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Unknown Input Reconstruction via Interval Observer and State and Unknown Input Compensation Feedback Controller Designs

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Abstract: In this paper, we investigate the state estimation, unknown input and measurement noise reconstruction problems and the feedback controller design issues for a linear discrete-time system with both unknown inputs and measurement noises. First, an augmented system is constructed and the state vector of the augmented system consists of the original system state and the measurement noise, and the preconditions between the original system and the augmented system is discussed in detail. Second, for the augmented system, a reduced-order observer is designed so that the original system state estimates and the measurement noise reconstruction can be obtained. Third, in order to get the asymptotical unknown input reconstruction, an interval observer for part of the measurable output is proposed and an unknown input reconstruction method based on the interval observer is developed. Finally, an observer-based state feedback and unknown input controller is designed and the closed-loop system stability is analyzed. We point out that the closed-loop system satisfies the so-called separation property. At last, two simulation examples are given to verify the effectiveness of the proposed methods.

Keywords: Compensation controller, interval observer, UIO, unknown input reconstruction.

1. INTRODUCTION

State feedback control and state estimation are two important issues in modern control theory and control engineering. Compare to the output feedback control, state feedback control shows the predominant superiority in that it can place the poles in any desired places as long as the system is a controllable linear time-invariant system. One of the drawbacks of the state feedback is its using of the system state information which is usually difficult or even impossible to be measured directly for many practical systems. To overcome this shortcoming of the sate feedback control, state estimation concept was first developed by Luenberger in 1960s [1] and since then the investigations to the state observers have never been stopped. For example, much excellent work has been done in designing observers for nonlinear systems especially T-S fuzzy systems [2, 3].

Since the system uncertain parameters, external disturbances and even the actuator faults can be regarded as system unknown inputs, state observers for systems with uncertain parameters and external disturbances are called unknown input observer (UIO) and, soon after the development of the Luenberger observer, UIO designs attracted the researchers' attentions. Indeed, the early work of the UIO design can be traced back to the end of the 1960s and in the early time, researchers focus only on state estimation problems by avoiding the negative influences of the unknown inputs without considering unknown input estimation issues. For example, Wang *et al*. [4] give an observer characterization in terms of transfer function matrices and the result is applied to directly decentralized control systems. In [5], the problems of the estimations of a linear function of the states are investigated for linear systems subjected to unknown or disturbance inputs. Even now, The UIO design for some complex control models with unknown input still follows this line of thinking [6,7]. For example, Zhang *et al*. present a systematical reduced-order observer design method for a class of switched descriptor systems containing unknown inputs in both the dynamic and the output equations [6].

UIO design with simultaneous state and unknown input estimation become one of the dominant research points in the field of observer designs since 1990s. Because of the fact that the reconstruction of the unknown input can makes the control designs much more convenient which is

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different from the passive fault tolerant control that usually eliminates fault by designing a controller which is robust to fault such as observer-based sliding mode controller designed in [8]. In fact, since then, many significant results about state estimation and unknown input reconstruction simultaneously have been reported in [9–22]. For example, for linear discrete-time systems with invariant zeros can be located anywhere in the complex plane, Marro and Zattoni [9] propose a way to solve the problems of the simultaneous state estimation and unknown input reconstruction. Kalsi *et al*. [16] and Zhu [17] cope with the simultaneous state and unknown input estimation problems for linear systems when the so-called observer matching condition is not satisfied. More recently, in [19], the UIO designs for a class of switched descriptor systems are considered and an algebraic unknown input reconstruction method is developed based on an asymptotical state reduced-order observer. An observer-based fuzzy adaptive controller for a class of uncertain nonstrict nonlinear systems with unknown control direction and unknown dead-zone is presented in [22] by using the estimation of unknown nonlinearities approximated by adaptive mechanism. Recently, UIO-based control designs are dealt in [23–25].

The interval observers, which can produces both the upper and lower boundary estimates of the system states, have drawn much attention in literature. To design an interval observer for system with unknown inputs, much information about the unknown inputs or/and the nonlinear terms, such as the Lipschitz condition, can be ignored. So, constructing an interval observer turns out to be much more convenient than designing a traditional Luenberger-like unknown input observer. Since the interval observer design method was first developed by Gouzé *et al*. [26] in the year 2000, many excellent results on interval observer designs have been reported in literature [27–37]. For example, an interval observer for a linear parameter-varying system subject to actuator faults is proposed and the fault-tolerant control issues are then considered in [31]. Ifqir *et al*. [32] investigate the problem of robust state estimation and unknown input interval reconstruction for uncertain switched linear systems. [34] proposes a novel interval observer design method for discrete-time linear systems with unknown but bounded disturbance and measurement noise. In a word, interval observer has become one of the most powerful alternative methods of UIO and nonlinear observer designs.

In the present paper, we are dedicate to develop an asymptotical unknown input reconstruction method based on interval observer and then investigate the observer-based state feedback and unknown input compensation controller designs for a class of discrete-time systems with both unknown inputs and measurement noise. Our method shows some advantages and novelties. 1) In order to estimate the states and measurement noise simultaneously, an augmented descriptor system is constructed first, and the augmented descriptor system is then transformed into a general augmented system. We prove that preconditions, specifically the minimum phase system condition and the observer matching condition, can be kept under these two system transformations. 2) A new unknown input reconstruction method is developed based on interval observer. Although the interval estimation of the output is introduced into the reconstruction, the reconstruction turns out be an asymptotical estimation of its actual unknown input. Moreover, the design of unknown input reconstruction is irrelative to the control signal, so it is convenient for us to design an unknown input compensation controller based on the reconstruction. 3) Based on the asymptotical state estimation and the unknown input reconstruction, a state feedback and unknown input compensation controller is designed and the stability of the closed-loop system is analyzed. We point out that the designs of the observer-based closed-loop system satisfy the so-called separation property.

The remainder of the paper is organized as follows: In Section 2, we present some preliminaries including assumptions, definitions and lemmas. In Section 3, a reduced-order observer is designed for the augmented system so that the state and the measurement noise can be estimated asymptotically and simultaneously. In Section 4, a simple interval observer is proposed first. And then, an unknown input reconstruction method is developed based on the interval observer. Moreover, the state feedback and unknown input compensation controller is given and the stability of the closed-loop system is analyzed. In Section 5, two simulation examples are given to verify the effectiveness of the proposed methods. Some conclusions are summarized in Section 6.

2. PRELIMINARIES

In this section, some basic assumptions are made and under these assumptions, the original discrete-time system is augmented into a descriptor system, and the augmented descriptor system is again transformed into a general augmented system. We prove that the preconditions can be kept under these two system transformations.

Consider a linear system with only unknown input without measurement noise first

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Dd(k), \\ y(k) = Cx(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbf{R}^n$, $u(k) \in \mathbf{R}^m$, $y(k) \in \mathbf{R}^p$, $d(k) \in \mathbf{R}^q$ are system state, control input, measurement output and unknown input vectors, respectively.

For system (1), we make the following two Assumptions.

Assumption 1:

$$\text{rank}\begin{bmatrix} sI-A & D \\ C & 0 \end{bmatrix} = n+q \tag{2}$$

holds for any complex number s, $|s| \geq 1$.

Assumption 2:

$$\text{rank}(CD) = \text{rank}(D) \tag{3}$$

holds.

Lemma 1: Assumption 2 holds if and only if there exists a nonsingular matrix $T \in \mathbf{R}^{n\times n}$ such that the state equation of system (1) is algebraically equivalent to $\{\hat{A},\hat{B},\hat{D}\}$, where $\hat{A} := TAT^{-1} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}$, $\hat{B} := TB = \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix}$, $\hat{D} := TD = \begin{bmatrix} I_q \\ 0 \end{bmatrix}$ and $\hat{A}_{11} \in R^{q\times q}$, $\hat{B}_1 \in R^{q\times m}$, $\hat{C}_{22} \in R^{(p-q)\times(n-q)}$. Besides, there exists a nonsingular matrix $S \in \mathbf{R}^{p\times p}$ such that $\hat{C} := SCT^{-1} = \begin{bmatrix} I_q & 0 \\ 0 & \hat{C}_{22} \end{bmatrix}$, when $p > q$, and when $p = q$, we have $\hat{C} = SCT^{-1} = \begin{bmatrix} I_p & 0 \end{bmatrix}$. Moreover, when $p > q$, Assumption 1 holds if and only if $(\hat{A}_{22},\hat{C}_{22})$ is detectable, and when $p = q$, $(\hat{A}_{22}, I_p)$ is detectable.

Proof: Please see Appendix A. □

Remark 1: A similar result to Lemma 1 has been presented in [38]. Here we still offer the proof of Lemma 1 because the proof given in the present paper is totally different from that of [38] and, moreover, new computation ways of the transformation matrices T and S are presented.

