d defines an unknown lumped disturbance of the target, and

$$|d| = \left|\frac{1}{r}a_t\cos(\theta_t - q)\right| \leq \frac{\max\{a_t\}}{r_0}. \quad (10)$$

Thus, the term d is bounded unknown external disturbance under the condition that the movement of the target is unknown.

As stated previously, the objective of this paper is to design a controller u to ensure the state x_1 can trend to a small enough neighborhood around zero with prescribed performance in the presence of external disturbance. The prescribed performance means the tracking error can converge to a predefined small residual set with convergence no less than a certain prospective value.

3. GUIDANCE LAW DESIGN

In this section, a guidance law with prescribed performance is proposed to force the LOS angle to converge to the reference command by compensating the disturbances by extended state observer (ESO). Prescribed performance control is used to transform the constrained original system to an unconstrained system, the transient and steady-state performance of the tracking error can be limited to a prescribed performance bound.

3.1. Observer design for disturbance estimation

The main advantage of the ESO is that it can estimate the total disturbances including the system model uncertainties and external disturbances without any information about the dynamics of the system [34]. In this paper, only d in (9) is unknown and the others are assumed to be measurable, a second-order ESO is established here. Consider the system (9), if the moving information of the target is unknown, we add the disturbance term d as an extended state, then the second-order ESO for the system (9) is constructed as

$$\begin{cases} e_v = z_1 - x_2, \\ \dot{z}_1 = z_2 - \beta_{01}e_v + f + gu, \\ \dot{z}_2 = -\beta_{02}fal(e_v, \alpha, \xi), \end{cases} \quad (11)$$

where e_v is the estimation error, z_1 and z_2 are the estimation value of the state x_2 and term d, respectively. The β_{01}, β_{02} are the observer gains. The function $fal(e_v, \alpha, \xi)$ is a continuous function [38]. The $\xi > 0$, $0 < \alpha < 1$ are extra parameters. If the related parameters are chosen properly, the estimated states can converge to the respective system states, i.e., $z_1 \to x_2$, $z_2 \to d$. This indicates that an improving performance can be achieved by introducing the estimated state z_2 into the controller design to compensate the disturbance d.

3.2. Prescribed performance control

For completeness and compactness of presentation, this subsection summarizes preliminary knowledge on prescribed performance concept which was originally proposed by Bechlioulis and Rovithakis [24,25]. The prescribed performance means that the minimum speed of the convergence, the maximum steady state error and the maximum allowable overshoot are set a priori. Generally, consider the tracking error $e(t)$, the prescribed performance can be achieved if the tracking error evolves strictly within predefined region that is bounded by a decreasing smooth function of time as follows:

$$-\underline{\delta}\rho(t) < e(t) < \bar{\delta}\rho(t),\ \forall t > 0, \tag{12}$$

where $\bar{\delta}$, $\underline{\delta}$ are chosen positive constants, and the function $\rho(t)$ is the prescribed performance function (PPF) which is formulated as follows:

$$\rho(t) = (\rho_0 - \rho_\infty)\mathrm{e}^{-\kappa t} + \rho_\infty, \tag{13}$$

where $\rho_0 > \rho_\infty$ and $\kappa > 0$. ρ_0, ρ_∞ denote the initial error bound and the maximum allowed steady error, respectively, and constant κ which is related to the decreasing rate of $\rho(t)$ influences the convergence rate of the tracking error. In order to transform the prescribed performance tracking control problem into an equivalent unconstrained form, an error transform function is introduced as

$$e(t) = \rho(t)T(\varepsilon(t)), \tag{14}$$

where $\varepsilon(t)$ is the transformed error, and $T(\cdot)$ is the transformed function, which possesses with the following properties:

1) $T(\cdot)$ is smooth and strictly increasing;
2) $-\underline{\delta} < T(\varepsilon) < \bar{\delta}$;
3) $\lim\limits_{\varepsilon\to+\infty} T(\varepsilon) = \bar{\delta}$, $\lim\limits_{\varepsilon\to-\infty} T(\varepsilon) = -\underline{\delta}$.

In this paper, we choose the function $T(\cdot)$ [32] as follows:

$$T(\varepsilon) = \frac{\bar{\delta}e^{\varepsilon} - \underline{\delta}e^{-\varepsilon}}{e^{\varepsilon} + e^{-\varepsilon}}. \tag{15}$$

Since the function $T(\cdot)$ is strictly monotonic increasing, its inverse function exists and the transformed error $\varepsilon(t)$ can be derived as

$$\varepsilon(t) = T^{-1}(\lambda) = \frac{1}{2}\ln\left(\frac{\underline{\delta}+\lambda}{\bar{\delta}-\lambda}\right), \tag{16}$$

where $\lambda = e(t)/\rho(t)$.

Meanwhile, the time derivatives of the normalized error λ can be obtained as

$$\dot{\lambda} = \frac{d(e/\rho)}{dt} = \frac{1}{\rho}(\dot{e} - \lambda\dot{\rho}) \tag{17}$$

Then, the time derivatives of the transformed error $\varepsilon(t)$ is given by

$$\dot{\varepsilon} = \frac{\partial T^{-1}}{\partial\lambda}\dot{\lambda} = \frac{1}{2}\left[\frac{1}{\lambda+\underline{\delta}} - \frac{1}{\lambda-\bar{\delta}}\right]\dot{\lambda} = \chi(\dot{e} - \lambda\dot{\rho}), \tag{18}$$

where $\chi = \frac{1}{2\rho}\left[\frac{1}{\lambda+\underline{\delta}} - \frac{1}{\lambda-\bar{\delta}}\right]$ can be calculated in terms of $e(t)$, $\rho(t)$ and fulfill $0 < \chi \le \chi_M$for positive constant χ_M.

It is worth to mention that if we can keep $\varepsilon(t)$ bounded for all $t \ge 0$, then the tracking error $e(t)$ can be limited to the prescribed performance bound as depicted in (12), namely, the transient and steady-state performance of $e(t)$ can be determined by tuning the performance function $\rho(t)$ as well as constants ρ_0, ρ_∞, κ, and $\bar{\delta}$, $\underline{\delta}$ appropriately.

