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Robust Fault Estimation Based on Proportional Differential (PD) Learning Observer for Linear Continuous-time Systems with State Time-varying Delay

Fuqiang You and Chao Wang*

Abstract: This article studies the fault estimation problem of linear systems with unkown state time-varying delay. A new PD learning observer (LO) is proposed to realize the simultaneous estimation of system states and actuator fault. Furthermore, the time-delay dependence criterion of the existence of the proposed observer is given through the technique of linear matrix inequalites (LMIs). Because the mismatch between the original system and the observer system in the time delay term has a bad influence on the fault estimation, the H_∞ performance index is introduced. At the same time, the problem of sensor fault estimation based on PD learning observer for time-delay systems is studied. Finally, three simulation examples are used to prove the effectiveness of the proposed method.

Keywords: Fault estimation, H_∞ performance index, linear continuous-time systems, linear matrix inequalities, PD learning observer.

1. INTRODUCTION

Modern industrial control systems are becoming more and more complex, so the possibility of system faults is increasing. The occurrence of actuator, sensor and component faults may degrade system performance or cause more serious consequences. Fault diagnosis and fault-tolerant control technology can effectively improve the reliability and security of systems [1]. Fault-tolerant control enables the closed-loop systems to be stable and have acceptable system performance even if faults occur [2]. Usually what we call fault diagnosis consists of three parts: fault detection, fault isolation and fault estimation (fault reconstruction) [3]. The function of fault detection and fault isolation is to determine whether a fault occurs and to determine the location of the fault. The task of fault estimation is to confirm the size of the fault and its change process. Many fault-tolerant control systems have a fault diagnosis subsystem that provides fault information.

In the last few decades, the fault diagnosis technology of dynamic systems based on observer has received extensive attention from scholars. In order to realize the fault estimation of the system, many observer algorithms are designed. For example, adaptive observer [4–6], sliding mode observer [7–9] and learning observer (LO) etc. [10–13]. In [4], the author solves the problem of actuator fault estimation in nonlinear cascade systems by designing an adaptive observer. In [5], The author realizes the state estimation and the reconstruction of unknown input based on the adaptive H_∞ observer. In [6], The author studies filter design problems for nonlinear systems with constraints such as time delay, actuator fault, and sensor fault. In this paper, a filter design method based on adaptive neural network is proposed. In [7], for a class of uncertain nonlinear systems, a sliding mode observer is proposed to realize system fault detection and fault estimation. In [8], the author proposes a high-order sliding mode observer to accurately estimate the observable states of multi-input and multi-output nonlinear systems with unknown inputs, and to estimate the unobservable states progressively. In [9], for a class of non-infinitely observable descriptor systems, the author uses a state and fault estimation scheme to estimate faults of system. In [10], the author uses a robust fuzzy learning observer to solve the problem of robust actuator fault estimation in the Takagi-Sugeno time-delay system with actuator fault and unknown input. In [11], considering the actuator fault that occur when the microsatellite is in orbit, the fault reconstruction of the satellite attitude control system based on the nonlinear learning observer is studied. In [12], The author uses the learning observer to reconstruct the actuator fault and sensor fault of the system respectively. In [13], the author realizes the fault estimation of the nonlinear system with state delay and external disturbances by learning the observer.

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Fuqiang You and Chao Wang are with the College of Information Science and Engineer, Northeastern University, 3 Wenhua Road, Heping District, Shenyang 110819, China (e-mails: youfuqiang@ise.neu.edu.cn, ChaoWang19941206@163.com).
* Corresponding author.

Time delay is widely present in industrial control systems [14]. The existence of time delay may make the performance of the system deteriorate or even cause the system to be unstable, which brings difficulties to the accurate analysis of the system. There are many results in research on time-delay systems [15–24]. In [15], the paper investigates the problem of fuzzy-approximation-based adaptive fault-tolerant tracking control for uncertain nonlinear time-varying delay systems. In [16], the author studies the fuzzy control problem of the uncertain time-delay active steering system with actuator fault. In [17], the author realizes the fault-tolerant control of uncertain time-delay system through adaptive sliding mode observer. In [18], the author studies the faults/states estimation and active fault-tolerant control of time-delay systems described by the T-S fuzzy model with external disturbances and actuator faults. In [19], the author uses the sliding mode observer to estimate the fault and states of the system with state and output delay. In [20], the authors studie the problem of simultaneously estimating the states and time delay of a nonlinear system with input delay. In [21], aiming at the unmanned ship with signal quantization and state time delay, the author studied its adaptive sliding mode fault-tolerant control problem. In [22], author studies an adaptive practical preassigned finite-time fault-tolerant control problem for a class of time-delay nonlinear systems. In [23], for a class of nonlinear systems , the author studied the problem of state estimation. The system in [23] suffers from unknown state delay and output delay. In [24], the author studies the distributed control problem of a class of uncertain nonlinear following time delay systems.

In recent years, the problem of fault estimation based on learning observers has attracted the attention of many scholars. The learning observer has the advantages of loose fault constraints and small calculations. The learning observer can also estimate the states of the system and the actuator faults at the same time. Based on the above advantages, it has strong practical significance to study the fault estimation problem of time-delay systems based on learning observer. In [25], the author uses iterative learning observer to realize the fault detection, estimation and compensation of time-delay nonlinear systems. Similar to Chen and Saif [25], Jia *et al*. [10], and Jia *et al*. [12] also use learning observers to implement fault estimation for time-delay systems. The above-mentioned fault estimation of the time-delay system uses proportional (P) learning observer and these articles assume that the time delay is constant or the time delay must be known in advance. However, the time delay of most systems is time-varying and unknowable. Therefore, the fault estimation problem of time-delay system with unknown time-varying delay based on proportional-differential (PD) learning observer studied in this paper has strong theoretical and practical significance. At present, the author has not seen the use of PD learning observer to realize the fault estimation result of linear system with unkown state time-varying delay. Compared with the P learning observer, the PD learning observer introduces the differential term of the output estimation error, so it is better than the P learning observer in the problem of fast-changing fault estimation. The main contributions of this article are as follows: 1) A novel PD learning observer is proposed for the system with unkown time-varying delay and the simultaneous estimation of actuator fault and system states is realized. 2) A design criterion of the PD learning observer is given, and the conditions of solving the gain matrices of the observer is given by the linear matrix inequalities (LMIs) technique. 3) Through the method of augmented system, the observer mentioned in the article is used to estimate the sensor fault.

Throughout the article, $A > 0 (A < 0)$ signifies that A is a positive (negative) definite matrix. $\lambda_{max}(\lambda_{min})$ is the maximum (minimum) eigenvalues of A. $*$ represents the symmetric term of the symmetric matrix. $||\cdot||$ and $||\cdot||_\infty$ signify euclidean norm and infinity norm of the vector or matrix, respectively. I_n represents the identity matrix and its dimension is n.

2. SYSTEM DESCRIPTION

Consider the following linear system with unknown state time-varying delay and actuator fault

$$\begin{cases} \dot{x}(t) = Ax(t) + A_\tau x(t-\tau(t)) + Bu(t) + Ef_a(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$ are system state, control input and measurement output vectors, respectively. $f_a(t) \in R^m$ denotes actuator fault, which can be constant or time-varying. Herein, A, A_τ, B, E, and C are known constant real matrices of appropriate dimensions. Suppose the matrix E is of full column rank, i.e. $rank(E) = m$ and (A,C) is observable. $\tau(t)$ is the unkown state time-varying delay satisfying $0 \le \tau(t) \le \tau$, $\dot{\tau}(t) \le \tau_m$, here τ and τ_m represent the upper limit of the time delay and the derivative of time delay, respectively.

Remark 1: In this article, we assume that delay $\tau(t)$ is differentiable, and we can know the maximum value of the delay and the maximum value of its reciprocal. The above assumptions are hold in many practical systems.

Three lemmas are introduced for the research behind this article:

Lemma 1: X and Y are matrices with appropriate dimensions. If the P is a positive-definite symmetric matrix, then the following inequality holds [26].

$$2X^TY \le X^TPX + Y^TP^{-1}Y. \tag{2}$$

Lemma 2: Integral inequality (3) holds when the following integral terms with respect to the vector function $x(s)$ are meaningful, where M is a arbitrary positive-definite symmetric constant matrix and $\gamma > 0$ [27].

$$\left(\int_0^{\gamma} x(s)ds\right)^T M \left(\int_0^{\gamma} x(s)ds\right) \leq \gamma \int_0^{\gamma} x^T(s)Mx(s)ds. \tag{3}$$

Lemma 3: Given a matrix X with appropriate dimensions satisfying $X^T \Pi X < 0$, there is a $\lambda > 0$ such that the following inequality holds [28].

$$X^T \Pi X < -2\lambda X - \lambda^2 \Pi^{-1}, \tag{4}$$

where Π is a negative matrix.

3. ACTUATOR FAULT ESTIMATION

3.1. Design of the PD learning observer

In order to estimate the system states and fault described by (1) at the same time, a PD learning observer is proposed as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_\tau \bar{x}(t) + Bu(t) + E\hat{f}_a(t) \\ \qquad + L(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t), \\ \hat{f}_a(t) = \hat{f}_a(t-d) + K(\sigma e_y(t) + \dot{e}_y(t-d)), \end{cases} \tag{5}$$

where $\hat{x}(t) \in R^n$ and $\hat{y}(t) \in R^p$ represent estimation of system state and output, respectively. $\bar{x}(t)$ is to be designed later. $\hat{f}_a(t)$ is the estimation of fault $f_a(t)$, which is decided by the $(t-d)$ moment $\hat{f}_a(t)$, current output estimation errors vector and its differential at the $(t-d)$ moment. The parameter d is called the learning interval. K and L are the observer gain matrices to be determined. δ is a positive number to be determined

According to [12], it can be known that the following assumptions are sufficient conditions for the existence of the proposed observer.

