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Reliable H_∞ Control on Stochastic Delayed Markovian Jump System with Asynchronous Jumped Actuator Failure

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Abstract: This paper studies the reliable H_∞ control on stochastic delayed Markovian jump systems (SDMJSs) with asynchronous jumped actuator failure and uncertain transition rates (TRs). It is assumed that the actuator failure occurs randomly under a Markov process with its jumping mode different from the system's one. A generalized functional Itô's formula for the closed-loop SDMJSs with mixed asynchronous Markovian jump modes (AMJMs) is successfully established. By the generalized functional Itô's formula and the mixed-mode-dependent Lyapunov functionals, a sufficient delay-dependent condition of the reliable H_∞ controller design for the SDMJSs with mixed AMJMs is proposed via matrix manipulation and a relaxation method. Finally, an example on VTOL (vertical take-off and landing) helicopter system is given to demonstrate the feasibility and effectiveness of the presented controller design scheme.

Keywords: Asynchronous jumped actuator failure, functional Itô's formula, Lyapunov functionals, reliable H_∞ control, stability analysis, stochastic delayed Markovian jump systems (SDMJSs).

1. INTRODUCTION

In the past two decades, the stochastic delayed systems (SDSs) have attracted great attention due to their wide range of significant applications in economics, finance, physics, biology, engineering and so on [1,2]. On the other hands, it is widely acknowledged that time delays in SDSs may cause instability or oscillation, which would be harmful to the applications of SDSs. Therefore, it is of great significance to study the stability and stabilization issues of SDSs, and a large number of results have been obtained on these issues, see, e.g. [1–3] and references therein. For the present study, Razumikhin method and Lyapunov functional are two primary techniques for stability analysis for SDSs [1–3]. However, different from its deterministic counterpart, the stochastic Razumikhin method has limited success [4]. By specific Lyapunov functionals, [2] and [5] examined stability and related issues of SDSs, but fundamental stability theory related to Lyapunov functionals for SDSs has not yet been fully developed. Furthermore, the solution processes to SDSs are no longer Markovian due to the existence of time delay which has defied bona fide operators as well as functional Itô's formulas in the past. Recently, Dupire extended Itô's formula to the functional circumstance via path-wise functional derivatives in [6], which was subsequently further developed to the case of càdlàg semi-martingales in [7]. This functional Itô's formula ensures us to achieve bona fide operators for SDSs and substantially ease the difficulties in using Lyapunov functionals to investigate stability and related issues for a much larger class of SDSs including SDSs with Markovian switching. Most recently, based on functional, Itô's formula and degenerate Lyapunov functional [4] established the moment and almost sure exponential stability criteria for SDSs, while [8] studied almost sure and L^p stability of SDSs with regime-switching by functional Itô's formula.

Meanwhile, since the Markov jump systems (MJSs) can be applied to describe numerous practical systems with structures subject to abrupt changes such as fluctuations in financial markets, sudden environmental changes and component failures [3, 9], researchers have paid considerable attention to analysis of the stability, stabilization, filtering and fault detection on stochastic delayed Markovian jump systems (SDMJSs) during the last two decades [3, 10–16]. On the other hand, for the dynamics of MJSs,

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it is generally difficult to obtain precise values of the transition rates (TRs) because they are determined by experimental tests or numerical simulations [17, 18]. Therefore, the uncertainty of TRs is an important issue in the literature. There are mainly three different types of uncertain TRs, i.e., element-wise uncertainty, polytopic uncertainty and general uncertainty. For the element-wise uncertainty, each element of transition rate matrix (TRM) with known error bound is assumed in practice [18–21]. For the polytopic uncertainty, the TRM is set to be in a convex hull subject to known vertices [22, 23]. For the general uncertainty, each TR is assumed to be (partially) unknown or only its estimate to be known [24–29]. It's noted that the TRs can be measured in practice, thus their estimate values and estimate error bounds can be given [18]. Therefore, in this paper, we mainly concern the element-wise uncertainty in the TRs of SDMJSs.

As is well-known, for the robust control of MJSs subject to uncertain TRs, how to appropriately bind the uncertain terms is a significant issue [18, 21]. By considering the relationships among the TRs, [20] provided a controller design scheme for the MJSs subject to uncertain TRs in terms of coupling LMIs. In [21], a tailored technique was proposed to bind the uncertain terms for reducing conservatism of the criteria on robust stabilization of MJSs with uncertain TRs. By the cone complementary linearization method, the finite-time stability and stabilization for stochastic MJSs subject to generally uncertain TRs were addressed in [28]. In [27], the stability and stabilization for singular semi-MJSs with generally uncertain TRs was studied. Recently, by applying the variable elimination technique, [29] derived the necessary and sufficient conditions of dynamic output-feedback stabilization for singular MJSs with partly unknown TRs.

Moreover, it's worth mentioning that the actuator failure can degrade performance of the closed-loop system since it is a major source of system instability. Thus the analysis and synthesis of control for MJSs subject to actuator failure have received considerable attention over recent decades [18, 30–33]. In practical applications, the actuator failure can usually be caused by some specific stochastic events [18, 33, 34], such as the computer-controlling actuator on a robotic mission which may be randomly hit by cosmic ray [18], the stochastic fault of multi-thrusters on the chaser spacecraft [34], the attack-angle transducer on the aircraft, and the digital flight control system [33]. The evolution of these jumped actuator failures can be generally modeled as Markovian chains [18, 33, 35]. Therefore, for the MJSs in the presence of asynchronous stochastic jumped actuator fault, it is rational to assume that the actuator failure is governed by a Markovian process with a different jumping mode from the system jumping mode [18]. Then, the closed-loop system can be represented as a mixed-mode MJSs subject to asynchronous jumped failure. In [36], the H_∞ estimation for discrete-time MJSs subject to time-varying transition probabilities has been investigated. [37] studied the reliable control for delayed systems with actuator saturation and asynchronous stochastic failure whose fault factor obeys certain probabilistic distribution. For the discrete-time stochastic MJS, [38] designed an asynchronous l_2-l_∞ filter such that the filtering error system is in the form of a stochastic mixed-mode MJS. For continuous-time linear semi-Markov jumping systems, the asynchronous H_∞ control is addressed in [39], where the asynchronous controller is governed by a stochastic process relating to the system's mode via conditional probability. More recently, [18] investigated the reliable H_∞ control of linear MJSs with uncertain TRs, actuator saturation and asynchronous jumping actuator failure.

However, to the best of our knowledge, owing to the complexity and mathematical difficulty, it lacks corresponding results in the existing literature on the reliable H_∞ control of SDMJSs subject to uncertain TRs and asynchronous jumping actuator failure that is governed by a Markovian process with a different jumping mode from the system's one. This motivates our present work and the aim is to close this gap via proposing a design scheme of asynchronous reliable H_∞ control on SDMJSs with uncertain TRs by applying functional Itô's formula. Compared with the previous results, the main contributions of this article are as follows:

(i) By representing the mixed Markovian processes as two Poisson integrals, a generalized functional Itô's formula for the closed-loop SDMJSs with mixed AMJMs is established.

(ii) Based on the generalized functional Itô's formula and mixed-mode-dependent Lyapunov functionals, the reliable H_∞ control problem on SDMJSs subject to uncertain TRs and asynchronous jumping actuator failure is addressed. Then, the mixed-mode-dependent controller gains can be constructed with minimizing H_∞ performance as an objective under LMIs constraints.

(iii) A relaxation method is presented to appropriately bind the uncertain terms related to TRs for reducing conservatism.

The rest of this article is organized as follows. Section 2 states necessary preliminaries and problems while Section 3 presents the main results. A numerical example and a concluding remark are given in Sections 4 and 5, respectively.

Notations: Unless otherwise stated, the following notations are adopted throughout this paper. Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\{e_i\}_{i=1}^n$ be the canonical basis in $\mathbb{R}^n$. $|\cdot|$ represents the Euclidean norm. For a matrix A, let a_{ij} be its (i, j)-th component and denote $A = [a_{ij}]$. Let the superscript "T" denote matrix transposition, and "*" denote the symmetry element of a matrix. If A is a square matrix, $\mathrm{He}\{A\} := A + A^{\mathrm{T}}$ and $\mathrm{tr}(A)$ denotes its trace. For square matrices A and B, $A > B$ ($A \geq B$) denotes that $A - B$ is positive definite (semidefinite). Let $L_2[0,\infty)$ be the space of square-integrable vector functions on $[0,\infty)$. Denote by $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ a complete probability space with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous with $\mathcal{F}_0$ containing all $\mathcal{P}$-null sets). For $\tau > 0$, let $C([-\tau,0];\mathbb{R}^n)$ denote the family of all continuous functions φ from $[-\tau,0]$ to $\mathbb{R}^n$ equipped with the norm $\|\varphi\| = \sup_{-\tau\leq s\leq 0}|\varphi(s)|$. Denote by $D([-\tau,0];\mathbb{R}^n)$ the family of all càdlàg (i.e., right continuous with left limits) functions φ from $[-\tau,0]$ to $\mathbb{R}^n$ endowed with the Skorokhod topology. $C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ stands for the family of all bounded $\mathcal{F}_0$-measurable, $C([-\tau,0];\mathbb{R}^n)$-valued random variables. For an $\mathbb{R}^n$-valued càdlàg stochastic process $x(t)$ on $[-\tau,\infty)$, let $x_t = \{x(t+\theta) : -\tau \leq \theta \leq 0\}$ for $t \geq 0$, then x_t represents a $D([-\tau,0];\mathbb{R}^n)$-valued stochastic process. $E(x)$ stands for the mathematical expectation of a random variable x. Let $\mathbf{1}_A(x)$ be the indicator function of a subset A of its domain.

2. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following SDMJSs:

$$\begin{aligned} dx(t) = {} & [A_{\sigma(t)}x(t) + A^d_{\sigma(t)}x(t-\tau) + B_{\sigma(t)}u(t) \\ & + C_{\sigma(t)}\omega(t)]dt \\ & + [D_{\sigma(t)}x(t) + D^d_{\sigma(t)}x(t-\tau) \\ & + E_{\sigma(t)}\omega(t)]dB(t), \qquad (1a) \\ z(t) = {} & G_{\sigma(t)}x(t) + G^d_{\sigma(t)}x(t-\tau) + F_{\sigma(t)}u(t) \\ & + H_{\sigma(t)}\omega(t), \qquad (1b) \end{aligned}$$

for $t \in \mathbb{R}^+$ with initial condition

$$x_0 = \phi \in C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n), \ \ \sigma(0) = \sigma_0, \qquad (2)$$

where $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^m$, $u(t) \in \mathbb{R}^p$ are respectively the state, control output and control input; $\tau \geq 0$ is the time delay and $\omega(t) \in \mathbb{R}^q$ is the disturbance input belonging to $L_2[0,\infty)$; $B(t)$ is a scalar Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$; $\{\sigma(t)\}_{t\geq 0}$ is a right-continuous Markovian chain on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with a finite state space $\mathcal{S} = \{1,\cdots,s\}$ and transition rate matrix (TRM) $[\lambda_{ij}]$ $(i,j \in \mathcal{S})$. The evolution of the Markovian process $\{\sigma(t)\}_{t\geq 0}$ is governed by the following transition probabilities:

$$P\{\sigma(t+\delta) = j \mid \sigma(t) = i\} = \begin{cases} \lambda_{ij}\delta + o(\delta), & j \neq i, \\ 1 + \lambda_{ii}\delta + o(\delta), & j = i, \end{cases} \qquad (3)$$

where $\lim_{\delta\to 0}(o(\delta)/\delta) = 0$; $\lambda_{ij} \geq 0$ is the transition rate (TR) from mode i at time t to mode j at time $t+\Delta$ for $j \neq i$, and $\lambda_{ii} = -\sum_{j\neq i}\lambda_{ij}$; The system matrices $A_{\sigma(t)}, A^d_{\sigma(t)}, \cdots, H_{\sigma(t)}$ are known with appropriate dimensions, which will be denoted by $A_i, A^d_i, \cdots, H_i$, respectively, for each $\sigma(t) = i \in \mathcal{S}$.

Assumption 1: It is assumed that the unexpected actuator failures with known mode-dependent actuator gains occur randomly and independently under a Markovian jumping mode that is different from the system's one.

The asynchronous jumped actuator failure model is given as follows:

$$u^F(t) = \Xi_{s(t)}u(t), \qquad (4)$$

where $\Xi_{s(t)} = \mathrm{diag}\left\{\varsigma^1_{s(t)}, \varsigma^2_{s(t)}, \cdots, \varsigma^p_{s(t)}\right\}$ with $\varsigma^1_{s(t)}, \cdots, \varsigma^p_{s(t)}$ being the failure scale factors of each channel. $\{s(t)\}_{t\geq 0}$ is another right-continuous Markovian chain on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with a finite state space $\mathcal{S}_f = \{1,\cdots,r\}$ and TRM $[\pi^{\sigma(t)}_{kl}]$ $(k,l \in \mathcal{S}_f, \sigma(t) \in \mathcal{S})$.

The evolution of the asynchronous Markovian process $\{s(t)\}_{t\geq 0}$ is governed by the following transition probabilities [18]:

$$P\{s(t+\delta) = l \mid s(t) = k\} = \begin{cases} \pi^{\sigma(t)}_{kl}\delta + o(\delta), & l \neq k, \\ 1 + \pi^{\sigma(t)}_{kk}\delta + o(\delta), & l = k, \end{cases} \qquad (5)$$

where the probability $P\{s(t+\delta) = l \mid s(t) = k\}$ is assumed to depend on the current system mode $\sigma(t)$ [18]. $\pi^{\sigma(t)}_{kl} \geq 0$ is the TR from mode k at time t to mode l at time $t+\Delta$ for $k \neq l$, and $\pi^{\sigma(t)}_{kk} = -\sum_{k\neq l}\pi^{\sigma(t)}_{kl}$.

Assumption 2: Suppose that $B(t)$ is independent of $\sigma(t)$ and $s(t)$. It is also assumed that $\{\sigma(t)\}_{t\geq 0}$ is independent on the σ-algebra given by $\sigma(s(\theta), 0 \leq \theta < t)$.

Assumption 3: Assume that the values of elements in the TRMs $[\lambda_{ij}]$ and $[\pi^i_{kl}]$ are uncertain, and only their admissible uncertainty domains are known [18, 21]:

$$\begin{aligned} \mathbb{D}_\Lambda &= \left\{\Lambda = \hat{\Lambda} + \Delta\Lambda : |\Delta\lambda_{ij}| \leq \delta_{ij} \leq |\hat{\lambda}_{ij}|, \delta_{ij} \geq 0\right\}, \\ \mathbb{D}^i_\Pi &= \{\Pi^i = \hat{\Pi}^i + \Delta\Pi^i : |\Delta\pi^i_{kl}| \leq \theta^i_{kl} < |\hat{\pi}^i_{kl}|, \theta^i_{kl} \geq 0\}, \end{aligned} \qquad (6)$$

where i, $j \in \mathcal{S}$, k, $l \in \mathcal{S}_f$ and

$$\begin{aligned} \hat{\lambda}_{ii} &= -\sum_{j\neq i}\hat{\lambda}_{ij}, \Delta\lambda_{ii} = -\sum_{j\neq i}\Delta\lambda_{ij}, \\ \hat{\pi}^i_{kk} &= -\sum_{l\neq k}\hat{\pi}^i_{kl}, \Delta\pi^i_{kk} = -\sum_{l\neq k}\Delta\pi^i_{kl}, \end{aligned} \qquad (7)$$

in which $\hat{\lambda}_{ij}$ ($\hat{\pi}^i_{kl}$) and $\Delta\lambda_{ij}$ ($\Delta\pi^i_{kl}$) denote the estimate value and the estimate error value of the TR λ_{ij} (π^i_{kl}), respectively. δ_{ij} (θ^i_{kl}) is the upper bound of $|\Delta\lambda_{ij}|$ ($|\Delta\pi^i_{kl}|$). Moreover, the values of $\hat{\lambda}_{ij}, \delta_{ij}, \hat{\pi}^i_{kl}$ and θ^i_{kl} are known a priori.

Remark 1: For the ιth ($\iota = 1,2,\ldots,p$) channel, the random variable $\varsigma^\iota_{s(t)}$ in (4) represents its failure scale factor, which indicates that the actuator fault is governed by the asynchronous Markov process $\{s(t)\}_{t\geq 0}$. Specifically, for each failure mode $s(t)=k\in\mathcal{S}_f$, when $\varsigma_k^\iota=1$ ($\varsigma_k^\iota=0$), the ιth channel is normal (outage); when $0<\varsigma_k^\iota<1$, it corresponds to the case of partial degradation of the ιth channel. Moreover, when $\mathcal{S}_f=\{1\}$, the asynchronous jumping actuator fault (4) becomes the classic type investigated in [20,21,40–43], i.e., the fault model in (4) is more general.

We adopt the following mixed mode-dependent state feedback controller for SDMJSs (1) subject to asynchronous jumping actuator failure (4)

$$u(t)=K(\sigma(t),s(t))x(t), \tag{8}$$

where $K(\sigma(t),s(t))\in\mathbb{R}^{p\times n}$ are controller gains to be designed.

By the controller (8), for each $\sigma(t)=i\in\mathcal{S}, s(t)=k\in\mathcal{S}_f$, the closed-loop system of (1) can be written as

$$dx(t)=f(x_t,t,i,k)dt+g(x_t,t,i,k)dB(t), \tag{9a}$$
$$z(t)=(G_i+F_i\Xi_kK_{ik})x(t)+G_i^dx(t-\tau)+H_i\omega(t), \tag{9b}$$

where $f(x_t,t,i,k)=(A_i+B_i\Xi_kK_{ik})x(t)+A_i^dx(t-\tau)+C_i\omega(t)$ and $g(x_t,t,i,k)=D_ix(t)+D_i^dx(t-\tau)+E_i\omega(t)$.

Remark 2: The mixed mode-dependent state feedback controller (8) that depends not only on the system's jumping mode but also on the actuator failure's asynchronous jumping mode is more general and applicable in the practice. To overcome the design challenges of the mixed mode-dependent controller (8), the mixed mode-dependent Lyapunov functionals and the generalized functional Itô's formula developed in Lemma 1 will be appropriately adopted.

