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Stability Analysis for Time-delay Systems via a Novel Negative Condition of the Quadratic Polynomial Function

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Abstract: This paper studies the stability analysis problem for time-varying delay systems. An appropriate Lyapunov-Krasovskii functional (LKF) is constructed where its derivative is a quadratic polynomial function of the delay. A novel negative condition of the mentioned quadratic function with two variable parameters is developed to ensure that the LKF derivative is negative, reducing conservatism on some similar results. Besides, an extended version of Bessel-Legendre inequality is introduced to be employed in the stability analysis of time-varying delay systems. Then, some stability criteria with less conservatism are derived for two kinds of the time-varying delay. Finally, the effectiveness of the proposed stability criteria is demonstrated through three examples.

Keywords: Bessel-Legendre inequality, quadratic polynomial inequality, stability, time-delay systems.

1. INTRODUCTION

Time-delay is an inevitable phenomenon in some real plants, such as network control systems, power systems, neural networks, and manufacturing systems [1–4]. Since time delays may affect the system stability, various significant studies have been performed about the stability analysis of time-delay systems [5–12].

In stability analysis of time-delay systems, it is important to ensure the asymptotic stability in a delay range, while the stability analysis conservatism depends on the maximum permissible delay value. Among all kinds of methodologies for stability analysis, the Lyapunov-Krasovskii functional (LKF) method has been known as the most popular approach for stability analysis of time-delay systems. Several works have been devoted to constructing a suitable LKF to reduce the stability analysis conservatism of time-delay systems, including the the delay-dependent LKF [13], the multiple integral LKF [14], the delay partitioning/decomposition LKF [15,16], and the delay-product-type LKF [17–20]. Note that the LKF constructed in [18–23] significantly different from the other ones. The LKF derivative is a quadratic polynomial function in terms of the time delay. This function can be formulated as $f(h(t)) = a_2h^2(t)+a_1h(t)+a_0$, where a_i $(i=0,1,2)$ are real matrixes independent of $h(t)$, while $h(t) \in [0,\widetilde{h}]$ is the time delay, and $\widetilde{h}$ is a constant. To ensure the stability for time-delay systems, $f(h(t))$ should be negative for $h(t)$ belonging to $[0,\widetilde{h}]$. Thus, it is vital to derive negativity conditions of the quadratic polynomial function to obtain a less conservative criterion. Recently, some necessary and sufficient conditions on the quadratic polynomial function based on Lemma 2 of literature [22] are reported in [24,25], but these conditions require too many decision variables. Although a novel quadratic-partitioning method with a small number of decision variables (NDVs) and a relaxed negative sufficient condition of the quadratic polynomial function was presented in [26,27], respectively, the conservatism of the mentioned stability criteria should be further studied. Making a balance between the stability criteria conservatism and the NDVs can be considered as an interesting research topic.

On the other hand, the inequality approach has been extensively utilized to reduce the stability analysis conservatism for time-delay systems. The first approach for estimating the integral term is Jensen's inequality [28]. The Wirtinger-based inequality, which is less conservation than Jensen's, has been proposed in [29]. Later, auxiliary function-based inequality [30,31], free-matrix-based inequality [32–34] and some reciprocally convex matrix inequalities [23,35–37] further promote the integral inequality development. The Bessel's inequality on Hilbert space and Legendre polynomials have been utilized in [38] to introduce a Bessel-Legendre inequality and obtain a generalized inequality. But, due to the complexity of the

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Legendre polynomials, applying them when the order is higher than two is difficult. As a result, two new styles of Bessel-Legendre inequality have been proposed in [39], involving only monomials without any need for Legendre polynomials. However, it has a weakness in the treatment of stability analysis for time-varying delay systems, which still requires a kind of reciprocally convex matrix inequalities.

The presents work investigates the stability problem of time-varying delay systems. The main contributions of the paper are summarized as follows: 1) A novel negative condition of the quadratic polynomial function with two variable parameters is developed, improving the negative conditions of the quadratic polynomial function for stability analysis of time-delay systems. 2) A new version of Bessel-Legendre inequality introduced without Legendre polynomials and extra reciprocally convex matrix inequalities, which is more appropriate for the stability analysis of time-varying delay systems. 3) An appropriate functional is constructed to obtain less conservative stability criteria in terms of linear matrix inequalities (LMIs) for the time-delay systems with two different time-varying delay functions.

Throughout this paper, $A>0$ denotes that the matrix A is symmetric and positive definite. A^{-1} and A^T mean the inverse and the transpose of matrix A, respectively. $\mathrm{Sym}\{A\}=A+A^T$. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\mathbb{S}^n$ (respectively, $\mathbb{S}^n_+$) denotes a set of symmetric matrices (respectively, positive definite matrices). I and 0 denote the identity matrix and the zero matrix, respectively. The symmetric terms in a symmetric matrix are denoted by $*$. $diag\{\cdots\}$ is a block-diagonal matrix. col is a column vector. $\mathbb{N}^+$ stands for the set of positive integers.

2. PRELIMINARIES

Consider the following linear system with a time-varying delay:

$$\begin{cases}\dot{x}(t)=Ax(t)+A_dx(t-h(t)),\\ x(t)=\varphi(t),\ \ t\in[-\widetilde{h},0],\end{cases} \tag{1}$$

where $\varphi(t)$ is the initial condition; the time-varying delay function $h(t)$ satisfying one of the following two cases:

Case I: the time-varying delay function $h(t)$ is differentiable with

$$0\leq h(t)\leq\widetilde{h},\ \ h_m\leq\dot{h}(t)\leq h_M<1. \tag{2}$$

Case II: the time-varying delay function $h(t)$ is not differentiable but continuous with

$$0\leq h(t)\leq\widetilde{h}, \tag{3}$$

where h_m, h_M and $\widetilde{h}$ are real constants.

The aim to develop the less conservative stability criteria, the following some lemmas are presented.