Assumption 1: From the observer expression (5), it can be seen that the differential term of output estimating error of the system is used. So we assume that the output of system (1) is differentiable.

Assumption 2: $rank(CE) = rank(E) = m$.

Assumption 3: $rank \begin{bmatrix} A - sI & E \\ C & 0 \end{bmatrix} = n + rank(E)$.

Assumption 4: The invariant zero point of (A, E, C) is in the left half plane of the open loop.

The state, measurable output and fault estimation error are defined as follows:

$$\begin{cases} e_x(t) = x(t) - \hat{x}(t), \\ e_y(t) = y(t) - \hat{y}(t), \\ e_f(t) = f_a(t) - \hat{f}_a(t), \end{cases} \tag{6}$$

then the error system are described by

$$\begin{aligned} \dot{e}_x(t) &= (A - LC)e_x(t) + A_\tau(x(t - \tau(t)) - \bar{x}(t)) \\ &\quad + E(f_a(t) - \hat{f}_a(t)), \\ e_y(t) &= Ce_x(t). \end{aligned} \tag{7}$$

Remark 2: Because the time delay $\tau(t)$ is unknowable, it is not available in the observer (5). This leads to the appearance of the mismatch term $x(t-\tau(t)) - \bar{x}(t)$ in the error system (7). The existence of $x(t-\tau(t)) - \bar{x}(t)$ brings challenges to the fault estimation of unknown time-delay systems. This motivates us to conduct research in this article.

To deal with $x(t-\tau(t)) - \bar{x}(t)$, the error system (7) will be rewritten. Here are the following definitions:

$$\omega(t) = \hat{x}(t - \tau(t)) - \bar{x}(t). \tag{8}$$

According to (8), the error system (7) can be rewritten as follows:

$$\begin{aligned} \dot{e}_x(t) &= (A - LC)e_x(t) + A_\tau e_x(t - \tau(t)) \\ &\quad + E(f_a(t) - \hat{f}(t)) + A_\tau \omega(t), \\ e_y(t) &= Ce_x(t), \end{aligned} \tag{9}$$

where $\omega(t)$ will be treated as an unknown bounded external disturbance.

In order to reduce the impact of $\omega(t)$ on fault estimation, the H_∞ performance index is introduced.

$$\begin{aligned} J = \int_0^\infty &(e_y^T(t)e_y(t) - (\eta_1 \vartheta(t))^T(\eta_1 \vartheta(t)) \\ &- (\eta_2 \vartheta(t))^T(\eta_2 \vartheta(t)))dt, \end{aligned} \tag{10}$$

where $\eta_1 = (\gamma_1 \;\; 0)$, $\eta_2 = (0 \;\; \gamma_2)$ and $\vartheta(t) = (\omega^T(t) \;\; \omega^T(t-d))^T$. $\gamma_1 > 0$ and $\gamma_2 > 0$ are constants indicating the degree of $\omega(t)$ attenuation level.

The objective of H_∞ fault estimation:

1) The error system expressed by (9) is progressively stable when $\vartheta(t) = 0$.

2) The given H_∞ performance index $J < 0$ holds when $\vartheta(t) \in L_2[0, \infty)$.

Next we discuss $\bar{x}(t)$ in Observer (5). Because we are interested in bounded systems, there are [23]

$$||\hat{x}(t_1) - \hat{x}(t_2)|| \leq \rho, \;\; |t_1 - t_2| \leq \rho_t. \tag{11}$$

In reality, many systems change slowly. This means that when ρ_t is small, ρ is a small constant. Therefore, for a slowly varying system with a time-varying delay $\tau(t)$ with a small upper limit τ, we have

$$||\hat{x}(t - \tau(t)) - \hat{x}(t - \bar{t})|| \leq \rho, \;\; \forall \bar{t} \in [0, \tau]. \tag{12}$$

Therefore, we can design $\bar{x}(t)$ into the following form:

$$\bar{x}(t) = \hat{x}(t - \tau/2). \tag{13}$$

The $\omega(t)$ in (9) can be written as follows:

$$\omega(t) = \hat{x}(t - \tau(t)) - \hat{x}(t - \tau/2). \tag{14}$$

Consider a linear system with unknown time delay $\tau(t) \in [0, \tau]$, where τ is a small value. Because the system works in a bounded situation, the design of $\bar{x}(t) = \hat{x}(t-\tau/2)$ can make $\omega(t) = \hat{x}(t-\tau(t)) - \hat{x}(t-\tau/2)$ satisfy $||\omega(t)|| \leq \rho$, where ρ is a small value.

We define

$$\omega_{max}(t) = max||\hat{x}(t-\bar{t}) - \hat{x}(t-\tau/2)|| \quad \bar{t} \in [0, \tau]. \tag{15}$$

From the definition of $\omega(t)$, $||\omega(t)|| \leq \omega_{max}$ $\forall t \geq 0$ can be obtained.

In order to estimate the states and fault simultaneously of the system (1), The following assumption hold.

Assumption 5: Suppose that $\left\|\tilde{f}_a(t)\right\|_\infty \leq k_f$, where $\tilde{f}_a(t) = f_a(t) - f_a(t-d)$ and k_f is a sufficiently small positive constant. $\tilde{f}_a(t)$ represents the difference of fault $f_a(t)$ within d time interval. Time interval d can be regarded as an unknown and adjustable number. The size of $\tilde{f}_a(t)$ can be controlled by choosing different d. The rules for selecting d are as follows: when the system fault $f_a(t)$ changes slowly, the larger d can be selected; when the system fault $f_a(t)$ changes quickly, the smaller d should be selected.

Remark 3: In [29], The author uses a adaptive fault estimation algorithm (FAFE) to implement fault estimation for time delay system. The requirement for fault in [29] is $||\dot{f}_a(t)|| \leq f_{max}$ where $0 \leq f_{max} \leq \infty$. Compared with [29], this paper has fewer constraints on faults because there is no requirement on the derivative of faults in this paper. Under Assumption 5, the proposed learning observer can realize the estimation of time-varying faults, especially for fast time-varying faults.

3.2. Stability analysis of the learning observer

In this part, refer to [30–33] on stability analysis of time-delay systems, the stability and convergence of the aforementioned learning observer will be demonstrated. The following theorem serves this purpose.

Theorem 1: For the given parameters γ_1, γ_2, τ, τ_m, ε, α, δ, μ if there are suitable positive definite symmetric matrices $P > 0$, $Q > 0$, $Q_1 > 0$, $Q_2 > 0$. $Q_3 > 0$, $Q_4 > 0$, $Q_5 > 0$, $R > 0$ and matrices $Y = PL$, K make the following equations (16), (17) and linear matrix inequalities (18), (19), (20), (21) and (22) are established, then when $\vartheta(t) = 0$, the error system (9) is progressively stable, and when $\vartheta(t) \in L_2[0,\infty)$, the error system (9) meets the given H_∞ performance index.

$$E^T P = KC, \tag{16}$$

$$KCE - I_3 = 0, \tag{17}$$

$$\begin{bmatrix} -Q & (A^TP - C^TY^T)E \\ * & -I_3 \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} -Q_3 & A_\tau^T C^T K^T \\ * & -I_3 \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} -Q_4 & (A^TP - C^TY^T)E \\ * & -2\alpha I + \alpha^2 E^T RE \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} -Q_5 & A_\tau^T C^T K^T \\ * & -2\alpha I + \alpha^2 E^T RE \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix}
\Omega_{11} & \Omega_{12} & 0 & PA_\tau & \Omega_{15} & \Omega_{16} & \Omega_{17} & 0 & 0 & \Omega_{110} & 0 \\
* & \Omega_{22} & R & 0 & 0 & \Omega_{26} & 0 & \Omega_{28} & 0 & 0 & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma_1^2 I & 0 & \Omega_{46} & 0 & 0 & \Omega_{49} & 0 & 0 \\
* & * & * & * & -\gamma_2^2 I & 0 & 0 & 0 & 0 & 0 & \Omega_{511} \\
* & * & * & * & * & \Omega_{66} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{88} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{99} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{1010} & 0 \\
* & * & * & * & * & * & * & * & * & * & \Omega_{1111}
\end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned}
\Omega_{11} &= PA + A^TP - C^TY^T - YC + Q + \varepsilon Q_1 + Q_2 \\
&\quad - R + \tau^2(16+4\delta)Q_4 + C^TC, \\
\Omega_{12} &= PA_\tau + R, \\
\Omega_{15} &= -PEKCA_\tau, \\
\Omega_{16} &= \Omega_{17} = \tau(A^TP - C^TY^T), \\
\Omega_{110} &= \sqrt{16\delta + 4\delta^2} C^T K^T E^T, \\
\Omega_{22} &= -\varepsilon(1-\tau_m)Q_1 - 2R + \frac{1}{(1-\tau_m)}Q_3 + \frac{\tau^2(16+\delta)}{1-\tau_m}Q_5, \\
\Omega_{26} &= \Omega_{28} = \tau A_\tau^T P, \\
\Omega_{33} &= -Q_2 - R, \\
\Omega_{46} &= \Omega_{49} = \tau A_\tau^T P, \\
\Omega_{511} &= \sqrt{16+4\delta} A_\tau^T C^T K^T E^T, \\
\Omega_{66} &= \Omega_{77} = \Omega_{88} = \Omega_{99} = -2\alpha P + \alpha^2 R, \\
\Omega_{1010} &= \Omega_{1111} = -2\alpha I + \alpha^2 R.
\end{aligned}$$

Then the gain matrix L of the learning observer can be obtained by the following formula (23).

$$L = P^{-1}Y. \tag{23}$$

Proof: The proof of Theorem 1 can be divide into the following three parts

Part 1: Construct a suitable Lyapunov-Krasovskii function.