Definition 1: A nonnegative matrix is a square matrix in which all its components are nonnegative. A square matrix is called stable matrix or Schur matrix if its eigenvalues are all located on inside the unit circle of the complex plane.

Lemma 2 [39]**:** Suppose that the matrix $A \in \mathbf{R}^{n\times n}$ is a nonnegative and Schur matrix in the following linear system $x(k+1) = Ax(k) + d(k)$, where $x(k) \in \mathbf{R}^n$ and $d(k) \in \mathbf{R}^n$ satisfies $d(k) \geq 0$ for all $k \geq 0$. If the initial condition satisfies $x(0) \geq 0$, then we have $x(k) \geq 0$ for all $k \geq 0$.

Lemma 3 [27]**:** Suppose that vector variables $\bar{x}(t)$, $\underline{x}(t)$, $x(t) \in \mathbf{R}^n$ satisfies $\underline{x}(t) \leq x(t) \leq \bar{x}(t)$ for all $t \geq 0$, then for any constant matrix $M \in \mathbf{R}^{m\times n}$, we have

$$M^+\underline{x}(t) - M^-\bar{x}(t) \leq Mx(t) \leq M^+\bar{x}(t) - M^-\underline{x}(t), \tag{4}$$

where $M^+ = \max\{M,0\}$, $M^- = \max(0,-M)$.

It should be declared that all the notations appearing in the above, including the notations in the proof of Lemma 1, are independent to the notations used in the following discussions.

The main purpose of the present paper is trying to develop observers which can estimate the system sates, the unknown inputs and measurement noise, and then design a controller for the following discrete-time linear system with both unknown input and measurement noise:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Dd(k), \\ y(k) = Cx(k) + Fw(k), \end{cases} \tag{5}$$

where $x(k) \in \mathbf{R}^n$, $u(k) \in \mathbf{R}^m$, $y(k) \in \mathbf{R}^p$, $d(k) \in \mathbf{R}^q$ and $w(k) \in \mathbf{R}^r$ are system state, control input, measurement output, unknown input and measurement noise vectors, respectively.

For system (5), we make the following Assumptions.

Assumption 3:

$$\text{rank}\begin{bmatrix} sI-A & 0 & D \\ C & F & 0 \end{bmatrix} = \bar{n}+q \tag{6}$$

holds for any complex number s with $|s| \geq 1$, where $\bar{n} = n+r$.

Assumption 4: Matrix C has full row column rank and both matrices D and F have full column rank.

Assumption 5: For any initial state $x(0)$, there exist two known vectors $x^-(0)$ and $x^+(0)$ such that $x^-(0) \leq x(0) \leq x^+(0)$. For the unknown input vector $d(k)$, there are two known vectors d^- and d^+ such that $d^- < d(k) < d^+$. For the unknown measurement noise $w(t)$, there are two known vectors w^- and w^+ such that $w^- < w(k) < w^+$.

If we make up an augmented state vector of $x_e(k)$, and introduce the notations E_e, A_e, C_e as follows:

$$x_e(k) = \begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix}^T C_e = \begin{bmatrix} C & F \end{bmatrix} \in \mathbf{R}^{p\times\bar{n}},$$
$$A_e = \begin{bmatrix} A & 0_{n\times r} \end{bmatrix} \in \mathbf{R}^{n\times\bar{n}}, \quad E_e = \begin{bmatrix} I_n & 0_{n\times r} \end{bmatrix} \in \mathbf{R}^{n\times\bar{n}}$$

an augmented system can be obtained as

$$\begin{cases} E_e x_e(k+1) = A_e x_e(k) + Bu(k) + Dd(k), \\ y(k) = C_e x_e(k). \end{cases} \tag{7}$$

Lemma 4: For matrices E_e and C_e, there exist two matrices $\Lambda \in \mathbf{R}^{\bar{n}\times p}$ and $G \in \mathbf{R}^{\bar{n}\times n}$ such that

$$GE_e + \Lambda C_e = I_{\bar{n}}. \tag{8}$$

Proof: First, (8) can be rewritten as $\begin{bmatrix} G & \Lambda \end{bmatrix} \begin{bmatrix} E_e \\ C_e \end{bmatrix} = I_{\bar{n}}$ Since $R := \begin{bmatrix} E_e \\ C_e \end{bmatrix} = \begin{bmatrix} I_n & 0_{n\times r} \\ C & F \end{bmatrix} \in \mathbf{R}^{(n+p)\times\bar{n}}$ is a matrix with full column rank, so $M := (R^T R)^{-1} \in \mathbf{R}^{\bar{n}\times\bar{n}}$ exists. Obviously,

$$\begin{bmatrix} G & \Lambda \end{bmatrix} = MR^T = M \begin{bmatrix} I_n & C^T \\ 0_{r\times n} & F^T \end{bmatrix}$$

is a solution of $\begin{bmatrix} G & \Lambda \end{bmatrix} \begin{bmatrix} E_e \\ C_e \end{bmatrix} = I_{\bar{n}}$. So, $G = M \begin{bmatrix} I_n \\ 0_{r\times n} \end{bmatrix}$ and $\Lambda = M \begin{bmatrix} C^T \\ F^T \end{bmatrix}$ are solutions of (8). □

Now based on (8), the augmented system (7) can be transformed into

$$\begin{cases} x_e(k+1) = GA_e x_e(k) + GBu(k) + \Lambda y(k+1) \\ \qquad\qquad + GDd(k), \\ y(k) = C_e x_e(k). \end{cases} \tag{9}$$

If we furthermore make a state shift transformation $z(k) = x_e(k) - \Lambda y(k)$ and denote $y_s(k) = (I_p - C_e\Lambda)y(k)$, then (9) becomes

$$\begin{cases} z(k+1) = GA_e z(k) + GBu(k) + GA_e\Lambda y(k) \\ \qquad\qquad + GDd(k), \\ y_s(k) = C_e z(k). \end{cases} \tag{10}$$

Lemma 5: Under the Assumption 3, we have

$$\text{rank}\begin{bmatrix} sI_{\bar{n}} - GA_e & GD \\ C_e & 0 \end{bmatrix} = \bar{n} + q \tag{11}$$

holds for any complex number s with $|s| \geq 1$.

Proof: Please see Appendix B. □

Lemma 6: Under Assumption 4, we have

$$\text{rank}(C_e GD) = \text{rank}(GD). \tag{12}$$

Proof: Please see Appendix C. □

Remark 2: Precondition (6) or (11) and condition (12) in Lemma 6, which is guaranteed by Assumption 3, are actually necessary and sufficient conditions for designing an unknown input observer [8]. Precondition (6) is called minimum phase system condition, while (12) is called the observer matching condition. In view of practical points, the two conditions may be a little restrictive. For this reason, many scholars tried to break through the restrictions. For example, Zhu [17] considered the problems of the simultaneous estimation of the system states and the unknown inputs when the so-called observer matching condition is not satisfied.

3. THE ESTIMATIONS OF THE STATES AND MEASUREMENT NOISE

In this section, we are dedicated to design a reduced-order observer for the augmented system (9) under Assumptions 3 and 4, such that the asymptotical estimation of the augmented state can be obtained. In this way, we can get the estimate of the original system and the reconstruction of the measurement noise. From the discussions in previous section, we known that under Assumption 3, (11) holds for any complex number s with $|s| \geq 1$ (Lemma 5). Moreover, (12) holds under Assumption 4 (Lemma 6). So, by Lemma 1, we know that under Assumptions 3 and 4, there exists an equivalent state transformation $\bar{z}(k) = Tz(k)$, where $T \in \mathbf{R}^{\bar{n}\times\bar{n}}$ is a nonsingular matrix, such that the sate equation of (10) is transformed into

$$\begin{cases} \bar{z}_1(k+1) = \bar{A}_{11}\bar{z}_1(k) + \bar{A}_{12}\bar{z}_2(k) + \bar{B}_1 u(k) \\ \qquad\qquad + \bar{\Pi}_1 y(k) + d(k), \\ \bar{z}_2(k+1) = \bar{A}_{22}\bar{z}_2(k) + \bar{A}_{21}\bar{z}_1(k) + \bar{B}_2 u(k) \\ \qquad\qquad + \bar{\Pi}_2 y(k), \end{cases} \tag{13}$$

and an equivalent output transformation $\bar{y}(k) = Sy_s(k)$ such that the output equation of (10) becomes

$$\begin{bmatrix} \bar{y}_1(k) \\ \bar{y}_2(k) \end{bmatrix} = \begin{bmatrix} I_q & 0 \\ 0 & \bar{C}_{22} \end{bmatrix} \begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix},$$

when $p > q$, where $\bar{\Pi}_1 = \begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)} \end{bmatrix} TGA_e\Lambda$, $\bar{\Pi}_2 = \begin{bmatrix} 0_{(\bar{n}-q)\times q} & I_{\bar{n}-q} \end{bmatrix} TGA_e\Lambda$, and

$$\bar{A} = T(GA_e)T^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix},$$

$$\bar{B} := T(GB) = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix},$$

$$\bar{D} := T(GD) = \begin{bmatrix} I_q \\ 0 \end{bmatrix}, \quad \bar{C} = SC_e T^{-1} = \begin{bmatrix} I_q & 0 \\ 0 & \bar{C}_{22} \end{bmatrix}. \tag{14}$$

Besides, $\bar{A}_{11} \in R^{q\times q}$, $\bar{B}_1 \in R^{q\times m}$ and $\bar{C}_{22} \in R^{(p-q)\times(\bar{n}-q)}$. When $p = q$, the output equation becomes

$$\bar{y}(k) = \begin{bmatrix} I_p & 0 \end{bmatrix} \bar{z}(k) = \bar{z}_1(k).$$

In the following discussions, we only consider the case when $p > q$. The case when $p = q$ can be discussed similarly and similar conclusions can also be obtained.

