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Distributed-observer-based Fault Tolerant Control Design for Nonlinear Multi-agent Systems

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Abstract: The problem of distributed adaptive fault tolerant control is investigated for nonlinear multi-agent systems with sensor faults in this paper. By utilizing radial basis function neural networks to approximate the unknown continuous nonlinear functions, a distributed-observer-based adaptive neural networks scheme is proposed to estimate each node state, which is unmeasured in the system. Then, a kind of distributed adaptive controller is proposed for each follower based on the sliding mode design technique and fault tolerant control technique. Based on graph and Lyapunov stability theory, it is proved that the tracking errors converge to a small neighborhood of the origin with all signals in the closed-loop system being bounded. Finally, simulation results are given to demonstrate the effectiveness of the control scheme proposed in this paper.

Keywords: Adaptive control, fault tolerant control, multiagent system.

1. INTRODUCTION

In the past two decades, the cooperative control problems of multi-agent systems (MASs) have received extensive attention within the control community. As stated in [1], due to wide applications in such areas as power systems, unmanned air vehicles (UAVs) and robot formation, these abundant results about MASs are especially attractive in modern intelligent control. Generally speaking, the research problem of the MASs can be divided into two main categories: the leaderless consensus problem and the leader-following consensus problem. For the former, it is called the cooperative problem, which aims to guarantee all nodes converge to a common value through constructing distributed controllers. For the latter, distributed control scheme is designed to guarantee that all the following nodes follow the leader. Current works on consensus problems can be found in [2–6].

In recent years, the fuzzy logic systems and the neural network (NN) have played an important role to solve the problem of adaptive tracking control for uncertain nonlinear systems, and a variety of adaptive control approaches have been proposed in [7–9]. However, the aforementioned schemes require that the system state variables are assumed to be measured. If the system states are immeasurable, these approaches cannot be applied to nonlinear systems. Up to now, abundant results have been developed by using the observer design for nonlinear systems [10–12]. In [10], a fuzzy adaptive control method was proposed for a class of SISO strict-feedback nonlinear systems with unmeasured states. In [11], the problem of adaptive fuzzy output feedback control was studied for a class of uncertain switched nonlinear systems, and a switched fuzzy state observer was introduced to estimate the immeasurable states. In [12], an adaptive backstepping neural-network control approach was investigated for a class of large-scale nonlinear output-feedback systems with completely unknown interconnections and unmeasured states. However, the above mentioned results cannot directly control those MASs.

In practical application, some common actuators and sensor faults inevitably occur in the components of the practical production systems, which result in performance degradation and process instability, or even catastrophic accidents. Therefore, it is a crucial step to develop effective fault tolerant control (FTC) methods to compensate for the faults and further maintain the acceptable system performance. Much attention has been paid to FTC and a bank of effective results have been achieved [13–16]. In [13], a novel fault diagnosis algorithm was proposed to detect and estimate the time-varying actuator fault in fuzzy systems. In [14], backstepping-based adaptive fuzzy FTC schemes were proposed for a class of nonlinear strict feedback systems with actuator faults. Although the above

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mentioned FTC results have solved the problem of actuator or sensor faults occurred in single systems, they cannot be directly applied to nonlinear multiagent systems. To solve the cooperative fault tolerant control problem, some effective fault tolerant control schemes have been proposed in [17–19]. In [17], a robust fault-tolerant consensus problem was studied for a class of nonlinear second-order leader-following multi-agent systems with multiple actuator faults and time-varying system uncertainties. In [18], an optimal fault tolerant control approach was proposed for multi-agent systems. In [19], the robust adaptive fault tolerant protocol was addressed for multi-agent systems with actuator failure and external disturbance. Although there are some achievements of the research on fault tolerant cooperative control for multi-agent systems, there still are some FTC problems of MASs with sensor faults that will be solved for the demands.

In this paper, based on the previous works, for a class of nonlinear multi-agent systems with sensor faults, we investigate the problem of adaptive FTC. Compared with the existing papers, the main contributions of the proposed solution should be emphasized:

1) A distributed-observer-based adaptive fault tolerant control scheme is proposed for multiagent systems with unmeasured states and sensor faults. A kind of adaptive distributed observer is constructed for each node to solve the state unmeasured problem;

2) The proposed control scheme combines the distributed observer design method and sliding-mode control, and the cooperative fault tolerant tracking control problem is solved.

The rest of this paper is organized as follows: Section 2 introduces some basic graph theory and formulates the problem. In Section 3, main results are given, which includes the fault tolerant control scheme and stability analysis. The simulation result is presented in Section 4. A conclusion is drawn in Section 5.

2. PROBLEM STATEMENT AND DESCRIPTION OF NEURAL NETWORKS

2.1. Graph theory and notations

Consider a weighted digraph $\mathscr{G} = (V,E)$ with a nonempty finite set of N nodes $V = \{v_1, v_2, \ldots, v_N\}$, a set of edges or arcs $E \subset V \times V$, and the associated adjacency matrix $G = [g_{ij}] \in R^{N\times N}$. An edge (v_j, v_i) denotes a link between node j and node i, which implies information flows from v_j to v_i. Node j is called a neighbor of node i if $(v_j, v_i) \in E$. The set of neighbors of node i is denoted as $N_i = \{j \mid (v_j, v_i)\} \in E$. Each entry g_{ij} of adjacency matrix is the weight associated with edge (v_j, v_i) and $g_{ij} > 0$ if $(v_j, v_i) \in E$. Otherwise, $g_{ij} = 0$. Throughout this paper, the digraph is assumed to be time-invariant, i.e., G is constant. Define the weighted in-degree matrix as $D = diag\{d_i\} \in R^{N\times N}$ with $d_i = \sum_{j=1}^{N} g_{ij}$. The Laplacian matrix L is defined as $L = D - G$. Let $\underline{\mathbf{1}} = [1, \ldots, 1]^T \in R^{N\times 1}$ with appropriate dimension, then $L\underline{\mathbf{1}} = 0$. A digraph contains a spanning tree if there is a node called root such that there exists a directed path from this node to every other node.

Notations: $I_n \in R^{n\times n}$ is the identity matrix. The Kronecker product of matrices $X \in R^{m\times n}$ and $Y \in R^{p\times q}$ is defined as

$$X \otimes Y = \begin{bmatrix} x_{11}Y & \cdots & x_{1n}Y \\ \vdots & \ddots & \vdots \\ x_{m1}Y & \cdots & x_{mn}Y \end{bmatrix},$$

which satisfies the following properties:

$$\begin{aligned} &||X\otimes Y|| = ||X||||Y||, \\ &X\otimes Y + X\otimes Z = X\otimes (Y+Z), \\ &(X\otimes Y)(M\otimes N) = (XM)\otimes (YN). \end{aligned}$$

2.2. Problem formulation

Consider the dynamics of ith node as follows

$$\begin{cases} \dot{x}_i = Ax_i + f_i(x_i) + B_u u_i, \\ y_i = Cx_i, \end{cases} \tag{1}$$

where $x_i = [x_{i,1}, \ldots, x_{i,n}]^T \in R^n$, $u_i \in R^m$, and $y_i \in R^p$, $i = 1, \ldots, N$ denote the state, control input and output of node i, respectively; $f_i(x_i) = [0, \ldots, f(x_i)]^T \in R^n$ is unknown smooth function. It is assumed that (A, B_u, C) is stabilizable and detectable. In this paper, we assume that each node state x_i is immeasurable, and only the output y_i can be measured.

