International Journal of Control, Automation and Systems 19(4) (2021) 1633-1639
http://dx.doi.org/10.1007/s12555-019-1063-8
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

An Improved Result on Stability Analysis of Delayed Load Frequency Control Power Systems

Shiyu Jiao, Jianwei Xia, Zhen Wang*, Xiangyong Chen*, Jing Wang, and Hao Shen

Abstract: This paper investigates the stability of power systems with load frequency control considering time delays (constant and time-varying delays). A new criterion for ensuring the stability of the system is proposed on the basis of Lyapunov stability theory and a further strengthened inequality. Finally, taking a single-area load frequency control scheme with the proportional-integral controller as an example, according to the stability criterion obtained, the relationship between the maximum allowable delay and the gain of proportional-integral controller is discussed. Besides, in case studies, the effectiveness of our method is also demonstrated.

Keywords: Load frequency control, power systems, proportional-integral controller, time delays.

1. INTRODUCTION

Load frequency control (LFC), because of its excellent ability to maintain frequency and power exchange with neighboring areas at predetermined values, has been used in power systems for many years and has also laid the foundation for the development of smart grids [1]. Subsequently, with the continuous development of smart grids, a lot of research emerged [2–4]. The LFC scheme model with communication channels is usually regarded as a typical delayed system [5]. In the LFC system, due to the introduction of the communication channel, there will be a constant delay in the signal transmission process. However, in most LFC designs and operations, because the value of the delay is relatively small, this delay is often ignored. In addition, it is also worth noting that the data packed dropout and disordering may occur in communication, which may cause time-varying delays, and the unavailability of data due to communication failure can also be equivalent to time-varying delays.

At present, the time delay phenomenon has become one of the most important unreliable factors, it will weaken the dynamic performance of the system, and even cause the instability of the LFC scheme. This means that the control area does not meet the control standards, resulting in deviation, which will have a negative impact on the stable operation of power systems. Therefore, finding the maximum allowable delay is of great significance for power systems with LFC schemes to maintain the stable operation [6].

There are two common methods for determining the maximum allowable delay, each of which has advantages and limitations. First, the critical eigenvalues and eigenroots can be obtained according to the characteristic equation of the system to calculate the accurate maximum allowable delay directly [7]. However, when the scale of the system increases, this method is very time-consuming and inefficient. And because it can only deal with constant delays, an indirect method on the basis of Lyapunov stability theory gradually replaced it as the main method for calculating the maximum allowable delay [8–10]. Because of the fact that the second method can handle both constant and time-varying delays, it has become a mainstream method, although this method is somewhat conservative. In recent years, many excellent methods have been proposed for the stability analysis of systems with time delays [11–16]. However, how to apply these methods to the stability analysis of power systems with LFC scheme still

Manuscript received December 20, 2019; revised June 17, 2020; accepted August 4, 2020. Recommended by Associate Editor Le Van Hien under the direction of Editor Young IL Lee. This work was supported by the National Natural Science Foundationof China (Nos. 61573008, 61973199) and Shandong University of Science and Technology Research Fund (2018 TDJH101).

Shiyu Jiao is with AnHui Province Key Laboratory of Special Heavy Load Robot and also the School of Electrical Engineering and Information, Anhui University of Technology, Anhui, 243032, China (e-mail: syjiao430@gmail.com). Jianwei Xia is with the College of Mathematic Science, Liaocheng university, Liaocheng, 252000, China (e-mail: njustxjw@126.com). Zhen Wang is the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China (e-mail: wangzhen_sd@126.com). Jing Wang is with the School of Electrical Engineering and Information, Anhui University of Technology, Anhui, 243032, China and also with the School of Information Science and Engineering, Chengdu University, Chengdu, 610106, China (e-mail: jingwang08@126.com). Xiangyong Chen is with School of Automation and Electrical Engineering, Linyi University, Linyi 276005, China (e-mail: cxy8305@163.com). Hao Shen is with AnHui Province Key Laboratory of Special Heavy Load Robot and also the School of Electrical Engineering and Information, Anhui University of Technology, Anhui, 243032, China (e-mail: haoshen10@gmail.com).
* Corresponding authors.

arouses the attention of many scholars. In order to reduce the limitation and conservatism of the results, a further strengthened inequality technique that is less conservative than the Jensen and Wirtinger-based integral inequality is adopted in this paper [17].

Also a fascinating topic is what control methods should be adopted when studying LFC schemes, many excellent control strategies have always attracted much attention [18–21]. Among them, it is worth mentioning that proportional-integral (PI) control, because of its simplicity and high economic benefits [22–25]. In this paper, time delays are taken into account during the design phase of the PI-type controller, which is an issue that has been ignored in many previous literatures. Moreover, the relationship between the maximum allowable delay and the controller gain is analyzed.