Theorem 1: Under Assumptions 3 and 4, the following system

$$\begin{cases} \hat{\bar{z}}_2(k+1) = \bar{A}_{22}\hat{\bar{z}}_2(k) + \bar{A}_{21}\bar{y}_1(k) + \bar{B}_2 u(k) \\ \qquad\qquad + \bar{\Pi}_2 y(k) - \bar{L}_{22}(\bar{y}_2(k) - \bar{C}_{22}\hat{\bar{z}}_2(k)), \\ \hat{x}_e(k) = \hat{z}(k) + \Lambda y(k) = T^{-1}\begin{bmatrix} \bar{y}_1(k) \\ \hat{\bar{z}}_2(k) \end{bmatrix} + \Lambda y(k) \end{cases} \tag{15}$$

is an asymptotical reduced-order observer of the augmented system (9) in that $\lim_{k\to\infty}(x_e(k) - \hat{x}_e(k)) = 0$, where the observer gain matrix is chosen such that $\bar{A}_{22} + \bar{L}_{22}\bar{C}_{22}$ is Schur stable.

Proof: The reduced-order observer error dynamic system can be obtained by subtracting the first equation of (15) from second equation of (13) and it is

$$\tilde{\bar{z}}_2(k+1) = (\bar{A}_{22} + \bar{L}_{22}\bar{C}_{22})\tilde{\bar{z}}_2(k), \tag{16}$$

where $\tilde{\bar{z}}_2(k) = \bar{z}_2(k) - \hat{\bar{z}}_2(k)$. If the observer gain matrix $\bar{L}_{22}$ is chosen such that $\bar{A}_{22} + \bar{L}_{22}\bar{C}_{22}$ is Schur stable, then we have $\lim_{k\to\infty}\tilde{z}_2(k) = 0$. On the other hand, $x_e(k) = z(k) + \Lambda y(k) = T^{-1}\begin{bmatrix} \bar{y}_1(k) \\ \bar{z}_2(k) \end{bmatrix} + \Lambda y(k)$, so we have

$\tilde{x}_e(k) = T^{-1}\begin{bmatrix} 0 \\ \tilde{\bar{z}}_2(k) \end{bmatrix}$ and this gives $\lim_{k\to\infty} \tilde{x}_e(k) = 0$, where $\tilde{x}_e(k) = x_e(k) - \hat{x}_e(k)$. □

After we have got the estimation of the augmented sate $x_e(k)$, i.e., $\hat{x}_e(k)$, the state estimate of the original system and the reconstruction of the measurement noise can be obtained as $\hat{x}(k) = \begin{bmatrix} I_n & 0_{n\times r} \end{bmatrix} \hat{x}_e(k)$ and $\hat{w}(t) = \begin{bmatrix} 0_{r\times n} & I_r \end{bmatrix} \hat{x}_e(k)$, respectively.

4. UNKNOWN INPUT RECONSTRUCTION AND COMPENSATION CONTROLLER DESIGN

In this section, a new asymptotical unknown input reconstruction based on an interval observer is developed. And based on the reconstructed information of unknown input, an observer-based state feedback and unknown input compensation controller is designed as well. For these purposes, an interval observer which can produce the upper and lower boundary estimations for the output variable $\bar{y}_1(k)$ is constructed. It should be emphasized in advance that the reconstruction is an asymptotical one although the interval estimation of the $\bar{y}_1(k)$ is introduced into the reconstruction. Moreover, the unknown input reconstruction is irrelative to the control signal of $u(k)$.

4.1. Interval observer design for $\bar{y}_1(k)$

Notice the fact that $\bar{z}_1(k) = \bar{y}_1(k)$ and from the first equation of (13), we know that the dynamic system of the output variable $\bar{y}_1(k)$ is actually governed by

$$\begin{aligned}\bar{y}_1(k+1) =& \bar{A}_{11}\bar{y}_1(k) + \bar{A}_{12}\bar{z}_2(k) + \bar{B}_1 u(k) \\ &+ \bar{\Pi}_1 y(k) + d(k). \end{aligned} \tag{17}$$

Theorem 2: Consider following system

$$\begin{cases} \hat{\bar{y}}_1^+(k+1) = \bar{A}_{11}\hat{\bar{y}}_1^+(k) + \bar{A}_{12}\hat{\bar{z}}_2(k) + \bar{B}_1 u(k) \\ \qquad + \bar{\Pi}_1 y(k) + L_s(\bar{y}_1(k) - \hat{\bar{y}}_1^+(k)) + d^+, \\ \hat{\bar{y}}_1^-(k+1) = \bar{A}_{11}\hat{\bar{y}}_1^-(k) + \bar{A}_{12}\hat{\bar{z}}_2(k) + \bar{B}_1 u(k) \\ \qquad + \bar{\Pi}_1 y(k) + L_s(\bar{y}_1(k) - \hat{\bar{y}}_1^-(k)) + d^-. \end{cases} \tag{18}$$

Suppose $\Gamma \in \mathbf{R}^{q\times q}$ is a Schur stable and nonnegative matrix which is chosen arbitrarily in advance, and determine the observer gain matrix L_s by $L_s = \bar{A}_{11} - \Gamma$, and moreover set the initial state of (18) as

$$\begin{cases} \hat{\bar{y}}_1^+(0) = Q_e^+ x_e^+(0) - Q_e^- x_e^-(0), \\ \hat{\bar{y}}_1^-(0) = Q_e^+ x_e^-(0) - Q_e^- x_e^+(0), \end{cases} \tag{19}$$

where $Q_e^+ = \max\{Q_e, 0\}$, $Q_e^- = \max(0, -Q_e)$, $Q_e = \begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)} \end{bmatrix} T(I_{\bar{n}} - \Lambda C_e) \in \mathbf{R}^{q\times\bar{n}}$, $x_e^+(0) = \begin{bmatrix} x^+(0) \\ w^+ \end{bmatrix}$ and $x_e^-(0) = \begin{bmatrix} x^-(0) \\ w^- \end{bmatrix}$, then system (18) is an interval observer of system (17) in the sense of: there exists a constant $K_0 \geq 0$ such that $\hat{\bar{y}}_1^-(k) \leq \bar{y}(k) \leq \hat{\bar{y}}_1^+(k)$ for all $k \geq K_0$.

Proof: Please see Appendix D. □

It should be emphasized that all the following results are given under the condition of $k \geq K_0$.

