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Stabilization of Nonlinear Switched Systems with Distributed Time-delay: The Discrete-time Case

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Abstract: This paper investigates the stabilization problem of nonlinear switched systems subject to the distributed time-delay. The considered nonlinear switched systems are quite general whose dynamics are affected by both exogenous noises and distributed time-delay. The purpose of the addressed problem is to propose a state feedback control law such that, the closed-loop system is exponentially stable in the mean square sense and meanwhile, the required weighted L_2 gain is achieved. By resorting to the Lyapunov functional method in combination with the average dwell time approach, sufficient conditions are provided for the existence of the desired control scheme in terms of the feasibility of certain Hamilton-Jacobi inequalities (HJIs). Within the established framework, the required feedback controller gains can be obtained by solving the series of HJIs. Finally, an illustrative numerical example is provided to demonstrate the effectiveness of the developed control algorithm.

Keywords: Distributed time-delay, Hamilton-Jacobi inequality, nonlinear switched systems, stabilisation.

1. INTRODUCTION

During the past decades, the switched systems have been garnering growing research attention within the systems science and control communities due to its ability to characterize the hybrid dynamics of certain widely utilized industrial systems [1–5]. Switched systems consist of a set of subsystems that are activated in turn regulated by a switching signal. In [6], the fundamental problems of stability has been investigated regarding to switched systems, which has been followed by many researchers who devoted a large amount of efforts to the examination of switched systems. So far, the stability issue has been attracting considerable research interest from both academia and industry, see [7–11] for example. Many approaches have been explored to deal with the analysis and synthesis issues, among which the most frequently used is the so-called average dwell-time approaches, see, for instance, [12–14] and the references therein. Note that the average dwell-time method is more appealing when handling the stability of switched systems due to the superiority that the switching instants are finite during the time interval of interest, thereby bringing much convenience in both theoretical study and practical application. As such, up to now, there have been plenty of research fruits available in literature concerning the average dwell-time approach and its utilization on various types of switched systems, see, e.g., [9,14–17].

It is worth mentioning that most of those aforementioned available results have been mainly concentrated on linear systems (e.g., [11,15,18]) or nonlinear systems with relatively simpler dynamics that always characterized in terms of linear dynamic with additional simple nonlinear disturbances (e.g., [19–22]). Moreover, due to its computing efficiency, the linear matrix inequality (LMI) method has been widely applied in coping with the dynamical analysis and control design problems of switched systems [20,23–26]. However, when it comes to more general nonlinear switched systems, the LMI-based framework is no longer applicable, which requires new paradigms for both analysis and synthesis. Unfortunately, up to now, despite some limited pioneering work, the relevant research has been far from adequate and the corresponding results have been scattered due probably to the substantial challenges stemming from the cross coupling between nonlinear dynamics and switching mechanism. This contributes the first motivation for us to conduct the current research.

As is well known, time-delays are widespread phenomena in many practical engineering systems especially those established upon networks where the data transmission would probably result in communication delays. Such delays might bring significant effects on the sys-

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tem performance and are usually the main sources of performance deterioration or even instability. Accordingly, when discussing the system analysis and synthesis, time-delays cannot be neglected and should be taken into account. Recently, one special type of time-delays, namely, the distributed time-delay has stirred particular attention because of its wide appearance in many practical systems including but not limited to, rocket engine combustion, neural networks and wireless sensor networks. To date, the stability analysis and the associated design problems subject to performance specifications have been widely studied for systems with distributed time-delays, and a few approaches have been developed, see, [4,23,26–28]. Some representative work can be summarized as follows: Specifically, the stability problem for systems with distributed delay has been solved in [29] by introducing an appropriate integral inequality in combination with Lyapunov theory. In [30], the stabilization issues has been dealt with by adopting the techniques of descriptor model transformation as well as discrete-delay term matrix decomposition. It is worth mentioning that, most of those aforementioned reported results have been focused on linear and/or continuous switched systems. To the best of the authors' knowledge, the corresponding control problem for discrete-time switched nonlinear systems with distributed time-delays has not yet been fully investigated, which presents our second motivation for the present study.

In this paper, it is our aim to investigate the exponentially stabilization problem for a class of general discrete-time nonlinear switched systems subject to distributed time-delays. A state feedback control law is proposed such that the closed-loop system achieve the desired exponential stability and the prespecified weighted L_2 gain simultaneously. The contributions of this paper can be highlighted as follows: i) *The considered nonlinear system is quite general that takes both switching dynamics and distributed time-delays into consideration*. ii) *A switched control law is proposed to reach the desired exponential stability and the prespecified weighted L_2 gain*. iii) *Sufficient conditions are derived for the existence of the desired controller in terms of the solvability of certain Hamilton-Jacobi inequalities*.

The rest of this paper is organized as follows: Section 2 formulates the exponential stabilization problem for nonlinear switched system with distributed time-delay. The main results are presented in Section 3 where sufficient conditions are given for the existence of the desired controller in terms of the feasibility of HJIs. Section 4 presents a numerical example to show the effectiveness of the provided control scheme and Section 5 outlines our conclusion.

2. PROBLEM PRELIMINARIES AND FORMULATION

Consider the following nonlinear discrete-time switched systems with distributed time-delay:

$$\begin{cases} x(k+1) = f_{\sigma_k}(x(k)) + B_{\sigma_k}u(k) + C_{\sigma_k}w(k) \\ \qquad\qquad + \sum_{p=1}^{\tau(k)} h_{\sigma_k}(x(k-p)), \\ y(k) = l_{\sigma_k}(x(k)), \\ x(k) = \varphi(k), \ \ k = -\tau_M, \ -\tau_M+1, \ ..., \ 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ represents the state vector, and its initial condition is $\varphi(k)$; $u(k) \in \mathbb{R}^\nu$ represents the control input, $y(k) \in \mathbb{R}^r$ describes the controlled output; $w(k) \in \mathbb{R}^p$ is the exogenous disturbance belonging to $l_2[0,\infty)$; $\sigma_k = \sigma(k)$: $\mathbb{Z}_{\geq 0} \longmapsto \mathcal{M} = \{1,2,\cdots,M\}$ is a switching signal. $f_{\sigma_k}(\cdot)$, $l_{\sigma_k}(\cdot)$ and $h_{\sigma_k}(\cdot)$ are vector-valued or matrix-valued nonlinear functions with compatible dimensions, all these nonlinear functions are assumed to satisfy the global Lipschitz condition; B_{σ_k} and C_{σ_k} are known real-valued matrix with compatible dimensions, $\tau(k)$ denotes the distributed time-delay satisfying $\tau_m \leq \tau(k) \leq \tau_M$.

