International Journal of Control, Automation and Systems 20(3) (2022) 769-779
http://dx.doi.org/10.1007/s12555-020-0283-2
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Adaptive Finite-time Consensus for Second-order Nonlinear Multi-agent Systems with Input Quantization

Jiabo Ren , Baofang Wang , and Mingjie Cai*

Abstract: In this paper, the adaptive finite-time consensus (FTC) control problem of second-order nonlinear multi-agent systems (MASs) with input quantization and external disturbances is studied. With the help of finite time control technology, a novel distributed adaptive control protocol is constructed to achieve FTC performance for second-order nonlinear MASs by using the recursive method. The control input is quantized through a hysteresis quantizer, which reduces the communication rate of arbitrary two agents. The unknown functions are approximated by adopting the radial basis function neural networks. Under the consensus protocols and adaptive laws, it can be proved that velocity errors of arbitrary two agents reach a small region of zero in finite time as well as position errors. Finally, the effectiveness of the proposed method is illustrated via a simulation example.

Keywords: Adaptive control, finite-time consensus, input quantization, multi-agent systems.

1. INTRODUCTION

During the past decades, many scholars have been a surge of interest in the consensus, because the consensus problem for MASs has broad applications in various fields. For example, formation control [1], synchronization [2], flocking [3], containment control [4] and so forth. Hence, it has been widely studied in the presence of consensus schemes. The objective of consensus is to design a control protocol such that a group of agents can reach an agreement. Therefore, the consensus of MASs implies that the states of all the agents can reach a same value through a suitable consensus protocol. Leaderless and leader-following consensus protocols have been reported in [5,6].

Recently, FTC becomes a popular topic due to its faster convergence and high performances. There has three major techniques to handle the FTC problems of second-order MASs, such as homogeneous method [7], terminal sliding mode technique [8] and adding a power integrator [9]. In [10], based on the distributed coordination control theory and the knowledge of fractional-order dynamics, a FTC protocol was investigated for fractional-order MASs. In [11], a distributed FTC was proposed for first-order MASs. By employing the terminal sliding mode technique, FTC control for second-order MASs without velocity measurements was studied in [12]. Du *et al*. addressed the problem of FTC algorithm for high-order MASs by using the adding a power integrator technique [13]. In [14], under fixed and switching undirected topologies, it was proved that all the agents can achieve the consensus in finite time. Under a directed topology, [15] addressed a FTC problem for second-order MASs with a positive odd power and nonsymmetric dead-zone. In addition, it can be pointed out that the nonlinear functions in MASs usually content linear growth condition. However, in fact, the nonlinear functions are often partly or totally unknown because of some constraints, such as unmodeled dynamics or unknown dynamic disturbances. Meanwhile, because of the approximation characteristic of fuzzy logic systems and neural networks, it can be used to deal with the problem of unknown nonlinear functions in [16,17]. In [18], a distributed consensus protocol was designed for second-order nonlinear MASs with unmodeled dynamics. He and Wang studied the problem of distributed finite-time leaderless consensus control for MASs with external disturbances in [19], where external disturbances were assumed to be known. To overcome the limitations, the unknown disturbances and uncertainties were assumed to be bounded by some positive functions in [20]. In [21], the problem of adaptive FTC control for MASs with parametric uncertainties was considered.

On the other hand, signal input quantization is a significant issue that should be considered for hybrid sys-

Manuscript received April 21, 2020; revised March 26, 2021; accepted May 18, 2021. Recommended by Associate Editor Xiangpeng Xie under the direction of Editor PooGyeon Park. This work is supported by the National Natural Science Foundation of China (61803215, 62103212), the Natural Science Foundation of Shandong Province (ZR2019BF038), the Science and Technology Support Plan for Youth Innovation of Universities in Shandong Province (2019KJN033), China Postdoctoral Science Foundation (2019M652323) and Qingdao Application Basic Research Project (18-2-2-40-jch).

Jiabo Ren, Baofang Wang and Mingjie Cai are with the School of Automation, Qingdao University, Qingdao 266071, China (e-mails: 331315082@qq.com, baofangtc@163.com, c_mj0810@163.com).
* Corresponding author.

tems, digital control systems, nonlinear uncertain systems and networked control systems [22–24]. In [25], it was the first time that Ceragiolia *et al*. proposed the hysteresis quantization to deal with chattering phenomena. On the basis of [25], [26] has been further studied by employing backstepping technique, where the strict feedback nonlinear systems with signal input quantization was studied and the designed method relaxed the stability condition in [24]. In [27], a new quantizer was first introduced to deal with uncertain nonlinear systems with input quantization, where the proposed method and novel quantizer removed the assumptions imposed in [26] that the nonlinearities of the system to be controlled should content global Lipschitz conditions with known Lipschitz constants. In addition, there are many papers considering the input quantization of MASs [28–30]. In [31], Zhang *et al*. proposed the leader-following consensus for linear and Lipschitz nonlinear MASs with uniform quantization, where an event-triggered control algorithm is proposed to reduce the communication burden. In [32], the quantized leaderless and leader-following consensus for high-order MASs with limited data rate were considered, which was a challenging issue because of data rate minimization for high-order systems. Based on the neural networks, [33] and [34] developed a fault tolerant consensus algorithm for high order MASs with input quantization and time-varying parameters and a distributed adaptive asymptotically consensus tracking control scheme for nonlinear MASs with input quantization and actuator faults, respectively. Recently, based on finite-time control theory, the cooperative finite-time control for stochastic MASs with input quantisation was investigated in [35]. Liu *et al*. developed a new finite-time event-triggered consensus algorithm for second-order MASs with the power of positive odd rational number and input quantization [36]. However, to the best of our knowledge, only few works pay attention to nonlinear FTC protocols for MASs with input quantization.

Therefore, according to the aforementioned observations, it can be observed that there are very few papers to consider the adaptive FTC for second-order MASs with input quantization and unknown disturbances. Due to the fact that digital communications are widely adopted and have attracted recurring interest, quantised consensus is a considerable problem in MASs. A hysteresis quantizer is used to avoid the chattering phenomenon. Compared with the existing results, the major contributions of this paper are given as follows: In this paper quantised problem is considered for nonlinear MASs, and a quantised control strategy is provided to guarantee the desired system performance in finite time. A novel distributed FTC protocols and adaptive laws are designed for second-order nonlinear MASs with input quantization. Compared with the proposed control schemes in [36], the second-order nonlinear MASs with input quantization and uncertain dynamics is further studied. The unknown nonlinearities are totally unknown in this paper and are approximated by using radial basis function neural networks. Instead of assuming the bound of unknown disturbance be known [19], an adaptive parameter is used to obtain the estimation of the unknown disturbances bound.

The rest of the paper is organized as follows: In Section 2, the problem description and preliminary results are presented. The main results are presented in Section 3. An example is designed to testify the proposed results and conclusions are given in Sections 4 and 5, respectively.

Notations: $[a_{ij}] \in R^{n\times n}$ denotes a matrix consisting of a_{ij}, $i, j = 1,2,\cdots,n$; $\mathrm{diag}\{\cdot\}$ denotes a block-diagonal matrix; $\|x\|$ represents the Euclidean norm of a vector x; $\mathrm{sign}(\cdot)$ stands for signum function; $\mathbf{0}$ is a vector representing all elements as 0 and $\mathbf{1}$ means a vector with all elements being 1.