3.3. Guidance law design

In this subsection, the impact angle constrained guidance law is developed for the system (9) by using the principles of the non-singular terminal sliding control theory, and the PPC technology is applied to the transient and steady-state performance of the tracking error. Meanwhile, the estimation of the external disturbance via ESO is introduced into the control loop to compensate the effect of disturbance. The design procedure can be processed as follows:

Define the tracking error as

$$e_1 = x_1 - x_{1c},\ \ e_2 = x_2 - x_{2c}, \tag{19}$$

where $x_{1c} = 0$ and $x_{2c} = 0$ is the desired command.

Then we design the transformed error of e_1 as

$$\varepsilon_1 = T^{-1}(\lambda_1) = \frac{1}{2}\ln\left(\frac{\underline{\delta}+\lambda_1}{\bar{\delta}-\lambda_1}\right), \tag{20}$$

where $\lambda_1 = e_1(t)/\rho_1(t)$ is the normalized output error by using a similar PPF defined in (13) as

$$\rho_1(t) = (\rho_{10} - \rho_{1\infty})e^{-l_1 t} + \rho_{1\infty}, \tag{21}$$

where ρ_{10}, $\rho_{1\infty}$ and l_1 are all positive constants.

Combining the non-singular TSMC and PPC technique, the sliding model surface is designed as

$$S = \varepsilon_1 + \eta\,|e_2|^{\gamma}\,\mathrm{sgn}(e_2), \tag{22}$$

where $\eta > 0$, $0 < \gamma < 1$ is the sliding model surface parameter, sgn() is the sign function, and the time derivative of S is obtained as

$$\begin{aligned}\dot{S} &= \dot{\varepsilon}_1 + \eta\gamma|e_2|^{\gamma-1}\dot{e}_2\\ &= \chi(\dot{x}_1 - \dot{x}_{1c} - \lambda_1\dot{\rho}_1) - \eta\gamma|x_2 - x_{2c}|^{\gamma-1}(\dot{x}_2 - \dot{x}_{2c})\\ &= \chi(x_2 - \lambda_1\dot{\rho}_1) + \eta\gamma|x_2|^{\gamma-1}(f + gu + d).\end{aligned} \tag{23}$$

Here, we select the double power reaching law

$$\dot{S} = -k_1|S|^{\alpha}\mathrm{sgn}(S) - k_2|S|^{\beta}\mathrm{sgn}(S), \tag{24}$$

where $\alpha > 1$, $0 < \beta < 1$, $k_1 > 0$, $k_2 > 0$ are the sliding model control gains.

If the moving dynamic of target is known, the controller u is obtained in terms of (23) and (24) as follows:

$$\begin{aligned}u =&[-\sigma\chi|x_2|\mathrm{sgn}(x_2) + \sigma\chi\lambda_1\dot{\rho}_1 - f - d]/g\\ &+ \sigma\left[-k_1|S|^{\alpha}\mathrm{sgn}(S) - k_2|S|^{\beta}\mathrm{sgn}(S)\right]/g,\end{aligned} \tag{25}$$

where $\sigma = |x_2|^{1-\gamma}/\eta\gamma$. If the moving dynamic of target is unknown, the controller u is given by

$$\begin{aligned}u =&[-\sigma\chi|x_2|\mathrm{sgn}(x_2) + \sigma\chi\lambda_1\dot{\rho}_1 - f - z_2]/g\\ &+ \sigma[-k_1|S|^{\alpha}\mathrm{sgn}(S) - k_2|S|^{\beta}\mathrm{sgn}(S)]/g,\end{aligned} \tag{26}$$

where z_2 is the estimation of disturbance d via ESO.

It should be pointed out that the guidance command generated by (25) or (26) is still discontinuous due to the existence of sign function $\mathrm{sgn}(S)$. In order to eliminate the control chattering, the above discontinuous sign function is approximated by a continuous saturation function [39] $\mathrm{sat}(S)$:

$$\mathrm{sat}(S) = \begin{cases} \mathrm{sgn}(S), & |S| > \Delta, \\ S/\Delta, & |S| \le \Delta, \end{cases} \tag{27}$$

where the $\Delta > 0$ is the thickness of boundary layer.

3.4. Stability analysis

In this subsection, the stability of the closed-loop system will be established by the following theorem.

Theorem 1: For system (9), extended state observer (11) and the proposed fault tolerant controller (25) and (26) with the prescribed performance function (13), there exists observer gains β_{01}, β_{02}, α, ξ such that the estimated states z_2 can converge to the disturbance term d, and closed-loop system output tracking error can be driven on the designed sliding surface and converge to a residual set of the origin asymptotically. Furthermore, then the output tracking error e_1 can be retained within a prescribed set, i.e., $-\underline{\delta}\rho_1(t) < e_1(t) < \bar{\delta}\rho_1(t)$.

Proof: Define the estimation errors $e_{v1} = z_1 - x_2$ and $e_{v2} = z_2 - d$, from (11), the error dynamics of the ESO are

$$\begin{cases} \dot{e}_{v1} = e_{v2} - \beta_{01} e_{v1}, \\ \dot{e}_{v2} = \dot{d} - \beta_{02} fal(e_{v1}, a, \xi), \end{cases} \tag{28}$$

where the term $\dot{d}$ is the derivative of fault term d, which is unknown but bounded. The stability of the ESO is satisfied under the condition $\beta_{01}^2 > 4\beta_{02}\xi^{a-1}$ [40]. When the ESO is stable, the time derivative of errors $\dot{e}_{v1} = 0$ and $\dot{e}_{v2} = 0$. Thus, the estimation errors can be expressed as

$$\begin{cases} e_{v2} = \beta_{01} e_{v1}, \\ fal(e_{v1}, a, \xi) = \dfrac{\dot{d}}{\beta_{02}}. \end{cases} \tag{29}$$

Then the estimation errors are rewritten as

$$\begin{cases} |e_{v1}| = |\dot{d}\delta^{1-a}|/\beta_{02}, & |e_{v1}| \le \xi, \\ |e_{v1}| = |\dot{d}/\beta_{02}|^{1/a}, & |e_{v1}| > \xi, \\ e_{v2} = \beta_{01} e_{v1}. \end{cases} \tag{30}$$

Consequently, we can verify that z_2 will converge into a neighborhood of disturbance d, i.e., the disturbance term d can be estimated by the ESO effectively.