Definition 1 [44, 45]**:** The nominal system (1) is said to be stochastically stabilizable if, when $\omega(t)\equiv 0$, there exists a control law (8), such that

$$\lim_{T\to\infty}E\left\{\int_0^T|x(t)|^2dt|\ \phi,\sigma_0\right\}<\infty, \tag{10}$$

for all initial mode $\sigma_0\in\mathcal{S}$ and finite initial condition $\phi(\theta)\in\mathbb{R}^n$ defined on $[-\tau,0]$.

Definition 2 [18, 40]**:** Given the control law (8) and a scalar $\gamma>0$, the nominal system (1) is said to be stochastically stabilizable with *γ-disturbance attenuation*, if (10) holds and the control output $z(t)$ satisfies

$$E\left\{\int_0^T z^{\mathrm{T}}(t)z(t)dt\right\}<\gamma^2E\left\{\int_0^T\omega^{\mathrm{T}}(t)\omega(t)dt\right\}, \tag{11}$$

for zero initial condition and all admissible disturbance input $\omega(t)\in L_2[0,\infty)$ and $T>0$.

As is known that the continuous-time Markovian process $\{\sigma(t)\}_{t\geq 0}$ ($\{s(t)\}_{t\geq 0}$) with its TRM $\Lambda=[\lambda_{ij}]$ ($\Pi^i=[\pi^i_{kl}], i\in\mathcal{S}$) can be rewritten as a stochastic integral with respect to (w.r.t.) corresponding Poisson random measure [3]. In fact, for $i\neq j\in\mathcal{S}$ ($k\neq l\in\mathcal{S}_f$), denote by Δ_{ij} (Δ^i_{kl}) the consecutive (w.r.t. the lexicographic ordering on $\mathcal{S}\times\mathcal{S}$ ($\mathcal{S}_f\times\mathcal{S}_f$)) left-closed right-open intervals of the real line with length λ_{ij} (π^i_{kl}). Let $\rho:\mathcal{S}\times\mathbb{R}\to\mathbb{R}$ and $\zeta:\mathcal{S}_f\times\mathbb{R}\to\mathbb{R}$ be $\rho(i,z)=\begin{cases} j-i, & \text{if } z\in\Delta_{ij}\\ 0, & \text{otherwise},\end{cases}$ and $\zeta(k,v)=\begin{cases} l-k, & \text{if } v\in\Delta^i_{kl}\\ 0, & \text{otherwise},\end{cases}$ respectively, where $i,j\in\mathcal{S}$, k, $l\in\mathcal{S}_f$. Then, $\{\sigma(t)\}_{t\geq 0}$ and $\{s(t)\}_{t\geq 0}$ can be respectively represented as the following two *Poisson integrals*:

$$\sigma(t)=\sigma_0+\int_0^t\int_{\mathbb{R}}\rho(\sigma(\theta^-),z)\nu(d\theta,dz), \tag{12}$$
$$s(t)=s_0+\int_0^t\int_{\mathbb{R}}\zeta(s(\theta^-),v)\mu(d\theta,dv), \tag{13}$$

where $\nu(dt,dz)$ ($\mu(dt,dv)$) is a Poisson random measure with intensity $dzdt$ ($dvdt$). Let $\tilde{\nu}(dt,dz)=\nu(dt,dz)-dzdt$ ($\tilde{\mu}(dt,dv)=\mu(dt,dv)-dvdt$), then $\tilde{\nu}(dt,dz)$ ($\tilde{\mu}(dt,dv)$) is a martingale measure [46, 47].

Next, we will introduce the functional Itô's formula for the solution of (9). For small enough $\delta\geq 0$, $\eta\in\mathbb{R}^n$ and each $\varphi\in D([-\tau,0];\mathbb{R}^n)$, the *horizontal extension* φ_δ and *vertical perturbation* φ^η of φ are defined as [6, 7, 48]:

$$\begin{cases}\varphi_\delta(\theta)=\varphi(\theta+\delta)\mathbf{1}_{[-\tau,-\delta]}(\theta)+\varphi(0)\mathbf{1}_{(-\delta,0]}(\theta),\\ \quad\forall\theta\in[-\tau,0],\\ \varphi^\eta(\theta)=\varphi(\theta)+\eta\mathbf{1}_{\{0\}}(\theta),\ \ \forall\,\theta\in[-\tau,0].\end{cases}$$

Let $C^{2,1}(D([-\tau,0];\mathbb{R}^n)\times\mathbb{R}^+\times\mathcal{S}\times\mathcal{S}_f;\mathbb{R}^+)$ be the family of nonnegative real-valued functionals which are continuously twice differentiable w.r.t. x and once differentiable w.r.t. t. For a given functional $V(\varphi,t,i,k)\in C^{2,1}(D([-\tau,0];\mathbb{R}^n)\times\mathbb{R}^+\times\mathcal{S}\times\mathcal{S}_f;\mathbb{R}^+)$, we define two kinds of path-wise derivatives for the non-anticipative functional V [7], i.e., the *horizontal derivative*, which is a derivative w.r.t. time, and the *vertical derivative*, which is a derivative w.r.t. the current value of underlying path x [6, 7]:

$$\mathcal{D}_tV(\varphi,t,i,k):=\lim_{\delta\to 0^+}\frac{V(\varphi_\delta,t+\delta,i,k)-V(\varphi,t,i,k)}{\delta},$$
$$\nabla_xV(\varphi,t,i,k):=(\partial_1V(\varphi,t,i,k),\cdots,\partial_nV(\varphi,t,i,k)),$$
$$\partial_{\boldsymbol{p}}V(\varphi,t,i,k):=\lim_{h\to 0}\frac{V(\varphi^{he_{\boldsymbol{p}}},t,i,k)-V(\varphi,t,i,k)}{h},$$
$$\nabla_{xx}V(\varphi,t,i,k):=(\partial_{\boldsymbol{pq}}V(\varphi,t,i,k))_{n\times n},$$
$$\partial_{\boldsymbol{pq}}V(\varphi,t,i,k):=\lim_{h\to 0}\frac{\partial_{\boldsymbol{q}}V(\varphi^{he_{\boldsymbol{p}}},t,i,k)-\partial_{\boldsymbol{q}}V(\varphi,t,i,k)}{h},$$

where $\boldsymbol{p},\boldsymbol{q}=1,\cdots,n$.

In what follows, we first establish a generalized functional Itô's formula (Lemma 1) for the closed-loop system

(9) by applying the well-known functional Itô's formula for càdlàg semi-martingale, and its proof will be given in the Appendix for the sake of readability.

Lemma 1: Let $(x(t),\sigma(t),s(t))$ be the jointly Markovian process defined by (9), (12) and (13), and $V\in C^{2,1}(D([-\tau,0];\mathbb{R}^n)\times\mathbb{R}^+\times\mathcal{S}\times\mathcal{S}_f;\mathbb{R}^+)$. Then, for $t>0$, with probability 1, we have

$$V(x_t,t,\sigma(t),s(t)) = V(x_0,0,\sigma_0,s_0)+\mathcal{M}(t) + \int_0^t \mathcal{L}V(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta, \tag{14}$$

where

$$\begin{aligned}&\mathcal{L}V(x_t,t,i,k)\\ &= \mathcal{D}_tV(x_t,t,i,k)+\nabla_xV(x_t,t,i,k)f(x_t,t,i,k)\\ &\quad+\frac{1}{2}\mathrm{tr}\{g^{\mathrm{T}}(x_t,t,i,k)\nabla_{xx}V(x_t,t,i,k)g(x_t,t,i,k)\}\\ &\quad+\sum_{j=1}^{s}\lambda_{ij}V(x_t,t,j,k)+\sum_{l=1}^{r}\pi_{kl}^{i}V(x_t,t,i,l),\\ &\quad\forall\, i\in\mathcal{S},k\in\mathcal{S}_f,\end{aligned}$$

and $\mathcal{M}(t)$ is a martingale given by

$$\begin{aligned}&\mathcal{M}(t)\\ &=\int_0^t\nabla_xV(x_\theta,\theta,\sigma(\theta),s(\theta))g(x_\theta,\theta,\sigma(\theta),s(\theta))dB(\theta)\\ &\quad+\int_0^t\int_{\mathbb{R}}[V(x_{\theta^-},\theta,\sigma(\theta^-)+\rho(\sigma(\theta^-),z),s(\theta^-))\\ &\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]\tilde{\nu}(d\theta,dz)\\ &\quad+\int_0^t\int_{\mathbb{R}}[V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-)+\zeta(s(\theta^-),v))\\ &\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]\tilde{\mu}(d\theta,dv).\end{aligned}\tag{15}$$

3. MAIN RESULTS

In this section, based on the generalized functional Itô's formula (Lemma 1) and mixed-mode-dependent Lyapunov functionals, our major purpose is to design the controller in the form of (8) for SDMJSs (1) with uncertain TRs and asynchronous jumped fault (4), such that the closed-loop SDMJSs (9) is stochastically stable with minimum H_∞ performance.

