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Event-triggered Consensus Control of Nonlinear Multi-agent Systems based on First-order Hold

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Abstract: In this paper, consensus problem is studied for the first-order nonlinear multi-agent systems under directed topology, and a novel event-triggered consensus protocol is proposed. Different from the existing results with general zero-order hold, the first-order hold is adopted to construct the event-triggered control signal so as to reduce the triggering rate. By using Lyapunov stability method and matrix theory, a sufficient consensus condition is derived for the agents to reach the asymptotic consensus, and Zeno-behavior can be avoided. Finally, a numerical example is presented to demonstrate the effectiveness of our proposed protocol.

Keywords: Consensus problem, directed topology, event-triggered control, first-order hold, nonlinear multi-agent systems.

1. INTRODUCTION

In the past decades, the distributed consensus problem of multi-agent systems (MASs) has received considerable attention due to its important applications in a wide range of engineering areas, such as sensor network [1, 2], vehicles [3, 4], power network [5] and so on, and it has been extensively analyzed and synthesized [6, 7].

In order to reduce the communication load of multi-agent network and the updating frequency of controller, recently, event-triggered control strategy has been applied to the consensus protocols [8–11, 23]. In the event-triggered consensus protocol, the event time instant of communication is determined by the predefined triggering function related closely to system measurement errors or performance level [12]. Up to now, event-triggered consensus protocols have been widely adopted, and there are many outstanding research achievements for MASs under undirected and symmetric topology [13–15]. However, information flow may usually be directed due to heterogeneity, non-uniform communication capabilities or sensing with limited visual fields, and consensus analysis under directed topology is more challenging than that under undirected topology.

Recently, a lot of event-triggered protocols have been developed to solve the consensus problem of MASs with directed topology [16–22]. For the second-order MASs with nonlinear dynamics, Dong and Gong [19] used the decentralized event-triggered strategies to deal with the consensus problem, and designed a time-variant triggering function to determine the event instants by switching mechanism. For the general linear MASs under directed graph, a distributed event-triggered consensus protocol was proposed to determine the control updating by using combinational measurements [20]. However, each agent needs to know the state of all neighbors at all times, which means that continuous communication is still necessary [20]. In order to reduce communication and congestion in the network, Liu *et al*. [16] proposed a novel distributed event-triggered consensus protocol without continuous communication. The triggering function in [16] depends on the agents' states, and each agent only needs to communicate with their neighbors at discrete time at which the event is triggered. Besides, Jian *et al*. [18] designed the Kx-functional observer-based output feedback event-triggered consensus protocol, which used the neighbors' states to predict the output of the controller, and the controller signal is updated within the event interval so as to reduce the triggering rate. Considering the consensus problem of nonlinear MASs under directed topology containing directed spanning trees, moreover, Li *et al*. [17] obtained the triggering function and event time intervals according to matrix theory and Lyapunov stability approach.

In the aforementioned results on event-triggered consensus problem of MASs with directed topology [17, 20–

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22], each agent communicates with its neighbors only when the event is triggered. The sampled-states obtained by the communication are used for updating the controller, and the event-triggered controller signal is kept unchanged by the zero-order hold (ZOH) until the next event time instant, which is easy to realize, but it provides no trend information. Thus, the information on changes between the consecutive event time instants is lost. One possible solution to improve the precision of the control signal is to use high-order hold, such as first-order hold (FOH) in [27,28], to provide trend information during the triggering interval. Experimental results indicate that triggering rate can be highly reduced by using FOH in [27].

Motivated by above discussions [17,20,27], a novel event-triggered protocol based on FOH is proposed to reduce the triggering rate in this paper. The contributions of our work are listed as follows:

1) An event-triggered consensus protocol for nonlinear MASs with directed topology is proposed. Compared with the existing results, this proposed protocol is based on piecewise linear signal (FOH) instead of the standard piecewise constant signal (ZOH), so as to decrease the updating frequency of controller and reduce the amount of communication.

2) On the basis of FOH implementation, the event-triggered condition is derived to guarantee that the asymptotic consensus of nonlinear MASs can be achieved, and Zeno-behavior is excluded. Experimental results show that the proposed event-triggered consensus protocol possesses the less triggering rate than the event-triggered consensus protocol based on ZOH.

The rest of this paper is organized as follows: In Section 2, we briefly introduce some notions on graph theory, and give several lemmas, definitions and make an assumption for later analysis. The event-triggered consensus protocol is designed in Section 3. The main theoretical results are given in Section 4. Section 5 presents a comparison between FOH and ZOH through a simulation example. Finally, Section 6 draws the conclusions.

2. PROBLEM FORMULATION

2.1. Notation

Throughout this paper, $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}^{N\times N}$ denote the set of positive natural numbers, the set of real numbers and $N\times N$ matrices, respectively. Let I_N be the identity matrix of order N, and 1_N be the vector with all elements being 1. $\|\cdot\|$ represents the Euclidean norm. In addition, A^T denotes the transpose of a real symmetric matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the maximum and the minimum eigenvalues of matrix A, respectively.