Part 2: When $\vartheta(t)=0$, the asymptotic stability of the error system (9) is proved.

Part 3: The observer gain matrices L, K are calculated so that the error system (9) meets the proposed H_∞ performance index when $\vartheta(t)\in L_2[0,\infty)$.

Part 1: The Lyapunov-Krasovskii function is selected as follows:

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t)+V_6(t), \tag{24}$$

where

$$V_1(t)=e_x{}^T(t)Pe_x(t)+\int_{t-d}^{t} e_x{}^T(s)Qe_x(s)ds, \tag{25}$$

$$\begin{aligned} V_2(t)=&\varepsilon\int_{t-\tau(t)}^{t} e_x{}^T(s)Q_1e_x(s)ds\\ &+\int_{t-\tau}^{\mathrm{t}} e_x{}^T(s)Q_2e_x(s)ds, \end{aligned}\tag{26}$$

$$V_3(t)=\tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot e_x^T(s)R\dot e_x(s)dsd\theta, \tag{27}$$

$$V_4(t)=\frac{1}{1-\tau_m}\int_{t-\tau(t)-d}^{t-\tau(t)} e_x{}^T(s)Q_3e_x(s)ds, \tag{28}$$

$$V_5(t)=\tau^2(16+4\delta)\int_{t-d}^{t} e_x{}^T(s)Q_4e_x(s)ds, \tag{29}$$

$$V_6(t)=\frac{1}{1-\tau_m}\tau^2(16+4\delta)\int_{t-\tau(t)-d}^{t-\tau(t)} e_x{}^T(s)Q_5e_x(s)ds. \tag{30}$$

Then, the derivative of $V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t)+V_6(t)$ respect time t is as follows:

$$\begin{aligned} \dot V_1(t)=&e_x{}^T(t)[P(A-LC)+(A-LC)^TP+Q]e_x(t)\\ &+2e_x{}^T(t)PA_\tau e_x(t-\tau(t))\\ &+2e_x{}^T(t)PEf_a(t)-2e_x{}^TPE\hat f_a(t)\\ &+2e_x(t)^TPA_\tau\omega(t)-e_x{}^T(t-d)Qe_x(t-d), \end{aligned}\tag{31}$$

$$\begin{aligned} \dot V_2(t)=&-\varepsilon(1-\dot\tau(t))e_x{}^T(t-\tau(t))Q_1e_x(t-\tau(t))\\ &+\varepsilon e_x{}^T(t)Q_1e_x(t)\\ &+e_x{}^T(t)Q_2e_x(t)-e_x{}^T(t-\tau)Q_2e_x(t-\tau)\\ \le&-\varepsilon(1-\tau_m)e_x{}^T(t-\tau(t))Q_1e_x(t-\tau(t))\\ &+\varepsilon e_x{}^T(t)Q_1e_x(t)\\ &+e_x{}^T(t)Q_2e_x(t)-e_x{}^T(t-\tau)Q_2e_x(t-\tau), \end{aligned}\tag{32}$$

$$\dot V_3(t)=\tau^2\dot e_x^T(t)R\dot e_x(t)-\tau\int_{t-\tau}^{t}\dot e_x^T(s)R\dot e_x(s)ds, \tag{33}$$

$$\begin{aligned} \dot V_4(t)=&\frac{1-\tau(t)}{1-\tau_m}e_x{}^T(t-\tau(t))Q_3e_x(t-\tau(t))\\ &-\frac{1-\dot\tau(t)}{1-\tau_m}e_x{}^T(t-\tau(t)-d)Q_3e_x(t-\tau(t)-d)\\ \le&\frac{1}{1-\tau_m}e_x{}^T(t-\tau(t))Q_3e_x(t-\tau(t))\\ &-e_x{}^T(t-\tau(t)-d)Q_3e_x(t-\tau(t)-d), \end{aligned}\tag{34}$$

$$\begin{aligned} \dot V_5(t)=&\tau^2(16+4\delta)[e_x{}^T(t)Q_4e_x(t)\\ &-e_x{}^T(t-d)Q_4e_x(t-d)], \end{aligned}\tag{35}$$

$$\begin{aligned} \dot V_6(t)=&\frac{1-\dot\tau(t)}{1-\tau_m}\tau^2(16+4\delta)e_x{}^T(t-\tau(t))\\ &\times Q_5e_x(t-\tau(t))\\ &-\frac{1-\dot\tau(t)}{1-\tau_m}\tau^2(16+4\delta)e_x{}^T(t-\tau(t)-d)\\ &\times Q_5e_x(t-\tau(t)-d)\\ \le&\frac{\tau^2(16+4\delta)}{1-\tau_m}e_x{}^T(t-\tau(t))Q_5e_x(t-\tau(t))\\ &-\tau^2(16+4\delta)e_x{}^T(t-\tau(t)-d)\\ &\times Q_5e_x(t-\tau(t)-d). \end{aligned}\tag{36}$$

Bring the expression of $\hat f_a(t)$ in formula (5) and the expression of $\tilde f_a(t)$ in Assumption 5 into $\dot V_1(t)$, the following formula can be obtained:

$$\begin{aligned} \dot V_1(t)=&e_x{}^T(t)[P(A-LC)+(A-LC)^TP\\ &+Q-2\delta PEKC]e_x(t)\\ &+2e_x{}^T(t)PA_\tau e_x(t-\tau(t))\\ &+2e_x{}^T(t)PE(I_3-KCE)f_a(t-d)\\ &+2e_x{}^T(t)PE(KCE-I_3)\hat f_a(t-d)\\ &-2e_x{}^T(t)PEKC(A-LC)e_x(t-d)\\ &-2e_x{}^T(t)PEKCA_\tau e_x(t-\tau(t)-d)\\ &-2e_x{}^T(t)PEKCA_\tau\omega(t-d)\\ &+2e_x{}^T(t)PE\tilde f_a(t)\\ &-e_x{}^T(t-d)Qe_x(t-d)\\ &+2e_x{}^T(t)PA_\tau\omega(t). \end{aligned}\tag{37}$$

Since (17) holds, (37) can be further simplified into the following form:

$$\begin{aligned} \dot V_1(t)=&e_x{}^T(t)[P(A-LC)+(A-LC)^TP\\ &+Q-2\delta PEKC]e_x(t)\\ &+2e_x{}^T(t)PA_\tau e_x(t-\tau(t))\\ &-2e_x{}^T(t)PEKC(A-LC)e_x(t-d)\\ &-2e_x{}^T(t)PEKCA_\tau e_x(t-\tau(t)-d)\\ &-2e_x{}^T(t)PEKCA_\tau\omega(t-d)\\ &+2e_x{}^T(t)PE\tilde f_a(t)\\ &-e_x{}^T(t-d)Qe_x(t-d)\\ &+2e_x{}^T(t)PA_\tau\omega(t). \end{aligned}\tag{38}$$

According to Lemma 1, the following inequality can be obtained

$$\dot v_1(t)\le e_x{}^T(t)[P(A-LC)+(A-LC)^TP+Q$$

$$
\begin{aligned}
&-2\delta PEKC+3PEE^TP]e_x(t)\\
&+2{e_x}^T(t)PA_\tau e_x(t-\tau(t))\\
&+{e_x}^T(t-d)[(A-LC)^TC^TK^TKC(A-LC)\\
&-Q]e_x(t-d)\\
&+e_x^T(t-\tau(t)-d)A_\tau^TC^TK^TKCA_\tau e_x(t-\tau(t)-d)\\
&+\tilde{f}_a^T(t)\tilde{f}_a(t)\\
&-2{e_x}^T(t)PA_\tau\omega(t)\\
&+2{e_x}^T(t)PEKCA_\tau\omega(t-d). \qquad (39)
\end{aligned}
$$

Next, we deal with the item $\dot{e}_x^T(t)R\dot{e}_x(t)$. According to the definition of formula (6), the error system (9) can be written in the following form.

$$
\begin{cases}
\dot{e}_x(t)=(A-LC)e_x(t)+A_\tau e_x(t-\tau(t))\\
\qquad\quad +Ee_f(t)+A_\tau\omega(t),\\
e_y(t)=Ce_x(t).
\end{cases} \qquad (40)
$$

We bring the error system (40) into term $\dot{e}_x^T(t)R\dot{e}_x(t)$, and the following formula (41) can be obtained:

$$
\begin{aligned}
&\dot{e}_x^T(t)R\dot{e}_x(t)\\
&={e_x}^T(t)(A-LC)^TR(A-LC)e_x(t)\\
&\quad+{e_x}^T(t)(A-LC)^TRA_\tau e_x(t-\tau(t))\\
&\quad+{e_x}^T(t)(A-LC)^TREe_f(t)\\
&\quad+{e_x}^T(t)(A-LC)^TRA_\tau\omega(t)\\
&\quad+{e_x}^T(t-\tau(t))A_\tau^TR(A-LC)e_x(t)\\
&\quad+{e_x}^T(t-\tau(t))A_\tau^TRA_\tau{e_x}^T(t-\tau(t))\\
&\quad+{e_x}^T(t-\tau(t))A_\tau^TREe_f(t)\\
&\quad+{e_x}^T(t-\tau(t))A_\tau^TRA_\tau\omega(t)\\
&\quad+{e_f}^T(t)E^TR(A-LC)e_x(t)\\
&\quad+{e_f}^T(t)E^TR(A-LC)e_x(t)\\
&\quad+{e_f}^T(t)E^TRA_\tau e_x(t-\tau(t))\\
&\quad+{e_f}^T(t)E^TREe_f(t)\\
&\quad+{e_f}^T(t)E^TRA_\tau\omega(t)\\
&\quad+\omega^T(t)A_\tau^TR(A-LC)e_x(t)\\
&\quad+\omega^T(t)A_\tau^TRA_\tau e_x(t-\tau(t))\\
&\quad+\omega^T(t)A_\tau^TREe_f(t)\\
&\quad+\omega^T(t)A_\tau^TRA_\tau\omega(t). \qquad (41)
\end{aligned}
$$