Next, choosing the following Lyapunov function

$$V = \frac{1}{2} S^2. \tag{31}$$

The time derivative of V along (23) is

$$\begin{aligned} \dot{V} &= S\dot{S} \\ &= S_1[\chi(x_2 - \lambda_1 \dot{\rho}_1) + \eta\gamma |x_2|^{\gamma-1}(f + gu + d)]. \end{aligned} \tag{32}$$

If the moving dynamic of target is known, based on the (25), we can obtain that

$$\begin{aligned} \dot{V} &= S[-k_1|S|^\alpha \mathrm{sgn}(S) - k_2|S|^\beta \mathrm{sgn}(S)] \\ &= S_1[\chi(x_2 - \lambda_1\dot{\rho}_1) + \eta\gamma|x_2|^{\gamma-1}[f + \cdots \\ &\quad + g\{[-\sigma\chi|x_2|\mathrm{sgn}(x_2) + \sigma\chi\lambda_1\dot{\rho}_1 - f - d]/g + \cdots \\ &\quad + \sigma[-k_1|S|^\alpha \mathrm{sgn}(S) - k_2|S|^\beta \mathrm{sgn}(S)]/g\} + d] \\ &= S[-k_1|S|^\alpha \mathrm{sgn}(S) - k_2|S|^\beta \mathrm{sgn}(S)] \\ &= -k_1|S|^{\alpha+1} - k_2|S|^{\beta+1}. \end{aligned} \tag{33}$$

Otherwise, the time derivative of V is given based on the (26),

$$\begin{aligned} \dot{V} &= S[-k_1|S|^\alpha \mathrm{sgn}(S) - k_2|S|^\beta \mathrm{sgn}(S)] \\ &= S_1[\chi(x_2 - \lambda_1\dot{\rho}_1) + \eta\gamma|x_2|^{\gamma-1}[f + \cdots \\ &\quad + g\{[-\sigma\chi|x_2|\mathrm{sgn}(x_2) + \sigma\chi\lambda_1\dot{\rho}_1 - f - z_2]/g \\ &\quad + \cdots + \sigma[-k_1|S|^\alpha \mathrm{sgn}(S) - k_2|S|^\beta \mathrm{sgn}(S)]/g\} + d] \\ &= S[-k_1|S|^\alpha \mathrm{sgn}(S) - k_2|S|^\beta \mathrm{sgn}(S) + (d - z_2)/\sigma] \\ &= -k_1|S|^{\alpha+1} - k_2|S|^{\beta+1} - Se_{v2}/\sigma, \end{aligned} \tag{34}$$

where $\sigma = |x_2|^{1-\gamma}/\eta\gamma$. As the estimation error e_{v2} can converge to zero. Thus if the control gains k_2 and k_2 which are positive parameters are selected appropriately such that $\dot{V} < 0$, i.e., the closed-loop system is stable whether or not the moving information of target is known.

In addition, from the definition and property of the error transformed function (16), we can obtain that

$$e^{2\varepsilon_1} = \frac{\underline{\delta} + \lambda_1}{\bar{\delta} - \lambda_1}. \tag{35}$$

The transformed errors $\varepsilon_1 = T^{-1}(\lambda_1)$ are bounded, i.e., $|\varepsilon_1| \le \varepsilon_{M1}$ holds for positive constants $\varepsilon_{M1} > 0$. This further implies

$$-\underline{\delta} < \frac{e^{-\varepsilon_{M1}}\bar{\delta} - \underline{\delta}}{e^{-\varepsilon_{M1}} + 1} \le \lambda_1 \le \frac{e^{\varepsilon_{M1}}\bar{\delta} - \underline{\delta}}{e^{\varepsilon_{M1}} + 1} < \bar{\delta}. \tag{36}$$

Consequently, from the fact $\lambda_1 = e_1(t)/\rho_1(t)$, one may verify that the output tracking error e_1 can be retained within a prescribed set, i.e., $-\underline{\delta}\rho_1(t) < e_1(t) < \bar{\delta}\rho_1(t)$ holds. This completes the proof. □

4. SIMULATION RESULTS

In this section, a missile is considered in its terminal guidance process to intercept a maneuvering target and extensive simulations are conducted for different kinds of scenarios to illustrate the effectiveness of the proposed

Table 1. The initial condition for missile and target.

Parameter	x_m	y_m	θ_m	v_m
Value	0 m	0 m	60 deg	600 m/s
Parameter	x_t	y_t	θ_t	v_t
Value	$2500\sqrt{3}$ m	2500 m	0 deg	300 m/s

nonsingular terminal sliding mode-based prescribed performance guidance law (NTSMC-PPC) and to validate the improvement of the control performance in comparison to nonsingular terminal sliding mode control guidance law (NTSMC) in [41]. The initial conditions for the missile and the target are shown in Table 1. The parameters of the ESO is chosen as $\beta_{01} = 300$, $\beta_{02} = 600$, $a = 0.5$, $\xi = 0.01$, the parameters of NTSMC method are selected as $\eta = 1/4$, $\gamma = 7/6$, $k_1 = k_2 = 3$, $\alpha = 3$, $\beta = 2/3$, $\Delta = 0.001$, and the PPF is designed as $\bar{\delta} = \underline{\delta} = 1$, $\rho_1(t) = (15-0.1)\mathrm{e}^{-1.5t} + 0.1$ deg, and the maximum acceleration of the missile is $40g$, g is the acceleration of gravity ($g = 10$ m/s^2).

4.1. Interception of known maneuvering target

In this part, we consider that the information of the target is known, i.e., the guidance law expressed in (25) is adopted into the simulation, and the desired final LOS angle q_d is selected as 20 deg. In order to validate the effectiveness of the proposed guidance law, three different target maneuvering cases which are constant maneuvering, step maneuvering and cosine maneuvering, are considered as listed below, and the simulation results are presented in Figs. 2-7.