2. PROBLEM FORMULATION

2.1. Problem formulation

The following class of second-order nonlinear MASs is given as

$$\begin{aligned} &\dot{x}_i = v_i,\\ &\dot{v}_i = q_i(u_i) + f_i(x_i,v_i) + d_i(t),\ \ i \in M = \{1,\ldots,m\}, \end{aligned} \tag{1}$$

where $x_i \in R$ denotes the position, $v_i \in R$ denotes the velocity, $u_i \in R$ is the control input to be designed, $q_i(u_i) \in R$ is the quantized control input, $f_i(x_i,v_i)$ is an unknown continuous function contenting $f_i(0,0) = 0$, $d_i(t)$ represents the external disturbances. Then, the following lemmas and assumptions need to be introduced.

Assumption 1: For arbitrary $i \in M$, the disturbance is bounded such that $|d_i(t)| \le \zeta_i$, where ζ_i is an unknown positive constant.

Definition 1: The FTC of second-order nonlinear MAS (1) can be reached if for any initial condition $\mathcal{P}_0 = [x_0, v_0]^T$, there exist $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $T(\mathcal{P}_0,\varepsilon_1,\varepsilon_2) < \infty$ such that position and velocity errors satisfy $|x_i - x_j| < \varepsilon_1, |v_i - v_j| < \varepsilon_2$, for all $t \ge T, i \in M, j \in M, i \ne j$.

Lemma 1 [37]**:** If there exists a continuous differentiable positive definite function $V(x)$ for a nonlinear system $\dot{x} = f(x)$, scalars $\gamma > 0$, $0 < \alpha < 1$ and $0 < \iota < \infty$ contenting $\dot{V}(x) \le -\gamma V^{\alpha}(x) + \iota$, then there exists a finite time T contents that $T \le V^{1-\alpha}(x_0)/(\gamma\theta_0(1-\alpha))$, such that when $t \ge T$, the trajectory of the system $\dot{x} = f(x)$ is bounded as $\mathrm{B} = \{x|V^{\alpha}(x) \le \iota/\gamma(1-\theta_0)\}$, where $0 < \theta_0 < 1$, x_0 is the initial state.

2.2. Graph theory

In this section, some knowledge about graph theory will be introduced. An undirected graph is given by $G = \{\mathcal{V}, E, \mathcal{A}\}$, which is composed of m agents, where $\mathcal{V} =$

$\{v_1, v_2, ..., v_m\}$ denotes the set of vertices, $E \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges, $\mathcal{A}$ denotes the weighted adjacency matrix. When there exists an edge between agent i and agent j, i.e., $(v_i, v_j) \in E$, then $a_{ij} = a_{ji} > 0$ and $a_{ij} = a_{ji} = 0$ otherwise. In addition, take care that self edges (v_i, v_i) are not permitted, therefore $(v_i, v_i) \notin E$, $a_{ii} = 0$. The set of neighbors of node v_i is defined as $N_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in E\}$. Denote $D = \text{diag}\{d_1, \ldots, d_m\} \in R^{m\times m}$ with $d_i = \sum_{j=1}^{m} a_{ij} = \sum_{j\in N_i} a_{ij}$ for $i \in M$ as a degree matrix of graph G, then the Laplacian of the weighted G is defined as $L = D - \mathcal{A}$. A path from v_i to v_j in graph G is a sequence of different vertices beginning with v_i and ending with v_j, such that the continuous vertices are adjacent. Therefore, there exists a path of arbitrary two agents $v_i, v_j \in \mathcal{V}$ if G is connected.

Assumption 2: Considering second-order nonlinear MAS (1), graph G is connected.

Lemma 2 [38]**:** According to a connected undirected graph, the following properties need to be introduced:

1) L is positive semi-definite;

2) 0 is a simple eigenvalue of L and $\mathbf{1}$ is the associated eigenvector, where $\mathbf{1}$ represents a vector with all elements being 1;

3) Supposing the eigenvalue of L is expressed as $0, \lambda_2, \ldots, \lambda_n$ contenting $0 \le \lambda_2 \le \cdots \le \lambda_n$, then the second smallest eigenvalue contents $\lambda_2 > 0$. What's more, if $\mathbf{1}^T x = 0$, then $x^T L x \ge \lambda_2 x^T x$.

2.3. Hysteretic quantizer

In this section, a hysteresis quantizer is employed to avoid the input disturbances phenomenon. According to [26], the hysteretic quantizer $q_i(u_i)$ is given as

$$q_i(u_i) = \begin{cases} u_{ik}\,\text{sgn}(u_i), & \dfrac{u_{ik}}{1+\delta_i} \le |u_i| \le u_{ik},\ \dot{u}_i < 0, \\ & \text{or } u_{ik} \le |u_i| \le \dfrac{u_{ik}}{1-\delta_i}, \\ & \dot{u}_i > 0, \\ u_{ik}(1+\delta_i)\,\text{sgn}(u_i), & u_{ik} \le |u_i| \le \dfrac{u_{ik}}{1-\delta_i},\ \dot{u}_i < 0, \\ & \text{or } \dfrac{u_{ik}}{1-\delta_i} < |u_i| \le \dfrac{u_i}{1-\delta_i}, \\ & \dot{u}_i > 0, \\ 0, & 0 \le |u_i| < \dfrac{\bar{u}_i}{1+\delta_i},\ \dot{u}_i < 0, \\ & \text{or } \dfrac{\bar{u}_i}{1+\delta_i} \le |u_i| < \bar{u}_i, \\ & \dot{u}_i > 0, \\ q_i(u_i(t^-)), & \dot{u}_i = 0, \end{cases} \tag{2}$$

where $u_{ik} = \rho_i^{1-k}\bar{u}_i$, $k = 1, 2, \ldots$, the constant $0 < \rho_i < 1$ represents a measure of quantization density, and $\bar{u}_i > 0$ represents the dead-zone size of the quantizer. $\delta_i = \frac{1-\rho_i}{1+\rho_i}$ and $0 < \delta_i < 1$. $q_i(u_i)$ is in the set $U_i = \{0, \pm u_{ik}, \pm u_{ik}(1+\delta_i)\}$.

In fact, the hysteretic quantizer $q_i(u_i)$ is divided into a linear term and a nonlinear term, its form is

$$q_i(u_i) = u_i + b_i, \tag{3}$$

where $b_i = q_i(u_i) - u_i \in R$, $i \in M$.

Remark 1: In this paper, for the hysteretic quantizer (2), the dead-zone size of the quantizer $\bar{u}_i$ and the unknown disturbance $d_i(t)$ will be estimated together.

2.4. Radial basis function neural networks (RBFNNs)

The unknown continuous function is approximated by adopting the RBFNN. In general, RBFNNs contain three layers, which are the input layer, the hidden layer, and the output layer, respectively.