From Lemma 1, the following inequality (42) can be obtained:

$$
\begin{aligned}
\dot{e}_x^T(t)R\dot{e}_x(t)\le&{e_x}^T(t)(A-LC)^TR(A-LC)e_x(t)\\
&+2{e_x}^T(t)(A-LC)^TRA_\tau e_x(t-\tau(t))\\
&+2{e_x}^T(t)(A-LC)^TRA_\tau\omega(t)\\
&+{e_x}^T(t-\tau(t))A_\tau^TRA_\tau e_x(t-\tau(t))\\
&+2{e_x}^T(t-\tau(t))A_\tau^TRA_\tau\omega(t)\\
&+\omega^T(t)A_\tau^TRA_\tau\omega(t)\\
&+{e_x}^T(A-LC)^TR(A-LC)e_x(t)\\
&+{e_x}^T(t-\tau(t))A_\tau^TRA_\tau e_x(t-\tau(t))\\
&+\omega^T(t)A_\tau^TRA_\tau\omega(t)\\
&+4{e_f}^T(t)E^TREe_f(t). \qquad (42)
\end{aligned}
$$

According to (16), $e_f(t)$ can be written as $e_f(t)=\tilde{f}_a(t)+\delta KCe_x(t)-KC(A-LC)e_x(t-d)-KCA_\tau e_x(t-\tau(t)-d)-KCA_\tau\omega(t-d)$. Further formula (43) can be obtained.

$$
\begin{aligned}
&{e_f}^T(t)E^TEe_f(t)\\
&=\tilde{f}_a^T(t)E^TRE\tilde{f}_a(t)\\
&\quad-2\delta{e_x}^T(t)C^TK^TE^TRE\tilde{f}_a(t)\\
&\quad-2\tilde{f}_a^T(t)E^TREKC(A-LC)e_x(t-d)\\
&\quad-2\tilde{f}_a^T(t)E^TREKCA_\tau e_x(t-\tau(t)-d)\\
&\quad-2\tilde{f}_a^T(t)E^TREKCA_\tau\omega(t-d)\\
&\quad+\delta^2{e_x}^T(t)C^TK^TE^TREKCe_x(t)\\
&\quad+2\delta{e_x}^T(t)C^TK^TE^TREKC(A-LC)e_x(t-d)\\
&\quad+2\delta{e_x}^T(t)C^TK^TE^TREKCA_\tau e_x(t-\tau(t)-d)\\
&\quad+2\delta{e_x}^T(t)C^TK^TE^TREKCA_\tau\omega(t-d)\\
&\quad+{e_x}^T(t-d)(A-LC)^TC^TK^TE^TREKC(A-LC)\\
&\quad\times e_x(t-d)\\
&\quad+2{e_x}^T(t-d)(A-LC)^TC^TK^TE^TREKCA_\tau\\
&\quad\times e_x(t-\tau(t)-d)\\
&\quad+2{e_x}^T(t-d)(A-LC)^TC^TK^TE^TREKCD\omega(t-d)\\
&\quad+{e_x}^T(t-\tau(t)-d){A_\tau}^TC^TK^TE^TREKCA_\tau\\
&\quad\times e_x(t-\tau(t)-d)\\
&\quad+2{e_x}^T(t-\tau(t)-d){A_\tau}^TC^TK^TE^TREKCA_\tau\omega(t-d)\\
&\quad+\omega^T(t-d)A_\tau^TC^TE^TREKCA_\tau\omega(t-d). \qquad (43)
\end{aligned}
$$

According to Lemma 1, formula (44) can be obtained.

$$
\begin{aligned}
&{e_f}^T(t)E^TREe_f(t)\\
&\le(4+\delta)\tilde{f}_a^T(t)E^TRE\tilde{f}_a(t)\\
&\quad+(4\delta+\delta^2){e_x}^T(t)C^TK^TE^TREKCe_x(t)\\
&\quad+(4+\delta){e_x}^T(t-d)(A-LC)^T\\
&\quad\times C^TK^TE^TREKC(A-LC)e_x(t-d)\\
&\quad+(4+\delta){e_x}^T(t-\tau(t)-d)\\
&\quad\times{A_\tau}^TC^TK^TE^TREKCA_\tau e_x(t-\tau(t)-d)\\
&\quad+(4+\delta)\omega^T(t-d)D^TC^TK^TE^TREKCD\omega(t-d).
\end{aligned} \qquad (44)
$$

Further formula (45) can be obtained.

$$
\begin{aligned}
&\dot{e}_x^T(t)R\dot{e}_x(t)\\
&\le{e_x}^T(t)(A-LC)^TR(A-LC)e_x(t)\\
&\quad+2{e_x}^T(t)(A-LC)^TRA_\tau e_x(t-\tau(t))
\end{aligned}
$$

$$
\begin{aligned}
&+2{e_x}^T(t)(A-LC)^T RD\omega(t)\\
&+{e_x}^T(t-\tau(t))A_\tau RA_\tau e_x(t-\tau(t))\\
&+2{e_x}^T(t-\tau(t))A_\tau RD\omega(t)\\
&+\omega^T(t)D^T RD\omega(t)\\
&+{e_x}^T(t)(A-LC)^T R(A-LC)e_x(t)\\
&+{e_x}^T(t-\tau(t))A_\tau RA_\tau e_x(t-\tau(t))\\
&+\omega^T(t)D^T RD\omega(t)\\
&+(16\delta+4\delta^2){e_x}^T(t)C^T E^T K^T E^T REKCe_x(t)\\
&+(16+4\delta){e_x}^T(t-d)(A-LC)^T\\
&\times C^T K^T E^T REKC(A-LC)e_x(t-d)\\
&+(16+4\delta){e_x}^T(t-\tau(t)-d)\\
&\times {A_\tau}^T C^T K^T E^T REKCA_\tau e_x(t-\tau(t)-d)\\
&+(16+4\delta)\omega^T(t-d)\\
&\times D^T C^T K^T E^T REKCD\omega(t-d)\\
&+(16+4\delta)\tilde{f}_a^T(t)E^T RE\tilde{f}_a(t).
\end{aligned}
\tag{45}
$$

According to Lemma 2, formula (46) can be obtained.

$$
\begin{aligned}
-\tau\int_{-\tau}^{t}\dot{e}_x^T(s)R\dot{e}_x(s)ds \le &-{e_x}^T(t)Re_x(t)\\
&+2{e_x}^T(t)Re_x(t-\tau(t))\\
&-2{e_x}^T(t-\tau(t))Re_x(t-\tau(t))\\
&+2{e_x}^T(t-\tau(t))Re_x(t-\tau)\\
&-{e_x}^T(t-\tau)Re_x(t-\tau).
\end{aligned}
\tag{46}
$$

According to Shure complement lemma: ${e_x}^T(t-d)\ [(A-LC)^T C^T K^T KC(A-LC)-Q]e_x(t-d)<0$ is equivalent to

$$
\begin{bmatrix} -Q & (A^T P - C^T Y^T)E \\ * & -I_3 \end{bmatrix} < 0. \tag{47}
$$

${e_x}^T(t-\tau(t)-d)[{A_\tau}^T C^T K^T KCA_\tau - Q_3]e_x(t-\tau(t)-d)<0$ is equivalent to

$$
\begin{bmatrix} -Q_3 & A_\tau^T C^T K^T \\ * & -I_3 \end{bmatrix} < 0. \tag{48}
$$

Here, it is assumed that $\delta \ge 1.5$ is satisfied, then formula (49) can be obtained.

$$
\begin{aligned}
\dot{V}(t) \le &{e_x}^T(t)[P(A-LC)+(A-LC)^T P+Q\\
&+\varepsilon Q_1+Q_2-R+\tau^2(16+4\delta)Q_4]e_x(t)\\
&+2{e_x}^T(t)[PA_\tau+R]e_x(t-\tau(t))\\
&+2{e_x}^T(t)PA_\tau\omega(t)\\
&-{e_x}^T(t-\tau(t))[\varepsilon(1-\tau_m)Q_1+2R\\
&-\frac{1}{1-\tau_m}Q_3-\frac{\tau^2(16+4\delta)}{1-\tau_m}Q_5]e_x(t-\tau(t))\\
&+2{e_x}^T(t-\tau(t))Re_x(t-\tau)\\
&-{e_x}^T(t-\tau)[Q_2+R]e_x(t-\tau)\\
&-2{e_x}^T(t)PEKCA_\tau\omega(t-d)\\
&+\tilde{f}_a^T(t)\tilde{f}_a(t)+\Xi,
\end{aligned}
\tag{49}
$$

where $\Xi=\tau^2\dot{e}_x^T(t)R\dot{e}_x(t)-\tau^2(16+4\delta){e_x}^T(t-d)Q_4e_x(t-d)-\tau^2(16+4\delta){e_x}^T(t-\tau(t)-d)Q_5e_x(t-\tau(t)-d)$. According to Shure complement lemma, we can get: $\tau^2(16+4\delta){e_x}^T(t-d)[(A-LC)^T C^T K^T E^T REKC(A-LC)-Q_4]e_x(t-d)<0$ is equivalent to

$$
\begin{bmatrix} -Q & (A^T P - C^T Y^T)E \\ * & -(E^T RE)^{-1} \end{bmatrix} < 0. \tag{50}
$$

$\tau^2(16+4\delta){e_x}^T(t-\tau(t)-d)[A_\tau^T C^T K^T E^T REKCA_\tau - Q_5]e_x(t-\tau(t)-d)<0$ is equivalent to

$$
\begin{bmatrix} -Q_3 & A_\tau^T C^T K^T \\ * & -(E^T RE)^{-1} \end{bmatrix} < 0. \tag{51}
$$