In summary, this paper discusses the stability of the delayed LFC scheme equipped with a PI-type controller, the main contributions are divided into the following three aspects. 1) The single-area LFC scheme is more comprehensive by considering the influence of time delays when designing the PI-type controller. 2) The stability criterion is improved by constructing a suitable Lyapunov-Krasovskii functional (LKF) and an enhancement inequality technique. 3) The relation between the controller gain and the maximum allowable delay is discussed from two aspects: constant delay and time-varying delay, which provides auxiliary conditions for the design and adjustment of the LFC scheme. And by comparison, it not only proves the effectiveness of our method, but also validates that our results are less conservative than [9] and [26].

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider time delays in the LFC scheme, as shown in Fig. 1 [1]. The LFC system model of a single-area can be expressed as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}u(t) + \tilde{D}_w w(t), \\ \tilde{y}(t) = \tilde{C}\tilde{x}(t), \end{cases} \tag{1}$$

where

$$\tilde{x}^T(t) = \begin{bmatrix} \Delta f & \Delta P_m & \Delta P_v \end{bmatrix},$$

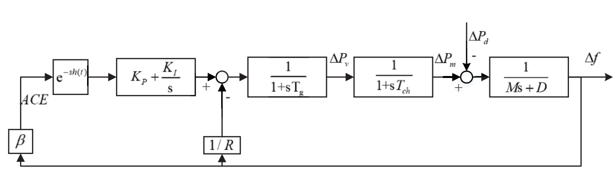


Fig. 1. The LFC scheme of the single-area.

Table 1. The explanation of terminologies.

Terminology	Meaning
ΔP_d	deviation of load
ΔP_m	deviation of generator mechanical output
ΔP_v	deviation of valve position
Δf	deviation of frequency
D	damping coefficient of generator
M	moment of inertia of generator
R	speed droop
T_g	time constant of governor
T_{ch}	time constant of turbine
β	frequency bias factor
ACE	area control error

$$\tilde{y}(t) = ACE, \quad \tilde{A} = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 \\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} \end{bmatrix},$$

$$\tilde{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix}, \quad \tilde{D}_w = \begin{bmatrix} -\frac{1}{M} \\ 0 \\ 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix}^T.$$

And explanations of some terms are shown in Table 1.

Since the power exchange phenomenon of the network tie-line does not occur in the LFC scheme of a single-area, the ACE can be expressed as

$$\tilde{y}(t) = ACE = \beta \Delta f, \tag{2}$$

in which, $\beta > 0$.

Considering ACE as the input if the controller, the form of PI-type LFC is as follows:

$$u(t) = -\mathcal{K}_P ACE - \mathcal{K}_I \int ACE, \tag{3}$$

where $\mathcal{K}_P$ and $\mathcal{K}_I$ represent the proportional gain and the integral gain of the controller respectively. Defining $y(t) \triangleq \begin{bmatrix} \tilde{y}(t) & \int \tilde{y}(t) \end{bmatrix}^T, \mathcal{K} \triangleq \begin{bmatrix} \mathcal{K}_P & \mathcal{K}_I \end{bmatrix}$, so expression (3) can be converted into

$$u(t) = -\mathcal{K}y(t).$$

However, it is not difficult to find from Fig. 1 that there are time delays in the transmission of the control signal, so time delays need to be taken into account in the controller, naturally

$$u(t) = -\mathcal{K}y(t - h(t)), \tag{4}$$

where $h(t)$ represents time delays, and $0 < h(t) < \bar{h}, \dot{h}(t) \leq \mu \leq 1$.

Define the new state variable as $x(t) \triangleq [\, \Delta f \;\; \Delta P_m \;\; \Delta P_v \;\; \int ACE \,]^T$. For ease of understanding, the PI-type control problem can be translated into output feedback control problem. By introducing the controller (4) into system (1), the closed-loop system in the following form can be obtained

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{A}_d x(t-h(t)) + \mathcal{D}_w w(t), \\ y(t) = \mathcal{C}x(t), \\ x(t) = \phi(t), \ \ t \in [-\bar{h}, 0], \end{cases} \tag{5}$$

where

$$\mathcal{A} = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} & 0 \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}, \ \mathcal{D}_w = \begin{bmatrix} -\frac{1}{M} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathcal{A}_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\mathcal{K}_P\beta}{T_g} & 0 & 0 & -\frac{\mathcal{K}_I}{T_g} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathcal{C} = \begin{bmatrix} \beta & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}^T.$$

Remark 1: When dealing with unknown external load disturbances in power systems, it can be modeled as a nonlinear perturbation in the current and delayed state vectors [26].

$$\mathcal{D}_w w(t) = g(x(t), x(t-h(t))), \tag{6}$$

meeting the following condition

$$||g(\cdot)|| \le \varepsilon ||x(t)|| + \theta ||x(t-h(t))||, \tag{7}$$

in which ε and θ are known non-negative scalars. A more generalized form of the condition is adopted, as follows:

$$\begin{aligned} g(\cdot)^T g(\cdot) \le & \varepsilon^2 x^T(t) E^T E x(t) \\ & + \theta^2 x^T(t-h(t)) F^T F x(t-h(t)), \end{aligned} \tag{8}$$

where E along with F are both known constant matrices with appropriate dimensions. The non-negative scalars ε, θ and the matrices E, F can be used to quantify the impact of load disturbances on power systems.

