

# Indirect Adaptive Robust Control Design for Course Tracking of Ships Subject to Unknown Control Coefficient and Disturbances

Jinbo Wu\* , Chenghao Zeng, and Yifei Hu

**Abstract:** For course control of ships with unknown control coefficient and model parameters, an indirect adaptive robust controller, in which the parameter estimation law and the control law are designed separately, is proposed. This design method can achieve not only excellent course control performance but also accurate parameter estimates for secondary purposes such as assisting in ship maneuvering decision. Firstly, a Nussbaum function is combined with the adaptive dynamic surface control method to design a strong robust controller which can ensure the stability of the closed-loop ship course control system in spite of parameter uncertainties, unknown control coefficient and disturbances. Secondly, the nonlinear model for ship steering is converted into linear form by using the X-swapping technique. And a modified least-squares identification algorithm is then proposed to estimate the unknown model parameters. The global uniform ultimate boundedness of all signals of the resulting closed-loop system is guaranteed via Lyapunov stability theory. Lastly, simulation results are executed to demonstrate the effectiveness of the proposed design method.

**Keywords:** Adaptive backstepping, course tracking, dynamic surface control, least-squares identification, Nussbaum function.

## 1. INTRODUCTION

The rising need for transportation services promotes the development of unmanned surface vehicles which is critical in providing cost-effective solutions to littoral, coastal and offshore problems [1,2]. Since the first mechanical autopilot was installed for automatic ship steering by Elmer Sperry, ship control systems have been an active area of research. While ship course control, which automatically steers the rudder to decrease the errors between the desired and the actual yaw angle, is always the main goal in navigation field.

The dynamics of ship steering motion are nonlinear in nature and are subject to a variety of disturbances such as waves, wind and ocean currents. Early research results show that the linear Nomoto model was adopted to describe the ship steering characteristics in control design, such as the model reference adaptive technique [3], neural network control [4] and fuzzy control [5]. Although these controllers based on linear steering model of ships can assure good course-keeping performance, they cannot satisfy the requirement for course changing maneuvers where the nonlinear characteristics of ship steering become non-ignorable. To deal with the nonlinear characteristics of ships, the feedback linearization techniques were

applied to course keeping [6,7]. But the model parameter uncertainties and external disturbances were not considered. The uncertainty and disturbance estimator (UDE)-based control methods were studied to deal with the model uncertainty and disturbances while the bandwidth information is required [8,9]. Sun *et al* [10] proposed a sliding mode approach to attenuate unmodeled dynamics and disturbances, but the chattering problem still was not effectively solved. It should be noted that backstepping has been widely used in the ship course control systems because it can often solve stabilization, tracking and robust control problems under conditions less restrictive than those encountered in other methods [11]. The neural network combined with the adaptive backstepping was introduced in [12] to solve a ship course tracking control problem with a prior knowledge of the sign of control coefficient. When the control coefficient is unknown, the Nussbaum-type gain was incorporated into the adaptive backstepping to construct the course control laws in [13,14]. Furthermore, the dynamic surface control was introduced into adaptive backstepping to obviate the problem of ‘explosion of terms’ existed in standard backstepping technique. This design idea has been successfully applied to course keeping [15] and course tracking [16] of ships.

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The above mentioned control strategies [11–16] are known as direct adaptive robust controller (DARC). Although the tracking performance is satisfactory, the identification accuracy of the unknown parameters exist in the nonlinear steering model is poor. To overcome the defects of DARC, the indirect adaptive robust controller (IARC) [17,18] was proposed, in which the controller and identifier can be designed separately. Therefore, advanced parameter identification algorithms can be applied to achieve accurate parameter estimation. The estimates can then be used for secondary purposes such as decision making in emergency control. [19] used a rate-limited projection type adaptation law to obtain more accurate parameter estimates, yet a priori knowledge of the bounds of unknown parameters is needed. In addition to the unknown model parameters that should be identified accurately for the nonlinear ship steering model, the rudder saturation constraint should also be considered in the control design. Only first-order linear model was studied in references [20,21] in presence of rudder saturation. In [22], an auxiliary system was utilized to reduce the influence of actuator saturation.

To deal with the nonlinear ship steering system with unknown control coefficient, uncertain model parameters, disturbances and rudder saturation constraint, this paper develops a course tracking control law under the framework of IARC. This control law can guarantee the global uniform ultimate boundedness of the closed-loop system and accurate identification of model parameters simultaneously. This paper is organized as follows: Section 2 presents the nonlinear ship steering model. In Section 3, an adaptive robust controller accompanying with Nussbaum function is introduced and an on-line parameter estimation law is presented to obtain accurate parameter estimates. Simulation results and comparisons are described in Section 4. Conclusions are drawn in Section 5.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

The following nonlinear Norrbin model is considered:

$$T\dot{\psi} + \psi + \alpha\psi^3 = K\delta + w, \quad (1)$$

where  $\psi$  is the yaw angle of ships and  $\delta$  is the rudder angle.  $K$ ,  $T$  are the gain and time parameters of the ship steering dynamic model, respectively.  $\alpha$  is the Norrbin coefficient. The model parameters  $K$ ,  $T$  and  $\alpha$  are unknown bounded constants.  $w$  is the disturbance term which is induced by waves and wind. It is generally considered that the ocean currents only affect the position of the ship's motion, so the ocean currents are ignored in the ship steering model (1).