Dynamics of the leader node are described by

$$\begin{cases} \dot{x}_0 = Ax_0 + f_0(x_0), \\ y_0 = Cx_0, \end{cases} \tag{2}$$

where $x_0 = [x_{0,1}, \ldots, x_{0,n}]^T \in R^n$ and $y_0 \in R^p$ denote the state and output, respectively. $f(x_0)$ is unknown smooth function and $f_0(x_0) = [0, \ldots, f(x_0)]^T \in R^n$. If there is an edge from node i to the leader, there exists a weight $b_i > 0$. Define the matrix B as $B = diag\{b_i\} \in R^{N\times N}$.

According to the knowledge of Kronecker product, the presentation of global leader node (2) can be rewritten as

$$\begin{cases} \dot{\underline{x}}_0 = (I_N \otimes A)\underline{x}_0 + \underline{f}_0, \\ \underline{y}_0 = (I_N \otimes C)\underline{x}_0, \end{cases} \tag{3}$$

where $\underline{x}_0 = [x_0^T, \ldots, x_0^T]^T \in R^{nN}$, $\underline{y}_0 = [y_0^T, \ldots, y_0^T]^T \in R^{pN}$, $\underline{f}_0 = [f_0^T(x_0), \ldots, f_0^T(x_0)]^T \in R^{nN}$.

Denote the global state as $x = [x_1^T, \ldots, x_N^T]^T \in R^{nN}$, $\underline{x}_0 = \underline{\mathbf{1}} \otimes x_0 \in R^{nN}$. Define the global tracking error as

$$\delta = x - \underline{x}_0 \in R^{nN}. \tag{4}$$

During the actual operation, the sensor may become faulty. The model of the sensor fault can be described as

$$y_{i,k}^{f} = y_{i,k} + f_{i,k}^{s}, \ \ i = 1, \dots, N, \ \ k = 1, \dots, p, \ \ t \geq t_f, \tag{5}$$

where $f_{i,k}^{s}$ is an unknown constant fault, the failure time t_f is unknown.

The aim of this paper is to design a distributed-observer-based adaptive fault tolerant control scheme for (1) such that tracking error δ converges to a small neighborhood of the origin and all signals in the closed-loop system are bounded.

The following assumptions of graph theory and observer design hold throughout this paper.

Assumption 1: The weighted digraph $\mathscr{G}$ contains a spanning tree. The weights of edges $g_{ij} \geq 0$ and there exists an unknown constant $\bar{g} > 0$ such that $\bar{g} \geq g_{ij}$, $i = 1$, $\dots$, N, $j = 1$, $\dots$, N.

Assumption 2: There exists an unknown constant $\bar{f}_{i,k}^{s} > 0 \in R$ such that $|f_{i,k}^{s}| \leq \bar{f}_{i,k}^{s}$, $i = 1$, $\dots$, N, $k = 1$, $\dots$, p.

Assumption 3: There exists a constant $M_{f_0} > 0 \in R$ such that $||f_0(x_0)|| \leq M_{f_0}$.

Lemma 1 [12]**:** Define $q = [q_{21}, \dots, q_{2N}] = (L+B)^{-1}\mathbf{1}$, $P_2 = diag\{p_{2i}\} = diag\{1/q_{2i}\}$, $Q_2 = P_2(L+B) + (L+B)^T P_2$, then $P_2 > 0$ and $Q_2 > 0$.

Lemma 2 [12]**:** $||\delta|| \leq ||z||/\underline{\sigma}(L+B)$, where $\underline{\sigma}(L+B)$ is a minimum singular value of matrix $L+B$.

2.3. Description of neural networks

Neural Networks (NNs) have been widely used in modeling and controlling of nonlinear systems because of their capabilities of nonlinear function approximation, learning and fault tolerance. In this paper, we choose the RBF neural networks [20] as the approximation NNs which is defined as follows:

$$h(Z) = W^T \varphi(Z), \tag{6}$$

where $Z = [z_1, \cdots, z_q]^T \in \Omega_z \in R^q$ is the input vector with q being the NN input dimension, $W = [w_1, \cdots, w_l]^T \in R^l$ is the weight vector and l is the number of the neural networks nodes, $\varphi(Z) = [\varphi_1(Z), \dots, \varphi_l(Z)]^T \in R^l$ denotes the radial basis function with $\varphi_i(Z)$ being chosen as follows

$$\varphi_i(Z) = \exp(-\frac{||z-\mu_i||^2}{d_i^2}), \tag{7}$$

where $d_i > 0$ and $\mu_i = [\mu_{i1}, \dots, \mu_{iq}]^T, i = 1, \dots, l$ are the width and center of the radial basis function, respectively.

Let

$$W^* = \arg\min_{W \in R^l} [\sup_{Z \in \Omega_Z} |h(Z) - W^T \varphi(Z)|], \tag{8}$$

where W^* denotes the ideal weight vector, Ω_z is a sufficiently large compact set.

For a continuous function $h(Z)$, it can be obtained

$$h(Z) = W^{*T} \varphi(Z) + \varepsilon(Z), \forall Z \in \Omega_Z, \tag{9}$$

where $\varepsilon(Z)$ denotes the optimal approximation error.

Assumption 4: There exists an unknown constant $\varepsilon^* > 0 \in R$ such that $|\varepsilon| \leq \varepsilon^*$ in a compact set.

3. MAIN RESULTS

3.1. Observer design

Since the states of system (1) are immeasurable, a neural network state observer will be designed. From (9), define the ideal parameter vector W_i^* as

$$W_i^* = \arg\min_{\hat{W}_i \in \Omega_i} [\sup |\hat{f}_i(\hat{x}_i|\hat{W}_i) - f(x_i)|], \tag{10}$$

where $x_i \in U_i, \hat{x}_i \in U_i^{'}$, Ω_i, U_i and $U_i^{'}$ are compact sets for $\hat{W}_i, x_i$ and $\hat{x}_i$, respectively.

Define the neural networks minimum approximation error as

$$\varepsilon_i = f(x_i) - \hat{f}_i(\hat{x}_i|W_i^*), \tag{11}$$

where $\hat{x}_i = [\hat{x}_{i,1}, \dots, \hat{x}_{i,n}]^T$ is the estimation of the state x_i. Assume that there exists a constant ε_i^* such that $|\varepsilon_i| \leq \varepsilon_i^*, i = 1, \dots N$.