4.2. The reconstruction of the unknown input $d(t)$

Denote

$$\hat{\bar{y}}_1^-(k) = \begin{bmatrix} \hat{\bar{y}}_{1,1}^-(k) \\ \vdots \\ \hat{\bar{y}}_{1,q}^-(k) \end{bmatrix}, \quad \bar{y}_1(k) = \begin{bmatrix} \bar{y}_{1,1}(k) \\ \vdots \\ \bar{y}_{1,q}(k) \end{bmatrix},$$
$$\hat{\bar{y}}_1^+(k) = \begin{bmatrix} \hat{\bar{y}}_{1,1}^+(k) \\ \vdots \\ \hat{\bar{y}}_{1,q}^+(k) \end{bmatrix}.$$

We then have $\hat{\bar{y}}_{1,i}^-(k) \leq \bar{y}_{1,i}(k) \leq \hat{\bar{y}}_{1,i}^+(k)$ $(i = 1, \cdots, q)$. So, there must exist $0 \leq \alpha_i(k) \leq 1$ such that $\bar{y}_{1,i}(k) = \alpha_i(k)\hat{\bar{y}}_{1,i}^+(k) + (1-\alpha_i(k))\hat{\bar{y}}_{1,i}^-(k)$, $(i = 1, \cdots, q)$ holds. Moreover, based on above equations, if we introduce notation of $\Phi(k) = \mathrm{diag}(\alpha_i(k))$, we will have

$$\bar{y}_1(k) = \Phi(k)\hat{\bar{y}}_1^+(k) + (I_q - \Phi(k))\hat{\bar{y}}_1^-(k), \tag{20}$$

or

$$\begin{aligned}\bar{y}_1(k+1) =& \Phi(k+1)\hat{\bar{y}}_1^+(k+1) \\ &+ (I_p - \Phi(k+1))\hat{\bar{y}}_1^-(k+1). \end{aligned} \tag{21}$$

Substituting (18) into (21) gives

$$\begin{aligned}\bar{y}_1(k+1) =& \Phi(k+1)(\bar{A}_{11} - L_s)\hat{\bar{y}}_1^+(k) \\ &+ (I_q - \Phi(k+1))(\bar{A}_{11} - L_s)\hat{\bar{y}}_1^-(k) \\ &+ \bar{A}_{12}\hat{\bar{z}}_2(k) + \bar{B}_1 u(k) + \bar{\Pi}_1 y(k) + L_s\bar{y}_1(k) \\ &+ \Phi(k+1)d^+ + (I_q - \Phi(k+1))d^-. \end{aligned} \tag{22}$$

Compare (22) with (17), we obtain

$$\begin{aligned} d(k) =& \Phi(k+1)(\bar{A}_{11} - L_s)\hat{\bar{y}}_1^+(k) \\ &+ (I_q - \Phi(k+1))(\bar{A}_{11} - L_s)\hat{\bar{y}}_1^-(k) \\ &+ (L_s - \bar{A}_{11})\bar{y}_1(k) - \bar{A}_{12}\tilde{\bar{z}}_2(k) \\ &+ (I_q - \Phi(k+1))d^- + \Phi(k+1)d^+. \end{aligned} \tag{23}$$

Now, based on (23), an unknown input reconstruction for $d(k)$ is given by

$$\begin{aligned} \hat{d}(k) =& \Phi(k+1)(\bar{A}_{11} - L_s)\hat{\bar{y}}_1^+(k) \\ &+ (I_q - \Phi(k+1))(\bar{A}_{11} - L_s)\hat{\bar{y}}_1^-(k) \\ &+ (L_s - \bar{A}_{11})\bar{y}_1(k) \\ &+ (I_q - \Phi(k+1))d^- + \Phi(k+1)d^+. \end{aligned} \tag{24}$$

Obviously, we have

$$\lim_{k\to\infty}(d(k) - \hat{d}(k)) = -\bar{A}_{12}\lim_{k\to\infty}\tilde{\bar{z}}_2(k) = 0.$$

Next, we are going to give the computation of $\Phi(k)$ or $\alpha(k)$. First, (20) can be rewritten as $\Phi(k)[\hat{\bar{y}}_1^+(k)-\hat{\bar{y}}_1^-(k)]=\bar{y}_1(k)-\hat{\bar{y}}_1^-(k)$ and it is equivalent to $\Omega(k)\alpha(k)=\bar{y}_1(k)-\hat{\bar{y}}_1^-(k)$ if we introduce the notation of $\Omega(k)=\text{diag}(\hat{\bar{y}}_1^+(k)-\hat{\bar{y}}_1^-(k))$. So, we have

$$\alpha(k)=\Omega^{-1}(k)[\bar{y}_1(k)-\hat{\bar{y}}_1^-(k)], \tag{25}$$

noticing the fact that $\Omega(k)$ is nonsingular for all $k\geq 0$ because of $\hat{\bar{y}}_1^+(k)\neq\hat{\bar{y}}_1^-(k)$ for all $k\geq 0$. Then $\Phi(k)=\text{diag}\{\Omega^{-1}(k)[\bar{y}_1(k)-\hat{\bar{y}}_1^-(k)]\}$.

Remark 3: It should be pointed out that although there are some papers dealing with the unknown reconstruction problems based on interval observers, the existing methods provide only interval estimations of the unknown inputs rather than asymptotic ones [40,41]. Moreover, in our method the unknown input reconstruction decouples from the control input $u(t)$ and this feature is significant because it will be convenient for us to design an unknown input compensation controller by introducing the reconstruction into the controller directly and this advantage will be tested in next section.

4.3. Observer-based unknown input compensation controller design

Based on the reduced-order observer (15) given in section 3 and the unknown input reconstruction (24) developed in Section 4.2, we are dedicated to investigate observer-based state feedback and unknown input compensation controller design problems. For this purpose, we assume that $B=D$ and (A,B) is controllable. The observer-based state feedback and unknown input compensation controller is designed as

$$u(k)=K\hat{x}(k)-\hat{d}(k). \tag{26}$$

Substituting the controller (26) into the original system (5) gives

$$\begin{aligned}x(k+1)=&(A+BK)x(k)-BK\tilde{x}(k)+B\tilde{d}(k)\\=&(A+BK)x(k)-BK\begin{bmatrix}I_n & 0_{n\times r}\end{bmatrix}\tilde{x}_e(k)+B\tilde{d}(k)\\=&(A+BK)x(k)+B\tilde{d}(k)\\&-BK\begin{bmatrix}I_n & 0_{n\times r}\end{bmatrix}T^{-1}\begin{bmatrix}0\\ \tilde{\bar{z}}_2(k)\end{bmatrix},\end{aligned}$$

where $\tilde{x}(t)=x(t)-\hat{x}(t)$. Now decompose the matrix T^{-1} into a block matrix $T^{-1}=\begin{bmatrix}* & T_{12}^*\\ * & *\end{bmatrix}$, where $T_{12}^*\in\mathbf{R}^{n\times(n-q)}$, we can get

$$x(k+1)=(A+BK)x(k)+\Psi\tilde{\bar{z}}_2(k)+B\tilde{d}(k), \tag{27}$$

where $\Psi=-BKT_{12}^*$ and $\tilde{d}(k)=d(k)-\hat{d}(k)$. Combining (16) and (27) leads to the dynamic closed-loop system under the observer-based state feedback and unknown input compensation controller:

$$\xi(k+1)=\mathcal{A}\xi(k)+\mathcal{B}\tilde{d}(k), \tag{28}$$

where $\xi(k)=\begin{bmatrix}x^T(k) & \tilde{\bar{z}}_2^T(k+1)\end{bmatrix}^T$ and

$$\mathcal{A}=\begin{bmatrix}A+BK & -\Psi_2\\ 0 & \bar{A}_{22}+\bar{L}_{22}\bar{C}_{22}\end{bmatrix},\ \mathcal{B}=\begin{bmatrix}B\\0\end{bmatrix}.$$

From the discussions in the previous sections, we know that, on the one hand, the unknown input reconstruction determined by (24) depends only on the reduced-order observer (15) and the interval observer (18). In fact, if the reduced-order observer (15) can guarantee that the asymptotical estimation of $\bar{z}_2(k)$ is successful, the interval observer (18) can insure that the interval estimation of $\bar{y}_1(k)$ is successful and this further guarantees that the asymptotical reconstruction of the unknown input $d(k)$ can be reached by (24). So, if we choose the reduced-order observer gain matrix $\bar{L}_{22}$ such that matrix $\bar{A}_{22}+\bar{L}_{22}\bar{C}_{22}$ is asymptotical stable, the asymptotical reconstruction of the unknown input $d(k)$ can be guaranteed. Moreover, if we further design the state feedback gain matrix K such that $A+BK$ is asymptotical stable, then we know that $\mathcal{A}$ is asymptotical stable. So, based on (28), we can conclude that the closed-loop system (28) is asymptotical stable. Now based on the above discussions, we can drew a conclusion which called separation property as follows:

Theorem 3: If we choose the state feedback gain matrix K and the reduced-order observer gain matrix $\bar{L}_{22}$ such that the eigenvalues of $A+BK$ and $\bar{A}_{22}+\bar{L}_{22}\bar{C}_{22}$ are all located on inside the unit circle, then the unknown input reconstruction $\hat{d}(k)$ determined by (24) can approach to the actual unknown input $d(k)$ asymptotically, and the closed-loop system (28) is asymptotically stable. Moreover, the inserting of the unknown input compensation and the reduced-order observer (15) does not affect the eigenvalue of the original state feedback, nor are the eigenvalue of the reduced-order observer affected by the connection. Thus, the designs of the state feedback and unknown input compensation controller, the unknown input reconstruction and the reduced-order observer can be carried out independently.

5. SIMULATION

In this section, two simulation examples are given to test the effectiveness of the proposed methods. One is used to verify the effects of the state asymptotical estimation with reduced-order observer and interval estimation of the interval observer, and the performance of the unknown input reconstruction. The other one is used to test the performance of the closed-loop system under the observer-based state feedback and unknown input compensation controller.

