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Adaptive Interconnection and Damping Assignment Passivity Based Control for Underactuated Mechanical Systems

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Abstract: In this paper, we present two adaptive control approaches to handle uncertainties caused by parametric and modeling errors in a class of nonlinear systems with uncertainties. The methods use the Port-controlled Hamiltonian (PCH) modelling framework and the interconnection and damping assignment passivity-based control (IDA-PBC) control design methodology being the most effectively applicable method to such models. The methods explore an extension on the classical IDA-PBC by adopting the state-transformation, yielding a dynamic state-feedback controller that asymptotically stabilizes a class of underactuated mechanical systems and preserves the PCH structure of the augmented closed-loop system. The results are applied to the underactuated mechanical systems that are a class of mechanical systems with broad applications and are more interesting as well as challenging control problems within this context. The results are illustrated with numerical simulations applied to two underactuated robotic systems; the Acrobot and non-prehensile planar rolling robotic (disk-on-disk) systems.

Keywords: Adaptive control, Hamiltonian systems, passivity-based control, underactuated mechanical systems.

1. INTRODUCTION

An important issue in the field of nonlinear control is the control of systems with uncertainties. The behavior of the control systems can be influenced by some externally acting signals (disturbances, noises, etc.) and model uncertainties which can be clearly noticed in the implementation phase. The control of nonlinear systems with uncertainties is traditionally approached as a robust or an adaptive control problem. Adaptive control has been proved to be a very useful method for controlling uncertain nonlinear systems. Most adaptive methods proposed in literature have adopted Lyapunov functions for the design and analysis of the control systems [1–4]. Recently, new results that adopted the two classical tools of nonlinear regulator theory and geometric nonlinear control (system) *immersion* and (manifold) *invariance* (I&I) have been developed in [5, 6]. In [7] passivity-based control (PBC) approaches have been proposed for systems with Lagrangian and Hamiltonian structures.

Adaptive control has also attracted the attention of the robotics research communities. Composite learning robot control with guaranteed parameter convergence has been proposed in [8] for serial-link robots. Without considering a stringent condition called persistent excitation, the method achieves fast parameter convergence using a composite adaptation law. As for mobile robots, the work in [9] has proposed an adaptive controller for the stabilization and tracking problem of a nonholonomic mobile robot with input saturation and disturbance.

Port-controlled Hamiltonian (PCH) model, together with Euler-Lagrange model, has been used widely to represent the dynamics of a large class of nonlinear systems, particularly those of mechanical and electro-mechanical systems. Among the advantages of the PCH model is that it is derived directly from the energy function of the system, thus provides direct information about the relation between the energy, particularly the kinetic and potential energy of the system, with the dynamic behavior. Knowing this relation is very useful from a control point of view, as this also provides information about the stability property of the system which, in most cases, is of the interest in control design. The fact that Hamiltonian system is energy conservative means the model is marginally stable, which is also quite a desirable property as a starting point for a controller design [10].

One of the popular passivity-based control methods that adopts the PCH formalism and has been successfully used to control a wide variety of physical systems/processes and practical applications is the interconnection and damping assignment passivity-based control (IDA-PBC). The main idea of the IDA-PBC method,

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which was first introduced in [11], is to obtain a state feedback controller via shaping the total energy of the system (the Hamiltonian) and modifying the interconnection structure [12]. This control law guarantees stability of the PCH system with the energy function qualifies as a Lyapunov function. Additionally, asymptotic stability of the closed-loop at the desired equilibrium is achieved by adding damping into the system via the damping-injection feedback controller [10].

While the PCH modeling framework and the IDA-PBC design method together create a plausible theoretical contribution of the field of control design, like many other design approaches, they are not readily useful in applications. The *classical* models of the PCH systems do not include the friction term which is an important component in the dynamical models. Hence, adopting the IDA-PBC design to control *classical* PCH models often results in a good performance observed in simulations while poor performance or even instability of the dynamical system observed when the controller is applied on the real plant [13, 14].

In PCH representation with physical damping, friction components are included in the open-loop dissipation matrix which is generally unknown/uncertain [10]. Moreover, uncertainty may appear in the energy function i.e the Hamiltonian H of the PCH structure. In particular, in the mass matrix $M(q)$ of the kinetic energy (KE) function $K(q,p)$ or in the potential energy (PE) function $V(q)$. The presence of such uncertainties in the model may cause inaccuracy in the controller designed based on the model, which may degrade the performance of the control system or even result in instability in the implementation.

Various solutions have been proposed to deal with the robustness issue of PCH systems. Some of the solutions, with focus on external disturbances, use integral control approach [15, 16], while the problem of robustification of IDA-PBC for *fully-actuated* mechanical systems has been recently addressed in [17]. As for adaptive control, which is the most useful method to deal with unmodeled dynamics and parameter uncertainty, [18] proposed a method to compensate for the input signal errors and the tracking of trajectories for fully actuated systems. A robust model reference adaptive control combined with the IDA-PBC method has been proposed in [19] for matched input disturbances.

It is the aim of this paper to provide some novel adaptive control design approaches utilizing the IDA-PBC method to handle modeling uncertainties for a class of underactuated PCH systems utilizing a state transformation approach. The state transformation (also known as the change of coordinates) approach transforms the PCH model into another one while preserving its PCH structure. This adds flexibility and enables stabilization of systems that cannot be stabilized using the classical methods [20], hence enlarging the class of systems that can be controlled/stabilized. The results in [21] adopted the state transformation to solve the stabilization and tracking problems within the PCH framework.

By combining the adaptive control and the classical IDA-PBC control, a dynamic control law that includes integral action and additional damping on some coordinates is produced to estimate and compensate for such uncertainties, i.e. friction and uncertainties in the energy function, hence ensuring a precise control of systems. The results are illustrated via two robotics benchmark examples: the Acrobot and nonprehensile planar rolling robotic (disk-on-disk) systems. The main contributions of this paper can be listed as:

- A characterization and detailed definition of uncertainties within the PCH framework.
- The adaptive control design uses a systematic approach based on IDA-PBC method with dynamic extension using the state-transformation to improve the stability and performance of systems subject to uncertainties.
- This approach adopts the state-transformation which add flexibility to the IDA-PBC design, while preserving the PCH structure as well as the IDA-PBC design effectiveness.
- The method achieve asymptotic stability of the states as well as the convergence of the estimated parameters.
- This novel design of the dynamic IDA-PBC controller can be applied to underactuated (robotic) mechanical systems with constant mass matrix.

This paper is organized as follows: Section 2 presents some reviews on the PCH framework and IDA-PBC design method, and Section 3 formulates the problem addressed in this paper. Section 4 illustrates the design of the dynamic adaptive IDA-PBC control law for friction estimation while Section 5 discusses the design of adaptive scheme to deal with uncertainties in the potential energy function. The control scheme for friction estimation is applied to Acrobot system in Section 6 with numerical simulations and for uncertainties in the energy function the adaptive controller is applied to the disk-on-disk systems as in Section 7. Section 8 provides the concluding remarks and directions for future work.

2. PRELIMINARIES

The set of real and natural numbers (including 0) are denoted respectively by $\mathbb{R}$ and $\mathbb{N}$. Given an arbitrary matrix G, we denote the transpose of G by $G^\top$. $G^\perp$ denotes the full rank left annihilator of G, i.e. $G^\perp G = 0$. We denote an $n \times n$ identity matrix with I_n. For a vector $x \in \mathbb{R}^n$ and a matrix $A \in \mathbb{R}^{n\times n}$, we denote the Euclidean norm as $|x|$ and $|A|$, respectively, where $|x|^2 = x^\top x$. Furthermore, the weighted norm is denoted as $\|x\|_A := x^\top Ax$. For any continuous function $H(i,j)$, the gradient is $\nabla_i H(i,j) := \partial H(i,j)/\partial i$. We also adopt the standard stability and passivity definitions for nonlinear systems from [1]. Due to space limit, the arguments of functions are often dropped whenever they are clear from the context.