2.2. Graph theory

Let $G=(V,E)$ denote a weighted directed graph of N order with the set of nodes $V=\{1,2,\cdots,N\}$, and $E\subseteq V\times V$ is the set of directed edges. If directed edge $(j,i)\in E$, then there is an edge from node j to node i and node j is called a neighbor of node i. $A=(a_{ij})\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix of N nodes where $a_{ij}>0$, if $(j,i)\in E$ and $a_{ij}=0$, otherwise. The neighboring set of node i is defined as $\mathcal{N}_i=\{j\in V:(j,i)\in E\}$, while $\bar{N}_i=|\mathcal{N}_i|$ indicates the number of neighbors belongs to node i. Define the in-degree of node i as $\deg_i^{in}=\sum_{j=1}^N a_{ij}$, and the in-degree matrix of G is defined as $\mathrm{Deg}^{in}=\mathrm{diag}\{\deg_1^{in},\ldots\deg_n^{in}\}$. The Laplacian matrix $L=[l_{ij}]\in\mathbb{R}^{N\times N}$ of the directed graph G is defined as $L=\mathrm{Deg}^{in}-A$. A directed graph G is called strongly connected if for any two distinct nodes i and j, there always exists a directed path from node i to node j.

2.3. Plant model

Consider the first-order MASs consisting of N identical agents, and the model of agent i is given by

$$\dot{x}_i(t)=f(x_i(t),t)+u_i(t),\ \ i=1,2,\cdots,N, \tag{1}$$

where $x_i\in\mathbb{R}$ and $u_i\in\mathbb{R}$ are the state variable and control input, respectively, of the ith agent, and $f(x_i(t),t)\in\mathbb{R}$ is a continuous nonlinear function.

In (1), the function $f(\cdot)$ describes the inherently nonlinear dynamics of each agent, and we make an assumption on this function as follows:

Assumption 1 [30]**:** Suppose that the nonlinear function $f(x_i(t),t)$ is locally Lipschitz continuous, and there exist a positive constant ρ such that

$$\|f(x_i(t),t)-f(x_j(t),t)\|\leq\rho\,\|x_i(t)-x_j(t)\|,\quad \forall x_i(t)\neq x_j(t).$$

Definition 1: Consensus of MASs (1) is said to be achieved asymptotically if for any initial states $x_i(0)\in\mathbb{R}$,

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\ \ \forall i,j=1,2,\ldots,N.$$

The following lemma will play an important role in the proof of main results.

Lemma 1 [26]**:** For arbitrary $p,q\in\mathbb{R}$ and $a>0$,

$$pq\leqslant\frac{a}{2}p^2+\frac{1}{2a}q^2$$

holds.

Lemma 2 [25]**:** If G is a strongly connected graph with Laplacian matrix L, then, $L1_N=0$ and there exists a positive vector $\xi=[\xi_1,\ \xi_2,\ \ldots,\ \xi_N]^T$ with positive elements and $\sum_{i=1}^N\xi_i=1$ such that $\xi^TL=0$. Moreover, let $R=(\Xi L+L^T\Xi)/2$ with $\Xi=\mathrm{diag}\{\xi_1,\ \xi_2,\ \ldots,\ \xi_N\}$, and then matrix R is symmetric and $\sum_{i=1}^N R_{ij}=\sum_{j=1}^N R_{ji}=0$, $\forall i,\ j=1,\ 2,\ \ldots,N$.

With Lemma 2, we have the following definition and lemma.

Definition 2 [25]**:** For a strongly connected network G with Laplacian matrix L, the general algebraic connectivity is defined as

$$\lambda_{\xi}(L) = \min_{x^T\xi=0, x\neq 0} \frac{x^T R x}{x^T \Xi x}.$$

Lemma 3 [25]**:** $\lambda_{\xi}(L)$ can be computed as follows:

$$\lambda_{\xi}(L) = \begin{cases} \max\ \delta, \\ \text{subject to } Q^T(R-\delta\Xi)Q \geqslant 0. \end{cases}$$

where $Q = \begin{pmatrix} I_{N-1} \\ -\frac{\hat{\xi}^T}{\xi_N} \end{pmatrix} \in \mathbb{R}^{N\times(N-1)}$ and $\hat{\xi} = (\xi_1, \xi_2, \cdots, \xi_{N-1})^T$.

3. DESIGN OF EVENT-TRIGGERED CONSENSUS PROTOCOL

In this section, we will construct the event-triggered input signal based on FOH, and then, propose a novel event-triggered function to determine the data communication. In addition, the event-triggered control framework of each agent i is shown in Fig. 1. In this framework, the event detector has the ability to receive the packages at the storage side. More specifically, the controller of agent i updates its control input by using the received information from the storage. Then, the output of the controller i is sent to the actuator with FOH, which is used to keep the control input of agent i piecewise linear until the next event time.