Lemma 1 [39]**:** Let $N\in\mathbb{N}$, any scalars $\alpha<\beta$, and x be a continuous and differentiable function x in $[\alpha,\beta]\to\mathbb{R}^n$. For a matrix $R\in\mathbb{S}^n_+$, the following inequality holds:

$$-\int_\alpha^\beta\dot{x}^T(s)R\dot{x}(s)ds\leq-\frac{1}{\beta-\alpha}\Omega_N^T\Psi_N^T\bar{\Phi}_N^T\bar{R}_N\bar{\Phi}_N\Psi_N\Omega_N, \tag{4}$$

where

$$\begin{aligned}
&\Omega_N=\begin{cases}col\{x(\beta),x(\alpha)\}, & N=0,\\ col\{\Omega_0,\frac{1}{\beta-\alpha}g^1,\dots,\ \frac{1}{\beta-\alpha}g^N\}, & N\geq1,\end{cases}\\
&g^N_{(\alpha,\beta)}=\int_\alpha^\beta(\frac{\beta-s}{\beta-\alpha})^Nx(s)ds,\ \ N\in\mathbb{N}^+,\\
&\Psi_N=\begin{cases}diag\{I,I\}, & N=0,\\ diag\{\Psi_0,I,2I,\dots,NI\}, & N\geq1,\end{cases}\\
&\bar{\Phi}_N=col\{\Phi_0,\Phi_1,\dots,\Phi_N\},\ \ N\in\mathbb{N},\\
&\Phi_N=\begin{cases}diag\{I,I\}, & N=0,\\ diag\{\delta_0^NI,-\sum\limits_{l=0}^N\delta_l^NI,\delta_l^1I,\dots,\delta_l^NI\}, & N\geq1,\end{cases}\\
&\delta_l^k=(-1)^l\binom{k}{l}\binom{k+1}{l},\ \ l,k\in\mathbb{N},\\
&\bar{R}_N=diag\{R,3R,\dots,(2N+1)R\},\ \ N\in\mathbb{N}.
\end{aligned}$$

Remark 1: According to (4), the Legendre polynomials of Bessel-Legendre inequality are replaced by simple polynomials, where the derivative of $g^N(t)$ is related to $x(t)$ and $g^1(t),\dots,\ g^{N-1}(t)$, simplifying the complex calculations in the stability analysis of time-delay systems. However, the stability conditions obtained from Lemma 1 for time-varying delay systems require extra reciprocally convex matrix inequalities, which can be considered a drawback.

Based on Lemma 1 and a simple basic inequality, a new version of Bessel-Legendre inequality is introduced as follow:

Lemma 2: Let $N\in\mathbb{N}$, any matrix $R\in\mathbb{S}^n_+$ and $G\in\mathbb{R}^{(N+1)n\times k}$, any scalars $\alpha<\beta$, x be a continuous and differentiable function x in $[\alpha,\beta]\to\mathbb{R}^n$, and a vector $\xi\in\mathbb{R}^k$, the following inequality holds:

$$\begin{aligned}&-\int_\alpha^\beta\dot{x}^T(s)R\dot{x}(s)ds\\ &\leq\ 2(\bar{\Phi}_N\Psi_N\Omega_N)^TG\xi+(\beta-\alpha)\xi^TG^T\bar{R}_N^{-1}G\xi,\end{aligned} \tag{5}$$

and some variables defined as same as Lemma 1.

Proof: Combining basic inequality $-2(\bar{\Phi}_N\Psi_N\Omega_N)^TG\xi\leq(\bar{\Phi}_N\Psi_N\Omega_N)^T\frac{\bar{R}_N}{\beta-\alpha}(\bar{\Phi}_N\Psi_N\Omega_N)+\xi^TG^T(\beta-\alpha)\bar{R}_N^{-1}G\xi$ with Lemma 1, Lemma 2 can be easily obtained, so the proof detail is omitted. □

Remark 2: Lemma 2 provides a new version of Bessel-Legendre inequality in which Legendre polynomials are remove. Unlike inequality (4), it is easy to deal with the integral interval in the right-hand side of inequality (5) by the Schur complement, and the reciprocally convex inequality is no required in the stability analysis for time-varying delay systems.

Lemma 3 [22]**:** A quadratic polynomial function $f(s) = a_2s^2+a_1s+a_0$ is considered, where $a_i \in \mathbb{R}$ $(i = 0, 1, 2)$, if following inequalities hold:

$$\text{(i) } f(0)<0,\ \text{(ii) } f(d)<0,\ \text{(iii) } f(0)-d^2a_2<0, \tag{6}$$

then $f(s)<0$ for all $s \in [0,d]$.

The following lemma presents a novel negative condition of the quadratic polynomial function.

Lemma 4: For a quadratic polynomial function $f(s) = a_2s^2+a_1s+a_0$, where $a_i \in \mathbb{R}$ $(i = 0,1,2)$, $f(s)<0$ holds for $c_1 \in [0,1]$, $c_2 \in [1,2]$ and all $s \in [0,d]$ if following the conditions hold:

$$f(0)<0, \tag{7}$$
$$f(d)<0, \tag{8}$$
$$-a_2(c_1\frac{d}{2})^2+f(0)<0, \tag{9}$$
$$-a_2((1-c_1)\frac{d}{2})^2+f(\frac{d}{2})<0, \tag{10}$$
$$-a_2((1-c_2)\frac{d}{2})^2+f(\frac{d}{2})<0, \tag{11}$$
$$-a_2((\frac{c_2}{2}-1)d)^2+f(d)<0. \tag{12}$$