Theorem 1: For the closed-loop SDMJSs (9) subject to asynchronous jumped actuator faults (4), given the mixed-mode-dependent control gains K_{ik}, TRMs $[\lambda_{ij}]$ and $[\pi_{kl}^i]$, if there exist a scalar $\gamma>0$, symmetric matrices $P_{ik}>0$, $Q>0$ and $Z>0$ with appropriate dimensions, such that for any $(i,k)\in\mathcal{S}\times\mathcal{S}_f$,

$$0>\Phi_{ik} = \begin{bmatrix}\Phi_{ik}^{11} & P_{ik}A_i^d & P_{ik}C_i & Z & D_i^{\mathrm{T}}P_{ik} & \tau D_i^{\mathrm{T}}Z & \Phi_{ik}^{17}\\ * & \Phi_{ik}^{22} & 0 & 0 & (D_i^d)^{\mathrm{T}}P_{ik} & \tau(D_i^d)^{\mathrm{T}}Z & (G_i^d)^{\mathrm{T}}\\ * & * & -\gamma^2I & 0 & E_i^{\mathrm{T}}P_{ik} & \tau E_i^{\mathrm{T}}Z & H_i^{\mathrm{T}}\\ * & * & * & -Z & 0 & 0 & 0\\ * & * & * & * & -P_{ik} & 0 & 0\\ * & * & * & * & * & -\tau Z & 0\\ * & * & * & * & * & * & -I\end{bmatrix}, \tag{16}$$

where $\Phi_{ik}^{11}=\mathrm{He}\{P_{ik}(A_i+B_i\Xi_kK_{ik})\}+Q-Z+\sum_{j=1}^{s}\lambda_{ij}P_{jk}+\sum_{l=1}^{r}\pi_{kl}^iP_{il}$, $\Phi_{ik}^{17}=(G_i+F_i\Xi_kK_{ik})^{\mathrm{T}}$ and $\Phi_{ik}^{22}=-Q+Z$, then the closed-loop system (9) is stochastically stable with H_∞ performance γ.

Proof: Construct the following mixed mode-dependent Lyapunov functionals for the closed-loop system (9):

$$\begin{aligned}&V(x_t,t,\sigma(t),s(t))\\ &=x^{\mathrm{T}}(t)P(\sigma(t),s(t))x(t)+\int_{t-\tau}^{t}x^{\mathrm{T}}(\theta)Qx(\theta)d\theta\\ &\quad+\int_{-\tau}^{0}\int_{t+v}^{t}g^{\mathrm{T}}(x_\theta,\theta,\sigma(\theta),s(\theta))Zg(x_\theta,\theta,\sigma(\theta),\\ &\quad s(\theta))d\theta dv.\end{aligned}\tag{17}$$

By the generalized functional Itô's formula (Lemma 1), along the trajectory of the system (9) with $\omega(t)=0$, for all $(i,k)\in\mathcal{S}\times\mathcal{S}_f$, we have

$$\begin{aligned}\mathcal{L}V(x_t,t,i,k) =& 2x^{\mathrm{T}}(t)P_{ik}[(A_i+B_i\Xi_kK_{ik})x(t)\\ &+A_i^dx(t-\tau)]+g^{\mathrm{T}}(x_t,t,i,k)P_{ik}g(x_t,t,i,k)\\ &+x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau)\\ &+\sum_{j=1}^{s}\lambda_{ij}x^{\mathrm{T}}(t)P_{jk}x(t)+\sum_{l=1}^{r}\pi_{kl}^ix^{\mathrm{T}}(t)P_{il}x(t)\\ &+\tau g^{\mathrm{T}}(x_t,t,i,k)Zg(x_t,t,i,k)\\ &-\int_{t-\tau}^{t}g^{\mathrm{T}}(x_\theta,\theta,\sigma(\theta),s(\theta))\\ &\times Zg(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta.\end{aligned}\tag{18}$$

Integrating (9a) on both sides from $t-\tau$ to t, we have

$$\begin{aligned}&x(t)-x(t-\tau)\\ &=\int_{t-\tau}^{t}f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta\\ &\quad+\int_{t-\tau}^{t}g(x_\theta,\theta,\sigma(\theta),s(\theta))dB(\theta),\ \ t\geq\tau.\end{aligned}\tag{19}$$

By Itô isometry and (19), it follows that

$$\begin{aligned}&E(\int_{t-\tau}^{t}g^{\mathrm{T}}(x_\theta,\theta,\sigma(\theta),s(\theta))Zg(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)\\ &=E((\int_{t-\tau}^{t}g(x_\theta,\theta,\sigma(\theta),s(\theta))dB(\theta))^{\mathrm{T}}Z\\ &\quad\times(\int_{t-\tau}^{t}g(x_\theta,\theta,\sigma(\theta),s(\theta))dB(\theta)))\end{aligned}$$

$$
\begin{aligned}
&= E((x(t)-x(t-\tau)-\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)^{\mathrm{T}} \\
&\quad \times Z(x(t)-x(t-\tau)-\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)) \\
&= E((x(t)-\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)^{\mathrm{T}} Z(x(t) \\
&\quad -\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)-2x(t-\tau)^{\mathrm{T}} Z(x(t) \\
&\quad -\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)+x(t-\tau)^{\mathrm{T}} Zx(t-\tau)).
\end{aligned} \tag{20}
$$

By using Lemma 1 in [49] and (19), we obtain

$$
\begin{aligned}
&E(-2x(t-\tau)^T Z(x(t)-\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)) \\
&= E(-2x(t-\tau)^T Z(x(t-\tau) \\
&\quad +\int_{t-\tau}^{t} g(x_\theta,\theta,\sigma(\theta),s(\theta))dB(\theta))) \\
&= E(-2x(t-\tau)^T Zx(t-\tau)).
\end{aligned} \tag{21}
$$

Substituting (21) into (20) gives

$$
\begin{aligned}
&E(\int_{t-\tau}^{t} g^{\mathrm{T}}(x_\theta,\theta,\sigma(\theta),s(\theta)) \\
&\quad \times Zg(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta) \\
&= E((x(t)-\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta)^{\mathrm{T}} \\
&\quad \times Z(x(t)-\int_{t-\tau}^{t} f(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta) \\
&\quad -x^{\mathrm{T}}(t-\tau)Zx(t-\tau)).
\end{aligned} \tag{22}
$$

By (18) and (22), we get

$$
E\mathcal{L}V(x_t,t,i,k) \le EX^{\mathrm{T}}(t)\check{\Phi}_{ik}X(t), \tag{23}
$$

where $X(t)=\left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau), \int_{t-\tau}^{t} f^{\mathrm{T}}(x_\theta,\theta,\sigma(\theta),s(\theta))\right.$ $\left.d\theta\right]^{\mathrm{T}}$ and $\check{\Phi}_{ik}=\begin{bmatrix}\Phi_{ik}^{11} & P_{ik}A_i^d & Z\\ * & \Phi_{ik}^{22} & 0\\ * & * & -Z\end{bmatrix}+\begin{bmatrix}D_i^{\mathrm{T}}\\ (D_i^d)^{\mathrm{T}}\\ 0\end{bmatrix}(P_{ik}$ $+\tau Z)\times\begin{bmatrix}D_i^{\mathrm{T}}\\ (D_i^d)^{\mathrm{T}}\\ 0\end{bmatrix}^{\mathrm{T}}$. On the other hand, it follows from (16) that

$$
\begin{bmatrix}
\Phi_{ik}^{11} & P_{ik}A_i^d & Z & D_i^{\mathrm{T}}P_{ik} & \tau D_i^{\mathrm{T}}Z\\
* & \Phi_{ik}^{22} & 0 & (D_i^d)^{\mathrm{T}}P_{ik} & \tau(D_i^d)^{\mathrm{T}}Z\\
* & * & -Z & 0 & 0\\
* & * & * & -P_{ik} & 0\\
* & * & * & * & -\tau Z
\end{bmatrix} < 0. \tag{24}
$$

Applying Schur complement, (24) implies that $\check{\Phi}_{ik}<0$, thus there exists a constant $\beta>0$ such that $\check{\Phi}_{ik}<-\beta I$. This toghther with Dynkin's formula, Fubini's theorem and (23) gives that

$$
\begin{aligned}
&E\{V(x_T,T,\sigma(T),s(T))\}-E\{V(\phi,0,\sigma_0,s_0)\} \\
&= E\left\{\int_0^T \mathcal{L}V(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta\right\} \\
&= \int_0^T E\mathcal{L}V(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta \\
&\le -\beta E\left\{\int_0^T |x(\theta)|^2 d\theta|\ \phi,\sigma_0\right\},
\end{aligned} \tag{25}
$$

which implies

$$
\begin{aligned}
&\lim_{T\to\infty} E\left\{\int_0^T |x(\theta)|^2 d\theta|\ \phi,\sigma_0\right\} \\
&\le \frac{1}{\beta}E\{V(\phi,0,\sigma_0,s_0)\}<\infty.
\end{aligned} \tag{26}
$$

Thus, it follows from (26) and Definition 1 that the closed-loop system (9) with $\omega(t)=0$ is stochastically stable.

