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Neural Network Observer Based Consensus Control of Unknown Nonlinear Multi-agent Systems with Prescribed Performance and Input Quantization

Zhengqing Shi, Chuan Zhou*, and Jian Guo

Abstract: This paper investigates the consensus tracking problem with predefined performances requirements for a class of unknown nonlinear multi-agent systems with hysteresis quantizer and external disturbances under a directed graph topology. Neural network observers are designed to estimate unmeasurable states and the the consensus tracking problem with performance requirements is transformed to a stabilization problem by prescribed performance error transformation schemes. The novel consensus protocol can be applied to a more general class of nonlinear multi-agent systems since the Lipschitz condition is avoided and state information is not required. It is strictly proved that all signals in the closed-loop systems are cooperatively uniformly ultimately bounded and both the transient and steady performances of the consensus tracking satisfy prescribed performance requirements. Finally, two numerical examples are presented to validate the effectiveness of the proposed strategy.

Keywords: Dynamic surface control, input quantization, neural network observer, prescribed performance, unknown nonlinear multi-agent system.

1. INTRODUCTION

Distributed consensus control for multi-agent systems (MASs) has been one of the most significant research directions due to the booming areas including satellite formation flying, mobile robot system and so on. Various recent results have been published for nonlinear MASs since nonlinearities are essential in practical systems. Neural networks and fuzzy logic systems are effective techniques due to their inherent properties of 'universal approximation' for more general nonlinear dynamics of MASs. In [1, 2], neural networks are utilized to approximate the characteristics of unknown nonlinearities. Moreover, the noise-induced uncertainty is addressed and noise-to-state practical stability is proved in existing literature [3, 4].

From a practical point of view, measuring all states of each agent is usually very difficult or impossible. Therefore, it is of great significance to investigate the observer-based consensus control protocol [5–7]. Neural network observer based consensus tracking control problem is studied in [8], by constructing the state observer and using backstepping techniques, the consensus tracking error and the observer error converge to a neighborhood of the origin. On the other hand, quantized control has received a great deal of interests in the past several years due to its potential in digital control systems [9–11].

It is worth noting that all the aforementioned papers solely focus on the consensus problem of MASs, transient and steady performances of the consensus tracking should be guaranteed to satisfy given performance indexes [12–16] rather than only reaching a consensus of behavior. For instances, the maximum overshoots is less than a prescribed value, the convergence rate is greater than a prescribed value given constant and the consensus tracking error stays in an arbitrarily small residual set. Moreover, the quantized control for nonlinear MASs with prescribed performance has been considered in some existing works [17, 18].

However, in existing results [17, 18], all states of follower agents are assumed to be available for nonlinear MASs with quantized input and prescribed performance, which is impractical for real applications. Moreover, some output feedback quantized controllers have been designed for MASs without considering prescribed performance [19–21]. Besides, the design of state observer is also difficult points. The major challenges lies in how to design consensus protocol meet the transient and steady performance requirements for the nonlinear MAS under some

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Zhengqing Shi, Chuan Zhou, and Jian Guo are with the School of Automation, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China (e-mails: shizhengqing888@163.com, njust_zc@126.com, guoj1002@mail.njust.edu.cn).
* Corresponding author.

non-negligible factors including estimation errors, quantization errors and external disturbances. To the best of our knowledge, the observer-based consensus tracking control for unknown MASs with prescribed performance and input quantization has not been investigated in existing literature.

Motivated by the above observations, a neural network observer-based distributed prescribed performance control protocol is designed for unknown nonlinear MASs with quantized controller and external disturbances. Based on radial basis function (RBF) neural networks, prescribed performance bound error transformation techniques and dynamic surface control schemes, the consensus conditions and consensus protocol design procedures are given. Then the main contributions are summarized as follows:

1) Compared with existing literature [17, 18], the proposed consensus protocol for consensus tracking problem with prescribed performance requirements of MAS with quantized controller utilizes only output information. Meanwhile, in contrast to existing results [11–13], the designed neural network observer does not require the unknown nonlinear dynamics of agents satisfying Lipschitz conditions. Therefore, the proposed consensus protocol can be can be applied to a more general class of MASs to solve the consensus tracking problem with prescribed performance.

2) The proposed consensus protocol improves the consensus tracking performance of nonlinear MAS. Different from existing results about practical consensus [5–8], the result in this paper can guarantee the convergence rate and tracking error satisfying predefined indexes. Furthermore, faster convergence rate and smaller consensus tracing error can be achieved by setting parameters of prescribed performance functions.

The rest of this paper is organized as follows: Section 2 gives the problem formulation. In Section 3, the main results are presented. Section 4 illustrates numerical examples. And section 5 concludes the whole paper.

Notations: Throughout this paper, $\mathbb{R}$ stands for the set of real numbers. I is an identity matrix with approximate dimensions. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of A, respectively. $A > 0$ is the matrix A is a positive definite matrix. $\|\cdot\|$ is the Euclidean norm.

2. PROBLEM FORMULATION

Consider a nonlinear MAS consists of one leader agent and m follower agents. The dynamics of follower agent i is described as

$$\begin{cases} \dot{x}_{ij}(t) = x_{i(j+1)}(t) + f_{ij}(\bar{x}_{ij}(t)) + d_{ij}(t), \\ \qquad\qquad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n_i - 1, \\ \dot{x}_{in_i}(t) = Q_i(u_i(t)) + f_{in_i}(x_i(t)) + d_{in_i}(t), \\ y_i(t) = x_{i1}(t), \end{cases} \tag{1}$$

where $x_{ij} \in \mathbb{R}$, $u_i(t) \in \mathbb{R}$ and $y_i(t) \in \mathbb{R}$ are the state variables, input and output of follower agent i, $Q_i(u_i(t))$ is the input quantization and $f_{ij}(\cdot)$ is the unknown nonlinear function. $d_{ij}(t)$ is the external disturbance and satisfies $|d_{ij}(t)| \leqslant d_{ij}^*$, where d_{ij}^* is an unknown positive constant. $\bar{x}_{ij}(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{ij}(t)]^T$, $x_i = [x_{i1}, \ldots, x_{in_i}]^T$ and only the output $y_i(t)$ can be measured.

The communication topology among m follower agents is described as a directed graph $G(V,E,A)$. $V = \{1,2,\ldots,m\}$ is the nonempty set of agents, $E = \{(i,j)\,|\,i,j \in V\}$ is the set of directed edges, and $A = [a_{ij}] \subseteq \mathbb{R}^{m\times m}$ with $a_{ij} > 0$ is the weighted adjacency matrix. If agent i can access the information from agent j, then $(i,j) \in E$ and $a_{ij} > 0$, otherwise, $a_{ij} = 0$ if $(i,j) \notin E$. Note $a_{ii} = 0$. The in-degree of agent i is defined as $d_i = \sum_{j=1}^{m} a_{ij}$ and the in-degree matrix $D = \operatorname{diag}\{d_i\}$. The Laplacian matrix L is $L = D - A$. The set of neighbor agents of agent i is denoted as $N_i = \{j \in V\,|\,(j,i) \in E, j \neq i\}$. Define the local consensus tracking error as

$$\tilde{y}_i(t) = \sum_{j=1}^{m} a_{ij}(y_i(t) - y_j(t)) + b_i(y_i(t) - y_d(t)), \tag{2}$$

where $y_d(t)$ is the trajectory of leader agent.

