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Dynamic Control Approach for Network Systems under Event-triggered Communication with Dual Triggers

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Abstract: This work studies the event-triggered control problem for networked control systems. The plant is controlled directly by a dynamic local controller, which receives the reference control signal from the remote controller. The measurement signal of the plant and the reference control signal of the remote controller have their separate event triggers, and thus the remote controller and the local controller can decide when to transmit signals on their own. It is proved that with the proposed control approach and the dual event triggers, the closed loop system is globally asymptotically stable, which has been illustrated by simulation results.

Keywords: Dynamic controller, event-triggered control, network systems, remote control.

1. INTRODUCTION

Networked control systems (NCSs) have been studied in many practical fields in the past decade [1,2]. Typical research directions in this field include switching systems [3], formation control [4], multi-agent coordination [5–8], coverage control of sensor networks [9–11]. A good many valuable analysis and synthesis approaches regarding the networked phenomena has been discussed, such as robust control, packet dropout, and quantization [12,13]. Due to the rapid development of electron and communication technique, more and more systems employ digital devices in the sensing, communication and control modules. However, traditional implementation of these digital devices for NCSs uses periodic sampling and communication schemes, which causes quite high redundant usage of rescouses such as communication bandwidth and power.

Recently, the event-triggered scheme has been employed in NCSs to deal with the high rescouses cost problem [14–19]. With the development of NCSs, the event-triggered control method has attracted more and more attention from scholars. Tabuada in [14] presents a event-triggered control approach for nonlinear control systems. It has been proved that the inter-event time of the proposed approach is strictly bounded from below if the state error satisfies certain Lipschitz continuity. In [20] the authors presented a self-triggered sampler for perturbed nonlinear systems ensuring uniformly ultimately boundedness of trajectories. To reduce conservativeness, a disturbance observer for the self-triggered sampler has been proposed. In [21], the exponential stabilization of linear NCSs with periodic event-triggered control was considered. Successive dropouts and a constant transmission delay are dealt with non-monotonic Lyapunov functions for discontinuous dynamical systems. More recently, event-triggered algorithm for the stabilization of switched linear systems has been studied in [22]. Pseudo-Lyapunov functions are designed to establish the event condition.

For many control systems the plant and the controller are located remotely. Not only the measurement signal of the plant should be transmitted to the controller, but also the control signal should be sent to the plant via the communication channel. Thus it is more natural and reasonable to design separate sampling schemes for the measurement signal of the plant and the control signal of the remote controller. Motivated by this, in this work we propose a local-remote control structure with event-triggered communication for NCSs. In this new structure, the plant is not controlled by the remote controller. Instead, it is directly controlled by its local dynamic controller, with the dynamic controller state updated by event-triggered control signal sent by the remote controller. Thus the role of the remote controller is not to directly control the plant, but to generate a reference control signal for the local controller. Using an appropriate method, we have designed event-triggered mechanisms for the plant

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measurement signal and the reference control signal of the remote controller separately. It has been proved that with the proposed local-remote control and the event trigger, the closed loop system can achieve globally asymptotically stable.

The contribution of this work is twofold. Firstly, for many practical remote control system, a local-remote control structure has been proposed, where the local controller can be considered as a realtime image of the remote controller. Thus the problem of imprecise control caused by the control input directly obtained by communication is solved. Secondly, a dual-trigger setup has been involved in the local-remote structure. As a result, the communication executions from the plant to the controller and from the controller to the plant need not to be synchronous. This brings a lot of freedom for the practical design of remote control law, and overcomes the problem of control quality degradation caused by the synchronous communication requirement.

The rest of this paper is organized as follows: Section 2 presents the problem formulation. Section 3 presents the local-remote control structure. In Section 4 we propose three different event-triggered communication scheme for the structure and prove the stability of the closed loop system under different schemes.

In the sequel of this paper, $\mathbb{R}$ and $\mathbb{R}^+$ represent the sets of all real numbers and all positive real numbers, respectively. $\mathbb{Z}_0^+$ represents the nonnegative integers. $\mathbb{R}^n$ is the n-dimensional space. $\|\cdot\|$ is the standard Euclid norm for vectors and also represents its matrix induced norm.

2. PROBLEM FORMULATION

In this section we revisit the results on output feedback control of linear systems. Consider a linear time invariant control system with the model given by

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, and $y(t) \in \mathbb{R}^p$ is the system output. If (A,B) is controllable, the system can be stabilized by the state feedback

$$u(t) \triangleq Kx(t),$$

with a proper designed feedback gain K.

In the control theory of linear systems, if the system is controllable and observable, then by applying the separation principle, the dynamic output feedback controller can be designed by using the combination of a state observer and the observer state feedback. Let $\hat{x}(t) \in \mathbb{R}^n$ be the observer state and $\hat{y}(t) \in \mathbb{R}^p$ be the observer output. Then the dynamic controller is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + F(\hat{y}(t) - y(t)), \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t), \tag{4}$$

with the control input being

$$u(t) = \hat{u}(t) \triangleq K\hat{x}(t). \tag{5}$$

Here F is the observer gain and can be designed if (A,C) is observable. To guarantee the effectiveness of this dynamic controller, it is expected that $\hat{x}(t)$ converges to $x(t)$ with a faster rate than that of $x(t)$. Denote the estimation error as

$$\tilde{x}(t) \triangleq \hat{x}(t) - x(t). \tag{6}$$

Then the dynamic of the error can be derived by using (1) and (3),

$$\begin{aligned}\dot{\tilde{x}}(t) &= \dot{\hat{x}}(t) - \dot{x}(t)\\ &= (A+FC)\tilde{x}(t).\end{aligned} \tag{7}$$