Moreover, for the system (9) with nonzero $\omega(t)\in L_2[0,\infty]$, define $J_T\triangleq E\left\{\int_0^T\left[z^{\mathrm{T}}(t)z(t)-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\right]dt\right\}$. Under zero initial condition, it follows from Dynkin's formula that $EV(x(T),T,\sigma(T),s(T))=E\Big\{\int_0^T \mathcal{L}V(x_t,t,\sigma(t),$ $s(t))dt\Big\}$. Then, for any nonzero $\omega(t)\in L_2[0,\infty]$, by Fubini's theorem, we have

$$
\begin{aligned}
J_T =& E\int_0^T [z^{\mathrm{T}}(t)z(t)-\gamma^2\omega^{\mathrm{T}}(t)\omega(t) \\
&+\mathcal{L}V(x_t,t,\sigma(t),s(t))]dt-EV(x(T),T,\sigma(T),s(T)) \\
\le& E\left\{\int_0^T \bar{X}^{\mathrm{T}}(t)\bar{\Phi}_{ik}\bar{X}(t)dt\right\},
\end{aligned}
$$

where

$$
\bar{X}(t)\triangleq\left[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\tau),\omega^{\mathrm{T}}(t),\int_{t-\tau}^{t} f^{\mathrm{T}}(x_\theta,\theta,\sigma(\theta),\right.
\left. s(\theta))d\theta\right]^{\mathrm{T}},
$$

and

$$
\begin{aligned}
\bar{\Phi}_{ik} =& \begin{bmatrix}
\Phi_{ik}^{11} & P_{ik}A_i^d & P_{ik}C_i & Z\\
* & -Q+Z & 0 & 0\\
* & * & -\gamma^2 I & 0\\
* & * & * & -Z
\end{bmatrix} \\
&+\begin{bmatrix}D_i^{\mathrm{T}}\\ (D_i^d)^{\mathrm{T}}\\ E_i^{\mathrm{T}}\\ 0\end{bmatrix}(P_{ik}+\tau Z)\begin{bmatrix}D_i^{\mathrm{T}}\\ (D_i^d)^{\mathrm{T}}\\ E_i^{\mathrm{T}}\\ 0\end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix}\Phi_{ik}^{17}\\ (G_i^d)^{\mathrm{T}}\\ H_i^{\mathrm{T}}\\ 0\end{bmatrix}\begin{bmatrix}\Phi_{ik}^{17}\\ (G_i^d)^{\mathrm{T}}\\ H_i^{\mathrm{T}}\\ 0\end{bmatrix}^{\mathrm{T}}.
\end{aligned}
$$

By Schur complement, (16) implies that $\bar{\Phi}_{ik}<0$. Therefore, for all $T>0$, $\omega(t)\in L_2[0,\infty)$, this gives that $J_T<0$. According to Definition 1 and Definition 2, the system (9) is stochastically stable with H_∞ performance γ. This completes the proof. □

Remark 3: In Theorem 1, the matrix inequalities (16) are difficult to be solved due to the existence of uncertainty in TRs. Therefore, the following theorem will develop a relaxation method to obtain a finite number of solvable LMIs from (16).

Theorem 2: For the closed-loop SDMJSs (9) with uncertain TRs and asynchronous jumped actuator faults (4), given the mixed-mode-dependent control gains K_{ik}, the estimated TRs $\hat{\lambda}_{ij}$, $\hat{\pi}^i_{kl}$ and their estimate errors $\delta_{ij} \geq 0$, $\theta^i_{kl} \geq 0$, if there exist a scalar $\gamma > 0$, symmetric matrices $P_{ik} > 0$, $Q > 0$, $Z > 0$ and matrices $V_{ijk} \geq 0, \tilde{V}_{ikl} \geq 0, T_{ik}, \tilde{T}_{ik}$ $(i, j \in \mathcal{S}, k, l \in \mathcal{S}_f)$ with appropriate dimensions, such that for any $(i,k) \in \mathcal{S}\times\mathcal{S}_f$,

$$P_{jk} - T_{ik} \leq V_{ijk},\ \forall j \in \mathcal{S}, j \neq i, \tag{27}$$

$$P_{il} - \tilde{T}_{ik} \leq \tilde{V}_{ikl},\ \forall l \in \mathcal{S}_f, l \neq k, \tag{28}$$

$$P_{ik} - T_{ik} \geq 0, P_{ik} - \tilde{T}_{ik} \geq 0, \tag{29}$$

$$0 > \hat{\Phi}_{ik} = \begin{bmatrix} \hat{\Phi}^{11}_{ik} & P_{ik}A^d_i & P_{ik}C_i & Z & D_i^{\mathrm{T}}P_{ik} & \tau D_i^{\mathrm{T}}Z & \hat{\Phi}^{17}_{ik} \\ * & \hat{\Phi}^{22}_{ik} & 0 & 0 & (D^d_i)^{\mathrm{T}}P_{ik} & \tau(D^d_i)^{\mathrm{T}}Z & (G^d_i)^{\mathrm{T}} \\ * & * & -\gamma^2 I & 0 & E_i^{\mathrm{T}}P_{ik} & \tau E_i^{\mathrm{T}}Z & H_i^{\mathrm{T}} \\ * & * & * & -Z & 0 & 0 & 0 \\ * & * & * & * & -P_{ik} & 0 & 0 \\ * & * & * & * & * & -\tau Z & 0 \\ * & * & * & * & * & * & -I \end{bmatrix}, \tag{30}$$

where

$$\begin{aligned}\hat{\Phi}^{11}_{ik} =& \mathrm{He}\{P_{ik}(A_i + B_i\Xi_k K_{ik})\} + Q - Z \\ &+ \sum_{j=1, j\neq i}^{s} [(\hat{\lambda}_{ij} - \delta_{ij})(P_{jk} - T_{ik}) + 2\delta_{ij}V_{ijk}] \\ &+ (\hat{\lambda}_{ii} + \delta_{ii})(P_{ik} - T_{ik}) + \sum_{l=1, l\neq k}^{r} [(\hat{\pi}^i_{kl} - \theta^i_{kl}) \\ &\times (P_{il} - \tilde{T}_{ik}) + 2\theta^i_{kl}\tilde{V}_{ikl}] + (\hat{\pi}^i_{kk} + \theta^i_{kk})(P_{ik} - \tilde{T}_{ik}), \\ \hat{\Phi}^{17}_{ik} =& (G_i + F_i\Xi_k K_{ik})^{\mathrm{T}},\ \ \hat{\Phi}^{22}_{ik} = -Q + Z,\end{aligned}$$

with $i,\ j \in \mathcal{S}$, $k,\ l \in \mathcal{S}_f$, then the closed-loop system (9) is stochastically stable with H_∞ performance γ.

Proof: For the closed-loop system (9) with asynchronous jumped actuator faults (4), given the estimated TRs $\hat{\lambda}_{ij}$, $\hat{\pi}^i_{kl}$ and their estimate errors $\delta_{ij} \geq 0$, $\theta^i_{kl} \geq 0$, we only need to prove that (30) guarantees the holding of (16). Noting (6) and (7), we have

$$\begin{aligned}&\sum_{j=1}^{s}\lambda_{ij}P_{jk} = \sum_{j=1}^{s}\lambda_{ij}(P_{jk} - T_{ik}), \\ &\sum_{l=1}^{r}\pi^i_{kl}P_{il} = \sum_{l=1}^{r}\pi^i_{kl}(P_{il} - \tilde{T}_{ik}).\end{aligned} \tag{31}$$

By (27)-(29) and (31), we get

$$\begin{aligned}\Phi^{11}_{ik} =& \mathrm{He}\{P_{ik}(A_i + B_i\Xi_k K_{ik})\} + Q - Z \\ &+ \sum_{j=1}^{s}\lambda_{ij}(P_{jk} - T_{ik}) + \sum_{l=1}^{r}\pi^i_{kl}(P_{il} - \tilde{T}_{ik}) \\ =& \mathrm{He}\{P_{ik}(A_i + B_i\Xi_k K_{ik})\} + Q - Z \\ &+ \sum_{j=1}^{s}(\hat{\lambda}_{ij} + \Delta\lambda_{ij})(P_{jk} - T_{ik}) \\ &+ \sum_{l=1}^{r}(\hat{\pi}^i_{kl} + \Delta\pi^i_{kl})(P_{il} - \tilde{T}_{ik}) \\ =& \mathrm{He}\{P_{ik}(A_i + B_i\Xi_k K_{ik})\} + Q - Z \\ &+ \sum_{j=1, j\neq i}^{s}(\hat{\lambda}_{ij} + \Delta\lambda_{ij})(P_{jk} - T_{ik}) \\ &+ (\hat{\lambda}_{ii} + \Delta\lambda_{ii})(P_{ik} - T_{ik}) \\ &+ \sum_{l=1, l\neq k}^{r}(\hat{\pi}^i_{kl} + \Delta\pi^i_{kl})(P_{il} - \tilde{T}_{ik}) \\ &+ (\hat{\pi}^i_{kk} + \Delta\pi^i_{kk})(P_{ik} - \tilde{T}_{ik}) \\ =& \mathrm{He}\{P_{ik}(A_i + B_i\Xi_k K_{ik})\} + Q - Z \\ &+ \sum_{j=1, j\neq i}^{s}[(\hat{\lambda}_{ij} - \delta_{ij}) + (\delta_{ij} + \Delta\lambda_{ij})](P_{jk} - T_{ik}) \\ &+ (\hat{\lambda}_{ii} + \Delta\lambda_{ii})(P_{ik} - T_{ik}) + (\hat{\pi}^i_{kk} + \Delta\pi^i_{kk})(P_{ik} - \tilde{T}_{ik}) \\ &+ \sum_{l=1, l\neq k}^{r}[(\hat{\pi}^i_{kl} - \theta^i_{kl}) + (\theta^i_{kl} + \Delta\pi^i_{kl})](P_{il} - \tilde{T}_{ik}) \\ \leq& \hat{\Phi}^{11}_{ik}.\end{aligned} \tag{32}$$