Let  $x_1 = \psi$ ,  $x_2 = \dot{\psi}$ , and  $u = \delta$ , then the nonlinear ship steering motion equation can be transformed into the fol-

lowing state space expression:

$$\dot{x}_1 = x_2, \quad (2)$$

$$\dot{x}_2 = \theta_0 u + \varphi^T(x)\theta + d(t), \quad (3)$$

where  $\theta_0 = K/T$  is the unknown control coefficient;  $\varphi^T(x) = [\varphi_1(x), \varphi_2(x)]$  where  $\varphi_1(x) = x_2$  and  $\varphi_2(x) = x_2^3$ ;  $\theta = [\theta_1, \theta_2]^T$  is a vector of unknown bounded parameters with  $\theta_1 = -1/T$  and  $\theta_2 = -\alpha/T$ ;  $d(t) = w/T$  is the external disturbance;  $u$  is the control input with saturation constrains.

**Remark 1:** If  $\theta_0$ ,  $\theta_1$ ,  $\theta_2$  can be estimated accurately, then  $K$ ,  $T$ ,  $\alpha$  can be calculated accordingly.  $K$  is the index of turning ability, while  $T$  is the index of quick response in steering [23]. In practical,  $K$  and  $T$  can be used to evaluate the ship's steering distance to the new heading and evaluate the ship's turning radius during constant turning. Accurate  $K$  and  $T$  can also provide reference information for emergency control. Therefore, accurate parameter estimates of  $\theta_0$ ,  $\theta_1$ ,  $\theta_2$  are valuable.

**Assumption 1:** The control coefficient  $\theta_0$  is nonzero, and its sign is unknown. Specially, we assume  $|\theta_0| \leq B$  with  $B$  being a known positive constant.

**Remark 2:** The values of  $T$  indicate the straight-line stability of ships [24], for ships with stable line-movement,  $T > 0$ , for ships with unstable line-movement,  $T < 0$ . Therefore, supposing the sign of  $\theta_0$  to be unknown is practical.

The wind and wave disturbance can be deemed as an equivalent rudder angle [25-26].

$$\delta_{wind} = K^0 \left( \frac{V_T}{U_T} \right)^2 \sin \gamma_R,$$

$$\delta_{wave} = \frac{M_w}{K_w U_T^2},$$

where  $K^0$  is the leeway coefficient,  $V_T$  is the relative wind speed,  $U_T$  denotes the ship speed, and  $\gamma_R$  is the wind angle on the bow.  $M_w$  denote the wave-induced yaw moment,  $K_w$  is the proportionality coefficient which is related to the size and the load of ships.

From the discussion above, the unknown external disturbance  $d(t)$  can be written as

$$d(t) = \frac{w}{T} = \frac{1}{T} [K(\delta_{wind} + \delta_{wave})], \quad (2)$$

we can conclude that  $d(t)$  is bounded by  $|d(t)| \leq p^* \varpi(t)$ , where  $p^*$  is an unknown positive constant and  $\varpi(t)$  is a known non-negative smooth function.

Consider the saturation characteristics of the ship rudder, we have  $|u| \leq u_{\max}$ .  $u_{\max}$  is the known limit of input saturation constraint. In practice, maximum rudder angle usually does not exceed 35 degrees. Consequently, the

control input is defined by

$$u = \text{sat}(u_c) \begin{cases} u_{\max}, & u_c > u_{\max}, \\ u_c, & |u_c| \leq u_{\max}, \\ -u_{\max}, & u_c < -u_{\max}, \end{cases} \quad (3)$$

where  $u_c$  is the control input which will be designed in Section 3.

The control objective of this paper is to design an indirect adaptive robust controller for the nonlinear ship steering system which can guarantee that the actual course  $\psi(t)$  tracks the desired course  $\psi_d(t)$  with arbitrarily small error, while all signals of the closed-loop system can be guaranteed as uniformly ultimately bounded. Also, the accurate parameter estimation should be realized.

**Assumption 2:** The desired course  $\psi_d(t)$  and its derivatives  $\dot{\psi}_d(t)$ ,  $\ddot{\psi}_d(t)$  are smooth and bounded.

**Definition 1 [27]:** A continuous function  $N(\zeta)$  can be called a Nussbaum-type function if the following properties hold for  $N(\zeta)$ :

$$\limsup_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty, \quad (4)$$

$$\liminf_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty. \quad (5)$$

In this paper, Nussbaum function  $e^{\zeta^2} \cos(\frac{\pi}{2}\zeta)$  is chosen and  $\zeta$  is its function variable.

**Lemma 1 [27]:** Let  $V(\cdot)$  and  $\zeta(\cdot)$  be smooth functions defined on  $[0, t_f]$  with  $V(t) \geq 0, \forall t \in [0, t_f]$ ,  $N(\zeta)$  be a smooth Nussbaum-type function. For  $\forall t \in [0, t_f]$ , if the following inequality holds:

$$V(t) \leq e^{-C_1 t} \int_0^t [\hbar N(\zeta) + 1] \dot{\zeta} e^{C_1 \tau} d\tau + C_2, \quad (6)$$

where the constant  $C_1 > 0$ ,  $\hbar$  is a nonzero constant,  $C_2$  represents a suitable constant. Then  $V(t)$ ,  $\zeta(t)$  and  $\int_0^t [\hbar N(\zeta) + 1] \dot{\zeta} e^{C_1 \tau} d\tau$  are bounded on  $[0, t_f]$ .