From (6) one has $\hat{x}(t) = x(t) + \tilde{x}(t)$. Substituting this into (1) by using the controller (5) yields

$$\dot{x}(t) = (A+BK)x(t) + BK\tilde{x}(t). \tag{8}$$

Let the aggregate state of the system with the dynamic controller be

$$z(t) \triangleq (x^T(t), \tilde{x}^T(t))^T.$$

Define a matrix G as

$$G \triangleq \begin{bmatrix} A+BK & BK \\ 0 & A+FC \end{bmatrix}. \tag{9}$$

Then from (7) and (8), one has

$$\dot{z}(t) = Gz(t).$$

In traditional control approaches, the periodic sampling of the state/output is employed. To further reduce the communication between the system and the remote controller, event-triggered control approach becomes an effective choice. By suitable design of the controller and the event generator, systems with event-triggered controller can reduce a lot of unnecessary sampling and thus reduce the amount of communication without significant performance degradation. Assume that the output of the system $y(t)$ is sampled and transmitted to the dynamic controller by communication to generate the control input $u(t)$. Let the sampling time instants, namely the event time instants if the event-triggered mechanism is involved, be

$$t_0 = 0, t_1, t_2, \ldots, t_k, \ldots, k \in \mathbb{Z}_0^+.$$

Then the sample-and-hold mechanism is developed for $y(t)$ and thus $u(t)$. Denote the sampled output as

$$y_s(t) = y(t_k), t \in [t_k, t_{k+1}). \tag{10}$$

Then the dynamic controller is given by

$$u(t) = \hat{u}_s(t) = K\hat{x}_s(t) \\ = K\hat{x}(t_k),$$

where $\hat{u}_s(t)$ is the observer control input and $\hat{x}_s(t)$ is the observer state at sampling time instants, respectively. $\hat{x}(t)$ is updated by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}_s(t) + F(\hat{y}(t) - y_s(t)).$$

This control input is transmitted from the controller to the plant by communication and holds on with the value $K\hat{x}(t_k)$ until the next event is triggered. It can be proved that the system is stabilizable if the matrix G is Hurwitz and the event-triggering condition for $y(t)$ and $u(t)$ is well designed.

3. LOCAL-REMOTE CONTROL STRUCTURE

In this section we develop a local-remote controller structure for event-triggered control systems. The purpose of this structure is to reduce more communication between the plant and the controller, as well as achieving a faster convergence rate. The control structure is illustrated in Fig. 1. In this structure, the local controller is co-located with the plant, and is responsible for updating the control input of the plant. The local controller is designed by

$$\dot{\hat{x}}_l(t) = (A + FC)\hat{x}_l(t) + Bu_s(t) - Fy_s(t), \tag{11}$$

$$u(t) = u_l(t) = K\hat{x}_l(t). \tag{12}$$

In this local controller, $\hat{x}_l(t)$ represents the dynamic state of the local controller. $y_s(t)$ is the event-triggered state generated by the event-triggered sampling of the plant output $y(t)$. Since this controller is co-located with the plant, it has the access to the realtime value of $y(t)$ by using the output sensors. Thus the local controller can decide the event time t_k and execute the sample-and-hold behavior

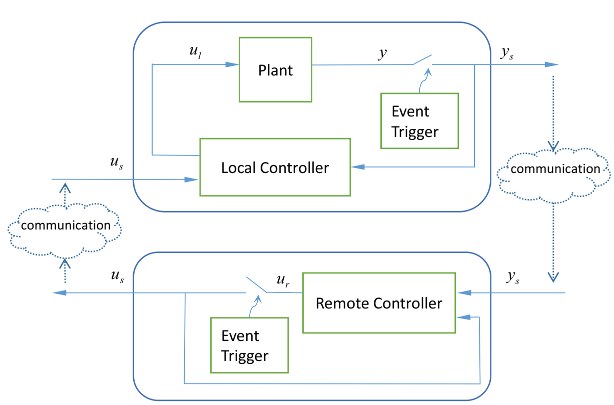

Fig. 1. Local-remote structure with event-triggered communication.

for the output $y(t)$ to generate the event-triggered value $y_s(t)$. $u_s(t)$ is the reference control input received from the remote controller. Thus this control input performs as the control instruction generated by the remote controller and then transmitted to the local dynamic controller at the event time instant t_k. The initial condition of the dynamic controller is denoted by $\hat{x}_l(0) = \hat{x}_{l0}$.

The event-triggered output $y_s(t)$ is also transmitted to the remote controller by communication to compute the reference control input $u_s(t)$. The computation is performed by using a dynamic remote controller given by

$$\dot{\hat{x}}_r(t) = (A + FC)\hat{x}_r(t) + Bu_s(t) - Fy_s(t), \tag{13}$$

$$u_r(t) = K\hat{x}_r(t), \tag{14}$$

$$u_s(t) = u_r(t_k),\ t \in [t_k, t_{k+1}). \tag{15}$$

Remark 1: In the proposed control structure, the plant measurement $y(t)$ and the controller output $u(t)$ should be sampled by event triggers. The sampled data $y_s(t)$ and $u_s(t)$ is transmitted through the communication network between the plant and the remote controller. However, if $u_s(t)$ is employed to control the plant, the control input cannot be precise enough since the event-triggered signal $u_s(t)$ is always lagging behind the realtime control input $u(t)$. Thus we have designed a local controller, located at the place of the plant, to tackle this problem. This local controller can be considered as an image of the remote controller. Its output, $u_l(t)$, will be a "realtime" estimation of the realtime control input $u(t)$. In order to achieve this goal, we let the local controller and the remote controller have the same dynamics by design. Their inputs are also the same, being $u_s(t)$ and $y_s(t)$, which happens to be information that can be obtained through communication.