3.1. Event-triggered control signal based on FOH

Discrete-time control signals need to be converted into continuous-time control signals by digital-to-converter [32], such as ZOH and FOH. Compared with the ZOH-based control signal, which is piecewise constant, the FOH-based control signal varies linearly during the triggering interval. By this difference, some results have shown the performance improvement of FOH technology [27, 28]. Hence, in this paper, we construct the control input $u_i(t)$ based on sampled data by FOH technology, then

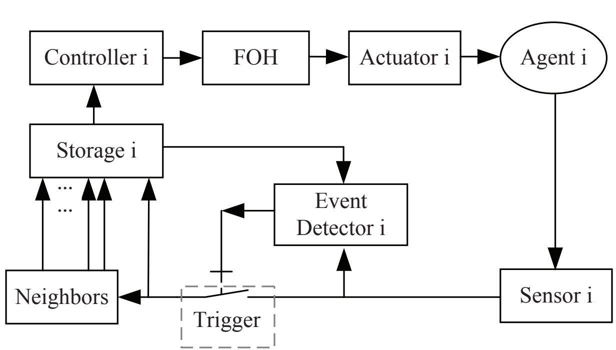

Fig. 1. Event-triggered control framework for agent i.

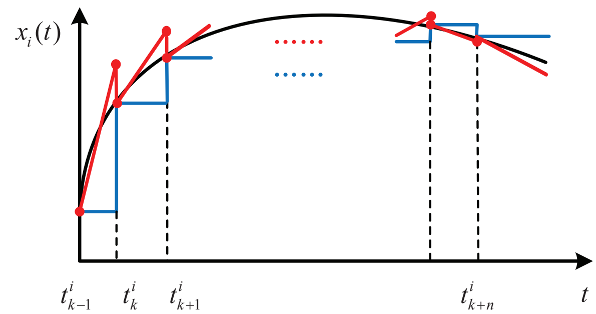

Fig. 2. State estimations of $x_i(t)$ based on FOH and ZOH.

the control signal of agent i is given by

$$u_i(t) = c\sum\nolimits_{j=1}^{N} a_{ij}\left(\hat{x}_j(t^i_k) - \hat{x}_i(t^i_k)\right),\ \ t > t^i_k, \tag{2}$$

in which

$$\hat{x}_j(t^i_k) = x_j(t^i_k) + \frac{x_j(t^i_k) - x_j(t^i_{k-1})}{t^i_k - t^i_{k-1}}(t - t^i_k),$$
$$\hat{x}_i(t^i_k) = x_i(t^i_k) + \frac{x_i(t^i_k) - x_i(t^i_{k-1})}{t^i_k - t^i_{k-1}}(t - t^i_k),$$

where $t \in [t^i_k, t^i_{k+1})$, $k \in \mathbb{N}$, t^i_k denotes the kth triggering instant of agent i, $\hat{x}_j(t^i_k)$ and $\hat{x}_i(t^i_k)$ denote state the estimations of $x_j(t)$ and $x_i(t)$, respectively, and $c > 0$ is the controller gain to be designed later. Fig. 2 shows the state estimations for ZOH (blue line) and FOH (red line) based on sampled data. As the black line is the continuous state $x_i(t)$ of agent i, $i = 1, 2, \cdots, N$. During the interval $[t^i_k, t^i_{k+1})$, $k \in \mathbb{N}$, the state estimations for ZOH is a constant $x_i(t^i_k)$, while that for FOH exhibits linear change.

Remark 1: Similar to that of [11, 15, 20], the protocol (2) is updated at its own event instants and only depends on the states at triggering time. Our proposed protocol (2) has been analyzed for the nonlinear agents under undirected and connected topology in our work [31]. However, this paper takes into account the proposed algorithm under directed and strongly connected topology, and it is well known that the convergence analysis under directed topology is much more challenging than that under undirected topology.

3.2. Event-triggered function for MASs

Define $e_i(t) = \hat{x}_i(t^i_k) - x_i(t)$, then the proposed event-triggered function is given by

$$E_i(t) = \|e_i(t)\| - \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\left\|\hat{x}_i(t^i_k) - \hat{x}_j(t^i_k)\right\| - \varphi_2 e^{-\gamma(t-t_0)}, \tag{3}$$

where $\varphi_1 > 0$, $\varphi_2 > 0$ and $\gamma > 0$. Therefore, the next event instant of agent i is determined by the following condition,

$$t^i_{k+1} = \inf\{t : t > t^i_k, E_i(t) \geqslant 0\}.$$

An event is triggered at $t = t^i_{k+1}$ when $E_i(t) \geqslant 0$, and then the dynamic error $\|e_i(t)\|$ is reset to zero.

Remark 2: Note that the proposed event-triggered condition (3) is dependent on the neighbors' discrete-time information $x_j(t^i_k)$ and $x_j(t^i_{k-1})$ rather than the continuous-time information $x_j(t), j \in \mathcal{N}_i$. This implies that each agent does not need continuous communication with their neighbors.

Remark 3: The proposed controller (2) and the triggering function (3) both depend on the same time variable t, i.e., all the agents evolve along the same time axis t, so that the hardware that reaches the clock synchronization is necessary. However, agents' event-triggered instants are determined by its triggering functions, i.e., the triggering times of one agent does not need to synchronize with that of other agents.