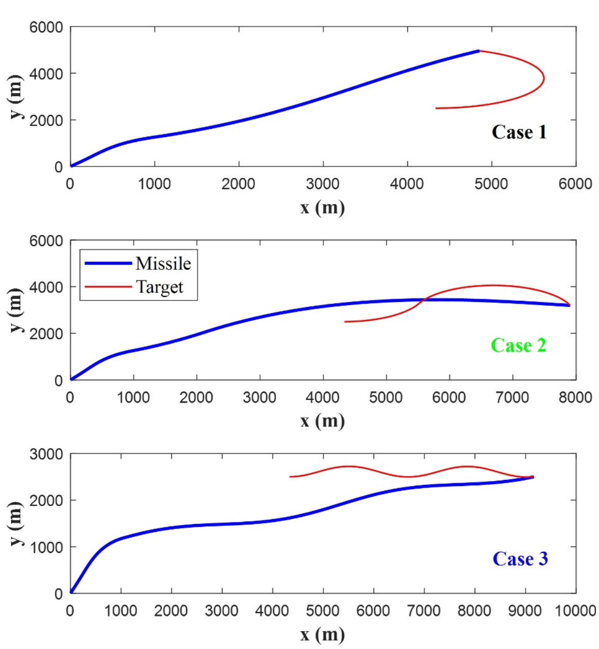

Fig. 2. Trajectories of missile and target.

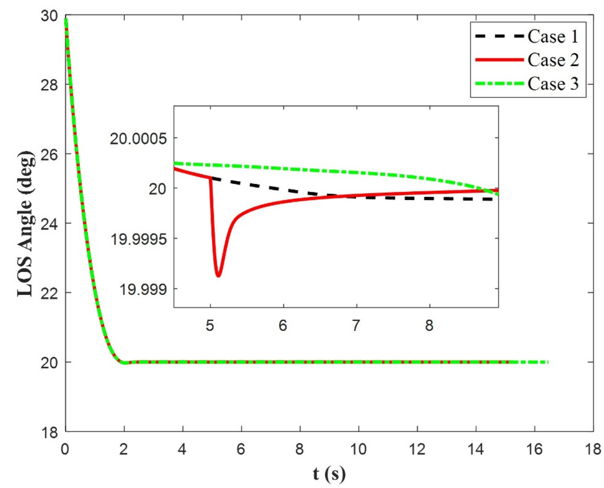

Fig. 3. Line-of-sight angle.

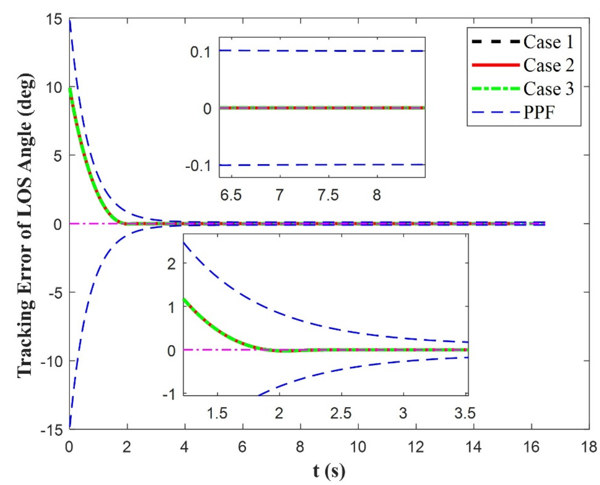

Fig. 4. Tracking error of line-of-sight angle.

Case 1: $a_t = 7g$ m/s^2,

Case 2: $a_t = 7g$ ($t < 5$ s) and $a_t = -7g$ m/s^2 ($t \geq 5$ s),

Case 3: $a_t = 7g\cos(\pi t/4)$ m/s^2,

where a_t is the target acceleration.

From Fig. 2, it can be seen that the missile can intercept the maneuvering target with the desired terminal LOS angle in any of the three cases successfully. Fig. 3 shows that the LOS angles can accurately and quickly track the desired command, and the tracking errors can converge to the small region around zero at a fast rate and a small overshoot as shown in Fig. 4. It can be seen from Fig. 5 that the LOS angular rates and can converge to zero fast in finite time with the proposed controllers shown in Fig. 6 which are within the reasonable bounds. Fig. 7 indicates that the sliding mode surface converges to zero roughly at 2.5 s under the proposed guidance law for the target acceleration profiles of Case 1 to Case 3.

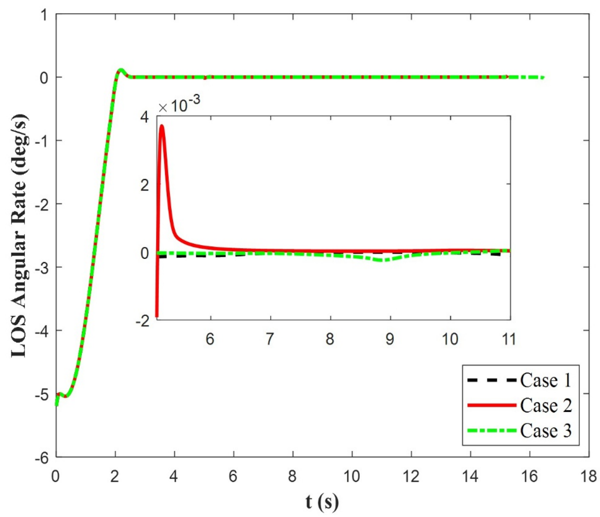

Fig. 5. Line-of-sight angular rate.

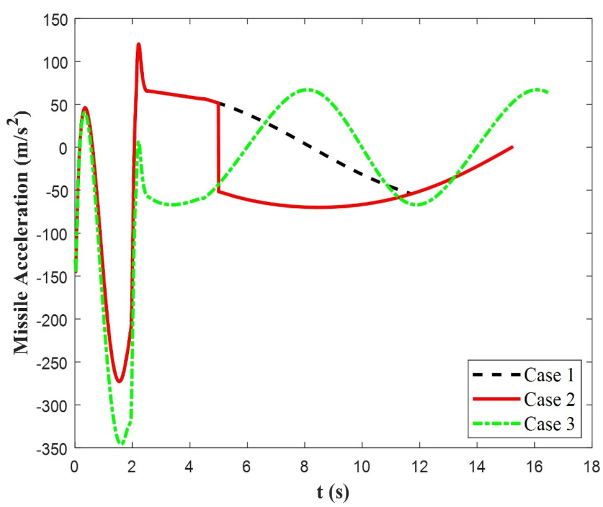

Fig. 6. Missile acceleration.