Lemma 1 [17]**:** Given a matrix $Z(\in \mathbb{R}^{n\times n}) > 0$, if there exists any matrix $H_i \in \mathbb{R}^{n\times m}$ $(i = 0, 1, 2, \cdots, N)$ and a vector $\delta \in \mathbb{R}^m$, for any continuous and differentiable function $x(\cdot)$ from $[a,b]$ to $\mathbb{R}^n$, and integer $N \in \mathbb{N}$, the following inequality holds

$$\begin{aligned} & -\int_a^b \dot{x}^T(s) Z \dot{x}(s) ds \\ & \le \sum_{k=0}^{N} [2\sigma_N^T \eta_N^T(k) H_k \delta + \frac{b-a}{(2k+1)} \delta^T H_k^T Z^{-1} H_k \delta], \end{aligned} \tag{9}$$

where

$$\sigma_N \triangleq \begin{cases} \left[\, x^T(b) \ \ x^T(a) \,\right]^T, & N = 0, \\ \left[\, x^T(b) \ \ x^T(a) \ \ \frac{1}{b-a}\Delta_0^T \ \cdots \ \frac{1}{b-a}\Delta_{N-1}^T \,\right]^T, & \\ & N > 0, \end{cases}$$

$$\eta_N(k) \triangleq \begin{cases} [\, I \ \ -I \,], & N = 0, \\ [\, I \ \ (-1)^{k+1} I \ \ \tau_{Nk}^0 I \ \cdots \ \tau_{Nk}^{N-1} I \,], & N > 0, \end{cases}$$

$$\tau_{Nk}^i \triangleq \begin{cases} -(2i+1)(1-(-1)^{k+i}), & i \le k, \\ 0, & i \ge k+1, \end{cases}$$

$$\Delta_k \triangleq \int_a^b Y_k(s) x(s) ds,$$

$$Y_k(s) \triangleq (-1)^k \sum_{e=0}^{k} \left[(-1)^e \binom{k}{e}\binom{k+e}{e}\right] \left(\frac{s-a}{b-a}\right)^e.$$

Remark 2: The stability of the LFC scheme is extremely important for the normal operation of power systems. When analyzing the stability of the LFC scheme with time delays, advanced inequality technology plays an important role. Compared with common inequalities, Jensen inequality and Wirtinger-based integral inequality, in order to effectively reduce the conservatism, a low-conservative inequality is applied this paper. Interestingly, as δ introduces more state variables, the less conservative it is.

3. MAIN RESULTS

To simplify the following calculation process, some relevant explanations are given here.

$$\Omega = \begin{bmatrix} I_n & -I_n & 0 & 0 \\ I_n & I_n & -2I_n & 0 \\ I_n & -I_n & 0 & -6I_n \end{bmatrix},$$

$$\varpi(t) = \left[\, \varpi_1^T(t) \ \ \varpi_2^T(t) \ \ \varpi_3^T(t) \,\right]^T,$$

$$\varpi_1(t) = \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-\bar{h}) \\ g(\cdot) \end{bmatrix}, \ P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix},$$

$$\varpi_2(t) = \begin{bmatrix} \frac{1}{h(t)} \int_{t-h(t)}^{t} x(s) ds \\ \frac{1}{h(t)} \int_{t-h(t)}^{t} (2\frac{s-t+h(t)}{h(t)} - 1) x(s) ds \end{bmatrix},$$

$$\varpi_3(t) = \begin{bmatrix} \frac{1}{\bar{h}-h(t)} \int_{t-\bar{h}}^{t-h(t)} x(s) ds \\ \frac{1}{\bar{h}-h(t)} \int_{t-\bar{h}}^{t-h(t)} (2\frac{s-t+\bar{h}}{\bar{d}-h(t)} - 1) x(s) ds \end{bmatrix},$$

$$Z_n = \text{diag}\{Z, 3Z, 5Z\},$$

$$c_i = [\, 0_{n\times(i-1)n} \ \ I_n \ \ 0_{n\times(9-i)n} \,], \ i = 1, 2, \ldots, 9,$$

$$l_i = [\, 0_{n\times(i-1)n} \ \ I_n \ \ 0_{n\times(8-i)n} \,], \ i = 1, 2, \ldots, 8.$$