Lemma 3 [42]**:** Given any unknown continuous function $h(o)$ defined on the compact set $\Upsilon \subset R^n$ and any precision ε_N, there exists an RBFNN $h_{nn}(o) = \pi^{*T}\omega(o)$ such that

$$h(o) = \pi^{*T}\omega(o) + \varepsilon(o), \quad \forall o \in \Upsilon,$$

where $\omega(o) = [\omega_1(o), \omega_2(o), \cdots, \omega_g(o)]^T$ represents known smooth vector function, $g > 1$ represents neural network node number, $\varepsilon(o)$ represents approximation error, π^* represents unknown parameter vector. The basis function $\omega_i(o)$ is selected as general Gaussian function as follows:

$$\omega_i(o) = \exp\left[-\frac{(o - z_i)^T (o - z_i)}{y_i^2}\right], \quad i = 1, 2, \ldots, g,$$

where $z_i = [z_{i1}, z_{i2}, \ldots, z_{in}]^T$ and y_i represent the center and width of the basis function $\omega_i(o)$, respectively. Choosing the optimal weight vector π^* as the value of π, which minimizes the values of $\varepsilon(o)$ for all $o \in \Upsilon$, i.e.,

$$\pi^* := \arg\min_{\pi\in R^g}\left\{\sup_{o\in\Upsilon}\left|h(o) - \pi^T\omega(o)\right|\right\}.$$

Assumption 3: Over a compact region $\Upsilon \subset R^n$, the approximation error contents

$$|\varepsilon(o)| \le \varepsilon_N, \quad \forall o \in \Upsilon,$$

where ε_N is an unknown bound.

Remark 2: Suppose that the bound of $\varepsilon(o)$ is unknown. Therefore, the bound of $\varepsilon(o)$ and unknown parameter vector π^* can be estimated together.

Lemma 4 [26]**:** The nonlinear term b_i contents the following inequalities:

$$\begin{aligned} |b_i| &\le \delta_i |u_i|, & |u_i| &\ge \bar{u}_i, \\ |b_i| &\le \bar{u}_i, & |u_i| &\le \bar{u}_i. \end{aligned}$$

Lemma 5 [39]**:** For $\alpha, \beta \in R$, if $0 < n = n_1/n_2 \le 1$, n_1 and n_2 are odd integers, then

$$|\alpha^n - \beta^n| \le 2^{1-n}|\alpha - \beta|^n.$$

Lemma 6 [40]**:** For $\alpha_i \in R$, $i \in M$, if $0 < n \le 1$, then

$$\left(\sum_{i=1}^{m}|\alpha_i|\right)^n \le \sum_{i=1}^{m}|\alpha_i|^n \le m^{1-n}\left(\sum_{i=1}^{m}|\alpha_i|\right)^n.$$

Lemma 7 [41]**:** For arbitrary two numbers $\chi_1 > 0$ and $\chi_2 > 0$, arbitrary real-valued function $\psi(\alpha,\beta) > 0$,

$$\begin{aligned}|\alpha|^{\chi_1}|\beta|^{\chi_2} \le& \frac{\chi_1}{\chi_1+\chi_2}\psi(\alpha,\beta)|\alpha|^{\chi_1+\chi_2}\\ &+\frac{\chi_2}{\chi_1+\chi_2}\psi^{-\chi_1/\chi_2}(\alpha,\beta)|\beta|^{\chi_1+\chi_2}.\end{aligned}$$

The paper will design an adaptive FTC control algorithm and adaptive laws for second-order nonlinear MAS (1) such that the FTC of system (1) can be reached.

3. MAIN RESULTS

3.1. Consensus protocols design

In this section, the adaptive FTC control algorithm for second-order nonlinear MASs will be designed. First of all, the virtual velocities are designed. Second, the control algorithm and adaptive laws will be designed.

Step 1: For any $i \in M$, define $\rho_i = \sum_{j\in N_i} a_{ij}(x_i - x_j)$, and $\rho = [\rho_1, \ldots, \rho_m]^T$.

Choose the following Lyapunov function:

$$V_1 = \frac{1}{4}\sum_{i=1}^{m}\sum_{j\in N_i} a_{ij}(x_i - x_j)^2, \tag{4}$$

the time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{m}\left[\sum_{j\in N_i} a_{ij}(x_i - x_j)\right]v_i\\ &= \sum_{i=1}^{m}\rho_i v_i^* + \sum_{i=1}^{m}\rho_i(v_i - v_i^*),\end{aligned} \tag{5}$$

where v_i^* denotes virtual velocity.

Next, v_i^* is designed as

$$v_i^* = -k_1\rho_i^{\kappa},\ \ i \in M, \tag{6}$$

where $k_1 > 0$ is a constant to be designed, κ is a ratio of odd integers contenting $0.5 < \kappa < 1$, one gets

$$\dot{V}_1 \le \sum_{i=1}^{m}\rho_i(v_i - v_i^*) - k_1\sum_{i=1}^{m}\rho_i^{1+\kappa}. \tag{7}$$

Step 2: A new variable will be defined, namely $\eta_i = v_i^{1/\kappa} - v_i^{*1/\kappa}$, $i \in M$. From Lemma 5, it yields

$$v_i - v_i^* \le |v_i - v_i^*| \le 2^{1-\kappa}|\eta_i|^{\kappa}. \tag{8}$$

Then according to Lemma 7, it gets

$$\begin{aligned}\rho_i(v_i - v_i^*) &\le 2^{1-\kappa}|\rho_i||\eta_i|^{\kappa}\\ &\le \frac{2^{1-\kappa}}{1+\kappa}\rho_i^{1+\kappa} + \frac{\kappa 2^{1-\kappa}}{1+\kappa}\eta_i^{1+\kappa}.\end{aligned} \tag{9}$$

Combining (7) and (9), we have

$$\dot{V}_1 \le -(k_1 - \frac{2^{1-\kappa}}{1+\kappa})\sum_{i=1}^{m}\rho_i^{1+\kappa} + \frac{2^{1-\kappa}\kappa}{1+\kappa}\sum_{i=1}^{m}\eta_i^{1+\kappa}. \tag{10}$$

Choose the following Lyapunov function candidate:

$$\begin{aligned}V_2 &= V_1 + \sum_{i=1}^{m}W_i,\\ W_i &= \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\int_{v_i^*}^{v_i}\left(s^{1/\kappa} - v_i^{*1/\kappa}\right)^{2-\kappa}ds.\end{aligned} \tag{11}$$

Combining (10) and (11), the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 \le& -(k_1 - \frac{2^{1-\kappa}}{1+\kappa})\sum_{i=1}^{m}\rho_i^{1+\kappa} + \frac{2^{1-\kappa}\kappa}{1+\kappa}\sum_{i=1}^{m}\eta_i^{1+\kappa}\\ &+\sum_{i=1}^{m}\dot{W}_i,\end{aligned} \tag{12}$$

where

$$\begin{aligned}\dot{W}_i =& \frac{\eta_i^{2-\kappa}q_i(u_i)}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} + \frac{\eta_i^{2-\kappa}f_i(x_i,v_i)}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\\ &+\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\eta_i^{2-\kappa}d_i(t)\\ &-\frac{1}{2^{1-\kappa}k_1^{1+1/\kappa}}\frac{dv_i^{*1/\kappa}}{dt}\int_{v_i^*}^{v_i}\left(s^{1/\kappa} - v_i^{*1/\kappa}\right)^{1-\kappa}ds.\end{aligned} \tag{13}$$

From the RBFNNs, take into account the term $\frac{\eta_i^{2-\kappa}}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}f_i(x_i,v_i)$. Since $f_i(x_i,v_i)$ is an unknown function, we can adopt a RBFNN to approximate it on the compact set Υ_i as follows:

$$f_i(x_i,v_i) = \pi_i^{*T}\omega_i(x_i,v_i) + \delta_i(x_i,v_i),\ \ \forall(x_i,v_i)\in\Upsilon_i, \tag{14}$$

where $\pi_i^* \in R^{g_i}$ represents optimal parameter vector, $\omega_i(x_i,v_i) \in R^{g_i}$ represents basis function vector, $\delta_i(x_i,v_i) \in R$ represents approximation error, $|\delta_i(x_i,v_i)| \le \varepsilon_{iN}$, $g_i > 1$ is the node number of neural network.