Then, it follows from (32) that (30) guarantees the holding of (16). This completes the proof. □

Remark 4: As is well-known, a significant issue of the analysis and synthesis of control for MJSs with uncertain TRs is to appropriately bind the uncertain items [18, 21]. Inspired by the work of [18], Theorem 2 has proposed a relaxation method without using traditional Young inequality to bind the uncertain terms by fully considering (a) the property of TRs as well as the characteristic of uncertainty domains (i.e., $\hat{\lambda}_{ii} + \Delta\lambda_{ii} < 0$, $\hat{\pi}^i_{kk} + \Delta\pi^i_{kk} < 0$, $\delta_{ij} + \Delta\lambda_{ij} \geq 0$ and $\theta^i_{kl} + \Delta\pi^i_{kl} \geq 0$), and (b) the introduction of the free matrices $\{T_{ik}, \tilde{T}_{ik}\}$ (via the facts that $\sum_{j=1}^{s}(\hat{\lambda}_{ij} + \Delta\lambda_{ij}) = 0$ and $\sum_{l=1}^{r}(\hat{\pi}^i_{kl} + \Delta\pi^i_{kl}) = 0$) to avoid the coupling items and reduce conservatism with the purpose of synthesis. Thus our results are more general and less conservative than that of [18, 20, 21, 50, 51], where the traditional Young inequality has be employed to bind the uncertain TRs.

Next, we will design the mixed-mode-dependent H_∞ controller for the closed-loop SDMJSs (9).

Theorem 3: For the closed-loop SDMJSs (9) with uncertain TRs and asynchronous jumped actuator faults (4), given the estimated TRs $\hat{\lambda}_{ij}, \hat{\pi}^i_{kl}$ and their estimate errors $\delta_{ij} \geq 0$, $\theta^i_{kl} \geq 0$, if there exist a scalar $\gamma > 0$, symmetric matrices $X_{ik} > 0, \mathcal{Q}_{ik} > 0, \mathcal{Z} > 0$ and matrices $\mathcal{V}_{ijk} \geq 0, \tilde{\mathcal{V}}_{ikl} \geq 0, \mathcal{T}_{ik}, \tilde{\mathcal{T}}_{ik}, Y_{ik}$ $(i, j \in \mathcal{S}, k, l \in \mathcal{S}_f)$ with appropriate dimensions, such that for any $(i,k) \in \mathcal{S}\times\mathcal{S}_f$,

$$\begin{bmatrix} -\mathcal{T}_{ik}-\mathcal{V}_{ijk} & X_{ik} \\ * & -X_{jk} \end{bmatrix} \le 0,\ \forall j \in \mathcal{S}, j \neq i, \tag{33}$$

$$\begin{bmatrix} -\tilde{\mathcal{T}}_{ik}-\tilde{\mathcal{V}}_{ikl} & X_{ik} \\ * & -X_{il} \end{bmatrix} \le 0,\ \forall l \in \mathcal{S}_f,\ l \neq k, \tag{34}$$

$$X_{ik}-\mathcal{T}_{ik} \ge 0,\ X_{ik}-\tilde{\mathcal{T}}_{ik} \ge 0, \tag{35}$$

$$\tilde{\Phi}_{ik} < 0, \tag{36}$$

where $\tilde{\Phi}_{ik}$ is given at the bottom with

$$\begin{aligned}
\tilde{\Phi}_{ik}^{11} &= \mathrm{He}\{A_iX_{ik}+B_i\Xi_kY_{ik}\}+\mathcal{Q}_{ik}+\sum_{j=1,j\neq i}^{s}[2\delta_{ij}\mathcal{V}_{ijk} \\
&\quad -(\hat{\lambda}_{ij}-\delta_{ij})\mathcal{T}_{ik})]+(\hat{\lambda}_{ii}+\delta_{ii})(X_{ik}-\mathcal{T}_{ik}) \\
&\quad +\sum_{l=1,l\neq k}^{r}[2\theta_{kl}^i\tilde{\mathcal{V}}_{ikl}-(\hat{\pi}_{kl}^i-\theta_{kl}^i)\tilde{\mathcal{T}}_{ik}] \\
&\quad +(\hat{\pi}_{kk}^i+\theta_{kk}^i)(X_{ik}-\tilde{\mathcal{T}}_{ik}), \\
\tilde{\Phi}_{ik}^{17} &= X_{ik}G_i^{\mathrm{T}}+Y_{ik}^{\mathrm{T}}\Xi_k^{\mathrm{T}}F_i^{\mathrm{T}}, \\
\tilde{\Phi}_{ik}^{19} &= \Big[\sqrt{\hat{\lambda}_{i1}-\delta_{i1}},\cdots,\sqrt{\hat{\lambda}_{i,i-1}-\delta_{i,i-1}}, \\
&\quad \sqrt{\hat{\lambda}_{i,i+1}-\delta_{i,i+1}},\cdots,\sqrt{\hat{\lambda}_{is}-\delta_{is}}\Big]X_{ik}, \\
\tilde{\Phi}_{ik}^{1,10} &= \Big[\sqrt{\hat{\pi}_{k1}^i-\theta_{k1}^i},\cdots,\sqrt{\hat{\pi}_{k,k-1}^i-\theta_{k,k-1}^i}, \\
&\quad \sqrt{\hat{\pi}_{k,k+1}^i-\theta_{k,k+1}^i},\cdots,\sqrt{\hat{\pi}_{kr}^i-\theta_{kr}^i}\Big]X_{ik}, \\
\tilde{\Phi}_{ik}^{99} &= -\mathrm{diag}\{X_{1k},\cdots,X_{i-1,k},X_{i+1,k},\cdots,X_{sk}\}, \\
\tilde{\Phi}_{ik}^{10,10} &= -\mathrm{diag}\{X_{i1},\cdots,X_{i,k-1},X_{i,k+1},\cdots,X_{ir}\},
\end{aligned}$$

then the stabilizing controller to provide $\gamma-$disturbance attenuation for the closed-loop system (9) can be constructed as

$$K_{ik}=Y_{ik}X_{ik}^{-1}. \tag{37}$$

Proof: Let $X_{ik}=P_{ik}^{-1}$, $\mathcal{Z}=Z^{-1}$, $\mathcal{Q}_{ik}=X_{ik}QX_{ik}$, $\mathcal{T}_{ik}=X_{ik}T_{ik}X_{ik}$, $\mathcal{V}_{ijk}=X_{ik}V_{ijk}X_{ik}$, $\tilde{\mathcal{T}}_{ik}=X_{ik}\tilde{T}_{ik}X_{ik}$, $\tilde{\mathcal{V}}_{ikl}=X_{ik}\tilde{V}_{ikl}X_{ik}$. Then, Applying the congruent transformation (CT) X_{ik} to both sides of inequalities (27) and (28) as well as using Schur complement, it follows that (27) and (28) are equivalent to (33) and (34), respectively.

Applying the CT X_{ik} to the left side of inequality (29), it follows that (29) is equivalent to (35).

Letting

$$\Omega_{ik}=\begin{bmatrix}
X_{ik} & 0 & 0 & X_{ik} & 0 & 0 & 0 & 0 \\
0 & X_{ik} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathcal{Z} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X_{ik} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathcal{Z} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & I
\end{bmatrix},$$

and applying the CT Ω_{ik} to the right side of (30) as well as using Schur complement, it follows that (30) is equivalent to (36) by letting $Y_{ik}=K_{ik}X_{ik}$. This completes the proof.□

Remark 5: The design scheme of the controllers with a minimum H_∞ performance in Theorem 3 can be expressed as the following optimal minimization problem:

$$\begin{aligned}
\text{OP:}\quad & \min \gamma^2 \\
\text{s.t.}\quad & \text{LMIs (33)-(36) with } X_{ik}>0,\ \mathcal{Q}_{ik}>0,\ \mathcal{Z}>0, \\
& \mathcal{V}_{ijk}\ge 0\ (j\neq i), \tilde{\mathcal{V}}_{ikl}\ge 0\ (l\neq k), \\
& \forall\, i,j\in\mathcal{S},\ \forall\, k,l\in\mathcal{S}_f.
\end{aligned} \tag{38}$$

Remark 6: Theorem 3 has proposed a new scheme to design the mixed-mode-dependent reliable H_∞ controllers for the closed-loop SDMJSs (9) subject to asynchronous jumping actuator failure and uncertain TRs. Besides, Theorem 3 has extended the related results of some existing literature [53–57] from the following aspects: (i) the methods in [53–57] cannot be applied to the asynchronous control problem on SDMJSs with mixed AMJMs and uncertain TRs. (ii) the random disturbance of Brownian motion on the system, which is usually a key factor causing system instability, was ignored in [53–57]; (iii) the time-delay as well as asynchronous control problem for the MJSs has not been taken into account in [53–57].