Further the formula (52) can be obtaine.

$$
\begin{aligned}
\Xi \le &\tau^2[{e_x}^T(t)(A-LC)^T R(A-LC)e_x(t)\\
&+2{e_x}^T(t)(A-LC)^T RA_\tau e_x(t-\tau(t))\\
&+2{e_x}^T(t)(A-LC)^T RA_\tau\omega(t)\\
&+{e_x}^T(t-\tau(t))A_\tau^T RA_\tau e_x(t-\tau(t))\\
&+2{e_x}^T(t-\tau(t))A_\tau^T RA_\tau\omega(t)+\omega^T(t)A_\tau^T RA_\tau\omega(t)\\
&+{e_x}^T(t)(A-LC)^T R(A-LC)e_x(t)\\
&+{e_x}^T(t-\tau(t))A_\tau^T RA_\tau e_x(t-\tau(t))\\
&+\omega^T(t)A_\tau^T RA_\tau\omega(t)+(16\delta\\
&+4\delta^2){e_x}^T(t)C^T K^T E^T REKCe_x(t)\\
&+(16+4\delta)\omega^T(t-d)A_\tau^T C^T K^T E^T REKCA_\tau\omega(t-d)\\
&+(16+4\delta)\tilde{f}_a^T(t)E^T RE\tilde{f}_a(t)].
\end{aligned}
\tag{52}
$$

Part 2: If $\vartheta(t)=0$ and $f_a(t)=0$ are established, (49) can be simplified to:

$$
\begin{aligned}
\dot{V}(t) \le &{e_x}^T(t)[P(A-LC)+(A-LC)^T P+Q\\
&+\varepsilon Q_1+Q_2-R+\tau^2(16+4\delta)Q_4]e_x(t)\\
&+2{e_x}^T(t)[PA_\tau+R]e_x(t-\tau(t))\\
&-{e_x}^T(t-\tau(t))\Bigg[\varepsilon(1-\tau_m)Q_1+2R-\frac{1}{1-\tau_m}Q_3\\
&-\frac{\tau^2(16+4\delta)}{1-\tau_m}Q_5\Bigg]e_x(t-\tau(t))\\
&+2{e_x}^T(t-\tau(t))Re_x(t-\tau)\\
&-{e_x}^T(t-\tau)[Q_2+R]e_x(t-\tau)+\Xi',
\end{aligned}
\tag{53}
$$

where

$$
\begin{aligned}
\Xi' \le &\tau^2[{e_x}^T(t)(A-LC)^T R(A-LC)e_x(t)\\
&+2{e_x}^T(t)(A-LC)^T RA_\tau e_x(t-\tau(t))\\
&+{e_x}^T(t-\tau(t))A_\tau RA_\tau e_x(t-\tau(t))\\
&+{e_x}^T(t)(A-LC)^T R(A-LC)e_x(t)
\end{aligned}
$$

$$+e_x{}^T(t-\tau(t))A_\tau RA_\tau e_x(t-\tau(t))$$
$$+(16\delta+4\delta^2)e_x{}^T(t)C^TK^TE^TREKCe_x(t)]. \quad (54)$$

Let $\xi'^T(t)=[e_x{}^T(t) \quad e_x{}^T(t-\tau(t)) \quad e_x{}^T(t-\tau)]$, further formula (53) can be written as

$$\dot{V}(t) \le \xi'^T(t)\{\Omega_1' + \tau^2[\Gamma_1'^T R\Gamma_1' + \Gamma_2'^T R\Gamma_2' + \Gamma_3'^T R\Gamma_3' + (16\delta+4\delta^2)\Gamma_4'^T R\Gamma_4']\}\xi'(t), \quad (55)$$

where

$$\Gamma_1{}' = [A-LC \quad A_\tau \quad 0],$$
$$\Gamma_2{}' = [A-LC \quad 0 \quad 0],$$
$$\Gamma_3{}' = [0 \quad A_\tau \quad 0],$$
$$\Gamma_4{}' = [EKC \quad 0 \quad 0],$$
$$\Omega_1' = \begin{bmatrix} \Omega_{11}' & PA_\tau+R & 0 \\ * & \Omega_{22}' & R \\ * & * & -Q_2-R \end{bmatrix},$$
$$\Omega'_{11} = PA + A^TP - C^TY^T - YC + Q + \varepsilon Q_1 + Q_2 - R + \tau^2(16+4\delta)Q_4,$$
$$\Omega'_{22} = -\varepsilon(1-\tau_m)Q_1 - 2R + \frac{1}{(1-\tau_m)}Q_3 + \frac{\tau^2(16+\delta)}{1-\tau_m}Q_5.$$

If there is a suitable solution to inequality (55), then $\dot{V}(t)<0$ holds when $\vartheta(t)=0$ and $f_a(t)=0$. According to the Lyapunov stability theory, the error system is asymptotically stable.

Part 3: When $f_a(t)\neq 0$ and $\vartheta(t)\in L_2[0,\infty)$, in order to suppress the influence of $\omega(t)$ on the fault estimation, we consider the following new Lyapunov-Krasovskii function $V_0(t)$.

$$V_0(t) = V(t) + e_y^T(t)e_y(t) - (\eta_1\vartheta(t))^T(\eta_1\vartheta(t)) - (\eta_2\vartheta(t))^T(\eta_2\vartheta(t)). \quad (56)$$

Let $\xi^T(t)=[e_x{}^T(t) \quad e_x{}^T(t-\tau(t)) \quad e_x{}^T(t-\tau) \quad \omega(t) \quad \omega(t-d)]$ and $\lambda=\lambda_{max}(E^TRE)$, (57) can be obtained.

$$\Xi \le \xi^T(t)\tau^2(\Gamma_1{}^T R\Gamma_1 + \Gamma_2{}^T R\Gamma_2 + \Gamma_3{}^T R\Gamma_3 + \Gamma_4{}^T R\Gamma_4 + (16\delta+4\delta^2)\Gamma_5{}^T R\Gamma_5 + (16+4\delta)\Gamma_6{}^T R\Gamma_6)\xi(t) + \tau^2(16+4\delta)\lambda\tilde{f}_a^T(t)\tilde{f}_a(t), \quad (57)$$

where

$$\Gamma_1 = [A-LC \quad A_\tau \quad 0 \quad A_\tau \quad 0],$$
$$\Gamma_2 = [A-LC \quad 0 \quad 0 \quad 0 \quad 0],$$
$$\Gamma_3 = [0 \quad A_\tau \quad 0 \quad 0 \quad 0],$$
$$\Gamma_4 = [0 \quad 0 \quad 0 \quad A_\tau \quad 0],$$
$$\Gamma_5 = [EKC \quad 0 \quad 0 \quad 0 \quad 0],$$
$$\Gamma_2 = [0 \quad 0 \quad 0 \quad 0 \quad EKCA_\tau],$$

then

$$\begin{aligned}\dot{V}_0(t) =& \dot{V}(t) + e_y^T(t)e_y(t) - (\eta_1\vartheta(t))^T(\eta_1\vartheta(t)) - (\eta_2\vartheta(t))^T(\eta_2\vartheta(t)) \\ \le& \xi^T(t)[\Omega_1 + \tau^2(\Gamma_1{}^T R\Gamma_1 + \Gamma_2{}^T R\Gamma_2 + \Gamma_3{}^T R\Gamma_3 + \Gamma_4{}^T R\Gamma_4 + (16\delta+4\delta^2)\Gamma_5{}^T R\Gamma_5 + (16+4\delta)\Gamma_6{}^T R\Gamma_6)]\xi(t) \\ &+ [1+\tau^2(16+4\delta)]\lambda\tilde{f}_a^T(t)\tilde{f}_a(t) \\ =& \xi^T(t)\Omega\xi(t) + [1+\tau^2(16+4\delta)]\lambda\tilde{f}_a^T(t)\tilde{f}_a(t),\end{aligned} \quad (58)$$

where

$$\Omega = \Omega_1 + \tau^2(\Gamma_1{}^T R\Gamma_1 + \Gamma_2{}^T R\Gamma_2 + \Gamma_3{}^T R\Gamma_3 + \Gamma_4{}^T R\Gamma_4 + (16\delta+4\delta^2)\Gamma_5{}^T R\Gamma_5 + (16+4\delta)\Gamma_6{}^T R\Gamma_6), \quad (59)$$
$$\Omega_1 = \begin{bmatrix} \Omega_{11} & PA_\tau+R & 0 & PA_\tau & -PEKCA_\tau \\ * & \Omega_{22} & R & 0 & 0 \\ * & * & -Q_2-R & 0 & 0 \\ * & * & * & -\gamma_1^2 I & 0 \\ * & * & * & * & -\gamma_2^2 I \end{bmatrix},$$
$$\Omega_{11} = PA + A^TP - C^TY^T - YC + Q + \varepsilon Q_1 + Q_2 - R + \tau^2(16+4\delta)Q_4 + C^TC,$$
$$\Omega_{22} = -\varepsilon(1-\tau_m)Q_1 - 2R + \frac{1}{(1-\tau_m)}Q_3 + \frac{\tau^2(16+4\delta)}{1-\tau_m}Q_5.$$

If $\Omega<0$ holds, then

$$\begin{aligned}\dot{V}_0(t) =& \dot{V}(t) + e_y^T(t)e_y(t) - (\eta_1\vartheta(t))^T(\eta_1\vartheta(t)) - (\eta_2\vartheta(t))^T(\eta_2\vartheta(t)) \\ \le& -\varphi||\xi(t)||_2^2 + \beta,\end{aligned} \quad (60)$$

where $\varphi=\lambda_{min}(-\Omega)$, $\beta=[1+\tau^2(16+4\delta)\lambda]K_f{}^2$. Therefore, when $\varphi||\xi(t)||_2^2>\beta$, $\dot{V}(t)+e_y^T(t)e_y(t)-(\eta_1\vartheta(t))^T(\eta_1\vartheta(t))-(\eta_2\vartheta(t))^T(\eta_2\vartheta(t))<0$ holds. According to Lyapunov stability theory, any trajectory in $\xi(t)$ outside the stable region

$$\Psi = \{\xi(t) \mid ||\xi(t)||_2^2 \le \frac{\beta}{\varphi}\}$$

will converge to Ψ. Further according to the LaSalle invariant set principle [34], the system state estimation errors $e_x(t)$ and the fault estimation errors $e_f(t)$ are finally uniformly bounded.