Letting $\bar{\pi}_i^{*T} = \left[\pi_i^{*T}, \varepsilon_{iN}\right]^T$, $\varpi_i(x_i,v_i) = \left[\omega_i^T(x_i,v_i), 1\right]^T$. In fact, basis function vector $\omega_i(x_i,v_i)$ contents $0 < \omega_i^T(x_i,v_i)\omega_i(x_i,v_i) \le g_i$, we have

$$\begin{aligned}f_i(x_i,v_i) &\le \bar{\pi}_i^{*T}\varpi_i(x_i,v_i) \le \left|\bar{\pi}_i^{*T}\varpi_i(x_i,v_i)\right|\\ &\le \|\bar{\pi}_i^*\|\|\varpi_i(x_i,v_i)\| \le \sqrt{g_i+1}\|\bar{\pi}_i^*\|.\end{aligned} \tag{15}$$

According to Lemma 7, we obtain

$$\begin{aligned}
&\eta_i^{2-\kappa} f_i(x_i,v_i) \le \left|\eta_i^{2-\kappa}\right| \sqrt{g_i+1}\,\|\bar{\pi}_i^*\| \\
&= \left(|\eta_i| \sqrt{g_i+1}\|\bar{\pi}_i^*\|^{\frac{1}{2-\kappa}}\right)^{2-\kappa} \cdot 1^{2\kappa-1} \\
&\le \frac{2-\kappa}{(2-\kappa)+(2\kappa-1)} \chi_i \eta_i^{(2-\kappa)+(2\kappa-1)} \cdot \\
&\quad \left(\sqrt{g_i+1}\,\|\bar{\pi}_i^*\|\right)^{((2-\kappa)+(2\kappa-1))/(2-\kappa)} \\
&\quad + \frac{2\kappa-1}{(2-\kappa)+(2\kappa-1)} \chi_i^{-(2-\kappa)/(2\kappa-1)} \cdot 1^{(2-\kappa)+(2\kappa-1)} \\
&\le \frac{2-\kappa}{1+\kappa} \chi_i (g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}} \eta_i^{1+\kappa} \Theta_i^* + \frac{2\kappa-1}{1+\kappa} \chi_i^{(\kappa-2)/(2\kappa-1)},
\end{aligned} \tag{16}$$

where $\Theta_i^* = \|\bar{\pi}_i^*\|^{(1+\kappa)/(2-\kappa)}$ is an unknown parameter and $\chi_i > 0$ is a constant.

Therefore, the term $\frac{\eta_i^{2-\kappa}}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} f_i(x_i,v_i)$ can be written as follows:

$$\begin{aligned}
&\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \eta_i^{2-\kappa} f_i(x_i,v_i) \\
&\le \frac{1}{2^{1-\kappa}(1+\kappa)k_1^{1+1/\kappa}} \chi_i (g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}} \eta_i^{1+\kappa} \Theta_i^* + \bar{\chi}_i,
\end{aligned} \tag{17}$$

where $\bar{\chi}_i = \frac{2\kappa-1}{(2-\kappa)(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \chi_i^{(\kappa-2)/(2\kappa-1)}$ is a constant.

Consider the term $-\frac{1}{2^{1-\kappa}k_1^{1+1/\kappa}} \frac{dv_i^{*1/\kappa}}{dt} \int_{v_i^*}^{v_i} \left(s^{1/\kappa} - v_i^{*1/\kappa}\right)^{1-\kappa} ds$. From the definition of η_i and Lemma 5, there has

$$\begin{aligned}
\left|\frac{dv_i^{*1/\kappa}}{dt}\right| &= \left| -\frac{d\left(\rho_i {k_1}^{1/\kappa}\right)}{dt} \right| \\
&= -{k_1}^{1/\kappa} \dot{\rho}_i \le {k_1}^{1/\kappa} \left(a|v_i| + b \sum_{j\in N_i} |v_j| \right),
\end{aligned} \tag{18}$$

where $a = \max_{i\in M}\left\{\sum_{j\in N_i} a_{ij}\right\}$ and $b = \max_{i,j\in M}\{a_{ij}\}$.

From (18) and Lemma 7, we have

$$\begin{aligned}
&\left| -\frac{1}{2^{1-\kappa}k_1^{1+1/\kappa}} \frac{dv_i^{*1/\kappa}}{dt} \int_{v_i^*}^{v_i} \left(s^{1/\kappa} - v_i^{*1/\kappa}\right)^{1-\kappa} ds \right| \\
&\le \frac{1}{k_1} \left(a|v_i| + b \sum_{j\in N_i} |v_j| \right) |\eta_i| \\
&\le \frac{a}{k_1} \left(\frac{2^{1-\kappa}+k_1}{1+\kappa} \eta_i^{1+\kappa} + \frac{2^{1-\kappa}\kappa}{1+\kappa} \eta_i^{1+\kappa} + \frac{k_1\kappa}{1+\kappa} \rho_i^{1+\kappa} \right) \\
&\quad + \frac{b}{k_1} \sum_{j\in N_i} \left(\frac{2^{1-\kappa}+k_1}{1+\kappa} \eta_i^{1+\kappa} + \frac{2^{1-\kappa}\kappa}{1+\kappa} \eta_j^{1+\kappa} + \frac{k_1\kappa}{1+\kappa} \rho_j^{1+\kappa} \right) \\
&= \tau_1 \eta_i^{1+\kappa} + \frac{a\kappa}{1+\kappa} \rho_i^{1+\kappa} + \frac{b}{k_1} \frac{2^{1-\kappa}\kappa}{1+\kappa} \sum_{j\in N_i} \eta_j^{1+\kappa} \\
&\quad + \frac{b\kappa}{1+\kappa} \sum_{j\in N_i} \rho_j^{1+\kappa},
\end{aligned} \tag{19}$$

where $\tau_1 = \frac{a}{k_1} \frac{2^{1-\kappa}+k_1+2^{1-\kappa}\kappa}{1+\kappa} + \frac{mb}{k_1} \frac{2^{1-\kappa}+k_1}{1+\kappa}$.

Combining (12), (13), (17), and (19), we can obtain $\dot{V}_2$ as follows:

$$\begin{aligned}
\dot{V}_2 \le &-\left(k_1 - \frac{2^{1-\kappa}}{1+\kappa}\right) \sum_{i=1}^m \rho_i^{1+\kappa} + \frac{2^{1-\kappa}\kappa}{1+\kappa} \sum_{i=1}^m \eta_i^{1+\kappa} \\
&+ \tau_1 \eta_i^{1+\kappa} + \frac{a\kappa}{1+\kappa} \rho_i^{1+\kappa} + \frac{b}{k_1} \frac{2^{1-\kappa}\kappa}{1+\kappa} \sum_{j\in N_i} \eta_j^{1+\kappa} \\
&+ \frac{b\kappa}{1+\kappa} \sum_{j\in N_i} \rho_j^{1+\kappa} + \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} d_i(t) \\
&+ \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} q_i(u_i) \\
&+ \frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \chi_i (g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}} \eta_i^{1+\kappa} \Theta_i^* + \bar{\chi},
\end{aligned} \tag{20}$$

further

$$\begin{aligned}
\dot{V}_2 \le &-\vartheta \sum_{i=1}^m \rho_i^{1+\kappa} + k_2 \sum_{i=1}^m \eta_i^{1+\kappa} \\
&+ \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} d_i(t) \\
&+ \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} (u_i + b_i) \\
&+ \frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \chi_i (g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}} \eta_i^{1+\kappa} \Theta_i^* + \bar{\chi},
\end{aligned} \tag{21}$$

where $\vartheta = k_1 - \frac{2^{1-\kappa}}{1+\kappa} - \frac{a\kappa}{1+\kappa} - \frac{mb\kappa}{1+\kappa} > 0$, $k_2 = \frac{\kappa 2^{1-\kappa}}{1+\kappa} + \tau_1 + \frac{mb}{k_1} \frac{2^{1-\kappa}\kappa}{1+\kappa}$.