Remark 7: It is worth pointing out that the number of decision variables in Theorem 3 is $sr(s+r+5)+2$, and the number of LMIs is $sr(s+r+1)$, where s and r denote the number of the finite state space $\mathcal{S}$ and $\mathcal{S}_f$, respectively.

$$\tilde{\Phi}_{ik}=\begin{bmatrix}
\tilde{\Phi}_{ik}^{11} & A_i^dX_{ik} & C_i & 0 & X_{ik}D_i^{\mathrm{T}} & \tau X_{ik}D_i^{\mathrm{T}} & \tilde{\Phi}_{ik}^{17} & 0 & \tilde{\Phi}_{ik}^{19} & \tilde{\Phi}_{ik}^{1,10} \\
* & -\mathcal{Q}_{ik} & 0 & 0 & X_{ik}(D_i^d)^{\mathrm{T}} & \tau X_{ik}(D_i^d)^{\mathrm{T}} & X_{ik}(G_i^d)^{\mathrm{T}} & X_{ik} & 0 & 0 \\
* & * & -\gamma^2 I & 0 & E_i^{\mathrm{T}} & \tau E_i^{\mathrm{T}} & H_i^{\mathrm{T}} & 0 & 0 & 0 \\
* & * & * & -\mathcal{Z} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -X_{ik} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\tau\mathcal{Z} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\mathcal{Z} & 0 & 0 \\
* & * & * & * & * & * & * & * & \tilde{\Phi}_{ik}^{99} & 0 \\
* & * & * & * & * & * & * & * & * & \tilde{\Phi}_{ik}^{10,10}
\end{bmatrix}.$$

4. NUMERICAL EXAMPLE

As an application of the controller design scheme presented in this paper, a simulation example for the VTOL helicopter system is given.

Example 1: Consider the following VTOL helicopter system [39, 52], which is modified for our purpose. For this system, the internal and external environments, such as airspeed, temperature, electromagnetic field and atmospheric pressure are assumed to be constantly changing with stochastic disturbances. Therefore, the VTOL helicopter system can be formulated as the SDMJS. Let $x_1(t), x_2(t), x_3(t)$ and $x_4(t)$ be the horizontal velocity, vertical velocity, pitch rate and pitch angle, respectively, then the VTOL helicopter system can be presented by system (1) with the following parameters:

$$A_{\sigma(t)} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & a^1_{\sigma(t)} & -0.707 & a^2_{\sigma(t)} \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_{\sigma(t)} = \begin{bmatrix} 0.4422 & b_{\sigma(t)} & -5.5200 & 0 \\ 0.1761 & -7.5922 & 4.4900 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$A^d_1 = 10^{-3} \times \begin{bmatrix} -1 & 0 & -2 & 0 \\ 0 & 5 & 4 & -3 \\ -7 & 3 & 2 & 0 \\ 0 & 5 & 0 & -3 \end{bmatrix},$$

$$A^d_2 = 10^{-3} \times \begin{bmatrix} 3 & 2 & -1 & 0 \\ 0 & 3 & 0 & -3 \\ 0 & 3 & 2 & 0 \\ 0 & 4 & 0 & -3 \end{bmatrix},$$

$$A^d_3 = 10^{-3} \times \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 3 & 2 & -3 \\ -4 & 3 & 2 & 0 \\ 0 & 2 & 3 & 0 \end{bmatrix},$$

$$C_1 = 10^{-2} \times \begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 3 & 1 & -3 \end{bmatrix}^{\mathrm{T}},$$

$$C_2 = 10^{-2} \times \begin{bmatrix} -1 & 3 & 1 & 1 \\ -3 & 0 & 0 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$C_3 = 10^{-2} \times \begin{bmatrix} 2 & 1 & 2 & 0 \\ 0 & 0 & -3 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$D_1 = 10^{-2} \times \begin{bmatrix} -7.6 & 1.5 & -2.9 & -4.7 \\ 1.8 & -2.5 & 5.5 & 1.6 \\ 3.7 & -2.6 & -4.3 & -1.9 \\ 0 & 0 & 3 & 0 \end{bmatrix},$$

$$D_2 = 10^{-2} \times \begin{bmatrix} -2.5 & 2.7 & 4.5 & -1.7 \\ -8.7 & -1.5 & 3.9 & 1.5 \\ 1.4 & -1.3 & -1.5 & -1.4 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D_3 = 10^{-2} \times \begin{bmatrix} -9.0 & 1.1 & 1.2 & 3.5 \\ 7.5 & -1.8 & 1.6 & 2.7 \\ 4.0 & -8.4 & -3.7 & -2.9 \\ 0 & 0 & 2 & 0 \end{bmatrix},$$

Table 1. The parameters depending on the airspeeds.

Airspeed (kn)	$\sigma(t)$	$a^1_{\sigma(t)}$	$a^2_{\sigma(t)}$	$b_{\sigma(t)}$
135	1	0.3681	1.4200	3.5446
60	2	0.0664	0.1198	0.9775
170	3	0.5047	2.5460	5.1120

$$E_1 = 10^{-2} \times \begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 3 & 1 & -3 \end{bmatrix}^{\mathrm{T}},$$

$$E_2 = 10^{-2} \times \begin{bmatrix} -1 & 3 & 1 & 1 \\ -3 & 0 & 0 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$E_3 = 10^{-2} \times \begin{bmatrix} 2 & 1 & 2 & 0 \\ 0 & 0 & -3 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$G_{\sigma(t)} = \begin{bmatrix} 0.4 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & 0.5 \end{bmatrix},$$

$$G^d_{\sigma(t)} = \begin{bmatrix} 0 & 0.2 & -0.4 & 0 \\ 0.3 & 0 & 0 & -0.2 \end{bmatrix},$$

$$F_{\sigma(t)} = \begin{bmatrix} 1 & 0.2 \\ 0.1 & -1 \end{bmatrix}, \; H_{\sigma(t)} = \begin{bmatrix} -0.6 & -0.12 \\ 0.23 & 0.8 \end{bmatrix},$$

$$D^d_{\sigma(t)} = 0.1 D_{\sigma(t)}, \; \tau = 0.5. \tag{39}$$

Here, $\sigma(t) \in \{1, 2, 3\}$ varies with different airspeeds 135 (nominal value), 60 and 170 knots. The behavior of $\sigma(t)$ can be formulated as a Markovian chain with three modes [39]. The parameters $a^1_{\sigma(t)}, a^2_{\sigma(t)}$ and $b_{\sigma(t)}$ are given in Table 1.

It is worth noting that the TRs among airspeeds are usually difficult to obtain if the external environment (i.e.,weather) changes. Therefore, the TRM is set to be uncertain and the estimated TRs is given by $[\hat{\lambda}_{ij}] = \begin{bmatrix} -1.3 & 0.7 & 0.6 \\ 0.5 & -1.3 & 0.8 \\ 0.7 & 0.5 & -1.2 \end{bmatrix}$ with its estimate error satisfying $|\Delta\lambda_{ij}| \le 0.1\hat{\lambda}_{ij} =: \delta_{ij}$. Assume that there are two modes for asynchronous jumped failure with $\Xi_1 = \mathrm{diag}\,\{1 \;\; 0.6\}$, $\Xi_2 = \mathrm{diag}\,\{0.4 \;\; 0.8\}$. The estimated TRs of faults are $[\pi^1_{kl}] = \begin{bmatrix} -0.6 & 0.6 \\ 0.5 & -0.5 \end{bmatrix}$, $[\pi^2_{kl}] = \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix}$ and $[\pi^3_{kl}] = \begin{bmatrix} -0.4 & 0.4 \\ 0.7 & -0.7 \end{bmatrix}$ with its estimate error values of TRs satisfying $|\Delta\pi^i_{kl}| \le 0.15\hat{\pi}^i_{kl} =: \theta^i_{kl}$.