Theorem 1: For given scalars $\varepsilon \ge 0$, $\theta \ge 0$, $\lambda \ge 0$, assuming $Q_2 > 0$, $Q_1 > 0$, $P > 0$, $Z > 0$, and any matrices $\tilde{M}_1^{3\times 9}$, $\tilde{M}_2^{3\times 9}$, the system (5) is asymptotically stable if the following conditions hold

$$\Theta_1 = \begin{bmatrix} \tilde{\Gamma}(0) & \tilde{M}_2^T \\ * & -Z_n \end{bmatrix}_{12\times 12} < 0, \tag{10}$$

$$\Theta_2 = \begin{bmatrix} \tilde{\Gamma}(\bar{h}) & \tilde{M}_1^T \\ * & -Z_n \end{bmatrix}_{12\times 12} < 0, \tag{11}$$

where

$$\tilde{\Gamma} = \sum_{i=1}^{8} \tilde{\Gamma}_i,$$
$$\tilde{\Gamma}_1 = sym\{c_1^T P_{11}\mathcal{A}c_1 + c_1^T P_{11}\mathcal{A}_d c_2 + c_1^T P_{11}c_4, + c_1^T P_{12}c_1 - c_1^T P_{12}c_3 + c_1^T P_{13}c_1\} - \varepsilon c_4^T I c_4,$$
$$\tilde{\Gamma}_2 = sym\{(\bar{h} - h(t))c_7^T P_{12}^T(\mathcal{A}c_1 + \mathcal{A}_d c_2 + c_4) + h(t)c_5^T P_{12}^T(\mathcal{A}c_1 + \mathcal{A}_d c_2 + c_4)\} + \varepsilon\varepsilon^2 c_1^T E^T E c_1,$$
$$\tilde{\Gamma}_3 = sym\{(\bar{h} - h(t))c_8^T P_{13}^T(\mathcal{A}c_1 + \mathcal{A}_d c_2 + c_4), + h(t)c_6^T P_{13}^T(\mathcal{A}c_1 + \mathcal{A}_d c_2 + c_4)\} + \varepsilon\theta^2 c_2^T F^T F c_2,$$
$$\tilde{\Gamma}_4 = sym\{[h(t)c_5^T + (\bar{h} - h(t))c_7^T]P_{22}(c_1 - c_3)\},$$
$$\tilde{\Gamma}_5 = sym\{[h(t)c_6^T + (\bar{h} - h(t))c_8^T]P_{23}^T(c_1 - c_3)\},$$
$$\tilde{\Gamma}_6 = sym\{(\bar{h} - h(t))c_7^T P_{23}c_1 + h(t)c_5^T P_{23}c_1 + (\bar{h} - h(t))c_8^T P_{33}c_1 + h(t)c_6^T P_{33}c_1\},$$
$$\tilde{\Gamma}_7 = c_1^T(Q_1 + Q_2)c_1 - (1 - \mu)c_2^T Q_1 c_2 - c_3^T Q_2 c_3 + sym\{\bar{h}\tilde{\Pi}_1^T \Omega^T M_1 + \bar{h}\tilde{\Pi}_2^T \Omega^T M_2\},$$
$$\tilde{\Gamma}_8 = \bar{h}c_1^T \mathcal{A}^T Z c_9 + \bar{h}c_2^T \mathcal{A}_d^T Z c_9 + \bar{h}c_4^T Z c_9 + c_9^T Z c_9.$$

Proof: Choose the appropriate LKF

$$\mathcal{V}(t) = \mathcal{V}_1(t) + \mathcal{V}_2(t) + \mathcal{V}_3(t), \tag{12}$$
$$\mathcal{V}_1(t) = \rho^T(t)P\rho(t), \tag{13}$$
$$\mathcal{V}_2(t) = \int_{t-h(t)}^{t} x^T(s)Q_1 x(s)ds + \int_{t-\bar{h}}^{t} x^T(s)Q_2 x(s)ds, \tag{14}$$
$$\mathcal{V}_3(t) = \bar{h}\int_{-\bar{h}}^{0}\int_{t+\gamma}^{t} \dot{x}^T(s)Z\dot{x}(s)dsd\gamma, \tag{15}$$

where

$$\rho(t) = \begin{bmatrix} x^T(t) & \int_{t-\bar{h}}^{t} x^T(s)ds & \int_{t-\bar{h}}^{t}(2\frac{s-t+\bar{h}}{\bar{h}} - 1)x^T(s)ds \end{bmatrix}^T.$$