Denote the switching sequence by $\{(k_0,\sigma_{k_0}),(k_1,\sigma_{k_1}),\cdots,(k_j,\sigma_{k_j}),\cdots,\}$ where $k_0 < k_1 < \cdots < k_j < \cdots$. When $m \in [k_r,k_{r+1})$, $\sigma_m = \sigma_{k_r}$. Moreover, it is assumed that the state of switched system (1) does not jump at switching instants. Furthermore, $\sigma_k = i$ indicates that the i-th subsystem $(f_i(\cdot),B_i,C_i,l_i(\cdot),h_i(\cdot))$ is activated at time step k.

The following assumptions and definitions are needed for later development.

Assumption 1: The nonlinear function $h_i(\cdot),(i \in \mathcal{M})$ is assumed to satisfy the following condition, i.e., $\|h_i(x(k))\| \leq \delta_i\|x(k)\|$, where δ_i $(i \in \mathcal{M})$ are known positive scalars.

Definition 1 [31]**:** Denote the switching times of signal σ_k on the interval $[k_0,k)$ by $N_\sigma(k_0,k)$. If $N_\sigma(k_0,k) \leq N_0 + \frac{k-k_0}{T_a}$ holds for $T_a > 0$ and $N_0 \geq 0$, then T_a is called the average dwell time, and N_0 is defined as the chatter bound. For simplicity and without loss of generality, we shall take $N_0 = 0$ in this paper.

By denoting $x_k(\ell) \triangleq x(k+\ell)$ for $\ell \in [-\tau_M,0]$, we now present the following stability definition for system (1).

Definition 2 [7]**:** System (1) is said to be exponentially stable in the mean square sense subject to switching function σ_k if the solution $x(k)$ with $w(k) = 0$ satisfies

$$\|x(k)\|^2 \leq \kappa\gamma^{k-k_0} \sup_{-\tau_M \leq \ell \leq 0} \|x_{k_0}(\ell)\|^2, \ \ \forall k \in \mathbb{Z}_{\geq k_0},$$

where the parameters κ and γ satisfy $\kappa \geq 1$ and $0 < \gamma < 1$, respectively.

We now proceed to give the definition of the weighted disturbance attenuation level of the investigated nonlinear switched system.

Definition 3: Given $\psi > 0$ and $\alpha > 0$. If there exists a switching function σ_k such that the following inequality

$$\sum_{s=0}^{\infty} e^{-\alpha s} y^{\mathrm{T}}(s)y(s) \leq \psi^2 \sum_{s=0}^{\infty} w^{\mathrm{T}}(s)w(s)$$

is satisfied under the zero initial condition, then, system (1) is said to have a weighted L_2 gain less than ψ.

3. MAIN RESULTS

In this section, the exponential stability as well as weighted disturbance attenuation performance of the system under investigation will be analysed separately. Then, sufficient conditions for the existence of desired state feedback controller will be established in terms of the solvability of a series of HJIs by solving which, we can obtain the required control parameters. First of all, we introduce the following lemmas which will be used in subsequent derivations.

Lemma 1 [31]**:** Let $M \in \mathbb{R}^{m\times m}$ be a positive semi-definite matrix, μ_i be a vector. If the series concerned is convergent, then the following holds:

$$(\sum_{i=1}^{l} \mu_i)^{\mathrm{T}} M (\sum_{i=1}^{l} \mu_i) \leq l \sum_{i=1}^{l} \mu_i^{\mathrm{T}} M \mu_i.$$

Lemma 2 [32]**:** For $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^n$, if matrix P^{-1} exists, then we have

$$a^{\mathrm{T}}Pa + a^{\mathrm{T}}b + b^{\mathrm{T}}a = (a+P^{-1}b)^T P(a+P^{-1}b) - b^T P^{-1} b.$$

Lemma 3 [33]**:** For any real-valued vectors a, b and matrix $P > 0$ of compatible dimensions, we have

$$a^{\mathrm{T}}Pb + b^{\mathrm{T}}Pa \leq \varepsilon a^{\mathrm{T}}Pa + \varepsilon^{-1} b^{\mathrm{T}}Pb,$$

where $\varepsilon > 0$ is a given constant.

3.1. Exponentially stable in the mean square sense

In this subsection, a sufficient condition is proposed for the unforced system (1) to reach the exponential stability in the mean square sense under the condition of $u(k) = 0$ and $w(k) = 0$. To this end, by setting $u(k) = 0$ and $w(k) = 0$ in system (1), we obtain the following unforced system:

$$\begin{cases} x(k+1) = f_{\sigma_k}(x(k)) + \sum_{p=1}^{\tau(k)} h_{\sigma_k}(x(k-p)), \\ y(k) = l_{\sigma_k}(x(k)), \\ x(k) = \varphi(k), \;\; k = -\tau_M, \; -\tau_M+1, \; \ldots, \; 0. \end{cases} \tag{2}$$

Theorem 1: Consider the unforced nonlinear switched time-delay system (2). For any given scalars $0 < \alpha < 1$ and $\rho \geq 1$, if there exist positive-definite matrices P_i and Q_i $(\forall i, j \in \mathcal{M})$ satisfying the following conditions:

$$T_a > T_a^* = -\frac{\ln \rho}{\ln \alpha}, \tag{3}$$

$$P_i \leq \rho P_j, Q_i \leq \rho Q_j, \tag{4}$$

$$\Xi_i = P_i - \frac{\alpha^{\tau_M}}{\tau_M} Q_i < 0, \tag{5}$$

$$\begin{aligned} \mathcal{H}_{1,i} = {} & f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k)) + h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k)) \\ & - \alpha x^{\mathrm{T}}(k)\Omega_i x(k) < 0, \end{aligned} \tag{6}$$

then system (2) is exponential stable in the mean square sense.