Hereafter, for simplicity, the explicit dependence on  $t$  in the notation will be dropped (that is, let  $V = V(t)$ ,  $\delta = \delta(t)$  and so on).

### 3. INDIRECT ADAPTIVE ROBUST CONTROLLER DESIGN

In this section the IARC scheme is developed. It consists of two parts: the adaptive robust course tracking controller design and the parameter estimation design. Unknown control coefficient, unknown model parameters, disturbance uncertainties and rudder saturation constraint are considered simultaneously in the control design to illustrate the robustness.

#### 3.1. Indirect adaptive robust control law

In this subsection, we design an indirect adaptive robust course tracking control law for the ship steering system. The design process consists of two steps.

**Step 1:** Define the heading error variable

$$\tilde{x}_1 = x_1 - \psi_d. \quad (7)$$

According to (2), the time derivative of  $\tilde{x}_1$  is

$$\dot{\tilde{x}}_1 = x_2 - \dot{\psi}_d. \quad (8)$$

Choose the virtual control function as

$$\alpha_1 = -k_1 \tilde{x}_1 + \dot{\psi}_d, \quad (9)$$

where  $k_1$  is a positive parameter to be designed.

Following the idea of dynamic surface control [28], let  $x_{2d}$  be generated by the filter

$$\dot{x}_{2d} = -\omega(x_{2d} - \alpha_1), \quad (10)$$

where  $\omega$  is a positive constant and the initial condition of the filter is  $x_{2d}(0) = \alpha_1(0)$ , which yields  $\dot{x}_{2d}(0) = 0$ .

Define error variable

$$\tilde{x}_2 = x_2 - x_{2d}. \quad (11)$$

To design a stabilizing adaptive control law, consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2} \tilde{x}_1^2 + \frac{1}{2} s_1^2, \quad (12)$$

where  $s_1 = x_{2d} - \alpha_1$ .

The time derivative of  $V_1$  is

$$\dot{V}_1 = \tilde{x}_1(x_2 - \dot{\psi}_d) + s_1(\dot{x}_{2d} - \dot{\alpha}_1). \quad (13)$$

By the definition of  $\tilde{x}_2$  in (11), the virtual control function (9) and first order filter (10), (13) can be written as

$$\begin{aligned} \dot{V}_1 &= \tilde{x}_1(\tilde{x}_2 + s_1 - k_1 \tilde{x}_1) + s_1[-\omega(x_{2d} - \alpha_1) + k_1 \tilde{x}_1 - \dot{\psi}_d] \\ &= \tilde{x}_1 \tilde{x}_2 + \tilde{x}_1 s_1 - k_1 \tilde{x}_1^2 - \omega s_1^2 \\ &\quad + s_1 k_1 (\tilde{x}_2 + s_1 - k_1 \tilde{x}_1) - \dot{\psi}_d s_1 \\ &\leq \frac{1}{4} \tilde{x}_1^2 + \tilde{x}_2^2 + \frac{1}{4} \tilde{x}_1^2 + s_1^2 - k_1 \tilde{x}_1^2 - \omega s_1^2 + k_1 s_1^2 + \frac{1}{4} \tilde{x}_2^2 \\ &\quad + k_1^4 s_1^2 + \frac{1}{4} \tilde{x}_1^2 + 4s_1^2 + \frac{1}{16} \dot{\psi}_d^2 \\ &\leq \frac{5}{4} \tilde{x}_2^2 + \frac{1}{16} \dot{\psi}_d^2 - \left(k_1 - \frac{3}{4}\right) \tilde{x}_1^2 \\ &\quad - (\omega - k_1^4 - k_1^2 - k_1 - 5) s_1^2. \end{aligned} \quad (14)$$

Let  $D_1 = \min\{k_1 - \frac{3}{4}, \omega - k_1^4 - k_1^2 - k_1 - 5\}$ , then choose the design parameters  $k_1 - \frac{3}{4} > 0$  and  $\omega - k_1^4 - k_1^2 - k_1 - 5 > 0$ .

We obtain

$$\dot{V}_1 \leq -D_1 V_1 + \frac{5}{4} \tilde{x}_2^2 + \frac{1}{16} \dot{\psi}_d^2. \quad (15)$$

**Step 2:** Consider (2) and differentiate  $\tilde{x}_2$  with respect to time yields, we have

$$\dot{\tilde{x}}_2 = \theta_0 u + \varphi^T(x) \theta + d(t) - \dot{x}_{2d}. \quad (16)$$

To handle the problem of input saturation of rudder, the following auxiliary design system is introduced [29]

$$\dot{e}_s = \begin{cases} -\gamma e_s - \frac{f(\cdot)}{\|e_s\|^2} \cdot e_s + \Delta u, & \|e_s\| \geq \varepsilon, \\ 0, & \|e_s\| < \varepsilon, \end{cases} \quad (17)$$

where  $f(\cdot) = f(\tilde{x}_2, \Delta u) = |\tilde{x}_2 \cdot B \cdot \Delta u| + 0.5\Delta u^2$ ,  $\Delta u = u - u_c$ ,  $\gamma > 0$  is a design parameter,  $\varepsilon > 0$  is a small design parameter and  $e_s$  is a variable which is introduced to facilitate the analysis of the effect of input saturation.