4. MAIN RESULTS

With control input (2), we obtain the following error dynamical system

$$\dot{x}_i(t) = f(x_i(t),t) - c\sum\nolimits_{j=1}^{N} L_{ij}\left(x_j(t) + e_j(t)\right). \tag{4}$$

Let $\bar{x}(t) = \sum_{j=1}^N \xi_j x_j(t)$ represent the weighed average trajectories of all agents with the nonnegative left eigenvector of L in Lemma 2. Define $y_i(t) = x_i(t) - \bar{x}(t)$, we further have

$$\begin{aligned}\dot{y}_i(t) =& f(x_i(t),t) - c\sum\nolimits_{j=1}^{N} L_{ij}\left(y_j(t) + e_j(t)\right)\\ &+ c\sum\nolimits_{l=1}^{N}\xi_l\sum\nolimits_{j=1}^{N} L_{ij}\left(y_j(t) + e_j(t)\right)\\ &- \sum\nolimits_{l=1}^{N}\xi_l f(x_l(t),t).\end{aligned} \tag{5}$$

Since $\xi^T L = 0$, we have

$$\begin{aligned}\sum\nolimits_{l=1}^{N}\xi_l\sum\nolimits_{j=1}^{N} L_{ij}\left(y_j(t) + e_j(t)\right) =& \xi^T L\left(y(t) + e(t)\right)\\ =& 0.\end{aligned}$$

Then, the dynamical system (5) is rewritten as

$$\begin{aligned}\dot{y}_i(t) =& f(x_i(t),t) - c\sum\nolimits_{j=1}^{N} L_{ij}\left(y_j(t) + e_j(t)\right)\\ &- \sum\nolimits_{l=1}^{N}\xi_l f(x_l(t),t).\end{aligned} \tag{6}$$

For the convenience of analysis, we express (6) in a vector form,

$$\dot{y}(t) = (I_N - 1_N\xi^T)F(x(t),t) - cLy(t) - cLe(t), \tag{7}$$

where $y(t) = (y_1(t), \ldots, y_N(t)) \in \mathbb{R}^N$, $F(x(t),t) = [f(x_1(t),t), \ldots, f(x_N(t),t)] \in \mathbb{R}^N$, $x(t) = (x_1(t), \ldots, x_N(t)) \in \mathbb{R}^N$ and $e(t) = (e_1(t), \ldots, e_N(t)) \in \mathbb{R}^N$.

Theorem 1: Suppose that Assumption 1 holds and the network G is strongly connected. Under the event-triggered function (3), the system (1) achieves the consensus asymptotically if

$$\lambda_\xi(L) > \frac{\rho+\gamma}{c} + \phi_1\frac{\|\Xi L\|}{\lambda_{\min}(\Xi)} + \phi_2\frac{\|\Xi L\|^2}{2a\lambda_{\min}(\Xi)} \tag{8}$$

holds, where $\rho > 0$, $\gamma > 0$, $c > 0$, $\phi_1 = \frac{\varphi_1(N_M+N)}{N_M - \varphi_1(N_M+N)} > 0$, $\phi_2 = \frac{\varphi_2 N_M N}{N_M - \varphi_1(N_M+N)} > 0$, $0 < \varphi_1 < \frac{N_M}{N_M+N}$, $\varphi_2 > 0$, $a > 0$, $N_M = \max\{\bar{N}_1, \bar{N}_2, \cdots, \bar{N}_N\}$. In addition, Zeno-behavior can be excluded.

Proof: Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}y^T(t)\Xi y(t). \tag{9}$$

Note that $V(t) \geqslant 0$ and $V(t) = 0$ if and only if $y(t) = 0$. Differentiating $V(t)$ along the trajectories of (7) gives

$$\begin{aligned}\dot{V}(t) =& y^T(t)\Xi\left[(I_N - 1_N\xi^T)F(x(t),t)\right.\\ &\left. - cLy(t) - cLe(t)\right].\end{aligned} \tag{10}$$

With Assumption 1, one has

$$\begin{aligned}&y^T(t)\Xi\left[(I_N - 1_N\xi^T)F(x(t),t)\right]\\ &= \sum\nolimits_{i=1}^{N} y_i(t)\xi_i\left[f(x_i(t),t) - f(\bar{x}(t),t)\right]\\ &\leqslant \sum\nolimits_{i=1}^{N}\|y_i(t)\|\xi_i\left(\rho\|x_i(t) - \bar{x}(t)\|\right)\\ &= \rho\sum\nolimits_{i=1}^{N}\xi_i\|y_i(t)\|^2.\end{aligned} \tag{11}$$

Taking (11) into (10), we have

$$\dot{V}(t) \leqslant \rho y^T(t)\Xi y(t) - cy^T(t)\Xi L y(t) - cy^T(t)\Xi L e(t). \tag{12}$$

From the event-triggered condition (3) with all $t \geqslant t_0$, we can conclude that $\|e_i(t)\|$ will not exceed the threshold, i.e.,

$$\|e_i(t)\| \leqslant \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\left\|\hat{x}_i(t^i_k) - \hat{x}_j(t^i_k)\right\| + \varphi_2 e^{-\gamma(t-t_0)}. \tag{13}$$