After applying Schur complements lemma to (59), equation (61) can be obtained.

$$\begin{bmatrix} \Omega_1 & \rho_{12} & \rho_{13} & \rho_{14} & \rho_{15} & \rho_{16} & \rho_{17} \\ * & \rho_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \rho_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \rho_{44} & 0 & 0 & 0 \\ * & * & * & * & \rho_{55} & 0 & 0 \\ * & * & * & * & * & -R^{-1} & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{61}$$

where

$$\begin{aligned} &\rho_{12} = \tau\Gamma_1^T P, \\ &\rho_{13} = \tau\Gamma_2^T P, \\ &\rho_{14} = \tau\Gamma_3^T P, \\ &\rho_{15} = \tau\Gamma_4^T P, \\ &\rho_{16} = (16\delta + 4\delta^2)\tau\Gamma_5^T, \\ &\rho_{17} = (16 + 4\delta)\tau\Gamma_6^T, \\ &\rho_{22} = \rho_{33} = \rho_{44} = \rho_{55} = -PRP^{-1}. \end{aligned}$$

According to Lemma 3, these inequalities can be obtained as follows:

$$-PR^{-1}P \le -2\alpha P + \alpha^2 R, \tag{62}$$

$$-R^{-1} \le -2\alpha I + \alpha^2 R, \tag{63}$$

$$-(E^T RE)^{-1} \le -2\alpha I + \alpha^2 E^T RE. \tag{64}$$

Then, (50) is equivalent to

$$\begin{bmatrix} -Q & (A^T P - C^T Y^T)E \\ * & -2\alpha I + \alpha^2 E^T RE \end{bmatrix} < 0, \tag{65}$$

equation (51) is equivalent to

$$\begin{bmatrix} -Q_3 & A_\tau^T C^T K^T \\ * & -2\alpha I + \alpha^2 E^T RE \end{bmatrix} < 0, \tag{66}$$

equation (61) is equivalent to

$$\begin{bmatrix} \Omega_1 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} & \Lambda_{17} \\ * & \Lambda_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Lambda_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Lambda_{44} & 0 & 0 & 0 \\ * & * & * & * & \Lambda_{55} & 0 & 0 \\ * & * & * & * & * & \Lambda_{66} & 0 \\ * & * & * & * & * & * & \Lambda_{77} \end{bmatrix} < 0, \tag{67}$$

where

$$\begin{aligned} &\Lambda_{12} = \tau\Gamma_1^T P, \\ &\Lambda_{13} = \tau\Gamma_2^T P, \\ &\Lambda_{14} = \tau\Gamma_3^T P, \\ &\Lambda_{15} = \tau\Gamma_4^T P, \\ &\Lambda_{16} = (16\delta + 4\delta^2)\tau\Gamma_5^T, \\ &\Lambda_{17} = (16 + 4\delta)\tau\Gamma_6^T, \\ &\Lambda_{22} = \Lambda_{33} = \Lambda_{44} = \Lambda_{55} = -2\alpha P + \alpha^2 R, \\ &\Lambda_{66} = \Lambda_{77} = -2\alpha I + \alpha^2 R. \end{aligned}$$

The proof is completed. □

Remark 4: Because of the existence of (16), it is difficult to solve Theorem 1 in matlab. Here we transform equation (16) into the following optimization problem.

$$\begin{aligned} &\min \\ &\text{s.t.} \begin{bmatrix} \mu I & E^T P - KC \\ * & \mu I \end{bmatrix} > 0. \end{aligned} \tag{68}$$

In order to make $E^T P$ approximate to KC with a satisfactory precision, a sufficiently small positive scalar μ should be selected to meet (16).

Based on Theorem 1, the design steps of the PD learning observer are given as follows:

1) According to equation (17), the observer gain matrix K is calculated.

2) Select the appropriate parameters μ and δ.

3) solve (18), (19), (20), (21), (22), and (68) using MATLAB LMI toolbox; then, matrices P, Q, Q_1, Q_2, Q_3, Q_4, Q_5 and Y can be obtained.

4) Calculate the observer gain matrix L by $L = P^{-1}Y$.

5) Choose an appropriate learning interval d, and then construct the PD learning observer shown in (5) according to the obtained observer gain matrices.

Based on Theorem 1, the following corollary can be obtained.

Corollary 1: For the given parameters γ_1, γ_2, τ, τ_m, ε, α, δ, μ, if there are suitable positive definite symmetric matrices $P > 0$, $Q > 0$, $Q_1 > 0$, $Q_2 > 0$. $Q_3 > 0$, $Q_4 > 0$, $Q_5 > 0$, $R > 0$ and matrices $Y = PL$, K make (16), (17) and linear matrix inequalities (18), (19), (20), (21), and (22) are established, then, the designed PD learning observer can reconstruct constant faults.

4. SENSOR FAULT ESTIMATION

As far as the author knows, there is no result of using PD learning observer to realize sensor faults estimation of systems with unkown state time-varying delay. This motivates us to extend the PD learning observer proposed in Section 3 to achieve the reconstruction of sensor faults in continuous-time systems. Consider the following linear system with sensor fault and unkown state time-varying delay.

$$\begin{cases} \dot{x}(t) = Ax(t) + A_\tau x(t - \tau(t)) + Bu(t), \\ y(t) = Cx(t) + Gf_s(t), \end{cases} \tag{69}$$

where $G \in R^{p\times r}$ and $f_s(t) \in R^r$ representing fault distribution matrix and sensor fault, respectively. The definitions of the remaining matrices and vectors are the same as in (1).

In [35], by constructing an augmented system, the problem of sensor fault estimation is transformed into the form of actuator fault estimation. With [35], we construct an augmented system and then designed an augmented PD learning observer for the augmented system to realize sensor fault estimation. To this end, consider a new state $x_x(t) \in R^p$ that is a filtered version of $y(t)$.

$$\dot{x}_x(t) = -A_x x_x(t) + A_x y(t), \tag{70}$$

where $-A_x$ is a Hurwitz matrix.

The following augmented system can be obtained by combining (69) and (70):

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_\tau \bar{x}(t-\tau(t)) + \bar{B}u(t) + \bar{G}f_s(t), \\ \bar{y}(t) = \bar{C}\bar{x}(t), \end{cases} \tag{71}$$

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ x_x(t) \end{bmatrix}, \ \bar{A} = \begin{bmatrix} A & 0 \\ A_x C & -A_x \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$
$$\bar{G} = \begin{bmatrix} 0 \\ A_x G \end{bmatrix}, \ \bar{x}(t-\tau(t)) = \begin{bmatrix} x(t-\tau(t)) \\ 0 \end{bmatrix},$$
$$\bar{A}_\tau = \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{C} = \begin{bmatrix} 0 & I_p \end{bmatrix}.$$

It is easy to verify that $(\bar{A}, \bar{C})$ is observable when (A, C) is observable.

In order to estimate the states and fault of the system (71), an augmented learning observer design is as follows:

$$\begin{cases} \dot{\hat{\bar{x}}}(t) = \bar{A}\hat{\bar{x}}(t) + \bar{A}_\tau \bar{x}_1(t) + \bar{B}u(t) + \bar{G}\hat{f}_s(t) \\ \qquad\quad + \bar{L}(\bar{y}(t) - \hat{\bar{y}}(t)), \\ \hat{\bar{y}}(t) = \bar{C}\hat{\bar{x}}(t), \\ \hat{f}_s(t) = \hat{f}_s(t-d) + \bar{K}(\bar{\delta}\bar{e}_y(t) + \dot{\bar{e}}_y(t-d)), \end{cases} \tag{72}$$

where $\hat{\bar{x}}(t) \in R^{n+p}, \hat{\bar{y}}(t) \in R^p$ and $\hat{f}_s(t) \in R^r$ are estimation of the augmented states, output and sensor fault. $\bar{K}$ and $\bar{L}$ are the observer gain matrices to be determined. $\bar{\delta}$ is a constant to be confirmed. $\bar{x}_1(t)$ has the same definition as $\bar{x}(t)$ in (5).

To ensure the stability and convergence of the learning observer the following Assumption 6 must be satisfied.

Assumption 6: Suppose that $\|\tilde{f}_s(t)\|_\infty \le k_s$, where $\tilde{f}_s(t) = f_s(t) - f_s(t-d)$ and k_s is a sufficiently small positive constant.