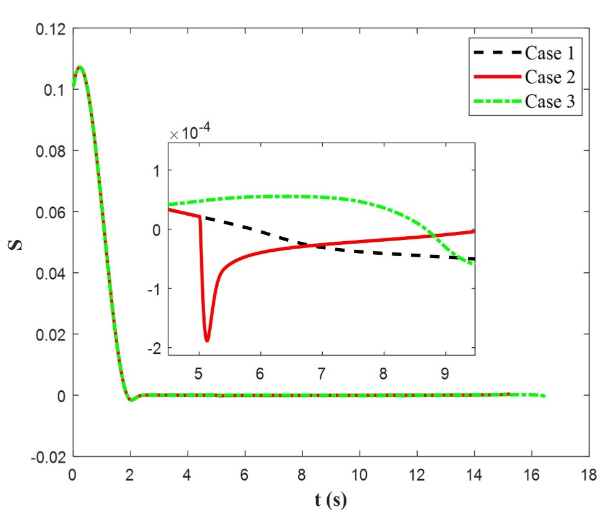

Fig. 7. Sliding mode surface.

4.2. Interception of unknown maneuvering target

In this subsection, we consider that the information of the target d is unknown, i.e., the guidance law (NTSMC-PPC) expressed in (6) is adopted into the simulation to illustrate the improvement of the control performance in comparison to nonsingular terminal sliding mode control guidance law (NTSMC) in [41]. The initial conditions of missile and target and the desired terminal LOS angle command ($q_d = 20\,\text{deg}$) are the same as the previous simulation. The target acceleration is chosen as $a_t = 7\,g\cos(\pi t/4)$ m/s^2, which is a cosine maneuvering, and the simulation results are depicted in Figs. 8-15.

As shown in Fig. 8, we can obtain that the missile can hit the cosine maneuvering target with the mentioned two guidance laws successfully. However, under the proposed NTSMC-PPC guidance law the intercepting time is 16.3 s, which is shorter than the intercepting time of the NTSMC guidance law. From Figs. 9 and 10, it can be observed that the LOS angle can accurately and quickly track the command with the external disturbance, and a fairly satisfactory tracking error response can be achieved with the proposed NTSMC-PPC guidance law. Furthermore, a faster rate and a much smaller overshoot can guarantee that the tracking error remains remarkably small and converges to the neighborhood of zero in approximately 2 s compared with the NTSMC guidance law. However, the convergence time of the tracking error with NTSMC guidance is approximately 4.5 s and the overshoot is unexpectedly 3 deg. That is to say, both the transient and steady-state performance regardless of external disturbance can be achieved by using the NTSMC-PPC guidance law with the help of the prescribed performance control technique. As shown in Fig. 11, it can be also observed that the convergence rate of the LOS angular rate under the NTSMC-PPC guidance law is much faster than that of the NTSMC scheme. From Fig. 13, there are acceleration saturations problem

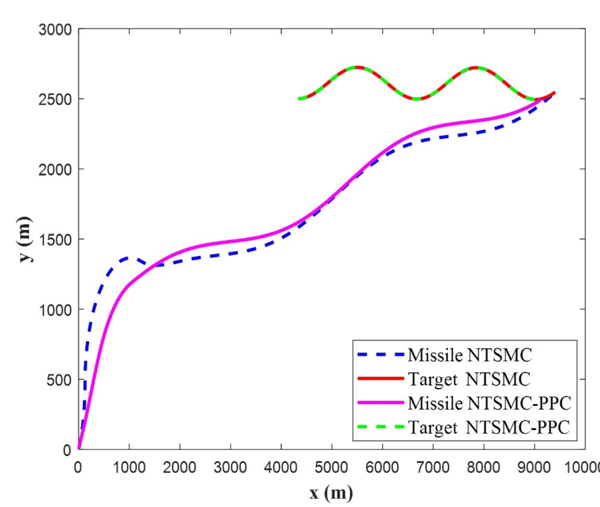

Fig. 8. Trajectories of missile and target.

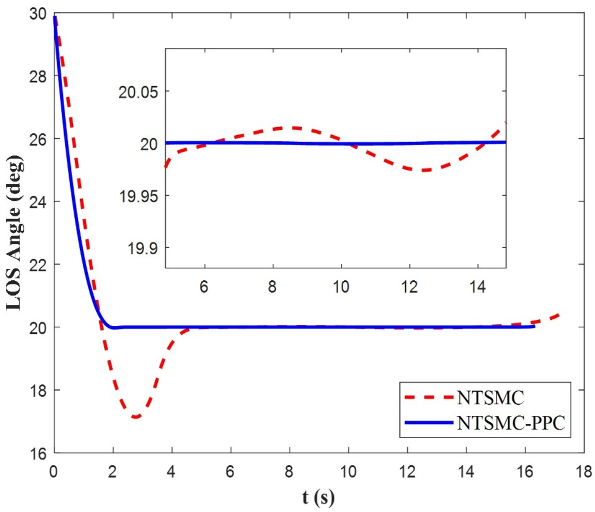

Fig. 9. Line-of-sight angle.

