International Journal of Control, Automation and Systems 19(5) (2021) 1962-1975
http://dx.doi.org/10.1007/s12555-020-0245-8
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Observer-based Adaptive Fuzzy Control for Strict-feedback Nonlinear Systems with Prescribed Performance and Dead Zone

Wen Zeng, Zhigang Li*, Chuang Gao, and Libing Wu

Abstract: This paper proposes an observer-based fuzzy controller for strict-feedback nonlinear systems. A fuzzy state observer is designed to estimate unmeasurable system states via a fuzzy logic system to approximate the unknown nonlinear functions of the system. A tangent prescribed performance function is utilized to gather the tracking error in a small neighborhood of the origin. The control input is designed to deal with the dead-zone property of the system. The proposed controller can guarantee that all the signals in the closed-loop system are semi-globally, uniformly, and ultimately bounded. Simulation results demonstrate the effectiveness of the proposed controller.

Keywords: Backstepping, fuzzy observer, prescribed performanc, strict-feedback nonlinear system.

1. INTRODUCTION

The backstepping method has become a general tool to construct controllers for dynamic nonlinear systems. It was first presented to obtain asymptotic tracking and global stability for single-input single-output (SISO) strict-feedback nonlinear systems [1]. This method effectively solves the nonlinear system controller design problem that does not satisfy the matching condition. Some improvements of the controller design for strict-feedback systems were developed for multi-input multi-output systems [2] and extended to switching systems [3] and pure feedback systems [4]. Some intelligent methods with universal approximation ability were combined into the approaches of the backstepping-based tracking controller, such as fuzzy logic systems (FLSs) [5–9] and neural networks (NNs) [10–13]. These developments have been widely applied to the control design of uncertain nonlinear systems.

Most system states are not measurable in real applications. Therefore, extensive attention has been paid to nonlinear output feedback control, and some significant results have been obtained. Adaptive fuzzy observers were designed for output feedback control of SISO nonlinear systems [15] and MIMO nonlinear systems [16]. By considering the fault-tolerant problem, an adaptive fuzzy control approach was investigated for nonlinear non-strict feedback systems [17,18]. A novel fuzzy observer scheme was proposed to solve unknown virtual control coefficients of nonlinear systems [19]. With the in-depth study of the control theory, the prescribed performance control theory proposed by Bechlioulis and Rovithakis has attracted wide attention [20]. The basic idea is to make the error of system response fall strictly within an area preset by the designer. Other forms of prescribed performance control were subsequently proposed. Funnel control was proposed by Ilchmann *et al*. [21], and the barrier Lyapunov function was considered in designing a controller [22]. A series of barrier Lyapunov function controls were achieved [23,24]. Based on previous work [22], a tan-type prescribed performance was proposed [25]. Further results based on a tan-type prescribed performance function were presented [26,27].

Prescribed performance control has become a major strategy in many nonlinear systems. A prescribed performance adaptive controller was designed for tracking problems for nonlinear systems with zero dynamics [28], and a prescribed performance controller with the method of command filters was designed to solve the problem of "explosion of complexity" [29]. An adaptive finite-time fuzzy funnel controller was proposed [30]. An event-triggered funnel controller for strict-feedback nonlinear systems with unknown parameters was constructed [31]. In industrial systems, many measuring and executing mechanisms

Manuscript received April 9, 2020; revised August 5, 2020; accepted August 20, 2020. Recommended by Associate Editor Jiuxiang Dong under the direction of Editor Guang-Hong Yang. This work is supported in part by the Ministry of education industry-school cooperative education program (Grant No. 201901018003) and Excellent Talents Training Program of University of Science and Technology Liaoning (Grant No. 2019RC09).

Wen Zeng, Zhigang Li, and Chuang Gao are with the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning 114051, P. R. China (e-mails: {azw262, li7275, 13500422153}@163.com). Libing Wu is with the School of Sciences, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China (e-mail: beyondwlb@163.com).
* Corresponding author.

include uncertain dead-zone phenomena to varying degrees due to their design processes and for manufacturing reasons. The existence of a dead zone will affect the output accuracy of the system. Scholars have done much to improve the effect of dead zones on system output. The dead zone is a common nonsmooth nonlinearity with a huge impact in many industrial processes, since it can severely limit system performance. Many control methods have been developed to overcome the impact of unknown dead zones in industrial systems [32–36]. An unknown dead zone in a MIMO nonlinear system was raised [32]. A control scheme was developed for an SISO nonlinear system [33]. A dynamic surface controller was designed for a pure-feedback nonlinear system [34]. An adaptive NN control design scheme was proposed for nonlinear discrete systems [35]. An unknown dead zone was designed for a nonlinear system with prescribed performance [36].

The use of adaptive fuzzy or neural network backstepping controllers is important and practical for nonlinear systems with unmeasured states and dead zones, and this motivates our research. Based on the study of nonlinear control theory, the exploration process of fuzzy observers has always been valuable, but there is scant literature on the steady-state performance of nonlinear systems with unknown dead zones. By considering the use of recursive design technology combined with a tan-type Lyapunov function, a fuzzy adaptive output feedback control scheme is developed to study the steady-state performance of a closed-loop system.

The main contributions of this paper include: 1) This is the first application of the tan-type Lyapunov function to strict-feedback systems with dead zones, so as to ensure that the tracking error of the system converges to a predetermined range, and the control scheme can be efficiently applied to practical systems. 2) Based on a nonlinear system including the information of the dead zone slope and the unmeasured system state, we design a state observer, and develop a new adaptive fuzzy backstepping output feedback control scheme. The controller ensures that the closed-loop system obtains better steady-state performance and convergence of the observer. 3) To overcome the constraint on the unmeasured state, we adopt the tan-type Lyapunov function, including prediction performance, to ensure the boundedness of tracking and observation errors, and to achieve good tracking performance.

The remainder of this paper is organized as follows: Section 2 describes the system and presents preliminaries. Section 3 introduces the design of a fuzzy state observer. Section 4 analyzes the design and stability of the controller. Section 5 presents simulation results. Section 6 summarizes this work and relates our conclusions.

2. PROBLEM FORMULATION AND PRELIMINARIES

A common form of strict-feedback nonlinear system is considered:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i), \\ \dot{x}_n = u + f_n(\bar{x}_n), \\ y = x_1, \ \ 1 \le i \le n-1, \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots x_n]^T \in R^n$ is the system state, $y \in R$ is the system output, the nonlinear function $f_i(\cdot)$ is presumed unknown and smooth, and $u \in R$ is the system input. The input dead zone is defined by

$$u = D(\Xi) = \begin{cases} m_r(\Xi), & \Xi \ge b_r, \\ 0, & b_l < \Xi < b_r, \\ m_l(\Xi), & \Xi \le b_l, \end{cases} \tag{2}$$

where Ξ is an intermediate control input, and b_r and b_l are respective design parameters for the positive and negative dead zones.

Assumption 1 [37]**:** Consider the unmeasurable states for the output of a dead zone. Assume the functions $m_r(\Xi)$ and $m_l(\Xi)$ are uncertain and smooth, and that there exist unknown positive constants h_{l0}, h_{l1}, h_{r0}, and h_{r1} such that

$$\begin{aligned} &0 < h_{l0} \le Z_l'(\Xi) \le h_{l1}, \ \ \forall \Xi \in (-\infty, b_l], \\ &0 < h_{r0} \le Z_r'(\Xi) \le h_{r1}, \ \ \forall \Xi \in [b_r, +\infty), \end{aligned} \tag{3}$$

where $Z_l'(\Xi) = dZ_l(\Xi)/d\Xi$ and $Z_r'(\Xi) = dZ_r(\Xi)/d\Xi$. Assume also that $\beta_0 \le \min\{h_{l0}, h_{r0}\}$ is an unknown parameter. The function of the dead zone can be expressed via Assumption 1 as

$$u(t) = D(\Xi) = H^T(t)\phi(t)\Xi + d(\Xi), \tag{4}$$

where $\phi(t) = [\varphi_r(t), \varphi_l(t)]^T, H(t) = [h_r(\Xi), h_l(\Xi)]^T$,

$$\begin{aligned} &\varphi_r(t) = \begin{cases} 1, & \Xi(t) > b_l, \\ 0, & \Xi(t) \le b_l, \end{cases} \\ &\varphi_l(t) = \begin{cases} 1, & \Xi(t) < b_r, \\ 0, & \Xi(t) \ge b_r, \end{cases} \\ &h_r(\Xi) = \begin{cases} 0, & \Xi \le b_l, \\ m_r'(g(\Xi)), & b_l < \Xi < +\infty, \end{cases} \\ &h_l(\Xi) = \begin{cases} m_l'(g(\Xi)), & -\infty < \Xi < b_r, \\ 0, & \Xi \ge b_r, \end{cases} \\ &d(\Xi) = \begin{cases} -m_r'(g_r(\Xi))b_r, & \Xi \ge b_r, \\ -[m_l'(g_l(\Xi)) + m_r'(g_r(\Xi))]\Xi, & b_l < \Xi < b_r, \\ -m_l'(g_l(\Xi))b_l, & \Xi \le b_l, \end{cases} \end{aligned} \tag{5}$$

where $g_l(\Xi) \in (\Xi, b_l)$ if $v < b_l$; $g_l(\Xi) \in (b_l, \Xi)$ if $b_l \le \Xi < b_r$; $g_r(\Xi) \in (b_r, \Xi)$ if $b_r < \Xi$; $g(\Xi) \in (\Xi, b_r)$ if $b_l < \Xi <$

b_r; $|d(\Xi)| \le \rho^*$; and ρ^* is an unknown positive constant, $\rho^* = (h_{r1} + h_{l1}) \max\{b_r, -b_l\}$.

Remark 1: (4) describes most of the dead-zone models that appear in practical problems, and $H^T(t)\phi(t) \in [\beta_0, h_{l0} + h_{r0}] \subset (0, +\infty)$ [35].

Our goal is to design a controller to make the system output y track a given reference signal y_d, and to ensure that all the signals in the closed-loop system are semi-globally, uniformly, and ultimately bounded.

Assumption 2: The desired signal $y_d(t)$ and its j-th order derivative $y_d^{(j)}(t)$ are continuous and bounded for $j = 1, \ldots, n$.

Assumption 3: For any $X, Y \in R^{i+1}$, there are known constants $\overline{m}_i$ such that

$$|f_i(X) - f_i(Y)| \le \overline{m}_i ||X - Y||, i = 1, \ldots, n. \tag{6}$$

2.1. Fuzzy logic systems

Fuzzy logic systems are affected by four factors: the knowledge base, fuzzifier, fuzzy inference engine, and defuzzifier. The knowledge base rests on some rules of fuzzy IF-THEN.