Theorem 2: For the given parameters γ_1, γ_2, τ, τ_m, ε, α, $\bar{\delta}$, μ if there are suitable positive definite symmetric matrices $\bar{P} > 0, \bar{Q} > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0$. $\bar{Q}_3 > 0, \bar{Q}_4 > 0, \bar{Q}_5 > 0$, $\bar{R} > 0$ and matrices $\bar{Y} = \bar{P}\bar{L}$, $\bar{K}$ make the following equations (73), (74) and linear matrix inequalities (75), (76), (77), (78), and (79) are established, then the PD learning observer (72) can estimate sensor fault.

$$\bar{G}^T\bar{P} = \bar{K}\bar{C}, \tag{73}$$

$$\bar{K}\bar{C}\bar{G} - I = 0, \tag{74}$$

$$\begin{bmatrix} -\bar{Q} & (\bar{A}^T\bar{P} - \bar{C}^T\bar{Y}^T)\bar{G} \\ * & -I \end{bmatrix} < 0, \tag{75}$$

$$\begin{bmatrix} -\bar{Q}_3 & \bar{A}_\tau^T\bar{C}^T\bar{K}^T \\ * & -I \end{bmatrix} < 0, \tag{76}$$

$$\begin{bmatrix} -\bar{Q}_4 & (\bar{A}^T\bar{P} - \bar{C}^T\bar{Y}^T)\bar{G} \\ * & -2\alpha I + \alpha^2\bar{G}^T\bar{R}\bar{G} \end{bmatrix} < 0, \tag{77}$$

$$\begin{bmatrix} -\bar{Q}_5 & \bar{A}_\tau^T\bar{C}^T\bar{K}^T \\ * & -2\alpha I + \alpha^2\bar{G}^T\bar{R}\bar{G} \end{bmatrix} < 0, \tag{78}$$

$$\begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & 0 & \bar{P}\bar{A}_\tau & \bar{\Omega}_{15} & \bar{\Omega}_{16} & \bar{\Omega}_{17} & 0 & 0 & \bar{\Omega}_{110} & 0 \\ * & \bar{\Omega}_{22} & \bar{R} & 0 & 0 & \bar{\Omega}_{26} & 0 & \bar{\Omega}_{28} & 0 & 0 & 0 \\ * & * & \bar{\Omega}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma_1^2 I & 0 & \bar{\Omega}_{46} & 0 & 0 & \bar{\Omega}_{49} & 0 & 0 \\ * & * & * & * & -\gamma_2^2 I & 0 & 0 & 0 & 0 & 0 & \bar{\Omega}_{511} \\ * & * & * & * & * & \bar{\Omega}_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \bar{\Omega}_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \bar{\Omega}_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\Omega}_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \bar{\Omega}_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \bar{\Omega}_{1111} \end{bmatrix} < 0, \tag{79}$$

where

$$\bar{\Omega}_{11} = \bar{P}\bar{A} + \bar{A}^T\bar{P} - \bar{C}^T\bar{Y}^T - \bar{Y}\bar{C} + \bar{Q} + \varepsilon\bar{Q}_1 + \bar{Q}_2 - \bar{R} + \tau^2(16 + 4\bar{\delta})\bar{Q}_4 + \bar{C}^T\bar{C},$$
$$\bar{\Omega}_{12} = \bar{P}\bar{A}_\tau + \bar{R},$$
$$\bar{\Omega}_{15} = -\bar{P}\bar{G}\bar{K}\bar{C}\bar{A}_\tau,$$
$$\bar{\Omega}_{16} = \bar{\Omega}_{17} = \tau(\bar{A}^T\bar{P} - \bar{C}^T\bar{Y}^T),$$
$$\bar{\Omega}_{110} = \sqrt{16\bar{\delta} + 4\bar{\delta}^2}C^T\bar{K}^T\bar{G}^T,$$
$$\bar{\Omega}_{22} = -\varepsilon(1 - \tau_m)\bar{Q}_1 - 2\bar{R} + \frac{1}{(1-\tau_m)}\bar{Q}_3 + \frac{\tau^2(16 + 4\bar{\delta})}{1 - \tau_m}\bar{Q}_5,$$
$$\bar{\Omega}_{26} = \bar{\Omega}_{28} = \tau\bar{A}_\tau^T\bar{P},$$
$$\bar{\Omega}_{33} = -\bar{Q}_2 - \bar{R},$$
$$\bar{\Omega}_{46} = \bar{\Omega}_{49} = \tau\bar{A}_\tau^T\bar{P},$$
$$\bar{\Omega}_{511} = \sqrt{16 + 4\bar{\delta}}\bar{A}_\tau^T\bar{C}^T\bar{K}^T\bar{G}^T,$$
$$\bar{\Omega}_{66} = \bar{\Omega}_{77} = \bar{\Omega}_{88} = \bar{\Omega}_{99} = -2\alpha\bar{P} + \alpha^2\bar{R},$$

$$\bar{\Omega}_{1010} = \bar{\Omega}_{1111} = -2\alpha I + \alpha^2 \bar{R}.$$

Then the gain matrix $\bar{L}$ of the learning observer can be obtained as follows:

$$\bar{L} = \bar{P}^{-1}\bar{Y}. \tag{80}$$

Proof: The proof of Theorem 2 is similar to the proof of Theorem 1, so omitted here. □

Corollary 2: For the given parameters γ_1, γ_2, τ, τ_m, ε, α, $\bar{\delta}$, μ if there are suitable positive definite symmetric matrices $\bar{P} > 0$, $\bar{Q} > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$. $\bar{Q}_3 > 0$, $\bar{Q}_4 > 0$, $\bar{Q}_5 > 0$, $\bar{R} > 0$ and matrices $\bar{Y} = \bar{P}\bar{L}$, $\bar{K}$ make (73), (74) and linear matrix inequalities (75), (76), (77), (78), and (79) are established, then the PD learning observer (81) can estimate constant sensor fault.

5. SIMULATION RESULTS

In this section, three examples are presented to show the effectiveness of the proposed method in this paper. The three simulation examples are the actuator fault estimation based on the PD learning observer, the sensor fault estimation based on the augmented PD learning observer and the sensor fault estimation based on the augmented P learning observer.

5.1. Example 1

Here we refer to the simulation example in [36]:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -10 & 1 & 2 \\ -48 & -2 & 0 \\ 1 & -1 & -20 \end{bmatrix} x(t) \\ \quad + \begin{bmatrix} 0.5 & 0 & -1 \\ -0.5 & 1 & 0.5 \\ 0.25 & 0 & 0.5 \end{bmatrix} x(t-\tau(t)) \\ \quad + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} f_a(t), \\ y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t), \end{cases} \tag{81}$$

where the control input $u(t)$ is the unit step function, It can be checked that the pair (A, C) is observable, $rank(CE) = 1$, and the triple (A, E, C) does not possess any invariant zeros in the right half plane. Therefore, the proposed PD learning observer for actuator fault estimation exists. The time-varying delay $\tau(t) = 0.1 + 0.1 sint$, so we can get $\tau = 0.2$, $\tau_m = 0.1$. We select parameters $\gamma_1 = \gamma_2 = 1$, $\alpha = 100$, $\delta = 1.5$, $\varepsilon = 1$, $\mu = 10^{-5}$.

When the system suffers from a time-varying actuator fault $f_{a1}(t)$, we choose the learning interval d of observer is 0.001.

$$f_{a1}(t) = \begin{cases} 0, & 0 \le t \le 2, \\ 5\sin(2t) + 4\cos t, & 2 < t \le 30. \end{cases}$$

By solving the Theorem 1, the following solutions and Figs. 1-4 can be obtained:

$$K = 1, \quad L = \begin{bmatrix} -9.1098 \\ -52.0783 \\ -22.0478 \end{bmatrix}.$$

When the system suffers from a constant fault $f_{a2}(t)$, we choose the learning interval $d = 0.05$.

$$f_{a2}(t) = \begin{cases} 0, & 0 \le t \le 2, \\ 1, & 2 < t \le 10. \end{cases}$$

By solving the conditions in Corollary 1, Figs. 5-8 can be obtained.

Figs. 1-3 and Figs. 5-7, respectively, represent the states and states estimation of the system when the system suffer from unkown state time-varying delay and actuator fault.

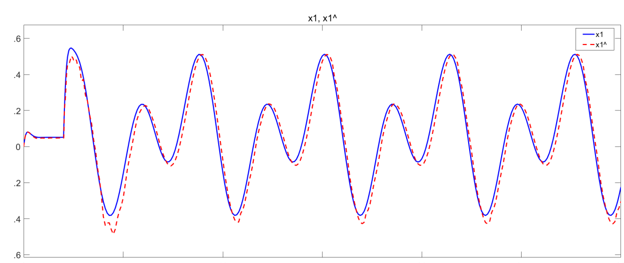

Fig. 1. System state $x_1(t)$ and its estimation with time-varying actuator fault.

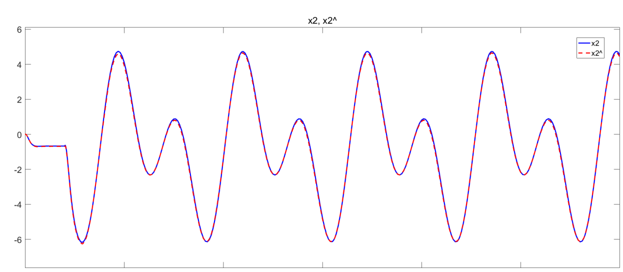

Fig. 2. System state $x_2(t)$ and its estimation with time-varying actuator fault.

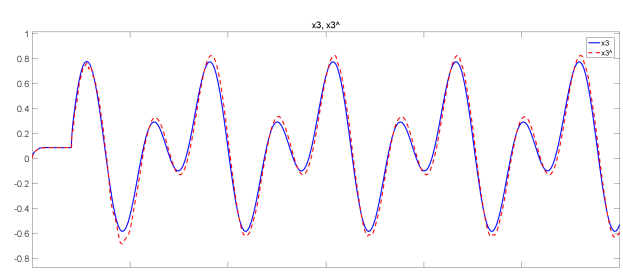

Fig. 3. System state $x_3(t)$ and its estimation with time-varying actuator fault.

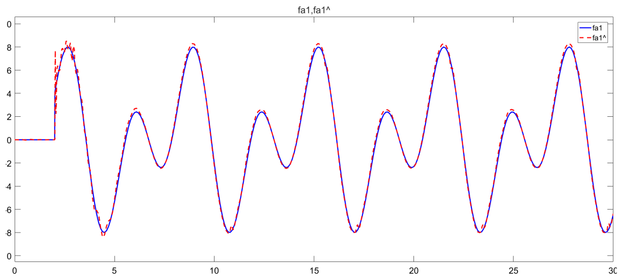

Fig. 4. Time-varying actuator fault $f_{a1}(t)$ and its estimation.