The advantage of the proposed local-remote control structure can be summarized as follows: Firstly, using the two dynamic controllers, the coupling between the plant and the remote controller can be greatly weakened. The communication transmitting of the output signal and the control signal can be designed separately. This will be useful in reducing the amount of communication. Secondly, notice that in the control input $u(t)$ of the plant there is no sample-and-hold mechanism. This is different from most existing works in literature. We replace the event-triggered control $u_s(t) = K\hat{x}_r(t_k)$ by the continuous one $u(t) = K\hat{x}_l(t)$. Actually, it is not necessarily introducing sample-and-hold mechanism for the plant control. By using the continuous control input, it is expected to reduce the amount of events and thus further save the communication. Moreover, continuous controller $\hat{u}(t)$ performs better in fast driving the system to equilibrium than the event-triggered one in general.

Remark 2: In event-triggered control systems, the event design approaches can be divided into traditional static trigger and dynamic trigger [16]. In the dynamic event-triggered control, the event condition is determined by an auxiliary dynamic variable and its related dynamic equation. This is different from the meaning of the term “dynamic” in this work. The term “dynamic” in this paper indicates that dynamic equations are embedded in the local-remote control laws, but not using dynamic equations in event design. Thus in fact this work discusses the combination of the dynamic control law and the event-triggered mechanism rather than the dynamic event-triggered control.

4. EVENT-TRIGGERED COMMUNICATION DESIGN

4.1. Basic event-triggering

To present the event-triggered controller for the system, the event condition should be designed. Let the output sample error and the reference control sample error be

$$e_y(t) = y_s(t) - y(t),$$
$$e_u(t) = u_s(t) - u(t).$$

Denote the aggregated sample error as

$$e(t) = (e_u^T(t), e_y^T(t))^T. \tag{16}$$

Then for the event condition design we propose the following result.

Theorem 1: Consider the system given by (1) and (2). Assume that (A,B) is controllable, (A,C) is observable, and G in (9) is Hurwitz. Let $Q \in \mathbb{R}^{(2n)\times(2n)}$ be a symmetric positive definite matrix and a matrix P be the solution of

$$G^T P + PG + Q = 0. \tag{17}$$

Let the event time instant be defined by the condition

$$t_{k+1} = \min\{t > t_k : f(z(t), e(t)) \geq 0\}, \tag{18}$$

with

$$f(z(t), e(t)) = v^T(t) M v(t),$$

where

$$v(t) = (z^T(t), e^T(t))^T,$$

and

$$M = \begin{bmatrix} -\sigma Q & PH \\ H^T P & 0 \end{bmatrix},$$

with $\sigma \in (0,1)$ being a parameter to be designed and

$$H = \begin{bmatrix} 0_{n\times m} & 0_{n\times p} \\ B & -F \end{bmatrix}.$$

Then the origin of the closed loop system is asymptotically stable under the local controller (12) with the filter reference input $u_s(t)$ provided by (15) and the plant measurement input $y_s(t)$ provided by (10).

Proof: Since the two dynamic filters (11) and (13) have the same system matrices and the same inputs $u_s(t)$ and $y_s(t)$, if $A+FC$ is Hurwitz by design, these two dynamic filters can be considered as one single filter in analysis and their initial conditions can be neglected. Denote them by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu_s(t) + F(\hat{y}(t) - y_s(t)), \tag{19}$$
$$\hat{y}(t) = C\hat{x}(t). \tag{20}$$

Then the dynamic of the filter state error is

$$\begin{aligned}\dot{\tilde{x}}(t) &= \dot{\hat{x}}(t) - \dot{x}(t) \\ &= (A+FC)\tilde{x}(t) + Be_u(t) - Fe_y(t).\end{aligned}$$

Combining this with (8) yields

$$\begin{aligned}\begin{bmatrix}\dot{x}(t)\\ \dot{\tilde{x}}(t)\end{bmatrix} &= \begin{bmatrix}(A+BK)x(t) + BK\tilde{x}(t) \\ (A+FC)\tilde{x}(t) + Be_u(t) - Fe_y(t)\end{bmatrix} \\ &= \begin{bmatrix} A+BK & BK \\ 0 & A+FC\end{bmatrix}\begin{bmatrix}x(t)\\ \tilde{x}(t)\end{bmatrix} \\ &\quad + \begin{bmatrix}0 & 0\\ B & -F\end{bmatrix}\begin{bmatrix}e_u(t)\\ e_y(t)\end{bmatrix}.\end{aligned}$$

Using the definitions of $z(t)$, G, H, and $e(t)$ previously one obtains

$$\dot{z}(t) = Gz(t) + He(t). \tag{21}$$

Since G is Hurwitz and Q is symmetric and positive definite, the solution P is a symmetric and positive definite matrix following the Lyapunov theory. Then we choose a Lyapunov functional candidate of the closed loop system as

$$V(z(t)) = z^T(t) P z(t). \tag{22}$$

Computing its time derivative and substituting (21) and (17) yields

$$\begin{aligned}\dot{V}(z(t)) &= z^T(t)G^T Pz(t) + z^T(t)PGz(t) + 2z^T(t)PHe(t) \\ &= -z^T(t)Qz(t) + 2z^T(t)PHe(t) \\ &= -(1-\sigma)z^T(t)Qz(t) \\ &\quad + \Big(-\sigma z^T(t)Qz(t) + 2z^T(t)PHe(t)\Big) \qquad (23) \\ &= -(1-\sigma)z^T(t)Qz(t) \\ &\quad + \begin{bmatrix} z^T(t) & e^T(t)\end{bmatrix}\begin{bmatrix}-\sigma Q & PH \\ H^T P & 0\end{bmatrix}\begin{bmatrix}z(t)\\ e(t)\end{bmatrix} \\ &= -(1-\sigma)z^T(t)Qz(t) + f(z(t), e(t)). \qquad (24)\end{aligned}$$