Take the derivative of the above functional, and the following expressions hold

$$\dot{\mathcal{V}}_1(t) = sym\{\rho^T(t)P\dot{\rho}(t)\}, \tag{16}$$
$$\dot{\mathcal{V}}_2(t) \le x^T(t)Q_1 x(t) + x^T(t)Q_2 x(t) - x^T(t-\bar{h})Q_2 x(t-\bar{h}) - (1-\mu)x^T(t-h(t))Q_1 x(t-h(t)), \tag{17}$$
$$\dot{\mathcal{V}}_3(t) = \bar{h}^2\dot{x}^T(t)Z\dot{x}(t) - \bar{h}\int_{t-\bar{h}}^{t}\dot{x}^T(s)Z\dot{x}(s)ds. \tag{18}$$

Setting $N = 2$, and according to Lemma 1, the following expressions hold

$$-\bar{h}\int_{t-\bar{h}}^{t}\dot{x}^T(s)Z\dot{x}(s)ds = -\bar{h}\int_{t-\bar{h}}^{t-h(t)}\dot{x}^T(s)Z\dot{x}(s)ds - \bar{h}\int_{t-h(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds, \tag{19}$$

$$-\bar{h}\int_{t-h(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds \le \varpi^T(t)[\bar{h}(sym\{\Pi_1^T\Omega^T M_1\} + h(t)M_1^T Z_n^{-1}M_1)]\varpi(t), \tag{20}$$

$$-\bar{h}\int_{t-\bar{h}}^{t-h(t)}\dot{x}^T(s)Z\dot{x}(s)ds \le \varpi^T(t)[\bar{h}(sym\{\Pi_2^T\Omega^T M_2\} + (\bar{h} - h(t))M_2^T Z_n^{-1}M_2)]\varpi(t), \tag{21}$$

where M_1 and M_2 are matrices with suitable dimensions, and

$$\tilde{M}_1 = \begin{bmatrix} M_1 & 0_{3\times1} \end{bmatrix}, \ \tilde{M}_2 = \begin{bmatrix} M_2 & 0_{3\times1} \end{bmatrix},$$
$$\Pi_1 = \begin{bmatrix} l_1^T & l_2^T & l_5^T & l_6^T \end{bmatrix}^T, \ \Pi_2 = \begin{bmatrix} l_2^T & l_3^T & l_7^T & l_8^T \end{bmatrix}^T,$$
$$\tilde{\Pi}_1 = \begin{bmatrix} c_1^T & c_2^T & c_5^T & c_6^T \end{bmatrix}^T, \ \tilde{\Pi}_2 = \begin{bmatrix} c_2^T & c_3^T & c_7^T & c_8^T \end{bmatrix}^T.$$

For any $\varepsilon > 0$, the inequality hold

$$-\varepsilon g^T(\cdot)g(\cdot) + \varepsilon\varepsilon^2 x^T(t)E^T E x(t) + \varepsilon\theta^2 x^T(t-h(t))F^T F x(t-h(t)) \ge 0. \tag{22}$$

By summarizing the above calculation, it is not difficult to get the following inequality

$$\dot{\mathcal{V}}(t) \le \varpi^T(t)\{\sum_{i=1}^{7}\Gamma_i(h(t)) + \bar{h}[h(t)M_1^T Z_n^{-1}M_1 + (\bar{h} - h(t))M_2^T Z_n^{-1}M_2]\}\varpi(t) + \bar{h}^2\dot{x}^T(t)Z\dot{x}(t), \tag{23}$$

where

$$\Gamma_1 = sym\{l_1^T P_{11}\mathcal{A}l_1 + l_1^T P_{11}\mathcal{A}_d l_2 + l_1^T P_{11}l_4 + l_1^T P_{12}l_1 - l_1^T P_{12}l_3 + l_1^T P_{13}l_1\} - \varepsilon l_4^T I l_4,$$
$$\Gamma_2 = sym\{(\bar{h} - h(t))l_7^T P_{12}^T(\mathcal{A}l_1 + \mathcal{A}_d l_2 + l_4) + h(t)l_5^T P_{12}^T(\mathcal{A}l_1 + \mathcal{A}_d l_2 + l_4)\} + \varepsilon\varepsilon^2 l_1^T E^T E l_1,$$
$$\Gamma_3 = sym\{(\bar{h} - h(t))l_8^T P_{13}^T(\mathcal{A}l_1 + \mathcal{A}_d l_2 + l_4) + h(t)l_6^T P_{13}^T(\mathcal{A}l_1 + \mathcal{A}_d l_2 + l_4)\} + \varepsilon\theta^2 l_2^T F^T F l_2,$$
$$\Gamma_4 = sym\{[h(t)l_5^T + (\bar{h} - h(t))l_7^T]P_{22}(l_1 - l_3)\},$$
$$\Gamma_5 = sym\{[h(t)l_6^T + (\bar{h} - h(t))l_8^T]P_{23}^T(l_1 - l_3)\},$$
$$\Gamma_6 = sym\{(\bar{h} - h(t))l_7^T P_{23}l_1 + h(t)l_5^T P_{23}l_1 + (\bar{h} - h(t))l_8^T P_{33}l_1 + h(t)l_6^T P_{33}l_1\},$$
$$\Gamma_7 = l_1^T(Q_1 + Q_2)l_1 - (1-\mu)l_2^T Q_1 l_2 - l_3^T Q_2 l_3 + sym\{\bar{h}\tilde{\Pi}_1^T\Omega^T M_1 + \bar{h}\tilde{\Pi}_2^T\Omega^T M_2\}.$$