In order to ensure the transient and steady performances of the local consensus tracking error $\tilde{y}_i(t)$ within the given performance indexes, on the basis of performance requirements, choose the corresponding prescribed performance function $\mu_i(t)$ for each follower agent i, which is a positive monotonic decreasing smooth function. A general form of the prescribed performance function is given as

$$\mu_i(t) = (\mu_i(0) - \mu_i(\infty))\, e^{-l_i t} + \mu_i(\infty), \tag{3}$$

where $\mu_i(\infty) > 0$ is a constant and refers to the desired steady state of local consensus tracking error for follower agent i, and $l_i > 0$ is defined as the desired convergence rate of local consensus tracking error. The following inequality should hold to ensure the given transient and steady performance requirements.

$$-\underline{\delta}_i \mu_i(t) < \tilde{y}_i(t) < \bar{\delta}_i \mu_i(t), \tag{4}$$

where $\underline{\delta}_i \in (0,1]$ and $\bar{\delta}_i \in (0,1]$ are constants to be chosen according to performance requirements.

Assumption 1: The leader's output signal y_d, $\dot{y}_d$ and $\ddot{y}_d$ are bounded, and only available to a part of follower agents.

Assumption 2: The graph G contains a spanning tree with the root node being the leader agent.

Remark 1: It should be noted that the follower agent (1) is a more general system than that investigated in existing results [9–13], which is an unknown nonlinear systems with unmatched nonlinearities in control input and does not satisfy Lipschitz conditions. Moreover, both Assumption 1 and Assumption 2 are common assumptions in existing results.

Remark 2: The prescribed performance functions represent the transient and steady performance requirements of consensus behavior. In the traditional control framework, it is difficult to design the control protocol that satisfies performance requirements. By using prescribed performance transformation, the consensus problem with constraints is transformed to the stabilization problem, which can be easily solved. The introduction of prescribed performance functions not only ensures that the transient and steady performance requirements are satisfied, but also simplifies the design process.

For system (1), the following hysteresis quantizer described in [9] is considered.

$$Q_i(u_i) = \begin{cases} u_{i,h}sgn(u_i), \ \dfrac{u_{i,h}}{1+g_i} < |u_i| \le u_{i,h}, \ \dot{u}_i < 0, \\ \text{or } u_{i,h} < |u_i| \le \dfrac{u_{i,h}}{1-g_i}, \ \dot{u}_i > 0, \\ u_{i,h}(1+g_i)sgn(u_i), \ u_{i,h} < |u_i| \le \dfrac{u_{i,h}}{1-g_i}, \ \dot{u}_i < 0, \\ \text{or } \dfrac{u_{i,h}}{1-g_i} < |u_i| \le \dfrac{u_{i,h}(1+g_i)}{1-g_i}, \ \dot{u}_i > 0, \\ 0, \ 0 < |u_i| \le \dfrac{u_{i,\min}}{1+g_i}, \ \dot{u}_i < 0, \\ \text{or } \dfrac{u_{i,\min}}{1+g_i} < |u_i| \le u_{i,\min}, \dot{u}_i > 0, \\ Q_i(u_i(t^{-1})), \ \text{otherwise}, \end{cases} \tag{5}$$

where $u_{i,h} = \rho_i^{1-h} u_{i,\min}$ with $h = 1,2,....,$ $0 < \rho_i < 1$ and $u_{i,\min} > 0$ is the size of dead zone of $Q_i(\cdot)$, $g_i = \frac{1-\rho_i}{1+\rho_i}$.

The hysteresis quantizer can be decomposed into two parts that a linear part and a nonlinear part as follows:

$$Q_i(u_i) = u_i + F_i(u_i),$$

where $F_i(u_i) = Q_i(u_i) - u_i \in \mathbb{R}$.

Lemma 1 [9]**:** The nonlinear function f_i satisfies the following inequalities:

$$F_i^2(u_i) \leqslant g_i^2 u_i^2, \forall |u_i| \geqslant u_{i,\min},$$
$$F_i^2(u_i) \leqslant u_{i,\min}^2, \forall |u_i| \leqslant u_{i,\min}.$$

Lemma 2 [21]**:** For any positive constant σ and any variable x, the following relationship holds:

$$0 \leqslant |x| - \frac{x^2}{\sqrt{x^2+\sigma^2}} < \sigma.$$

The objective of this paper is to design a consensus protocol for a class of unknown nonlinear MAS with hysteresis quantizer and external disturbances under a directed graph topology to achieve consensus and meet the prescribed performance requirements.

3. DESIGN OF OBSERVER AND PRESCRIBED PERFORMANCE CONTROL PROTOCOL

In this section, a consensus protocol will be proposed to achieve the consensus and guarantee prescribed performances for the whole MAS. Since only output information of each follower agent is available, neural network state observers are designed to estimate states for follower agents. Then the prescribed performance consensus protocol is designed based on dynamic surface control schemes.

According to the universal approximation property of neural networks, RBF neural networks can be used to approximate unknown nonlinear functions. For any continuous function $f(z)$ defined on a compact set $z \in \Omega_z \subseteq R^n$ and any predefined accuracy $\bar{\varepsilon} > 0$, there exists

$$f(z) = W^{*T}\Phi(z) + \varepsilon(z), \tag{6}$$

where $W^* = [W_1^*, ..., W_N^*]^T \in \mathbb{R}^N$ is the unknown ideal weight vector and the approximation error $|\varepsilon(z)| \leqslant \bar{\varepsilon}$. $\Phi(z) = [\Phi_1(z), ..., \Phi_N(z)]^T \in \mathbb{R}^N$ is the basis vector chosen as the Gaussian function, i.e.,

$$\Phi_i(z) = \exp\left[-\frac{(z-\mu_i)^T(z-\mu_i)}{\eta_i} \right], \tag{7}$$

where $\mu_i = [\mu_{i1}, ..., \mu_{il}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. Obviously, the basis function is bounded, and there exists $\bar{\Phi}_i > 0$ such that $\|\Phi_i(z)\| \leqslant \bar{\Phi}_i$.

3.1. State observer design

The following neural network observer is designed to estimate unmeasurable states of follower agent i described as (1). For simplicity, the term 't' is omitted in the following sections.

$$\begin{cases} \dot{\hat{x}}_{ij} = \hat{x}_{i(j+1)} + \hat{W}_{ij}^T \Phi_{ij}\left(\hat{\bar{x}}_{ij}\right) + l_{ij}\left(x_{i1} - \hat{x}_{i1}\right), \\ \qquad i = 1,2,...,m, \ \ j = 1,2,...,n_i - 1, \\ \dot{\hat{x}}_{in_i} = Q_i(u_i) + \hat{W}_{in_i}^T \Phi_{in_i}(\hat{x}_i) + l_{in_i}(x_{i1} - \hat{x}_{i1}), \end{cases} \tag{8}$$

where $\hat{x}_{ij}$ is the estimation of x_{ij}, $\hat{\bar{x}}_{ij} = [\hat{x}_{i1}, ..., \hat{x}_{ij}]^T$ and $\hat{x}_i = [\hat{x}_{i1}, ..., \hat{x}_{in_i}]^T$. Since $f_{ij}(\bar{x}_{ij}) = W_{ij}^{*T}\Phi_{ij}(\bar{x}_{ij}) + \varepsilon_{ij}(\bar{x}_{ij})$ with the unknown ideal weight vector W_{ij}^*, we use $\hat{W}_{ij}$ to approximate W_{ij}^*. l_{ij} is the parameter to be chosen. $Q_i(u_i)$ is the quantized control input.