The closed loop system is asymptotically stable if

$$f(z(t), e(t)) < 0.$$

Notice that at the event time instant t_k, $y_s(t)$ and $u_s(t)$ is updated by $y(t_k)$ and $u(t_k) = K\hat{x}(t_k)$. Thus $e_y(t)$ and $e_u(t)$ are reset to 0. If the event time is determined by (25), then $f(z(t), e(t)) < 0$ will be enforced and from (24) it is concluded that $\dot{V}(z(t)) < 0$ holds for all t, which implies that the closed loop system is asymptotically stable. □

4.2. Single trigger setup

Although the event-triggered controller can drive the closed loop system to asymptotically stable, the event condition in (25) requires $z(t)$ and thus the plant state $x(t)$, which cannot be directly measured under the output based control assumption. Thus to develop a workable event trigger, the event design should be based only on available signal, e.g., $\hat{x}(t)$ and $y(t)$. Since both the plant and the local controller have the access to $\hat{x}_l(t)$ and the remote controller have the access to $\hat{x}_r(t)$, we present improvement for the event condition such that only $\hat{x}_l(t)$ is employed,

$$t_{k+1} = \{t > t_k : \|e(t)\| \geq \frac{\sigma\lambda_m}{2\sqrt{2}\|PH\|}\|\hat{x}_l(t)\|\}, \tag{25}$$

where λ_m is the smallest eigenvalue of Q.

Employing the proposed event condition, the communication scheme between the plant and the remote controller is as follows: The event trigger is co-located with the plant and using (25) to generate events. When an event triggers, the output $y(t)$ is sampled and $y_s(t)$ is obtained. $y_s(t)$ will be transmitted to local controller and remotely transmitted to the remote controller by communication. Then once the remote controller receives $y_s(t)$, it generates the sampling control signal $u_s(t)$, and transmits it to the remote controller and also transmits it back to the plant.

We have the following theorem for the system with this event-trigger.

Theorem 2: Consider the system given by (1) and (2) with the same conditions as presented in Theorem 1, under the proposed local-remote event-triggered controller. If the event time determined by condition (25), the origin of the closed loop system is asymptotically stable under the local controller (12) with $y_s(t)$ provided by (10) and $u_s(t)$ provided by (15).

Proof: Choose the same Lyapunov function as in (22), and compute its time derivative following (23) yielding

$$\begin{aligned}\dot{V}(z(t)) &\leq -(1-\sigma)z^T(t)Qz(t)\\ &\quad -\Big(\sigma\lambda_m\|z(t)\|^2 - 2\|z(t)\|\|PH\|\|e(t)\|\Big)\\ &= -(1-\sigma)z^T(t)Qz(t)\\ &\quad -\|z(t)\|\Big(\sigma\lambda_m\|z(t)\| - 2\|PH\|\|e(t)\|\Big).\end{aligned} \tag{26}$$

Now from the norm of $z(t)$ one has

$$\begin{aligned}\|z(t)\|^2 &= z^T(t)z(t)\\ &= \begin{bmatrix} x^T(t) & \hat{x}^T(t) - x^T(t)\end{bmatrix}\begin{bmatrix} x(t)\\ \hat{x}(t)-x(t)\end{bmatrix}\\ &= 2\|x(t)\|^2 + \|\hat{x}(t)\|^2 - 2x^T(t)\hat{x}(t) \qquad (27)\\ &\geq 2\|x(t)\|^2 + \|\hat{x}(t)\|^2 - 2\|x(t)\|\|\hat{x}(t)\|. \qquad (28)\end{aligned}$$

Notice that $\|x(t)\|$ and $\|\hat{x}(t)\|$ are 0 if and only if $x(t)$ and $\hat{x}(t)$ are 0. Then without loss of generality we denote $\|x(t)\| = \eta\|\hat{x}(t)\|$ and substitute it into (28), and yield

$$\begin{aligned}\|z(t)\| &\geq \|\hat{x}(t)\|\sqrt{2(\eta-\frac{1}{2})^2+\frac{1}{2}}\\ &\geq \frac{1}{\sqrt{2}}\|\hat{x}(t)\|.\end{aligned} \tag{29}$$

Substituting this into (26) yields

$$\begin{aligned}\dot{V}(z(t)) &\leq -(1-\sigma)z^T(t)Qz(t)\\ &\quad -\|z(t)\|\Big(\frac{\sigma\lambda_m}{\sqrt{2}}\|\hat{x}(t)\| - 2\|PH\|\|e(t)\|\Big).\end{aligned} \tag{30}$$

If $\frac{\sigma\lambda_m}{\sqrt{2}}\|\hat{x}(t)\| - 2\|PH\|\|e(t)\| > 0$, one has $\dot{V}(z(t)) < 0$ and the system is asymptotically stable. This give rise to the event trigger as in (25) and the proof completes. □

4.3. Dual-trigger setup

In the event condition proposed in Subsection 4.2, it is assumed that the reference control input $u_s(t)$ is sampled once the remote controller received the event-triggered output $y_s(t)$. Although this can be realized separately by the local and remote controllers in practice, they share a single event trigger theoretically. In this subsection we propose a dual-trigger setup for the system and thus the local and remote controllers can decide the transmission of $y_s(t)$ and $u_s(t)$ independently.