Proof: First, we introduce a Lyapunov functional as follows:

$$V_{\sigma_k}(k) = V_{1,\sigma_k}(k) + V_{2,\sigma_k}(k) + V_{3,\sigma_k}(k), \tag{7}$$

where

$$\begin{aligned} V_{1,\sigma_k}(k) &= x^{\mathrm{T}}(k)P_{\sigma_k}x(k), \\ V_{2,\sigma_k}(k) &= \sum_{d=1}^{\tau(k)} \sum_{p=k-d}^{k-1} \alpha^{k-1-p} h_{\sigma_k}^{\mathrm{T}}(x(p)) Q_{\sigma_k} h_{\sigma_k}(x(p)), \\ V_{3,\sigma_k}(k) &= \sum_{l=\tau_m+1}^{\tau_M} \sum_{d=1}^{l-1} \sum_{p=k-d}^{k-1} \alpha^{k-1-p} h_{\sigma_k}^{\mathrm{T}}(x(p)) Q_{\sigma_k} h_{\sigma_k}(x(p). \end{aligned}$$

For denotation simplicity, we assume that $\sigma_k = \sigma_{k_r} = i$. For any $k \in [k_r, k_{r+1})$, define $\triangle V_{m,i}(k) = V_{m,i}(k+1) - \alpha V_{m,i}(k)$ $(m = 1,2,3)$, then it follows that

$$\begin{aligned} \triangle V_{1,i}(k) = {} & V_{1,i}(k+1) - \alpha V_{1,i}(k) \\ = {} & x^{\mathrm{T}}(k+1)P_i x(k+1) - \alpha x^{\mathrm{T}}(k)P_i x(k) \\ = {} & f_i^{\mathrm{T}}(x(k))P_i f_i(x(k)) + \big(B_i u(k)\big)^{\mathrm{T}} P_i \big(B_i u(k)\big) \\ & + \big(C_i w(k)\big)^{\mathrm{T}} P_i \big(C_i w(k)\big) \\ & + 2\big(B_i u(k)\big)^{\mathrm{T}} P_i C_i w(k) \\ & + \big(\sum_{p=1}^{\tau(k)} h_i(x(k-p))\big)^{\mathrm{T}} P_i \big(\sum_{p=1}^{\tau(k)} h_i(x(k-p))\big) \\ & + 2f_i^{\mathrm{T}}(x(k))P_i B_i u(k) + 2f_i^{\mathrm{T}}(x(k))P_i C_i w(k) \\ & + 2f_i^{\mathrm{T}}(x(k))P_i \sum_{p=1}^{\tau(k)} h_i(x(k-p)) \\ & - \alpha x^{\mathrm{T}}(k)P_i x(k) \\ & + 2\big(B_i u(k)\big)^{\mathrm{T}} P_i \sum_{p=1}^{\tau(k)} h_i(x(k-p)) \\ & + 2\big(C_i w(k)\big)^{\mathrm{T}} P_i \sum_{p=1}^{\tau(k)} h_i(x(k-p)), \end{aligned} \tag{8}$$

$$\begin{aligned} \triangle V_{2,i}(k) = {} & V_{2,i}(k+1) - \alpha V_{2,i}(k) \\ = {} & \sum_{d=1}^{\tau(k+1)} \sum_{p=k-d+1}^{k} \alpha^{k-p} h_i^{\mathrm{T}}(x(p)) Q_i h_i(x(p)) \\ & - \sum_{d=1}^{\tau(k)} \sum_{p=k-d}^{k-1} \alpha^{k-p} h_i^{\mathrm{T}}(x(p)) Q_i h_i(x(p)) \end{aligned}$$

$$\begin{aligned}
&\le \sum_{d=1}^{\tau_M}\sum_{p=k-d+1}^{k}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p))\\
&\quad -\sum_{d=1}^{\tau_m}\sum_{p=k-d}^{k-1}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p))\\
&\quad -\sum_{d=1}^{\tau(k)}\alpha^{d}h_i^{\mathrm{T}}(x(k-d))Q_ih_i(x(k-d))\\
&=\sum_{d=1}^{\tau_M}h_i^{\mathrm{T}}(x(k))Q_ih_i(x(k))\\
&\quad +\sum_{d=1}^{\tau_M}\sum_{p=k-d+1}^{k}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p))\\
&\quad -\sum_{d=1}^{\tau_m}\sum_{p=k-d+1}^{k-1}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p))\\
&\quad -\sum_{d=1}^{\tau(k)}\alpha^{d}h_i^{\mathrm{T}}(x(k-d))Q_ih_i(x(k-d))\\
&\le \tau_M h_i^{\mathrm{T}}(x(k))Q_ih_i(x(k))\\
&\quad +\sum_{d=\tau_m+1}^{\tau_M}\sum_{p=k-d+1}^{k}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p))\\
&\quad -\alpha^{\tau_M}\sum_{d=1}^{\tau(k)}h_i^{\mathrm{T}}(x(k-d))Q_ih_i(x(k-d)),
\end{aligned}\tag{9}$$

and

$$\begin{aligned}
\triangle V_{3,i}(k) &= V_{3,i}(k+1)-\alpha V_{3,i}(k)\\
&=\sum_{l=\tau_m+1}^{\tau_M}\sum_{d=1}^{l-1}\sum_{p=k-d+1}^{k}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p)\\
&\quad -\sum_{l=\tau_m+1}^{\tau_M}\sum_{d=1}^{l-1}\sum_{p=k-d}^{k-1}\alpha^{k-p}h_i^{\mathrm{T}}(x(p))Q_ih_i(x(p))\\
&=\frac{1}{2}(\tau_M-\tau_m)(\tau_M+\tau_m-1)h_i^{\mathrm{T}}(x(k))Q_ih_i(x(k))\\
&\quad -\sum_{l=\tau_m+1}^{\tau_M}\sum_{d=1}^{l-1}\alpha^{d}h_i^{\mathrm{T}}(x(k-d)Q_ih_i(x(k-d)).
\end{aligned}\tag{10}$$