Proof: When $a_2>0$, $f(s)<0$ is guaranteed if (7) and (8) hold. When $a_2<0$, first, a tangent function $y(s) = f'(c)(s-c)+f(c)$ in $f(s)$ is given, where $c \in [0,d]$. if $y(s)<0$ holds, one has $f(s)<y(s)<0$. second, the interval $[0,d]$ is divided into $[0,\frac{d}{2}]$ and $[\frac{d}{2},d]$. In the interval $[0,\frac{d}{2}]$, let $c = c_1\frac{d}{2}$ with $c_1 \in [0,1]$. In the interval $[\frac{d}{2},d]$, let $c = c_2\frac{d}{2}$ with $c_2 \in [1,2]$. Third, since $y(s)$ is a linear function, $y(s)<0$ is guaranteed if $y(0)<0$, $y(\frac{d}{2})<0$, and $y(d)<0$ hold. Thus, (9)−(12) lead to $f(s)<0$ in the case of $a_2<0$. This completes the proof. □

Remark 3: It can be concluded during the proof procedure that Lemma 3 and Lemma 1 in [24] can be obtained when $c=d$ and $c=\frac{d}{2}$ for $s=\{0,d\}$, respectively. Therefore, Lemma 4 provides a more generalized negative sufficient condition for the quadratic polynomial function. Besides, in Lemma 1 of [26], the interval $[0,d]$ has been uniformly divided into $N(N \in \mathbb{N}^+)$ subintervals with fixed values of c in these subintervals, Thus, complex calculations are required for large values of N when using Lemma 1 in [26]. Two variable parameters and the delay interval decomposition method are employed to reduce the negative condition conservatism of the quadratic polynomial function compared with [22,24,26,27], while less conservative stability criteria of time-varying delay systems can be obtained through Lemma 4.

3. MAIN RESULTS

In this section, some less conservative stability criteria are developed. To simply the vector and matrix representation, the following notations are denoted:

$$
\begin{aligned}
&\sigma_N(t) = g^N_{(t-\widetilde{h},t-h(t))}(t),\\
&\varsigma_N(t) = g^N_{(t-h(t),t)}(t),\\
&\zeta_0(t) = \left[x^T(t),\ x^T(t-h(t)),\ x^T(t-\widetilde{h})\right]^T,\\
&\zeta_1(t,s) = \Big[\zeta_0^T(t), x^T(s), \dot{x}^T(s), \int_{t-h(t)}^{s} x^T(v)dv,\\
&\qquad \int_s^t x^T(v)dv\Big]^T,\\
&\zeta_2(t,s) = \Big[\zeta_0^T(t), x^T(s), \dot{x}^T(s), \int_{t-\widetilde{h}}^{s} x^T(v)dv,\\
&\qquad \int_s^{t-h(t)} x^T(v)dv\Big]^T,\\
&\zeta_3(t) = \Big[\frac{\sigma_0^T(t)}{\widetilde{h}-h(t)}, \frac{\sigma_1^T(t)}{\widetilde{h}-h(t)}, \frac{\varsigma_0^T(t)}{h(t)}, \frac{\varsigma_1^T(t)}{h(t)}\Big]^T,\\
&\zeta_4(t) = \left[\sigma_0^T(t),\ \sigma_1^T(t),\ \varsigma_0^T(t),\ \varsigma_1^T(t)\right]^T,\\
&\eta(t,s) = \Big[x^T(t),\ x^T(s), \int_{t-\widetilde{h}}^{s} x^T(v)dv, \int_s^t x^T(v)dv\Big]^T,\\
&\bar{\xi}(t) = \left[\zeta_0^T(t),\ \zeta_3^T(t)\right]^T,\\
&\xi(t) = \left[\bar{\xi}^T(t),\ \dot{x}^T(t-h(t)),\ \dot{x}^T(t-\widetilde{h})\right]^T,\\
&e_i = \left[0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(9-i)n}\right],\ i = 1,2,\cdots,9,\\
&\bar{e}_i = \left[0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(7-i)n}\right],\ i = 1,2,\cdots,7.
\end{aligned}
$$

When the time-varying delay function $h(t)$ is Case I, we present our first stability criterion based on a novel negative condition of the quadratic polynomial function and a new version of Bessel-Legendre inequality.

Theorem 1: For given $\widetilde{h}$, h_m, h_M, $c_1 \in [0,1]$ and $c_2 \in [1,2]$, the system (1) with delay function $h(t)$ satisfying (2) is asymptotically stable if there exist matrices $Q_1, Q_2 \in \mathbb{S}^{7n}_+$, $R \in \mathbb{S}^n_+$, and any matrices $N_1, N_2 \in \mathbb{R}^{3n\times 9n}$, such that the following LMIs are hold for $\dot{h}(t) \in \{h_m, h_M\}$:

$$\begin{bmatrix} \Gamma(0,\dot{h}(t)) & \sqrt{\widetilde{h}}N_1^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \Gamma(\widetilde{h},\dot{h}(t)) & \sqrt{\widetilde{h}}N_2^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} -(\frac{c_1\widetilde{h}}{2})^2\Xi(\dot{h}(t))+\Gamma(0,\dot{h}(t)) & \sqrt{\widetilde{h}}N_1^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} -(\frac{(1-c_1)\widetilde{h}}{2})^2\Xi(\dot{h}(t)) + \Gamma(\frac{\widetilde{h}}{2},\dot{h}(t)) & \sqrt{\frac{\widetilde{h}}{2}}N_1^T & \sqrt{\frac{\widetilde{h}}{2}}N_2^T \\ * & -\widetilde{R} & 0 \\ * & * & -\widetilde{R} \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} -(\frac{(1-c_2)\widetilde{h}}{2})^2\ \Xi(\dot{h}(t)) + \Gamma(\frac{\widetilde{h}}{2},\dot{h}(t)) & \sqrt{\frac{\widetilde{h}}{2}}N_1^T & \sqrt{\frac{\widetilde{h}}{2}}N_2^T \\ * & -\widetilde{R} & 0 \\ * & * & -\widetilde{R} \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} -((\frac{c_2}{2}-1)\widetilde{h})^2\Xi(\dot{h}(t)) + \Gamma(\widetilde{h},\dot{h}(t)) & \sqrt{\widetilde{h}}N_2^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \quad (18)$$