Rewrite (1) into the following form:

$$\begin{cases} \dot{x}_i = \bar{A}x_i + My_i + B_u u_i + W_i^{*T} \varphi_i(\hat{x}_i) + \varepsilon_i, \\ y_i = Cx_i, \end{cases} \tag{12}$$

where $W_i^{*T} \varphi_i(\hat{x}_i) = [0, \dots, 0, W_i^* \varphi(\hat{x}_i)]^T \in R^n$, $\varepsilon_i \in R^n$, $M \in R^{n \times p}$, $\bar{A} = A - MC$. Choose matrix M such that $\bar{A}$ is a Hurwitz matrix.

According to the knowledge of Kronecker product, the presentation of global MASs (12) is described by

$$\begin{cases} \dot{x} = (I_N \otimes \bar{A})x + (I_N \otimes M)y + (I_N \otimes B_u)u \\ \quad\quad + W^{*T} \varphi(\hat{x}) + \varepsilon, \\ y = (I_N \otimes C)x, \end{cases} \tag{13}$$

where $x = [x_1^T, \dots, x_N^T]^T \in R^{nN}$, $u = [u_1^T, \dots, u_N^T]^T \in R^{mN}$, $W^{*T} \varphi(\hat{x}) = [W_1^{*T} \varphi_1(\hat{x}_1), W_2^{*T} \varphi_2(\hat{x}_2) \dots, W_N^{*T} \varphi_N(\hat{x}_N)]^T \in R^{nN}$, $\varepsilon = [\varepsilon_1^T, \dots, \varepsilon_N^T]^T \in R^{nN}$, $y = [y_1^T, \dots, y_N^T] \in R^{pN}$.

To estimate the system states, the distributed observer is constructed as

$$\begin{cases} \dot{\hat{x}}_i = \bar{A}\hat{x}_i + My_i + B_u u_i + \hat{W}_i^T \varphi_i(\hat{x}_i) + sgn(e_{yi}^{fT} F_i^T)\hat{\varepsilon}_i \\ \quad\quad + K\zeta_i + N\hat{\bar{g}}_k K_p(y_i^f - \hat{y}_i^f), \\ \hat{y}_i^f = C\hat{x}_i, \end{cases} \tag{14}$$

where $\hat{x}_i = [\hat{x}_{i,1}, \dots \hat{x}_{i,n}]^T \in R^n$, $\hat{W}_i$ and $\hat{\varepsilon}_i$ denote the estimate of state x_i, W_i^* and ε_i^* of the ith node, respectively;

$K \in R^{n\times p}$ and $K_p \in R^{n\times p}$ denote the observer gain; $N \in R$ is the number of nodes; $\bar{g}_k = \bar{g}\bar{k}$ is an unknown positive constant, $\hat{\bar{g}}_k$ is the estimate of $\bar{g}_k$, which is an unknown constant and will be defined later, ζ_i is a neighborhood output estimation error of node i, which is defined as

$$\zeta_i = \sum_{j\in N_i} g_{ij}((y_i^f - \hat{y}_i^f) - (y_j^f - \hat{y}_j^f)), \tag{15}$$

where g_{ij} is the element of the adjacency matrix G.

From (14) and (15), we can further obtain

$$\begin{aligned}\dot{\hat{x}}_i =& \bar{A}\hat{x}_i + My_i + B_u u_i + \hat{W}_i^T \varphi_i(\hat{x}_i) + sgn(e_{yi}^{fT} F_i^T)\hat{\varepsilon}_i \\ &+ (g_{i1} + \ldots + g_{iN_i})K(y_i^f - \hat{y}_i^f) - g_{i1}K(y_1^f - \hat{y}_1^f) \\ &- \ldots - g_{iN_i}K(y_{N_i}^f - \hat{y}_{N_i}^f) + N\hat{\bar{g}}_k K_p(y_i^f - \hat{y}_i^f).\end{aligned} \tag{16}$$

Similar to (13), the global observer is

$$\begin{cases}\dot{\hat{x}} = (I_N\otimes\bar{A})\hat{x} + (I_N\otimes M)y + (I_N\otimes B_u)u + \hat{W}^T\varphi(\hat{x}) \\ \qquad + sgn(e_y^{fT}F^T)\hat{\varepsilon} + (L\otimes K)(y^f - \hat{y}^f) \\ \qquad + N\hat{\bar{g}}_k K_P(y^f - \hat{y}^f), \\ \hat{y}^f = (I_N\otimes C)\hat{x},\end{cases} \tag{17}$$

where $\hat{x} = [\hat{x}_1^T,\ldots,\hat{x}_N^T]^T$, $\hat{W}^T = diag\{\hat{W}_1^T,\ldots,\hat{W}_N^T\}$, $\hat{W}_i^T$, $i = 1,\ldots,N$ are the estimates of W_i^{*T}, $\hat{\varepsilon} = [\hat{\varepsilon}_1^T,\ldots,\hat{\varepsilon}_N^T]^T$, $sgn(e_y^{fT}F^T) = diag\{sgn(e_{y1}^{fT}F_1^T),\ldots,sgn(e_{yN}^{fT}F_N^T)\}$, $u = [u_1^T,\ldots,u_N^T]^T$, $K_P = I_N\otimes K_p$, $y = [y_1^T,\ldots,y_N^T]^T$.

Thus, given a positive definite matrix Q_1, there exist real matrices $K, P_{1i} = P_{1i}^T > 0$ such that

$$\begin{aligned}&[I_N\otimes\bar{A} - D\otimes KC]^T P_1 + P_1[I_N\otimes\bar{A} - D\otimes KC] \\ &\le -Q_1,\end{aligned} \tag{18}$$

$$P_1 = (FC_m)^T, \tag{19}$$

where $P_1 = diag\{P_{11},\ldots,P_{1N}\}$, $F = diag\{F_1,\ldots,F_N\} \in R^{nN\times pN}$, $C_m = (I_N\otimes C) \in R^{pN\times nN}$.

Define the observer error of the ith node $e_{xi} = x_i - \hat{x}_i$, $e_{yi} = y_i - \hat{y}_i$. Then, let us define e_x, e_y and e_y^f as

$$\begin{aligned}&e_x = [e_{x1}^T,\ldots,e_{xN}^T]^T, \ \ e_y = [e_{y1}^T,\ldots,e_{yN}^T]^T, \\ &e_y^f = [e_{y1}^{fT},\ldots,e_{yN}^{fT}]^T.\end{aligned} \tag{20}$$

From (13) and (17), we obtain the observer error dynamics

$$\begin{cases}\dot{e}_x = (I_N\otimes\bar{A})e_x + \tilde{W}^T\varphi(\hat{x}) + \varepsilon - sgn(e_y^{fT}F^T)\hat{\varepsilon} \\ \qquad - (L\otimes K)(y^f - \hat{y}^f) - N\hat{\bar{g}}_k K_P(y^f - \hat{y}^f), \\ \dot{e}_y = (I_N\otimes C)e_x,\end{cases} \tag{21}$$

where $\tilde{W} = W^* - \hat{W}$.