R^r: If x_1 is L_1^r, x_2 is L_2^r, ..., and x_n is L_n^r, then Y is P^r, $r = 1, 2, \cdots, N$, where $X = [X_1, X_2, \ldots, X_n]^T$ is the FLS input and Y is the FLS output. The fuzzy sets L_i^r and P^r are associated with the fuzzy membership functions $v_{L_i^r}(X_i)$ and $v_{P^r}(Y)$, respectively, and N is the number of IF-THEN rules.

Based on the singleton fuzzifier, the center average defuzzification, and product inference, the FLS can be described as

$$Y(X) = \frac{\sum_{r=1}^N \bar{Y}_r \Pi_{i=1}^n v_{L_i^r}(X_i)}{\sum_{r=1}^N \left[\Pi_{i=1}^n v_{L_i^r}(X_i)\right]}, \tag{7}$$

where $\bar{Y}_l = \max_{Y \in R} v_{P^r}(Y)$.

The fuzzy basis functions can be defined as

$$S_r(X) = \frac{\Pi_{i=1}^n v_{L_i^r}(X_i)}{\sum_{l=1}^N \left[\Pi_{i=1}^n v_{L_i^r}(X_i)\right]}. \tag{8}$$

Define $S(X) = [S_1(X), S_2(X), \ldots, S_N(X)]^T$ and $W^T = [\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_N] = [T_1, T_2, \ldots, T_N]$. Then the FLS (7) can be expressed as

$$y(X) = T^T S(X). \tag{9}$$

Lemma 1 [38]**:** $f(X)$ is defined to be continuous on a compact set Ω. Using the approximation method of FLSs, $f(X)$ can attain arbitrary positive accuracy ϖ, and

$$\sup_{X \in \Omega} \left| f(X) - T^{*T} S(X) \right| \le \varpi, \tag{10}$$

where $|\varpi| < \gamma$ is the approximation accuracy, which can be arbitrarily small, and T^* is the desired weight vector.

3. FUZZY STATE OBSERVER DESIGN

To estimate the unknown states for system (1), a fuzzy state observer is designed as follows:

$$\begin{cases} \dot{\zeta}_1 = \zeta_2 + l_1(y - \zeta_1) + \omega_1^T \psi_1(x_1), \\ \dot{\zeta}_i = \zeta_{i+1} + l_i(y - \zeta_1) + \omega_i^T \psi_i(\bar{\zeta}_i), \\ \dot{\zeta}_n = u + l_n(y - \zeta_1) + \omega_n^T \psi_n(\bar{\zeta}_n), \end{cases} \tag{11}$$

where $\bar{\zeta}_i = [\zeta_1, \zeta_2, \ldots, \zeta_i]^T$, $1 \le i \le n$, ζ_i is the estimate of x_i, and l_i is a parameter of the observer. Furthermore, ω_i is the estimate of ω_i^*, and it is an ideal constant vector for the FLSs $\omega_i^T \psi_i(\bar{\zeta}_i)$ to approximate the unknown functions $f_i(\bar{\zeta}_i)$, which can be defined by

$$f_1(x_1) = \omega_1^{*T} \psi_1(x_1) + \delta_1(x_1), |\delta_1(x_1)| \le \varepsilon_1, \tag{12}$$

$$f_i(\bar{\zeta}_i) = \omega_i^{*T} \psi_i(\bar{\zeta}_i) + \delta_i(\bar{\zeta}_i), |\delta_i(\bar{\zeta}_i)| \le \varepsilon_i, \quad 2 \le i \le n, \tag{13}$$

where $\delta_1(x_1)$ and $\delta_i(\bar{\zeta}_i)$ are the approximation errors and $\varepsilon_i > 0$.

Then define $\tilde{\omega}_i^T = \omega_i^{*T} - \omega_i^T$ and the state estimation error $e_i = x_i - \zeta_i$, $1 \le i \le n$. The time derivatives of e_i can be derived as

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{\zeta}_1 \\ &= e_2 - l_1 e_1 + \tilde{\omega}_1^T \psi_1(x_1) + \delta_1(x_1), \end{aligned} \tag{14}$$

and

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{\zeta}_i \\ &= e_{i+1} - l_i e_1 + f_i(\bar{x}_i) - \omega_i^T \psi_i(\bar{\zeta}_i). \end{aligned} \tag{15}$$

Using the mid-value theorem and (13), it can be seen that $f_i(\bar{x}_i) - \omega_i^T \psi_i(\bar{\zeta}_i)$ is equivalent to

$$\begin{aligned} & f_i(\bar{x}_i) - \omega_i^T \psi_i(\bar{\zeta}_i) \\ &= f_i(\bar{x}_i) - f_i(\bar{\zeta}_i) + f_i(\bar{\zeta}_i) \\ &\quad - \omega_i^{*T} \psi_i(\bar{\zeta}_i) + \omega_i^{*T} \psi_i(\bar{\zeta}_i) - \omega_i^T \psi_i(\bar{\zeta}_i) \\ &= \frac{\partial f_i}{\partial x} \bar{e}_i + \delta_i(\bar{\zeta}_i) + \tilde{\omega}_i^T \psi_i(\bar{\zeta}_i), \end{aligned} \tag{16}$$

where $\partial f_i / \partial x = [\partial f_i / \partial x_1, \ldots, \partial f_i / \partial x_i]$ and $\bar{e}_i = [e_1, e_2, \ldots, e_i]^T$.

Assumption 4 [19]**:** Assume that f_is are smooth functions and there exist constants d_{lij} and d_{uij} satisfying

$$d_{lij} \le \frac{\partial f_i}{\partial x_j} \le d_{uij}, \quad 1 \le i \le n, \quad 1 \le j \le n. \tag{17}$$

Substituting (16) in (15) gives

$$\dot{e}_i = e_{i+1} - l_i e_i + \frac{\partial f_i}{\partial x} \bar{e}_i + \delta_i(\bar{\zeta}_i) + \tilde{\omega}_i^T \psi_i(\bar{\zeta}_i). \tag{18}$$

Similarly, $\dot{e}_n$ can be derived as

$$\begin{aligned}\dot{e}_n =&\dot{x}_n-\dot{\zeta}_n=u+f_n(\bar{x}_n)-u-l_ne_1-\omega_n^T\psi_n(\bar{\zeta}_n)\\ =&-l_ne_1+\frac{\partial f_n}{\partial x}e+\delta_n(\bar{\zeta}_n)+\tilde{\omega}_n^T\psi_n(\bar{\zeta}_n),\end{aligned} \tag{19}$$

where $e=[e_1,e_2,\dots,e_n]^T$.

Then

$$\dot{e}=(K-LB)e+Je+\delta(\bar{\zeta})+\tilde{\omega}^T\psi(\bar{\zeta}), \tag{20}$$

where

$$K=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\ddots&\vdots\\ \vdots&\ddots&\ddots&\ddots&0\\0&0&\ddots&0&1\\0&0&\cdots&0&0\end{bmatrix}, \tag{21}$$

$$L=[l_1,\cdots,l_n]^T, \tag{22}$$

$$B=\left[1,\underbrace{0,\cdots,0}_{n-1}\right], \tag{23}$$

$$J=\left[\bar{0}^T,\left(\frac{\partial f_2}{\partial x}\right)^T,\dots,\left(\frac{\partial f_n}{\partial x}\right)^T\right]^T, \tag{24}$$

$$\bar{0}=\left[\underbrace{0,\cdots,0}_{n}\right], \tag{25}$$

$$\delta(\bar{\zeta})=\left[\delta_1(x_1),\delta_2(\bar{\zeta}_2),\cdots,\delta_n(\bar{\zeta}_n)\right]^T, \tag{26}$$

$$\tilde{\omega}^T\psi(\bar{\zeta})=\left[\tilde{\omega}_1^T\psi(x_1),\cdots,\tilde{\omega}_n^T\psi(\bar{\zeta}_n)\right]^T. \tag{27}$$

The Lyapunov function is chosen as

$$V_e=e^T\bar{P}e, \tag{28}$$

where $\bar{P}$ is a symmetric and positive definite matrix.

Then the time derivative of V_e can be obtained as

$$\begin{aligned}\dot{V}_e=&e^T(((K-LB)^T)\bar{P}+\bar{P}(A-LB)+J^T\bar{P}\\ &+\bar{P}J)e+2e^T(\bar{P}\tilde{\omega}^T\psi(\bar{\zeta})+\delta(\bar{\zeta})).\end{aligned} \tag{29}$$

According to Young's inequality, $\delta_i(\zeta_i)\le\varepsilon_i$ and $0<\psi(\bar{\zeta})\psi^T(\bar{\zeta})\le 1$, we can obtain

$$\begin{aligned}2e^T\bar{P}\delta(\bar{\zeta})\le&\kappa e^T\bar{P}e+\frac{1}{\kappa}||\bar{P}||\sum_{i=1}^n\delta_i^2(\bar{\zeta})\\ \le&\kappa e^T\bar{P}e+\frac{1}{\kappa}||\bar{P}||\sum_{i=1}^n\varepsilon_i^2,\end{aligned} \tag{30}$$

$$\begin{aligned}2e^T\bar{P}\tilde{\omega}^T\psi(\bar{\zeta})\le&\kappa e^T\bar{P}e+\frac{1}{\kappa}||\bar{P}||\tilde{\omega}^T\psi(\bar{\zeta})\psi(\bar{\zeta})^T\tilde{\omega}\\ \le&\kappa e^T\bar{P}e+\frac{1}{\kappa}||\bar{P}||\tilde{\omega}^T\tilde{\omega},\end{aligned} \tag{31}$$

where $\kappa>0$ is a design parameter. Substituting (30) and (31) in (29) gives

$$\begin{aligned}\dot{V}_e\le&e^T(((K-LB)^T)\bar{P}+\bar{P}(K-LB)+J^T\bar{P}\\ &+\bar{P}J+2\kappa\bar{P})e+\frac{1}{\kappa}||\bar{P}||\sum_{i=1}^n\varepsilon_i^2\\ &+\frac{1}{\kappa}||\bar{P}||\tilde{\omega}^T\tilde{\omega}.\end{aligned} \tag{32}$$

4. CONTROLLER DESIGN AND STABILITY ANALYSIS

We present a computationally and structurally efficient controller.

Define the state errors as

$$z_i=\zeta_i-\alpha_{i-1}, \tag{33}$$

where α_i is the virtual control signal with $\alpha_0=y_d$.

Then the system can be rewritten as

$$\begin{cases}\dot{z}_1=\zeta_2+l_1e_1+\omega_1^T\psi_1(x_1)-\dot{y}_d,\\ \dot{z}_i=\zeta_{i+1}+l_ie_1+\omega_i^T\psi_i(\bar{\zeta}_i)-\dot{\alpha}_{i-1},\\ \dot{z}_n=u+l_ne_1+\omega_n^T\psi_n(\bar{\zeta}_n)-\dot{\alpha}_{n-1}.\end{cases} \tag{34}$$

The design process for the controller is carried out in steps. For simplicity, let $\psi_i(\bar{\zeta}_n)=\psi_i$ and $f_i(\bar{\zeta}_i)=f_i$.