Then the actual control  $u_c$  appears. Let the control input be design as

$$u_c = N(\zeta_2) \eta_2, \quad (18)$$

with

$$\eta_2 = k_2(\tilde{x}_2 - e_s) + \varphi^T \hat{\theta} - \dot{x}_{2d} + v_2, \quad (19)$$

$$\dot{\zeta}_2 = \tilde{x}_2 \eta_2, \quad (20)$$

$$v_2 = \frac{1}{4} k \tilde{x}_2 (\varphi^T \varphi + \varpi^2), \quad (21)$$

where  $k$  and  $k_2$  are positive parameters to be designed.

Consider the following Lyapunov function candidate:

$$V_2 = \frac{1}{2} \tilde{x}_2^2 + \frac{1}{2} e_s^2. \quad (22)$$

Invoking (16), the time derivative of  $V_2$  is

$$\dot{V}_2 = \tilde{x}_2 [\theta_0 u + \varphi^T \theta + d(t) - \dot{x}_{2d}] + e_s \cdot \dot{e}_s. \quad (23)$$

It is clear that

$$e_s \cdot \dot{e}_s \leq -\gamma e_s^2 - \frac{|\tilde{x}_2 \cdot \theta_0 \cdot \Delta u| + \frac{1}{2} \Delta u^2}{\|e_s\|^2} \cdot e_s^2 + \Delta u \cdot e_s, \quad (24)$$

$$\Delta u \cdot e_s \leq \frac{1}{2} \Delta u^2 + \frac{1}{2} e_s^2. \quad (25)$$

Substituting (24) and (25) into (23), we have

$$\begin{aligned} \dot{V}_2 &\leq \tilde{x}_2 [\theta_0 u + \varphi^T \theta + d(t) - \dot{x}_{2d}] - (\gamma - 0.5) e_s^2 \\ &\quad - |\tilde{x}_2 \cdot \theta_0 \cdot (u - u_c)| \\ &\leq \tilde{x}_2 [\theta_0 u_c + \varphi^T \theta + d(t) - \dot{x}_{2d}] - (\gamma - 0.5) e_s^2. \end{aligned} \quad (26)$$

Adding and subtracting  $\dot{\zeta}_2$  on the right side of (26) and considering (18)-(21), we have

$$\begin{aligned} \dot{V}_2 &\leq \theta_0 N(\zeta_2) \tilde{x}_2 \eta_2 + \dot{\zeta}_2 - k_2 \tilde{x}_2^2 - \tilde{x}_2 \varphi^T \hat{\theta} + \tilde{x}_2 \dot{x}_{2d} - \tilde{x}_2 v_2 \\ &\quad + \tilde{x}_2 \varphi^T \theta + \tilde{x}_2 d(t) - \tilde{x}_2 \dot{x}_{2d} + k_2 \tilde{x}_2 e_s - (\gamma - 0.5) e_s^2 \\ &= [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 - k_2 \tilde{x}_2^2 - \tilde{x}_2 \varphi^T \tilde{\theta} + \tilde{x}_2 d(t) \\ &\quad - \tilde{x}_2 v_2 + k_2 \tilde{x}_2 e_s - (\gamma - 0.5) e_s^2 \\ &\leq -k_2 \tilde{x}_2^2 + [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 - \tilde{x}_2 \varphi^T \tilde{\theta} - \frac{1}{4} k \tilde{x}_2 \varphi^T \varphi \end{aligned}$$

$$\begin{aligned} &+ |\tilde{x}_2| p^* \varpi - \frac{1}{4} k \tilde{x}_2^2 \varpi^2 + \frac{1}{2} k_2 \tilde{x}_2^2 \\ &+ \frac{1}{2} k_2 e_s^2 - (\gamma - 0.5) e_s^2, \end{aligned} \quad (27)$$

where  $\tilde{\theta} = \hat{\theta} - \theta$  is the parameter estimation error vector.

By completing the squares, we have

$$\begin{aligned} &-\frac{1}{4} k \tilde{x}_2^2 \varpi^2 + |\tilde{x}_2| p^* \varpi \leq \frac{1}{k} p^{*2}, \\ &-\frac{1}{4} k \tilde{x}_2^2 \varphi^T \varphi - \tilde{x}_2 \varphi^T \tilde{\theta} \leq \frac{1}{k} \tilde{\theta}^T \tilde{\theta}. \end{aligned}$$

By choosing  $\gamma - \frac{1}{2} k_2 - 0.5 > \frac{1}{2} k_2$ , (27) can be written as

$$\dot{V}_2 \leq -k_2 V_2 + [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 + \frac{1}{k} \tilde{\theta}^T \tilde{\theta} + \frac{1}{k} p^{*2}. \quad (28)$$

**Lemma 2:** For the nonlinear ship steering system (2), when Assumptions 1-2 are satisfied, if an estimation law can be adopted to guarantee the boundedness of the unknown parameters, then our proposed control law (18) guarantees that the actual course  $\psi$  can track the desired course  $\psi_d$  with arbitrary small error.

**Proof:** If the boundedness of  $\tilde{\theta}$  can be guaranteed via an estimation law. Then, (28) can be written as

$$\dot{V}_2 \leq -k_2 V_2 + [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 + M, \quad (29)$$

where  $M$  is a constant with  $M > \frac{1}{k} \tilde{\theta}^T \tilde{\theta} + \frac{1}{k} p^{*2}$ .