where

$$\begin{aligned}
&\Gamma(h(t),\dot{h}(t)) \\
&= [\kappa_{11}+h(t)\kappa_{12}]^T Q_1[\kappa_{11}+h(t)\kappa_{12}] \\
&\quad -(1-\dot{h}(t))[\kappa_{21}+h(t)\kappa_{22}]^T Q_1[\kappa_{21}+h(t)\kappa_{22}] \\
&\quad +(1-\dot{h}(t))[\kappa_{41}+(\widetilde{h}-h(t))\kappa_{42}]^T \\
&\quad \times Q_2[\kappa_{41}+(\widetilde{h}-h(t))\kappa_{42}] \\
&\quad -[\kappa_{51}+(\widetilde{h}-h(t))\kappa_{52}]^T Q_2[\kappa_{51}+(\widetilde{h}-h(t))\kappa_{52}] \\
&\quad +\widetilde{h}\kappa_0^T R\kappa_0 + \mathrm{Sym}\{\Pi_1^T(\dot{h}(t)) \\
&\quad \times Q_1[\kappa_{30}+h(t)\kappa_{31}+h^2(t)\times\kappa_{32}] \\
&\quad +\Pi_2^T(\dot{h}(t))Q_2[\kappa_{60}+(\widetilde{h}-h(t))\kappa_{61}+(\widetilde{h}-h(t))^2\kappa_{62}] \\
&\quad +\Lambda_1^T N_1+\Lambda_2^T N_2\},\\
&\Xi(\dot{h}(t)) = \mathrm{Sym}\{\Pi_1^T(\dot{h}(t))Q_1\kappa_{32}+\Pi_2^T(\dot{h}(t))Q_2\kappa_{62}\} \\
&\qquad +\kappa_{12}^T Q_1\kappa_{12}-(1-\dot{h}(t))\kappa_{22}^T Q_1\kappa_{22} \\
&\qquad -\kappa_{52}^T Q_2\kappa_{52}+(1-\dot{h}(t))\kappa_{42}^T Q_2\kappa_{42},\\
&\Pi_1(\dot{h}(t)) = \left[\kappa_0^T,\ (1-\dot{h}(t))e_8^T,\ e_9^T,\ 0,\ 0,\ (\dot{h}(t)-1)e_2^T,\ e_1^T\right]^T,\\
&\Pi_2(\dot{h}(t)) = \left[\kappa_0^T,\ (1-\dot{h}(t))e_8^T,\ e_9^T,\ 0,\ 0,\ -e_3^T,\ (1-\dot{h}(t))e_2^T\right]^T,\\
&\kappa_0 = Ae_1+A_de_2,\\
&\kappa_{11} = \left[e_1^T,\ e_2^T,\ e_3^T,\ e_1^T,\ \kappa_0^T,\ 0,\ 0\right]^T,\\
&\kappa_{12} = \left[0,\ 0,\ 0,\ 0,\ 0,\ e_6^T,\ 0\right]^T,\\
&\kappa_{21} = \left[e_1^T,\ e_2^T,\ e_3^T,\ e_2^T,\ e_8^T,\ 0,\ 0\right]^T,\\
&\kappa_{22} = \left[0,\ 0,\ 0,\ 0,\ 0,\ 0,\ e_6^T\right]^T,\\
&\kappa_{30} = \left[0,\ 0,\ 0,\ 0,\ e_1^T-e_2^T,\ 0,\ 0\right]^T,\\
&\kappa_{31} = \left[e_1^T,\ e_2^T,\ e_3^T,\ e_6^T,\ 0,\ 0,\ 0\right]^T,\\
&\kappa_{32} = \left[0,\ 0,\ 0,\ 0,\ 0,\ e_7^T,\ e_6^T-e_7^T\right]^T,\\
&\kappa_{41} = \left[e_1^T,\ e_2^T,\ e_3^T,\ e_2^T,\ e_8^T,\ 0\ 0\right]^T,\\
&\kappa_{42} = \left[0,\ 0,\ 0,\ 0,\ 0,\ e_4^T,\ 0\right]^T,\\
&\kappa_{51} = \left[e_1^T,\ e_2^T,\ e_3^T,\ e_3^T,\ e_9^T,\ 0\ 0\right]^T,\\
&\kappa_{52} = \left[0,\ 0,\ 0,\ 0,\ 0,\ 0,\ e_4^T\right]^T,\\
&\kappa_{60} = \left[0,\ 0,\ 0,\ 0,\ e_2^T-e_3^T,\ 0,\ 0\right]^T,\\
&\kappa_{61} = \left[e_1^T,\ e_2^T,\ e_3^T,\ e_4^T,\ 0,\ 0,\ 0\right]^T,\\
&\kappa_{62} = \left[0,\ 0,\ 0,\ 0,\ 0,\ e_5^T,\ e_4^T-e_5^T\right]^T,\\
&\widetilde{R} = diag\{R,3R,5R\},\\
&\Lambda_1 = \bar{\Phi}_2\Psi_2\Omega_{21},\quad \Lambda_2 = \bar{\Phi}_2\Psi_2\Omega_{22},\\
&\bar{\Phi}_2 = \begin{bmatrix} I & -I & 0 & 0 \\ I & I & -2I & 0 \\ I & -I & -6I & 6I \end{bmatrix},\\
&\Psi_2 = diag\{I,\ I,\ I,\ 2I\},\\
&\Omega_{21} = \left[e_2^T,\ e_3^T,\ e_4^T,\ e_5^T\right]^T,\\
&\Omega_{22} = \left[e_1^T,\ e_2^T,\ e_6^T,\ e_7^T\right]^T.
\end{aligned}$$

Proof: A suitable LKF candidate is established as

$$V(t) = \int_{t-h(t)}^{t} \zeta_1^T(t,s)Q_1\zeta_1(t,s)ds + \int_{t-\widetilde{h}}^{t-h(t)} \zeta_2^T(t,s) \times Q_2\zeta_2(t,s)ds + \int_{t-\widetilde{h}}^{t}\int_{\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta. \quad (19)$$