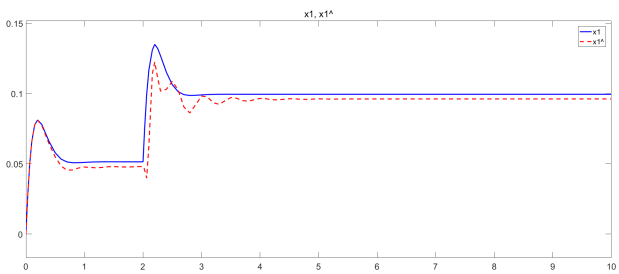

Fig. 5. System state $x_1(t)$ and its estimation with constant actuator fault.

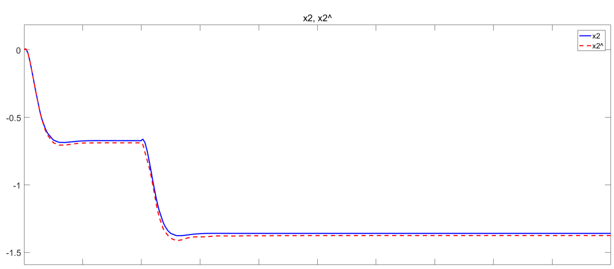

Fig. 6. System state $x_2(t)$ and its estimation with constant actuator fault.

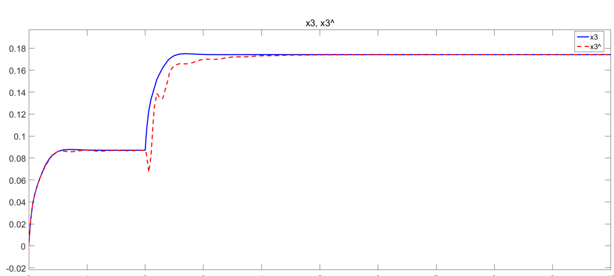

Fig. 7. System state $x_3(t)$ and its estimation with constant actuator fault.

Figs. 4 and 8 represent time-varying and constant actuator fault and their estimation, respectively. It can be seen from the Figures that the previously designed PD learning observer can realize the simultaneous estimation of system states and actuator fault. We can see that the observer

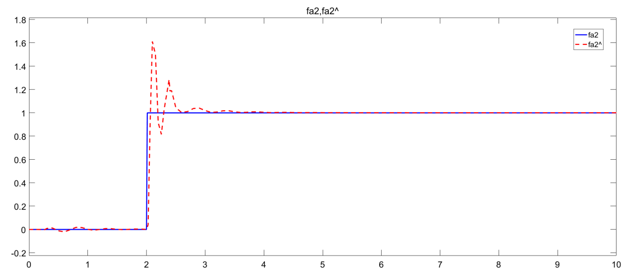

Fig. 8. Constant actuator fault $f_{a2}(t)$ and its estimation.

designed in this paper can realize the error-free estimation of constant faults and has good tracking performance for time-varying actuator faults.

5.2. Example 2

Here we refer to the simulation example in [37]:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0.25 & 0 \\ -0.5 & 0.5 \end{bmatrix} x(t-\tau(t)) \\ \qquad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f_s(t). \end{cases} \tag{82}$$

Let $A_x = I_2$, the augmented system can be obtained:

$$\begin{cases} \dot{\bar{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0.5 & 1 & 0 & -1 \end{bmatrix} \bar{x}(t) \\ \qquad + \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ -0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \bar{x}(t-\tau(t)) \\ \qquad + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f_s(t), \\ y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bar{x}(t), \end{cases}$$

where $u(t)$ is the unit step signal. The remaining parameters are the same as in Example 1.

When the sensor fault $f_{s1}(t)$ is a time-varying fault, we choose the learning interval $d = 0.001$.

$$f_{s1}(t) = \begin{cases} 0, & 0 \le t \le 2, \\ 2\sin(5t) + \cos(7t), & 2 < t \le 10. \end{cases}$$

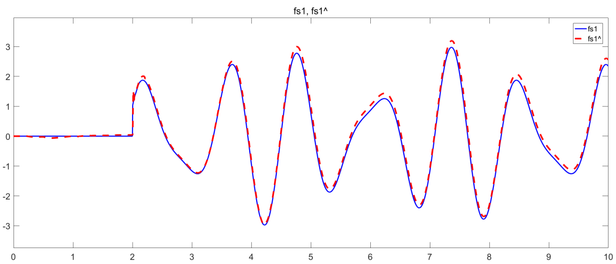

Fig. 9. Time-varying sensor fault $f_{s1}(t)$ and its estimation.

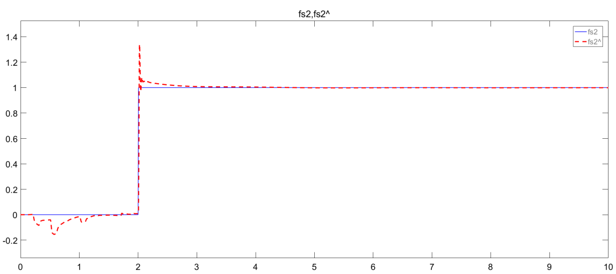

Fig. 10. Constant sensor fault $f_{s2}(t)$ and its estimation.

By solving the conditions in Theorem 2, the following solutions can be obtained:

$$\bar{K} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ \bar{L} = \begin{bmatrix} 1.1023 \times 10^4 & 0.0171 \\ 2.7935 \times 10^4 & 0.0269 \\ 875.8503 & 8.3030 \times 10^{-4} \\ 0.7952 & -4.2938 \end{bmatrix}.$$

Using MATLAB simulation Fig. 9 can be obtained:

When the system suffers from a constant fault $f_{s2}(t)$, we choose the learning interval $d = 0.05$.

$$f_{s2}(t) = \begin{cases} 0, & 0 \le t \le 2, \\ 1, & 2 < t \le 20. \end{cases}$$

Through Corollary 2, Fig. 10 can be obtained.

It can be seen from Figs. 9 and 10 that the augmented learning observer designed in the article can realize the estimation of system sensor fault. And when the system suffers from a constant value sensor fault, no difference estimation can be achieved.

5.3. Example 3

Considering the influence of unkown state time-varying delay, we use the P learning observer designed in [12] to achieve system fault estimation for system model in Example 2. When the system suffers from time-varying fault $f_{s1}(t)$ and constant value fault $f_{s2}(t)$, we choose the learning interval $d = 0.001$ and 0.05 respectively. Using Matlab simulation Figs. 11 and 12 can be obtained.

By comparing Figs. 9, 10, and Figs. 11, 12 we can know that under the same learning interval d, the P learning observer and PD learning observer have good results for the estimation of constant faults. However, in the case of fast time-varying faults estimation, PD learning observer is better than P learning observer.

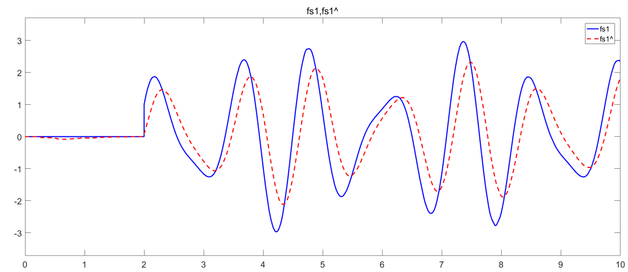

Fig. 11. $f_{s1}(t)$ and its estimation based on P learning observer.

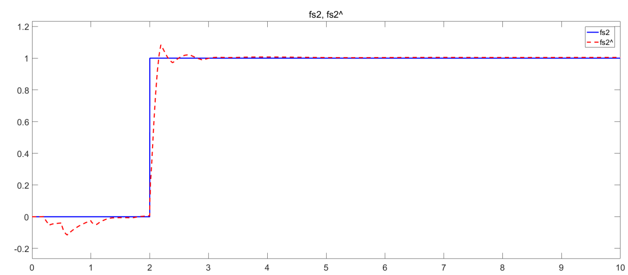

Fig. 12. $f_{s2}(t)$ and its estimation based on P learning observer.

6. CONCLUSION

This paper designs a PD learning observer for linear systems with unkown state time-varying delay and actuator fault. The designed observer can realize the simultaneous estimation of system states and actuator fault. By building an augmented system, the sensor fault estimation can be converted into the form of actuator fault estimation form. Later, by constructing an augmented PD learning observer, the system states and sensor fault can be estimated simultaneously. The introduction of H_∞ performance index can effectively suppress the impact of time-delay mismatch on state estimation and fault estimation. The learning interval d can be selected according to the speed of the fault change: for constant value and slow-change faults, a larger d can be selected; for fast-change faults, a smaller learning interval d should be selected. In the estimation of constant faults or slowly varying faults, both the proportional learning observer and the proportional differential learning observer have good performance. However, In terms of fast-changing fault estimation, the PD learning observer is better than the P learning observer.

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Fuqiang You received his B.Eng. and M.Eng. degrees in automation from Harbin Institute of Technology, China, in 2000 and 2002, respectively, and received a Ph.D. degree in control science and engineering from School of Electronic Information and Electronic Engineering of Shanghai Jiaotong University, China in 2005. Currently, he is an associate professor in College of Information Science and Engineer, Northeastern University, China. His research interests include fault diagnostic and fault-tolerant control.

Chao Wang received his B.Eng. degree in automation from College of electric power of Inner Mongol University of Technology, China in 2017. His is pursuing an M.Eng. degree in control engineering in Northeastern University, China. His research interests include fault diagnostic and fault-tolerant control.