3.2. Controller design and stability analysis

For node i, the neighborhood synchronization error is given by

$$z_i = \sum_{j\in N_i} g_{ij}(\hat{x}_j - \hat{x}_i) + b_i(x_0 - \hat{x}_i). \tag{22}$$

Define $z = [z_1,\ldots,z_N]^T \in R^{n\times N}$, one has

$$z = -(L+B)(\hat{x} - \underline{x}_0). \tag{23}$$

The dynamic of global tracking error z is

$$\begin{aligned}\dot{z} =& -(L+B)[(I_N\otimes A)\hat{x} + \hat{W}^T\varphi(\hat{x}) + \hat{\varepsilon} \\ &+ (I_N\otimes B_u)u - (I_N\otimes A)\underline{x}_0 - \underline{f}_0].\end{aligned} \tag{24}$$

Define the error s_i for node i as

$$s_i = H_i z_i - \int_0^t H_i(A + B_u E)z_i(\tau)d\tau, \tag{25}$$

where $s_i \in R$, $H_i \in R^{1\times n_i}$, let $H_i = B_u^T/||B_u||^2$, result in $H_i B_u = 1$, i.e., $H_i = B_u^+$, $E \in R^{1\times n_i}$ satisfy $max(Re[\lambda(A + B_u E)]) < 0$, here $Re[\lambda(*)]$ denotes the real part of λ, and $\lambda(*)$ expresses the eigenvalue of matrix $*$. From (25), we have

$$\dot{s}_i = H_i\dot{z}_i(t) - H_i(A + B_u E)z_i(t). \tag{26}$$

Now, define $S(t) = [s_1(t), s_2(t),\ldots,s_N(t)]^T$. Then, from (26), the dynamic of S is

$$\dot{S} = -H(L+B)[\hat{W}^T\varphi(\hat{x}) + \hat{\varepsilon} + (I_N\otimes B_u)u - \underline{f}_0] - \gamma, \tag{27}$$

where $H = diag\{H_1,\ldots,H_N\} \in R^{n\times nN}$, $\gamma = (I_N\otimes E)z$.

Define the Lyapunov function as

$$V = V_1 + V_2 + V_3, \tag{28}$$

where $V_1 = e_x^T P_1 e_x$, $V_2 = S^T P_2 S$, $V_3 = \frac{1}{2\eta_{g_k}}\tilde{\bar{g}}_k^2 + \frac{1}{2\eta_f}\tilde{f}^{sT}\tilde{f}^s + \frac{1}{2\eta_w}tr\{\tilde{W}^T\tilde{W}\} + \frac{1}{2\eta_\varepsilon}\tilde{\varepsilon}^T\tilde{\varepsilon} + \frac{1}{2\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\tilde{M}_{\underline{f}_0}$, $\eta_{g_k} > 0 \in R$, $\eta_f > 0 \in R$, $\eta_w > 0 \in R$, $eta_\varepsilon > 0 \in R$, $\eta_{\underline{f}_0} > 0 \in R$ are design parameters.

From (18), (21) and (28), differentiating V_1 with respect to time t, one has

$$\begin{aligned}\dot{V}_1 \le& -e_x^T Q_1 e_x + 2e_x^T P_1(G\otimes K)e_y + 2e_x^T P_1(G\otimes K)\tilde{f}^s \\ &- 2e_x^T P_1(D\otimes K)\tilde{f}^s + 2e_x^T P_1[\tilde{W}^T\varphi(\hat{x}) + \varepsilon \\ &- sgn(e_y^{fT}F^T)\hat{\varepsilon}] - 2e_x^T P_1 N\hat{\bar{g}}_k K_P e_y^f.\end{aligned} \tag{29}$$

Now, let us consider the second and third term in (29), since $e_y^f = e_y + \tilde{f}^s$, one has

$$\begin{aligned}&2e_x^T P_1(G\otimes K)e_y + 2e_x^T P_1(G\otimes K)\tilde{f}^s \\ &= 2e_x^T P_1(G\otimes K)e_y^f.\end{aligned} \tag{30}$$

From the Kronecker product properties, we can obtain

$$2e_x^T P_1(G\otimes K)e_y^f = 2e_x^T P_1(G\otimes K_G)K_P e_y^f, \tag{31}$$

where $K_G \in R^{n\times n}$, $K_P = I_N\otimes K_p$. Since $P_1 = P_1^T > 0$, we further have

$$2e_x^T P_1(G\otimes K)K_P e_y^f \le 2e_x^T P_1||G\otimes K_G||K_P e_y^f. \tag{32}$$

From Assumption 1 and the Kronecker property: $||G\otimes K_G|| = ||G||||K_G||$ and $||G|| = \sqrt{tr(G^TG)} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij}^2} \leq N\bar{g}$. Since the matrix K is satisfied with (18), there exists a real unknown constant $\bar{k}$ such that $||K_G|| \leq \bar{k}$. We can further obtain $0 \leq ||G||||K_G|| \leq N\bar{g}\bar{k} = N\bar{g}_k$, where $\bar{g}_k = \bar{g}\bar{k}$.

Hence, from (32), it follows that

$$2e_x^T P_1(G\otimes K)e_y^f \leq 2e_x^T P_1 N\bar{g}_k K_P e_y^f. \tag{33}$$

Substituting (33) into (29), one has

$$\begin{aligned}\dot{V}_1 \leq &- e_x^T Q_1 e_x + 2e_x^T P_1 N\tilde{\bar{g}}_k K_P e_y^f - 2e_x^T P_1(D\otimes K)\tilde{f}^s \\ &+ 2e_x^T P_1[\tilde{W}^T\varphi(\hat{x}) + \varepsilon - sgn(e_y^{fT}F^T)\hat{\varepsilon}]. \end{aligned}\tag{34}$$

Consider the second term in (34), since $P_1 = (FC_m)^T$ and $e_y = e_y^f - \tilde{f}^s$, we have

$$\begin{aligned}&2e_x^T P_1 N\tilde{\bar{g}}_k K_P e_y^f \\ &= 2(e_y^{fT} - \tilde{f}^{sT})F^T N\tilde{\bar{g}}_k K_P e_y^f \\ &= 2e_y^{fT} F^T N\tilde{\bar{g}}_k K_P e_y^f - 2e_y^{fT} K_P^T N\tilde{\bar{g}}_k F\tilde{f}^s. \end{aligned}\tag{35}$$

Then, using Young's inequality, one has

$$\begin{aligned}2e_x^T P_1 N\tilde{\bar{g}}_k K_P e_y^f \leq &2e_y^{fT} F^T N\tilde{\bar{g}}_k K_P e_y^f \\ &+ e_y^{fT} K_P^T N^2\tilde{\bar{g}}_k^2 K_P e_y^f + \tilde{f}^{sT}F^TF\tilde{f}^s. \end{aligned}\tag{36}$$