The proposed event trigger for the local controller, called as the local trigger, is

$$t_{k+1}^y = \{t > t_k^y : \|e_y(t)\| \geq \frac{\sigma\lambda_m\zeta}{2\sqrt{2}\|PH\|}\|\hat{x}_l(t)\|\}, \tag{31}$$

and the event trigger for the remote controller, called as the remote trigger, is

$$t_{k+1}^u = \{t > t_k^u : \|e_u(t)\| \geq \frac{\sigma\lambda_m(1-\zeta)}{2\sqrt{2}\|PH\|}\|\hat{x}_r(t)\|\}, \tag{32}$$

where $\zeta \in (0,1)$ is a parameter to be determined. Accordingly, $y_s(t)$ and $u_s(t)$ will be generated by

$$y_s(t) = y(t_k^y), t \in [t_k^y, t_{k+1}^y), \tag{33}$$

$$u_s(t) = u_r(t_k^u), t \in [t_k^u, t_{k+1}^u). \tag{34}$$

For the dual-trigger setup we have the following result.

Theorem 3: Consider the system given by (1) and (2) with the same conditions as presented in Theorem 1, under the proposed local-remote event-triggered controller. If the event time sequences for the local controller and the remote controller are determined by conditions (31) and (32), respectively, the origin of the closed loop system is asymptotically stable under the local controller (12) with $y_s(t)$ provided by (33) and $u_s(t)$ provided by (34).

Proof: Choose the same Lyapunov function as in (22). From the definition of $e(t)$ in (16), and considering the dual triggers in (31) and (32), we have

$$\begin{aligned}\|e(t)\| &= \left\|\begin{matrix} e_u(t) \\ e_y(t) \end{matrix}\right\| \\ &\le \|e_u(t)\| + \|e_y(t)\| \qquad (35) \\ &< \frac{\sigma\lambda_m(1-\zeta)}{2\sqrt{2}\|PH\|}\|\hat{x}(t)\| + \frac{\sigma\lambda_m\zeta}{2\sqrt{2}\|PH\|}\|\hat{x}(t)\| \\ &= \frac{\sigma\lambda_m}{2\sqrt{2}\|PH\|}\|\hat{x}(t)\|. \qquad (36)\end{aligned}$$

This implies that $\frac{\sigma\lambda_m}{\sqrt{2}}\|\hat{x}(t)\| - 2\|PH\|\|e(t)\| > 0$. Then from (30), one has $\dot{V}(z(t)) < 0$ and thus the closed loop system is asymptotically stable. □

5. EVENT ANALYSIS

In the proposed event-triggered controller, the implicit definition of the event time raises the question of the existence of a lower bound for the inter-event time $t_{k+1} - t_k$. Otherwise the event-triggered control scheme will require faster and faster updates and consequently cannot be implemented by digital platforms [15]. This is called as the Zeno behavior for a hybrid system. It has been proved in [14] that a minimal inter-event time exists for most linear systems and even nonlinear systems with state feedback. For the proposed local-remote dynamic control system in this work, we have the following similar results.

The following result provides an estimated lower bound for the event time interval $t_{k+1} - t_k$.

Theorem 4: Consider the system given in (1) and (2) with the same conditions as presented in Theorem 1, under the proposed local-remote event-triggered controller. For the event time determined by the event condition (25), there exists a strictly positive time length $\tau^* \in \mathbb{R}^+$ such that $t_{k+1} - t_k > \tau^*$ for all $k \in \mathbb{Z}_0^+$.

Proof: It has been proved that the closed loop system is globally asymptotically stable. Since $A+BK$, $A+FC$ and G are Hurwitz, then from (1), (19) and (21), by employing the Lyapunov theory and the ISS Theory for LTI systems we conclude that there exists $\rho \in \mathbb{R}^+$ such that $\|x(t)\| \le \rho\|\hat{x}(t)\|$. Then from (27) one has

$$\begin{aligned}\|z(t)\|^2 &\le 2\|x(t)\|^2 + \|\hat{x}(t)\|^2 + 2\|x(t)\|\|\hat{x}(t)\| \\ &\le (2\rho^2 + 2\rho + 1)\|\hat{x}(t)\|^2,\end{aligned}$$

and thus

$$\chi\|z(t)\| \le \|\hat{x}(t)\|, \qquad (37)$$

with $\chi \triangleq \frac{1}{\sqrt{2\rho^2+2\rho+1}}$.

For event condition (25), $t_{k+1} - t_k$ is the time length for $\frac{\|e(t)\|}{\|\hat{x}(t)\|}$ to evolute from 0 to $\frac{\sigma\lambda_m}{2\sqrt{2}\|PH\|}$. From (37) one has

$$\frac{\|e(t)\|}{\|\hat{x}(t)\|} \le \frac{\|e(t)\|}{\chi\|z(t)\|},$$

for all t. Thus if we denote the time for $\frac{\|e(t)\|}{\|z(t)\|}$ to evolute from 0 to $\frac{\sigma\lambda_m\chi}{2\sqrt{2}\|PH\|}$ as τ, then

$$t_{k+1} - t_k > \tau.$$

To obtain an estimation of τ, we compute

$$\begin{aligned}\frac{d}{dt}\frac{\|e(t)\|}{\|z(t)\|} &= \frac{e^T(t)\dot{e}(t)}{\|e(t)\|\|z(t)\|} - \frac{\|e(t)\|z^T(t)\dot{z}(t)}{\|z(t)\|^3} \\ &\le \frac{\|\dot{e}(t)\|}{\|z(t)\|} + \frac{\|e(t)\|}{\|z(t)\|}\frac{\|\dot{z}(t)\|}{\|z(t)\|}.\end{aligned}$$