Multiplying  $e^{k_2 t}$  on both sides of (29) leads to

$$\frac{d}{dt} (V_2 e^{k_2 t}) \leq [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 e^{k_2 t} + M e^{k_2 t}. \quad (30)$$

Integrating over  $[0, t]$  and multiplying both sides by  $e^{-k_2 t}$ , we obtain

$$\begin{aligned} V_2 &\leq e^{-k_2 t} V_2(0) + e^{-k_2 t} \int_0^t [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 e^{k_2 \sigma} d\sigma + \frac{M}{k_2} \\ &= C + e^{-k_2 t} \int_0^t [\theta_0 N(\zeta_2) + 1] \dot{\zeta}_2 e^{k_2 \sigma} d\sigma, \end{aligned} \quad (31)$$

where  $C = e^{-k_2 t} V_2(0) + \frac{M}{k_2}$  is bounded.

According to Lemma 1, we can conclude that  $V_2$  and  $\zeta_2$  are bounded. From the definition of  $V_2$ , we can conclude that  $\tilde{x}_2$  and  $e_s$  are bounded.

Notice Assumption 2, then (15) can be written as

$$V_1 \leq -D_1 V_1 + A, \quad (32)$$

where  $A$  is a constant which satisfies  $A > \frac{5}{4} \tilde{x}_2^2 + \frac{1}{16} \tilde{\psi}_d^2$ .

We can further conclude

$$V_1 \leq \frac{A}{D_1} + \left[ V_1(0) - \frac{A}{D_1} \right] e^{-D_1 t}. \quad (33)$$

From (12) and (33), we obtain

$$|\tilde{x}_1| \leq \sqrt{\frac{2A}{D_1} + \left[ V_1(0) - \frac{A}{D_1} \right] e^{-D_1 t}}. \quad (34)$$

For given arbitrary  $\mu > \sqrt{2A/D_1}$ , there exist a constant  $T > 0$  that for any  $t > T$ , the tracking error  $\|\tilde{x}_1\| \leq \mu$ . By choosing the design parameters  $k_1, k_2$  and  $\omega$  appropriately, we can make  $\sqrt{2A/D_1}$  arbitrary small. Namely, the actual course  $\psi$  can track the desired course  $\psi_d$  with arbitrary small error.  $\square$

### 3.2. Parameter estimation algorithm

In this subsection, a modified on-line LS identification algorithm [30] is adopted with the help of the X-swapping technique [8].

Equation (2) can be rewritten in the following form:

$$\begin{aligned} \dot{x} &= H(x) + F(x, u)^T \theta_b + \Delta, \\ F(x, u)^T &= \begin{bmatrix} 0 & 0 & 0 \\ x_2 & x_2^3 & u \end{bmatrix}, \end{aligned} \quad (35)$$

where  $\dot{x} = [\dot{x}_1, \dot{x}_2]^T$ ,  $H(x) = [x_2, 0]^T$ ,  $\theta_b = [\theta_1, \theta_2, \theta_0]^T$  and  $\Delta = [0, d(t)]^T$ .

The following filters are constructed:

$$\dot{\psi}_0 = N(x, t)(\psi_0 + x) - H(x), \quad \psi_0 \in \mathbb{R}^2, \quad (36)$$

$$\dot{\psi}^T = N(x, t) \psi^T + F(x, u)^T, \quad \psi \in \mathbb{R}^{2 \times 3}, \quad (37)$$

where  $N(x, t)$  is exponentially stable for each  $x$  continuous in  $t$ . To guarantee the boundedness of  $\psi$ , a particular choice of  $N(x, t)$  is [11]

$$N(x, t) = N_0 - \lambda F(x, u)^T F(x, u) Q, \quad (38)$$

where  $\lambda > 0$  and  $N_0$  is a negative constant matrix which satisfies:  $QN_0 + N_0^T Q = -I$ ,  $Q = Q^T > 0$ .

Define  $Y = x + \psi_0$ , we have

$$\dot{Y} = N(x, t)Y + F(x, u)^T \theta_b + \Delta(x, t). \quad (39)$$

We can further verify that

$$Y = \psi^T \theta_b + \eta. \quad (40)$$

The nonlinear model (2) is thus transformed into a static form which is linear in  $\theta_b$ . The unknown disturbance can be written as  $\eta = x + \psi_0 - \psi^T \theta_b$ .

For the model (40), when Assumptions 1-2 are satisfied and the control law (18) is chosen, as in [18], we can prove that  $\psi_0, Y, \psi, \eta \in L_\infty[0, \infty)$ .

Before putting forward the identification algorithm, we first define

$$P_d(t) = \beta P(t) - P(t) \psi(t) \psi(t)^T P(t) + \mu I, \quad (41)$$

$$\begin{aligned} Q(t) &= \frac{d(\lambda_{\max}(P(t)))}{dt} \\ &= \frac{\text{tr}(\text{adj}(P(t)) - \lambda_{\max}(P(t)) \cdot I) \cdot P_d(t)}{\text{tr}(\text{adj}(P(t)) - \lambda_{\max}(P(t)) \cdot I)}, \end{aligned} \quad (42)$$

where is the covariance matrix which is symmetrical with, is the forgetting factor and is a positive constant.  $\lambda_{\max}(\cdot)$  denote the maximum eigenvalues of the corresponding matrices.  $\text{tr}(\cdot)$  and  $\text{adj}(\cdot)$  separately denote the trace and the adjoint of the corresponding matrix.