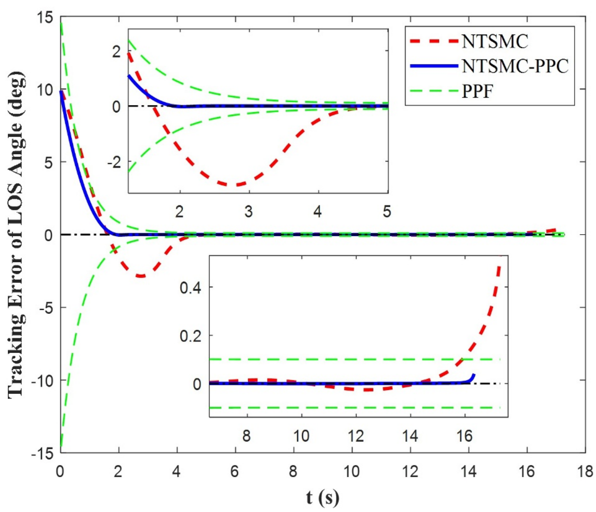

Fig. 10. Tracking error of line-of-sight angle.

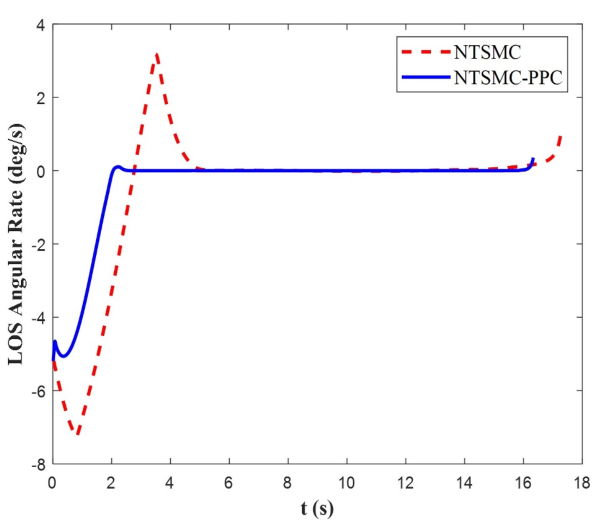

Fig. 11. Line-of-sight angular rate.

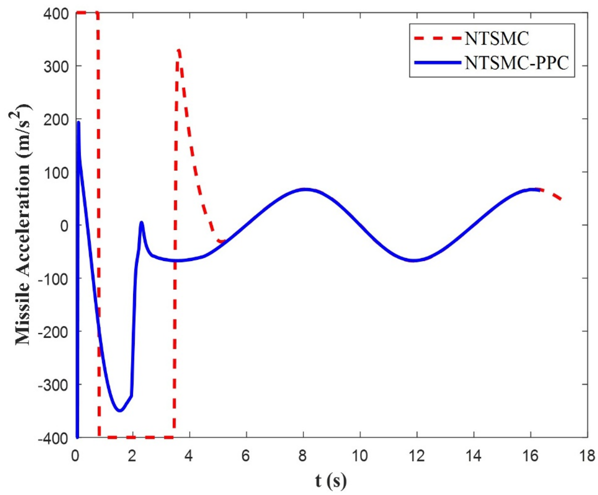

Fig. 12. Missile acceleration.

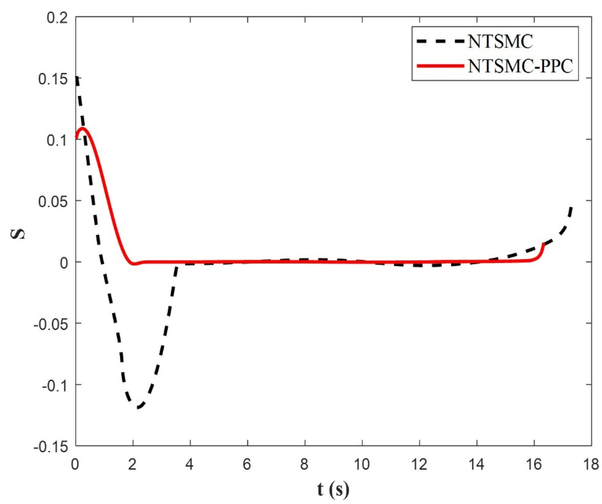

Fig. 13. Sliding mode surface.

under the NTSMC guidance law owing to that the curve of the LOS angular rate has peaks in initial phase. The sliding mode surface of NTSMC-PPC guidance law is fairly smoother than that of NTSMC guidance law. The results of the estimation disturbance are exhibited in Figs. 14 and 15, and we can obtain that the designed ESO can precisely and rapidly estimate the external disturbance of the system.

4.3. Interception with loss of control effectiveness

In actual flight of the missile, the actuator failure may occur which will lead to the loss of control effectiveness. In order to validate the effectiveness of the proposed guidance law in the extreme condition, a contrast simulation is presented with the above NTSMC and NTSMC-PPC guidance law. We assumed that the information of the target is known and the actuator failure is the loss of control effectiveness, i.e., $u = u_c(1-k)$, k is the degraded control

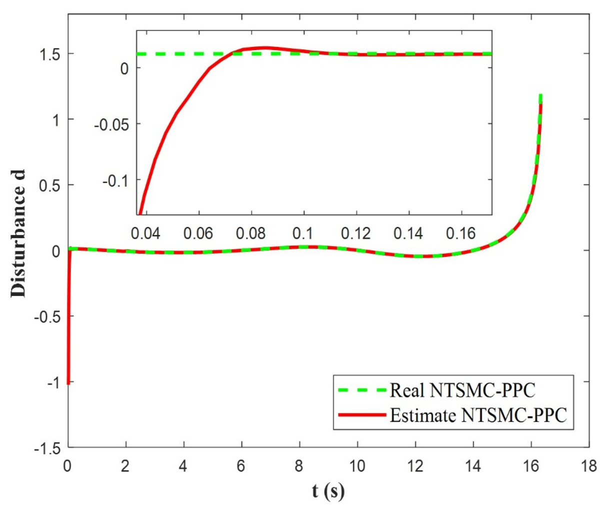

Fig. 14. Estimation of the disturbance.