It can be deduced from Lemma 1 that

$$\begin{aligned}
&-\alpha^{\tau_M}\sum_{d=1}^{\tau(k)}h_i^{\mathrm{T}}(x(k-d))Q_ih_i(x(k-d))\\
&\le -\frac{\alpha^{\tau_M}}{\tau(k)}\big(\sum_{d=1}^{\tau(k)}h_i(x(k-d))\big)^{\mathrm{T}}Q_i\big(\sum_{d=1}^{\tau(k)}h_i(x(k-d))\big)\\
&\le -\frac{\alpha^{\tau_M}}{\tau_M}\big(\sum_{d=1}^{\tau(k)}h_i(x(k-d))\big)^{\mathrm{T}}Q_i\big(\sum_{d=1}^{\tau(k)}h_i(x(k-d))\big).
\end{aligned}\tag{11}$$

Consequently, it follows readily from (8)-(11) that

$$\begin{aligned}
\triangle V_i(k) &= V_i(k+1)-\alpha V_i(k)\\
&\le f_i^{\mathrm{T}}(x(k))P_if_i(x(k))+\big(B_iu(k)\big)^{\mathrm{T}}P_i\big(B_iu(k)\big)\\
&\quad +\big(C_iw(k)\big)^{\mathrm{T}}P_i\big(C_iw(k)\big)+2f_i^{\mathrm{T}}(x(k))P_iB_iu(k)\\
&\quad +\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))\big)^{\mathrm{T}}P_i\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))\big)\\
&\quad +2f_i^{\mathrm{T}}(x(k))P_iC_iw(k)+2\big(B_iu(k)\big)^{\mathrm{T}}P_iC_iw(k)\\
&\quad +2f_i^{\mathrm{T}}(x(k))P_i\sum_{p=1}^{\tau(k)}h_i(x(k-p))\\
&\quad +2\big(B_iu(k)\big)^{\mathrm{T}}P_i\sum_{p=1}^{\tau(k)}h_i(x(k-p))\\
&\quad +2\big(C_iw(k)\big)^{\mathrm{T}}P_i\sum_{p=1}^{\tau(k)}h_i(x(k-p))\\
&\quad +\tau_Mh_i^{\mathrm{T}}(x(k))Q_ih_i(x(k))\\
&\quad +\frac{1}{2}(\tau_M-\tau_m)(\tau_M+\tau_m-1)h_i^{\mathrm{T}}(x(k))Q_ih_i(x(k))\\
&\quad -\frac{\alpha^{\tau_M}}{\tau_M}\big(\sum_{d=1}^{\tau(k)}h_i(x(k-d))\big)^{\mathrm{T}}Q_i\big(\sum_{d=1}^{\tau(k)}h_i(x(k-d))\big)\\
&\quad -\alpha x^{\mathrm{T}}(k)P_ix(k)\\
&\le f_i^{\mathrm{T}}(x(k))P_if_i(x(k))+\big(B_iu(k)\big)^{\mathrm{T}}P_i\big(B_iu(k)\big)\\
&\quad +\big(C_iw(k)\big)^{\mathrm{T}}P_i\big(C_iw(k)\big)-\alpha x^{\mathrm{T}}(k)P_ix(k)\\
&\quad +h_i^{\mathrm{T}}(x(k))\Theta_ih_i(x(k))+2f_i^{\mathrm{T}}(x(k))P_iB_iu(k)\\
&\quad +2f_i^{\mathrm{T}}(x(k))P_iC_iw(k)+2\big(B_iu(k)\big)^{\mathrm{T}}P_iC_iw(k)\\
&\quad +2\Sigma_i^{\mathrm{T}}\sum_{p=1}^{\tau(k)}h_i(x(k-p))\\
&\quad +\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))\big)^{\mathrm{T}}\Xi_i\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))\big),
\end{aligned}\tag{12}$$

where

$$\begin{aligned}
\Xi_i &= P_i-\frac{\alpha^{\tau_M}}{\tau_M}Q_i,\\
\Theta_i &= \tau_M+\frac{1}{2}(\tau_M-\tau_m)(\tau_M+\tau_m-1),\\
\Sigma_i &= P_i^{\mathrm{T}}\big(f_i(x(k))+B_iu(k)+C_iw(k)\big).
\end{aligned}$$

Moreover, it can be obtained from Lemma 2 that

$$\begin{aligned}
&2\Sigma_i^{\mathrm{T}}\sum_{p=1}^{\tau(k)}h_i(x(k-p))\\
&\quad +\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))\big)^{\mathrm{T}}\Xi_i\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))\big)\\
&=\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))-h^*\big)^{\mathrm{T}}\Xi_i\big(\sum_{p=1}^{\tau(k)}h_i(x(k-p))-h^*\big)\\
&\quad -\Sigma_i^{\mathrm{T}}\Xi_i\Sigma_i,
\end{aligned}\tag{13}$$

where

$$h^* = -\Xi_i^{-1}\Sigma_i.$$

Noticing the condition (5), one further has

$$\begin{aligned}\triangle V_i(k) =& V_i(k+1) - \alpha V_i(k)\\ \leq& f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k)) + \big(B_i u(k)\big)^{\mathrm{T}}\Omega_i\big(B_i u(k)\big)\\ &+ \big(C_i w(k)\big)^{\mathrm{T}}\Omega_i\big(C_i w(k)\big) - \alpha x^{\mathrm{T}}(k)\Omega_i x(k)\\ &+ h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k)) + 2 f_i^{\mathrm{T}}(x(k))\Omega_i B_i u(k)\\ &+ 2 f_i^{\mathrm{T}}(x(k))\Omega_i C_i w(k) + 2\big(B_i u(k)\big)^{\mathrm{T}}\Omega_i C_i w(k)\\ =& f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k)) + u^{\mathrm{T}}(k)\Phi_i u(k)\\ &+ h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k)) - \alpha x^{\mathrm{T}}(k)\Omega_i x(k)\\ &+ \Gamma_{1,i} u(k) + \Gamma_{2,i} w(k) + u^{\mathrm{T}}(k)\Gamma_{3,i} w(k)\\ &+ w^{\mathrm{T}}(k)\Psi_i w(k),\end{aligned} \tag{14}$$