Calculating the derivative of $V(t)$ along the solution of (1), one has

$$\begin{aligned}
\dot{V}(t) =& \zeta_1^T(t,t)Q_1\zeta_1(t,t) - (1-\dot{h}(t))\zeta_1^T(t,t-h(t))Q_1 \\
&\times\zeta_1(t,t-h(t)) + 2\int_{t-h(t)}^{t}\zeta_1^T(t,s)Q_1\frac{\partial\zeta_1^T(t,s)}{\partial t}ds \\
&+(1-\dot{h}(t))\zeta_2^T(t,t-h(t))Q_2\zeta_2(t,t-h(t)) \\
&-\zeta_2^T(t,t-\widetilde{h})Q_2\zeta_2^T(t,t-\widetilde{h}) + 2\int_{t-\widetilde{h}}^{t-h(t))}\zeta_2^T(t,s) \\
&\times Q_2\frac{\partial\zeta_2^T(t,s)}{\partial t}ds + \widetilde{h}\dot{x}^T(t)R\dot{x}(t) + J(t),
\end{aligned}$$

where

$$\begin{aligned}
&\zeta_1(t,t) = (\kappa_{11}+h(t)\kappa_{12})\xi(t),\\
&\zeta_1(t,t-h(t)) = (\kappa_{21}+h(t)\kappa_{22})\xi(t),\\
&\zeta_2(t,t-h(t)) = (\kappa_{41}+(\widetilde{h}-h(t))\kappa_{42})\xi(t),\\
&\zeta_2(t,t-\widetilde{h}) = (\kappa_{51}+(\widetilde{h}-h(t))\kappa_{52})\xi(t),\\
&2\int_{t-h(t)}^{t}\zeta_1^T(t,s)Q_1\frac{\partial\zeta_1^T(t,s)}{\partial t}ds \\
&= \xi^T(t)\mathrm{Sym}\{\Pi_1^T(\dot{h}(t))Q_1[\kappa_{30}+h(t)\kappa_{31} \\
&\quad +h^2(t)\kappa_{32}]\}\xi(t),\\
&2\int_{t-\widetilde{h}}^{t-h(t))}\zeta_2^T(t,s)Q_2\frac{\partial\zeta_2^T(t,s)}{\partial t}ds \\
&= \xi^T(t)\mathrm{Sym}\{\Pi_2^T(\dot{h}(t))Q_2[\kappa_{60}+(\widetilde{h}-h(t))\kappa_{61} \\
&\quad +(\widetilde{h}-h(t))^2\kappa_{62}]\}\xi(t),\\
&J(t) = -\int_{t-h(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\widetilde{h}}^{t-h(t)}\dot{x}^T(s)R\dot{x}(s)ds.
\end{aligned}$$

By applying Lemma 2 to $J(t)$, it can be deduced that

$$-\int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le \xi^T(t)(\mathrm{Sym}\{\Lambda_2^T N_2\} + h(t)N_2^T\widetilde{R}^{-1}N_2)\xi(t),$$

$$-\int_{t-\widetilde{h}}^{t-h(t)} \dot{x}^T(s)R\dot{x}(s)ds \le \xi^T(t)(\mathrm{Sym}\{\Lambda_1^T N_1\} + (\widetilde{h}-h(t))N_1^T\widetilde{R}^{-1}N_1)\xi(t).$$

Summing the previous proof process, one has

$$\dot{V}(t) \le \xi^T(t)(\Gamma(h(t),\dot{h}(t)) + h(t)N_2^T\widetilde{R}^{-1}N_2 + (\widetilde{h}-h(t))N_1^T\widetilde{R}^{-1}N_1)\xi(t).$$

In view of observations from $\dot{V}(t)$ that is a quadratic polynomial function with respect to $h(t)$, then, by applying Lemma 4 and Schur complement, if (13)$-$(18) are satisfied, we get that $\dot{V}(t) < 0$. This completes the proof. □

Remark 4: Theorem 1 presents a new stability criterion for time-delay systems with the first type of time-varying delay function. Two free choose variable parameters are given in a decomposition interval of time delay, by means of the proposed a novel negative condition of the quadratic polynomial function plays critical roles in reducing the conservatism for stability analysis of the time-varying delay systems. Furthermore, a new version of Bessel-Legendre inequality is introduced. Taking advantages of Lemma 2 and Lemma 4, it can be predicted that a less conservative stability criterion can be obtained by Theorem 1.

Based on Lemma 3, the following Corollary can be derived.

Corollary 1: For given scalars $\widetilde{h}, h_m, h_M$, the system (1) with delay function $h(t)$ satisfying (2) is asymptotically stable if there exist matrices $Q_1, Q_2 \in \mathbb{S}_+^{7n}$, $R \in \mathbb{S}_+^n$, and any matrices $N_1, N_2 \in \mathbb{R}^{3n\times 9n}$, such that (13)$-$(14) and the following condition hold for $\dot{h}(t) \in \{h_m, h_M\}$:

$$\begin{bmatrix} -\widetilde{h}^2\Xi(\dot{h}(t)) + \Gamma(0,\dot{h}(t)) & \sqrt{\widetilde{h}}N_1^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{20}$$

where all terms are defined in Theorem 1.

Remark 5: Corollary 1 is given to show the superiority of Lemma 4. It can be concluded from Remark 3 that Corollary 1 is a particular case of Theorem 1 for some certain parameters are kept fixed. Furthermore, when the time-varying delay function $h(t)$ satisfies Case II, the following theorem can be presented.