2.1. Port-controlled Hamiltonian systems

The equations of motion for the nonlinear PCH systems are described by the form [10]:

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}0 & I_n\\ -I_n & 0\end{bmatrix}\begin{bmatrix}\nabla_q H\\ \nabla_p H\end{bmatrix} + \begin{bmatrix}0\\ G(q)\end{bmatrix}u,$$
$$y = G^\top \nabla_p H, \tag{1}$$

where $q \in \mathbb{R}^n,\ p \in \mathbb{R}^n$ are the configuration and momenta states, respectively. u and $y \in \mathbb{R}^m$, $m \le n$, are the control input and output variables, respectively. If $m = n$ the system is called *fully-actuated*, whereas if $m < n$ it is called *underactuated*. The Hamiltonian function is the total energy stored in the system, being the sum of the kinetic energy and the potential energy

$$H(q,p) = K(q,p) + V(q) = \frac{1}{2}p^\top M^{-1}(q)p + V(q), \tag{2}$$

where $V(q)$ is the potential energy function and $M(q) = M(q)^\top > 0$ is the inertia matrix.

2.2. Background on IDA-PBC design

The main idea of the IDA-PBC design [11], is to transform via a state-feedback controller the open-loop PCH model (1) into a closed-loop preserved PCH model in the form

$$\begin{bmatrix}\dot q\\ \dot p\end{bmatrix} = \begin{bmatrix}0 & M^{-1}M_d\\ -M_dM^{-1} & J_2 - R_d\end{bmatrix}\begin{bmatrix}\nabla_q H_d\\ \nabla_p H_d\end{bmatrix},$$
$$y_d = G^\top(q)\nabla_p H_d, \tag{3}$$

where

$$H_d = \frac{1}{2}p^\top M_d^{-1}(q)p + V_d(q) \tag{4}$$

is the desired energy function stratifying

$$q_e = \arg\min H_d(q) = \arg\min V_d(q), \tag{5}$$

at the desired equilibrium point q_e. $M_d = M_d^\top > 0$ is the desired inertia matrix, $J_2 = -J_2^\top$ is a free parameter, and

$$R_d = GK_vG^\top \ge 0, \tag{6}$$

with $K_v = K_v^\top > 0$ is the desired dissipation (damping) matrix. The equivalency of the closed-loop (3) and open-loop (1) systems are satisfied with the IDA-PBC controller

$$u_{ida} = u_{es} + u_{di}, \tag{7}$$

where u_{es} is the energy shaping controller

$$u_{es} = G^\ddagger\left(\nabla_q H - M_dM^{-1}\nabla_q H_d + J_2M_d^{-1}p\right), \tag{8}$$

with

$$G^\ddagger = (G^\top G)^{-1}G^\top, \tag{9}$$

and u_{di} is the damping injection controller

$$u_{di} = -K_vG^\top\nabla_p H_d, \quad K_v > 0. \tag{10}$$

3. PROBLEM FORMULATION

The application of IDA-PBC method in general assumes that all the system parameters are known. However in reality, the presence of modeling uncertainties is inevitable. Ignoring it in model based controller design might cause poor performance or even instability when applying the controller to real systems. In PCH model (1), uncertainty may appear in the energy function or the Hamiltonian function H, in particular in the mass matrix $M(q)$ of the kinetic energy (KE) function $K(q,p)$ or in the potential energy (PE) function $V(q)$.

As we will show in Subsection 5.1, the classical IDA-PBC may guarantee only the stability of PCH system in the presence of small uncertainty, but it does not guarantee asymptotic stability which is usually required in applications. In this paper, we deploy the change of coordinates (a.k.a state transformation) method that gives flexibility in controlling the system while preserving its PCH structure and stability. This approach has been successfully applied in [16, 17, 22], to improve robustness of some mechanical systems against external disturbances.

Furthermore, to conform with the energy conservation/balancing property of Hamiltonian dynamics, the natural (physical) damping is excluded from a PCH model structure [14]. As a consequence, for mechanical systems, the PCH model neglects non-conservative forces (e.g. friction). Thus, applying the IDA-PBC controller alone is not enough to stabilize such systems in the hardware implementation, as the effects of friction are not taken into account in the controller design. The friction is considered as a physical damping, while in the IDA-PBC methodology the damping is injected by applying the controller u_{di}. To precisely tune the damping, i.e choosing the proper gain K_v, we must know the physical damping i.e. friction as to avoid the accumulation of excess damping. In addition, if the cancellation of friction effect is considered, the friction terms must be known or estimated otherwise [23]. We discuss in Section 4 a method to estimate the viscous friction.

Assumption 1: The input matrix G, and the inertia matrix M and desired inertia matrix M_d are constant.

This assumption basically emphasizes the class of systems we consider in this paper which was the same one adopted in [22]. While disturbance rejection has been proposed in [22], we formulate our results for the adaptive control approach, which extends the result.

Assumption 2: We assume a stabilizing IDA-PBC controller (7) has been obtained for the underactuated PCH system (1), i.e. the system (1) is (*asymptotically*) stable with the state feedback controller (7), and the desired (closed-loop) energy function is given by (4). The asymptotic stability proof is established by calculating the time derivative of (4) along the trajectories of (3), which satisfies

$$\dot{H}_d \leq -\lambda_{min}\{K_v\}|G^\top M_d^{-1} p|^2 \leq 0. \tag{11}$$

Thus, asymptotic stability is concluded using the arguments used in the proof of [24, Proposition 1] and [25, Proposition 1] by applying the detectability condition and invoking Barbashin-Krasovskii's theorem [10].

Assumption 3: Assumption 2 implies that the partial differential equations (PDEs), also called the matching equations, have already been solved. Thus, the matching conditions have been satisfied when solving the standard IDA-PBC problem, i.e the desired potential energy function V_d and desired inertia matrix M_d were obtained and used on the Hamiltonian function. Therefore, we emphasize that *no* solution to the PDEs is required throughout this paper.

It is well-known (see for instance [24]) that the main result of the solution of a partial differential equation (PDE) — as a result of the choice of the desired subsystems interconnections and damping — is used to determine the closed-loop energy function (potential and kinetic energies).

Remark 1: The gradient of the energy function H and the desired energy function H_d w.r.t. the position q are

$$\nabla_q H = \nabla_q V + \frac{1}{2}\sum_{i=1}^{n} e_i p^\top \nabla_{q_i} M^{-1} p,$$
$$\nabla_q H_d = \nabla_q V_d + \frac{1}{2}\sum_{i=1}^{n} e_i p^\top \nabla_{q_i} M_d^{-1} p,$$

respectively. Following Assumption 1 where the inertia matrix is constant, then $\nabla_q H = \nabla_q V$ and $\nabla_q H_d = \nabla_q V_d$.

Problem 1: The open-loop PCH system for the mechanical system (1) with friction is formulated as

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}0 & I_n\\ -I_n & -R_f\end{bmatrix}\begin{bmatrix}\nabla_q H\\ \nabla_p H\end{bmatrix} + \begin{bmatrix}0\\ G\end{bmatrix} u, \tag{12}$$

with the constant viscous friction matrix

$$R_f = G K_f G^\top, \quad K_f = \mathrm{diag}\{r\} > 0. \tag{13}$$

The control design objective is to find u, the dynamic adaptive IDA-PBC, to compensate for the unknown R_f.

Remark 2: To preserve the PCH structure when the open-loop systems is matched with the closed-loop one, the structure of real viscous damping R_f must coincide with that of the desired one R_d, given in (6), which is obtained in the procedure of adding damping to the system via the controller (10).

Problem 2: Given the PCH system (1) with uncertain energy function H, find a dynamic adaptive IDA-PBC controller u, to estimate this energy function.

4. ADAPTIVE IDA-PBC FOR FRICTION ESTIMATION

The PCH systems are well-identified by their first basic "energy-conserving/ energy-balancing" physical property [10]. To preserve this property, the PCH formulation does not take into account the effects of the natural dissipative forces (natural damping such as friction). While the effect of friction does not appear in the simulation stage, it will be obvious in the real implementation stage; compromising the stability and/or performance of the controlled system [26]. To encounter such effects, aiming at improving the performance of the system, friction compensations are the most widely used methods. In recent works of [23, 27, 28], friction compensation within the PCH framework has been considered. Here, we utilize the change of coordinates method as follows.

The open-loop PCH system for the mechanical system (1) with friction is formulated as (12).