According to Schur complement, the expressions Θ_1 and Θ_2 hold, since the condition $\tilde{\Gamma}(0) + \bar{h}^2 M_2^T Z_n^{-1} M_2 < 0$ and the condition $\tilde{\Gamma}(\bar{h}) + \bar{h}^2 M_1^T Z_n^{-1} M_1 < 0$ can guarantee the establishment of the condition $\tilde{\Gamma}(h(t)) +$

Table 2. Parameters of single-area LFC system.

Parameter	β	R	D	$M(s)$	$T_{ch}(s)$	$T_g(s)$
*Area*1	21	0.05	1.0	1.0	0.3	0.1

Table 3. The maximum allowable delay $\bar{h}$ for LFC system when $\mu = 0$.

Methods		$\mu = 0$		
$\mathcal{K}_P$	$\mathcal{K}_I$	Theorem1	[9]	[26]
0	0.2	11.70	7.33	6.69
0	0.4	6.15	3.38	3.12
0	0.6	4.17	2.04	1.91
0.1	0.2	10.96	7.79	6.94
0.1	0.4	5.83	3.61	3.29
0.1	0.6	4.05	2.19	2.02

Table 4. The maximum allowable delay $\bar{h}$ for LFC system when $\mu = 0.9$.

Methods		$\mu = 0.9$		
$\mathcal{K}_P$	$\mathcal{K}_I$	Theorem1	[9]	[26]
0	0.2	9.98	6.43	6.25
0	0.4	4.44	2.91	2.85
0	0.6	2.80	1.71	1.68
0.1	0.2	9.17	6.59	5.93
0.1	0.4	4.31	3.11	2.87
0.1	0.6	2.83	1.84	1.75

$\bar{h}[h(t)M_1^T Z_n^{-1} M_1 + (\bar{h} - h(t))M_2^T Z_n^{-1} M_2] < 0$ for $\forall\ h(t) \in [0, \bar{h}]$. Based on Schur complement, inequalities (10) and (11) can be derived. This completes the proof of Theorem 1. □

4. CASE STUDIES

In this section, taking a single-area LFC system as an example, the superiority of our results is proved by comparison with the previous literature [9] and [26]. The parameters of the system are shown in Table 2.

Suppose time delays $h(t)$ as two cases : constant or time-varying delays ($\mu = 0/0.9$). In these two cases, analyze the maximum allowable delay for the normal operation of the system. Consider the load disturbance $\mathcal{D}_w w(t)$ as formula (8) and set the parameters $\varepsilon = 0$, $\theta = 0$, $E = F = 0.1I_4$. With the help of Matlab, the maximum allowable delay under diverse values of controller gains ($\mathcal{K}_P$, $\mathcal{K}_I$) can be solved. The specific data are shown in Table 3 and Table 4.

It can be found from Table 3 that when considering $h(t)$ as constant delay ($\mu = 0$), under the same controller gain, the maximum allowable delay calculated by our method is greater than than the values in [9] and [26]. And Fig. 2 shows the state response of the single-area LFC system at $\mathcal{K}_P = 0$, $\mathcal{K}_I = 0.2$ when $\mu = 0$. It is proved that the system finally achieves asymptotic stability. Table 4 shows the maximum allowable delay obtained under different controller gains when $h(t)$ is time-varying delay ($\mu = 0.9$). By comparing Table 3 and Table 4 longitudinally, it is not difficult to see that under the premise of ensuring the same controller gain, the maximum allowable upper bound of constant delay is generally higher than that of time-varying delay.

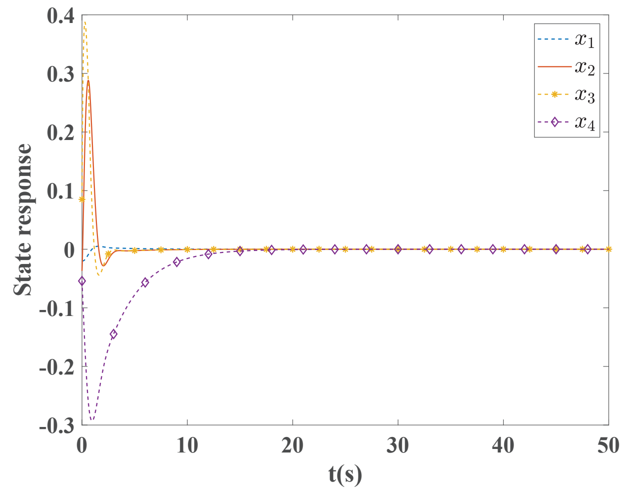


Fig. 2. The state response trajectory of LFC system.