Theorem 2: For given scalars $\widetilde{h}$, $c_1 \in [0,1]$ and $c_2 \in [1,2]$, the system (1) with delay function $h(t)$ satisfying (3) is asymptotically stable if there exist matrices $S \in \mathbb{S}_+^{4n}$, $R \in \mathbb{S}_+^n$, and any matrices $\bar{N}_1$ and $\bar{N}_2 \in \mathbb{R}^{3n\times 7n}$, such that

$$\begin{bmatrix} \bar{\Gamma}(0) & \sqrt{\widetilde{h}}N_1^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} \bar{\Gamma}(\widetilde{h}) & \sqrt{\widetilde{h}}N_2^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} -(\frac{c_1\widetilde{h}}{2})^2\Delta + \bar{\Gamma}(0) & \sqrt{\widetilde{h}}N_1^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -(\frac{(1-c_1)\widetilde{h}}{2})^2\Delta + \bar{\Gamma}(\frac{\widetilde{h}}{2}) & \sqrt{\frac{\widetilde{h}}{2}}N_1^T & \sqrt{\frac{\widetilde{h}}{2}}N_2^T \\ * & -\widetilde{R} & 0 \\ * & * & -\widetilde{R} \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -(\frac{(1-c_2)\widetilde{h}}{2})^2\Delta + \bar{\Gamma}(\frac{\widetilde{h}}{2}) & \sqrt{\frac{\widetilde{h}}{2}}N_1^T & \sqrt{\frac{\widetilde{h}}{2}}N_2^T \\ * & -\widetilde{R} & 0 \\ * & * & -\widetilde{R} \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} -((\frac{c_2}{2}-1)\widetilde{h})^2\Delta + \bar{\Gamma}(\widetilde{h}) & \sqrt{\widetilde{h}}N_2^T \\ * & -\widetilde{R} \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}
&\bar{\Gamma}(h(t)) = (\nu_{11} + h(t)\nu_{12})^T S(\nu_{11} + h(t)\nu_{12}) \\
&\quad - (\nu_{21} + h(t)\nu_{22})^T S(\nu_{21} + h(t)\nu_{22}) + \widetilde{h}\nu_0^T R\nu_0 \\
&\quad + \mathrm{Sym}\{\Theta^T S(\nu_{30} + h(t)\nu_{31} + h^2(t)\nu_{32}) + \bar{\Lambda}_1^T\bar{N}_1 \\
&\quad + \bar{\Lambda}_2^T\bar{N}_2\}, \\
&\Theta = \begin{bmatrix} \nu_0^T, & 0, & -\bar{e}_3^T, & \bar{e}_1^T \end{bmatrix}^T, \quad \nu_0 = A\bar{e}_1 + A_d\bar{e}_2, \\
&\nu_{11} = \begin{bmatrix} \bar{e}_1^T, & \bar{e}_1^T, & \widetilde{h}\bar{e}_4^T, & 0 \end{bmatrix}^T, \nu_{12} = \begin{bmatrix} 0, & 0, & \bar{e}_6^T - \bar{e}_4^T, & 0 \end{bmatrix}^T, \\
&\nu_{21} = \begin{bmatrix} \bar{e}_1^T, & \bar{e}_3^T, & 0, & \widetilde{h}\bar{e}_4^T \end{bmatrix}^T, \\
&\nu_{22} = \begin{bmatrix} 0, & 0, & 0, & \bar{e}_6^T - \bar{e}_4^T \end{bmatrix}^T, \\
&\nu_{30} = \begin{bmatrix} \widetilde{h}\bar{e}_1^T, & \widetilde{h}\bar{e}_4^T, & \widetilde{h}^2\bar{e}_5^T, & \widetilde{h}^2(\bar{e}_4^T - \bar{e}_5^T) \end{bmatrix}^T, \\
&\nu_{31} = \begin{bmatrix} 0, \bar{e}_6^T - \bar{e}_4^T, \widetilde{h}(\bar{e}_4^T - 2\bar{e}_5^T), \widetilde{h}(\bar{e}_6^T - 2(\bar{e}_4^T - \bar{e}_5^T)) \end{bmatrix}^T, \\
&\nu_{32} = \begin{bmatrix} 0, & 0, & \bar{e}_5^T - \bar{e}_4^T + \bar{e}_7^T, & \bar{e}_4^T - \bar{e}_5^T - \bar{e}_7^T \end{bmatrix}^T, \\
&\bar{\Lambda}_1 = \bar{\Phi}_2\Psi_2\bar{\Omega}_{21}, \quad \bar{\Lambda}_2 = \bar{\Phi}_2\Psi_2\bar{\Omega}_{22}, \\
&\bar{\Omega}_{21} = \begin{bmatrix} \bar{e}_2^T, & \bar{e}_3^T, & \bar{e}_4^T, & \bar{e}_5^T \end{bmatrix}^T, \\
&\bar{\Omega}_{22} = \begin{bmatrix} \bar{e}_1^T, & \bar{e}_2^T, & \bar{e}_6^T, & \bar{e}_7^T \end{bmatrix}^T, \\
&\Delta = \mathrm{Sym}\{\Theta^T S\nu_{32}\} + \nu_{12}^T S\nu_{12} - \nu_{22}^T S\nu_{22},
\end{aligned}$$

and other notations are defined in the same as Theorem 1.