Note that the unforced system with AMJMs is unstable (see Fig. 1, which shows the state responses of the open-loop system). However, the unforced system is stochastically stabilizable according to Theorem 3, and by using the MATLAB LMI toolbox to solve the optimization problem (38), we can obtain $\gamma_{\min} = 0.7948$ as well as the following controller gains:

$$K_{11} = \begin{bmatrix} -0.9492 & 0.1267 & 0.5691 & 1.5274 \\ -0.3252 & 0.3065 & -0.1560 & 0.0904 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -0.9655 & 0.1558 & 0.6152 & 1.5130 \\ -0.0158 & 0.2961 & -0.2964 & -0.3114 \end{bmatrix},$$
$$K_{31} = \begin{bmatrix} -0.8361 & 0.1084 & 0.4967 & 1.4602 \\ -0.4189 & 0.3084 & -0.1548 & 0.2415 \end{bmatrix},$$
$$K_{12} = \begin{bmatrix} -1.3639 & 0.1868 & 0.8145 & 2.1959 \\ -0.2494 & 0.2306 & -0.1114 & 0.0724 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -1.4350 & 0.2335 & 0.9103 & 2.2446 \\ 0.0019 & 0.2269 & -0.2286 & -0.2570 \end{bmatrix},$$
$$K_{32} = \begin{bmatrix} -1.2664 & 0.1692 & 0.7499 & 2.1831 \\ -0.3323 & 0.2365 & -0.0973 & 0.1944 \end{bmatrix}.$$

For the convenience of simulation, under zero initial condition, let the disturbance input $\omega(t) = [\frac{50}{1+t^{3/2}} \ \frac{-30}{2+t^{3/2}}]^{\mathrm{T}}$ and the initial modes be $\sigma_0 = 1$ and $s_0 = 1$. Then, By employing the Euler-Maruyama method with step size 0.002, the simulation results of the closed-loop SDMJSs (9) with (39) are given in Figs. 2-4. Fig. 2 shows the reliable control input while Fig. 3 describes the state responses of the closed-loop system with AMJMs. Moreover, Fig. 4 gives the state responses of the closed-loop system in 100 random samplings (RSs). It is thus clear from Figs. 1-4 that the designed controller can stabilize the unforced system as well as achieving the prescribed level of disturbance attenuation.

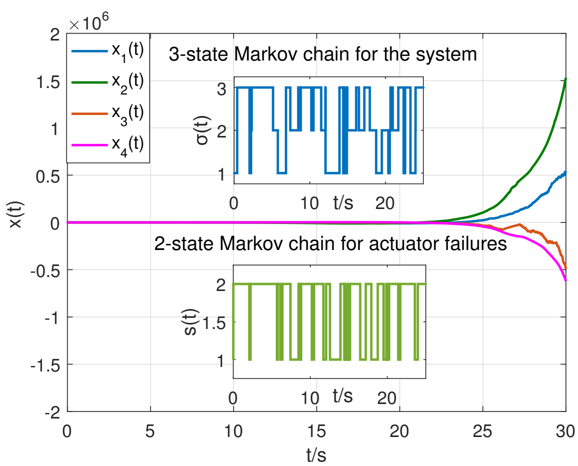

Fig. 1. The sate responses of the open-loop system with AMJMs.

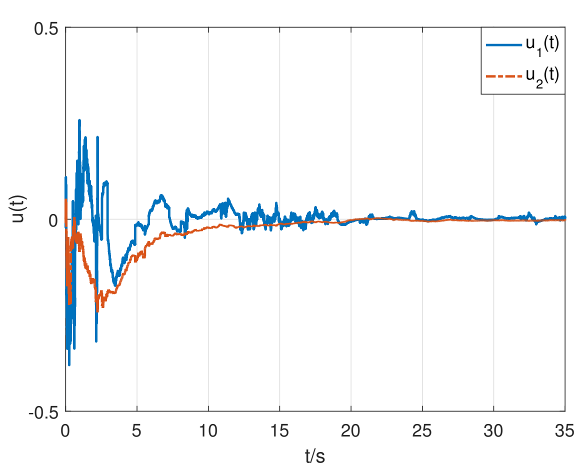

Fig. 2. The reliable control input of the closed-loop system.

5. CONCLUSIONS

This paper has mainly concerned with the reliable H_∞ control for SDMJSs with asynchronous jumped actuator failure and uncertain TRs. By representing the mixed Markovian processes as two Poisson integrals, a generalized functional Itô's formula for the closed-loop SDMJSs with mixed AMJMs has been successfully established. By the generalized functional Itô's formula, the mixed-mode-dependent Lyapunov functionals and matrix manipulation, a delay-dependent design scheme of the asynchronous reliable H_∞ control for SDMJSs is proposed. Then, the mixed-mode-dependent controller gains can be achieved by solving a convex optimization problem under LMIs constraints. Meanwhile, a relaxation method is presented to bind the uncertain terms appropriately for reducing the conservatism. Finally, the controller design scheme has been verified via a simulation example on the VTOL helicopter system.

APPENDIX A

In this appendix, the proof of Lemma 1 is given.

Proof of Lemma 1: With the inspiration of [47], the proof is given as follows. For the joint Markovian process $(x(t), \sigma(t), s(t))$ given by by (9), (12) and (13), applying the functional Itô's formula for càdlàg semi-martingales

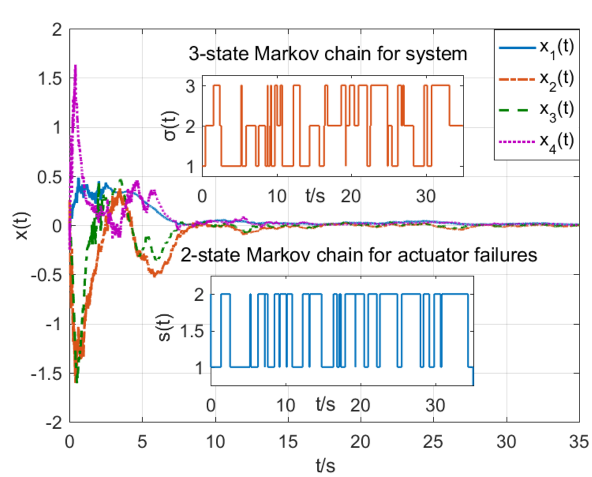

Fig. 3. The state response of the closed-loop system with AMJMs.

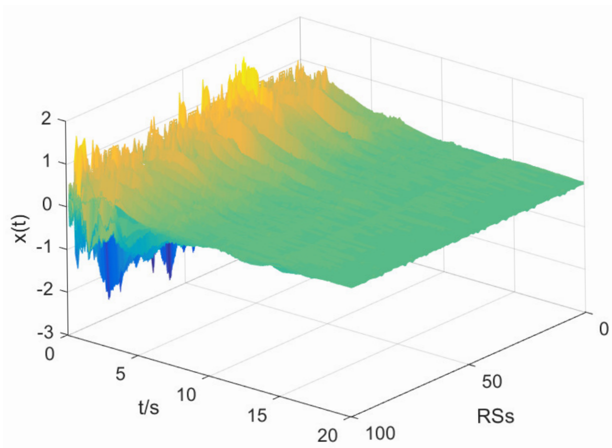

Fig. 4. The state response of the closed-loop system with 100 RSs.

(see [7], Proposition 6) to functional $V(x_t,t,\sigma(t),s(t))$ on the interval $[0,t]$, we have

$$\begin{aligned}
&V(x_t,t,\sigma(t),s(t))\\
&=V(x_0,0,\sigma_0,s_0)\\
&\quad+\int_0^t \mathcal{D}_t V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))d\theta\\
&\quad+\int_0^t \nabla_x V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times f(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))d\theta\\
&\quad+\int_0^t \nabla_x V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times g(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))dB(\theta)\\
&\quad+\frac{1}{2}\int_0^t \mathrm{tr}\{g^{\mathrm{T}}(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times \nabla_{xx} V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times g(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\}d\theta\\
&\quad+\int_0^t\int_{\mathbb{R}}[V(x_{\theta^-},\theta,\sigma(\theta^-)+\rho(\sigma(\theta^-),z),s(\theta^-))\\
&\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]dzd\theta\\
&\quad+\int_0^t\int_{\mathbb{R}}[V(x_{\theta^-},\theta,\sigma(\theta^-)+\rho(\sigma(\theta^-),z),s(\theta^-))\\
&\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]\tilde{v}(d\theta,dz)\\
&\quad+\int_0^t\int_{\mathbb{R}}[V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-)+\zeta(s(\theta^-),v))\\
&\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]dvd\theta\\
&\quad+\int_0^t\int_{\mathbb{R}}[V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-)+\zeta(s(\theta^-),v))\\
&\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]\tilde{\mu}(d\theta,dv)\\
&=V(x_0,0,\sigma_0,s_0)+\mathcal{M}(t)\\
&\quad+\int_0^t \mathcal{D}_t V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))d\theta\\
&\quad+\int_0^t \nabla_x V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times f(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))d\theta\\
&\quad+\frac{1}{2}\int_0^t \mathrm{tr}\{g^{\mathrm{T}}(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times \nabla_{xx} V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\\
&\quad\times g(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))\}d\theta\\
&\quad+\int_0^t \sum_{j=1,j\neq\sigma(\theta^-)}^{s}[V(x_{\theta^-},\theta,j,s(\theta^-))\\
&\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]\lambda_{\sigma(\theta^-),j}d\theta\\
&\quad+\int_0^t \sum_{l=1,l\neq s(\theta^-)}^{r}[V(x_{\theta^-},\theta,\sigma(\theta^-),l)\\
&\quad-V(x_{\theta^-},\theta,\sigma(\theta^-),s(\theta^-))]\pi_{s(\theta^-),l}^{\sigma(\theta^-)}d\theta\\
&=V(x_0,0,\sigma_0,s_0)+\int_0^t \mathcal{L}V(x_\theta,\theta,\sigma(\theta),s(\theta))d\theta\\
&\quad+\mathcal{M}(t). \qquad\qquad (\mathrm{A}.1)
\end{aligned}$$

The proof is completed. □

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