Then the following on-line modified least-squares identification algorithm is adopted:

$$\begin{aligned} \dot{P}(t) &= \begin{cases} P_d(t), & \text{if } \lambda_{\max}(P(t)) < P_U \text{ or if} \\ & \lambda_{\max}(P(t)) = P_U \text{ and } Q(t) < 0, \\ 0, & \text{otherwise,} \end{cases} \\ \dot{\hat{\theta}}_b(t) &= \begin{cases} P(t) \psi(t) e(t), & \text{if } \lambda_{\max}(P(t)) < P_U \text{ or if} \\ & \lambda_{\max}(P(t)) = P_U \text{ and } Q(t) < 0, \\ P(t) \psi(t) e(t) - \sigma P(t) \hat{\theta}_b(t), & \text{otherwise,} \end{cases} \end{aligned} \quad (43)$$

where  $\hat{\theta}_b(t)$  is the estimate of the unknown parameters,  $P_U$  is a design scalar chosen as  $P(0) < P_U I$ ,  $\sigma > 0$  represents the fixed -modification and  $e(t)$  is the prediction error defined as

$$e(t) = y(t) - \hat{y}(t) = -\psi^T(t) \tilde{\theta}_b(t) + w(t), \quad (44)$$

where  $\tilde{\theta}_b(t) = \hat{\theta}_b(t) - \theta_b(t)$  is the estimation error.

As shown in [31], we can state the following lemma:

**Lemma 3:** For the linear model (40) with  $\psi_0, Y, \psi, \eta \in L_\infty[0, \infty)$ , the adopted estimation algorithm (43) guarantees that

(a)  $P, P^{-1}, e, \hat{\theta}_b, \dot{\hat{\theta}}_b \in L_\infty[0, \infty)$ .

(b) If  $\psi$  satisfies the following persistent excitation (PE) condition:

$$\exists T_0, \alpha_0 > 0, \text{ s.t. } \frac{1}{T_0} \int_t^{t+T_0} \psi(\tau) \psi^T(\tau) d\tau \geq \alpha_0 I, \quad \forall t, \quad (45)$$

then the parameter estimates  $\hat{\theta}_b$  converge to their true values.

### 3.3. Performance result

**Theorem 1:** For the nonlinear ship steering system (2) with unknown control coefficients, uncertain disturbances and rudder saturation constraint, when Assumptions 1-2 are satisfied, if we apply the control law (18) with virtual control (9), first order filter (10), auxiliary design system (17) and the identification algorithm (37), then all signals of the closed-loop ship course control system are globally uniformly ultimately bounded. And the actual course can track the desired course with arbitrary small error by choosing the design parameters appropriately.

**Proof:** From Lemma 3, the boundedness of the unknown parameters can be guaranteed by the identification algorithm (43). Then according to Lemma 2, the proof can be easily completed.  $\square$



#### 4. SIMULATION AND COMPARISON STUDIES

In this section, simulation results are presented to show the effectiveness of our proposed control law and parameter estimation algorithm. Also, to illustrate the superiority of the proposed IARC scheme, the direct adaptive robust control (DARC) approach [16] is chosen to make a comparison.

A small ship of 45 m in length is taken as the test bed. Its dynamic parameters are  $K = 0.5 \text{ s}^{-1}$ ,  $T = 31 \text{ s}$  and  $\alpha = 0.4 \text{ s}^2$  at a forward speed of  $U = 5 \text{ m/s}$  [31].

The desired course signal  $\psi_d$  is chosen as:  $\psi_d = 25 \sin(t/12)$ , we can easily conclude that the desired course  $\psi_d$  and its derivatives  $\dot{\psi}_d$ ,  $\ddot{\psi}_d$  are smooth and bounded which meets Assumption 2.

**Remark 3:** The chosen  $\psi_d$  actually presents a similar shape to the trajectory of a zigzag test. Zigzag test is a commonly used method to obtain the value of  $K$  and  $T$ , but it has limitations such as the high requirement for experimental waters and cumbersome operation. However, our method does not suffer from these drawbacks. Choose the trajectory similar to zigzag test and apply advanced identification law, we can identify the parameters accurately and the value of  $K$  and  $T$  can then be obtained for other purposes. In addition, the method proposed in this paper could further be extended to coordinated formation control if cooperated with other strategies like game theory [32].

For wind and wave disturbances, corresponding to the sea state 3, we choose  $K^0 = 1.47$ ,  $V_T = 3.4 \text{ m/s}$ ,  $U_T = 5 \text{ m/s}$ ,  $\gamma_R = 10^\circ$ ,  $K_w = 3500$ . is calculated by the same method in [16] with parameters chosen as the same.

In simulation, the initial conditions are chosen as  $[x_1(0), x_2(0)] = [0.5^\circ \ 2^\circ/s]$ ,  $\zeta_2(0) = 1.9$ ,  $e_s(0) = 0$ ,  $\varepsilon = 0.01$ ,  $\psi_0(0) = [0, 0]^T$ ,  $\psi(0) = [0, -2, 0; 0, 0, 0]$ ,  $\hat{\theta}_b(0) = [0, 0, 0]$ ,  $P(0) = [5, 0, 0; 0, 7.5, 0; 0, 0, 5]$ . For control design parameters, we choose  $k = 1$ ,  $k_1 = 1$ ,  $k_2 = 10$ ,  $\gamma = 15$  and  $\varpi(t) = 1$ . The coefficient of the first-order filter (10) is chosen as . For formula (38), choose  $N_0 = [-30, 0; 0, -30]$ ,  $\lambda = 0.4$  and  $Q = 0.025I$ . The adopted parameter estimation law is employed with design parameters  $P_U = 10^4$ ,  $\sigma = 10^{-4}$ ,  $\beta = 0.05$ , and  $\mu = 0.1$ .