Step 1:

Considering both the prescribed performance control and fuzzy state observer, the Lyapunov function has the form

$$V_{z1}=\frac{1}{2}\tan^2\left(\frac{\pi}{2}\frac{z_1}{\varkappa}\right)+\frac{1}{2r_1}\tilde{\theta}_1^2+\frac{1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1, \tag{35}$$

where $\varkappa(t)=(k_0-k_\infty)e^{-\tau t}+k_\infty$ is the prescribed performance function; k_0, k_∞, and τ are positive constants satisfying $k_0>k_\infty$; and r_1 and η_1 are positive design parameters. The approximation error is $\tilde{\theta}$, $\tilde{\theta}_1=\theta_1^*-\theta_1$, and θ_1 is the estimate of θ_1^*. Define

$$D(t)=\frac{\tan\left(\frac{\pi}{2}\frac{z_i}{\varkappa}\right)}{\cos^2\left(\frac{\pi}{2}\frac{z_i}{\varkappa}\right)}. \tag{36}$$

It is worth noting that the prescribed performance function $\varkappa(t)$ and its j-th order derivative $\varkappa^{(j)}(t)$ are continuous and bounded for $j=1,\dots,n$.

The time-derivative of V_{z1} is

$$\begin{aligned}\dot{V}_{z1}=&\rho D\left(\dot{z}_1-\frac{\dot{\varkappa}}{\varkappa}z_1\right)-\frac{1}{r_1}\tilde{\theta}_1\dot{\theta}_1-\frac{1}{\eta_1}\tilde{\omega}_1^T\dot{\omega}_1\\ =&\rho D(\zeta_2+l_1e_1+\omega_1^{*T}\psi_1(x_1)-\tilde{\omega}_1^T\psi_1(x_1)\\ &-\dot{y}_d-\frac{\dot{\varkappa}}{\varkappa}z_1+\frac{1}{2}\rho D)-\frac{1}{r_1}\tilde{\theta}_1\dot{\theta}_1\\ &-\frac{1}{\eta_1}\tilde{\omega}_1^T\dot{\omega}_1-\frac{1}{2}\rho^2D^2\end{aligned}$$

$$
\begin{aligned}
=&\rho D\zeta_2-\rho D\tilde{\omega}_1^T\psi_1(x_1)-\frac{1}{r_1}\tilde{\theta}_1\dot{\theta}_1\\
&-\frac{1}{\eta_1}\tilde{\omega}_1^T\dot{\omega}_1-\frac{1}{2}\rho^2D^2+\rho D(l_1e_1-\dot{y}_d\\
&-\frac{\dot{\varkappa}}{\varkappa}z_1+\omega_1^{*T}\psi_1(x_1)+\frac{1}{2}\rho D),
\end{aligned}\tag{37}
$$

where $\rho=\frac{\pi}{2\varkappa}$.

Define an unknown packaged function as $F_1(x_1,\zeta_1,y_d,\dot{y}_d,\varkappa,\dot{\varkappa})=l_1e_1-\dot{y}_d-\frac{\dot{\varkappa}}{\varkappa}z_1+\omega_1^{*T}\psi_1(x_1)+\frac{1}{2}\rho D$. By substituting $z_2=\zeta_2-\alpha_1$ into (37), $\dot{V}_{z1}$ can be described as

$$
\begin{aligned}
\dot{V}_{z1}\leq\ &\rho Dz_2+\rho D\alpha_1-\rho D\tilde{\omega}_1^T\psi_1(x_1)+\rho DF_1\\
&-\frac{1}{r_1}\tilde{\theta}_1\dot{\theta}_1-\frac{1}{\eta_1}\tilde{\omega}_1^T\dot{\omega}_1-\frac{1}{2}\rho^2D^2.
\end{aligned}\tag{38}
$$

According to Lemma 1, the unknown function F_1 can be approximated by FLS as

$$
F_1=W_1^{*T}S_1+\varpi_1,|\varpi_1|\leq\gamma_1.\tag{39}
$$

It follows from Young's inequality that

$$
\begin{aligned}
\rho DF_1\leq&\frac{1}{2a_1^2}\rho^2D^2\theta_1^*S_1^TS_1+\frac{1}{2}a_1^2\\
&+\frac{1}{2}\rho^2D^2+\frac{1}{2}\gamma_1^2,
\end{aligned}\tag{40}
$$

where $\theta_1^*=||W_1||^2$, and $a_1>0$ is a design parameter. Substituting (40) in (38) gives

$$
\begin{aligned}
\dot{V}_{z1}\leq&\frac{1}{2a_1^2}\rho^2D^2\theta_1S_1^TS_1+\frac{1}{2a_1^2}\rho^2D^2\tilde{\theta}_1S_1^TS_1\\
&+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2+\rho Dz_2+\rho D\alpha_1\\
&-\frac{1}{r_1}\tilde{\theta}_1\dot{\theta}_1-\frac{1}{\eta_1}\tilde{\omega}_1^T(\dot{\omega}_1+\eta_1\rho D\psi_1(x_1)).
\end{aligned}\tag{41}
$$

Define the following virtual control:

$$
\alpha_1=-\frac{c_1}{\rho}\tan(\rho z_1)\cos^2(\rho z_1)-\frac{1}{2a_1^2}\rho D\theta_1S_1^TS_1,\tag{42}
$$

where $c_1>0$ is a design parameter.

Substituting (42) in (41) gives

$$
\begin{aligned}
\dot{V}_{z1}\leq&-c_1\tan^2(\rho z_1)+\frac{1}{2a_1^2}\rho^2D^2\tilde{\theta}_1S_1^TS_1\\
&-\frac{1}{\eta_1}\tilde{\omega}_1^T(\dot{\omega}_1+\eta_1\rho D\psi_1(x_1))-\frac{1}{r_1}\tilde{\theta}_1\dot{\theta}_1\\
&+\rho Dz_2+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2.
\end{aligned}\tag{43}
$$

The adaptive laws $\dot{\theta}_1$ and $\dot{\omega}_1$ can be designed as

$$
\dot{\theta}_1=\frac{r_1}{2a_1^2}\rho^2D^2S_1^TS_1-m_1\theta_1\tag{44}
$$

and

$$
\dot{\omega}_1=-\eta_1\rho D\psi_1(x_1)-q_1\omega_1,\tag{45}
$$

where $m_1>0$ and $q_1>0$ are design parameters.

Substituting (44) and (45) in (43) , we obtain

$$
\begin{aligned}
\dot{V}_{z1}\leq&-c_1\tan^2(\rho z_1)+\rho Dz_2+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2\\
&+\frac{m_1}{r_1}\theta_1\tilde{\theta}_1+\frac{q_1}{\eta_1}\omega_1\tilde{\omega}_1^T.
\end{aligned}\tag{46}
$$

Since

$$
\begin{aligned}
\theta_1\tilde{\theta}_1=&\tilde{\theta}_1(\theta_1^*-\tilde{\theta}_1)\\
\leq&\frac{1}{2}\theta_1^{*2}+\frac{1}{2}\tilde{\theta}_1^2-\tilde{\theta}_1^2\\
=&\frac{1}{2}\theta_1^{*2}-\frac{1}{2}\tilde{\theta}_1^2,
\end{aligned}\tag{47}
$$

and

$$
\omega_1\tilde{\omega}_1^T\leq\frac{1}{2}\omega_1^{*T}\omega_1^*-\frac{1}{2}\tilde{\omega}_1^T\tilde{\omega}_1,\tag{48}
$$

we can rewrite (46) as

$$
\begin{aligned}
\dot{V}_{z1}\leq&-c_1\tan^2(\rho z_1)-\frac{m_1}{2r_1}\tilde{\theta}_1^2-\frac{q_1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1\\
&+\rho Dz_2+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2+\frac{m_1}{2r_1}\theta_1^{*2}\\
&+\frac{q_1}{2\eta_1}\omega_1^{*T}\omega_1^*.
\end{aligned}\tag{49}
$$

Step 2:

Construct the Lyapunov function as

$$
V_{z2}=V_{z1}+\frac{1}{2}z_2^2+\frac{1}{2r_2}\tilde{\theta}_2^2+\frac{1}{2\eta_2}\tilde{\omega}_2^T\tilde{\omega}_2.\tag{50}
$$

The time yields of V_{z2} can be described as

$$
\begin{aligned}
\dot{V}_{z2}=&\dot{V}_{z1}+z_2(\zeta_3+l_2e_1+\omega_2^T\psi_2(\bar{\zeta}_2)-\dot{\alpha}_1)\\
&-\frac{1}{r_2}\tilde{\theta}_2\dot{\theta}_2-\frac{1}{\eta_2}\tilde{\omega}_2^T\dot{\omega}_2.
\end{aligned}\tag{51}
$$

By substituting (49) in (51), we can write

$$
\begin{aligned}
\dot{V}_{z2}=&-c_1\tan^2(\rho z_1)-\frac{m_1}{2r_1}\tilde{\theta}_1^2-\frac{q_1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1\\
&+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2+\frac{m_1}{2r_1}\theta_1^{*2}+\frac{q_1}{2\eta_1}\omega_1^{*T}\omega_1^*\\
&+z_2(\zeta_3+l_2e_1+\omega_2^T\psi_2(\bar{\zeta}_2)-\dot{\alpha}_1)\\
&-\frac{1}{r_2}\tilde{\theta}_2\dot{\theta}_2-\frac{1}{\eta_2}\tilde{\omega}_2^T\dot{\omega}_2+\rho Dz_2.
\end{aligned}\tag{52}
$$

Substituting $z_3=\zeta_3-\alpha_2$ into (52) gives

$$
\dot{V}_{z2}=-c_1\tan^2(\rho z_1)-\frac{m_1}{2r_1}\tilde{\theta}_1^2-\frac{q_1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1+z_2\alpha_2
$$

$$
\begin{aligned}
&+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2+\frac{m_1}{2r_1}\theta_1^{*2}+\frac{q_1}{2\eta_1}\omega_1^{*T}\omega_1^*+z_2z_3\\
&-\frac{1}{r_2}\tilde{\theta}_2\dot{\theta}_2-\frac{1}{\eta_2}\tilde{\omega}_2^T\dot{\omega}_2-\frac{1}{2}z_2^2-z_2\tilde{\omega}_2^T\psi_2\left(\bar{\zeta}_2\right)\\
&+z_2(\frac{1}{2}z_2+l_2e_1+\omega_2^{*T}\psi_2\left(\bar{\zeta}_2\right)\\
&-\dot{\alpha}_1+\rho D).
\end{aligned}\tag{53}
$$

Define $F_2\big(x_1,\zeta_1,\zeta_2,\theta_1,\chi,\chi^{(2)},\chi^{(3)},y_d,y_d^{(1)},y_d^{(2)}\big)=\rho D+l_2e_1+\omega_2^{*T}\psi_2\left(\bar{\zeta}_2\right)-\dot{\alpha}_1+\frac{1}{2}z_2$, which can be approximated by FLS as

$$
F_2=W_2^{*T}S_2+\varpi_2,|\varpi_2|\leq\gamma_2.\tag{54}
$$

Then we can obtain the inequality

$$
\begin{aligned}
z_2F_2=&z_2W_2^{*T}S_2+z_2\varpi_2\\
\leq&\frac{1}{2a_2^2}z_2^2\theta_2^*S_2^TS_2+\frac{1}{2}a_2^2+\frac{1}{2}z_2^2+\frac{1}{2}\gamma_2^2,
\end{aligned}\tag{55}
$$

where $\theta_2^*=||W_2||^2$, and $a_2>0$ is a design parameter.