Similar to (35) and (36), it follows that

$$\begin{aligned}&2e_x^T P_1[\tilde{W}^T\varphi(\hat{x}) + \varepsilon - sgn(e_y^{fT}F^T)\hat{\varepsilon}] \\ &\leq 2e_y^{fT}F^T\tilde{W}^T\varphi(\hat{x}) + \tilde{f}^{sT}F^TF\tilde{f}^s + \varphi^T\tilde{W}\tilde{W}^T\varphi \\ &\quad + 2e_y^{fT}F^T sgn(e_y^{fT}F^T)\tilde{\varepsilon} + \tilde{f}^{sT}F^TF\tilde{f}^s + ||\varepsilon^*||^2 \\ &\quad + \tilde{f}^{sT}F^TF\tilde{f}^s + sgn(e_y^{fT}F^T)\hat{\varepsilon}^T sgn(e_y^{fT}F^T)\hat{\varepsilon}. \end{aligned}\tag{37}$$

Now, consider the third term in (35), one has

$$\begin{aligned}&-2e_x^T P_1(D\otimes K)\tilde{f}^s \\ &= -2(e_y^{fT} - \tilde{f}^{sT})F^T(D\otimes K)\tilde{f}^s \\ &= -2\tilde{f}^{sT}(D\otimes K)^T F e_y^f + 2\tilde{f}^{sT}F^T(D\otimes K)\tilde{f}^s. \end{aligned}\tag{38}$$

Furthermore, from (36), (37) and (38), we can obtain

$$\begin{aligned}\dot{V}_1 \leq &- e_x^T Q_1 e_x + 2e_y^{fT}F^T N\tilde{\bar{g}}_k K_P e_y^f + 2e_y^{fT}F^T\tilde{W}^T\varphi(\hat{x}) \\ &+ 2e_y^{fT}F^T sgn(e_y^{fT}F^T)\tilde{\varepsilon} - 2\tilde{f}^{sT}(D\otimes K)^T F e_y^f \\ &+ e_y^{fT}K_P^T N^2\tilde{\bar{g}}_k^2 K_P e_y^f + 4\tilde{f}^{sT}F^TF\tilde{f}^s \\ &+ 2\tilde{f}^{sT}F^T(D\otimes K)\tilde{f}^s + \mu_e, \end{aligned}\tag{39}$$

where $\mu_e = \varphi^T\tilde{W}\tilde{W}^T\varphi + ||\varepsilon^*||^2 + \hat{\varepsilon}^T\hat{\varepsilon}$.

Now, differentiating V_2 with respect to t, one has

$$\begin{aligned}\dot{V}_2 = &-2S^T P_2 H(L+B)[\hat{W}^T\varphi(\hat{x}) + sgn(e_y^{fT}F^T)\hat{\varepsilon} - \underline{f}_0] \\ &- 2S^T P_2 H(L+B)(I_N\otimes B_u)u - 2S^T P_2\gamma. \end{aligned}\tag{40}$$

The control law is given by

$$\begin{aligned}u = &-(I_N\otimes B_u)^+[\hat{W}^T\varphi(\hat{x}) + sgn(e_y^{fT}F^T)\hat{\varepsilon}] \\ &+ (I_N\otimes B_u)^+ sgn(S^T P_2 H(D+B))\hat{M}_{\underline{f}_0} \\ &- (I_N\otimes B_u)^+(D+B)^{-1}H^+\gamma + h(I_N\otimes B_u)^+ S, \end{aligned}\tag{41}$$

where $\hat{W}^T = diag\{\hat{W}_1^T, \hat{W}_2^T, \ldots, \hat{W}_N^T\}$, $\hat{\varepsilon} = [\hat{\varepsilon}_1^T, \ldots, \hat{\varepsilon}_N^T]^T$, $sgn(e_y^{fT}F^T) = diag\{sgn(e_{y1}^{fT}F_1^T), \ldots, sgn(e_{yN}^{fT}F_N^T)\}$, $\varphi(\hat{x}) = [\varphi_1^T(\hat{x}_1), \ldots, \varphi_N^T(\hat{x}_N)]^T$, $\hat{M}_{\underline{f}_0} = [\hat{M}_{f_0}, \ldots, \hat{M}_{f_0}]^T$ is an estimate of $M_{\underline{f}_0} = [M_{f_0}, \ldots, M_{f_0}]^T$, $h > 0$ is represented as

$$\begin{aligned}&h\lambda_{\min}(HQ_2H^T) - 6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)r\lambda_{\min}(HH^T) \\ &\qquad - \lambda_{\min}(H_D)/2r > 0. \end{aligned}$$

Substituting (41) into (40), we have

$$\begin{aligned}\dot{V}_2 \leq &- 2hS^T P_2 H(L+B)S \\ &+ 2S^T P_2 H(D+B)sgn(S^T P_2 H(D+B))\tilde{M}_{\underline{f}_0} \\ &+ 2S^T P_2 HGsgn(S^T P_2 H(D+B))\hat{M}_{\underline{f}_0} \\ &- 2S^T P_2 HG\underline{f}_0 - 2S^T P_2 HG(D+B)^{-1}H^+\gamma, \end{aligned}\tag{42}$$

where $\tilde{M}_{\underline{f}_0} = M_{\underline{f}_0} - \hat{M}_{\underline{f}_0}$.

From Young's inequality, it follows that

$$\begin{aligned}&2S^T P_2 HG\text{sgn}(S^T P_2 H(D+B))\hat{M}_{\underline{f}_0} \\ &\leq 2\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^T HH^T S + \hat{M}_{\underline{f}_0}^T\hat{M}_{\underline{f}_0}/2r, \end{aligned}\tag{43}$$

$$\begin{aligned}&-2S^T P_2 HG\underline{f}_0 \leq 2\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^T HH^T S \\ &\qquad + M_{\underline{f}_0}^T M_{\underline{f}_0}/2r, \end{aligned}\tag{44}$$

$$\begin{aligned}&-2S^T P_2 HG(D+B)^{-1}H^+\gamma \\ &\leq 2\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^T HH^T S \\ &\quad + \gamma^T(H^+)^T(D+B)^{-2}H^+\gamma/2r, \end{aligned}\tag{45}$$

where $r > 0$ is a design parameter, $\bar{\sigma}$ is the maximum singular value of the matrix.

Substituting (43)-(45) into (42), one has

$$\begin{aligned}\dot{V}_2 \leq &- 2hS^T P_2 H(L+B)S \\ &+ 2S^T P_2 H(D+B)\text{sgn}(S^T P_2 H(D+B))\tilde{M}_{\underline{f}_0} \\ &+ 6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^T HH^T S \\ &+ \mu_s\gamma^T(H^+)^T(D+B)^{-2}H^+\gamma/2r, \end{aligned}\tag{46}$$

where $\mu_s = (\hat{M}_{\underline{f}_0}^T\hat{M}_{\underline{f}_0} + M_{\underline{f}_0}^T M_{\underline{f}_0})/2r$.

From Assumption 4, $M_{\underline{f}_0}$ is bounded. The adaptive law (52) ensures that $\hat{M}_{\underline{f}_0}$ is also bounded. Because $M_{\underline{f}_0}$ and $\hat{M}_{\underline{f}_0}$ are bounded, $r > 0 \in R$ is a design parameter, then μ_s is bounded.