Denote $L = \begin{bmatrix} K & K \\ C & 0_{p\times n}\end{bmatrix}$. From the definition of $e(t)$ one has

$$\begin{aligned}\|\dot{e}(t)\| &= \left\|\begin{matrix}\dot{u}(t) \\ \dot{y}(t)\end{matrix}\right\| \\ &= \left\|\begin{matrix}K\dot{\hat{x}}(t) \\ C\dot{x}(t)\end{matrix}\right\| \\ &= \left\|\begin{bmatrix} K & K \\ C & 0_{p\times n}\end{bmatrix}\begin{bmatrix}\dot{x}(t) \\ \dot{\hat{x}}(t) - \dot{x}(t)\end{bmatrix}\right\| \\ &= \|L\dot{z}(t)\| \\ &\le \|L\|\|\dot{z}(t)\|. \qquad (38)\end{aligned}$$

On the other hand, from (21) we have

$$\begin{aligned}\|\dot{z}(t)\| &= \|Gz(t) + He(t)\| \\ &\le \|G\|\|z(t)\| + \|H\|\|e(t)\|.\end{aligned}$$

Then we can obtain

$$\begin{aligned}\frac{d}{dt}\frac{\|e(t)\|}{\|z(t)\|} &\le \|L\|\frac{\|\dot{z}(t)\|}{\|z(t)\|} + \frac{\|e(t)\|}{\|z(t)\|}\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\ &= \left(\|L\| + \frac{\|e(t)\|}{\|z(t)\|}\right)\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\ &\le \left(\|L\| + \frac{\|e(t)\|}{\|z(t)\|}\right)\frac{\|G\|\|z(t)\| + \|H\|\|e(t)\|}{\|z(t)\|} \\ &= \left(\|L\| + \frac{\|e(t)\|}{\|z(t)\|}\right)\left(\|G\| + \|H\|\frac{\|e(t)\|}{\|z(t)\|}\right).\end{aligned}$$

Now consider the solution of the differential function

$$\dot{\phi}(t) = a\phi^2(t) + b\phi(t) + c, \phi(0) = \phi_0,\ a,b,c > 0.$$

Denote its solution trajectory as $\phi_{\phi_0}(t)$. Since in $[t_k, t_{k+1})$, $\frac{\|e(t)\|}{\|z(t)\|}$ starts from 0, we consider the case of $\phi_0 = 0$ and thus the solution is $\phi_0(t)$. Define the function

$$\mathcal{T}_{a,b,c}(\cdot) \triangleq \phi_0^{-1}(\cdot).$$

Denote $a_1 = \|H\|$, $b_1 = \|G\| + \|H\|\|L\|$, and $c_1 = \|G\|\|L\|$. Then since

$$\frac{d}{dt}\frac{\|e(t)\|}{\|z(t)\|} \le a_1\Big(\frac{\|e(t)\|}{\|z(t)\|}\Big)^2 + b_1\frac{\|e(t)\|}{\|z(t)\|} + c_1,$$

one can conclude that

$$\tau \ge \tau^* \triangleq \mathcal{T}_{a_1,b_1,c_1}\Big(\frac{\sigma\lambda_m\chi}{2\sqrt{2}\|PH\|}\Big) > 0,$$

which completes the proof. □

Now we provide estimations of the lower bounds for the event time intervals $t_{k+1}^y - t_k^y$ and $t_{k+1}^u - t_k^u$.

Theorem 5: Consider the system given in (1) and (2) with the same conditions as presented in Theorem 1, under the proposed local-remote event-triggered controller. For the event time sequences determined by the event conditions (31) and (32), there exist strictly positive time lengths $\tau_y^*, \tau_u^* \in \mathbb{R}^+$ such that $t_{k+1}^y - t_k^y > \tau_y^*$ and $t_{k+1}^u - t_k^u > \tau_u^*$ for all $k \in \mathbb{Z}_0^+$.

Proof: Denote $\gamma_y = \frac{\sigma\lambda_m\zeta}{2\sqrt{2}\|PH\|}$ and $\gamma_u = \frac{\sigma\lambda_m(1-\zeta)}{2\sqrt{2}\|PH\|}$. From the event conditions (31) and (32) we have

$$\|e_y(t)\| < \gamma_y\|\hat{x}(t)\|,$$
$$\|e_u(t)\| < \gamma_u\|\hat{x}(t)\|.$$

From (37) one notice that if we denote the time for $\frac{\|e_u(t)\|}{\|z(t)\|}$ to evolute from 0 to $\gamma_u\chi$ as τ_u, then

$$t_{k+1}^u - t_k^u > \tau_u.$$

Denote $L_u = \begin{bmatrix} K & K\end{bmatrix}$ and similar to (38) one has

$$\|u(t)\| \le \|L_u\|\|z(t)\|,$$
$$\|\dot{e}_u(t)\| \le \|L_u\|\|\dot{z}(t)\|.$$

Using (29) and (35) one can compute

$$\begin{aligned}\frac{d}{dt}\frac{\|e_u(t)\|}{\|z(t)\|} &\le \frac{\|\dot{e}_u(t)\|}{\|z(t)\|} + \frac{\|e_u(t)\|}{\|z(t)\|}\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\ &\le \|L_u\|\frac{\|\dot{z}(t)\|}{\|z(t)\|} + \frac{\|e_u(t)\|}{\|z(t)\|}\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\ &= \Big(\|L_u\| + \frac{\|e_u(t)\|}{\|z(t)\|}\Big)\frac{\|\dot{z}(t)\|}{\|z(t)\|} \\ &\le \Big(\|L_u\| + \frac{\|e_u(t)\|}{\|z(t)\|}\Big) \\ &\quad\cdot \frac{\|G\|\|z(t)\| + \|H\|\|e_u(t)\| + \|H\|\|e_y(t)\|}{\|z(t)\|} \\ &\le \Big(\|L_u\| + \frac{\|e_u(t)\|}{\|z(t)\|}\Big) \\ &\quad\cdot \Big(\|G\| + \|H\|\frac{\|e_u(t)\|}{\|z(t)\|} + \|H\|\frac{\gamma_y\|\hat{x}(t)\|}{\|z(t)\|}\Big) \\ &\le \Big(\|L_u\| + \frac{\|e_u(t)\|}{\|z(t)\|}\Big) \\ &\quad\times \Big(\|G\| + \|H\|\frac{\|e_u(t)\|}{\|z(t)\|} + \sqrt{2}\gamma_y\|H\|\Big).\end{aligned}$$