Design the virtual control

$$
\alpha_2=-c_2z_2-\frac{1}{2a_2^2}z_2\theta_2S_2^TS_2,\tag{56}
$$

where $c_2>0$ is a design parameter.
Combining (55) and (56), we can derive

$$
\begin{aligned}
\dot{V}_{z2}\leq&-c_1\tan^2\left(\rho z_1\right)-c_2z_2^2-\frac{m_1}{2r_1}\tilde{\theta}_1^2-\frac{q_1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1\\
&+\frac{1}{2a_2^2}z_2^2\tilde{\theta}_2S_2^TS_2-\frac{1}{r_2}\tilde{\theta}_2\dot{\theta}_2+z_2z_3\\
&-z_2\tilde{\omega}_2^T\psi_2\left(\bar{\zeta}_2\right)-\frac{1}{\eta_2}\tilde{\omega}_2^T\dot{\omega}_2+\frac{1}{2}a_1^2\\
&+\frac{1}{2}\gamma_1^2+\frac{1}{2r_1}\theta_1^{*2}+\frac{1}{2\eta_1}\omega_1^{*T}\omega_1^*\\
&+\frac{1}{2}a_2^2+\frac{1}{2}\gamma_2^2.
\end{aligned}\tag{57}
$$

Construct the adaptive laws $\dot{\theta}_2$ and $\dot{\omega}_2$ as

$$
\dot{\theta}_2=\frac{r_2}{2a_2^2}z_2^2S_2^TS_2-m_2\theta_2,\tag{58}
$$

$$
\dot{\omega}_2=-\eta_2z_2\psi_2\left(\bar{\zeta}_2\right)-q_2\omega_2,\tag{59}
$$

where $m_2>0$ and $q_2>0$ are design parameters.
Moreover, (57) can be transformed to

$$
\begin{aligned}
\dot{V}_{z2}\leq&-c_1\tan^2\left(\rho z_1\right)-c_2z_2^2-\frac{1}{2r_1}\tilde{\theta}_1^2-\frac{1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1\\
&-\frac{m_2}{r_2}\tilde{\theta}_2\theta_2-\frac{q_2}{\eta_2}\tilde{\omega}_2^T\omega_2+z_2z_3\\
&+\frac{1}{2}a_1^2+\frac{1}{2}\gamma_1^2+\frac{m_1}{2r_1}\theta_1^{*2}\\
&+\frac{q_1}{2\eta_1}\omega_1^{*T}\omega_1^*+\frac{1}{2}a_2^2+\frac{1}{2}\gamma_2^2.
\end{aligned}\tag{60}
$$

Similarly, it can be seen that

$$
\tilde{\theta}_2\theta_2\leq\frac{1}{2}\theta_2^{*2}-\frac{1}{2}\tilde{\theta}_2^2,\tag{61}
$$

$$
\tilde{\omega}_2^T\omega_2\leq\frac{1}{2}\omega_2^{*T}\omega_2^*-\frac{1}{2}\tilde{\omega}_2^T\tilde{\omega}_2.\tag{62}
$$

Therefore, we can write

$$
\begin{aligned}
\dot{V}_{z2}\leq&-c_1\tan^2\left(\rho z_1\right)-c_2z_2^2-\frac{m_1}{2r_1}\tilde{\theta}_1^2-\frac{m_2}{2r_2}\tilde{\theta}_2^2\\
&-\frac{q_2}{2\eta_2}\tilde{\omega}_2^T\tilde{\omega}_2+\frac{1}{2}\sum_{i=1}^2a_i^2\\
&+\frac{1}{2}\sum_{i=1}^2\gamma_i^2+z_2z_3-\frac{q_1}{2\eta_1}\tilde{\omega}_1^T\tilde{\omega}_1\\
&+\sum_{i=1}^2\frac{m_i}{2r_i}\theta_i^{*2}+\sum_{i=1}^2\frac{q_i}{2r_i}\omega_i^{*T}\omega_i^*.
\end{aligned}\tag{63}
$$

Step *i*:

For the *i*-th subsystem, we choose the Lyapunov function

$$
V_{zi}=V_{zi-1}+\frac{1}{2}z_i^2+\frac{1}{2r_i}\tilde{\theta}_i^2+\frac{1}{2\eta_i}\tilde{\omega}_i^T\tilde{\omega}_i.\tag{64}
$$

The time derivative of V_{zi} can be obtained as

$$
\begin{aligned}
\dot{V}_{zi}=&\dot{V}_{zi-1}+z_i\left(\zeta_{i+1}+l_ie_1+\omega_i^T\psi_i\left(\bar{\zeta}_i\right)-\dot{\alpha}_{i-1}\right)\\
&-\frac{1}{r_i}\tilde{\theta}_i\dot{\theta}_i-\frac{1}{\eta_i}\tilde{\omega}_i^T\dot{\omega}_i.
\end{aligned}\tag{65}
$$

Since $z_{i+1}=\zeta_{i+1}-\alpha_i$, we can derive that

$$
\begin{aligned}
\dot{V}_{zi}=&-c_1\tan^2\left(\rho z_1\right)-\sum_{j=2}^{i-1}c_jz_j^2-\sum_{j=1}^{i-1}\frac{m_j}{2r_j}\tilde{\theta}_j^2+z_i\alpha_i\\
&-\sum_{j=1}^{i-1}\frac{q_j}{2\eta_j}\tilde{\omega}_j^T\tilde{\omega}_j+\sum_{j=1}^{i-1}\frac{q_j}{2\eta_j}\omega_j^{*T}\omega_j^*\\
&+\frac{1}{2}\sum_{j=1}^{i-1}a_j^2+\frac{1}{2}\sum_{j=1}^{i-1}\gamma_j^2\\
&+\sum_{j=1}^{i-1}\frac{m_j}{2r_j}\theta_j^{*2}-\frac{1}{2}z_i^2+z_iz_{i+1}\\
&+z_i(z_{i-1}+l_ie_1+\omega_i^{*T}\psi_i\left(\bar{\zeta}_i\right)\\
&-\dot{\alpha}_{i-1}+\frac{1}{2}z_i)-\frac{1}{r_i}\tilde{\theta}_i\dot{\theta}_i\\
&-\frac{1}{\eta_i}\tilde{\omega}_i^T\dot{\omega}_i-z_i\tilde{\omega}_i^T\psi_i\left(\bar{\zeta}_i\right).
\end{aligned}\tag{66}
$$

Define $F_i\Big(x_1,\zeta_1,\cdots,\zeta_i,\theta_1,\cdots,\theta_{i-1},\varkappa,\cdots,\varkappa^{(i)},y_d,\cdots,y_d^{(i)}\Big)=z_{i-1}+l_ie_1+\omega_i^{*T}\psi_i\left(\bar{\zeta}_i\right)-\dot{\alpha}_{i-1}+\frac{1}{2}z_i$, which can be approximated by FLS as

$$
F_i=W_i^{*T}S_i+\varpi_i,|\varpi_i|\leq\gamma_i.\tag{67}
$$

Similar to Step 1, we can obtain

$$
\begin{aligned}
z_iF_i =& z_iW_i^{*T}S_i + z_i\varpi_i \\
\leq& \frac{1}{2a_i^2}z_i^2\theta_i^*S_i^TS_i + \frac{1}{2}a_i^2 + \frac{1}{2}z_i^2 + \frac{1}{2}\gamma_i^2,
\end{aligned} \tag{68}
$$

where $\theta_i^* = ||W_i||^2$ and $a_i > 0$ is a positive design parameter.

Then a virtual control α_i is designed as

$$
\alpha_i = -c_iz_i - \frac{1}{2a_i^2}z_i\theta_iS_i^TS_i, \tag{69}
$$

where c_i is a design parameter.

Combining (68) and (69) produces

$$
\begin{aligned}
\dot{V}_{zi} \leq& -c_1\tan^2(\rho z_1) - \sum_{j=2}^{i-1}c_jz_i^2 - \sum_{j=1}^{i-1}\frac{m_j}{2r_j}\tilde{\theta}_j^2 \\
&- \sum_{j=1}^{i-1}\frac{q_j}{2\eta_j}\tilde{\omega}_j^T\tilde{\omega}_j + \sum_{j=1}^{i-1}\frac{m_j}{2r_j}\theta_j^{*2} \\
&+ \frac{1}{2}\sum_{j=1}^{i}a_j^2 + \frac{1}{2}\sum_{j=1}^{i}\gamma_j^2 \\
&+ \sum_{j=1}^{i-1}\frac{q_j}{2\eta_j}\omega_j^{*T}\omega_j^* + z_iz_{i+1} \\
&- z_i\tilde{\omega}_i^T\psi_i(\bar{\zeta}_i) + \frac{1}{2a_i^2}z_i^2\tilde{\theta}_iS_i^TS_i \\
&- \frac{1}{r_i}\tilde{\theta}_i\dot{\theta}_i - \frac{1}{\eta_i}\tilde{\omega}_i^T\dot{\omega}_i.
\end{aligned} \tag{70}
$$

Construct the adaptive laws $\dot{\theta}_2$ and $\dot{\omega}_2$ as

$$
\dot{\theta}_i = \frac{r_i}{2a_i^2}z_i^2S_i^TS_i - m_i\theta_i, \tag{71}
$$

$$
\dot{\omega}_i = -\eta_iz_i\psi_i(\bar{\zeta}_i) - q_i\omega_i, \tag{72}
$$

where $m_i > 0$ and $q_i > 0$ are design parameters.