The estimation error is defined as

$$e_{ij} = x_{ij} - \hat{x}_{ij}. \tag{9}$$

From (1) and (8), we have

$$\dot{e}_i = A_{ic}e_i + \tilde{W}_i^T\Phi_i(\hat{x}_i) + W_i^{*T}(\Phi_i(x_i) - \Phi_i(\hat{x}_i)) + \varepsilon_i(x_i) + d_i, \tag{10}$$

where $e_i = [e_{i1}, e_{i2}, ..., e_{in_i}]^T$, $\tilde{W}_i = diag\{\tilde{W}_{i1}, \tilde{W}_{i2}, ..., \tilde{W}_{in_i}\}$, $\Phi_i(x_i) = [\Phi_{i1}(x_{i,1})^T, \Phi_{i2}(\bar{x}_{i,2})^T, \cdots, \Phi_{i,n_i}(\bar{x}_{i,n_i})^T]^T$, $\Phi_i(\hat{x}_i) = [\Phi_{i1}(\hat{x}_{i,1})^T, \Phi_{i2}(\hat{\bar{x}}_{i,2})^T, \cdots, \Phi_{i,n_i}(\hat{\bar{x}}_{i,n_i})^T]^T$, and

$$A_{ic} = \begin{bmatrix} -l_{i1} & 1 & 0 & \cdots & 0 \\ -l_{i2} & 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ -l_{i,n_i-1} & \cdots & & 0 & 1 \\ -l_{i,n_i} & \cdots & & & 0 \end{bmatrix}.$$

Since $W_i^{*T}(\Phi_i(x_i) - \Phi_i(\hat{x}_i)) + \varepsilon_i(x_i)$ is bounded, there exists $\bar{\xi}_i > 0$ such that $\|W_i^{*T}(\Phi_i(x_i) - \Phi_i(\hat{x}_i)) + \varepsilon_i(x_i)\| \leq \bar{\xi}_i$. The Lyapunov function candidate is chosen as

$$V_{i0} = e_i^T P_i e_i, \tag{11}$$

where $P_i > 0$. Since $\|\Phi_i(\hat{x}_i)\| \leqslant \bar{\Phi}_i$, by using Young's inequality, we obtain

$$2e_i^T P_i \left(W_i^{*T}(\Phi_i(x_i) - \Phi_i(\hat{x}_i)) + \varepsilon_i(x_i)\right) \leq c_{i,01} e_i^T P_i P e_i + c_{i,01}^{-1} \bar{\xi}_i^2, \tag{12}$$

$$2e_i^T P_i d_i \leq c_{i,02} e_i^T P_i P_i e_i + c_{i,02}^{-1} d_i^{*2}, \tag{13}$$

$$2e_i^T P_i \left(\tilde{W}_i^T \Phi_i(\hat{x}_i)\right) \leqslant c_{i,03} e_i^T P_i P_i e_i + c_{i,03}^{-1} \bar{\Phi}_i^2 \sum\nolimits_{j=1}^{n_i} \tilde{W}_{ij}^T \tilde{W}_{ij}, \tag{14}$$

where $c_{i,01} > 0$, $c_{i,02} > 0$ and $c_{i,03} > 0$. The time derivative of V_{i0} satisfies

$$\begin{aligned} \dot{V}_{i0} \leq & e_i^T \left(P_i A_{ic} + A_{ic}^T P_i + (c_{i,01} + c_{i,02} + c_{i,03}) P_i P_i\right) e_i \\ & + c_{i,01}^{-1} \bar{\xi}_i^2 + c_{i,02}^{-1} d_i^{*2} + c_{i,03}^{-1} \bar{\Phi}_i^2 \sum\nolimits_{j=1}^{n_i} \tilde{W}_{ij}^T \tilde{W}_{ij}. \end{aligned} \tag{15}$$

3.2. Prescribed performance control protocol design

According to the property (4), if we set $\tilde{y}_i = \mu_i S_i(\varepsilon_i)$ and $S_i(\varepsilon_i) = \frac{\bar{\delta}_i e^{\varepsilon_i} - \underline{\delta}_i e^{-\varepsilon_i}}{e^{\varepsilon_i} + e^{-\varepsilon_i}}$, since $S_i(\varepsilon_i)$ is strictly monotonic increasing and its inverse function exists, i.e. $S_i^{-1}(\tilde{y}_i/\mu_i) = \frac{1}{2}\ln\frac{\underline{\delta}_i + \tilde{y}_i/\mu_i}{\bar{\delta}_i - \tilde{y}_i/\mu_i}$. The transformed error is given as

$$s_{i1} = S_i^{-1}(\tilde{y}_i/\mu_i) - \frac{1}{2}\ln\left(\underline{\delta}_i/\bar{\delta}_i\right). \tag{16}$$

From the above procedure, the consensus tracking problem is transformed into a stabilization problem described as (16), if s_{i1} is bounded, the given performance requirements of the consensus tracking error is satisfied and the tracking consensus is achieved. Define n_i error surfaces as follows.

$$\begin{cases} s_{i1} = S_i^{-1}(\tilde{y}_i/\mu_i) - \frac{1}{2}\ln\left(\underline{\delta}_i/\bar{\delta}_i\right), \\ s_{i2} = \hat{x}_{i2} - \bar{v}_{i2}, \\ \vdots \\ s_{in_i} = \hat{x}_{in_i} - \bar{v}_{in_i}, \end{cases} \quad i = 1, 2, ..., m, \tag{17}$$

where $\bar{v}_{ij}$, $j = 2, ..., n_i$ is a stabilizing function and obtained through a first-order filter with respect to the virtual control v_{ij}. The first-order filter is designed as

$$\beta_{ij}\dot{\bar{v}}_{ij} + \bar{v}_{ij} = v_{ij}, \bar{v}_{ij}(0) = v_{ij}(0), \tag{18}$$

where $\beta_{ij} > 0$ is a parameter to be chosen.

The virtual control law v_{ij}, $j = 2, ..., n_i$ for follower agent i during the design procedure of prescribed performance consensus protocol are constructed as follows:

$$\begin{cases} v_{i2} = -\frac{k_{i1}}{r_i q_i} s_{i1} + \frac{1}{q_i}\left(b_i \dot{y}_d + \frac{\tilde{y}_i \dot{\mu}_i}{\mu_i} + \sum\nolimits_{j=1}^{m} a_{ij}\hat{x}_{j2}\right) \\ \quad - \frac{1}{r_i q_i}\left(\frac{c_{i11} + c_{i12} + c_{i13} + c_{i14} + c_{i17}}{2}(r_i q_i)^2 \right. \\ \quad \left. + \frac{c_{i15} + c_{i16}}{2} m r_i^2\right) s_{i1} - \hat{W}_{i1}^T \Phi_{i1}(\hat{x}_{i1}) \\ \quad - \hat{M}_i^T \Psi_i(\chi_i), \\ v_{i3} = -\left(k_{i2} + \frac{c_{i21} + c_{i22}}{2}\right) s_{i2} - r_i q_i s_{i1} \\ \quad + l_{i2}(x_{i1} - \hat{x}_{i1}) + \dot{\bar{v}}_{i2}, \\ v_{i(j+1)} = -\left(k_{ij} + \frac{c_{ij1} + c_{ij2}}{2}\right) s_{ij} - s_{i(j-1)} \\ \quad + l_{ij}(x_{i1} - \hat{x}_{i1}) + \dot{\bar{v}}_{ij}, \\ \quad (i = 1, 2, ..., m;\ \ 3 \leqslant j \leqslant n_i - 1), \end{cases} \tag{19}$$

where $c_{i1k} > 0, k = 1, 2, ..., 7, c_{ijk} > 0, j = 2, ..., n_i, k = 1, 2$, $k_{i1} > 0$ and $k_{ij} > 0$ are parameters to be chosen later, and $\hat{W}_i$, $\hat{M}_i$ and ϖ_i will be defined later. The actual control law is constructed as

$$u_i = -\frac{s_{in_i}\varpi_i^2}{(1 - g_i)\sqrt{s_{in_i}^2 \varpi_i^2 + \sigma^2}}. \tag{20}$$