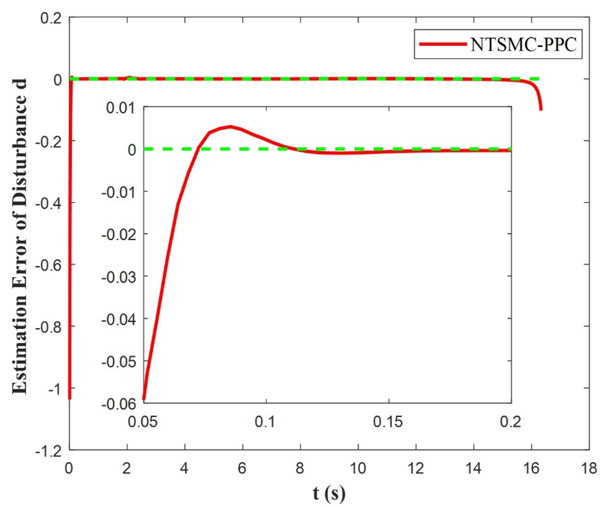

Fig. 15. Estimation error of the disturbance.

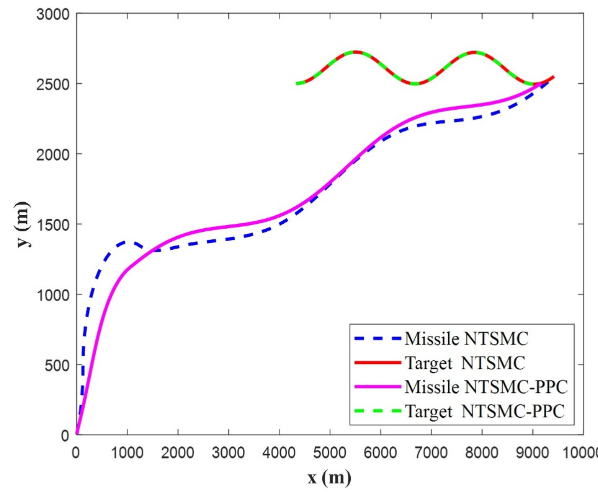

Fig. 16. Trajectories of missile and target.

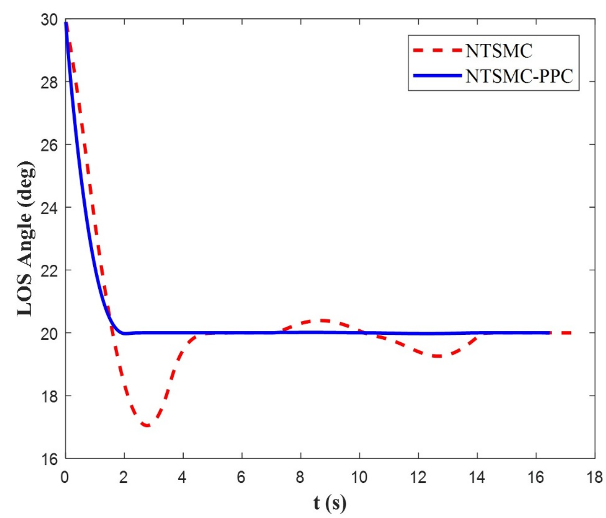

Fig. 17. Line-of-sight angle.

coefficient. Corresponding to the derived system (9), the dynamic of the system can be rewritten as

$$f = -2\frac{\dot{r}}{r}x_2 + \frac{1}{r}a_t\cos(\theta_t - q), \;\; g = -\frac{1}{r}\cos(\theta_m - q),$$
$$u = a_m, \;\; d = -k(ga_m). \tag{37}$$

The degraded control coefficient is selected as

$$k = \begin{cases} 0.0, & 0\text{ s} \le t < 7\text{ s},\ t > 14\text{ s}, \\ 0.4, & 7\text{ s} \le t \le 14\text{ s}. \end{cases} \tag{38}$$

The initial conditions of missile and target and the desired terminal LOS angle command ($q_d = 20\,\text{deg}$) are the same as the previous simulation. The target acceleration is chosen as $a_t = 7g\cos(\pi t/4)$ m/s^2, which is a cosine maneuvering, and the simulation results are depicted in Figs. 16-23.

As shown in Fig. 16, we can obtain that the missile can attack the cosine maneuvering target with the mentioned two guidance laws successfully in presence of unknown fault. Figs. 17-19 provide the response of the guidance system state with the two guidance laws. One may find that the proposed NTSMC-PPC guarantee the prescribed transient and steady-state performance. Notably, when the loss of control effectiveness occurs, there exists a significant fluctuation in the tracking error of the LOS angle which is out of the designed performance bound. Fig. 18 illustrates the missile acceleration under the two guidance laws. As a comparison, the sliding mode surface still converges more rapidly to a small region around zero in finite time under NTSMC-PPC guidance law as shown in Fig. 21. As shown in Figs. 22 and 23, the maximum estimation error is approximately 0.002 during the loss of control effectiveness occurs, that is to say, the designed ESO can also precisely and rapidly estimate the fault term.

All above simulation results illustrate that the effectiveness of the suggested NTSMC-PPC guidance law to in-

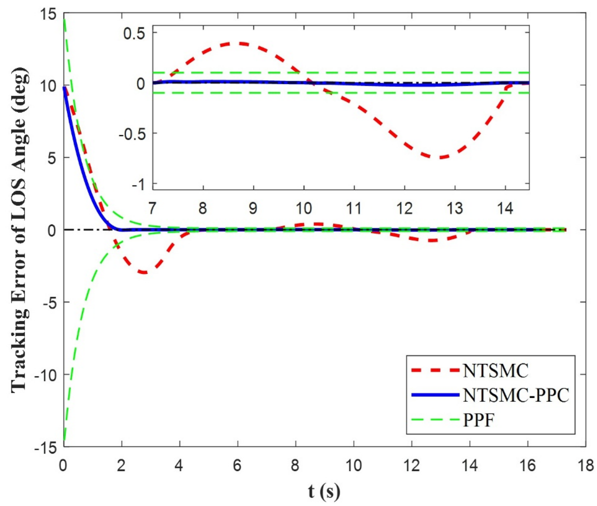

Fig. 18. Tracking error of line-of-sight angle.