5.1. Example 1

Consider a system in form (5) with

$$A=\begin{bmatrix}0.25 & 0 & 0 & 0 & 0.25\\ 0.05 & 0.35 & 0.05 & 0.05 & 0.05\\ 0.1 & 0.2 & 0.35 & 0.1 & -0.15\\ 0.05 & 0.1 & 0.05 & 0.3 & -0.2\\ 0 & 0 & 0 & 0 & 0.5\end{bmatrix},$$

$$B=D=\begin{bmatrix}3 & 1\\ 1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix},\quad F=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}.$$

We assume $w(k)=0.5\sin(k)$ and

$$u(k)=\begin{bmatrix}\sin(k)\\ \cos(k)\end{bmatrix},\quad d(k)=\begin{bmatrix}2\sin(0.8k+2)\\ 0.5\cos(k+1)\end{bmatrix}.$$

First, the augmented matrices E_e, A_e and C_e can be constructed easily and then the solution to (8) can be computed out as

$$G=\begin{bmatrix}0.72727 & -0.18182 & 0.090909 & 0 & 0\\ -0.18182 & 0.54545 & -0.27273 & 0 & 0\\ 0.090909 & -0.27273 & 0.63636 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ -0.36364 & 0.090909 & -0.54545 & 0 & 0\end{bmatrix},$$

$$\Lambda=\begin{bmatrix}0.27273 & -0.090909 & -0.27273\\ 0.18182 & 0.27273 & -0.18182\\ -0.090909 & 0.36364 & 0.090909\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0.36364 & -0.45455 & 0.63636\end{bmatrix}.$$

Based on Lemma 1, the transformation matrices T and S which can transform the system (10) into the canonical form (14) are

$$T=\begin{bmatrix}0.5 & 3.5 & 3 & 0 & 0 & 0\\ 0 & -11 & -11 & 0 & 0 & 0\\ 0 & 0.44721 & 0.89443 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0.44721 & 0 & 0 & 0 & 0 & 0.89443\end{bmatrix},$$

$$S=\begin{bmatrix}0.5 & 3 & -0.5\\ 0 & -11 & 0\\ 0.70711 & 0 & 0.70711\end{bmatrix},$$

and furthermore,we have

$$\bar{A}_{11}=\begin{bmatrix}0.3 & 0.045455\\ -0.6 & -0.022727\end{bmatrix},$$

$$\bar{A}_{12}=\begin{bmatrix}0 & 0.15 & -0.1 & 0\\ 0 & -0.55 & 0.7 & 0\end{bmatrix},$$

$$\bar{A}_{21}=\begin{bmatrix}0.089443 & 0.030492\\ 0.1 & 0.022727\\ 0 & 0\\ -0.089443 & -0.030492\end{bmatrix},$$

$$\bar{A}_{22}=\begin{bmatrix}0.25 & 0.044721 & -0.067082 & 0\\ 0 & 0.3 & -0.2 & 0\\ 0 & 0 & 0.5 & 0\\ -0.25 & -0.044721 & 0.067082 & 0\end{bmatrix},$$

and $\bar{C}_{22}=\begin{bmatrix}1.5811 & 0 & 0 & 1.5811\end{bmatrix}$. Since the matrix $\bar{A}_{22}$ is a Schur stable matrix with eigenvalues of 0, 0.25, 0.3 and 0.5, so the reduced-order observer gain matrix can be choose as $\bar{L}_{22}=0_{4\times 1}$. Now the reduced-order observer, which can produce the asymptotical state estimates of the original system and the asymptotical reconstruction of the measurement noise, can be constructed by (15), and the estimating performances are shown in Figs. 1 and 2.

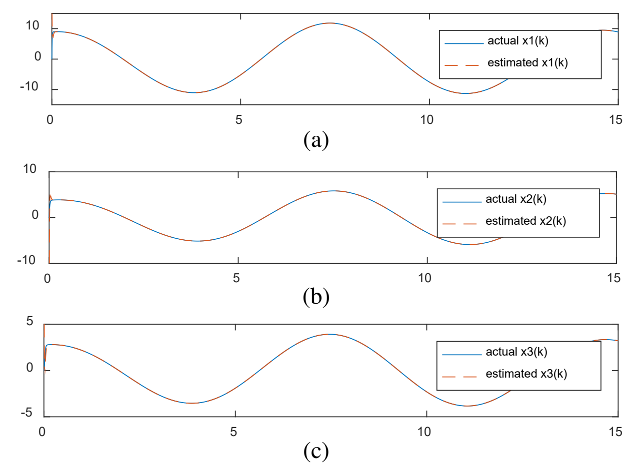

Fig. 1. The state estimation for $x_1 - x_3$.

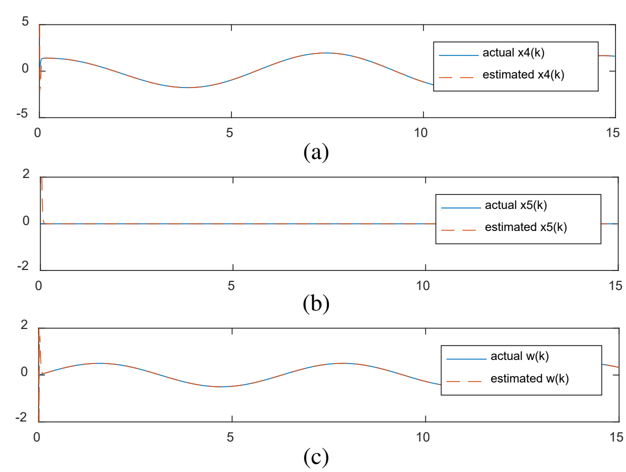

Fig. 2. The state estimation for x_4, x_5 and w.

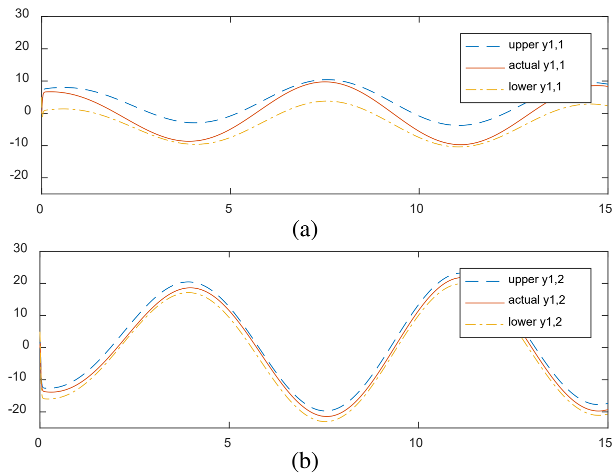

Fig. 3. The interval estimation for variable $\bar{y}_1(k)$.

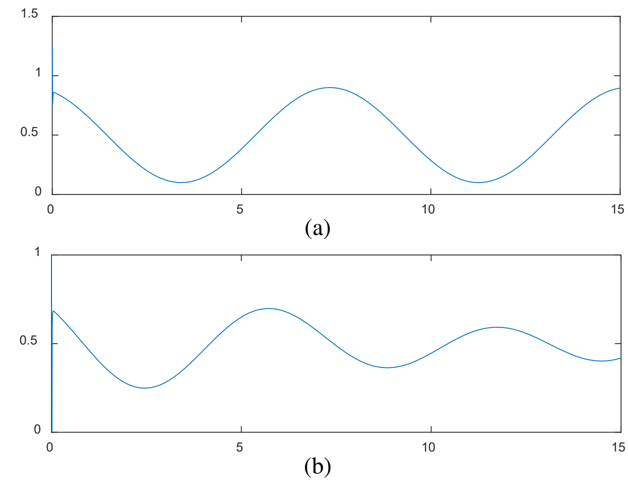

Fig. 4. The signals of $\alpha_1(k)$ and $\alpha_2(k)$.

To design the interval observer, we need to compute out $\bar{\Pi}_1$ and $\bar{\Pi}_2$ and they are

$$\bar{\Pi}_1 = \begin{bmatrix} 0.10455 & 0.28182 & -0.10455 \\ -0.27727 & -1.0409 & 0.27727 \end{bmatrix},$$

$$\bar{\Pi}_2 = \begin{bmatrix} 0.01423 & 0.077246 & -0.01423 \\ 0.027273 & 0.040909 & -0.027273 \\ 0 & 0 & 0 \\ -0.01423 & -0.077246 & 0.01423 \end{bmatrix}.$$

Choose $\Gamma = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.2 \end{bmatrix}$ as the nonnegative and Schur stable matrix and then the gain matrix L_s can be obtained by $L_s = \bar{A}_{11} - \Gamma$ and it is $L_s = \begin{bmatrix} 0.05 & 0.045455 \\ -0.7 & -0.22273 \end{bmatrix}$.