Then it can be seen that

$$
\tilde{\theta}_i\theta_i \leq \frac{1}{2}\theta_i^{*2} - \frac{1}{2}\tilde{\theta}_i^2, \tag{73}
$$

$$
\tilde{\omega}_i^T\omega_i \leq \frac{1}{2}\omega_i^{*T}\omega_i^* - \frac{1}{2}\tilde{\omega}_i^T\tilde{\omega}_i. \tag{74}
$$

Moreover, $\dot{V}_{zi}$ can be expressed as

$$
\begin{aligned}
\dot{V}_{zi} \leq& -c_1\tan^2(\rho z_1) - \sum_{j=2}^{i}c_iz_i^2 - \sum_{j=1}^{i}\frac{m_j}{2r_j}\tilde{\theta}_j^2 \\
&- \sum_{j=1}^{i}\frac{q_j}{2\eta_j}\tilde{\omega}_j^T\tilde{\omega}_j + \frac{1}{2}\sum_{j=1}^{i}a_j^2 \\
&+ \frac{1}{2}\sum_{j=1}^{i}\gamma_j + z_iz_{i+1} \\
&+ \sum_{j=1}^{i}\frac{m_j}{2r_j}\theta_j^{*2} + \sum_{j=1}^{i}\frac{q_j}{2\eta_j}\omega_j^{*T}\omega_j^*.
\end{aligned} \tag{75}
$$

Step *n*:

Consider the Lyapunov function candidate

$$
V_{zn} = V_{n-1} + \frac{1}{2}z_n^2 + \frac{\beta_0}{2r_n}\tilde{\theta}_n^2 + \frac{1}{2\eta_n}\tilde{\omega}_n^T\tilde{\omega}_n. \tag{76}
$$

The time yields of V_{zn} can be described as

$$
\begin{aligned}
\dot{V}_{zn} =& \dot{V}_{n-1} + z_n\dot{z}_n - \beta_0\tilde{\theta}_n\dot{\theta}_n - \tilde{\omega}_n^T\dot{\omega}_n \\
=& -c_1\tan^2(\rho z_1) - \sum_{j=2}^{n-1}c_jz_j^2 - \sum_{j=1}^{n-1}\frac{m_j}{2r_j}\tilde{\theta}_j^2 \\
&- \sum_{j=1}^{n-1}\frac{q_j}{2\eta_j}\tilde{\omega}_j^T\tilde{\omega}_j + \frac{1}{2}\sum_{j=1}^{n-1}a_j^2 \\
&+ \frac{1}{2}\sum_{j=1}^{n-1}\gamma_j^2 - z_n\tilde{\omega}_n^T\psi_n(\bar{\zeta}_n) + \sum_{j=1}^{n-1}\frac{m_j}{2r_j}\theta_j^{*2} \\
&+ \sum_{j=1}^{n-1}\frac{q_j}{2\eta_j}\omega_j^{*T}\omega_j^* + z_n\left(H^T(t)\phi(t)v + d(v)\right) \\
&+ z_n\left(z_{n-1} + l_ne_1 + \omega_n^{*T}\psi_n(\bar{\zeta}_n) - \dot{\alpha}_{n-1} + z_n\right) \\
&- \frac{\beta_0}{r_n}\tilde{\theta}_n\dot{\theta}_n - \frac{1}{\eta_n}\tilde{\omega}_n^T\dot{\omega}_n - z_n^2.
\end{aligned} \tag{77}
$$

Define $F_n\Big(x_1, \zeta_1, \cdots, \zeta_n, \theta_1, \cdots, \theta_{n-1}, \varkappa, \cdots, \varkappa^{(n)}, y_d, \cdots, y_d^{(n)}\Big) = z_{n-1} + l_ne_1 + \omega_n^{*T}\psi_n(\bar{\zeta}_n) - \dot{\alpha}_{n-1} + z_n$, which can be approximated by FLS as

$$
F_n = W_n^{*T}S_n + \varpi_n, |\varpi_n| \leq \gamma_n. \tag{78}
$$

Similar to Step 1, we can obtain the inequality

$$
\begin{aligned}
z_nF_n =& z_nW_n^{*T}S_n + z_n\varpi_n \\
\leq& \frac{\beta_0}{2a_n^2}z_n^2\theta_n^*S_n^TS_n + \frac{1}{2}a_n^2 \\
&+ \frac{1}{2}z_n^2 + \frac{1}{2}\gamma_n^2,
\end{aligned} \tag{79}
$$

where $\theta_n^* = ||W_n||^2/\beta_0$, and a_n is a positive design parameter.

Define the intermediate control law

$$
\Xi = -c_0z_n - \frac{1}{2a_n^2}z_n\theta_nS_n^TS_n, \tag{80}
$$

where $c_0 > 0$ is a design parameter.

According to Remark 1, we have $H^T(t)\phi(t) \geq \beta_0$ and $|d(\Xi)| < \rho^*$, hence

$$
z_nH^T(t)\phi(t)\Xi \leq -c_0\beta_0z_n^2 - \frac{\beta_0}{2a_n^2}z_n^2\theta_nS_n^TS_n, \tag{81}
$$

$$
z_nd(\Xi) \leq |z_n||d(\Xi)| \leq \frac{1}{2}z_n^2 + \frac{1}{2}\rho^{*2}. \tag{82}
$$

Combining (79), (80), (81), (82), and (77) yields

$$
\dot{V}_{zn} \leq -c_1\tan^2(\rho z_1) - \sum_{i=2}^{n-1}c_iz_i^2 - \sum_{i=1}^{n-1}\frac{m_i}{2r_i}\tilde{\theta}_i^2
$$

$$
\begin{aligned}
&-c_0\beta_0 z_n^2+\frac{1}{2}\sum_{i=1}^{n-1}a_i^2\\
&+\frac{1}{2}\sum_{i=1}^{n-1}\gamma_i^2-\frac{\beta_0}{r_n}\tilde{\theta}_n\dot{\theta}_n\\
&-\frac{1}{\eta_n}\tilde{\omega}_n^T\dot{\omega}_n-z_n\tilde{\omega}_n^T\psi_n\left(\bar{\zeta}_n\right)\\
&-\sum_{i=1}^{n-1}\frac{q_i}{2\eta_i}\tilde{\omega}_i^T\tilde{\omega}_i+\frac{\beta_0}{2a_n^2}z_n^2\tilde{\theta}_n S_n^T S_n\\
&+\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\theta_i^{*2}+\sum_{i=1}^{n-1}\frac{q_i}{2\eta_i}\omega_i^{*T}\omega_i^*\\
&+\frac{1}{2}a_n^2+\frac{1}{2}\gamma_n^2+\frac{1}{2}\rho^{*2},
\end{aligned}
\tag{83}
$$

where $c_n=c_0\beta_0$.

The adaptive laws $\dot{\theta}_n$ and $\dot{\omega}_n$ are constructed as

$$\dot{\theta}_n=\frac{r_n}{2a_n^2}z_n^2S_n^TS_n-m_n\theta_n, \tag{84}$$

$$\dot{\omega}_n=-r_{2n}z_n\psi_n\left(\bar{\zeta}_n\right)-q_n\omega_n, \tag{85}$$

where $m_n>0$ and $q_n>0$ are design parameters.
Similarly, $\dot{V}_{zn}$ can be expressed as

$$
\begin{aligned}
\dot{V}_{zn}\leq&-c_1\tan^2(\rho z_1)-\sum_{i=2}^{n}c_iz_i^2-\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\tilde{\theta}_i^2\\
&-\frac{m_n\beta_0}{2r_n}\tilde{\theta}_n^2-\sum_{i=1}^{n}\frac{q_i}{2\eta_i}\tilde{\omega}_i^T\tilde{\omega}_i\\
&+\sum_{i=1}^{n}\frac{q_i}{2\eta_i}\omega_i^{*T}\omega_i^*+\frac{1}{2}\sum_{i=1}^{n}a_i^2\\
&+\frac{1}{2}\sum_{i=1}^{n}\gamma_i^2+\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\theta_i^{*2}\\
&+\frac{m_n\beta_0}{2r_n}\theta_n^{*2}+\frac{1}{2}\rho^{*2}.
\end{aligned}
\tag{86}
$$

Define $V=V_e+V_{zn}$. Then we can obtain

$$
\begin{aligned}
\dot{V}=&\dot{V}_e+\dot{V}_{zn}\\
\leq&e^T\left(\left((K-LB)^T\right)\bar{P}+P(K-LB)\right.\\
&\left.+J^T\bar{P}+\bar{P}J+2\kappa\bar{P}\right)e+\frac{1}{\kappa}||\bar{P}||\sum_{i=1}^{n}\varepsilon_i^2\\
&+\frac{1}{\kappa}||\bar{P}||\tilde{\omega}^T\tilde{\omega}-c_1\tan^2(\rho z_1)\\
&-\sum_{i=2}^{n-1}c_iz_i^2-\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\tilde{\theta}_i^2\\
&-\frac{m_n\beta_0}{2r_n}\tilde{\theta}_n^2-\sum_{i=1}^{n}\frac{q_i}{2\eta_i}\tilde{\omega}_i^T\tilde{\omega}_i\\
&+\frac{1}{2}\sum_{i=i}^{n}a_i^2+\frac{1}{2}\sum_{i=i}^{n}\gamma_i^2\\
&+\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\theta_i^{*2}+\frac{m_n\beta_0}{2}\theta_n^{*2}\\
&+\sum_{i=1}^{n}\frac{q_i}{2\eta_i}\omega_i^{*T}\omega_i^*+\frac{1}{2}\rho^{*2}.
\end{aligned}
\tag{87}
$$

To ensure the stability of the error dynamics, it must hold that $(K-LB)^T\bar{P}+\bar{P}(K-LB)+J^T\bar{P}+\bar{P}J+2\kappa\bar{P}+M=-Q$. Then L and $\bar{P}$ can be determined with Q being a positive matrix, and

$$
\begin{aligned}
\dot{V}\leq&-e^TQe+\frac{1}{\kappa}||P||\sum_{i=1}^{n}\varepsilon_i^2+\frac{1}{\kappa}||P||\tilde{\omega}^T\tilde{\omega}\\
&-c_1\tan^2(\rho z_1)-\sum_{i=2}^{n-1}c_iz_i^2-\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\tilde{\theta}_i^2\\
&-\frac{m_n\beta_0}{2r_n}\tilde{\theta}_n^2-\sum_{i=1}^{n}\frac{q_i}{2\eta_i}\tilde{\omega}_i^T\tilde{\omega}_i\\
&+\sum_{i=1}^{n}\frac{q_i}{2\eta_i}\omega_i^{*T}\omega_i^*+\frac{1}{2}\sum_{i=i}^{n}a_i^2+\frac{1}{2}\sum_{i=i}^{n}\gamma_i^2\\
&+\sum_{i=1}^{n-1}\frac{m_i}{2r_i}\theta_i^{*2}+\frac{m_n\beta_0}{2}\theta_n^{*2}+\frac{1}{2}\rho^{*2}.
\end{aligned}
\tag{88}
$$

According to Assumption 3, there exists $0<\sigma_{0ij}<1$ such that

$$\frac{\partial f_i}{\partial x_j}=\sigma_{0ij}d_{lij}+(1-\sigma_{0ij})\,d_{uij}. \tag{89}$$

To ensure that (89) holds, we must have

$$(K-LB)^T\bar{P}+P(K-LB)+J_\sigma^T\bar{P}+\bar{P}J_\sigma+2\kappa\bar{P}<0, \tag{90}$$

where

$$[J_\sigma]_{ij}=\begin{cases}d_{lij}\text{ or }d_{uij},\ j\leq i,\ 1\leq i\leq n,\ 1\leq j\leq n,\\ 0,\ j>i,\ 1\leq i\leq n,\ 1\leq j\leq n.\end{cases} \tag{91}$$

The controller design procedure is complete, with the following main results.