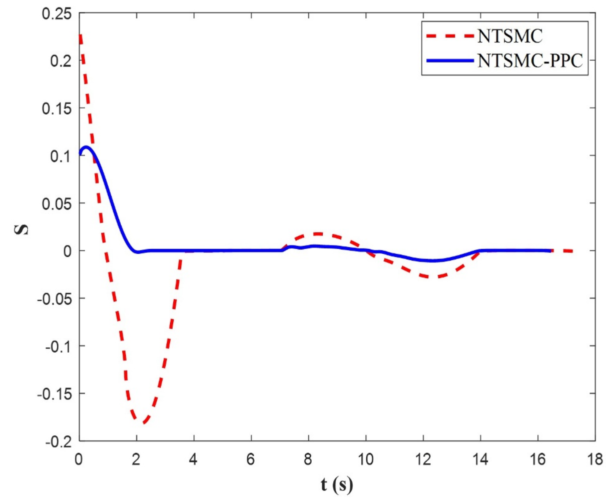

Fig. 21. Sliding mode surface.

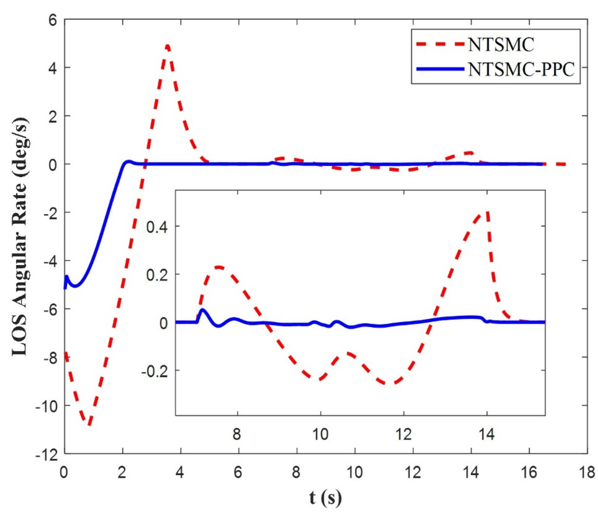

Fig. 19. Line-of-sight angular rate.

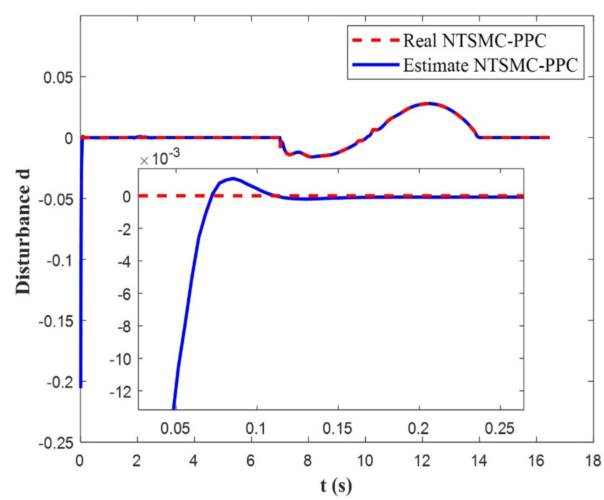

Fig. 22. Estimation of the disturbance.

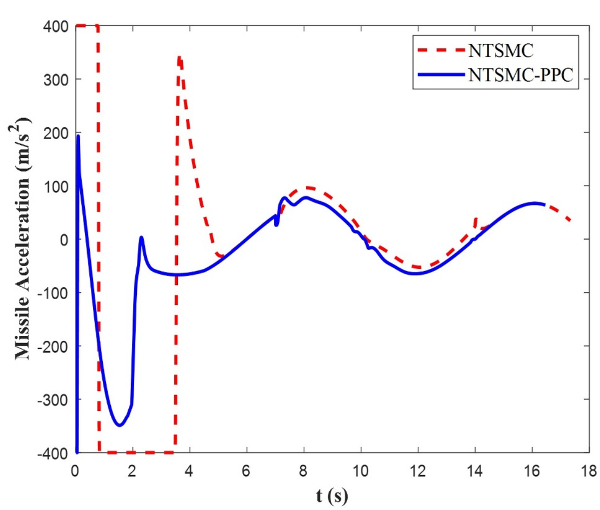

Fig. 20. Missile acceleration.

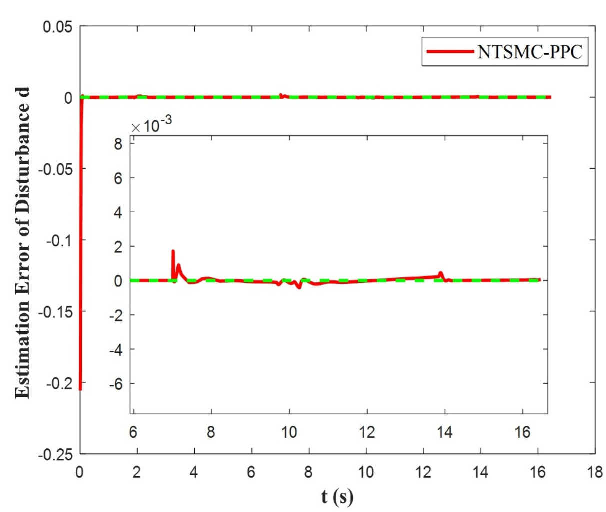

Fig. 23. Estimation error of the disturbance.

tercept maneuvering target with impact angle constrained regardless of the unknown dynamic or the external disturbance of the guidance system. Furthermore, the improvement of the transient and steady-state performance can be achieved owing to the prescribed performance control technique.

5. CONCLUSION

This paper is concerned with the prescribed performance guidance law design for missiles intercepting the maneuvering targets subjected to impact angle constraint. The chief feature of this design is the prescribed performance function is introduced into the nonsingular terminal sliding mode control, which can guarantee both the transient and steady-state control performance against the unknown bounded external disturbances. And the extended state observer is constructed to estimate the external disturbance which includes the unknown target maneuvering and the estimation is compensated into the guidance law to alleviate the chattering phenomenon. Theoretical analysis and contrast simulations conducted to illustrate the effectiveness and the superiority of the proposed guidance law. Future work will focus on the integrated guidance and control design with impact angle constraint based on the proposed theory.

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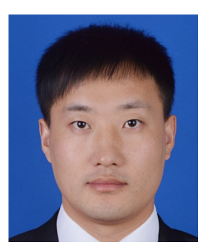

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