Remark 3: The parameters l_i and $\mu_i(\infty)$ of the prescribed performance functions are chosen according to the transient and steady performance requirements, where l_i is associated with the convergence rate and $\mu_i(\infty)$ is related to the consensus tracking errors. Larger l_i will bring faster convergence rates and smaller $\mu_i(\infty)$ will bring smaller consensus tracking errors, thus better transient and steady performances will be achieved by setting parameters of prescribed performance functions. However, too large l_i or too small $\mu_i(\infty)$ will bring other threatens to the systems. Therefore, it is important to choose appropriate parameters of the prescribed performance functions.

Theorem 1: Consider nonlinear MAS (1) under Assumption 1 and Assumption 2, if there exists a matrix $P_i = P_i^T > 0$ and $Q_i > 0$ such that $P_i A_{ic} + A_{ic}^T P_i + (c_{i01} + c_{i02} + c_{i03}) P_i P_i + \left(\frac{1}{2c_{i11}} + \sum\nolimits_{j=1}^{m} \frac{a_{ji}^2}{2c_{j15}}\right) I < -Q_i$ for $\forall$ $i = 1, 2, ..., m$, the adaptive laws are given as (23), (24), (41) and (48), and the control is given by (20), then all signals of the closed loop system are bounded and the local consensus tracking error $\tilde{y}_i$ satisfies the prescribed performance requirements.

Proof: Step 1: The Lyapunov function is chosen as

$$V_{i1} = \frac{1}{2}s_{i1}^2 + \frac{1}{2\vartheta_{i1}}\tilde{W}_{i1}^T\tilde{W}_{i1} + \frac{1}{2\psi_{i1}}\tilde{M}_i^T\tilde{M}_i + \frac{1}{2}z_{i2}^2 \tag{21}$$

where $\tilde{M}_i = M_i^* - \hat{M}_i$, $\tilde{W}_{i1} = W_{i1}^* - \hat{W}_{i1}$, $\psi_{i1} > 0$, $\vartheta_{i1} > 0$. $z_{i2} = \bar{v}_{i2} - v_{i2}$ and $\hat{M}_i$ is the estimation weight vector of M_i^*.

From (1), (16), the definition of e_{i2}, z_{i2} and e_{j2}, the time derivative of the first error surface is given as

$$\begin{aligned}\dot{s}_{i1} =& r_i\big(q_i(s_{i2}+e_{i2}+v_{i2}+z_{i2}+f_{i1}(\hat{x}_{i1})+\Gamma_i(\chi_i)+d_{i1}) \\ & -\sum_{j=1}^m a_{ij}(\hat{x}_{j2}+e_{j2}+d_{j1}) - b_i\dot{y}_d - \frac{\tilde{y}_i\dot{\mu}_i}{\mu_i}\big),\end{aligned} \tag{22}$$

where $q_i = \sum_{j=1}^m a_{ij} + b_i$, $r_i = \frac{1}{2}\left[\frac{1}{\tilde{y}_i+\underline{\delta}_i\mu_i} - \frac{1}{\tilde{y}_i-\bar{\delta}_i\mu_i}\right]$ with $-\underline{\delta}_i\mu_i(0) < \tilde{y}_i(0) < \bar{\delta}_i\mu_i(0)$, $\Gamma_i(\mathcal{X}_i) = f_{i1}(x_{i1}) - f_{i1}(\hat{x}_{i1}) - \frac{\sum_{j=1}^m a_{ij}f_{j1}(x_{j1})}{q_i}$ and $\mathcal{X}_i = [x_{i1}, \hat{x}_{i1}, x_{j1}], \forall j \in N_i$.

Construct RBF neural networks to approximate f_{i1} and $\Gamma_i(\chi_i)$ such that $f_{i1}(\hat{x}_{i1}) = W_{i1}^{*T}\Phi_{i1}(\hat{x}_{i1}) + \varepsilon_{i1}(\hat{x}_{i1})$, $\Gamma_i(\chi_i) = M_i^{*T}\Psi_i(\chi_i) + \tau_i(\chi_i)$, W_{i1}^* and M_i^* are unknown weight vectors, and there exists $\bar{\tau}_i > 0$ such that $\|\tau_i(\chi_i)\| \le \bar{\tau}_i$. The $\Psi(\cdot)$ is a Gaussian function defined as (7), and the adaptive laws of $\hat{M}_i$ and $\hat{W}_{i1}$ are designed as

$$\dot{\hat{M}}_i = \psi_i r_i q_i s_{i1}\Psi_i(\chi_i) - \gamma_i\hat{M}_i, \tag{23}$$

$$\dot{\hat{W}}_{i1} = \vartheta_{i1} r_i q_i s_{i1}\Phi_{i1}(x_{i1}) - \eta_{i1}\hat{W}_{i1}. \tag{24}$$

We define the compact set $\Omega_{i2} = \Big\{\sum_{j=1}^m(e_j^TP_je_j + \frac{1}{2}s_{j1}^2 + \frac{1}{2}s_{j2}^2 + \frac{1}{2}z_{j2}^2 + \frac{1}{2}\tilde{W}_{j1}^T\tilde{W}_{j1} + \frac{1}{2}\tilde{W}_{j2}^T\tilde{W}_{j2} + \frac{1}{2}\tilde{M}_j^T\tilde{M}_j) \leqslant \mu\Big\}$ and $\Pi = \{y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leqslant R\}$, where μ, R are constants. From (18), we have

$$\dot{z}_{i2} = -\frac{z_{i2}}{\beta_{i2}} + B_{i2}, \tag{25}$$

where $B_{i2} = -\dot{v}_{i2}$ is a continuous function and has a maximum $\bar{B}_{i2}$ over $\Omega_{i2}\times\Pi$. By using Young's inequality, we have

$$s_{i1}r_iq_ie_{i,2} \le \frac{c_{i11}}{2}(r_iq_i)^2s_{i1}^2 + \frac{e_{i2}^2}{2c_{i11}}, \tag{26}$$

$$s_{i1}r_iq_i\tau_i(\chi_i) \le \frac{c_{i12}}{2}(r_iq_i)^2s_{i1}^2 + \frac{\bar{\tau}_i^2}{2c_{i12}}, \tag{27}$$

$$s_{i1}r_iq_i\varepsilon_{i1}(\hat{x}_{i1}) \leqslant \frac{c_{i13}}{2}(r_iq_i)^2s_{i1}^2 + \frac{\bar{\varepsilon}_{i1}^2}{2c_{i13}}, \tag{28}$$

$$s_{i1}r_iq_id_{i1} \le \frac{c_{i14}}{2}(r_iq_i)^2s_{i1}^2 + \frac{d_{i1}^{*2}}{2c_{i14}}, \tag{29}$$