It follows from (13) that

$$\begin{aligned}\|e_i(t)\| \leqslant& \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\|x_i(t) + e_i(t) - x_j(t) - e_j(t)\\ &- \bar{x}(t) + \bar{x}(t)\| + \varphi_2 e^{-\gamma(t-t_0)}\\ =& \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\|y_i(t) + e_i(t) - y_j(t) - e_j(t)\|\\ &+ \varphi_2 e^{-\gamma(t-t_0)}\\ \leqslant& \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\|e_i(t)\| + \frac{\varphi_1}{\bar{N}_i}\|e(t)\| + \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\|y_i(t)\|\\ &+ \frac{\varphi_1}{\bar{N}_i}\|y(t)\| + \varphi_2 e^{-\gamma(t-t_0)}\end{aligned}$$

$$\leqslant \varphi_1 \|e_i(t)\| + \frac{\varphi_1}{N_M}\|e(t)\| + \varphi_1 \|y_i(t)\| + \frac{\varphi_1}{N_M}\|y(t)\| + \varphi_2 e^{-\gamma(t-t_0)}. \tag{14}$$

Then, by (14), we further have

$$\|e(t)\| \leqslant \left(\varphi_1 + \frac{N\varphi_1}{N_M}\right)\|e(t)\| + \left(\varphi_1 + \frac{N\varphi_1}{N_M}\right)\|y(t)\| + \varphi_2 N e^{-\gamma(t-t_0)},$$

i.e.,

$$\left[1 - \frac{\varphi_1(N_M+N)}{N_M}\right]\|e(t)\| \leqslant \frac{\varphi_1(N_M+N)}{N_M}\|y(t)\| + \varphi_2 N e^{-\gamma(t-t_0)}.$$

It follows from above inequality that

$$\begin{aligned}\|e(t)\| \leqslant& \frac{\varphi_1(N_M+N)}{N_M - \varphi_1(N_M+N)}\|y(t)\| + \frac{\varphi_2 N_M N}{N_M - \varphi_1(N_M+N)} e^{-\gamma(t-t_0)} \\ \triangleq& \phi_1 \|y(t)\| + \phi_2 e^{-\gamma(t-t_0)},\end{aligned}$$

where $\phi_1 > 0, \phi_2 > 0, 0 < \varphi_1 < \frac{N_M}{N_M+N}$. Substituting above expression into (12), with Lemma 1, one has

$$\begin{aligned}\dot{V}(t) \leqslant& \rho y^T(t)\Xi y(t) - c y^T(t)\Xi L y(t) + c\phi_1 \|\Xi L\| \|y(t)\|^2 + c\phi_2 \|y(t)\| \|\Xi L\| e^{-\gamma(t-t_0)} \\ \leqslant& \rho y^T(t)\Xi y(t) - c y^T(t)\Xi L y(t) + c\phi_1 \|\Xi L\| \|y(t)\|^2 + \frac{1}{2a} c\phi_2 \|y(t)\|^2 \|\Xi L\|^2 + \frac{a}{2} c\phi_2 e^{-2\gamma(t-t_0)} \\ \leqslant& 2\rho V(t) - 2c\lambda_\xi(L)V(t) + 2c\phi_1 \frac{\|\Xi L\|}{\lambda_{\min}(\Xi)} V(t) + c\phi_2 \left[\frac{\|\Xi L\|^2}{a\lambda_{\min}(\Xi)}V(t) + \frac{a}{2}e^{-2\gamma(t-t_0)}\right] \\ =& -2c\left(\lambda_\xi(L) - \frac{\rho}{c} - \phi_1 \frac{\|\Xi L\|}{\lambda_{\min}(\Xi)} - \phi_2 \frac{\|\Xi L\|^2}{2a\lambda_{\min}(\Xi)}\right)V(t) + c\phi_2 \frac{a}{2} e^{-2\gamma(t-t_0)}. \end{aligned} \tag{15}$$

If condition (8) holds, let

$$\sigma = \lambda_\xi(L) - \frac{\rho}{c} - \phi_1 \frac{\|\Xi L\|}{\lambda_{\min}(\Xi)} - \phi_2 \frac{\|\Xi L\|^2}{2a\lambda_{\min}(\Xi)} > \frac{\gamma}{c} > 0,$$

and we obtain that

$$\dot{V}(t) \leqslant -2c\sigma V(t) + \eta e^{-2\gamma(t-t_0)},$$

where $\eta = ac\phi_2/2$. Thus

$$V(t) \leqslant \left(V(t_0) + \frac{\eta}{2c\sigma - 2\gamma}\right) e^{-2\gamma(t-t_0)}. \tag{16}$$

Furthermore, it comes to

$$\|y(t)\| \leqslant \sqrt{\frac{2}{\lambda_{\min}(\Xi)}\left(V(t_0) + \frac{\eta}{2c\sigma - 2\gamma}\right)} e^{-\gamma(t-t_0)}. \tag{17}$$

Equation (17) implies that $\|y(t)\| \to 0$, as $t \to \infty$, and the consensus of MASs (1) is achieved under the proposed consensus protocol.