Therefore, in the presence of the friction, the term

$$-R_f \nabla_p H = -R_f M^{-1} p, \tag{14}$$

must be added to the controller (7). Let us define the inverse of the mass (inertia) matrix and the unknown viscous friction as

$$M^{-1} = \begin{bmatrix}m_1 & m_2\\ m_2 & m_3\end{bmatrix} \text{ and } \begin{bmatrix}r_1\\ r_2\end{bmatrix} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \theta,$$

respectively, with θ the new state to be estimated. Then (14) can be formulated as

$$\begin{aligned} R_f \nabla_p H = R_f M^{-1} p &= \begin{bmatrix}\theta_1 & 0\\ 0 & \theta_2\end{bmatrix}\begin{bmatrix}m_1 & m_2\\ m_2 & m_3\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix}\\ &= \begin{bmatrix}m_1 p_1 \theta_1 + m_2 p_2 \theta_1\\ m_2 p_1 \theta_2 + m_3 p_2 \theta_2\end{bmatrix} = \begin{bmatrix}(m_1 p_1 + m_2 p_2)\theta_1\\ (m_2 p_1 + m_3 p_2)\theta_2\end{bmatrix}. \end{aligned} \tag{15}$$

In the closed-loop system, as the friction term to be estimated represents the new state(s) which is a vector in PCH formulation, the term $\nabla_p H = M^{-1} p$ must be represented in a matrix form as

$$\mathrm{diag}\{M^{-1} p\} = \begin{bmatrix}(m_1 p_1 + m_2 p_2) & 0\\ 0 & (m_2 p_1 + m_3 p_2)\end{bmatrix}. \tag{16}$$

Thus, (15) can be rewritten as

$$R_f \nabla_p H = \mathrm{diag}\{\nabla_p H\}\theta = \mathrm{diag}\{M^{-1} p\}\theta. \tag{17}$$

By multiplying (16) by the estimation vector $\hat{\theta} = \left[\hat{\theta}_1 \ \hat{\theta}_2\right]^\top$, we obtain the expression

$$\mathrm{diag}\{M^{-1}p\}\hat{\theta} = \begin{bmatrix}(m_1p_1+m_2p_2)\hat{\theta}_1\\(m_2p_1+m_3p_2)\hat{\theta}_2\end{bmatrix}, \quad (18)$$

which is similar to (15). Now we can state the proposition:

Proposition 1: Consider the PCH system (12) with unknown constant friction R_f, in closed-loop with the adaptive controller

$$u = G^{\ddagger}\Big(\nabla_q V - M_d M^{-1}\nabla_q V_d - R_d M_d^{-1}\Psi_1\nabla_q V_d - R_d M_d^{-1}p - \Psi_1 \mathrm{diag}\{\nabla_q^2 V_d\}M^{-1}p + \Psi_2\hat{\theta}\Big) \quad (19)$$

with the update law

$$\dot{\hat{\theta}} = -\Psi_2 M_d^{-1}p - \Psi_2 M_d^{-1}\Psi_1\nabla_q V_d, \quad (20)$$

with

$$\Psi_1 > 0, \ \ \Psi_2 = G\mathrm{diag}\{\nabla_p H\}G^\top, \quad (21)$$

and the desired Hamiltonian function

$$H_z = \frac{1}{2}z_2^\top M_d^{-1}z_2 + \frac{1}{2}|\tilde{\theta}|^2 + V_d(z_1). \quad (22)$$

Then $(q_e, 0, \theta)$ is an *asymptotically* stable equilibrium point of the closed-loop system, provided that the detectability condition of the output is satisfied. The closed-loop dynamics can be represented as

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}-M^{-1}\Psi_1 & M^{-1}M_d & 0\\ -M_d M^{-1} & -R_d & \Psi_2\\ 0 & -\Psi_2^\top & 0\end{bmatrix}\begin{bmatrix}\nabla_{z_1}H_z\\ \nabla_{z_2}H_z\\ \nabla_{z_3}H_z\end{bmatrix}, \quad (23)$$

via the state transformation

$$z_1 = q, \ \ z_2 = p + \Psi_1\nabla_{z_1}H_z, \ \ z_3 = \tilde{\theta}, \quad (24)$$

with $\tilde{\theta}$ the estimation error, which is

$$\tilde{\theta} = z_3 = \hat{\theta} - \theta. \quad (25)$$

Remark 3: The adaptive control law (19) consists of the IDA-PBC control law (7) and the physical damping (friction) term $\mathrm{diag}\{M^{-1}p\}\hat{\theta}$ which needs to be estimated via the update law (20). It also contains additional derivative terms $\Psi_1\mathrm{diag}\{\nabla_q^2 H_d\}M^{-1}p$ w.r.t. $\nabla_q H_d$ and the integral action is introduced via the integration of the update law for some terms in the adaptive control law. The dissipation (damping) have been assigned (injected) onto some coordinates via the terms $M^{-1}\Psi_1, R_d$ ensuring asymptotic stability. Thus, the controller can be viewed as a PID-type controller, which was first introduced in [16] within the IDA-PBC framework.

Proof of Proposition 1: Consider the desired Hamiltonian function (22) as a candidate Lyapunov function. Its time derivative along the trajectories of (23) is

$$\begin{aligned}\dot{H}_z =& \nabla_{z_1}H_z^\top \dot{z}_1 + \nabla_{z_2}H_z^\top \dot{z}_2 + \nabla_{z_3}H_z^\top \dot{z}_3\\ =& \nabla_{z_1}H_z^\top(-M^{-1}\Psi_1\nabla_{z_1}H_z + M^{-1}M_d\nabla_{z_2}H_z)\\ &+\nabla_{z_2}H_z^\top(-M_dM^{-1}\nabla_{z_1}H_z - R_d\nabla_{z_2}H_z + \Psi_2\nabla_{z_3}H_z)\\ &-\nabla_{z_3}H_z^\top(\Psi_2\nabla_{z_2}H_z).\end{aligned} \quad (26)$$

Canceling equivalent terms with opposite signs in (26) yields

$$\dot{H}_z = -\nabla_{z_1}H_z^\top M^{-1}\Psi_1\nabla_{z_1}H_z - \nabla_{z_2}H_z^\top R_d\nabla_{z_2}H_z. \quad (27)$$

Using the norm-notation defined in Section 2, (27) can be expressed as the following inequality

$$\dot{H}_z = -\|M^{-1}\nabla_{z_1}H_z\|^2_{\Psi_1} - \|G^\top M_d^{-1}z_2\|^2_{K_v} \leq 0. \quad (28)$$

This proves that the desired equilibrium $(q_e, 0, \theta)$ is stable. Furthermore, *asymptotic* stability will be satisfied imposing that the output

$$y_d = \begin{bmatrix}M^{-1}\nabla_{z_1}H_z\\ G^\top M_d^{-1}z_2\end{bmatrix},$$

of the closed-loop system (23) is *detectable* (the detectability condition has been used in [22] although not exactly in the same context). Next, we verify the coincidence of the states of the open-loop dynamics (12) with the closed-loop dynamics (23) that generates the adaptive control law (19). Using the state transformation (24) we have for the position states,

$$\begin{aligned} q &= z_1 \implies \dot{q} = \dot{z}_1,\\ \dot{z}_1 &= -M^{-1}\Psi_1\nabla_{z_1}H_z + M^{-1}M_d\nabla_{z_2}H_z\\ &= -M^{-1}\Psi_1\nabla_{z_1}H_z + M^{-1}(p + \Psi_1\nabla_{z_1}H_z)\\ &= M^{-1}p \equiv \dot{q},\end{aligned}$$

and for the momenta state, we have

$$\begin{aligned} z_2 &= p + \Psi_1\nabla_{z_1}H_z \implies\\ \dot{p} &= \dot{z}_2 - \Psi_1\frac{d}{dt}(\nabla_{z_1}H_z), \quad \left(\frac{d}{dt}(\nabla_{z_1}H_z) = \nabla_{z_1}^2 H_z\dot{z}_1\right)\\ \dot{p} &= \dot{z}_2 - \Psi_1\nabla_{z_1}^2 H_z\dot{z}_1\\ &\quad -\nabla_q H - R_f\nabla_p H + Gu = -M_dM^{-1}\nabla_{z_1}H_z\\ &\quad - R_d\nabla_{z_2}H_z + \Psi_2\nabla_{z_3}H_z - \Psi_1\mathrm{diag}\{\nabla_{z_1}^2 H_z\}M^{-1}p,\end{aligned}$$

where $\dot{z}_3 = \dot{\tilde{\theta}} = \dot{\hat{\theta}}$ as θ is constant. Extending the terms, using (13)-(17) and rearranging

$$\begin{aligned}Gu =& \nabla_q V - M_dM^{-1}\nabla_q V_d - R_dM_d^{-1}p\\ &- R_dM_d^{-1}\Psi_1\nabla_q V_d - \Psi_1\mathrm{diag}\{\nabla_q^2 H_d\}M^{-1}p\\ &+ \underbrace{G\mathrm{diag}\{\nabla_p H\}G^\top\theta + \Psi_2\tilde{\theta}.}_{\text{terms with uncertainty}}\end{aligned} \quad (29)$$