$$-s_{i1}r_i\sum_{j=1}^m a_{ij}e_{j2} \le \frac{c_{i15}}{2}mr_i^2s_{i1}^2 + \frac{1}{2c_{i15}}\sum_{j=1}^m a_{ij}^2e_{j2}^2, \tag{30}$$

$$-s_{i1}r_i\sum_{j=1}^m a_{ij}d_{j1} \le \frac{c_{i16}}{2}mr_i^2s_{i1}^2 + \frac{1}{2c_{i16}}\sum_{j=1}^m a_{ij}^2d_{j1}^{*2}, \tag{31}$$

$$s_{i1}r_iq_iz_{i2} \leqslant \frac{c_{i17}}{2}(r_iq_i)^2s_{i1}^2 + \frac{1}{2c_{i17}}z_{i2}^2, \tag{32}$$

$$z_{i2}\bar{B}_{i2} \leqslant \frac{1}{2c_{i18}}z_{i2}^2 + \frac{c_{i18}}{2}\bar{B}_{i2}^2, \tag{33}$$

where $c_{i1k} > 0$, $k = 1,2,\ldots,8$. From the definition of v_{i2} in (19), the time derivative of V_{i1} is

$$\begin{aligned}\dot{V}_{i1} \leqslant & -k_{i1}s_{i1}^2 - \frac{\gamma_i}{2\psi_i}\tilde{M}_i^T\tilde{M}_i - \frac{\eta_{i1}}{2\vartheta_{i1}}\tilde{W}_{i1}^T\tilde{W}_{i1} \\ & + \left(\frac{1}{2c_{i17}} + \frac{1}{2c_{i18}} - \frac{1}{\beta_{i2}}\right)z_{i2}^2 + \frac{e_i^Te_i}{2c_{i11}} \\ & + \frac{\sum_{j=1}^m a_{ij}^2e_j^Te_j}{2c_{i15}} + \frac{\bar{\tau}_i^2}{2c_{i12}} + \frac{\bar{\varepsilon}_{i1}^2}{2c_{i13}} + \frac{d_{i1}^{*2}}{2c_{i14}} \\ & + \frac{1}{2c_{i16}}\sum_{j=1}^m a_{ij}^2d_{j1}^{*2} + \frac{c_{i18}}{2}\bar{B}_{i2}^2 + s_{i1}r_iq_is_{i2} \\ & + \frac{\gamma_i}{2\psi_i}M_i^{*T}M_i^* + \frac{\eta_{i1}}{2\vartheta_{i1}}W_{i1}^{*T}W_{i1}^*.\end{aligned} \tag{34}$$

Choose positive parameters η_{i1}, ϑ_{i1}, β_{i2}, c_{i03}, c_{i17} and c_{i18} such that $\frac{\eta_{i1}}{2\vartheta_{i1}} - c_{i03}^{-1}\bar{\Phi}_i > 0$ and $\frac{2}{\beta_{i2}} - \frac{1}{c_{i17}} - \frac{1}{c_{i18}} > 0$.

Step j ($2 \le j < n_i$): The Lyapunov function candidate is chosen as

$$V_{ij} = \frac{1}{2}s_{ij}^2 + \frac{1}{2\vartheta_{ij}}\tilde{W}_{ij}^T\tilde{W}_{ij} + \frac{1}{2}z_{i(j+1)}^2, \tag{35}$$

where $\tilde{W}_{ij} = W_{ij}^* - \hat{W}_{ij}$ and $z_{i(j+1)} = \bar{v}_{i(j+1)} - v_{i(j+1)}$. The time derivative of s_{ij} is

$$\begin{aligned}\dot{s}_{ij} =& s_{i(j+1)} + v_{i(j+1)} + z_{i(j+1)} + \hat{W}_{ij}^T\Phi_{ij}(\hat{\bar{x}}_{ij}) \\ & + l_{ij}(x_{i1} - \hat{x}_{i1}) - \dot{\bar{v}}_{ij}.\end{aligned} \tag{36}$$

We define the compact set $\Omega_{i(j+1)} = \{\sum_{k=1}^m[e_k^TP_ke_k + \sum_{l=1}^{j+1}(\frac{1}{2}s_{kl}^2 + \frac{1}{2}\tilde{W}_{kl}^T\tilde{W}_{kl}) + \sum_{f=2}^{j+1}(\frac{1}{2}z_{kf}^2) + \frac{1}{2}\tilde{M}_k^T\tilde{M}_k] \leqslant \mu\}$, $\mu > 0$ is a constant. From (18), we have

$$\dot{z}_{i(j+1)} = -\frac{z_{i(j+1)}}{\beta_{i(j+1)}} + B_{i(j+1)}, \tag{37}$$

where $B_{i(j+1)} = \dot{v}_{i(j+1)}$ is a continuous function and has a maximum $\bar{B}_{i(j+1)}$ over $\Omega_{i(j+1)}\times\Pi$. By using Young's inequality, we have

$$s_{ij}z_{i(j+1)} \leqslant \frac{c_{ij1}}{2}s_{ij}^2 + \frac{1}{2c_{ij1}}z_{i(j+1)}^2, \tag{38}$$

$$s_{ij}W_{ij}^{*T}\Phi_{ij}(\hat{\bar{x}}_{ij}) \leqslant \frac{c_{ij2}}{2}s_{ij}^2 + \frac{1}{2c_{ij2}}\bar{\Phi}_i^2W_{ij}^{*T}W_{ij}^*, \tag{39}$$

$$z_{i(j+1)}B_{i(j+1)} \leqslant \frac{c_{ij3}}{2}z_{i(j+1)}^2 + \frac{1}{2c_{ij3}}\bar{B}_{i(j+1)}^2, \tag{40}$$

where c_{ij1}, c_{ij2} and c_{ij3} are positive parameters.

The adaptive law of estimation vector $\hat{W}_{ij}$ is chosen as

$$\dot{\hat{W}}_{ij} = -\vartheta_{ij}s_{ij}\Phi_{ij}(\hat{\bar{x}}_{ij}) - \eta_{ij}\hat{W}_{ij}, \tag{41}$$

where $\vartheta_{ij} > 0$ and $\eta_{ij} > 0$ are design parameters. From the Lyapunov function (35) and the virtual control law (19), if $j = 2$, then

$$\begin{aligned}\dot{V}_{ij} \leqslant& s_{ij}s_{i(j+1)} - k_{ij}s_{ij}^2 - r_iq_is_{ij}s_{i(j-1)} \\ &- \left(\frac{1}{\beta_{i(j+1)}} - \frac{1}{2c_{ij1}} - \frac{c_{ij3}}{2}\right) z_{i(j+1)}^2 - \frac{\eta_{ij}}{2\vartheta_{ij}}\tilde{W}_{ij}^T\tilde{W}_{ij} \\ &+ \frac{1}{2c_{ij3}}\bar{B}_{i(j+1)}^2 + \left(\frac{1}{2c_{ij2}}\bar{\Phi}_i^2 + \frac{\eta_{ij}}{2\vartheta_{ij}}\right) W_{ij}^{*T}W_{ij}^*. \end{aligned} \tag{42}$$