For the algorithm to be compared, Most initial value choices are consistent with those in [16].

The simulation results are shown in Figs. 1-8. We can see from Figs. 1-3 that IARC and DARC approaches can both achieve satisfactory control performance and our proposed IARC method leads to smaller errors. From Fig. 3, we can obtain that the ship actual control rudder angle  $\delta$  is practical. Fig. 4 shows that the actual heading rate does not exceed the restraint of  $|\dot{\psi}(t)|_{\max} = 3^\circ/\text{s}$ . Fig. 5 presents that the Nussbaum function and its argument are bounded.

Figs. 6-8 present the estimation effect of the unknown parameters. It is obviously seen that the DARC approach in [16] cannot trace the actual value accurately. On the

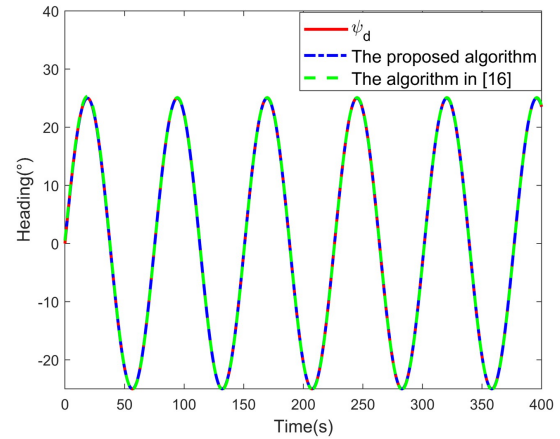


Fig. 1. Actual course angles under different algorithms.

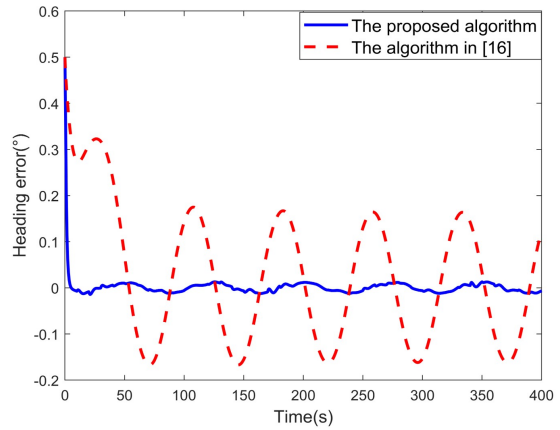


Fig. 2. Tracking errors under different algorithms.

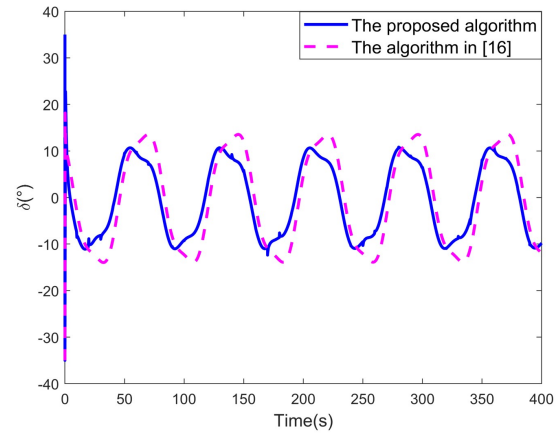


Fig. 3. Control rudder angles under different algorithms.

contrary, the adopted LS identification algorithm achieves satisfactory parameter estimates.

It should be noted that the DARC algorithms typically

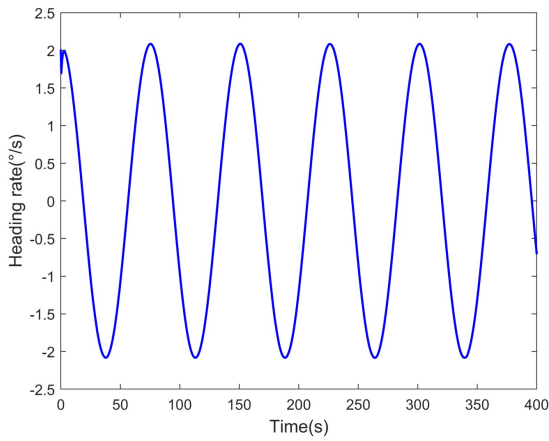


Fig. 4. Time curve of heading rate under the proposed algorithm.

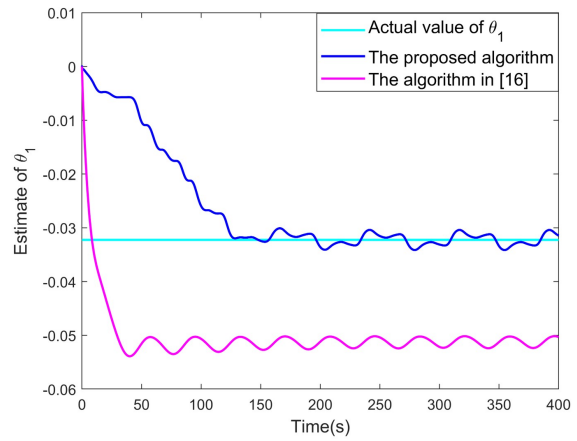


Fig. 7. Estimation effect of parameter  $\theta_1$ .