Theorem 1: Based on system (1) with Assumptions 1-3, the controller was designed as (80), the state observer was designed as (11), and the adaptive control laws (44), (45), (71), and (72) ensure that: i) all the signals of the stable closed-loop system are semi-globally, uniformly, and ultimately bounded; ii) the tracking error z_1 and prescribed performance function $\varkappa$ satisfy $|z_1|<\varkappa$; and iii) the tracking error z_1 converges to a small neighborhood around zero under the prescribed performance.

Proof:

$\dot{V}$ can be rewritten as

$$\dot{V}\leq-a_0V+b_0. \tag{92}$$

Define

$$a_0=\min\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)},2c_i,m_i,q_i,i=1,\cdots,n\right\}, \tag{93}$$

and

$$b_0 = \frac{1}{2}\sum_{i=1}^{n} a_i^2 + \frac{1}{2}\sum_{i=1}^{n}\gamma_i^2 + \sum_{i=1}^{n-1}\frac{m_i}{2r_i}\theta_i^{*2} + \frac{m_n\beta_0}{2r_n}\theta_n^{*2} + \sum_{i=1}^{n}\frac{q_i}{2\eta_i}\omega_i^{*T}\omega_i^* + \frac{1}{2}\rho^{*2}. \tag{94}$$

Equation (92) implies that for all $t \geq 0$,

$$V(t) \leq V(0)e^{-a_0 t} + \frac{b_0}{a_0}. \tag{95}$$

Therefore, z_i, e_i, θ_i, and ω_i, the signals of the closed-loop system, are semi-globally, uniformly, and ultimately bounded. Similarly, ζ_i and x_1 are obviously bounded.

In addition, it follows from (95) that

$$\frac{1}{2}\tan^2\left(\frac{\pi}{2}\frac{z_1}{\varkappa}\right) \leq V(0)e^{-a_0 t} + \frac{b_0}{a_0}. \tag{96}$$

Then it holds that

$$|z_1| \leqslant \frac{2}{\pi}\arctan\left(\left(2\left(V(0)e^{-a_0(t-t_0)} + \frac{b_0}{a_0}\right)\right)^{\frac{1}{2}}\right)\varkappa < \varkappa.$$

This shows that $|z_1| < \varkappa$, hence the tracking error $|z_1|$ converges to a small area of the origin within the prescribed performance function $\varkappa$. This completes the proof.

5. SIMULATION RESULTS

We provide two examples to verify the proposed approach. To more clearly illustrate the effectiveness of the proposed method, we consider a simulation comparison for examples with the control method proposed in [39].

Example 1: Consider a 2nd-order strict feedback nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + 0.5x^3, \\ \dot{x}_2 = u + \sin x_1 \cos x_2, \\ y = x_1. \end{cases} \tag{97}$$

Assume that $-1 \leq \frac{\partial f_2}{\partial x_2} \leq 1$ for $i = 1, 2$. Moreover,

$$K = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{98}$$

and

$$J_0 \in \left\{ \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}. \tag{99}$$

Select $\kappa = \frac{1}{3}$, and solve LMI (90) to obtain

$$\bar{P} = \begin{bmatrix} 79.298 & -52.901 \\ -52.901 & 50.714 \end{bmatrix} \text{ and } L = \begin{bmatrix} 37.78 \\ 78.63 \end{bmatrix}. \tag{100}$$

To approximate the packaged unknown nonlinear functions, nine fuzzy sets are adopted over the interval $[-3, 3]$. The fuzzy membership functions of this system are defined as

$$s_i(X_i) = \exp\left(-0.5\,(X_i + p_j)^2\right), \tag{101}$$

where $1 \leq i \leq 10$, $1 \leq j \leq 9$, $p_j \in \{3,\ 0,\ -3\}$ are the partitioning points, and $X_1 = x_1$, $X_2 = y_d$, $X_3 = \dot{y}_d$, $X_4 = \varkappa$, $X_5 = \dot{\varkappa}$, $X_6 = \zeta_1$, $X_6 = \zeta_2$, $X_8 = \theta_1$, $X_9 = y_d^{(2)}$, and $X_{10} = \varkappa^{(2)}$.
Then $S_1 = [S_1^1, \ldots, S_1^{3^6}]^T$ and $S_2 = [S_2^1, \ldots, S_2^{3^{10}}]^T$ can be calculated by

$$S_1^a = \prod_{i=1}^{6}(s_j(X_i)) \Big/ \sum_{j=1}^{a}\left(\prod_{i=1}^{6} s_j(X_i)\right), \quad a = 1, \cdots, 3^6,$$

$$S_2^b = \prod_{i=1}^{10}(s_j(X_i)) \Big/ \sum_{j=1}^{b}\left(\prod_{i=1}^{10} s_j(X_i)\right), \quad b = 1, \cdots, 3^{10}.$$

Similarly, for the observer, five fuzzy sets are adopted over the interval $[-5, 5]$, with fuzzy membership functions

$$h_i(\zeta_i) = \exp\left(-0.5\,(\zeta_i + q_j)^2\right), \tag{102}$$

where $1 \leq i \leq 2$, $1 \leq j \leq 5$, and $q_j \in \{5,\ 2,\ 0,\ -2,\ -5\}$ are the partitioning points.
Then $\psi_1 = [\psi_1^1, \ldots, \psi_1^5]^T$ and $\psi_2 = [\psi_2^1, \ldots, \psi_2^{25}]^T$ can be given by

$$\psi_1^c = h_j(x_1) \Big/ \sum\nolimits_{j=1}^{c} h_j(x_1), \quad c = 1, \cdots, 5,$$

$$\psi_2^d = h_j(x_1)h_j(\zeta_2) \Big/ \sum_{j=1}^{d}(h_j(x_1)h_j(\zeta_2)), \ d = 1, \cdots, 25.$$

The virtual control α_1 and intermediate control law Ξ are determined as

$$\alpha_1 = -\frac{c_1}{\rho}\tan(\rho z_1)\cos^2(\rho z_1) - \frac{1}{2a_1^2}\rho D\theta_1 S_1^T S_1, \tag{103}$$

and

$$\Xi = -\left(c_0 + \frac{1}{2}\right)z_n - \frac{1}{2a_n^2}z_n\theta_n S_n^T S_n. \tag{104}$$

The model of the dead-zone input u is

$$u = D(\Xi) = \begin{cases} (1 - 0.2\sin(\Xi))(\Xi - 1.25), & \Xi \geq 1.25, \\ 0, & -2.5 < \Xi < 1.25, \\ (0.8 - 0.2\cos(\Xi))(\Xi + 2.5), & \Xi \leq -2.5. \end{cases}$$

The adaptive laws are constructed as

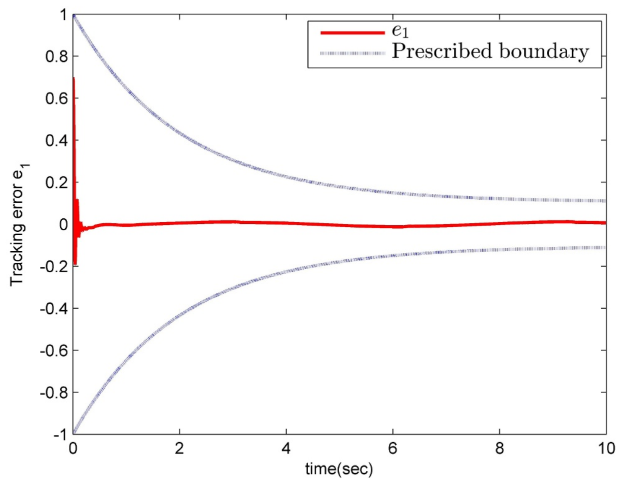

Fig. 1. Trajectory of tracking error.

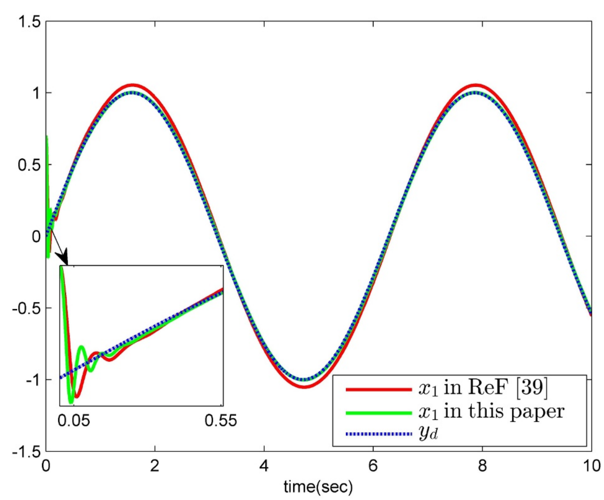

Fig. 2. Tracking performance of system.

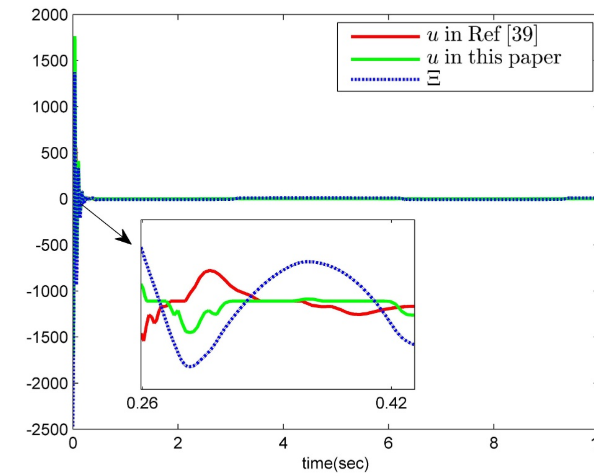

Fig. 3. Control input u and intermediate control input Ξ.