Choose the following control input:

$$\begin{aligned}
u_i = &-\frac{1}{1-\delta_i} \eta_i^{2\kappa-1} \Big((2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa} (\vartheta + k_2) \\
&+ \frac{2-\kappa}{1+\kappa} \sigma_i \sqrt{1+\hat{D}_i^2} \\
&+ \frac{2-\kappa}{1+\kappa} \chi_i (g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}} \sqrt{1+\hat{\Theta}_i^2} \Big),
\end{aligned} \tag{22}$$

where $\sigma_i > 0$ is a constant, $\hat{D}_i$ is the estimation of D_i^* that will be defined later, $\hat{\Theta}_i$ is the estimation of Θ_i^*.

Consider the $\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} b_i$ term. According to Lemma 4, by noting that $\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} u_i \le 0$, we can obtain that

$$\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}} \sum_{i=1}^m \eta_i^{2-\kappa} b_i$$

$$\leq \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\left(\sum_{i=1}^{m}|\eta_i|^{2-\kappa}\delta_i|u_i|+\sum_{i=1}^{m}|\eta_i|^{2-\kappa}\bar{u}_i\right)$$
$$\leq \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\left(-\sum_{i=1}^{m}\eta_i^{2-\kappa}\delta_i u_i+\sum_{i=1}^{m}|\eta_i|^{2-\kappa}\bar{u}_i\right). \tag{23}$$

Combining (21) and (23), we have

$$\begin{aligned}\dot{V}_2 \leq & -\vartheta\sum_{i=1}^{m}\rho_i^{1+\kappa}+\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\eta_i^{2-\kappa}(1-\delta_i)u_i\\ &+\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}|\eta_i|^{2-\kappa}\bar{u}_i\\ &+\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\eta_i^{2-\kappa}d_i(t)\\ &+\frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\chi_i(g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}}\eta_i^{1+\kappa}\Theta_i^*+\bar{\chi}.\end{aligned} \tag{24}$$

Apparently, there exists an upper bound for $\bar{u}_i$ and $d_i(t)$. Hence, before the adaptive laws are designed, we can define

$$D_i = \bar{u}_i + \zeta_i. \tag{25}$$

According to Assumption 1 and Lemma 7, one gets

$$\begin{aligned}&\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\eta_i^{2-\kappa}d_i(t)\\ &+\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}|\eta_i|^{2-\kappa}\bar{u}_i\\ \leq &\frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}|\eta_i|^{2-\kappa}D_i\\ \leq &\frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\eta_i^{1+\kappa}\sigma_i D_i^*\\ &+\frac{2\kappa-1}{(2-\kappa)(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\sigma_i^{(2-\kappa)/(2\kappa-1)},\end{aligned} \tag{26}$$

where $D_i^* = D_i^{(1+\kappa)/(2-\kappa)}$ is an unknown constant.

Defining $\tilde{\Theta}_i = \Theta_i^* - \hat{\Theta}_i$ and $\tilde{D}_i = D_i^* - \hat{D}_i$ as the estimation errors. From (24) and (26), one obtains

$$\begin{aligned}\dot{V}_2 \leq & -\vartheta\sum_{i=1}^{m}\left(\rho_i^{1+\kappa}+\eta_i^{1+\kappa}\right)\\ &+\frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\sigma_i\eta_i^{1+\kappa}\tilde{D}_i\\ &+\frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\chi_i(g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}}\eta_i^{1+\kappa}\tilde{\Theta}_i+\varepsilon,\end{aligned} \tag{27}$$

where $\varepsilon = \bar{\chi} + \frac{2\kappa-1}{(2-\kappa)(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sum_{i=1}^{m}\sigma_i^{(2-\kappa)/(2\kappa-1)}$.

Choose the following Lyapunov function:

$$V(\xi) = V_2 + \frac{1}{2}\sum_{i=1}^{m}\tilde{\Theta}_i^2 + \frac{1}{2}\sum_{i=1}^{m}\tilde{D}_i^2, \tag{28}$$

where $\xi_i = \left[\rho_i, \eta_i, \tilde{D}_i, \tilde{\Theta}_i\right]^T, i \in M, \xi = \left[\xi_1^T, \ldots, \xi_m^T\right]^T$.

Choose the following adaptive laws:

$$\begin{aligned}\dot{\hat{D}}_i &= \frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sigma_i\eta_i^{1+\kappa} - p_{0i}\hat{D}_i,\\ \dot{\hat{\Theta}}_i &= \frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\chi_i(g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}}\eta_i^{1+\kappa} - p_{1i}\hat{\Theta}_i,\end{aligned} \tag{29}$$

where $p_{0i} > 0$ and $p_{1i} > 0$ are parameters to be designed.

Then from (27) and (29), the time derivative of $V(\xi)$ is

$$\begin{aligned}\dot{V}(\xi) \leq & -\vartheta\sum_{i=1}^{m}\left(\rho_i^{1+\kappa}+\eta_i^{1+\kappa}\right)+\sum_{i=1}^{m}p_{0i}\tilde{D}_i\hat{D}_i\\ &+\sum_{i=1}^{m}p_{1i}\tilde{\Theta}_i\hat{\Theta}_i+\varepsilon.\end{aligned} \tag{30}$$

This completes the design procedure.

4. CONSENSUS ANALYSIS

Theorem 1: Suppose Assumptions 1-3 are contented, take into account the second-order nonlinear MAS (1) under the consensus protocols (22) and the adaptive laws (29), the FTC in the sense of Definition 1 can be achieved.

Proof: According to the Lyapunov function $V(\xi)$ in (30). Since $Lx = [\rho_1, \ldots, \rho_m]^T$, we can obtain $\sum_{i=1}^{m}\rho_i^2 = (Lx)^T Lx = x^T L^2 x$. Let $L^{1/2}\mathbf{1} = h = [h_1, \ldots, h_m]^T$, then $h^T h = (L^{1/2}\mathbf{1})^T L^{1/2}\mathbf{1} = \mathbf{1}^T L\mathbf{1}$. According to Lemma 2, we get $L\mathbf{1} = \mathbf{0}$. Thus $h^T h = 0$, which means that $h^T = \mathbf{0}^T$, then $h^T x = 0$, so $\mathbf{1}^T L^{1/2} x = 0$. From Lemma 2, one obtains

$$\begin{aligned}\sum_{i=1}^{m}\rho_i^2 &= (L^{1/2}x)^T L(L^{1/2}x) \geq \lambda_2 x^T Lx\\ &= \frac{\lambda_2}{2}\sum_{i=1}^{m}\sum_{j=N_i}a_{ij}(x_i-x_j)^2 = 2\lambda_2 V_1.\end{aligned} \tag{31}$$