Denote $a_u = a_1 = \|H\|$, $b_u = \|G\| + \sqrt{2}\gamma_y\|H\| + \|H\|\|L_u\|$, and $c_u = \|G\|\|L_u\| + \sqrt{2}\gamma_y\|H\|\|L_u\|$. Then

$$\frac{d}{dt}\frac{\|e_u(t)\|}{\|z(t)\|} \le a_u\Big(\frac{\|e_u(t)\|}{\|z(t)\|}\Big)^2 + b_u\frac{\|e_u(t)\|}{\|z(t)\|} + c_u,$$

and we conclude that

$$\tau_u \ge \tau_u^* \triangleq \mathcal{T}_{a_u,b_u,c_u}(\gamma_u\chi) > 0.$$

Now denote $L_y = \begin{bmatrix} C & 0_{p\times n}\end{bmatrix}$, $a_y = a_1 = \|H\|$, $b_y = \|G\| + \sqrt{2}\gamma_u\|H\| + \|H\|\|L_y\|$, and $c_y = \|G\|\|L_y\| + \sqrt{2}\gamma_u\|H\|\|L_y\|$. Then by similar calculation one also have

$$\tau_y \ge \tau_y^* \triangleq \mathcal{T}_{a_y,b_y,c_y}(\gamma_y\chi) > 0,$$

and thus the proof completes. □

6. SIMULATIONS

In this section we present simulation results to verify the results in the previous sections.

Consider a linear system with matrices given by

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 5 & 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 3 \\ 1 & 2 \\ 3 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}.$$

Matrices

$$K = \begin{bmatrix} -10.7215 & 16.9241 & -6.1899 \\ 0.2532 & -4.3418 & 0.6456 \end{bmatrix},$$

and

$$F = \begin{bmatrix} -4.5965 & 0.3567 \\ 2.3180 & -2.2799 \\ -4.4659 & 0.3367 \end{bmatrix}$$

are chosen such that $A + BK$ and $A + FC$ are Hurwitz. For simplicity let $Q = I_6$ and thus $\lambda_m = 1$. Solving (17) yields a proper P. Choose parameter $\sigma = 0.8$. Let the initial conditions of the plant be $x_0 = [2, 1, 2]^T$.

The simulation result for the single trigger setup is shown in Figs. 2, 3, 4 and 5. From Fig. 2 one can notice that the proposed local-remote controller with the single trigger setup is effective in driving the plant to steady state. The communication transmission of $u_s(t)$ and $y_s(t)$ is event-triggered, which is shown in Figs. 3 and 4. Since there is only a single trigger, the plant output and the remote controller share the same event time sequence, which is shown in Fig. 5.

For the same system dynamic and parameter setup, simulation for the dual-trigger case has also been completed.

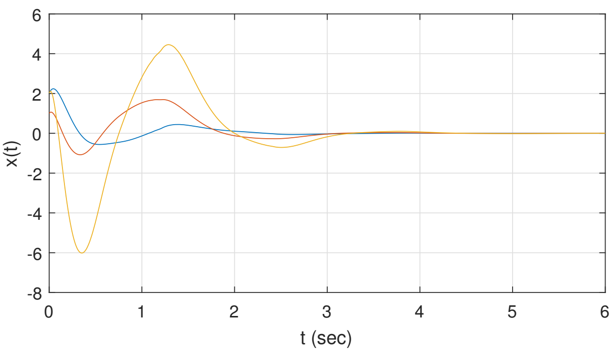

Fig. 2. Plant state under single trigger setup.

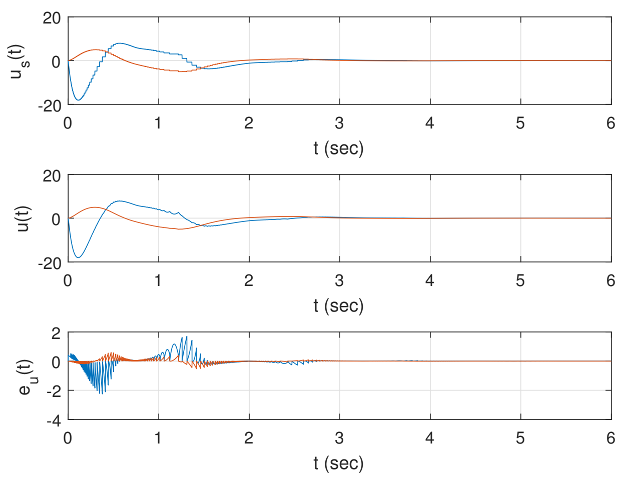

Fig. 3. Event-triggered input, plant input, and the input error under single trigger setup.

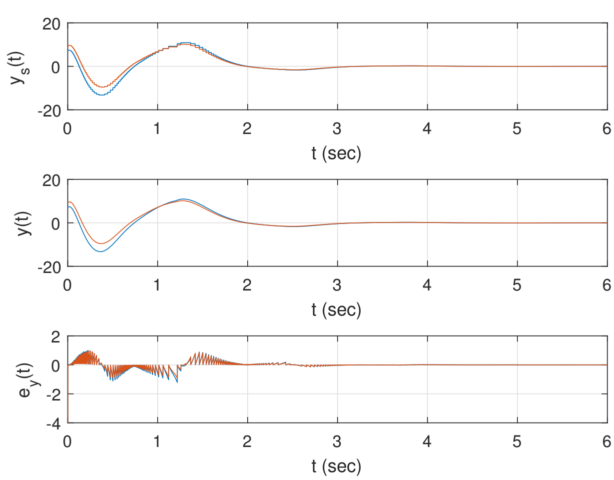

Fig. 4. Event-triggered output, plant output, and the output error under single trigger setup.