Remark 4: The designed parameters φ_1, φ_2 and γ of event-triggered function (3) and control gain c are closely related to the convergence rate and the inter-event interval. Intuitively, the smaller γ and the larger φ_1 and φ_2 lead to the longer triggering interval. On the other hand, the smaller the control gain c is, the longer triggering interval is. Therefore, triggering interval can be pre-designed by selecting these parameters properly. From (16), moreover, it is found that increasing the parameter γ or the control gain c can make the convergence rate increase.

In the following, we will show that the inter-event interval for each agent is strictly positive, $\{t^i_{k+1} - t^i_k\} \geq \tau_i > 0$ for all $k \in \mathbb{N}$ and $i = 1, 2, \cdots, N$, such that the proposed consensus protocol has no Zeno-behavior.

Suppose that the events of agent i occur at time instants $\{t^i_k\}_{k=0}^{\infty}$, that is, $e_i(t^i_k) = 0$. During the interval $t \in [t^i_k, t^i_{k+1}]$, for agent i in network G, one has

$$\begin{aligned}\|\dot{e}_i(t)\| \leqslant& \|u_i(t)\| + \|f(x_i(t),t)\| + \left\|\Delta(t^i_k, t^i_{k-1})\right\| \\ \leqslant& \rho \|x_i(t)\| + \|u_i(t)\| + \left\|\Delta(t^i_k, t^i_{k-1})\right\| \\ \leqslant& \rho\left(\|e_i(t)\| + \left\|\hat{x}_i(t^i_k)\right\|\right) + U + \left\|\Delta(t^i_k, t^i_{k-1})\right\|,\end{aligned}$$

where $\Delta(t^i_k, t^i_{k-1}) = \left(x(t^i_k) - x(t^i_{k-1})\right) / \left(t^i_k - t^i_{k-1}\right)$ and $U = \max_{t\in[t^i_k, t^i_{k+1})}\{\|u_i(t)\|\}$. Let $\chi(t^i_k, t^i_{k-1}) = \rho\left\|\hat{x}_i(t^i_k)\right\| + U + \left\|\Delta(t^i_k, t^i_{k-1})\right\|$, then we have

$$\|e_i(t)\| \leqslant \frac{\chi(t^i_k, t^i_{k-1})}{\rho} e^{\rho(t-t^i_k)} - \frac{\chi(t^i_k, t^i_{k-1})}{\rho}.$$

Taking $t = t^{i-}_{k+1}$, one has

$$\left\|e_i(t^{i-}_{k+1})\right\| \leqslant \frac{\chi(t^i_k, t^i_{k-1})}{\rho}\left(e^{\rho(t^i_{k+1}-t^i_k)} - 1\right). \tag{18}$$

According to the event-triggered condition (3), for agent i, we can see that the next triggering time instant t^i_{k+1} happens to the moment when

$$\left\|e_i(t^{i-}_{k+1})\right\| = \frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\left\|\hat{x}_i(t^i_k) - \hat{x}_j(t^i_k)\right\| + \varphi_2 e^{-\gamma(t^i_{k+1}-t_0)}. \tag{19}$$

Combining (18) and (19), we have

$$\frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\left\|\hat{x}_i(t^i_k) - \hat{x}_j(t^i_k)\right\| + \varphi_2 e^{-\gamma(t^i_{k+1}-t_0)}$$

$$\leqslant \frac{\chi(t_k^i, t_{k-1}^i)}{\rho}\left(e^{\rho(t_{k+1}^i - t_k^i)} - 1\right).$$

For some $i \in V$ and $k \in \mathbb{N}$, if there exists time instant t_{k+1}^i such that $\tau_i = t_{k+1}^i - t_k^i = 0$, then we have

$$\frac{\varphi_1}{\bar{N}_i}\sum_{j\in\mathcal{N}_i}\left\|\hat{x}_i(t_k^i) - \hat{x}_j(t_k^i)\right\| + \varphi_2 e^{-\gamma(t_k^i - t_0)} = 0,$$

which implies that $\varphi_1 = \varphi_2 = 0$. This contradicts with the fact that $\varphi_1 > 0$ and $\varphi_2 > 0$. Consequently, Zeno-behavior is excluded for all $k \in \mathbb{N}$ and $i = 1, 2, \cdots, N$. Theorem 1 is thus completed. $\square$

5. SIMULATION EXAMPLE

To illustrate the effectiveness of the proposed method in this paper, we consider the nonlinear MASs consisting of six agents, and the directed topology is plotted in Fig. 3. In addition, the Laplacian matrix L is chosen as

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 2 & 0 \\ 0 & -1 & 0 & 0 & -2 & 3 \end{bmatrix}.$$

The nonlinear dynamics of agent i is described by

$$\dot{x}_i(t) = u_i(t) + f(x_i(t), t), \quad i = 1, 2, \cdots, 6,$$

where $f(x_i(t), t) = \frac{0.1\sin t}{t}x_i(t) + 0.2\sin(t)$. By computation, one has $\rho = 0.1$. By Lemma 2, we have the nonnegative left eigenvector as $\xi = [0.3,\, 0.3,\, 0.1,\, 0.1\; 0.1,\, 0.1]^T$. According to Definition 2 and Lemma 2, one has the general algebraic connectivity degree as $\lambda_\xi(L) = 0.6045$. Selecting $c = 0.8$, $\varphi_1 = 0.01$, $\varphi_2 = 0.03$, $\gamma = 0.1$ and $a = 100$, we check the condition (8) in Theorem 1, $\lambda_\xi(L) > \frac{\rho+\gamma}{c} + \phi_1\frac{\|\Xi L\|}{\lambda_{\min}(\Xi)} + \phi_2\frac{\|\Xi L\|^2}{2a\lambda_{\min}(\Xi)} = 0.5826$.