Next, from W_i in (11) and Lemma 5, one gets

$$\begin{aligned}W_i &\leq \frac{1}{(2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}|v_i-v_i^*|\left|v^{1/\kappa}-v_i^{*1/\kappa}\right|^{2-\kappa}\\ &\leq \frac{1}{(2-\kappa)k_1^{1+1/\kappa}}\eta_i^2.\end{aligned} \tag{32}$$

Combining (31) and (32), we have

$$\begin{aligned}V_2 &\leq \frac{1}{2\lambda_2}\sum_{i=1}^{m}\rho_i^2 + \frac{1}{(2-\kappa)k_1^{1+1/\kappa}}\sum_{i=1}^{m}\eta_i^2\\ &\leq \vartheta_1\sum_{i=1}^{m}(\rho_i^2+\eta_i^2),\end{aligned} \tag{33}$$

where $\vartheta_1 = \max\left\{1/2\lambda_2, 1/(2-\kappa)k_1^{1+1/\kappa}\right\}$, from Lemma 6, we have

$$V_2^{(1+\kappa)/2} \le \vartheta_1^{(1+\kappa)/2}\sum_{i=1}^{m}(\rho_i^{1+\kappa}+\eta_i^{1+\kappa}). \tag{34}$$

Combining (30), (33) and (34), we can obtain

$$\begin{aligned}\dot V(\xi) \le& -\vartheta\sum_{i=1}^{m}(\rho_i^{1+\kappa}+\eta_i^{1+\kappa})+\sum_{i=1}^{m}p_{0i}\tilde D_i\hat D_i\\ &+\sum_{i=1}^{m}p_{1i}\tilde\Theta_i\hat\Theta_i+\varepsilon\\ \le& -\frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}V^{(1+\kappa)/2}(\xi)\\ &+\frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}\left(\frac12\sum_{i=1}^{m}\tilde\Theta_i^2+\frac12\sum_{i=1}^{m}\tilde D_i^2\right)^{(1+\kappa)/2}\\ &+\sum_{i=1}^{m}p_{0i}\tilde D_i\hat D_i+\sum_{i=1}^{m}p_{1i}\tilde\Theta_i\hat\Theta_i+\varepsilon.\end{aligned} \tag{35}$$

From Lemma 7, we have

$$\begin{aligned}\left(\frac12\tilde D_i^2\right)^{(1+\kappa)/2} &\le \frac12\tilde D_i^2+\frac{1-\kappa}{2}\left(\frac{1+\kappa}{2}\right)^{(1+\kappa)/(1-\kappa)},\\ \left(\frac12\tilde\Theta_i^2\right)^{(1+\kappa)/2} &\le \frac12\tilde\Theta_i^2+\frac{1-\kappa}{2}\left(\frac{1+\kappa}{2}\right)^{(1+\kappa)/(1-\kappa)}.\end{aligned} \tag{36}$$

According to $\tilde\Theta_i = \Theta_i^* - \hat\Theta_i$, $\tilde D_i = D_i - \hat D_i$, it has the following inequality:

$$\begin{aligned}p_{0i}\tilde D_i\hat D_i &= p_{1i}\tilde D_i(-\tilde D_i+D_i^*) \le p_{0i}\left(-\frac12\tilde D_i^2+\frac12 D_i^{*2}\right),\\ p_{1i}\tilde\Theta_i\hat\Theta_i &= p_{1i}\tilde\Theta_i(-\tilde\Theta_i+\Theta_i^*) \le p_{1i}\left(-\frac12\tilde\Theta_i^2+\frac12\Theta_i^{*2}\right).\end{aligned} \tag{37}$$

Choosing $p_{0i} \ge \frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}$ and $p_{1i} \ge \frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}$, combining (35)-(37), we have

$$\begin{aligned}&\frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}\left(\frac12\sum_{i=1}^{m}\tilde\Theta_i^2+\frac12\sum_{i=1}^{m}\tilde D_i^2\right)^{(1+\kappa)/2}+\sum_{i=1}^{m}p_{0i}\tilde D_i\hat D_i\\ &+\sum_{i=1}^{m}p_{1i}\tilde\Theta_i\hat\Theta_i\\ \le& \frac12\frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}\sum_{i=1}^{m}\tilde\Theta_i^2+\frac12\frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}\sum_{i=1}^{m}\tilde D_i^2\\ &+\frac{2m\vartheta}{\vartheta_1^{(1+\kappa)/2}}\frac{1-\kappa}{2}\left(\frac{1+\kappa}{2}\right)^{(1+\kappa)/(1-\kappa)}-\frac12\sum_{i=1}^{m}p_{0i}\tilde D_i^2\\ &+\frac12\sum_{i=1}^{m}p_{0i}D_i^{*2}-\frac12\sum_{i=1}^{m}p_{1i}\tilde\Theta_i^2+\frac12\sum_{i=1}^{m}p_{1i}\Theta_i^{*2}\\ \le& \frac12\sum_{i=1}^{m}p_{0i}D_i^{*2}+\frac12\sum_{i=1}^{m}p_{1i}\Theta_i^{*2}\\ &+\frac{2m\vartheta}{\vartheta_1^{(1+\kappa)/2}}\frac{1-\kappa}{2}\left(\frac{1+\kappa}{2}\right)^{(1+\kappa)/(1-\kappa)}.\end{aligned} \tag{38}$$

Letting

$$\begin{aligned}\bar\varepsilon =& \varepsilon+\frac12\sum_{i=1}^{m}p_{0i}D_i^{*2}+\frac12\sum_{i=1}^{m}p_{1i}\Theta_i^{*2}\\ &+\frac{2m\vartheta}{\vartheta_1^{(1+\kappa)/2}}\frac{1-\kappa}{2}\left(\frac{1+\kappa}{2}\right)^{(1+\kappa)/(1-\kappa)},\end{aligned}$$

and substituting (38) into (35), we have

$$\dot V(\xi) \le -\frac{\vartheta}{\vartheta_1^{(1+\kappa)/2}}V^{(1+\kappa)/2}(\xi)+\bar\varepsilon. \tag{39}$$

This completes the proof of Theorem 1. □

Remark 3: According to Lemma 1, we can know there exists a finite time T contenting $T \le 2\vartheta_1^{(1+\kappa)/2}V^{(1-\kappa)/2}(\xi(0))/(\vartheta\theta_0(1-\kappa))$, such that when $t \ge T$, the trajectories of the MAS are bounded as

$$\Omega = \{\xi | V(\xi) \le (\bar\varepsilon\vartheta_1^{(1+\kappa)/2}/(\vartheta(1-\theta_0)))^{2/(1+\kappa)}\}.$$

Remark 4: From the expression of $\bar\varepsilon$, it is worth mentioning that if appropriate parameters $p_{0i}, p_{1i}, g_i, \chi_i, \sigma_i$ and ϑ are selected, the constant $\bar\varepsilon$ can make arbitrarily small. However, considering the convergence rate of state and parameter estimations, the parameters $p_{0i}, p_{1i}, g_i, \chi_i, \sigma_i$ and ϑ cannot be selected too small.

5. NUMERICAL EXAMPLE

A simulation example is designed to further testify the effectiveness of the proposed method. Take into account second-order nonlinear MAS (1), the unknown functions $f_i(x_i, v_i)$ and external disturbances $d_i(t)$ are set as $f_1 = x_1+v_1$, $f_2 = x_2^3+2v_2$, $f_3 = 0.5x_3^3+v_3$, $f_4 = 0.5x_4+v_4^3$, $f_5 = \sin(x_5^3+v_5^2)+v_5$, $d_1 = 0.5\sin(x_1)+\sin(v_1)$, $d_2 = \sin(x_2)$, $d_3 = 0.5\sin(x_3)$, $d_4 = 0.8\cos(x_4)$, $d_5 = 0.7\cos(x_5+v_5)$. An undirected interconnected topology is viewed, as shown in Fig. 1.