In addition, according to Table 3 and Table 4, it is obvious that compared with literature [9] and [26], whether $h(t)$ is constant delay ($\mu = 0$) or time-varying delay ($\mu = 0.9$), the maximum allowable delay obtained by our method is larger. This proves that our results are less conservative than those in [9] and [26], and the effectiveness and superiority of our method are also confirmed, directly.

5. CONCLUSIONS

In this paper, the stability of frequency load control scheme with time delays has been studied. By Lyapunov stability theory and an improved inequality technology, the stability criterion has been further optimized. In addition, the maximum allowable delay under different controller gain values has been calculated, and the relationship between the two has been discussed. By comparing the examples, the validity and excellence of our method have been fully confirmed whether under the constant delay or time-varying delay.

REFERENCES

[1] H. Bevrani, *Robust Power System Frequency Control*, Springer, 2014.

[2] T. N. Pham, H. Trinh, and L. V. Hien, "Load frequency control of power systems with electric vehicles and diverse

transmission links using distributed functional observers," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 238-252, Jan. 2015.

[3] T. N. Pham, L. V. Hien, H. Trinh, and K. Wong, "An improved stability criterion for time-delay power systems with electric vehicles and high voltage direct current power links," *Proc. of 10th International Conference on Advances in Power System Control, Operation & Management (APSCOM)*, 2015.

[4] T. N. Pham, S. Nahavandi, L. V. Hien, H. Trinh, and K. P. Wong, "Static output feedback frequency stabilization of time-delay power systems with coordinated electric vehicles state of charge control," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3862-3874, Sep. 2017.

[5] S. Wang, X. Meng, and T. Chen, "Wide-area control of power systems through delayed network communication," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 2, pp. 495-503, Mar. 2011.

[6] L. Jiang, W. Yao, Q. Wu, J. Wen, and S. Cheng, "Delay-dependent stability for load frequency control with constant and time-varying delays," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 932-941, May 2011.

[7] Z. Liu, C. Zhu, and Q. Jiang, "Stability analysis of time delayed power system based on cluster treatment of characteristic roots method," *Proc. of IEEE Power and Energy Soc. General Meeting*, pp. 1-6, Jul. 2008.

[8] L. Xiong, H. Li, and J. Wang, "LMI based robust load frequency control for time delayed power system via delay margin estimation," *Int. J. Electr. Power Energy Syst.*, vol. 100, pp. 91-103, 2018.

[9] F. Yang, J. He, and J. Wang, "Novel stability analysis of delayed LFC power systems by infinite-series-based integral inequality," *Proc. of IEEE Conference on Control Technology and Applications (CCTA)*, pp. 1384-1389, Aug. 2017.

[10] R. Saravanakumar, M. S. Ali, H. Huang, J. Cao, and Y. H. Joo, "Robust H_∞ state-feedback control for nonlinear uncertain systems with mixed time-varying delays," *Int. J. Control, Automation and Systems*, vol. 16, no. 1, pp. 225-233, 2018.

[11] L. V. Hien and H. Trinh, "An enhanced stability criterion for time-delay systems via a new bounding technique," *J. Franklin Inst.*, vol. 352, no. 10, pp. 4407-4422, 2015.

[12] L. V. Hien and H. Trinh, "Refined Jensen-based inequality approach to stability analysis of time-delay systems," *IET Control Theory & Appl.*, vol. 9, no. 14, pp. 2188-2194, 2015.

[13] L. V. Hien and H. Trinh, "Exponential stability of time-delay systems via new weighted integral inequalities," *Appl. Math. Comput.*, vol. 275, pp. 335-344, 2016.

[14] H. Shen, F. Li, H. C. Yan, H. R. Karimi, and H. K. Lam, "Finite-time event-triggered $\mathcal{H}_\infty$ control for T-S fuzzy Markov jump systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 3122-3135, Dec. 2018.

[15] H. Shen, M. Chen, Z.-G. Wu, J. Cao, and J. H. Park, "Reliable event-triggered asynchronous passive control for semi-Markov jump fuzzy systems and its application," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1708-1722, Aug. 2020.

[16] H. Shen, S. Jiao, T. Huang, and J. Cao, "An improved result on sampled-data synchronization of Markov jump delayed neural networks," *IEEE Trans. Syst., Man, Cybern., Syst.*, 2019. DOI:10.1109/TSMC.2019.2931533

[17] H.-B. Zeng, X.-G. Liu, W. Wang, and S.-P. Xiao, "New results on stability analysis of systems with time-varying delays using a generalized free-matrix-based inequality," *J. Franklin Inst.*, vol. 356, no. 13, pp. 7312-7321, 2019.