Now based on (18) we can design the interval observer for variable $\bar{y}_1(k)$ to produce the upper and lower boundary estimations. The interval estimation for variable $\bar{y}_1(k)$ is plotted in Fig. 3.

In order to get the reconstruction of the unknown input, we need to compute the $\alpha_1(k)$ and $\alpha_2(k)$ based on (25) first and they are reflected in Fig. 4 and from the figure, we find that calculations to them are as our expectation in that $0 \le \alpha_i(k) \le 1$ $(i = 1, 2)$. Now we can offer the unknown input reconstruction by (24) and the two unknown input reconstructions are plotted in Figs. 5(a) and 5(b), respectively. From the figures, we find that the unknown input reconstruction performances are satisfactory.

5.2. Example 2

Consider system in form (5) with

$$A = \begin{bmatrix} 0.81472 & 0.09754 & 0.15761 & 0.14189 & 0.65574 \\ 0.90579 & 0.2785 & 0.97059 & 0.42176 & 0.035712 \\ 0.12699 & 0.54688 & 0.95717 & 0.91574 & 0.84913 \\ 0.91338 & 0.95751 & 0.48538 & 0.79221 & 0.93399 \\ 0.63236 & 0.96489 & 0.80028 & 0.95949 & 0.67874 \end{bmatrix},$$

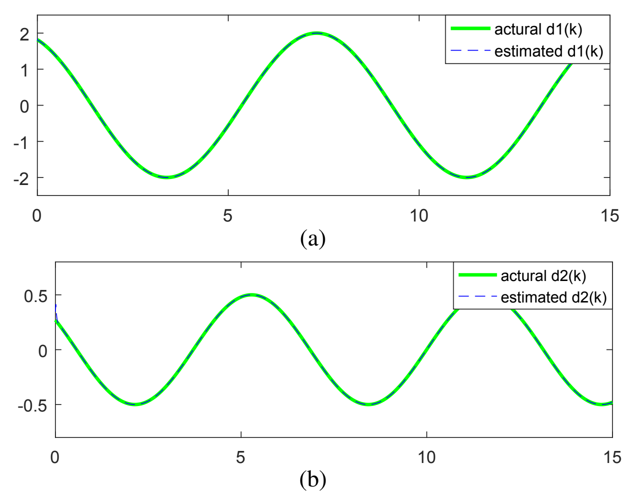

Fig. 5. The unknown input reconstruction of $d(k)$.

$$B = D = \begin{bmatrix} 0.75774 & 0.70605 \\ 0.74313 & 0.031833 \\ 0.39223 & 0.27692 \\ 0.65548 & 0.046171 \\ 0.17119 & 0.097132 \end{bmatrix}, \quad F = \begin{bmatrix} 0.27603 \\ 0.6797 \\ 0.6551 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.82346 & 0.95022 & 0.38156 & 0.18687 & 0.64631 \\ 0.69483 & 0.034446 & 0.76552 & 0.48976 & 0.70936 \\ 0.3171 & 0.43874 & 0.7952 & 0.44559 & 0.75469 \end{bmatrix}.$$

We assume $d(k) = \begin{bmatrix} 2\sin(0.8k+2) \\ 0.5\cos(k+1) \end{bmatrix}$ and $w(k) = 0.5\sin(k)$.

First, we should emphasize that the open-loop system is unstable because the matrix A contains an unstable eigenvalue of 3.2037. So, we are going to design a state feedback and unknown input compensation controller in form (26) such that the closed-loop system is asymptotical stable. To do so, we need to get the asymptotical state esti-

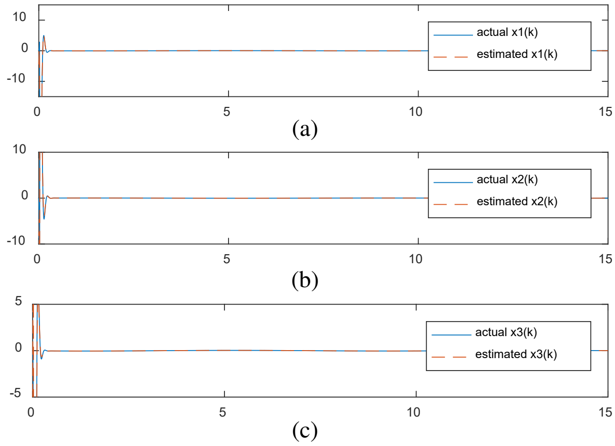

Fig. 6. The state estimation for $x_1 - x_3$.

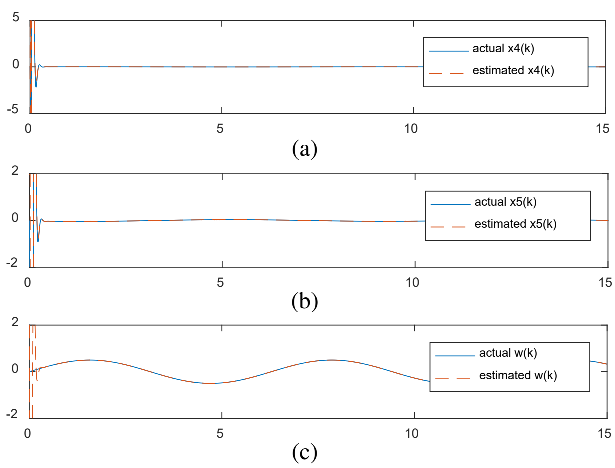

Fig. 7. The state estimation for x_4, x_5 and w.

mation $\hat{x}(k)$ via the reduced-order observer of (15) and the asymptotical unknown input reconstruction $\hat{d}(k)$ based on (24). The state estimations are reflected in Figs. 6 and 7.

In order to get the asymptotical unknown input reconstruction $\hat{d}(k)$, we have to design an interval observer for the variable $\bar{y}_1(k)$ and this need us to compute out $\alpha_1(k)$ and $\alpha_2(k)$ in advance. The interval estimation for variable $\bar{y}_1(k) = \begin{bmatrix} \bar{y}_{1,1}(k) & \bar{y}_{1,2}(k) \end{bmatrix}^T$ is given in Fig. 8, where Fig. 8 (a) is for $\bar{y}_{1,1}(k)$ and Fig. 8 (b) for $\bar{y}_{1,2}(k)$. The curves of $\alpha_1(k)$ and $\alpha_2(k)$ are plotted in Fig. 9 (Fig. 9(a) shows the $\alpha_1(k)$ and Fig. 9(b) the $\alpha_2(k)$).

Now, with the interval estimation for $\bar{y}_1(k)$ at hand and the calculation of $\alpha(k)$ being completed, the unknown input reconstruction can be implemented base on (24) and the reconstruction performances are shown in Figs. 10(a) and 10(b). After we have obtained the $\hat{x}(k)$ and $\hat{d}(k)$, we can carry out the state feedback and unknown input compensation controller design. The feedback gain matrix is designed as

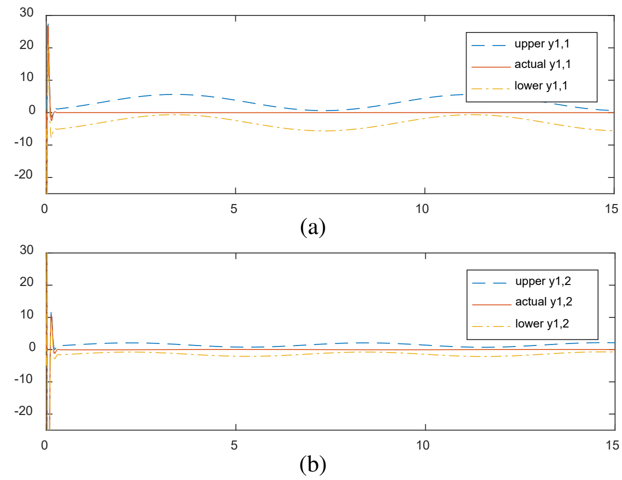

Fig. 8. The interval estimation for variable $\bar{y}_1(k)$.

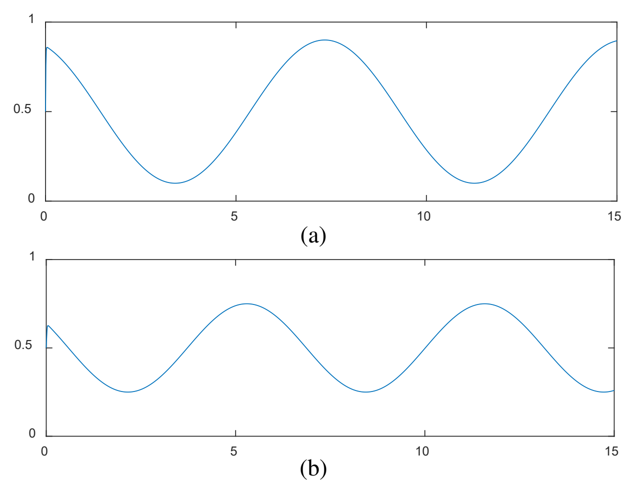

Fig. 9. The signals of $\alpha_1(k)$ and $\alpha_2(k)$.