The simulation results for both the proposed consensus protocol and the ZOH-based consensus protocol [17] are shown in Fig. 4 to Fig. 9. The state trajectories of nonlinear MASs (1) under the proposed protocol is indicated by

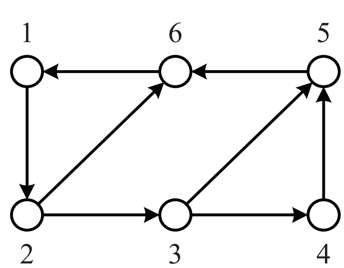

Fig. 3. The network topology.

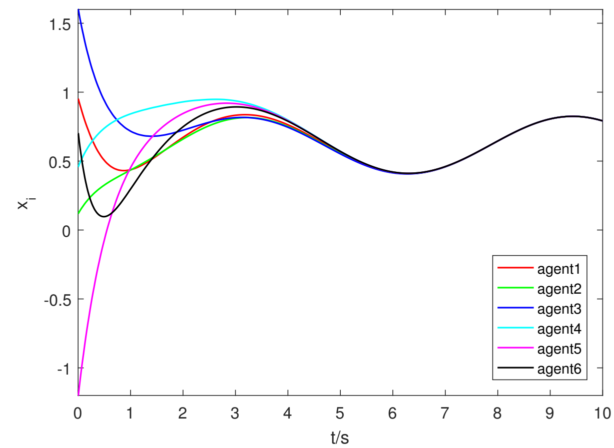

Fig. 4. State trajectories of agents based on FOH.

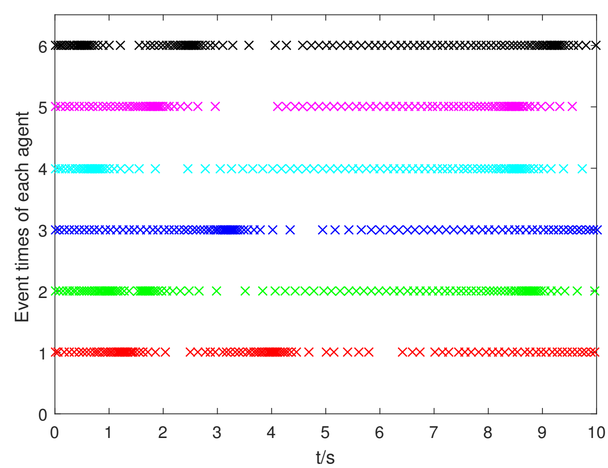

Fig. 5. Event time instants of agents based on FOH.

Fig. 4. It indicates that the global consensus of MASs (1) is achieved. In addition, Fig. 5 and Fig. 6 show the triggering instants of six agents by the proposed method (3) and the ZOH-based one, respectively. During $[0, 20]$ (s), the triggering times by proposed protocol for agent 1-6 are 328, 324, 300, 290, 305, 345, respectively, while the triggering times by ZOH-based protocol for agent 1-6 are 587, 650, 521, 587, 603, 599, respectively. In this example, we come to the conclusion that the triggering frequency is reduced by our proposed protocol.

From the dynamical trajectory of control signal of FOH-based in Fig. 7 and ZOH-based in Fig. 8, we can see that the control signal of FOH-based is piecewise linear, while that of ZOH-based is piecewise constant. Define the total consensus error as

$$\delta(t) = \sqrt{\sum_{i=1}^{6}\left(x_i(t) - \sum_{j=1}^{6}\xi_j x_j(t)\right)^2},$$

and we have compared consensus converging time for the proposed protocol and the existing ZOH-based protocol in

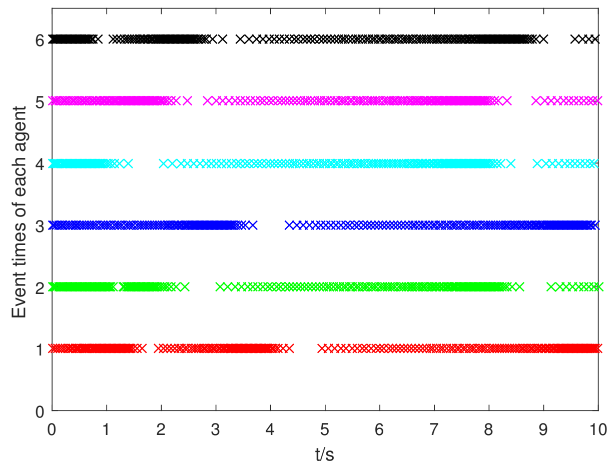

Fig. 6. Event time instants of agents based on ZOH.