The parameter ζ for dual-trigger design is chosen to be 0.5. Similar results hold for the dual-trigger setup, which are illustrated in Figs. 6, 7, 8, and 9. Since the remote con-

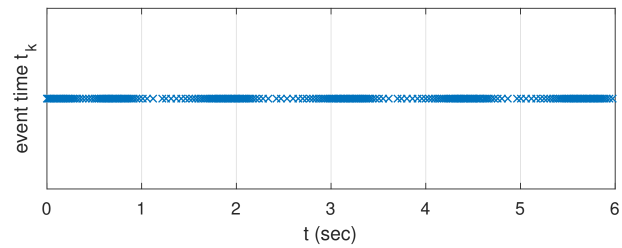

Fig. 5. Event time instants under single trigger setup.

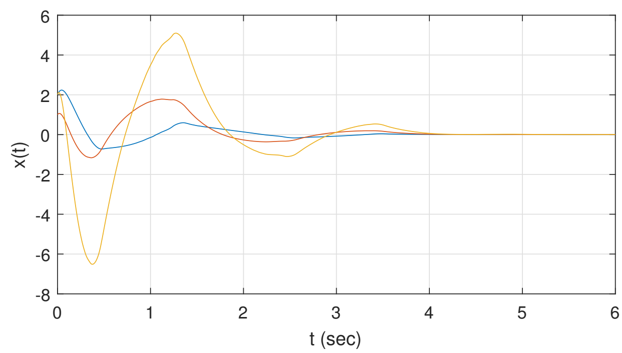

Fig. 6. Plant state under dual-trigger setup.

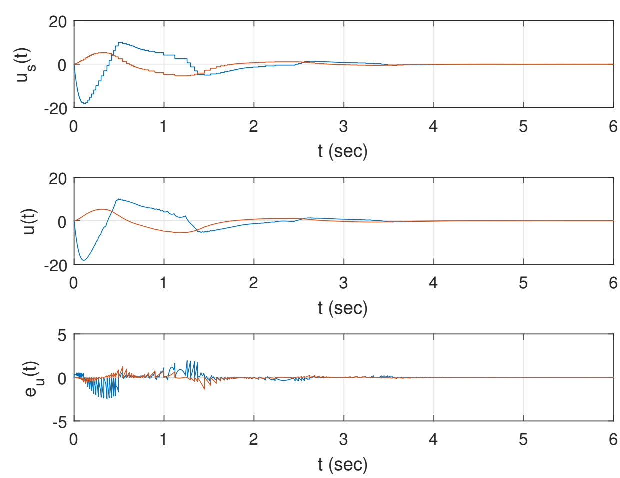

Fig. 7. Event-triggered input, plant input, and the input error under dual-trigger setup.

troller and the plant output have their own event triggers, their event time sequences are different. This avoids the requirement of synchronous communication by the controller and the plant in practical control systems over communication network.

7. CONCLUSIONS

In this work the event-triggered control for linear systems have been investigated. We have proposed a local-remote control structure for the system. The output of the plant and the reference control input generated by the re-

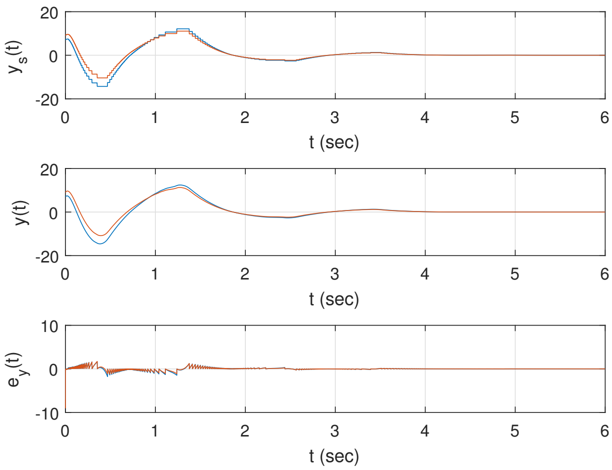

Fig. 8. Event-triggered output, plant output, and the output error under dual-trigger setup.

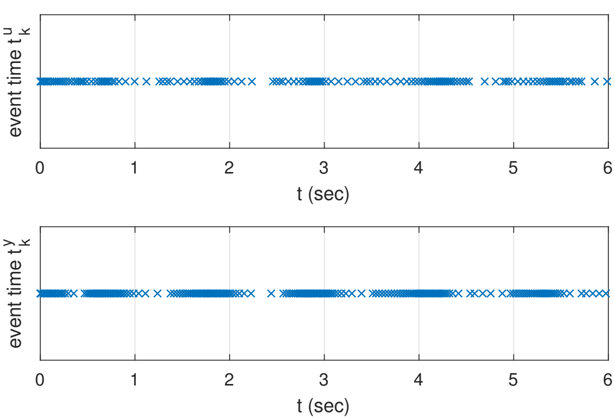

Fig. 9. Event time instants under dual-trigger setup.

mote controller are event-triggered. It is noted that the output and the reference input can be triggered synchronously or separately, and both proposed setups can drive the plant to steady state. The future research of this work include extending the local-remote controller to plant with nonlinear dynamics, and with network environment such as time delay, saturation, package dropout, and quantization.

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