If $j = 3, ..., n_i - 1$, then

$$\begin{aligned}\dot{V}_{ij} \leqslant& s_{ij}s_{i(j+1)} - k_{ij}s_{ij}^2 - s_{ij}s_{i(j-1)} \\ &- \left(\frac{1}{\beta_{i(j+1)}} - \frac{1}{2c_{ij1}} - \frac{c_{ij3}}{2}\right) z_{i(j+1)}^2 - \frac{\eta_{ij}}{2\vartheta_{ij}}\tilde{W}_{ij}^T\tilde{W}_{ij} \\ &+ \frac{1}{2c_{ij3}}\bar{B}_{i(j+1)}^2 + \left(\frac{1}{2c_{ij2}}\bar{\Phi}_i^2 + \frac{\eta_{ij}}{2\vartheta_{ij}}\right) W_{ij}^{*T}W_{ij}^*. \end{aligned} \tag{43}$$

Step n_i: The Lyapunov function candidate is chosen as

$$V_{in_i} = \frac{1}{2}s_{in_i}^2 + \frac{1}{2\vartheta_{in_i}}\tilde{W}_{in_i}^T\tilde{W}_{in_i}, \tag{44}$$

where $\tilde{W}_{in_i} = W_{in_i}^* - \hat{W}_{in_i}$. Let

$$\begin{aligned}\varpi_i =& s_{i(n_i-1)} + \left(k_{in_i} + \frac{1}{2c_{in_i}}\right) s_{in_i} + l_{in_i}(x_{i1} - \hat{x}_{i1}) \\ &- \dot{\bar{v}}_{in_i} + \frac{u_{i\min}^2 s_{in_i}}{\sqrt{u_{i\min}^2 |s_{in_i}|^2 + \sigma^2}}. \end{aligned} \tag{45}$$

The time derivative of s_{in_i} is transformed into

$$\begin{aligned}\dot{s}_{in_i} =& u_i + F_i(u_i) - \tilde{W}_{in_i}^T\Phi_{in_i}(\hat{x}_i) + W_{in_i}^{*T}\Phi_{in_i}(\hat{x}_i) \\ &- \left(k_{in_i} + \frac{1}{2c_{in_i1}}\right) s_{in_i} - s_{i(n_i-1)} \\ &- \frac{u_{i\min}^2 s_{in_i}}{\sqrt{u_{i\min}^2 |s_{in_i}| + \sigma^2}} + \varpi_i. \end{aligned} \tag{46}$$

From Lemma 1, we have

$$s_{in_i}F_i(u_i) \leqslant -g_is_{in_i}u_i + u_{i\min}|s_{in_i}|. \tag{47}$$

The adaptive law $\hat{W}_{in_i}$ is chosen as

$$\dot{\hat{W}}_{in_i} = -\vartheta_{in_i}s_{in_i}\Phi_{in_i}(\hat{x}_{in_i}) - \eta_{in_i}\hat{W}_{in_i}, \tag{48}$$

where $\vartheta_{in_i} > 0$ and $\eta_{in_i} > 0$ are design parameters. From the control law (20), the time derivative of V_{in_i} is

$$\begin{aligned}\dot{V}_{in_i} \leqslant& -k_{in_i}s_{in_i}^2 - s_{i(n_i-1)}s_{in_i} - \frac{\eta_{in_i}}{2\vartheta_{in_i}}\tilde{W}_{in_i}^T\tilde{W}_{in_i} \\ &+ \left(\frac{c_{in_i1}}{2}\bar{\Phi}_i^2 + \frac{\eta_{in_i}}{2\vartheta_{in_i}}\right) W_{in_i}^{*T}W_{in_i}^* + 2\sigma. \end{aligned} \tag{49}$$

The overall Lyapunov function for follower agent i is chosen as

$$V_i = V_{i0} + \sum\nolimits_{j=1}^{n_i} V_{ij}. \tag{50}$$

The time derivative of (50) is

$$\begin{aligned}\dot{V}_i \leqslant& e_i^T\left(P_iA_{ic} + A_{ic}^TP_i + (c_{i01} + c_{i02} + c_{i03})P_iP_i + \frac{e_i^Te_i}{2c_{i11}}I\right)e_i \\ &+ \frac{\sum_{j=1}^m a_{ij}^2e_j^Te_j}{2c_{i15}} + \left(c_{i03}^{-1}\bar{\Phi}_i^2 - \frac{\eta_{i1}}{2\vartheta_{i1}}\right)\sum\nolimits_{j=1}^{n_i}\tilde{W}_{ij}^T\tilde{W}_{ij} \\ &- \frac{\gamma_i}{2\psi_i}\tilde{M}_i^T\tilde{M}_i - \sum_{j=1}^{n_i} k_{ij}s_{ij}^2 - \left(\frac{1}{\beta_{i2}} - \frac{1}{2c_{i17}} - \frac{1}{2c_{i18}}\right) z_{i2}^2 \\ &- \sum_{j=2}^{n_i-1}\left(\frac{1}{\beta_{i(j+1)}} - \frac{1}{2c_{ij1}} - \frac{c_{ij3}}{2}\right) z_{i(j+1)}^2 \\ &+ c_{i01}^{-1}\bar{\xi}_i^2 + c_{i02}^{-1}d_i^{*2} + \frac{\bar{\tau}_i^2}{2c_{i12}} + \frac{\bar{\varepsilon}_{i1}^2}{2c_{i13}} + \frac{d_{i1}^{*2}}{2c_{i14}} \\ &+ \frac{1}{2c_{i16}}\sum_{j=1}^m a_{ij}^2d_{j1}^{*2} + \frac{c_{i18}}{2}\bar{B}_{i2}^2 + \frac{\gamma_i}{2\psi_i}M_i^{*T}M_i^* \\ &+ \sum_{j=2}^{n_i-1}\left(\frac{1}{2c_{ij3}}\bar{B}_{i(j+1)}^2 + \left(\frac{1}{2c_{ij2}}\bar{\Phi}_i^2 + \frac{\eta_{ij}}{2\vartheta_{i1}}\right)W_{ij}^{*T}W_{ij}^*\right) \\ &+ \frac{\eta_{i1}}{2\vartheta_{i1}}W_{i1}^{*T}W_{i1}^* + \left(\frac{c_{in_i1}}{2}\bar{\Phi}_i^2 + \frac{\eta_{in_i}}{2\vartheta_{in_i}}\right)W_{in_i}^{*T}W_{in_i}^* + 2\sigma. \end{aligned} \tag{51}$$

For the whole MAS, the Lyapunov function is chosen as

$$V(t) = \sum\nolimits_{i=1}^m V_i(t). \tag{52}$$

From (51), we have

$$\dot{V}(t) = \sum\nolimits_{i=1}^m \dot{V}_i(t) \leqslant -\zeta V + \Delta, \tag{53}$$

where $\zeta = \min\{\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}, \frac{\eta_{i1}}{\vartheta_{i1}} - 2c_{i03}^{-1}\bar{\Phi}_i^2, 2k_{ij}, \frac{2}{\beta_{i(j+1)}} - \frac{1}{c_{ij1}} - c_{ij3}, \frac{2}{\beta_{i(j+1)}} - \frac{1}{c_{ij7}} - \frac{1}{c_{ij8}}, \frac{\gamma_{i1}}{\psi_{i1}}\}$, $\Delta = \sum_{i=1}^m(c_{i01}^{-1}\bar{\xi}_i^2 + c_{i02}^{-1}d_i^{*2} + \frac{\bar{\tau}_i^2}{2c_{i12}} + \frac{\bar{\varepsilon}_{i1}^2}{2c_{i13}} + \frac{d_{i1}^{*2}}{2c_{i14}} + \frac{1}{2c_{i16}}\sum_{j=1}^m a_{ij}^2d_{j1}^{*2} + \frac{c_{i18}}{2}\bar{B}_{i2}^2 + \frac{\gamma_i}{2\psi_i}M_i^{*T}M_i^* + \sum_{j=2}^{n_i-1}(\frac{1}{2c_{ij3}}\bar{B}_{i(j+1)}^2 + \left(\frac{1}{2c_{ij2}}\bar{\Phi}_i^2 + \frac{\eta_{ij}}{2\vartheta_{i1}}\right)W_{ij}^{*T}W_{ij}^*) + \frac{\eta_{i1}}{2\vartheta_{i1}}W_{i1}^{*T}W_{i1}^* + (\frac{c_{in_i1}}{2}\bar{\Phi}_i^2 + \frac{\eta_{in_i}}{2\vartheta_{in_i}})W_{in_i}^{*T}W_{in_i}^* + 2\sigma)$.