Using Ψ_2 as defined in (21) and the estimation as in (25), the term containing the unknown friction θ is canceled and we are left with a term containing the estimate $\hat{\theta}$. By multiplying both sides with $G^{\ddagger}$ we obtain the controller (19). Finally the update law (20) is obtained as

$$\dot{z}_3 = \dot{\hat{\theta}} = -\Psi_2 \nabla_{z_2} H_z \quad (\dot{z}_3 = \dot{\tilde{\theta}} = \dot{\hat{\theta}})$$
$$= -\Psi_2 M_d^{-1} z_2 = -\Psi_2 M_d^{-1} p - \Psi_2 M_d^{-1} \Psi_1 \nabla_q V_d.$$

Remark 4 (The matching condition verification)**:** The matching condition in (29) can be verified by substituting $R_d = GK_vG^\top$, (21) and selecting $\Psi_1 = G\Xi$ where Ξ is a controller design parameter to be selected, then (29) can be rewritten as

$$\begin{aligned} Gu =& \nabla_q V - M_d M^{-1} \nabla_q V_d - GK_v G^\top M_d^{-1} \Psi_1 \nabla_q V_d \\ &- GK_v G^\top M_d^{-1} p - G\Xi \mathrm{diag}\{\nabla_q^2 V_d\} M^{-1} p \\ &+ G\mathrm{diag}\{\nabla_p H\} G^\top \hat{\theta}. \end{aligned} \tag{30}$$

It is now necessary to verify the following matching condition (by multiplying (30) with $G^\perp$):

$$\begin{aligned} G^\perp Gu =& G^\perp \Big(\nabla_q V - M_d M^{-1} \nabla_q V_d \\ &- GK_v G^\top M_d^{-1} \Psi_1 \nabla_q V_d - GK_v G^\top M_d^{-1} p \\ &- G\Xi \mathrm{diag}\{\nabla_q^2 V_d\} M^{-1} p + G\mathrm{diag}\{\nabla_p H\} G^\top \hat{\theta} \Big). \end{aligned} \tag{31}$$

As $G^\perp G = 0$, the expression (31) is reduced to

$$0 = G^\perp \Big(\nabla_q V - M_d M^{-1} \nabla_q V_d \Big), \tag{32}$$

which is the original PDE, which, using Assumption 2, has already been solved.

5. ADAPTIVE IDA-PBC CONTROL FOR POTENTIAL ENERGY FUNCTION ESTIMATION

By inspecting the standard PCH system (1), we notice that uncertainties may exist in the kinetic energy function $K(p,q)$ and potential energy function $V(q)$, as the interconnection matrix contains the identity matrix only and G is usually constant and known. The implementation of the IDA-PBC controller (7) requires the exact knowledge of the system parameters, essentially the inertia matrix $M(q)$ (or its inverse M^{-1}) for the kinetic energy function and the potential energy function $V(q)$. As we deal with *constant* uncertainties only in this paper, we limit our discussion to the class of systems with $K(p,q)$ comprises a constant inertia matrix M and the potential energy function $V(q)$. Thus, we use the following assumption.

Here, we consider the gradient of the potential energy function $\nabla_q V(q,\theta)$ that can be represented as a linearly parametrized expression of the form

$$\nabla_q V(q,\theta) = F(q)\theta, \tag{33}$$

where the matrix function $F(q)$ is known and the constant vector θ contains the unknown parameters.

Assumption 4: Similarly, the *desired* potential energy function $V_d(q)$ is expressed as

$$\nabla_q V_d(q,\theta) = \nabla_q V(q,\theta) + S(q) = F(q)\theta + S(q), \tag{34}$$

where $S(q)$ is known, as V_d is chosen by design through solving the matching equations. Furthermore, the matrix function $F(q)$ in (34) is symmetric, i.e. $F(q) = F(q)^\top$ such that the potential energy function $V(q)$ is of the form $V(q) = \sum_{i=1}^{n} V(q_i)$.

In the following, we propose two adaptive IDA-PBC control approaches; in the first approach, we show that using classical IDA-PBC without state-transformation, only stability can be concluded, whereas in the second one, which includes a change of coordinates and adding an integral action control, asymptotic stability is guaranteed using the dynamic adaptive control law.

Remark 5: Instead of dealing with single unknown parameters, e.g. mass m, length l etc., we consider unknown terms in the potential energy function as done in [29] (see expressions (2.31)-(2.42) in [29]).

5.1. Adaptive control design using classical IDA-PBC

Proposition 2: Consider the system (1) satisfying Assumption 2, with uncertainty in the PE functions whose gradients, (33) and (34) respectively, satisfy Assumption 4. The adaptive controller

$$u = G^{\ddagger}(F\hat{\theta} - M_d M^{-1} F\hat{\theta} - M_d M^{-1} S - R_d M_d^{-1} p), \tag{35}$$

with the update law

$$\dot{\hat{\theta}} = -(K_a F)^\top \nabla_p H_d, \tag{36}$$

and the desired Hamiltonian function

$$H_d(q,p,\tilde{\theta}) = \frac{1}{2} p^\top M_d^{-1} p + \frac{1}{2} |\tilde{\theta}|^2 + V_d(q,\theta), \tag{37}$$

stabilizes the equilibrium $(q_e, 0, \theta)$ of the system. The closed-loop system can then be represented as the augmented system

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\hat{\theta}} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d & 0 \\ -M_d M^{-1} & -R_d & K_a F \\ 0 & -(K_a F)^\top & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \\ \nabla_{\tilde{\theta}} H_d \end{bmatrix}, \tag{38}$$

with $\hat{\theta}$ the estimate of θ, $\tilde{\theta}$ the estimation error and $K_a = I_n - M_d M^{-1}$.

Proof of Proposition 2: Consider the desired Hamiltonian function (37) as a candidate Lyapunov function. Its time derivative along the trajectories of (38) satisfies

$$\dot{H}_d = p^\top M_d^{-1} \dot{p} + \tilde{\theta}^\top \dot{\tilde{\theta}} + \nabla_q V_d^\top \dot{q}$$

$$
\begin{aligned}
&= -p^\top M^{-1}\nabla_q V_d - p^\top M_d^{-1} R_d M_d^{-1} p + p^\top M_d^{-1} K_a F\tilde{\theta} \\
&\quad - \tilde{\theta}^\top F^\top K_a^\top M_d^{-1} p + \nabla_q V_d^\top M^{-1} p \qquad (39)\\
&= -p^\top M^{-1}\nabla_q V_d + (p^\top M^{-1}\nabla_q V_d)^\top + p^\top M_d^{-1} K_a F\tilde{\theta} \\
&\quad - (p^\top M_d^{-1} K_a F\tilde{\theta})^\top - p^\top M_d^{-1} R_d M_d^{-1} p \\
&= -p^\top M_d^{-1} R_d M_d^{-1} p \le -\|(M_d^{-1} p)^\top G\|_{K_v}^2 \le 0,
\end{aligned}
$$

i.e. the equilibrium $(q_e, 0, \theta)$ of the closed-loop system is *stable*, with $\frac{\partial H_d}{\partial x} f^*(x(t)) \le 0$, with $f^*(x(t))$ is the nominal system without uncertainty. As thoroughly discussed in [30], if $\frac{\partial H_d}{\partial x} f^*(x(t)) < 0$, for all $x \neq x_e$, then LaSalle's invariance principle is sufficient to conclude the convergence of the states to their equilibrium. Alternatively, it is necessary to add the following detectability assumption to complete the stability proof [6, 30].

Assumption 5: The PCH system (1) with uncertainty can be written as a general input affine nonlinear system of the form

$$
\Sigma \,:\, \begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \qquad (40)
$$

with the uncertainty included in $f(x)$, such that $\dot{x} = f^*(x(t))$ represents the nominal system (without uncertainty) with the states $x = \begin{bmatrix} q & p \end{bmatrix}^\top$ and $x_e = (q_e, 0)$ is the equilibrium of the nominal system. We assume that a stabilizing IDA-PBC controller (7) has already been obtained for the nominal system such that the trajectories of the closed-loop system are such that $\frac{\partial H_d}{\partial x} f^*(x(t)) \equiv 0$ implies $\lim_{t\to\infty} x(t) = x_e$.