where

$$\begin{aligned}\Omega_i &= P_i - P_i\Xi_i^{-1}P_i^{\mathrm{T}},\\ \Phi_i &= B_i^{\mathrm{T}}\Omega_i B_i,\\ \Psi_i &= C_i^{\mathrm{T}}\Omega_i C_i,\\ \Gamma_{1,i} &= 2 f_i^{\mathrm{T}}(x(k))\Omega_i B_i,\\ \Gamma_{2,i} &= 2 f_i^{\mathrm{T}}(x(k))\Omega_i C_i,\\ \Gamma_{3,i} &= 2 B_i^{\mathrm{T}}\Omega_i C_i.\end{aligned}$$

Setting $u(k) = 0$ and $w(k) = 0$, we arrive at

$$\begin{aligned}\triangle V_i(k) =& V_i(k+1) - \alpha V_i(k)\\ \leq& f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k)) + h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\\ &- \alpha x^{\mathrm{T}}(k)\Omega_i x(k).\end{aligned} \tag{15}$$

Note that (15) in combination with $\mathcal{H}_{1,i} \leq 0$ implies that

$$V_{\sigma_k}(k+1) \leq \alpha V_{\sigma_k}(k). \tag{16}$$

Therefore, we can acquire that

$$\begin{aligned}V_{\sigma_k}(k) &\leq \alpha^{k-k_r} V_{\sigma_{k_r}}(k_r)\\ &\leq \rho\alpha^{k-k_r} V_{\sigma_{k_{r-1}}}(k_r)\\ &\leq \rho\alpha^{k-k_{r-1}} V_{\sigma_{k_{r-1}}}(k_{r-1})\\ &\leq \rho^{N_\sigma(k_0,k)}\alpha^{k-k_0} V_{\sigma_{k_0}}(k_0)\\ &\leq \big(\rho^{\frac{1}{T_a}}\alpha\big)^{k-k_0} V_{\sigma_{k_0}}(k_0).\end{aligned} \tag{17}$$

Subsequently, by virtue of (7), it is implied that

$$\begin{aligned}\zeta_1\|x(k)\|^2 &\leq V_{\sigma_k}(k),\\ V_{\sigma_{k_0}}(k_0) &\leq \zeta_2 \sup_{-\tau_M\leq\ell\leq 0}\|x_{k_0}(\ell)\|^2,\end{aligned} \tag{18}$$

where

$$\begin{aligned}\zeta_1 &= \min_{i\in\mathcal{M}}\lambda_{\min}(P_i),\\ \zeta_2 &= \max_{i\in\mathcal{M}}\lambda_{\max}(P_i)\\ &\quad + \frac{1}{2}\delta_i^2\tau_M(\tau_M+1)(\tau_M-\tau_m+1)\max_{i\in\mathcal{M}}\lambda_{\max}(Q_i).\end{aligned}$$

Thus, (17) and (18) result in

$$\|x(k)\|^2 \leq \frac{\zeta_2}{\zeta_1}\big(\rho^{\frac{1}{T_a}}\alpha\big)^{k-k_0}\sup_{-\tau_M\leq\ell\leq 0}\|x_{k_0}(\ell)\|^2. \tag{19}$$

Therefore, according to Definition 2, system (2) is exponential stability in the mean square sense with $\kappa = \frac{\zeta_2}{\zeta_1}$ and $\gamma = \rho^{\frac{1}{T_a}}\alpha$, where $\kappa \geq 1$ and $0 < \gamma < 1$. The proof is now complete. □

3.2. Weighted L_2 gain

In this subsection, we shall proceed to investigate the weighted L_2 gain specification defined in Definition 3. A sufficient condition is presented by means of the Hamilton-Jacobi inequality approach. First, consider the following nonlinear switched time-delay system subject to disturbance $w(k) \in L_2[0,\infty)$:

$$\begin{cases} x(k+1) = f_{\sigma_k}(x(k)) + C_{\sigma_k}w(k) + \displaystyle\sum_{p=1}^{\tau(k)} h_{\sigma_k}(x(k-p)),\\ y(k) = l_{\sigma_k}(x(k)),\\ x(k) = \varphi(k),\ \ k = -\tau_M,\ -\tau_M+1,\ \ldots,\ 0.\end{cases} \tag{20}$$

Theorem 2: Consider the nonlinear switched time-delay system (20). For any given scalars $0 < \alpha < 1$, $\rho > 1$ and $\psi > 0$, if there exist positive-definite matrices P_i and Q_i ($\forall i, j \in \mathcal{M}$) satisfying the following conditions:

$$T_a > T_a^* = -\frac{\ln\rho}{\ln\alpha}, \tag{21}$$

$$P_i \leq \rho P_j, Q_i \leq \rho Q_j, \tag{22}$$

$$\Xi_i = P_i - \frac{\alpha^{\tau_M}}{\tau_M}Q_i < 0, \tag{23}$$

$$\Psi_i - \psi^2 < 0, \tag{24}$$

$$\begin{aligned}\mathcal{H}_{2,i} =& f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k)) + h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\\ &- \alpha x^{\mathrm{T}}(k)\Omega_i x(k) + l_i^{\mathrm{T}}(x(k)) l_i(x(k))\\ &+ \frac{\Gamma_{2,i}(\Psi_i - \psi^2)^{-1}\Gamma_{2,i}^{\mathrm{T}}}{4} < 0,\end{aligned} \tag{25}$$

then system (20) is exponential stable in the mean square sense and achieves the weighted L_2 gain performance defined in Definition 3.