By the above design procedure, the consensus protocols (22) and adaptive laws (29) are given by

$$u_i = -\frac{1}{1-\delta_i}\eta_i^{2\kappa-1}\Big((2-\kappa)2^{1-\kappa}k_1^{1+1/\kappa}(\vartheta+k_2)$$

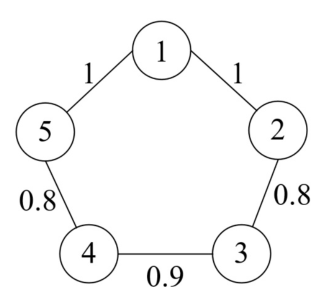


Fig. 1. Interconnected topology.

$$
\begin{aligned}
&+\frac{2-\kappa}{1+\kappa}\sigma_i\sqrt{1+\hat{D}_i^2}\\
&+\frac{2-\kappa}{1+\kappa}\chi_i(g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}}\sqrt{1+\hat{\Theta}_i^2}\Bigg),\\
\dot{\hat{D}}_i &= \frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\sigma_i\eta_i^{1+\kappa}-p_{0i}\hat{D}_i,\\
\dot{\hat{\Theta}}_i &= \frac{1}{(1+\kappa)2^{1-\kappa}k_1^{1+1/\kappa}}\chi_i(g_i+1)^{\frac{1+\kappa}{2(2-\kappa)}}\eta_i^{1+\kappa}-p_{1i}\hat{\Theta}_i,
\end{aligned}
\tag{40}
$$

where $i\in\underline{5}=\{1,2,3,4,5\}$.

According to Fig. 1, we can obtain $m=5$, $a=2.7$, $b=1$ and $\lambda_2=1.1642$. In the simulation, the parameters of the hysteretic quantizer are $\delta_i=0.01$, $\bar{u}_i=0.2$. The parameters of consensus protocols and adaptive laws are set as

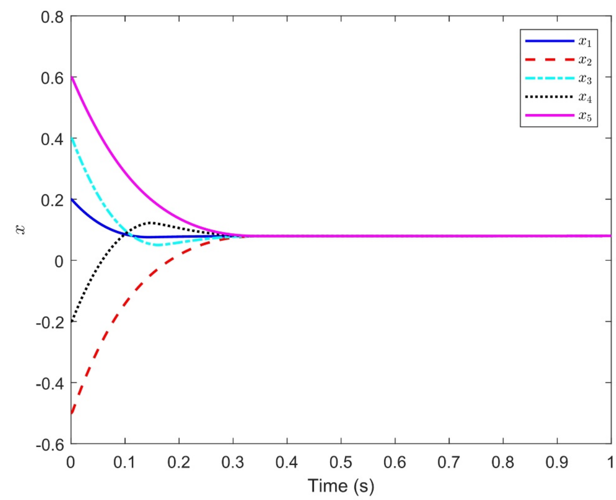
$$
\begin{aligned}
&\kappa=\frac{3}{5},\ k_1=1.1\left(\frac{2^{1-\kappa}}{1+\kappa}+\frac{a\kappa}{1+\kappa}+\frac{mb\kappa}{1+\kappa}\right),\ \vartheta=0.1k_1,\\
&g_i=25,\ \chi_i=25,\ \sigma_i=120,\\
&k_2=\frac{\kappa 2^{1-\kappa}}{1+\kappa}+\tau_1+\frac{m\beta}{k_1}\frac{2^{1-\kappa}\kappa}{1+\kappa},\\
&\tau_1=\frac{a}{k_1}\frac{2^{1-\kappa}+k_1+2^{1-\kappa}\kappa}{1+\kappa}+\frac{mb}{k_1}\frac{2^{1-\kappa}+k_1}{1+\kappa},\\
&\vartheta_1=\max\left\{1/2\lambda_2,1/(2-\kappa)k_1^{1+1/\kappa}\right\}=1/2\lambda_2,\\
&p_{01}=7.6,\ p_{02}=8.3,\ p_{03}=9.1,\ p_{04}=7.9,\ p_{05}=9.9,\\
&p_{11}=8.3,\ p_{12}=9.2,\ p_{13}=7.8,\ p_{14}=8.5,\ p_{15}=7.9.
\end{aligned}
$$

Besides, the initial conditions are set as

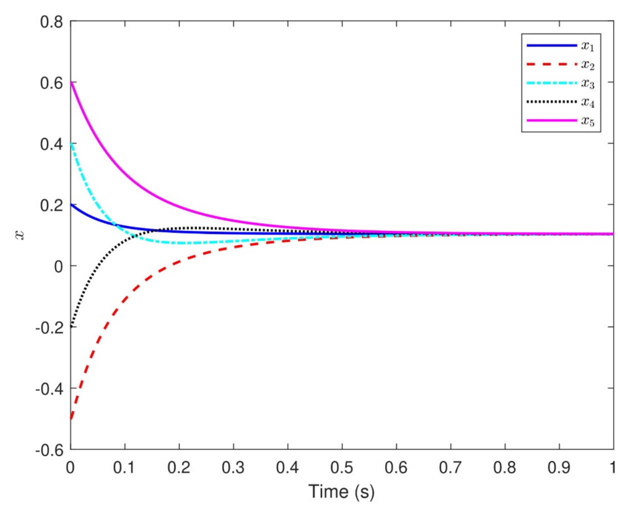
$$
\begin{aligned}
&\mathcal{P}_1(0)=[0.2,-0.3]^T,\ \mathcal{P}_2(0)=[-0.5,0.2]^T,\\
&\mathcal{P}_3(0)=[0.4,-0.3]^T,\ \mathcal{P}_4(0)=[-0.2,0.1]^T,\\
&\mathcal{P}_5(0)=[0.6,-0.5]^T,\ \hat{\Theta}_i(0)=0,\ \hat{D}_i(0)=0,\ i\in\underline{5}.
\end{aligned}
$$

To further demonstrate the effectiveness of the proposed scheme, the proposed finite-time consensus protocol is compared with the traditional asymptotic convergence method with input quantization. In the compared method, the control parameters are chosen as $\delta_i=0.3$, $\bar{u}_i=0.25$, $g_i=30$, $\chi_i=30$ and $\sigma_i=12$.

Figs. 2-6 show the simulation results. In Fig. 2, the trajectories of x between any two agents is bounded in finite time. It can be seen that the proposed control algorithm can guarantee the trajectories of x have fast convergence performance. The trajectories of v are shown in Fig. 3, from which we can see that the errors of v between arbitrary two agents can reach a region of zero in finite time. However, it is obvious that the proposed control algorithm can guarantee the trajectories of v have fast convergence performance. The parameter estimations $\hat{\Theta}_i$ and $\hat{D}_i$ are shown in Figs. 4-5, it is obvious that $\hat{\Theta}_i$ and $\hat{D}_i$ can achieve a region of zero in finite time. In Fig. 6, the value of the quantized control input for each agent is bounded. Therefore, simulation results testify the effectiveness of the designed method.



(a) Trajectory of velocity x under the proposed control algorithm.