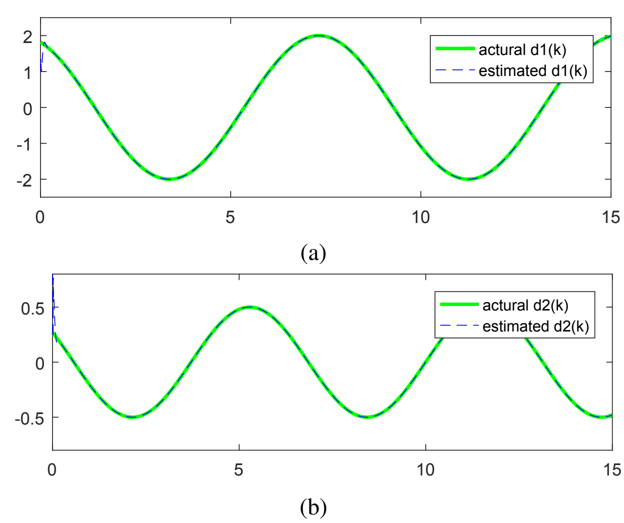

Fig. 10. The unknown input reconstruction of $d(k)$.

$$K = \begin{bmatrix} -1.5421 & -1.9352 & -1.8365 & -1.9659 & -1.4987 \\ 0.6901 & 2.7953 & 1.7956 & 2.6059 & 1.6167 \end{bmatrix}$$

such that the eigenvalues of $A+BK$ are placed to 0, 0.2, 0.4, -0.2, -0.4 which are all on the inside the unit circle. Again, from Fig. 6 and Fig. 7, we can see that all the states are driven to zero asymptotically under the deigned state feedback and unknown input compensation controller.

6. CONCLUSIONS

In this paper, two major issues, specifically, the unknown or unmeasured information estimations and feedback controller constructed using the estimations, are investigated for a class of discrete-time systems. In order to estimate the unmeasured states and the unknown measurement noises, an augmented descriptor system is constructed and a reduced-order observer is designed for it. We prove that the preconditions can be kept under the construction. In order to estimate the unknown inputs, an interval observer for parts of the outputs is designed and then a new unknown input reconstruction method is developed by using the interval estimation. The unknown input reconstruction can approach to its actual values asymptotically although it is given based on interval observation. Besides, the unknown input reconstruction is irrelative to the known control input and this is a kind of benefit to the controller designed by introducing the reconstruction into the controller. Finally, we design a state and unknown input feedback controller based on the state estimation and the unknown input reconstruction. Moreover, we analyze the stability of the closed-loop system and point out that the closed-loop system satisfies the so-called separation property. How to apply the proposed methods to FDI and FTC designs will be our next considerations.

APPENDIX A

First, (3) implies that $p \geq q$. Since $\text{rank}(D) = q$, so

$$T_0^{-1} = \begin{bmatrix} \left(D^T D\right)^{-1} D^T \\ D^{\perp} \end{bmatrix} \in \mathbf{R}^{n\times n}$$

exists and is nonsingular, and obviously $T_0^{-1}D = \begin{bmatrix} I_q \\ 0 \end{bmatrix}$. Now denote $CT_0 = \begin{bmatrix} \hat{C}_1 & \hat{C}_2 \end{bmatrix}$, where $\hat{C}_1 \in \mathbf{R}^{p\times q}$ and $\hat{C}_2 \in \mathbf{R}^{p\times(n-q)}$, we have $CD = CT_0T_0^{-1}D = \begin{bmatrix} \hat{C}_1 & \hat{C}_2 \end{bmatrix} \begin{bmatrix} I_q \\ 0 \end{bmatrix} = \hat{C}_1$. When $p > q$, since $\text{rank}(\hat{C}_1) = \text{rank}(CD) = q$ which means that $\hat{C}_1$ has full column rank, so we know that

$$S = \begin{bmatrix} \left(\left(\hat{C}_1\right)^T \hat{C}_1\right)^{-1} \left(\hat{C}_1\right)^T \\ \left(\hat{C}_1\right)^{\perp} \end{bmatrix} \in \mathbf{R}^{p\times p}$$

exists and is nonsingular and $S\hat{C}_1 = \begin{bmatrix} I_q \\ 0 \end{bmatrix} \in \mathbf{R}^{p\times q}$. Denote $S\hat{C}_2 = \begin{bmatrix} \hat{C}_{21} \\ \hat{C}_{22} \end{bmatrix} \in \mathbf{R}^{p\times(n-q)}$, where $\hat{C}_{21} \in \mathbf{R}^{q\times(n-q)}$ and $\hat{C}_{22} \in \mathbf{R}^{(p-q)\times(n-q)}$, then

$$SCT_0 = S\begin{bmatrix} \hat{C}_1 & \hat{C}_2 \end{bmatrix} = \begin{bmatrix} S\hat{C}_1 & S\hat{C}_2 \end{bmatrix} = \begin{bmatrix} I_q & \hat{C}_{21} \\ 0 & \hat{C}_{22} \end{bmatrix}.$$

If we set $T = \begin{bmatrix} I_q & \hat{C}_{21} \\ 0 & I_{n-q} \end{bmatrix} T_0^{-1}$, we will have

$$\begin{aligned} \hat{C} &:= SCT^{-1} = SCT_0 \begin{bmatrix} I_q & -\hat{C}_{21} \\ 0 & I_{n-q} \end{bmatrix} \\ &= \begin{bmatrix} I_q & \hat{C}_{21} \\ 0 & \hat{C}_{22} \end{bmatrix} \begin{bmatrix} I_q & -\hat{C}_{21} \\ 0 & I_{n-q} \end{bmatrix} = \begin{bmatrix} I_q & 0 \\ 0 & \hat{C}_{22} \end{bmatrix}, \\ \hat{D} &:= TD = \begin{bmatrix} I_q & \hat{C}_{21} \\ 0 & I_{n-q} \end{bmatrix} T_0^{-1} D \\ &= \begin{bmatrix} I_q & \hat{C}_{21} \\ 0 & I_{n-q} \end{bmatrix} \begin{bmatrix} I_q \\ 0 \end{bmatrix} = \begin{bmatrix} I_q \\ 0 \end{bmatrix}. \end{aligned}$$

Obviously, when $p = q$, $\hat{C}$ is reduced to $\hat{C} = \begin{bmatrix} I_p & 0 \end{bmatrix}$. Besides, when $p > q$, because for any complex s, we have

$$\begin{aligned} &\text{rank} \begin{bmatrix} sI_n - A & D \\ C & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} T & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} sI_n - A & D \\ C & 0 \end{bmatrix} \begin{bmatrix} T^{-1} & 0 \\ 0 & I_q \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} sI_n - TAT^{-1} & TD \\ SCT^{-1} & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} sI_q - \hat{A}_{11} & -\hat{A}_{12} & I_q \\ -\hat{A}_{21} & sI_{n-q} - \hat{A}_{22} & 0 \\ I_q & 0 & 0 \\ 0 & \hat{C}_{22} & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} 0 & 0 & I_q \\ 0 & sI_{n-q} - \hat{A}_{22} & 0 \\ I_q & 0 & 0 \\ 0 & \hat{C}_{22} & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} 0 & I_q & 0 \\ I_q & 0 & 0 \\ 0 & 0 & sI_{n-q} - \hat{A}_{22} \\ 0 & 0 & \hat{C}_{22} \end{bmatrix} \\ &= 2q + \text{rank} \begin{bmatrix} sI_{n-q} - \hat{A}_{22} \\ \hat{C}_{22} \end{bmatrix}. \end{aligned}$$

So, (2) holds for any complex s with $|s| \geq 1$ if and only if

$$\text{rank} \begin{bmatrix} sI_{n-q} - \hat{A}_{22} \\ \hat{C}_{22} \end{bmatrix} = n - q \tag{A.1}$$

holds for any complex s with $|s| \geq 1$, that is the pair $(\hat{A}_{22}, \hat{C}_{22})$ is detectable. When $p = q$, (A.1) is reduced to $\text{rank}\begin{bmatrix} sI_{n-q} - \hat{A}_{22} \end{bmatrix} = n - q$ and it is equivalent to

$$\text{rank} \begin{bmatrix} sI_{n-p} - \hat{A}_{22} \\ I_p \end{bmatrix} = n - p,$$

which means that $(\hat{A}_{22},\ I_p)$ is detectable.