Let $\zeta > \Delta/\mu$, $\dot{V}(t) \leqslant 0$ on $V(t) = \mu$. Thus, $V(t) \leqslant \mu$ is an invariant set, i.e., if $V(0) \leqslant \mu$, then $V(t) \leqslant \mu$, $\forall t \geqslant 0$. Therefore, all signals in the closed-loop systems are cooperative semi-globally uniformly ultimately bounded. Then we have

$$s_1^2 \leqslant 2V(t) \leqslant 2e^{-\zeta t}V(0) + \frac{2\Delta}{\zeta}\left(1 - e^{-\zeta t}\right), \tag{54}$$

which means $\|s_1\|_1 \leqslant \sqrt{2e^{-\zeta t}V(0) + \frac{2\Delta}{\zeta}\left(1 - e^{-\zeta t}\right)}$, $s_1 = [s_{11}, s_{21}, ..., s_{m1}]^T$. Since $\lim_{t\to\infty} e^{-\zeta t} = 0$, $\lim_{t\to\infty}\|s_1\|_1 \leqslant \sqrt{\frac{2\Delta}{\zeta}}$. From the aforementioned analysis, we know that the local consensus tracking error satisfy the given performance requirements (4). □

4. NUMERICAL EXAMPLES

In this section, we will give two numerical examples to validate the effectiveness of the designed control protocol.

Example 1: A two-order nonlinear MAS is considered, the communication graph of the considered nonlinear MAS is shown in Fig. 1 and the dynamics of the follower agent is described in (55).

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) + f_{i1}(x_{i1}, t) + d_{i1}(t), \\ \dot{x}_{i2}(t) = Q_i(u_i) + f_{i2}(x_{i1}, x_{i2}, t) + d_{i2}(t), \\ y_i(t) = x_{i1}(t), \end{cases} \tag{55}$$

where $f_{i1}(x_{i1}, t) = -\frac{x_{i1}(t)}{1+x_{i1}^2(t)}$, $f_{i2}(x_{i1}, x_{i2}, t) = x_{i1}^2(t) + x_{i2}(t)$, $d_{i1}(t) = -0.9\sin(t)$ and $d_{i2}(t) = 0.9\cos(t)$.

The dynamics of the leader agent is described as $y_d = \sin(t)$, the performance functions are chosen as $\mu_i(t) = 2e^{-5t} + 0.25$ and $\underline{\delta}_i = \bar{\delta}_i = 1$.

The initial states of the four follower agents are chosen as $x_1(0) = [0.1, 0.1]^T$, $x_2 = [0.2, 0.2]^T$, $x_3(0) = [0.3, 0.3]^T$, $x_4(0) = [0.4, 0.4]^T$. $c_{i01} = c_{i02} = c_{i03} = c_{i11} = \cdots = c_{i18} = c_{i21} = c_{i22} = c_{i23} = 1$, $\forall i \in \{1, 2, 3, 4\}$. The parameters of the neural network observer are chosen as $l_{i1} = 10$, $l_{i2} = 20$, $\forall i \in \{1, 2, 3, 4\}$. The parameters of the neural networks are selected as $\eta_{i1} = \eta_{i2} = 10$, $\vartheta_{i1} = \vartheta_{i2} = 0.01$, and $\gamma_i = \psi_i = 1, \forall i \in \{1, 2, 3, 4\}$. $k_{11} = 12$, $k_{12} = 0.5$, $k_{21} = 3$, $k_{22} = 2$, $k_{31} = 1$, $k_{32} = 3$, $k_{41} = 0.4$, $k_{42} = 3$. The design parameters of the first-order filter are $\beta_{i1} = \beta_{i2} = 0.01$, $\forall i \in \{1, 2, 3, 4\}$. By using schur complement lemma and LMI toolbox, the symmetric and positive definite matrix P_i for each agent i is

$$P_i = \begin{bmatrix} 6.2410 & -2.2854 \\ -2.2854 & 1.2177 \end{bmatrix}. \tag{56}$$

The simulation results are shown in Figs. 2-3, Fig. 2 depicts the trajectory of y_i, Fig. 3 depicts the trajectory of local consensus tracking error $\tilde{y}_i$. From these figures, we can see the output signals of all follower agents converge to the leader and the consensus is achieved, furthermore, the local consensus tracking error signals $\tilde{y}_i$ satisfy the performance requirements.

Example 2: In order to further illustrate the effectiveness of the designed control protocol, a three-order nonlinear MAS is considered that consists of one leader agent and four follower agents. The communication graph is given in Fig.4, and the dynamics of the follower agent is described as

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) + f_{i1}(x_{i1}(t)) + d_{i1}(t), \\ \dot{x}_{i2}(t) = x_{i3}(t) + f_{i2}(x_{i1}(t), x_{i2}(t)) + d_{i2}(t), \\ \dot{x}_{i3}(t) = Q_i(u_i(t)) + f_{i3}(x_{i1}(t), x_{i2}(t), x_{i3}(t)) + d_{i3}(t), \\ y_i(t) = x_{i1}(t), \end{cases} \tag{57}$$

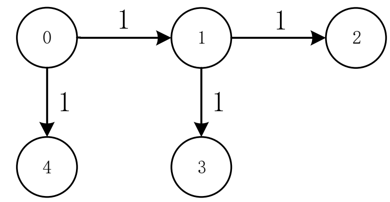

Fig. 1. Communication topology graph.

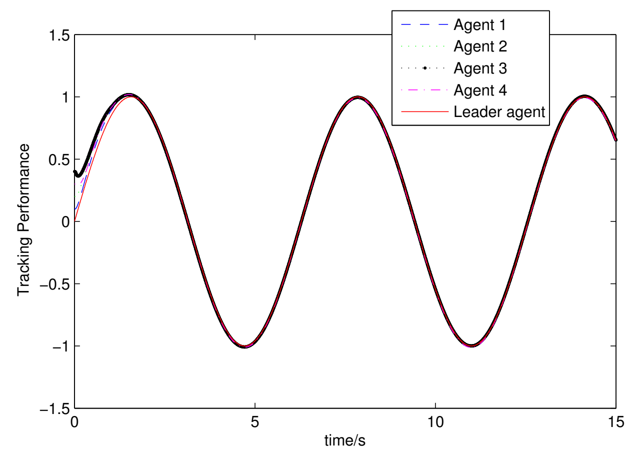

Fig. 2. Tracking performance.