Since

$$\gamma^T(H^+)^T(D+B)^{-2}H^+\gamma/2r \leq S^T H_D S/2r, \tag{47}$$

where

$$H_D = (H^+)^T(I_N \otimes E)^T(H^+)^T(D+B)^{-2}H^+(I_N \otimes E)H^+.$$

Then, (46) can be rewritten as

$$\begin{aligned}\dot{V}_2 \le &-2hS^TP_2H(L+B)S\\ &+2S^TP_2H(D+B)\text{sgn}(S^TP_2H(D+B))\tilde{M}_{\underline{f}_0}\\ &+6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^THH^TS+S^TH_DS/2r+\mu_s.\end{aligned} \tag{48}$$

From (28), the derivative of V_3 is

$$\begin{aligned}\dot{V}_3 = &-\frac{1}{\eta_{g_k}}\tilde{\bar{g}}_k\dot{\hat{\bar{g}}}_k - \frac{1}{\eta_f}\tilde{f}^{sT}\dot{\hat{f}}^s - \frac{1}{\eta_w}tr\{\tilde{W}^T\dot{\hat{W}}\}\\ &-\frac{1}{\eta_\varepsilon}\tilde{\varepsilon}^T\dot{\hat{\varepsilon}} - \frac{1}{\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\dot{\hat{M}}_{\underline{f}_0}.\end{aligned} \tag{49}$$

Define the adaptive laws as follows:

$$\dot{\hat{\bar{g}}}_k = 2\eta_g N e_y^{fT}F^TK_Pe_y^f - r_{g_k}\hat{\bar{g}}_k, \tag{50}$$

$$\dot{\hat{f}}^s = -2\eta_f(D\otimes K)^TFe_y^f - r_f\hat{f}^s, \tag{51}$$

$$\dot{\hat{W}} = 2\eta_w\varphi Fe_y^f - r_w\hat{W}, \tag{52}$$

$$\dot{\hat{\varepsilon}} = 2\eta_\varepsilon Fe_y^f sgn(e_y^{fT}F^T) - r_\varepsilon\hat{\varepsilon}, \tag{53}$$

$$\begin{aligned}\dot{\hat{M}}_{\underline{f}_0} = &2\eta_{\underline{f}_0}S^TP_2H(D+B)sgn(S^TP_2H(D+B))\\ &-r_{\underline{f}_0}\hat{M}_{\underline{f}_0}.\end{aligned} \tag{54}$$

Theorem 1: Consider system (1) and (2), Assumptions 1-4, control law (41) and adaptive laws (50)-(54), if there exist appropriate parameters h, r, η_{g_k}, η_f, η_w, η_ε, $\eta_{\underline{f}_0}$, then all nodes in graph $\mathcal{G}$ synchronize to the leader node 0, the global tracking error δ and all signals in the closed-loop system belong to a small adjustable set Ω defined as

$$\Omega = \left\{\begin{aligned}&(e_x,S,\tilde{\bar{g}}_k,\tilde{f}^s,\tilde{W},\tilde{\varepsilon},\tilde{M}_{\underline{f}_0},\delta)\ |\ ||e_x||\le\sqrt{\alpha/\lambda_{\min}(P_1)},\\ &||S||\le\sqrt{\alpha/\lambda_{\min}(P_2)},||\tilde{M}_{\underline{f}_0}||\le\sqrt{2\eta_{\underline{f}_0}\alpha},\\ &||\tilde{W}||\le\sqrt{2\eta_w\alpha},||\tilde{\varepsilon}||\le\sqrt{2\eta_\varepsilon\alpha},|\tilde{\bar{g}}_k|\le\sqrt{2\eta_{g_k}\alpha},\\ &||\tilde{f}^s||\le\sqrt{2\eta_f\alpha},\delta\le\sqrt{\alpha/\lambda_{\min}(P_2)}/\underline{\sigma}(L+B)\end{aligned}\right\}.$$

Proof: Choose the Lyapunov function as

$$\begin{aligned}V = &e_x^TP_1e_x+S^TP_2S+\frac{1}{2\eta_{g_k}}\tilde{\bar{g}}_k^2+\frac{1}{2\eta_f}\tilde{f}^{sT}\tilde{f}^s\\ &+\frac{1}{2\eta_w}tr\{\tilde{W}^T\tilde{W}\}+\frac{1}{2\eta_\varepsilon}\tilde{\varepsilon}^T\tilde{\varepsilon}+\frac{1}{2\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\tilde{M}_{\underline{f}_0}.\end{aligned} \tag{55}$$

Differentiating V with respect to time t and from (41), (50)-(54), one has

$$\begin{aligned}\dot{V} \le &-e_x^TQ_1e_x+e_y^{fT}K_P^TN^2\tilde{\bar{g}}_k^2K_Pe_y^f+4\tilde{f}^{sT}F^TF\tilde{f}^s\\ &+2\tilde{f}^{sT}F^T(D\otimes K)\tilde{f}^s+\mu_e-hS^THQ_2H^TS\\ &+6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^THH^TS+S^TH_DS/2r+\mu_s\\ &+\frac{r_{g_k}}{\eta_{g_k}}\tilde{\bar{g}}_k\hat{\bar{g}}_k+\frac{r_f}{\eta_f}\tilde{f}^{sT}\hat{f}^s+\frac{r_w}{\eta_w}tr\{\tilde{W}^T\hat{W}\}+\frac{r_\varepsilon}{\eta_\varepsilon}\tilde{\varepsilon}^T\hat{\varepsilon}\\ &+\frac{r_{\underline{f}_0}}{\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\hat{M}_{\underline{f}_0}.\end{aligned} \tag{56}$$

From Young's inequality, one has

$$\frac{r_{g_k}}{\eta_{g_k}}\tilde{\bar{g}}_k\hat{\bar{g}}_k \le -\frac{r_{g_k}}{2\eta_{g_k}}\tilde{\bar{g}}_k^2+\frac{r_{g_k}}{2\eta_{g_k}}\bar{g}_k^2, \tag{57}$$

$$\frac{r_f}{\eta_f}\tilde{f}^{sT}\hat{f}^s \le -\frac{r_f}{2\eta_f}\tilde{f}^{sT}\tilde{f}^s+\frac{r_f}{2\eta_f}\bar{f}^{sT}\bar{f}^s, \tag{58}$$

$$\begin{aligned}&\frac{r_w}{\eta_w}tr\{\tilde{W}^T\hat{W}\}\\ &\le -\frac{r_w}{2\eta_w}tr\{\tilde{W}^T\tilde{W}\}+\frac{r_w}{2\eta_w}tr\{W^{*T}W^*\},\end{aligned} \tag{59}$$

$$\frac{r_\varepsilon}{\eta_\varepsilon}\tilde{\varepsilon}^T\hat{\varepsilon} \le -\frac{r_\varepsilon}{2\eta_\varepsilon}\tilde{\varepsilon}^T\tilde{\varepsilon}+\frac{r_\varepsilon}{2\eta_\varepsilon}||\varepsilon^*||^2, \tag{60}$$

$$\frac{r_{\underline{f}_0}}{\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\hat{M}_{\underline{f}_0} \le -\frac{r_{\underline{f}_0}}{2\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\tilde{M}_{\underline{f}_0}+\frac{r_{\underline{f}_0}}{2\eta_{\underline{f}_0}}||M_{\underline{f}_0}||^2, \tag{61}$$

where $\bar{f}^s = [\bar{f}_1^s,\dots,\bar{f}_N^s]^T$, $\bar{f}_i^s = [\bar{f}_{i,1}^s,\dots,\bar{f}_{i,p}^s]^T \in R^p$, $i = 1, \dots, N$.