Proof: First, it can be seen that if the set of HJIs (6) hold, then system (20) is exponentially stable. By utilizing the similar technique, we can easily obtain that

$$\begin{aligned}\triangle V_i(k) =& V_i(k+1) - \alpha V_i(k)\\ \leq& f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k)) + h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\end{aligned}$$

$$-\alpha x^{\mathrm{T}}(k)\Omega_i x(k)+w^{\mathrm{T}}(k)\Psi_i w(k)+\Gamma_{2,i}w(k). \tag{26}$$

Adding the term $z(k)=y^{\mathrm{T}}(k)y(k)-\psi^2 w^{\mathrm{T}}(k)w(k)$ to (26) yields

$$\begin{aligned}
&V_i(k+1)-\alpha V_i(k)+z(k)\\
&\le f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\\
&\quad -\alpha x^{\mathrm{T}}(k)\Omega_i x(k)-w^{\mathrm{T}}(k)(\psi^2-\Psi_i)w(k)\\
&\quad +l_i^{\mathrm{T}}(x(k))l_i(x(k))+\Gamma_{2,i}w(k)\\
&\le f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\\
&\quad -\alpha x^{\mathrm{T}}(k)\Omega_i x(k)+l_i^{\mathrm{T}}(x(k))l_i(x(k))\\
&\quad +\frac{\Gamma_{2,i}(\psi^2-\Psi_i)^{-1}\Gamma_{2,i}^{\mathrm{T}}}{4}\\
&\quad -\bar{w}^{\mathrm{T}}(k)(\psi^2-\Psi_i)^{-1}\bar{w}(k),
\end{aligned} \tag{27}$$

where

$$\bar{w}(k)=(\psi^2-\Psi_i)w(k)-\frac{\Gamma_{2,i}}{2}.$$

Subsequently, it can be known from (25) that

$$V_{\sigma_k}(k+1)\le \alpha V_{\sigma_k}(k)-z(k). \tag{28}$$

Thus, for $k\in[k_r,k_{r+1})$, it is obtained that

$$\begin{aligned}
V_{\sigma_k}(k)&\le \alpha^{k-k_r}V_{\sigma_{k_r}}(k_r)-\sum_{s=k_r}^{k-1}\alpha^{k-s}z(s)\\
&\le \rho\alpha^{k-k_r}V_{\sigma_{k_{r-1}}}(k_r)-\sum_{s=k_r}^{k-s}\alpha^{k-s}z(s)\\
&\le \rho\alpha^{k-k_r}\big(\alpha^{k_r-k_{r-1}}V_{\sigma_{k_{r-1}}}(k_{r-1})\\
&\qquad -\sum_{s=k_{r-1}}^{k_r-1}\alpha^{k-s}z(s)\big)-\sum_{s=k_r}^{k-1}\alpha^{k-s}z(s)\\
&\le \rho^k\alpha^k V_{\sigma_0}(0)-\sum_{s=k_r}^{k-1}\alpha^{k-s}z(s)\\
&\qquad -\rho\sum_{s=k_{r-1}}^{k_r-1}\alpha^{k-s}z(s)-\cdots-\rho^k\sum_{s=0}^{k_1-1}\alpha^{k-s}z(s)\\
&\le \rho^k\alpha^k V_{\sigma_0}(0)-\sum_{s=0}^{k-1}\alpha^{k-s}e^{N_\sigma(s,k)\ln\rho}z(s),
\end{aligned} \tag{29}$$

where $N_\sigma(s,k)$ represents the switching instants between $[s,k]$ as defined in Definition 1.

Taking the zero initial condition into account, the inequality (29) results in

$$\sum_{s=0}^{k-1}\alpha^{k-s}e^{N_\sigma(s,k)\ln\rho}z(s)\le 0, \tag{30}$$

which means

$$\begin{aligned}
&\sum_{s=0}^{k-1}\alpha^{k-s}e^{-N_\sigma(s,k)\ln\rho}y^{\mathrm{T}}(s)y(s)\\
&\le \sum_{s=0}^{k-1}\alpha^{k-s}e^{-N_\sigma(s,k)\ln\rho}\psi^2 w^{\mathrm{T}}(s)w(s).
\end{aligned} \tag{31}$$

Noticing that $N_\sigma(0,k)=\frac{s}{T_a}\le\frac{\alpha s}{\ln\rho}$, we acquire

$$\sum_{s=0}^{k-1}\alpha^{k-s}e^{-\alpha s}y^{\mathrm{T}}(s)y(s)\le\sum_{s=0}^{k-1}\alpha^{k-s}\psi^2 w^{\mathrm{T}}(s)w(s). \tag{32}$$

Then, summing both sides of (32) with respect to s from 0 to ∞, we can immediately arrive at

$$\sum_{s=0}^{\infty}e^{-\alpha s}y^{\mathrm{T}}(s)y(s)\le\sum_{s=0}^{\infty}\psi^2 w^{\mathrm{T}}(s)w(s), \tag{33}$$

which indicates that the pre-specified weighted L_2 gain is achieved. The proof is now complete. □

3.3. Controller design

In this subsection, based on the results obtained previously, we shall give the sufficient condition for the existence of the desired control strategy that is capable of ensuring both expected exponential stability in the mean square sense and pre-specified weighted noise attenuation level. First, applying $u(k)=k_\sigma(x(k))$ to system (1), we obtain the following closed-loop system:

$$\begin{cases}
x(k+1)=f_{\sigma_k}(x(k))+B_{\sigma_k}k_\sigma(x(k)),\\
\qquad\qquad +C_{\sigma_k}w(k)+\sum_{p=1}^{\tau(k)}h_{\sigma_k}(x(k-p))\\
y(k)=l_{\sigma_k}(x(k)),\\
x(k)=\varphi(k),\ \ k=-\tau_M,\ -\tau_M+1,\ \ldots,\ 0.
\end{cases} \tag{34}$$