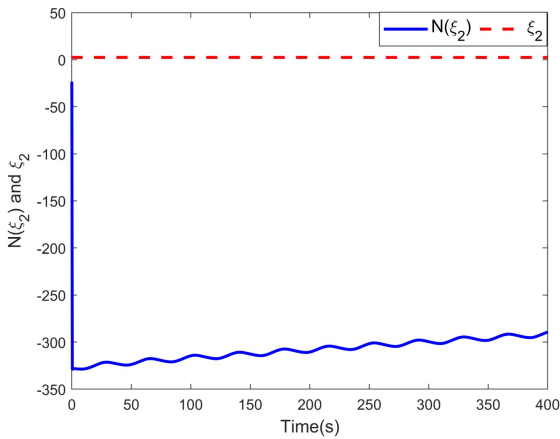


Fig. 5. Nussbaum function  $N(\xi_2)$  and its argument  $\xi_2$ .

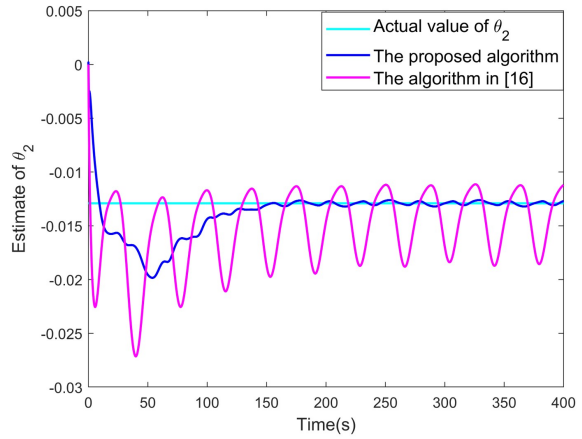


Fig. 8. Estimation effect of parameter  $\theta_2$ .

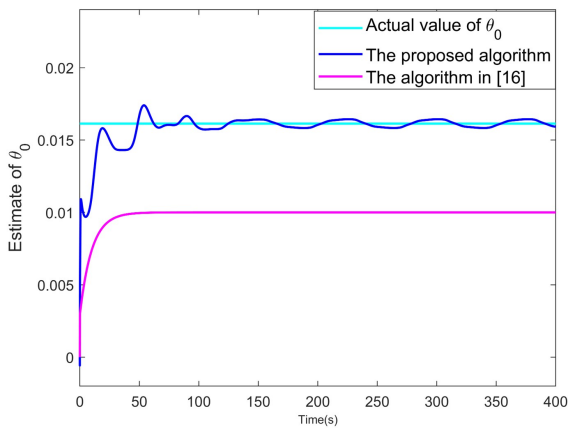


Fig. 6. Estimation effect of parameter  $\theta_0$ .

use tracking errors as the driving signals which are normally small, so the parameter adaptation is tend to be cor-

rupted by noise or other factors [17]. To further explain why the algorithm in [16] cannot achieve good estimation effect, let's take a close look at the parameter adaptive laws in [16]

$$\begin{aligned} \dot{\hat{\theta}}_0 &= \lambda_{\theta_0} \rho \Delta u - \lambda_{\theta_0} \sigma_{\theta_0} (\hat{\theta}_0 - \theta_0^0), \\ \dot{\hat{\theta}}_j &= \lambda_{\theta_j} \rho \varphi_j - \lambda_{\theta_j} \sigma_{\theta_j} (\hat{\theta}_j - \theta_j^0), \quad j = 1, 2. \end{aligned} \quad (46)$$

In fact, the performance of parameter adaptive laws in [16] depends on the design parameters  $\theta_0^0$ ,  $\theta_1^0$  and  $\theta_2^0$ . The accurate estimations are based on the conditions that the values of  $\theta_0^0$ ,  $\theta_1^0$  and  $\theta_2^0$  are set close to the actual values. Therefore, a priori knowledge is required to set appropriate preset values. In practice, however, such priori knowledge is hardly available. The proposed method does not depend on such prior knowledge, and can track the actual value accurately. The relatively more accurate estimates of IARC approach can then be used for other secondary purpose (e.g., decision making in emergency control).

## 5. CONCLUSION

In this paper, an indirect adaptive robust controller has been presented for course control of ships with unknown control coefficient, unknown model parameters, disturbance uncertainties and rudder saturation constraint. Based on the backstepping technique, the Nussbaum function is exploited to handle the problem of unknown control coefficient, DSC technique is utilized to escape the problem of ‘explosion of complexity’ and the auxiliary design system is introduced to deal with the rudder saturation constraint. The global uniform ultimate boundedness of the closed-loop system signals can be achieved by using the proposed IARC scheme. Furthermore, a modified on-line LS identification algorithm is introduced to achieve good estimation performance. Simulation results illustrate the effectiveness and superiority of the proposed method. Further, the design process proposed in this paper can be extended to course tracking of ships with Time-varying parameters.

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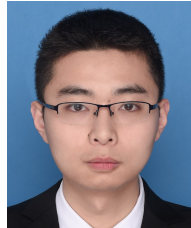
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