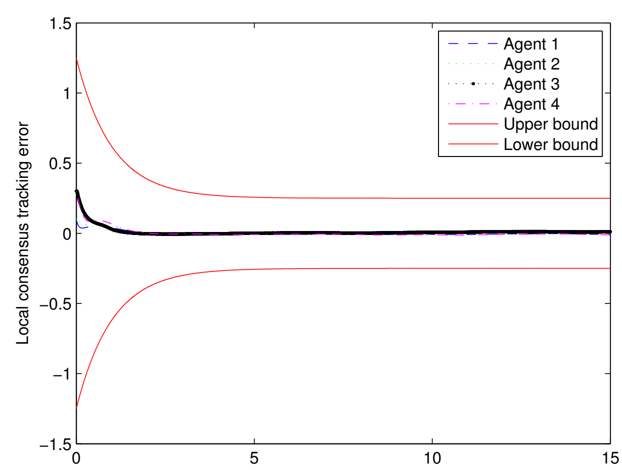

Fig. 3. Local consensus tracking error.

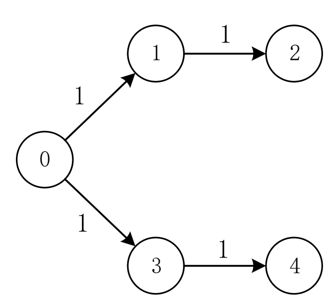

Fig. 4. Communication topology graph.

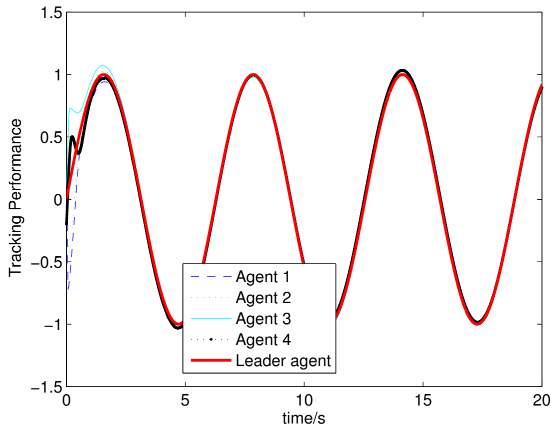

Fig. 5. Tracking performance.

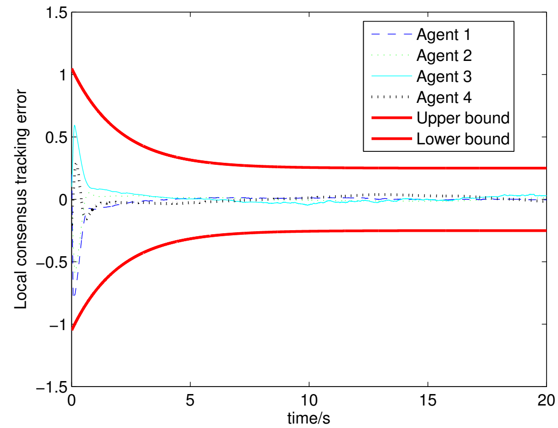

Fig. 6. Local consensus tracking error.

where $f_{i1}(x_{i1}(t))=\sin(x_{i1}(t))$, $f_{i2}(x_{i1}(t),x_{i2}(t))=\sin(x_{i1}(t)+x_{i2}(t))$, $f_{i3}(x_{i1}(t),x_{i2}(t),x_{i3}(t))=-\sin(x_{i1}(t))\sin(x_{i2}(t))+\cos(x_{i3}(t))$, $d_{i1}(t)=d_{i2}(t)=d_{i3}(t)=0.1\sin(t)$.

The dynamics of the leader agent is descried as $y_d(t)=\sin(t)$, the prescribed performance function is chosen as $\mu_i(t)=0.8e^{-t}+0.25$, $\underline{\delta}_i=\bar{\delta}_i=1$.

The initial states of the four follower agents are chosen as $x_1=[0.2,\ 0.2,\ 0.2]^T$, $x_2=[-0.1,\ -0.1,\ -0.1]^T$, $x_3=[0.1,\ 0.1,\ 0.1]^T$ and $x_4=[-0.2,\ -0.2,\ -0.2]^T$. The parameters are chosen as $c_{i01}=c_{i02}=c_{i03}=0.03$, $c_{i11}=\cdots=c_{i18}=c_{i21}=c_{i22}=c_{i23}=c_{i31}=c_{i32}=c_{i33}=1$, $\forall i\in\{1,\ 2,\ 3,\ 4\}$. The parameters of the neural network observer are chosen as $l_{i1}=20$, $l_{i2}=30$, $l_{i3}=20$, $\forall i\in\{1,\ 2,\ 3,\ 4\}$. The parameters of neural networks are chosen as $\eta_{i1}=\eta_{i2}=\eta_{i3}=10$, $\vartheta_{i1}=\vartheta_{i2}=\vartheta_{i3}=0.01$, $\gamma_i=\psi_i=1$, $\forall i\in\{1,\ 2,\ 3,\ 4\}$. $k_{11}=2$, $k_{12}=5$, $k_{13}=3$, $k_{21}=2.5$, $k_{12}=3$, $k_{13}=3$, $k_{31}=1$, $k_{32}=2$, $k_{33}=3$, $k_{41}=2.5$, $k_{42}=3$, $k_{43}=2$. The design parameters of the first-order filter are $\beta_{i1}=\beta_{i2}=\beta_{i3}=0.01$, $\forall i\in\{1,2,3,4\}$. By using schur complement lemma and LMI toolbox, the symmetric and positive definite matrix P_i for each agent i is

$$P_i=\begin{bmatrix}26.7774 & -3.8887 & -1.2019\\ -3.8887 & 4.9814 & -3.8624\\ -1.2019 & -3.8624 & 6.6709\end{bmatrix}. \tag{58}$$

The simulation results are shown in Figs. 5-6, Fig. 5 depicts the trajectory of y_i, Fig. 6 depicts the trajectory of local consensus tracking error $\tilde{y}_i$. From these figures, we can see the output signals of all follower agents converge to the leader and the consensus is achieved, furthermore, the local consensus tracking error signals $\tilde{y}_i$ satisfy the performance requirements.

5. CONCLUSIONS

In this paper, the observer-based prescribed performance consensus control protocol is proposed for unknown nonlinear MASs with quantized controller and external disturbances under directed communication topology. Effects of estimation errors, external disturbances and quantization errors on the performances of consensus are solved in this paper. Therefore, the designed control protocol can be applied to a more general class of practical engineering systems. Based on Lyapunov stability theory, it is proved that all signals in the closed-loop system are cooperatively ultimately uniformly bounded and both the transient and steady performances of the consensus tracking errors satisfy prescribed performance requirements.

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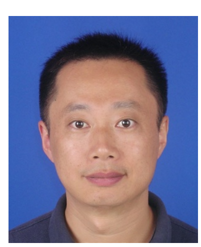
Zhengqing Shi is a Ph.D. candidate of Nanjing University of Science and Technology, China. His research interests include optimal consensus control and prescribed performance consensus control of multi-agent systems.

Chuan Zhou received his Ph.D. from Nanjing University of Aeronautics and Astronautics, China in 1999. Since 2001, he has been with Nanjing University of Science and Technology and he is a Professor in the School of Automation. His research interests include network control system, intelligent control and multi-agent systems.



Jian Guo received his Ph.D. from Nanjing University of Science and Technology, China in 2002. He is a Professor in the School of Automation. His research interests include robotic system and high performance servo system.