Theorem 3: Consider the closed-loop system (34). For any given scalars $0<\alpha<1$, $\rho>1$ and $\psi>0$, if there exist positive-definite matrices P_i and Q_i ($\forall i,j\in\mathcal{M}$) satisfying the conditions (21)–(23) and

$$\Psi_i+\eta^{-1}\Gamma_{3,i}-\psi^2<0, \tag{35}$$

$$\begin{aligned}
\mathcal{H}_{3,i}=&f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\\
&-\alpha x^{\mathrm{T}}(k)\Omega_i x(k)+l_{\sigma_k}^{\mathrm{T}}(x(k))l_{\sigma_k}(x(k))\\
&-\frac{\Gamma_{1,i}\breve{\Phi}_i^{-1}\Gamma_{1,i}^{\mathrm{T}}}{4}-\frac{\Gamma_{2,i}\breve{\Psi}_i^{-1}\Gamma_{2,i}^{\mathrm{T}}}{4}<0,
\end{aligned} \tag{36}$$

then, the controller of the form

$$k_i(x(k))=-\frac{\breve{\Phi}_i^{-1}\Gamma_{1,i}^{\mathrm{T}}}{2} \tag{37}$$

stabilize system (1) exponentially in the mean square sense with guarantee of the predetermined weighted L_2 gain.

Proof: First, based on the previous derivation, one can easily obtain that

$$\triangle V_i(k)=V_i(k+1)-\alpha V_i(k)$$

$$\begin{aligned}&\leq f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+k_\sigma^{\mathrm{T}}(x(k))\Phi_i k_\sigma(x(k))\\&\quad+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))-\alpha x^{\mathrm{T}}(k)\Omega_i x(k)\\&\quad+\Gamma_{1,i}k_\sigma(x(k))+\Gamma_{2,i}w(k)+k_\sigma^{\mathrm{T}}(x(k))\Gamma_{3,i}w(k)\\&\quad+w^{\mathrm{T}}(k)\Psi_i w(k).\end{aligned}\tag{38}$$

For a given $\eta>0$, it follows from Lemma 3 that

$$\begin{aligned}\triangle V_i(k)=&V_i(k+1)-\alpha V_i(k)\\&\leq f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+k_\sigma^{\mathrm{T}}(x(k))\tilde{\Phi}_i k_\sigma(x(k))\\&\quad+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))-\alpha x^{\mathrm{T}}(k)\Omega_i x(k)\\&\quad+\Gamma_{1,i}k_\sigma(x(k))+\Gamma_{2,i}w(k)\\&\quad+w^{\mathrm{T}}(k)\tilde{\Psi}_i w(k),\end{aligned}\tag{39}$$

where

$$\begin{aligned}\tilde{\Phi}_i&=\Phi_i+\frac{1}{4}\eta\Gamma_{3,i},\\\tilde{\Psi}_i&=\Psi_i+\eta^{-1}\Gamma_{3,i}.\end{aligned}$$

Adding the term $z(k)=y^{\mathrm{T}}(k)y(k)-\psi^2w^{\mathrm{T}}(k)w(k)$ to both sides of (39) yields

$$\begin{aligned}&V_i(k+1)-\alpha V_i(k)+z(k)\\&\leq f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+w^{\mathrm{T}}(k)\tilde{\Psi}_i w(k)\\&\quad+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))+k_\sigma^{\mathrm{T}}(x(k))\tilde{\Phi}_i k_\sigma(x(k))\\&\quad+\Gamma_{1,i}k_\sigma(x(k))+\Gamma_{2,i}w(k)+l_{\sigma_k}^{\mathrm{T}}(x(k))l_{\sigma_k}(x(k))\\&\quad-\psi^2w^{\mathrm{T}}(k)w(k)-\alpha x^{\mathrm{T}}(k)\Omega_i x(k)\\&\leq f_i^{\mathrm{T}}(x(k))\Omega_i f_i(x(k))+h_i^{\mathrm{T}}(x(k))\Theta_i h_i(x(k))\\&\quad-\alpha x^{\mathrm{T}}(k)\Omega_i x(k)+l_{\sigma_k}^{\mathrm{T}}(x(k))l_{\sigma_k}(x(k))\\&\quad-\frac{\Gamma_{1,i}\tilde{\Phi}_i^{-1}\Gamma_{1,i}^{\mathrm{T}}}{4}-\frac{\Gamma_{2,i}\breve{\Psi}_i^{-1}\Gamma_{2,i}^{\mathrm{T}}}{4}\\&\quad+\tilde{k}_\sigma^{\mathrm{T}}(x(k))\tilde{\Phi}_i\tilde{k}_\sigma(x(k))\\&\quad+\tilde{w}^{\mathrm{T}}(k)\breve{\Psi}_i\tilde{w}(k),\end{aligned}\tag{40}$$

where

$$\begin{aligned}\breve{\Psi}_i&=\tilde{\Psi}_i-\psi^2,\\\tilde{k}_\sigma(x(k))&=k(x(k))+\frac{\tilde{\Phi}_i^{-1}\Gamma_{1,i}^{\mathrm{T}}}{2},\\\tilde{w}(k)&=w(k)+\frac{\breve{\Psi}_i^{-1}\Gamma_{2,i}^{\mathrm{T}}}{2}.\end{aligned}$$

Taking into account (36) and (37), we obtain

$$V_{\sigma_k}(k+1)\leq\alpha V_{\sigma_k}(k)-z(k),\tag{41}$$

which, from Theorem 2, indicates that both the exponential stability and the weighted L_2 gain are ensured for the closed-loop system (34). The proof is now complete. □

4. SIMULATION RESULTS

In this section, a numerical example is provided to demonstrate the effectiveness of the proposed feedback control strategy. The system under consideration described by (1) is with parameters as follows:

$$\begin{aligned}&f(x,1)=\begin{bmatrix}3x_1+3x_1x_2^2\\0.1x_1\end{bmatrix},\ f(x,2)=\begin{bmatrix}0.2x_2\\2x_2+2x_2x_1^2\end{bmatrix},\\&B_1=\begin{bmatrix}3\\0\end{bmatrix},\ B_2=\begin{bmatrix}0\\4\end{bmatrix},\ C_1=\begin{bmatrix}0\\1\end{bmatrix},\ C_2=\begin{bmatrix}1\\0\end{bmatrix},\\&h(x,1)=\begin{bmatrix}0.5(|\ x_1x_2\ |)^{\frac{1}{2}}\\0.5x_1\end{bmatrix},\ h(x,2)=\begin{bmatrix}0.12x_2\\0.5(|\ x_1x_2\ |)^{\frac{1}{2}}\end{bmatrix},\\&l(x,1)=\begin{bmatrix}0.5(|\ x_1x_2\ |)^{\frac{1}{2}}\\0.5x_1\end{bmatrix},\ l(x,2)=\begin{bmatrix}0.5x_1\\0.5(|\ x_1x_2\ |)^{\frac{1}{2}}\end{bmatrix}.\end{aligned}$$