Note that from (39), we have $\dot{H}_d = 0 \implies p = 0$. Furthermore, $p \equiv 0 \implies \dot{p} \equiv 0$. Thus, under the dynamics (38) we have:

$$
\begin{aligned}
\dot{p} &= -M_d M^{-1}\nabla_q V_d - R_d \nabla_p H_d + K_a F(q)\nabla_{\tilde{\theta}} H_d \\
&= -M_d M^{-1}(F\theta + S(q)) - R_d M_d^{-1} p + K_a F\tilde{\theta} \\
&= -M_d M^{-1}(F\theta + S) + (I_n - M_d M^{-1})F(\hat{\theta} - \theta) \\
&= -M_d M^{-1}(S + F\hat{\theta}) + F(\hat{\theta} - \theta) = 0. \qquad (41)
\end{aligned}
$$

From (37) and (39), $p \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $q, \tilde{\theta} \in \mathcal{L}_\infty$. Therefore, the zero momentum (velocity) may guarantee boundedness of $\tilde{\theta}$, however asymptotic stability is not guaranteed. This motivates establishing a new adaptive control law using states-transformation as follows. ■

5.2. Adaptive control design using IDA-PBC with state-transformation

It has been shown in the previous subsection that asymptotic stability can not be assured in case of uncertainty using the classical IDA-PBC. Here, we deploy a change of coordinates similar to those proposed in the previous section, aiming at asymptotically stabilizing the system (1) with uncertainty, at the equilibrium point $(q_e, 0, \theta)$.

Proposition 3: Consider the system (1) satisfying Assumption 5, with uncertainty in the PE function (33), (34) satisfying Assumption 4, in closed-loop with the adaptive controller

$$
\begin{aligned}
u &= G^\ddagger \Big[\Gamma_1\hat{\theta} - \Gamma_2 S - \Psi_1 \mathrm{diag}\{M^{-1}p\}\nabla S \\
&\qquad - R_d M_d^{-1} p - \Psi_2\dot{\hat{\theta}}\Big], \\
\Gamma_1 &= (I - M_d M^{-1} - R_d M_d^{-1}\Psi_1)F \\
&\quad - \Psi_1 \mathrm{diag}\{M^{-1}p\}\nabla F \\
\Gamma_2 &= M_d M^{-1} + R_d M_d^{-1}, \qquad (42)
\end{aligned}
$$

with the update law

$$
\begin{aligned}
\dot{\hat{\theta}} &= (M^{-1}\Psi_2 F + \Psi_3 M_d^{-1}\Psi_1 F)\hat{\theta} + M^{-1}\Psi_2 S \\
&\quad + \Psi_3 M_d^{-1}\Psi_1 S + \Psi_3 M_d^{-1} p,
\end{aligned} \qquad (43)
$$

and the desired Hamiltonian function

$$
H_z = \frac{1}{2} z_2^\top M_d^{-1} z_2 + \frac{1}{2}|\tilde{\theta}|^2 + V_d(z_1). \qquad (44)
$$

Then, $(q_e, 0, \theta)$ is an *asymptotically* stable equilibrium point of the closed-loop system. The closed-loop dynamics can be represented as

$$
\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -M^{-1}\Psi_1 & M^{-1}M_d & -M^{-1}\Psi_2 \\ -M_d M^{-1} & -R_d & -\Psi_3 \\ (M^{-1}\Psi_2)^\top & \Psi_3^\top & -\Psi_4 \end{bmatrix} \begin{bmatrix} \nabla_{z_1} H_z \\ \nabla_{z_2} H_z \\ \nabla_{z_3} H_z \end{bmatrix}, \quad (45)
$$

via the state transformation

$$
z_1 = q,\; z_2 = p + \Psi_1\nabla_{z_1} H_z + \Psi_2 z_3,\; z_3 = \tilde{\theta} = \hat{\theta} - \theta, \quad (46)
$$

with

$$
\begin{aligned}
\Psi_3 &= (R_d M_d^{-1}\Psi_1 + M_d M^{-1} - I)F \\
&\qquad + \Psi_1 \mathrm{diag}\{M^{-1}p\}\nabla F - R_d M_d^{-1}\Psi_2,, \qquad (47)\\
\Psi_4 &= \Psi_3 M_d^{-1}\Psi_2 - (M^{-1}\Psi_2 + \Psi_3 M_d^{-1}\Psi_1)F, \qquad (48)
\end{aligned}
$$

and Ψ_1, Ψ_2 are chosen such that $\Psi_4 > 0$, $\hat{\theta}$ is the estimate of θ and the estimation error is (25).

Proof of Proposition 3: The proof is established by: (i) verifying the coincidence of the position and momenta states of system (1) with their corresponding states in (45); (ii) proving that the expression of the update law $\dot{z}_3$ does not depend on θ; (iii) proving that the proposed method achieves asymptotic stability.

(i) Using the state-transformation (46), for the position states q, we have

$$
\begin{aligned}
q &= z_1 \implies \dot{q} = \dot{z}_1 \\
\dot{z}_1 &= -M^{-1}\Psi_1\nabla_{z_1} H_z + M^{-1}M_d\nabla_{z_2} H_z - M^{-1}\Psi_2\nabla_{z_3} H_z \\
&= -M^{-1}\Psi_1\nabla_{z_1} H_z + M^{-1}M_d M_d^{-1} z_2 - M^{-1}\Psi_2 z_3
\end{aligned}
$$

$$= -M^{-1}\Psi_1\nabla_{z_1}H_z + M^{-1}(p+\Psi_1\nabla_{z_1}H_z+\Psi_2 z_3) - M^{-1}\Psi_2 z_3 = M^{-1}p \equiv \dot{q},$$

and for the momenta p,

$$\begin{aligned}
&z_2 = p+\Psi_1\nabla_{z_1}H_z+\Psi_2 z_3 \implies\\
&\dot{p} = \dot{z}_2 - \Psi_1\nabla^2_{z_1}H_z\dot{z}_1 - \Psi_2\dot{z}_3\\
&-\nabla_q H + Gu = -M_dM^{-1}\nabla_{z_1}H_z - R_d\nabla_{z_2}H_z - \Psi_3\nabla_{z_3}H_z\\
&\qquad -\Psi_1\mathrm{diag}\{M^{-1}p\}\nabla^2_{z_1}H_z - \Psi_2\dot{\hat{\theta}}\\
&-\nabla_q H + Gu = -M_dM^{-1}\nabla_{z_1}H_z - R_dM_d^{-1}z_2 - \Psi_3 z_3\\
&\qquad -\Psi_1\mathrm{diag}\{M^{-1}p\}\nabla^2_{z_1}H_z - \Psi_2\dot{\hat{\theta}}.
\end{aligned}$$

Since M_d is constant from Assumption 1, then

$$\nabla_q H = \nabla_q V,\ \ \nabla_{z_1}H_z = \nabla_q H_d = \nabla_q V_d.$$

Therefore, using (33), (34) we have:

$$\begin{aligned}
&\nabla_q H = \nabla_q V = F\theta\\
&\nabla_{z_1}H_z = \nabla_q H_d = \nabla_q V_d = F\theta + S\\
&\nabla^2_{z_1}H_z = \nabla F\theta + \nabla S \qquad (\text{as } \theta \text{ is constant}).
\end{aligned} \tag{49}$$

Extending the terms, using (49) and rearranging

$$\begin{aligned}
Gu =& F\theta - M_dM^{-1}F\theta - M_dM^{-1}S\\
&- R_dM_d^{-1}(p+\Psi_1\nabla_{z_1}H_z+\Psi_2 z_3)\\
&- \Psi_3\tilde{\theta} - \Psi_1\mathrm{diag}\{M^{-1}p\}(\nabla F\theta+\nabla S) - \Psi_2\dot{\hat{\theta}}\\
=& -(M_dM^{-1}+R_dM_d^{-1})S - \Psi_1\mathrm{diag}\{M^{-1}p\}\nabla S\\
&- R_dM_d^{-1}p - \Psi_2\dot{\hat{\theta}}\\
&\underbrace{+F\theta - M_dM^{-1}F\theta - R_dM_d^{-1}\Psi_1 F\theta}_{\text{terms with uncertainty}}\\
&\underbrace{-\Psi_1\mathrm{diag}\{M^{-1}p\}\nabla F\theta - R_dM_d^{-1}\Psi_2\tilde{\theta} - \Psi_3\tilde{\theta}}_{\text{terms with uncertainty}}.
\end{aligned}$$