[18] K. Vrdoljak, N. Perić, and I. Petrović, "Sliding mode based load-frequency control in power systems," *Elect. Power Syst. Res.*, vol. 80, no. 5, pp. 514-527, 2010.

[19] K. Lu, W. Zhou, G. Zeng, and Y. Zheng, "Constrained population extremal optimization-based robust load frequency control of multi-area interconnected power system," *Int. J. Electr. Power Energy Syst.*, vol. 105, pp. 249-271, 2019.

[20] K. Eltag, M. S. Aslamx, and R. Ullah, "Dynamic stability enhancement using fuzzy PID control technology for power system," *Int. J. Control, Automation and Systems*, vol. 17, no. 1, pp. 234-242, 2019.

[21] I. Kasireddy, A. W. Nasir, and A. K. Singh, "IMC based controller design for automatic generation control of multi area power system via simplified decoupling," *Int. J. Control, Automation and Systems*, vol. 16, no. 3, pp. 994-1010, 2018.

[22] V. Celik, M. T. Özdemir, and K. Y. Lee, "Effects of fractional-order PI controller on delay margin in single-area delayed load frequency control systems," *J. Modern Power Syst. Clean Energy*, vol. 7, no. 2, pp. 380-389, 2019.

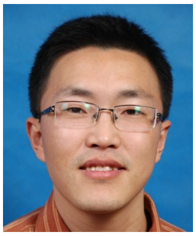
[23] S. Saxena and Y. V. Hote, "PI controller based load frequency control approach for single-area power system having communication delay," *IFAC-PapersOnLine*, vol. 51, no. 4, pp. 622-626, 2018.

[24] H. Shen, F. Li, S. Xu, and V. Sreeram, " Slow state variables feedback stabilization for semi-Markov jump systems with singular perturbations," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2709-2714, Aug. 2018.

[25] H. Shen, T. Wang, J. Cao, G. Lu, Y. Song, and T. Huang, "Non-fragile dissipative synchronization for Markovian memristive neural networks: A Gain-scheduled control scheme," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 6, pp. 1841-1853, Jun. 2019.

[26] K. Ramakrishnan and G. Ray, "Stability criteria for nonlinearly perturbed load frequency systems with time-delay," *IEEE J. Emerg. Sel. Topic Circuits Syst.*, vol. 5, no. 3, pp. 383-392, Sep. 2015.

Shiyu Jiao received her M.S. degree in control theory and control engineering from Anhui University of Technology, Ma'anshan, China, in 2020. She is currently pursuing a Ph.D. degree in control science and engineering with the School of Automation, Nanjing University of Science and Technology, Nanjing, China. Her current research interests include Markov jump systems, power systems, and sampled-data control.



Jianwei Xia received his Ph.D. degree in automatic control from Nanjing University of Science and Technology in 2007. From 2010 to 2012, he worked as a Postdoctoral Research Associate in the School of Automation, Southeast University, Nanjing, China. From 2013 to 2014, he worked as a Postdoctoral Research Associate in the Department of Electrical Engineering, Yeungnam University, Kyongsan, Korea. His research topics are robust control,stochastic systems and neural networks, etc.

Zhen Wang received his Ph.D. degree in the School of Automation, Nanjing University of Science and Technology, Nanjing, China in 2014. Since 2004, he has been with Shandong University of Science and Technology, Qingdao 266590, China, where he is currently a Professor and a Doctoral Supervisor. His current research interests include nonlinear control, neural networks, fractional order systems and multi-agent systems.

Xiangyong Chen received his Ph.D. degree in control theory and control engineering from the Northeastern University, China, in 2008 and 2012, respectively. He is an Associate Professor of College of Automation and Electrical Engineering at the Linyi University. From July 2014 to April, 2019, he was a Post-doctoral student in the Department of Mathematics, Southeast University, China. From April 1, 2016 to December 31, 2017, he was a Visiting Scholar in the Department of Electrical Engineering at the Yeungnam University, Korea. His current research interests include complex dynamic system and complex networks, synchronization control of chaotic systems.

Jing Wang received her Ph.D. degree in electric power system and automation from Hohai University in 2019. Since 2011, she has been with Anhui University of Technology, China, where she is currently an Associate Professor. Her current research interests include nonlinear control, complex networks, power systems.

Hao Shen received his Ph.D. degree in control theory and control engineering from Nanjing University of Science and Technology, Nanjing, China, in 2011. From February 2013 to March 2014, he was a Post-Doctoral Fellow with the Department of Electrical Engineering, Yeungnam University, Korea. Since 2011, he has been with Anhui University of Technology, China, where he is currently a Professor and a Doctoral Supervisor. His current research interests include stochastic hybrid systems, complex networks, fuzzy systems and control, nonlinear control. Prof. Shen was a recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) in 2019.