Proof: We choose the simple LKF as

$$\widetilde{V}(t) = \int_{t-\widetilde{h}}^{t} \eta^T(t,s)S\eta(t,s)ds + \int_{t-\widetilde{h}}^{t}\int_{\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta. \tag{27}$$

Taking the time-derivative of $\widetilde{V}(t)$, one obtains

$$\dot{\widetilde{V}}(t) = \eta^T(t,t)S\eta(t,t) - \eta^T(t,t-\widetilde{h})S\zeta(t,t-\widetilde{h}) + 2\int_{t-\widetilde{h}}^{t} \eta^T(t,s)S\frac{\partial\eta^T(t,s)}{\partial t}ds + \widetilde{h}\dot{x}^T(t)R\dot{x}(t)$$

$$-\int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\widetilde{h}}^{t-h(t)} \dot{x}^T(s)R\dot{x}(s)ds,$$

where

$$\eta(t,t) = (\nu_{11} + h(t)\nu_{12})\bar{\xi}(t),$$
$$\zeta(t,t-\widetilde{h}) = (\nu_{21} + h(t)\nu_{22})\bar{\xi}(t),$$
$$2\int_{t-\widetilde{h}}^{t} \eta^T(t,s)S\frac{\partial \eta^T(t,s)}{\partial t}ds$$
$$= \bar{\xi}^T(t)\mathrm{Sym}\{\Theta^T S[\nu_{30} + h(t)\nu_{31} + h^2(t)\nu_{32}]\}\bar{\xi}(t).$$

Similar to the proof of Theorem 1, one has

$$\dot{\tilde{V}}(t) \leq \bar{\xi}^T(t)(\bar{\Gamma}(h(t)) + h(t)\bar{N}_2^T\widetilde{R}^{-1}\bar{N}_2 + (\widetilde{h} - h(t)) \times \bar{N}_1^T\widetilde{R}^{-1}\bar{N}_1)\bar{\xi}(t).$$

Finally, by applying Lemma 4 and Schur complement, if (21)−(26) are satisfied, we get that $\dot{\tilde{V}}(t) < 0$. This completes the proof. □

Remark 6: Theorem 2 develops a sufficient condition for stability of time-varying delay systems with the second type of time-varying delay function, which does not require too many decision variables, but some simple LMIs conditions. The simulation results in the next section will demonstrate that less conservative results can be obtained through Theorem 2 compared with similar results.

4. NUMERICAL EXAMPLES

In this section, three examples are provided to verify the effectiveness and merits of proposed stability criteria.

Example 1: Consider the linear time-delay system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \; A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

and the delay function $h(t)$ satisfies Case I.

Considering the tradeoff between the admissible delay upper bounds (ADUBs) and the NDVs, assume $c_1 \in \{0.00, 0.05, \ldots, 0.95, 1.00\}$ and $c_2 \in \{1.00, 1.05, \ldots, 1.95, 2.00\}$. The ADUBs and the NDVs obtained by Theorem 1, Corollary 1, and some previously-published results for different values of $d = h_M = -h_m$ are given in Table 1, where the values of c_1 and c_2 in Theorem 1 are chosen $c_1 = 0.25 \sim 1, c_2 = 1.75 (d = 0.1)$, $c_1 = 0.45 \sim 1, c_2 = 1.55 (d = 0.5)$ and $c_1 = 0.6, c_2 = 1.4 (d = 0.8)$, respectively. It can be concluded from Table 1 that the quadratic-partitioning approaches of [26] is more effective compared with other existing works, and the ADUBs values obtained by Theorem 1 are higher than the Corollary 1 and the results of [10,22,23,26,29,32,38], demonstrating the advantages of the proposed novel negative condition for the quadratic polynomial function and the new version of Bessel-Legendre inequality.

Table 1. The ADUBs $\widetilde{h}$ for various $d = h_M = -h_m$ (Example 1).

d	0.1	0.5	0.8	NDVs
[29](Th.7)	4.703	2.420	2.137	46
[22](Th.1)	4.753	2.429	2.183	124
[32](Th.1)	4.788	3.055	2.615	282
[38](Th.8,$N=2$)	4.93	3.09	2.66	263
[10](Th.2,N=2)	4.90	3.16	2.73	276
[23](Prop.1)	4.910	3.216	2.789	231
[26](Th.1,C3)	4.939	3.298	2.869	446
Corollary 1	4.937	3.287	2.867	429
Theorem 1	4.967	3.376	2.922	429

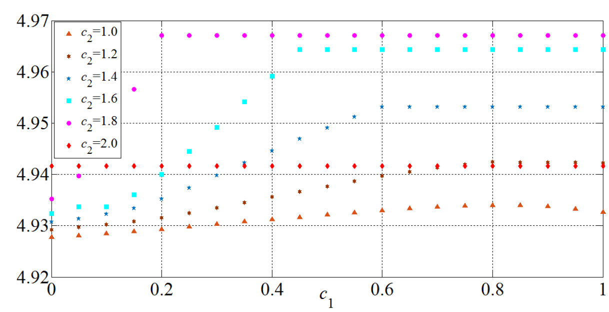

Fig. 1. The ADUBs $\widetilde{h}$ for $d = 0.1$ and various c_1 when $c_2 = 1.0$, $c_2 = 1.2$, $c_2 = 1.4$, $c_2 = 1.6$, $c_2 = 1.8$, and $c_2 = 2.0$ (Example 1).

For given different c_1, the ADUBs when $c_2 \in \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$ are depicted in Fig. 1 under $d = 0.1$, respectively. Fig. 1 shows that superior ADUBs can be obtained by appropriate choice of the variable parameters c_1 and c_2, which confirms the effectiveness of Theorem 1.

Example 2: Consider the linear time-delay system (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -10 & -1 \end{bmatrix}, \; A_d = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}.$$

and the delay function $h(t)$ satisfies Case II.

For given various $c_1 \in \{0.00, 0.05, \ldots, 0.95, 1.00\}$ and $c_2 \in \{1.00, 1.05, \ldots, 1.95, 2.00\}$, the ADUBs and the NDVs are shown in Table 2 by utilizing Theorem 2 and some methods in the literature, where $c_1 = 0.65$, $c_2 = 1.35$. From the Table 2, Theorem 2 provides a lager ADUBs than other literature. Furthermore, the NDVs in Theorem 2 is fewer than those of [27,33,34,36]. It means

Table 2. The ADUBs $\widetilde{h}$ for Example 2.