Then, (56) can be rewritten as

$$\begin{aligned}\dot{V} \le &-e_x^TQ_1e_x-hS^THQ_2H^TS\\ &+6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^THH^TS+S^TH_DS/2r\\ &+\tilde{f}^{sT}[4F^TF+2F^T(D\otimes K)-\frac{r_f}{2\eta_f}I_N]\tilde{f}^s\\ &+[N^2e_y^{fT}K_P^TK_Pe_y^f-\frac{r_{g_k}}{2\eta_{g_k}}]\tilde{\bar{g}}_k^2-\frac{r_w}{2\eta_w}tr\{\tilde{W}^T\tilde{W}\}\\ &-\frac{r_\varepsilon}{2\eta_\varepsilon}\tilde{\varepsilon}^T\tilde{\varepsilon}-\frac{r_{\underline{f}_0}}{2\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\tilde{M}_{\underline{f}_0}+\mu,\end{aligned} \tag{62}$$

where $\mu = \mu_e+\mu_s+\frac{r_{g_k}}{2\eta_{g_k}}\bar{g}_k^2+\frac{r_f}{2\eta_f}\bar{f}^{sT}\bar{f}^s+\frac{r_{\underline{f}_0}}{2\eta_{\underline{f}_0}}||M_{\underline{f}_0}||^2+\frac{r_w}{2\eta_w}tr\{W^{*T}W^*\}+\frac{r_\varepsilon}{2\eta_\varepsilon}||\varepsilon^*||^2$.

Let $r_{g_k} = 1+2\eta_{g_k}N^2e_y^{fT}K_P^TK_Pe_y^f$, $r_w = r_\varepsilon = r_{\underline{f}_0} = 1$, $r_fI_N = I_N+8\eta_fF^TF+4\eta_fF^T(D\otimes K)$, we have

$$\begin{aligned}\dot{V} \le &-e_x^TQ_1e_x-hS^THQ_2H^TS\\ &+6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)rS^THH^TS+S^TH_DS/2r\\ &-\frac{1}{2\eta_{g_k}}\tilde{\bar{g}}_k^2-\frac{1}{2\eta_f}\tilde{f}^{sT}\tilde{f}^s-\frac{1}{2\eta_w}tr\{\tilde{W}^T\tilde{W}\}\\ &-\frac{1}{2\eta_\varepsilon}\tilde{\varepsilon}^T\tilde{\varepsilon}-\frac{1}{2\eta_{\underline{f}_0}}\tilde{M}_{\underline{f}_0}^T\tilde{M}_{\underline{f}_0}+\mu.\end{aligned} \tag{63}$$

Therefore, (63) can be further obtained

$$\dot{V} \le -\kappa V+\mu, \tag{64}$$

where $\kappa = \min\{1, \lambda_{\min}(Q_1)/\lambda_{\max}(P_1), [h\lambda_{\min}(HQ_2H^T) - 6\bar{\sigma}(GG^T)\bar{\sigma}^2(P_2)r\lambda_{\min}(HH^T) - \lambda_{\min}(H_D)/2r]/\lambda_{\max}(P_2)\}$.

From (64), it follows that

$$0 \le V(t) \le \frac{\mu}{\kappa} + (V(0) - \frac{\mu}{\kappa})e^{-\kappa t} \le \frac{\mu}{\kappa} + V(0). \quad (65)$$

Let $\alpha = \mu/\kappa + V(0)$, one has $||e_x|| \le \sqrt{\alpha/\lambda_{\min}(P_1)}, ||S|| \le \sqrt{\alpha/\lambda_{\min}(P_2)}, |\tilde{\bar{g}}_k| \le \sqrt{2\eta_{g_k}\alpha}, ||\tilde{f}^s|| \le \sqrt{2\eta_f\alpha}, ||\tilde{W}|| \le \sqrt{2\eta_w\alpha}, ||\tilde{\varepsilon}|| \le \sqrt{2\eta_\varepsilon\alpha}, ||\tilde{M}_{\underline{f}_0}|| \le \sqrt{2\eta_{\underline{f}_0}\alpha}$, which implies the boundedness of all signals in the closed-loop system. Further, from (23), the synchronization error z is bounded.

From Lemma 2, we have

$$\delta \le \sqrt{\alpha/\lambda_{\min}(P_2)}/\underline{\sigma}(L+B). \quad (66)$$

Thus, the consensus error δ is uniform ultimate boundedness, and all signals in the closed-loop system belong to the adjustable set Ω. □

3.3. Simulation results

To demonstrate the effectiveness of the proposed approach, a power system with 4 buses is used in the simulation study. Power systems can be seen as an example of multi-agent systems, where each bus denotes a node in the system. For $i = 1, \ldots, N$, the behavior of node i can be represented by the swing equation

$$m_i\ddot{\delta}_i(t) + d_i\dot{\delta}_i(t) - P_{mi}(t) = -\sum_{j\in N_i} P_{ij}(t),$$

where δ_i is the phase angle of node i, m_i is the inertia coefficients of motors, d_i is the damping coefficients of generators, P_{mi} denotes the mechanical input power, P_{ij} denotes the active power flow from v_i to v_j, and N_i is the neighborhood set of node i where node j and i share a common route.

Now, let $\xi_i(t) = \delta_i(t)$ and $\zeta_i(t) = \dot{\delta}_i(t)$, the dynamics of node i can be rewritten as

$$\begin{aligned}\dot{\xi}_i(t) &= \zeta_i(t),\\ \dot{\zeta}_i(t) &= -\frac{d_i}{m_i}\zeta_i(t) - \frac{1}{m_i}\sum_{j\in N_i}[\omega_{ij}^1\cos(\xi_i(t) - \xi_j(t))\\ &\quad + \omega_{ij}^2\sin(\xi_i(t) - \xi_j(t))] + \frac{P_{mi}(t)}{m_i}.\end{aligned}$$

Consider the system in this paper, the leader is represented by (2), and the following nodes are represented by second-order systems in the shape of (1), where the state $x = [x_{i,1}, x_{i,2}]^T = [\xi_i, \zeta_i]^T$, the input $u_i \in R$, $f_i(x_i) = [0, f_i(x_i)]^T$, and

$$A = \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}.$$

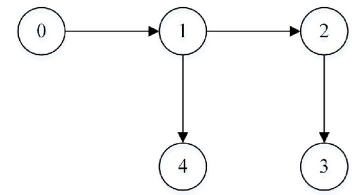

Fig. 1. Communication topology.