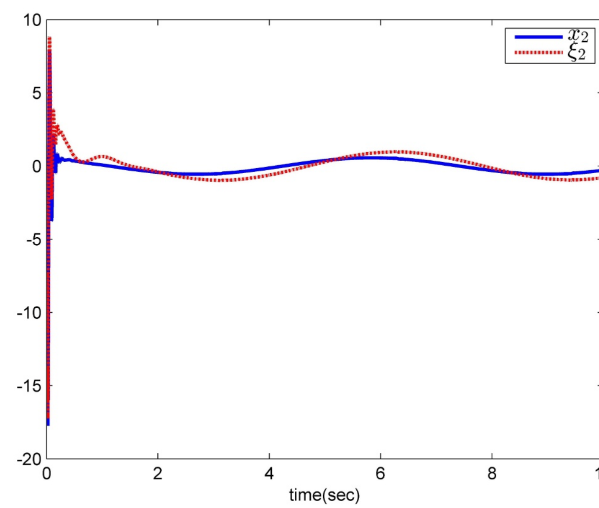

Fig. 4. Trajectory of x_2 and its observer state.

$$\begin{cases} \dot{\theta}_1 = \dfrac{\rho^2 r_1}{2a_1^2} D^2 S_1^T S_1 - m_1\theta_1, \\ \dot{\theta}_2 = \dfrac{r_2}{2a_2^2} z_2^2 S_2^T S_2 - m_2\theta_2, \\ \dot{\omega}_1 = -\eta_1 \rho D \psi_1\left(\bar{\zeta}_1\right) - q_1\omega_1, \\ \dot{\omega}_2 = -\eta_2 z_2 \psi_2\left(\bar{\zeta}_2\right) - q_2\omega_2. \end{cases} \tag{105}$$

The relative parameters are chosen through constant adjustment as follows: $x_1(0) = 0.7$, $x_2(0) = 0.2$, $\zeta_1(0) = \zeta_2(0) = 0$, $\theta_1(0) = 0$, $\theta_2(0) = 0.8$, $\omega_1(0) = [\underbrace{0,\ldots,0}_{5}]^T$, $\omega_2(0) = [\underbrace{0,\ldots,0}_{25}]^T$, $k_0 = 1.5$, $k_\infty = 0.105$, $\tau = 2$, $a_1 = 6$, $a_2 = 100$, $r_1 = 15$, $r_2 = 20$, $\eta_1 = 15$, $\eta_2 = 20$, $m_1 = m_2 = 0.1$, $q_1 = q_2 = 0.1$, $c_1 = 85$, and $c_0 = 60$. The target reference trajectory is given as $y_d = \sin t$.

The tracking performance and tracking error e_1 of the proposed controller are shown in Fig. 1. The results show that the system has good tracking performance and the tracking error e_1 trajectory is limited under the specified performance, eventually converging from 0.5 to a small neighborhood around zero within 0.8 second. Fig. 2 shows the good tracking performance of the system output x_1 and the trajectory of the reference signal y_d. The results show that the reference signal is well tracked in about 0.18 second. By comparison to [39], it can be seen that the tracking speed is slightly slower. The tracking effect of the control method in this paper is more accurate than that of reference [39]. Fig. 3 shows the trajectories of control input u in this paper, control input u in [39], and intermediate control law Ξ. The results show that the control input proposed in this paper has good control performance in overcoming the influence of the dead zone. The trajectories of the system state x_2 and its observer state ζ_2 are shown in Fig. 4, indicating that the proposed fuzzy state observer well solves the problem of the unmeasured state of the system. Fig. 5 shows that the adaptive laws θ_1 and θ_2 are ultimately restricted. The final simulation results show that the proposed controller is reasonable.

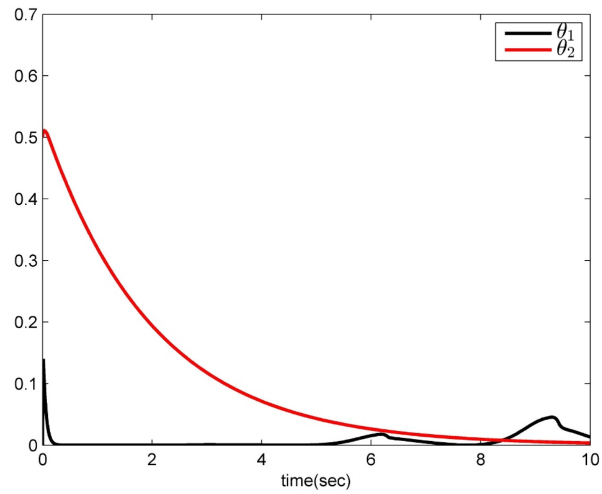

Fig. 5. Trajectory of adaptive parameters θ_1 and θ_2.

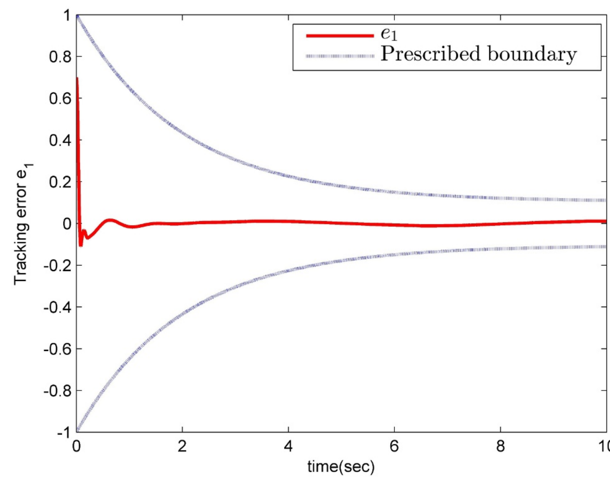

Fig. 6. Trajectory of tracking error.

Example 2: To more clearly illustrate the effectiveness of the proposed method, we consider a simulation comparison for a model with the control method proposed in [39],

$$\begin{cases} K\ddot{\vartheta} + \dfrac{1}{2} m_n GN \sin\vartheta = u, \\ y = \vartheta, \end{cases} \tag{106}$$

where ϑ is the angle of the system, u is the input torque of the system, K is the moment of inertia, G is the acceleration of gravity, m_n is the mass of the linkage, and N is the length of the linkage. We choose parameter values $m_n = 1, N = 1, K = 2$, and $G = 9.8$. Rewrite the single-link robot equation as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \dfrac{1}{K}\left(u - \dfrac{m_n GN \sin\vartheta}{2}\right), \\ y = x_1, \end{cases} \tag{107}$$

where $x_1 = \vartheta, x_2 = \dot{\vartheta}$. The above conditions are chosen to be $x_1(0) = 0.7, x_2(0) = 0.3$.

The matrix P, L, the fuzzy membership functions of FLSs, the virtual control law, the model of the dead-zone input, and the adaptive laws are the same as in the above simulation. The other parameters are selected as follows: $\zeta_1(0) = \zeta_2(0) = 0$, $\theta_1(0) = 0$, $\theta_2(0) = 0.4$, $\omega_1(0) = [\underbrace{0,\ldots,0}_{5}]^T$, $\omega_2(0) = [\underbrace{0,\ldots,0}_{25}]^T$, $k_0 = 1.5$, $k_\infty = 0.105$, $\tau = 2$, $a_1 = 6$, $a_2 = 70$, $r_1 = 15$, $r_2 = 20$, $\eta_1 = 10$, $\eta_2 = 22$, $m_1 = m_2 = 2$, $q_1 = q_2 = 0.5$, $c_1 = 80$, and $c_0 = 60$.

Fig. 6 shows the trajectory of the tracking error e_1, from which it can be seen that the tracking error converges from 0.9 to a small neighborhood near the origin within about 0.2 second within the specified performance range. In addition, the tracking error is under the bound of the prescribed performance. Fig. 7 shows the tracking performance of the output x_1 of the system, the output x_1 of the system in [39], and the trajectory of the reference signal y_d. The results show that x_1 in [39] takes about 0.4 second to track the reference signal, and the control method proposed in this paper takes about 0.2 second. In terms of tracking accuracy, the results show that the proposed control method is better. Fig. 8 shows the trajectories of the control input u in this paper, the control input u in [39], and the intermediate control law Ξ. The results show that the control method proposed in this paper has good control performance in overcoming the influence of the dead zone. Fig. 9 shows the trajectory of system state x_2 and its observer state ζ_2. From the figure, the observer state ζ_2 can estimate the system state with a small error. This shows the effectiveness of the fuzzy state observer. Fig. 10 shows the adaptive parameters θ_1 and θ_2. With the same results as the above example, they finally became bounded. These

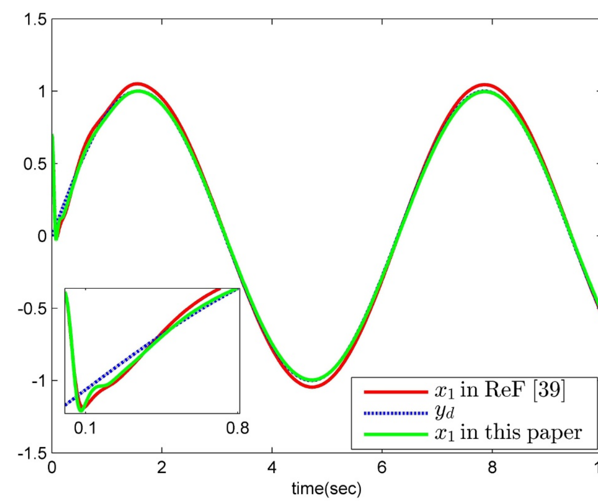

Fig. 7. Tracking performance of system.

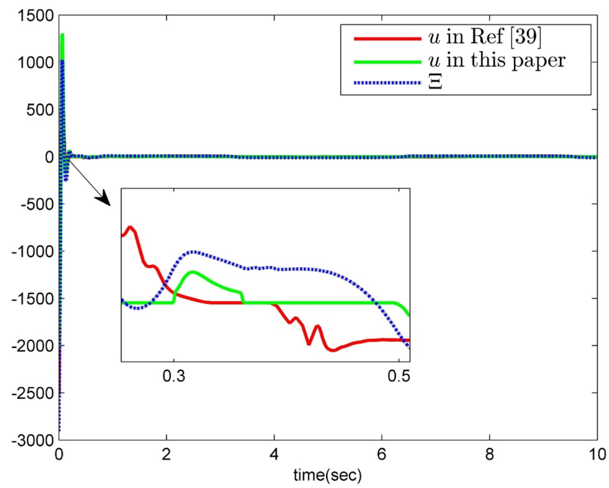

Fig. 8. Control input u and intermediate control input Ξ.

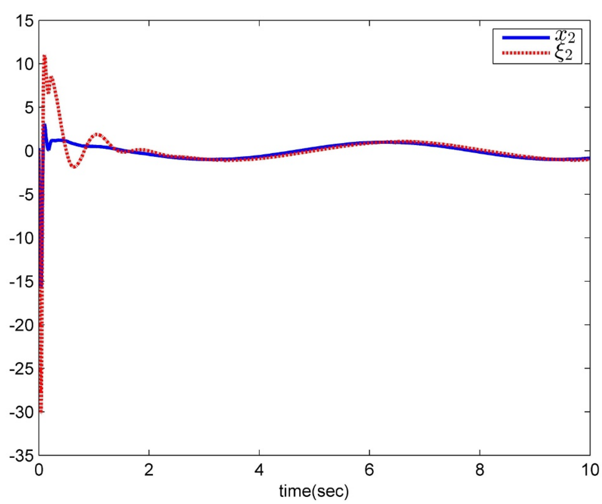

Fig. 9. Trajectory of x_2 and its observer state.

simulation results show that the proposed controller is effective.

6. CONCLUSIONS

We introduced an observer-based adaptive fuzzy control scheme for a class of strict-feedback nonlinear systems with prescribed performance and a dead zone. The proposed scheme ensures the stability of a closed-loop system, and the tracking error falls within a preset boundary. Simulation results confirm its feasibility.