Utilizing the proposed algorithm, we have $\alpha=\frac{3}{4}$, $\rho=2$, $\psi=\frac{5}{2}$, $\eta=1$ and we can acquire that $T_a^*=-\frac{\ln\rho}{\ln\alpha}=2.409$, $T_a=3>T_a^*$, $P_1=P_2=\begin{bmatrix}\frac{5}{4}&0\\0&\frac{5}{4}\end{bmatrix}$, $Q_1=Q_2=\begin{bmatrix}8&0\\0&8\end{bmatrix}$. Consequently, we can verify that

$$\begin{aligned}\mathcal{H}_1(1)&=-(\frac{1}{2}x_1-x_2)^2-\frac{569}{704}x_1^2-\frac{71}{64}x_2^2<0,\\\mathcal{H}_1(2)&=-(\frac{1}{2}x_1-x_2)^2-\frac{103}{64}x_1^2-\frac{5537}{8800}x_2^2<0.\end{aligned}\tag{42}$$

Therefore, the desired controller $u_k=k(x_k,i)$ can be determined by

$$\begin{aligned}k(x_k,1)&=-x_1-x_1x_2^2,\\k(x_k,2)&=-x_2-x_1^2x_2.\end{aligned}\tag{43}$$

Setting the initial value $x_0=[1\ \ -0.8]^{\mathrm{T}}$, we obtain the simulation results shown in Figs. 1 and 2. Specifically, Fig. 1 shows the trajectory of the open-loop system while Fig.

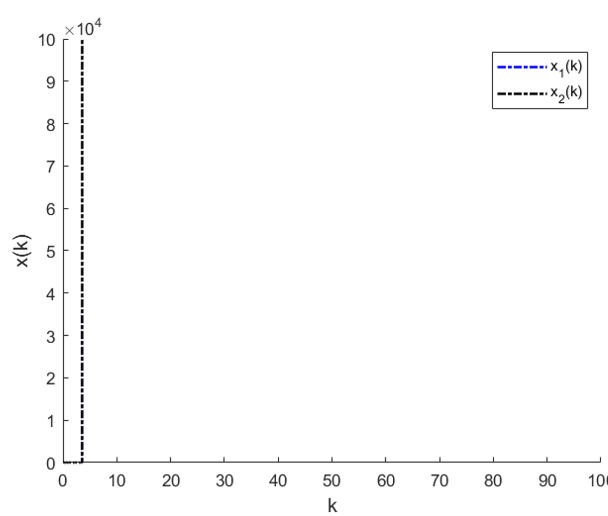

Fig. 1. The dynamics of open-loop system.

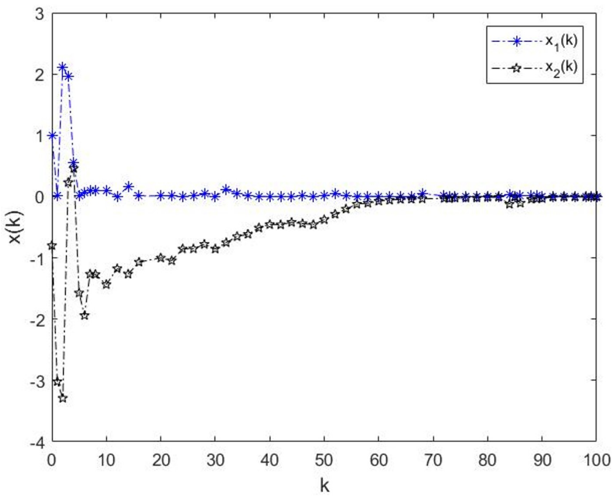

Fig. 2. The dynamics of closed-loop system.

2 depicts the corresponding closed-loop case by implementing the proposed control strategy. It can be seen that the proposed algorithm is effective that can exponentially stabilize the original instable systems.

Remark 1: It can be seen from (21) that the lower bound T_a^* of average dwell time is monotonic increasing with respect to α and ρ. Therefore, in order to obtain a small value of T_a^*, we should choose α and ρ as small as possible under the feasibility of the (22)-(24) and a set of Hamilton-Jacobi inequalities in Theorem 3.

Remark 2: So far, a unified framework has been established for the exponential stabilization of general nonlinear switched system subject to distributed time-delay. The advantages of our proposed algorithm can be highlighted as follows: a) the system under investigation is quite general, which is modeled by the delayed nonlinear stochastic difference equation; b) distributed time-delay is considered with in a unified framework based on the HJI approach; c) based on the proposed framework, we can easily utilize our methodology to investigate systems with more performance requirements, such as dissipativity [34,35] and guaranteed cost [5,36].

5. CONCLUSION

In this paper, the control problem has been investigated for discrete-time nonlinear switched system subject to distributed time-delays. By resorting to the Lyapunov functional approach, sufficient conditions have been established, in terms of a set of HJIs, for the existence of the desired state feedback control scheme capable of stabilizing the unstable system exponentially in the mean square sense while satisfying the predetermined weighted disturbance attenuation level. The explicit form of the desired controller has been formulated which can be obtained via solving the corresponding set of HJIs. An illustrative simulation example has been presented to show the applicability and correctness of the proposed control algorithm.

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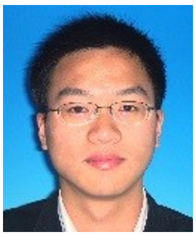

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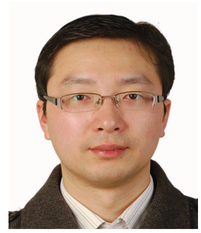

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