These variables' specification is given as follows: ξ_i and ζ_i denote the phase and frequency of the ith node, respectively; $f_i(x_i)$ denotes the power flow of the ith node.

Consider a 4-node digraph $\mathscr{G}$ and a leader node, the communication topology is described in Fig. 1. As shown in Fig. 1, we know that the matrix $B = diag\{1,0,0,0\}$. We choose the following value: $f_0(x_0) = -x_{0,1} - 2x_{0,2} - (x_{0,1} + x0,2)(x_{0,1} + 4x_{0,2}) + 0.5\sin(2t)$, $f_1(x_1) = -x_{1,1} + 0.5x_{1,2} - x_{1,1}^2 x_{1,2}$, $f_2(x_2) = -0.5x_{2,1} + 0.5x_{2,2} - x_{2,1}^2 x_{2,2}$, $f_3(x_3) = x_{3,1} + 0.25x_{3,2} - x_{3,1}^2 x_{3,2}$, $f(x_4) = x_{4,1} + x_{4,2} - 0.5x_{4,1}^2 x_{4,2}$. The values for initial conditions are $x_{1,1}(0) = -0.1$, $x_{1,2}(0) = -0.05$, $x_{2,1}(0) = 0.15$, $x_{2,2}(0) = -0.01$, $x_{3,1}(0) = 0.05$, $x_{3,2}(0) = -0.15$, $x_{4,1}(0) = 0.1, x_{4,2}(0) = 0.2$. The parameter values of the neural networks are selected as $q = 10$, $\mu_{i,j} = 0.1 \times (j-5)$, $d_{i,j} = 1$, $j = 1, 2, \ldots, q$, $i = 1$. We assume that only node 2 becomes faulty, and the sensor fault is described as $y_2^f = y_2 + 0.1\sin(t)$.

The simulation results are given in Figs. 2-6. In fault-free case, the observer error is shown in Fig. 2, which implies that the observers designed in this paper have good performance. In Fig. 3, it is seen that the observer error significantly deviates from the origin without fault compensation. From Fig. 4, in faulty case, the tracking errors asymptotically converge to a small adjustable neighborhood of the origin with the proposed fault tolerant control design method. In addition, the boundedness of control signals are shown in Fig. 5. Fig. 6 shows the trajectories of $||\hat{W}_i||_2$, $i = 1, 2, 3, 4$.

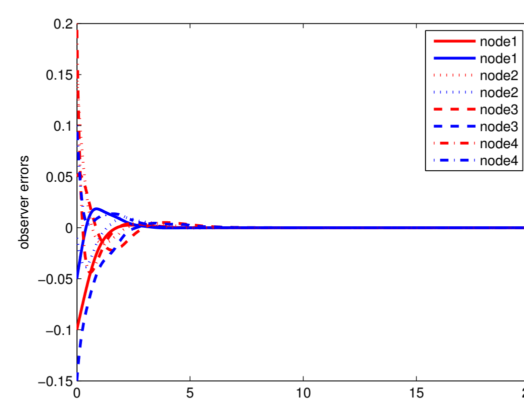

Fig. 2. The observer error in the fault-free case.

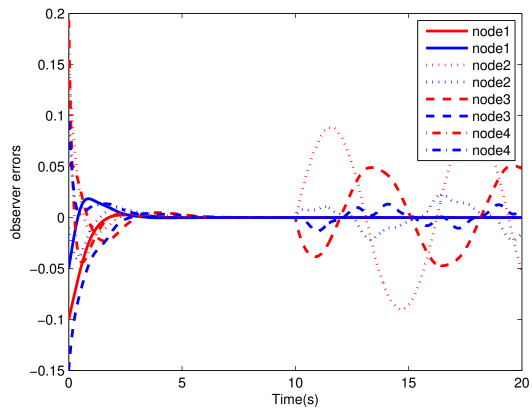

Fig. 3. The observer error in the fault case.

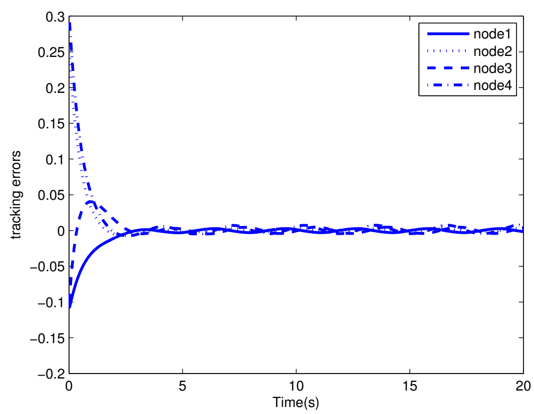

Fig. 4. Tracking errors.

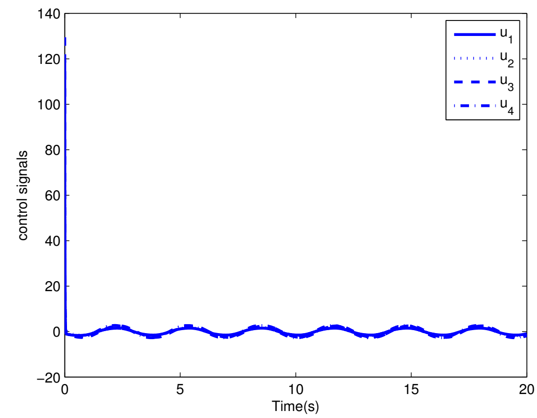

Fig. 5. Control signals.

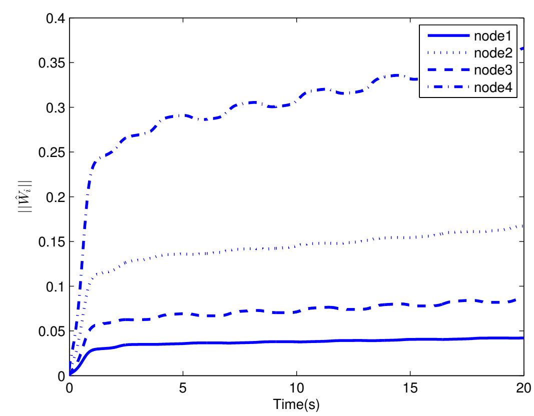

Fig. 6. Trajectories of $||\hat{W}_i||_2$.

4. CONCLUSIONS

In this paper, the adaptive fault tolerant control problem is investigated for a class of nonlinear multi-agent systems. Based on the distributed observer design method and sliding mode control technique, a novel adaptive neural networks control scheme is proposed. By graph and Lyapunov theory analysis, the cooperative tracking error converges exponentially to a small adjustable neighborhood of the origin with all signals in the closed-loop system being bounded.

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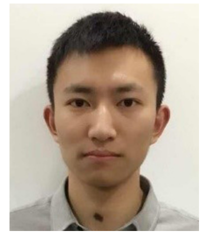

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