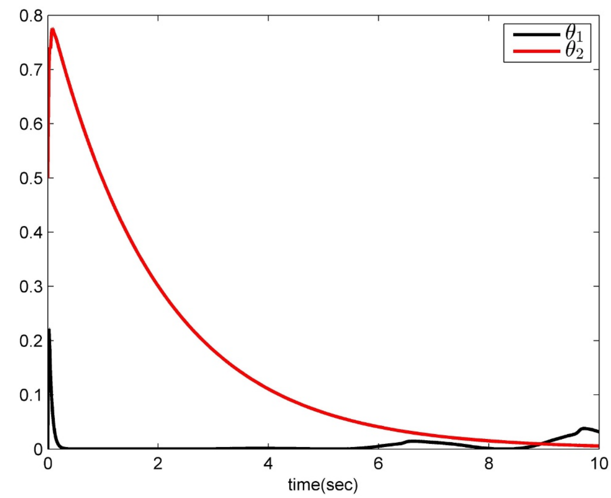

Fig. 10. Trajectory of adaptive parameters θ_1 and θ_2.

REFERENCES

[1] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *Proc. of American Control Conference*, pp. 649-654, June 1991.

[2] M. Chen, S. S. Ge, and B. V. E. How, "Robust adaptive neural network control for a class of uncertain mimo nonlinear systems with input nonlinearities," *IEEE Transactions on Neural Networks*, vol. 21, no. 5, pp. 796-812, March 2010.

[3] S. Sivrioglu, "Adaptive backstepping for switching control active magnetic bearing system with vibrating base," *IET Control Theory & Applications*, vol. 1, no. 4, pp. 1054-1059, July 2007.

[4] A. M. Zou, Z. G. Hou, and M. Tan, "Adaptive control of a class of nonlinear pure-feedback systems using fuzzy backstepping approach," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 4, pp. 886-897, August 2008.

[5] S. Tong and Y. Li, "Robust adaptive fuzzy backstepping output feedback tracking control for nonlinear system with dynamic uncertainties," *Science China Information Sciences*, vol. 53, no. 2, pp. 307–324, February 2010.

[6] F. Wang, B. Chen, Y. Sun, Y. Gao, and C. Lin, "Finite-time fuzzy control of stochastic nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 50, no. 6, pp. 2617-2626, June 2019.

[7] F. Wang and X. Zhang, "Adaptive finite time control of nonlinear systems under time-varying actuator failures," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 9, pp. 1845-1852, September 2019.

[8] L. B. Wu, and J. H. Park, "Adaptive fault-tolerant control of uncertain switched nonaffine nonlinear systems with actuator faults and time delays," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 9 pp. 3470-3480, 2020.

[9] L. B. Wu, J. H. Park, and N. N. Zhao, "Robust adaptive fault-tolerant tracking control for nonaffine stochastic nonlinear systems with full-state constraints," *IEEE Transactions on Cybernetics*, vol 50, no. 8, pp. 3793-3805, 2020.

[10] T. Sun, H. Pei, Y. Pan, H. Zhou, and C. Zhang, "Neural network-based sliding mode adaptive control for robot manipulators," *Neurocomputing*, vol. 74, no. 14-15, pp. 2377-2384, July 2011.

[11] T. Zhang, and S. S. Ge, "Adaptive neural control of mimo nonlinear state time-varying delay systems with unknown dead-zones and gain signs," *Automatica*, vol. 43, no. 6, pp. 1021-1033, June 2007.

[12] W. He, Y. Dong, and C. Sun, "Adaptive neural network control of unknown nonlinear affine systems with input deadzone and output constraint," *ISA Transactions*, vol. 58, pp. 96-104, September 2015.

[13] S. Zhou, M. Chen, C. J. Ong, and P. C. Chen, "Adaptive neural network control of uncertain mimo nonlinear systems with input saturation," *Neural Computing and Applications*, vol. 27, no. 5, pp. 1317-1325, June 2016.

[14] S. Tong and Y. Li, "Observer-based adaptive fuzzy backstepping control of uncertain nonlinear pure-feedback systems," *Science China Information Sciences*, vol. 57, pp. 1–14, Springer, May 2014.

[15] Q. Zhou, C. Wu, and P. Shi, "Observer-based adaptive fuzzy tracking control of nonlinear systems with time delay and input saturation," *Fuzzy Sets and Systems*, vol. 316, pp. 49-68, June 2017.

[16] J. Kim, G. Caire, and A. F. Molisch, "Quality-aware streaming and scheduling for device-to-device video delivery," *IEEE/ACM Transactions on Networking*, vol. 24, no. 4, pp. 2319-2331, August 2016.

[17] C. Wu, J. Liu, Y. Xiong, and L. Wu, "Observer-based adaptive fault-tolerant tracking control of nonlinear nonstrict-feedback systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 7, pp. 3022-3033, July 2017.

[18] S. Tong, X. Min, and Y. Li, "Observer-based adaptive fuzzy tracking control for strict-feedback nonlinear systems with unknown control gain functions," *IEEE Transactions on Cybernetics*, vol. 50, no. 9, pp. 3903-3913, 2020.

[19] B. Chen, X. Liu, and C. Lin, "Observer and adaptive fuzzy control design for nonlinear strict-feedback systems with unknown virtual control coefficients," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 3, pp. 1732-1743, June 2018.

[20] C. P. Bechlioulis and G. A. Rovithakis, "Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance," *IEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2090-2099, October 2008.

[21] A. Ilchmann, E. P. Ryan, and C. J. Sangwin, "Tracking with prescribed transient behaviour," *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 7, pp. 471-493, July 2002.

[22] W. Sun, S. F. Su, G. Dong, and W. Bai, "Reduced adaptive fuzzy tracking control for high-order stochastic nonstrict feedback nonlinear system with full-state constraints," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2019. DOI: 10.1109/TSMC.2019.2898204

[23] C. Wang, Y. Wu, and J. Yu, "Barrier Lyapunov functions-based adaptive control for nonlinear pure-feedback systems with time-varying full state constraints," *International Journal of Control, Automation and Systems*, vol. 15, no. 6, pp. 2714-2722, October 2017.

[24] J. Zhang, "Integral barrier Lyapunov functions-based neural control for strict-feedback nonlinear systems with multi-constraint," *International Journal of Control, Automation and Systems*, vol. 16, no. 4, pp. 2002-2010, July 2018.

[25] J. X. Xu and X. Jin, "State-constrained iterative learning control for a class of MIMO systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 5, pp. 1322-1327, May 2013.

[26] W. Sun, Y. Wu, and Z. Sun, "Command filter-based finite-time adaptive fuzzy control for uncertain nonlinear systems with prescribed performance," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 12, pp. 311-3170, 2020.

[27] J. X. Zhang and G. H. Yang, "Prescribed performance fault-tolerant control of uncertain nonlinear systems with unknown control directions," *IEEE Transactions on Automatic Control*, vol. 62, no. 12, pp. 6529-6535, December 2017.

[28] C. Liu, H. Wang, X. Liu, Y. Zhou, and S. Lu, "Adaptive prescribed performance tracking control for strict-feedback nonlinear systems with zero dynamics," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 18, pp. 6507-6521, October 2019.

[29] W. Sun, F. S. Su, J. Xia, and G. Zhuang, "Command filter-based adaptive prescribed performance tracking control for stochastic uncertain nonlinear systems," *IEEE Transactions on Fuzzy Systems*, 2020. DOI: 10.1109/TSMC.2019.2963220

[30] C. Liu, H. Wang, X. Liu, and Y. Zhou, "Adaptive finite-time fuzzy funnel control for nonaffine nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2019. DOI: 10.1109/TSMC.2019.2917547

[31] C. Liu, X. Liu, H. Wang, Y. Zhou, S. Lu, and B. Xu, "Event-triggered adaptive tracking control for uncertain nonlinear systems based on a new funnel function," *ISA Transactions*, vol. 99, pp. 130-138, April 2020.

[32] S. Tong and Y. Li, "Adaptive fuzzy output feedback control of mimo nonlinear systems with unknown dead-zone inputs," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 1, pp. 134-146, February 2012.

[33] W. Li, "Approaches to decision making with interval-valued intuitionistic fuzzy information and their application to enterprise financial performance assessment,"*Journal of Intelligent & Fuzzy Systems*, vol. 27, no. 1, pp. 1-8, February 2014.

[34] T. P. Zhang and S. S. Ge, "Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form," *Automatica*, vol. 44, no. 7, pp. 1895-1903, July 2008.

[35] Y. J. Liu and S. Tong, "Adaptive NN tracking control of uncertain nonlinear discrete-time systems with nonaffine dead-zone input," *IEEE Transactions on Cybernetics*, vol. 45, no. 3, pp. 497-505, March 2015.

[36] J. Na, "Adaptive prescribed performance control of nonlinear systems with unknown dead zone," *International Journal of Adaptive Control and Signal Processing*, vol. 27, no. 5, pp. 426-446, July 2012.

[37] T. Zhang and S. S. Ge, "Adaptive neural control of mimo nonlinear state time-varying delay systems with unknown dead-zones and gain signs," *Automatica*, vol. 43, no. 6, pp. 1021-1033, June 2007.

[38] L. X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 1, no. 2, pp. 146-155, May 1993.

[39] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, pp. 918–927, April, 2009.

Wen Zeng was born in Liaoning province, China in 1994, received his B.S. degree from University of Science and Technology Liaoning. Now he is a graduate student of control science and engineering in Liaoning University of Science and Technology, engaged in nonlinear system research.

Zhigang Li received his B.S. degree from Xi'an Jiaotong University, his M.S. degrees from Liaoning University of Technology. Now he is an associate professor and master's supervisor of University of Science and Technology Liaoning. His research interests include artificial intelligence and pattern recognition.

Chuang Gao received his B.S. degree in Electronic and Communication Engineering from Warwick University, Coventry, UK, the M.S. degree in Digital Signal Processing from King's College London, UK and a Ph.D. degree in Iron and Steel Metallurgy from University of Science and Technology Liaoning, Anshan, P. R. China, in 2005, 2007, and 2020, respectively. He has authored over 20 research papers indexed by SCI and EI. His research interests include nonlinear system control, machine learning and intelligent control.

Libing Wu received his B.S. and M.S. degrees from the Department of Mathematics, Jinzhou Normal College, Jinzhou, China, in 2004, and in Basic Mathematics from Northeastern University, Shenyang, China, in 2007, respectively, and a Ph.D. degree in Control Theory and Control Engineering from Northeastern University, Shenyang, China, in 2016. He is currently an Associate Professor at the School of Science, University of Science and Technology Liaoning, and also as a Postdoctoral Fellow at the Department of Electrical Engineering, Yeungnam University. His research interests include adaptive control, fault-tolerant control, nonlinear control and fault estimation.