Method	[29]	[30]	[33]	[27]	[36]	[34]	Theorem 2
$\widetilde{h}$	1.59	1.64	1.80	1.977	2.39	2.40	2.485
NDVs	28	96	237	371	627	209	207

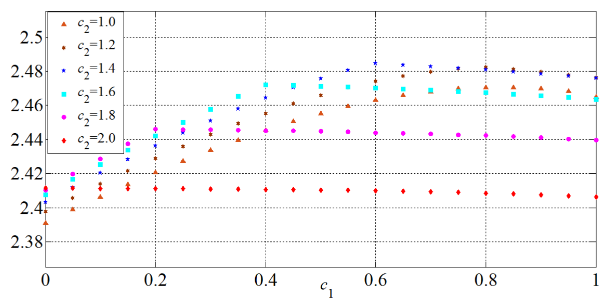

Fig. 2. The ADUBs $\widetilde{h}$ for various c_1 when $c_2 = 1.0$, $c_2 = 1.2$, $c_2 = 1.4$, $c_2 = 1.6$, $c_2 = 1.8$, and $c_2 = 2.0$ (Example 2).

that Theorem 2 is a good balance between the ADUBs and the NDVs. For given different c_1, the ADUBs when $c_2 \in \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$ are presented in Fig. 2, respectively. Fig. 2 shows the validity of the proposed Theorem 2.

Example 3: Consider the load frequency control of single power system [40] modeled with (Practical example of the system (1))

$$x(t) = [\Delta f \ \ \Delta P_m \ \ \Delta P_v \ \ \int ACE\ ds]^T,$$

$$A = \begin{bmatrix} \frac{\mathcal{D}}{\mathcal{M}} & \frac{1}{\mathcal{M}} & 0 & 0 \\ 0 & -\frac{1}{\mathcal{T}_t} & \frac{1}{\mathcal{T}_t} & 0 \\ -\frac{1}{\mathcal{R}\mathcal{T}_g} & 0 & -\frac{1}{\mathcal{T}_g} & 0 \\ \rho & 0 & 0 & 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\mathcal{K}_p\rho}{\mathcal{T}_g} & 0 & 0 & -\frac{\mathcal{K}_i}{\mathcal{T}_g} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where Δf, ΔP_m, ΔP_v and ACE are respectively the deviation of frequency, the generator mechanical output, valve position and area control error. $\mathcal{D}$, $\mathcal{M}$, $\mathcal{T}_g$, $\mathcal{T}_t$, $\mathcal{R}$ and ρ are the generator damping coefficient, the moment of inertia of the generator, the time constants of the governor, the turbine, the speed drop and the frequency bias factor, respectively. In addition, $\mathcal{K}_p$ and $\mathcal{K}_i$ are the gains of PI controller (See reference [27,40] for more details).

For given $\mathcal{D} = 1.0$, $\mathcal{M} = 10$, $\mathcal{T}_g = 0.1$, $\mathcal{T}_t = 0.3$, $\mathcal{R} = 0.05$, $\rho = 21$, $\mathcal{K}_p = 0.05$, $\mathcal{K}_i = 0.15$, $c_1 \in \{0.00, 0.05, 0.10, \ldots, 0.95, 1.00\}$ and $c_2 \in \{1.00, 1.05, 1.1, \ldots, 1.95, 2.00\}$, the values of ADUBs computed by the literature [27] and Theorem 2 are 7.649 ($\beta_0 = 0.75$) and 8.785 ($c_1 = 0.5, c_2 = 1.5$), respectively. One can find that the Theorem 2 present a significantly better stability criterion. For given different c_1, the ADUBs when $c_2 \in \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$ are depicted in Fig. 3, respectively. From the Fig. 3, it is observed that the some suitable values of c_1 and c_2 can derive less conservatism results. In addition, the state responses of the system under $\widetilde{h} = 8.785$ is

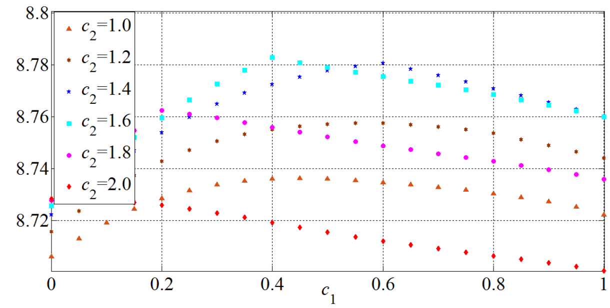

Fig. 3. The ADUBs $\widetilde{h}$ for various c_1 when $c_2 = 1.0$, $c_2 = 1.2$, $c_2 = 1.4$, $c_2 = 1.6$, $c_2 = 1.8$, and $c_2 = 2.0$ (Example 3).

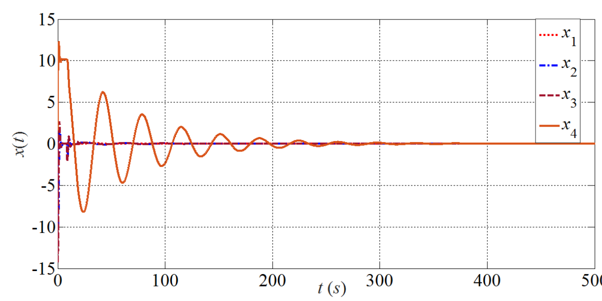

Fig. 4. Responses of system states.

plotted in Fig. 4, it can be found that the system is asymptotically stable.

5. CONCLUSION

In this paper, the stability analysis problem of time-varying delay systems with two types of time-varying delay function was investigated. A novel LKF with some integral terms was established. Furthermore, a new version of Bessel-Legendre inequality and a novel negative condition of the quadratic polynomial function were utilized to develop a less conservative theorem for the stability analysis of time-varying delay systems. Two commonly used numerical examples and a practical example were provided to evaluate the effectiveness and advantages of the proposed approaches. The future work will focus on applications of the proposed approach to delayed chaotic systems and delayed neural networks.

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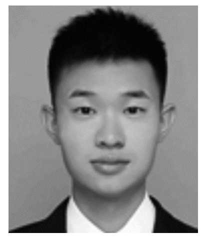

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