Using (25) and selecting Ψ_3 as in (47), the term containing the unknown friction θ is canceled and we are left with a term containing the estimate $\hat{\theta}$. By multiplying both sides with $G^{\ddagger}$ we obtain the controller (42).
(ii) The update law (43) is obtained as

$$\begin{aligned}
\dot{z}_3 =& \dot{\hat{\theta}} = (M^{-1}\Psi_2)^\top\nabla_{z_1}H_z + \Psi_3^\top\nabla_{z_2}H_z - \Psi_4\nabla_{z_3}H_z\\
=& M^{-1}\Psi_2\nabla_{z_1}H_z + \Psi_3M_d^{-1}p + \Psi_3M_d^{-1}\Psi_1\nabla_{z_1}H_z\\
&+ \Psi_3M_d^{-1}\Psi_2\tilde{\theta} - \Psi_4\tilde{\theta}\\
=& M^{-1}\Psi_2F\theta + M^{-1}\Psi_2S + \Psi_3M_d^{-1}p + \Psi_3M_d^{-1}\Psi_1F\theta\\
&+ \Psi_3M_d^{-1}\Psi_1S + \Psi_3M_d^{-1}\Psi_2\tilde{\theta} - \Psi_4\tilde{\theta}\\
=& (M^{-1}\Psi_2 + \Psi_3M_d^{-1}\Psi_1)S + \Psi_3M_d^{-1}p\\
&+\underbrace{M^{-1}\Psi_2F\theta + \Psi_3M_d^{-1}\Psi_1F\theta + \Psi_3M_d^{-1}\Psi_2\tilde{\theta} - \Psi_4\tilde{\theta}}_{\text{terms with uncertainty}}.
\end{aligned}$$

Furthermore, choosing Ψ_4 as in (48), we obtain the update law (43).

(iii) Consider the desired Hamiltonian function (44) as a candidate Lyapunov function. Its time derivative along the trajectories of (45) is

$$\begin{aligned}
\dot{H}_z =& \nabla_{z_1}H_z^\top\dot{z}_1 + \nabla_{z_2}H_z^\top\dot{z}_2 + \nabla_{z_3}H_z^\top\dot{z}_3\\
=& \nabla_{z_1}H_z^\top\big(-M^{-1}\Psi_1\nabla_{z_1}H_z + M^{-1}M_d\nabla_{z_2}H_z\\
&\quad - M^{-1}\Psi_2\nabla_{z_3}H_z\big)\\
&+\nabla_{z_2}H_z^\top(-M_dM^{-1}\nabla_{z_1}H_z - R_d\nabla_{z_2}H_z - \Psi_3\nabla_{z_3}H_z)\\
&+\nabla_{z_3}H_z^\top(M^{-1}\Psi_2\nabla_{z_1}H_z + \Psi_3\nabla_{z_2}H_z - \Psi_4\nabla_{z_3}H_z).
\end{aligned} \tag{50}$$

Canceling equivalent terms with opposite signs then (50) is reduced to

$$\begin{aligned}
\dot{H}_z =& -\nabla_{z_1}H_z^\top M^{-1}\Psi_1\nabla_{z_2}H_z - \nabla_{z_2}H_z^\top R_d\nabla_{z_2}H_z\\
&- \nabla_{z_3}H_z^\top\Psi_4\nabla_{z_3}H_z\\
=& -\|M^{-1}\nabla_q V_d\|^2_{\Psi_1} - \|G^\top M_d^{-1}z_2\|^2_{K_v} - \|\nabla_{z_3}H_z\|^2_{\Psi_4}\\
\le& \,0.
\end{aligned} \tag{51}$$

Thus, the system (45) has a stable equilibrium at $(q_e, 0, \theta)$. Finally, asymptotic stability is ensured imposing the following detectability condition as done in [22].

Condition 1: The closed-loop system (45) is *detectable* from the output:

$$y_d = \begin{bmatrix} M^{-1}\nabla_q V_d\\ G^\top M_d^{-1}z_2\\ \nabla_{z_3}H_z \end{bmatrix}. \tag{52}$$

With this condition, we can conclude that the closed-loop system (45) has an *asymptotically stable* equilibrium at $(q_e, 0, \theta)$. This completes the proof of the proposition.

Remark 6 (The matching condition verification)**:** The verification follows the same procedures as in Remark 4, that is:

$$\begin{aligned}
G^\perp Gu =& G^\perp\Big[(F - M_dM^{-1}F - R_dM_d^{-1}\Psi_1F\\
&- \Psi_1\mathrm{diag}\{M^{-1}p\}\nabla F)\hat{\theta}\\
&- (M_dM^{-1}+R_dM_d^{-1})S\\
&+ \Psi_1\mathrm{diag}\{M^{-1}p\}\nabla S - R_dM_d^{-1}p - \Psi_2\dot{\hat{\theta}}\Big].
\end{aligned} \tag{53}$$

Substituting $R_d = GK_vG^\top$ and selecting $\Psi_1 = G\Xi_1, \Psi_2 = G\Xi_2$, then (53) is reduced to

$$0 = G^\perp(\underbrace{F\hat{\theta}}_{\nabla_q V} - \underbrace{M_dM^{-1}F\hat{\theta} - M_dM^{-1}S}_{M_dM^{-1}\nabla_q V_d}), \tag{54}$$

which is the original PDE, which, using Assumption 2, has already been solved.

6. EXAMPLE 1: THE ACROBOT SYSTEM

We use the dynamical model of the well-known Acrobot system shown in Fig. 1, as taken from [31]. The IDA-PBC controller (7), assuming no friction, was also borrowed from [22, 31].

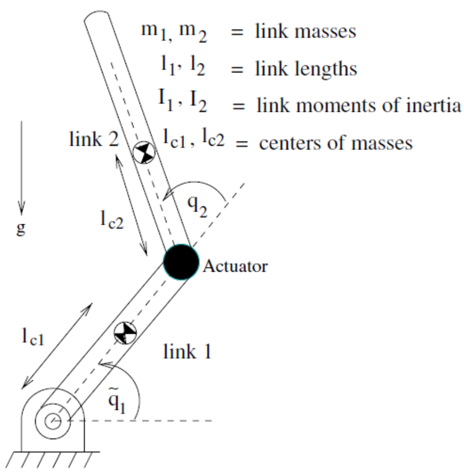

Fig. 1. The Acrobot system [31].

6.1. The Acrobot model and IDA-PBC controller

The open-loop system of the Acrobot can be represented by the dynamic model (1) with $n=2, m=1$, $G=\begin{bmatrix}0 & 1\end{bmatrix}^\top$ and

$$M=\begin{bmatrix} c_1+c_2+2c_3\cos(q_2) & c_2+2c_3\cos(q_2) \\ c_2+2c_3\cos(q_2) & c_2 \end{bmatrix}, \quad (55)$$

$$V(q_1)=g\Big(c_4\cos(q_1)+c_5\cos(q_1+q_2)\Big). \quad (56)$$

The parameters of the model $g, c_i, i=1,\cdots,5$ are the same as in [22]. The stabilizing controller (7) proposed in [22, 31] is given as

$$\begin{aligned} u_{ida}=&\nabla_{q_2}V+\frac{1}{2}p^\top\nabla_{q_2}(M)^{-1}p-\begin{bmatrix}0 & 1\end{bmatrix}M_dM^{-1}\nabla_qV_d \\ &+\frac{k_v}{m_1m_3-m_2^2}(m_2p_1-m_1p_2), \end{aligned} \quad (57)$$

where m_1, m_2, m_3 are the elements of the desired mass matrix

$$M_d=\begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix}, \quad (58)$$

and ∇_qV_d is the gradient of the desired potential energy function with

$$\begin{aligned} \nabla_{q_1}V_d=&-k_0\sin(q_1-\mu q_2)-b_1\sin(q_1)-b_2\sin(q_1+q_2) \\ &-b_3\sin(q_1+2q_2)-b_4\sin(q_1-q_2) \\ &+k_u(q_1-\mu q_2) \\ \nabla_{q_2}V_d=&k_0\mu\sin(q_1-\mu q_2)-b_2\sin(q_1+q_2) \\ &-k_u\mu(q_1-\mu q_2) \\ &-2b_3\sin(q_1+2q_2)+b_4\sin(q_1-q_2). \end{aligned}$$

The constant parameters b_1, b_2, b_3, b_4, k_0, k_u, and μ are given in [22].