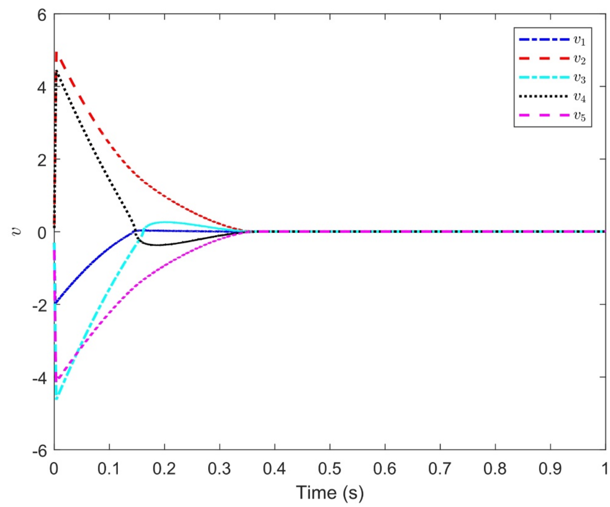


(b) Trajectory of velocity x using compared method.

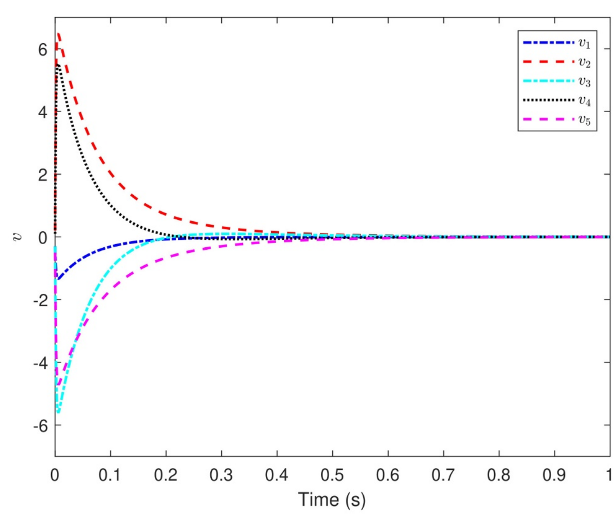
Fig. 2. Trajectory of position x.

6. CONCLUSION

The adaptive FTC for second-order nonlinear MASs with input quantization, unknown dynamics and unknown disturbances has been presented in this paper. By employing a hysteretic quantizer, the quantization input signals have been obtained. Meanwhile, the quantization input signals and unknown disturbances have been handled together by using recursive method. On the basis of RBFNNs theories, the unknown functions have been approximated. Under the consensus protocols and adaptive laws, it has been proved that consensus could be reached in finite time. Finally, the effectiveness of the designed scheme has been illustrated by the obtained simulation results.Our future work will focus on the research of fixed-time consensus for high-order MASs and multiple Euler-Lagrange systems with input quantization.



(a) Trajectory of velocity v under the proposed control algorithm.



(b) Trajectory of velocity v using compared method.

Fig. 3. Trajectory of velocity v.

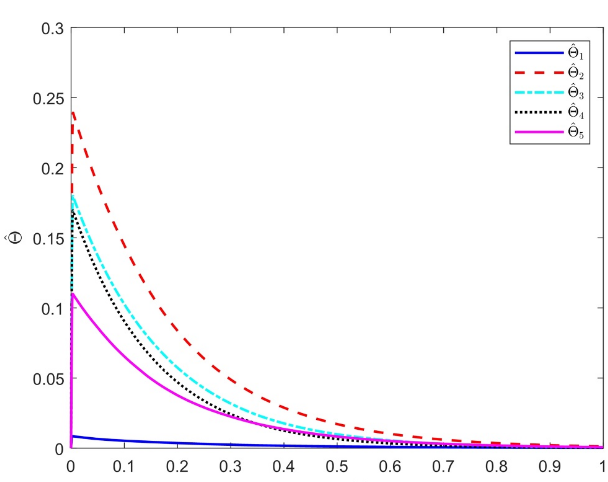


Fig. 4. Parameter estimation $\hat{\Theta}$.

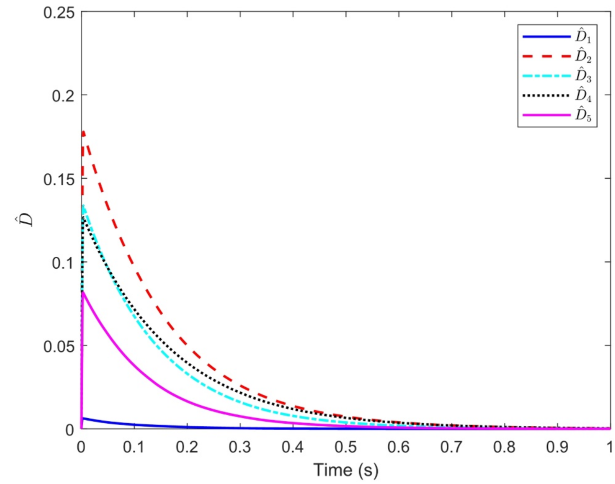


Fig. 5. Parameter estimation $\hat{D}$.

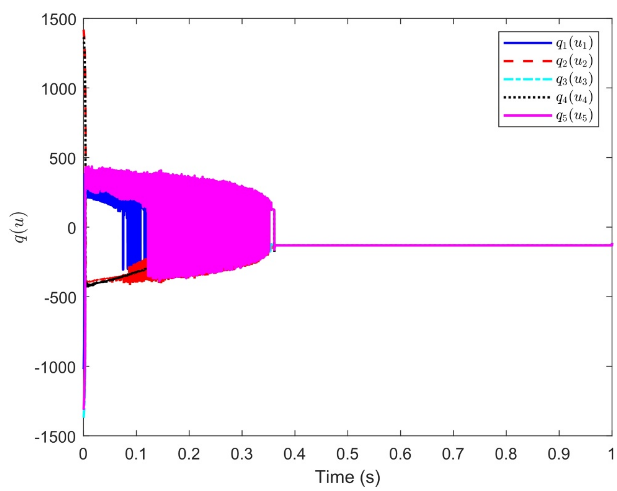


Fig. 6. Quantized control inputs $q(u)$.

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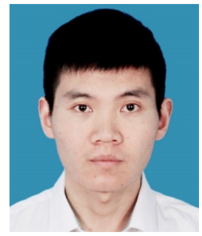
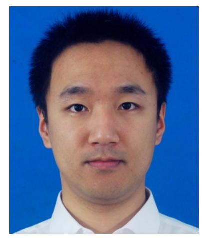


Jiabo Ren received his B.Sc. degree from the Faculty of Electronic Information Engineering, Huaiyin Institute of Technology, Huaian, China, in 2018. He is currently pursuing an M.S. degree with the School of Automation, Qingdao University, Qingdao, China. His current research interests include distributed control of multi-agent systems and nonlinear system control.



Baofang Wang received his B.Sc. degree in Automation from Nanjing University of Science and Technology, Nanjing, China, in 2012, and a Ph.D. degree in Control Theory and Control Engineering from Nanjing University of Science and Technology, Nanjing, China, in 2018. He is currently a Lecturer in the School of Automation, Qingdao University. His current research interests include robot intelligent control, servo system and nonlinear system control.

Mingjie Cai received her B.Sc. degree in Automation from Nanjing University of Science and Technology, Nanjing, China, in 2012, and a Ph.D. degree in Control Theory and Control Engineering from Nanjing University of Science and Technology, Nanjing, China, in 2017. She is currently an Assistant Professor in the School of Automation, Qingdao University. Her current research interests include distributed control of multi-agent systems and nonlinear system control.