APPENDIX B

$$\begin{aligned}
&\text{rank}\begin{bmatrix} sI_{\bar{n}}-GA_e & GD \\ C_e & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} sI_{\bar{n}}-M\begin{bmatrix} I_n \\ 0_{r\times n}\end{bmatrix}A_e & M\begin{bmatrix} I_n \\ 0_{r\times n}\end{bmatrix}D \\ C_e & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} sI_{\bar{n}}-M\begin{bmatrix} A_e \\ 0_{r\times \bar{n}}\end{bmatrix} & M\begin{bmatrix} D \\ 0_{r\times q}\end{bmatrix} \\ C_e & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} M & 0_{\bar{n}\times p} \\ 0_{p\times \bar{n}} & I_p\end{bmatrix}\begin{bmatrix} sM^{-1}-\begin{bmatrix} A_e \\ 0_{r\times \bar{n}}\end{bmatrix} & \begin{bmatrix} D \\ 0_{r\times q}\end{bmatrix} \\ C_e & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} sM^{-1}-\begin{bmatrix} A_e \\ 0_{r\times \bar{n}}\end{bmatrix} & \begin{bmatrix} D \\ 0_{r\times q}\end{bmatrix} \\ C_e & 0_{p\times q}\end{bmatrix},
\end{aligned}$$

It should be noticed that

$$\begin{aligned}
M^{-1} &= R^TR = \begin{bmatrix} I_n & C^T \\ 0_{r\times n} & F^T\end{bmatrix}\begin{bmatrix} I_n & 0_{n\times r} \\ C & F\end{bmatrix}\\
&= \begin{bmatrix} I_n+C^TC & C^TF \\ F^TC & F^TF\end{bmatrix}.
\end{aligned}$$

So,

$$\begin{aligned}
&\text{rank}\begin{bmatrix} sI_{\bar{n}}-GA_e & GD \\ C_e & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} \begin{bmatrix} s(I_n+C^TC) & sC^TF \\ sF^TC & sF^TF\end{bmatrix} - \begin{bmatrix} A & 0_{n\times r} \\ 0_{r\times n} & 0_{r\times r}\end{bmatrix} & \begin{bmatrix} D \\ 0_{r\times q}\end{bmatrix} \\ \bar{C} & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} s(I_n+C^TC)-A & sC^TF & D \\ sF^TC & sF^TF & 0_{r\times q} \\ C & F & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} I_n & 0_{n\times r} & -sC^T \\ 0_{r\times n} & I_r & -sF^T \\ 0_{p\times n} & 0_{p\times r} & I_p\end{bmatrix}\\
&\quad\times\begin{bmatrix} s(I_n+C^TC)-A & sC^TF & D \\ sF^TC & sF^TF & 0_{r\times q} \\ C & F & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} sI_n-A & 0_{n\times r} & D \\ 0_{r\times n} & 0_{r\times r} & 0_{r\times q} \\ C & F & 0_{p\times q}\end{bmatrix}\\
&=\text{rank}\begin{bmatrix} sI_n-A & 0_{n\times r} & D \\ C & F & 0_{p\times q}\end{bmatrix}.
\end{aligned}$$

The above equation means that (6) in Assumption 3 holds for any complex s with $|s|\geq 1$ if and only if (11) in Lemma 5 holds for any complex s with $|s|\geq 1$.

APPENDIX C

Notice that $\begin{bmatrix} G & 0 \\ 0_{p\times n} & I_p\end{bmatrix}$ has full column rank $n+p$, so we have

$$\begin{aligned}
\bar{n}+q &= \text{rank}\begin{bmatrix} E_e & D \\ C_e & 0_{p\times q}\end{bmatrix}\\
&= \text{rank}\begin{bmatrix} G & 0 \\ 0_{p\times n} & I_p\end{bmatrix}\begin{bmatrix} E_e & D \\ C_e & 0_{p\times q}\end{bmatrix}\\
&= \text{rank}\begin{bmatrix} GE_e & GD \\ C_e & 0_{p\times q}\end{bmatrix}\\
&= \text{rank}\begin{bmatrix} I_{\bar{n}} & \Lambda \\ 0_{p\times \bar{n}} & I_p\end{bmatrix}\begin{bmatrix} GE_e & GD \\ C_e & 0_{p\times q}\end{bmatrix}\\
&= \text{rank}\begin{bmatrix} GE_e+\Lambda C_e & GD \\ C_e & 0_{p\times q}\end{bmatrix}\\
&= \text{rank}\begin{bmatrix} I_{\bar{n}} & GD \\ C_e & 0_{p\times q}\end{bmatrix}\\
&= \text{rank}\left\{\begin{bmatrix} I_{\bar{n}} & 0_{\bar{n}\times p} \\ -C_e & I_p\end{bmatrix}\begin{bmatrix} I_{\bar{n}} & GD \\ C_e & 0_{p\times q}\end{bmatrix}\right\}\\
&= \text{rank}\begin{bmatrix} I_{\bar{n}} & 0 \\ 0 & C_eGD\end{bmatrix} = \bar{n}+\text{rank}(C_eGD).
\end{aligned}$$

This gives $\text{rank}(C_eGD)=q=\text{rank}(D)=\text{rank}(GD)$, that is (12) holds.

APPENDIX D

$$\begin{aligned}
\begin{bmatrix} \tilde{\bar{y}}_1^+(k+1) \\ \tilde{\bar{y}}_1^-(k+1)\end{bmatrix} &= \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma\end{bmatrix}\begin{bmatrix} \tilde{\bar{y}}_1^+(k) \\ \tilde{\bar{y}}_1^-(k)\end{bmatrix}\\
&\quad+\begin{bmatrix} -\bar{A}_{12} \\ \bar{A}_{12}\end{bmatrix}\tilde{\bar{z}}_2(k)+\begin{bmatrix} d^+-d(k) \\ d(k)-d^-\end{bmatrix},
\end{aligned}$$

where $\tilde{\bar{y}}_1^+(k)=\hat{\bar{y}}_1^+(k)-\bar{y}_1(k)$ and $\tilde{\bar{y}}_1^-(k)=\bar{y}_1(k)-\hat{\bar{y}}_1^-(k)$. On the one hand, since

$$\begin{bmatrix} d^+-d(k) \\ d(k)-d^-\end{bmatrix}>0,\quad \begin{bmatrix} -\bar{A}_{12} \\ \bar{A}_{12}\end{bmatrix}\tilde{\bar{z}}_2(k)\to 0,$$

so there exists a constant $K_0\geq 0$ such that

$$\begin{bmatrix} -\bar{A}_{12} \\ \bar{A}_{12}\end{bmatrix}\tilde{\bar{z}}_2(k)+\begin{bmatrix} d^+-d(k) \\ d(k)-d^-\end{bmatrix}>0,$$

for all $k\geq K_0$. On the other hand, we know that

$$\begin{aligned}
\bar{y}_1(k) &= \bar{z}_1(k)=\begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)} \ |!\end{bmatrix}\bar{z}(k)\\
&= \begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)}\end{bmatrix}Tz(k)\\
&= \begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)}\end{bmatrix}T(x_e(k)-\Lambda y(k))\\
&= \begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)}\end{bmatrix}T(x_e(k)-\Lambda C_ex_e(k))\\
&= \begin{bmatrix} I_q & 0_{q\times(\bar{n}-q)}\end{bmatrix}T(I_{\bar{n}}-\Lambda C_e)x_e(k)=Q_ex_e(k),
\end{aligned}$$

which means $\bar{y}_1(0)=Q_ex_e(0)$. We have

$$Q_e^+x_e^-(0)-Q_e^-x_e^+(0)\leq \bar{y}_1(0)=Q_ex_e(0)$$

$$\leq Q_e^+ x_e^+(0) - Q_e^- x_e^-(0),$$

because of (4). So, if the initial state of system (18) is set as by (19), then $\bar{y}_1^-(0) \leq \bar{y}_1(0) \leq \bar{y}_1^+(0)$ which means that both $\tilde{\bar{y}}_1^+(0) \geq 0$ and $\tilde{\bar{y}}_1^-(0) \geq 0$. Consider the fact that Γ is a not only nonnegative but also Schur stable matrix, we conclude by Lemma 2 that both $\tilde{\bar{y}}_1^+(0) \geq 0$ and $\tilde{\bar{y}}_1^-(0) \geq 0$ holds for all $k \geq K_0$ and this equivalent to say that $\hat{\bar{y}}_1^-(k) \leq \bar{y}_1(k) \leq \hat{\bar{y}}_1^+(k)$ for all $k \geq K_0$.

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