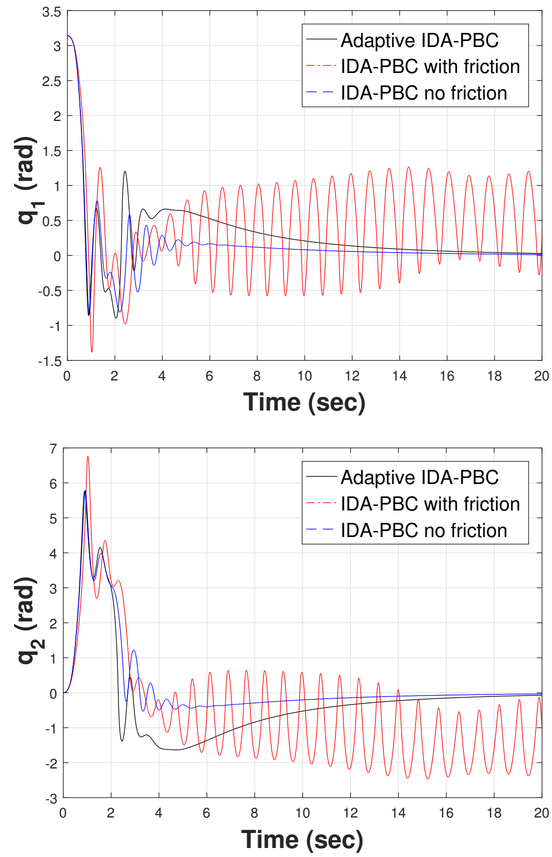

Fig. 2. State trajectories (Acrobot angles).

6.2. The Acrobot simulations with friction

To assess the effect of friction, three sets of simulations were carried out. The model of the Acrobot was first simulated using the classical IDA-PBC assuming both the cases of no friction and with friction. Then the model was also tested with the proposed adaptive controller, at the presence of friction. The values of the model parameters used in the simulations are [22] $g=9.8, c_1=2.3333, c_2=5.3333, , c_3=2, c_4=3, c_5=2$. The IDA-PBC design parameters are chosen as: $m_1=0.3386, m_2=1, m_3=5.9073, \mu=-0.6019, k_0=-350, k_u=10$, and $k_v=12$. The simulations are performed assuming the parameters of the friction R_f are $r=2$ and the initial conditions are defined as $q_1(0)=\pi, q_2(0)=0, p_1(0)=0$, and $p_2(0)=0$. To compare the response with the non-adaptive case, the unknown friction was fixed to $\frac{r}{2}$ when applied to classical IDA-PBC only.

Fig. 2 and Fig. 3 plot the states (angles and momenta) for the three cases. These plots show that without the friction (dashed blue line), classical IDA-PBC asymptotically stabilizes the Acrobot at its equilibrium. However, in the presence of friction, a significant steady-state error appears, which shifted the equilibrium from the desired one (dash-dot red line). Oscillation about this shifted equilib-

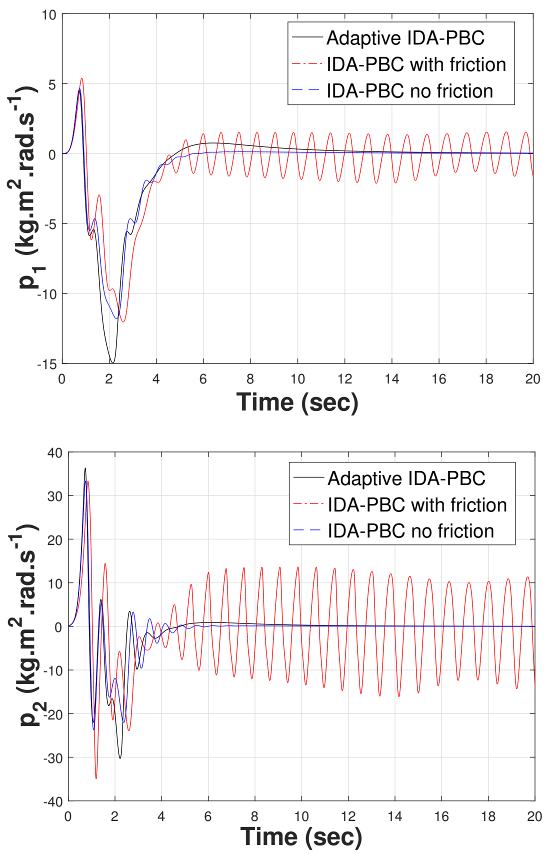

Fig. 3. State trajectories (Acrobot momenta).

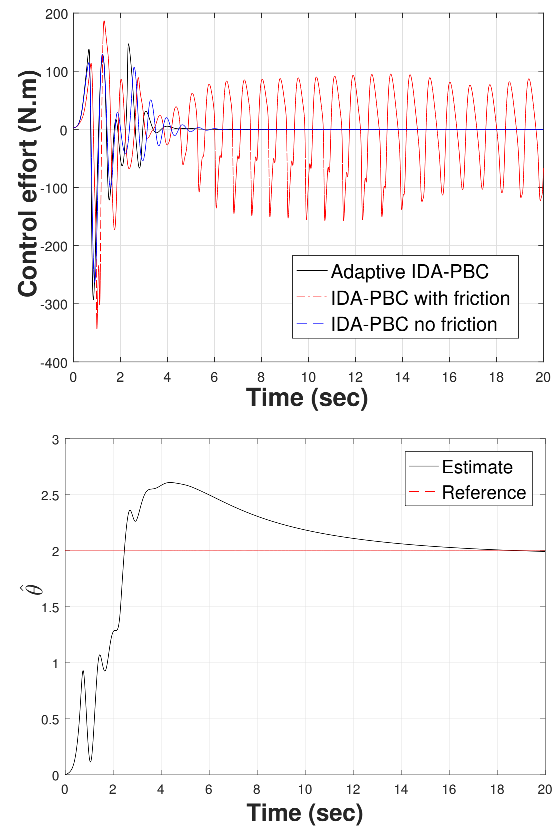

Fig. 4. Control effort and estimation.

rium is also noticed due to the effect of friction which exist in the form (14); $R_f \nabla_p H = R_f M^{-1} p$, and as long as the states p have not been converged, we expect oscillations about the offset equilibrium. By adding the adaptive controller along with IDA-PBC the states (solid black line) converged to their desired equilibrium though the presence of the friction as shown in Fig. 2 and Fig. 3.

Finally, Fig. 4 (top) shows the control effort for the three cases while Fig. 4 (bottom) depicts the convergence of the estimate R_f to its true value (reference value), demonstrating the effectiveness of the proposed adaptive controller.

7. EXAMPLE 2: THE DISK-ON-DISK SYSTEM

In this section, the disk-on-disk module shown in Fig. 5 is used to illustrate our proposed results. The module consists of two circular disks; an unactuated disk with no-slipping rolling on the actuated disk. The angular position of the actuated disk q_1 and the angular position of the unactuated disk q_2 represent the general coordinates of the system which has two degrees-of-freedom. Starting from arbitrary initial conditions, the control objective is to drive both disks to the desired equilibrium $q_e = (0,0)$.

7.1. The system model and IDA-PBC controller

The disk-on-disk dynamics is described by the PCH representation (1) with $n = 2$, $m = 1$ and [22]

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}, \; G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ and} \tag{59}$$

$$V(q_2) = m_2 g(r_1 + r_2)\cos(q_2), \tag{60}$$

with $m_{11} = r_1^2(m_1 + m_2)$, $m_{12} = -m_2 r_1(r_1 + r_2)$, $m_{22} = 2m_2(r_1 + r_2)$, where the model parameters r_1, r_2 are the radii of the two disks and m_1, m_2 are their masses. Assuming no uncertainties in the model, the classical IDA-PBC controller proposed in [22] is obtained as

$$u_{IDA} = \alpha_1 m_2 g(r_1 + r_2)\sin(q_2) - \alpha_2(q_1 - \alpha_3 q_2) - \frac{k_v}{\Delta_{M_d}}(k_3 p_1 - k_2 p_2), \tag{61}$$

by assigning the desired inertia and potential energy function as

$$M_d = \begin{bmatrix} k_1 & k_2 \\ k_2 & k_3 \end{bmatrix} > 0, \tag{62}$$

$$V_d(q) = -\alpha_4 m_2 g(r_1 + r_2)\cos(q_2) + \frac{k_p}{2}(q_1 - \alpha_3 q_2)^2. \tag{63}$$

The controller parameters are defined as:

$$\alpha_1 = \frac{k_2 m_{11} - k_1 m_{12}}{k_3 m_{11} - k_2 m_{12}}, \quad \alpha_2 = k_p \frac{\Delta_{M_d}}{k_3 m_{11} - k_2 m_{12}},$$

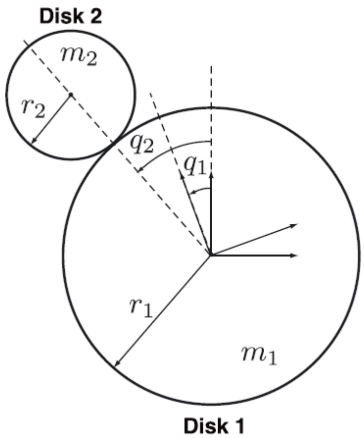

Fig. 5. Idealized schematic of the disk-on-disk [22].

$$\alpha_3 = \frac{k_2 m_{22} - k_3 m_{12}}{k_3 m_{11} - k_2 m_{12}}, \quad \alpha_4 = \frac{k_3 m_{11} - k_2 m_{12}}{\Delta_M},$$
$$\Delta_M = m_{11} m_{22} - m_{12}^2, \quad \Delta_{M_d} = k_1 k_3 - k_2^2,$$

with $k_v > 0$, $k_p > 0$ the damping injection gain and energy-shaping gain, respectively.