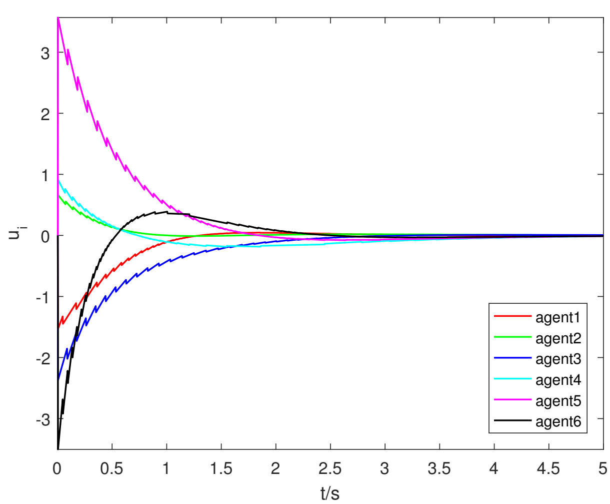

Fig. 7. State trajectories of $u_i(t)$ based on FOH.

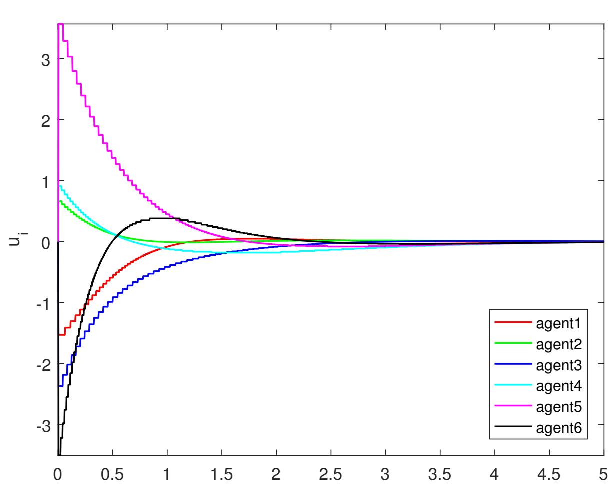

Fig. 8. Trajectories of $u_i(t)$ based on ZOH.

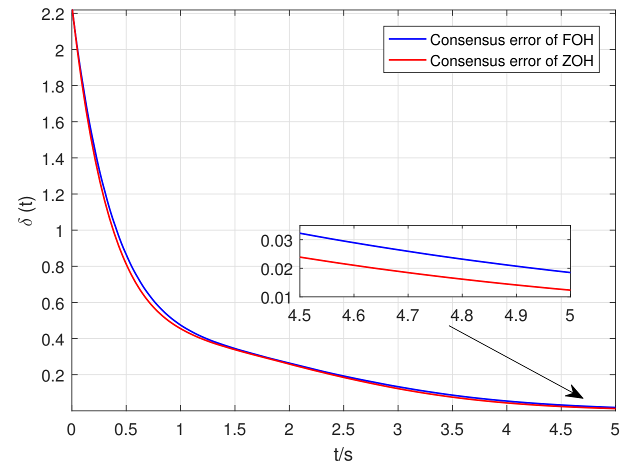

Fig. 9. Consensus error of the proposed and ZOH-based protocols.

Table 1. Comparison on the average triggering interval T_{at} (s) and the consensus converging time t_{ct} (s).

Control gain c	0.8	1	1.5	2	3
T_{at} (Proposed)	0.0634	0.0576	0.0446	0.0341	0.0264
T_{at} [17]	0.0305	0.0287	0.0213	0.0170	0.0151
t_{ct} (Proposed)	5.5050	4.4175	2.9525	2.2125	1.4625
t_{ct} [17]	5.1675	4.1425	2.7774	2.0700	1.3750

Fig. 9. By choosing $\delta(t) < 10^{-2}$, the consensus converging time of or proposed protocol is 5.5050 (s), and that of the ZOH-based protocol is 5.1675 (s).

In addition, we have also compared the average triggering intervals for the proposed FOH-based and the existing ZOH-based protocols under different control gain c, and the results are presented in Table 1. It is observed that the average triggering interval obtained by the proposed protocol is longer than that obtained by the ZOH-based protocol. Evidently, since the control inputs of the agents under the FOH-based protocol are updated less frequently, the convergence rate is slower than that for the ZOH-based protocol.

6. CONCLUSION

In this paper, we designed a distributed event-triggered consensus protocol to solve the consensus problem of the first-order nonlinear MASs with directed topology. Our proposed protocol uses the FOH instead of the standard ZOH to construct the event-triggered control signal. The stability of the closed-loop system has been analyzed by constructing Lyapunov function. In addition, it is also proved that there is no Zeno-behavior when the proposed FOH-based protocol is applied. Experimental results indicate that triggering rate can be highly reduced by using the FOH strategy. In our future work, we will focus on the event-triggered consensus control for second-order nonlinear MASs under denial-of-service attack.

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