7.2. The simulations of the disk-on-disk system

Following the discussion in Subsection 5.2, here we show the design of an adaptive controller to compensate for the uncertainty in the potential energy function $V(q)$. Consider the potential energy function of disk-on-disk system (60). The gradient of this function is a

$$\nabla_q V = -m_2 g(r_1 + r_2)\sin(q_2), \tag{64}$$

which can be linearly parametrized as (33), with $F(q_2) = -\sin(q_2)$ and $\theta = m_2 g(r_1 + r_2)$ the uncertain term. Thus, classical IDA-PBC controller (61) is rewritten as

$$\hat{u}_{ida} = \alpha_1 \theta \sin(q_2) - \alpha_2(q_1 - \alpha_3 q_2) - \frac{k_v}{\Delta_{M_d}}(k_3 p_1 - k_2 p_2).$$

By applying Proposition 3, the adaptive IDA-PBC controller (42) can be used to compensate for the uncertainty θ. The model parameters, taken from [22], are $m_1 = 0.235$ kg, $m_2 = 0.0216$ kg, $r_1 = 0.15$ m, and $r_1 = 0.075$ m. The parameters of the classical IDA-PBC controller are slightly modified from [22] and have been selected as follows: $k_1 = 0.4$, $k_2 = -0.03$, $k_3 = 0.003$, $kp = 0.00001$, and $k_v = 0.5$. Furthermore, we have adjusted the uncertain term $\theta = m_2 g(r_1 + r_2)$ as $\theta = \vartheta + \zeta$, with ζ is a fixed estimate. This enables us to compare this method with the non-adaptive one. Given the value of $\theta = m_2 g(r_1 + r_2) = 0.0477$, we have selected $\zeta = \theta/2$ for this case.

Fig. 6 and Fig. 7 show the time histories of the states (angles and momenta) for both cases. As shown, without adaptation law (dashed blue line) the uncertainty in $V(q)$ results in a relatively large steady-state error in q_1 (around 10 degrees) and poor transients. In contrast, applying the adaptation law (solid black line), the trajectories of the disk-on-disk system converge to their desired states with excellent performance. Finally, Fig. 8 shows the convergence of the estimate $\hat{\theta}$ to the true value of θ, which confirms that the effectiveness of the proposed adaptive method.

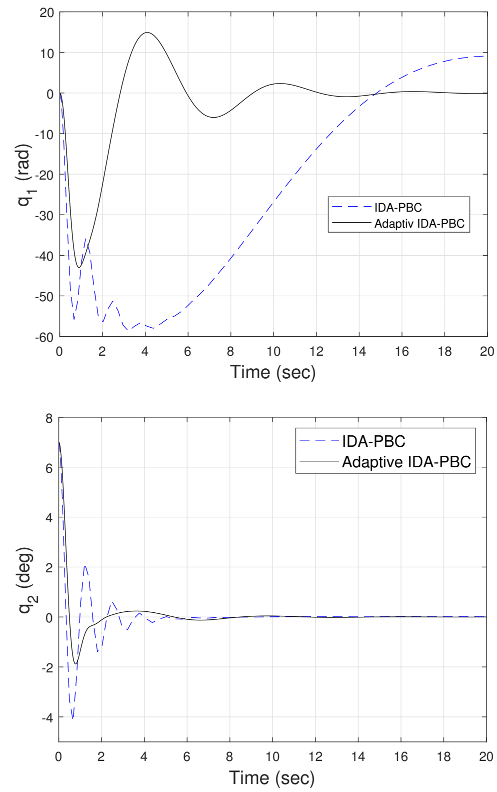

Fig. 6. State trajectories (disk-on-disk angles).

8. CONCLUSION

We have proposed an adaptive IDA-PBC design method to deal with several robustness-related issues (friction estimation and uncertainties in the energy function) for underactuated mechanical systems within PCH framework. In particular, the adaptive control law has been combined with the classical IDA-PBC control. Adopting the state-transformation approach, this dynamic state-feedback controller has employed an integral control action to further reduce the steady-state error. The effectiveness of the proposed adaptive control scheme has been verified with two underactuated mechanical systems; the Acrobot and disk-on-disk systems. Future research direction includes a general adaptive control scheme that considers wider classes of robotic and underactuated mechanical systems.

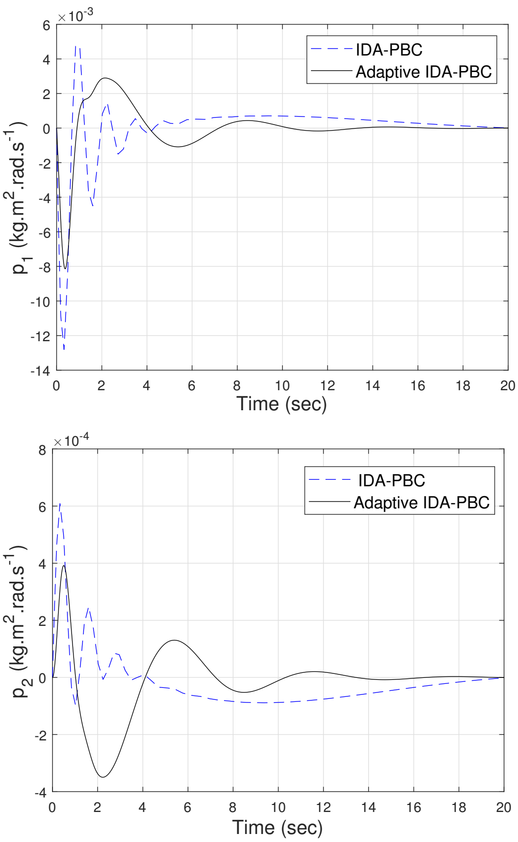

Fig. 7. State trajectories (disk-on-disk momenta).

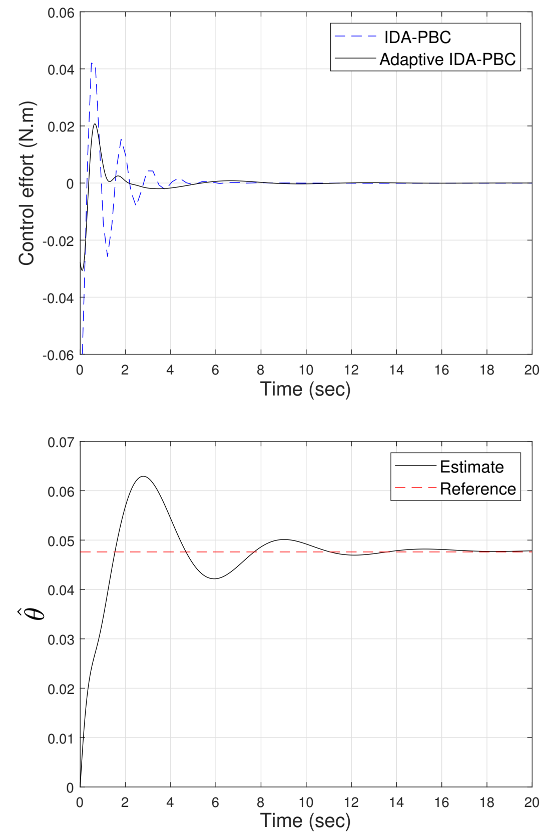

Fig. 8. Control effort and estimation.

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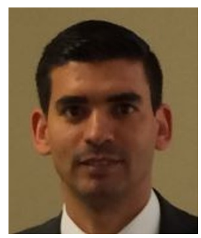

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