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Observer-based Finite-time Control of Stochastic Non-strict-feedback Nonlinear Systems

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Abstract: This paper investigates the observer-based adaptive finite-time neural control issue of stochastic non-strict-feedback nonlinear systems. By establishing a state observer and utilizing the approximation property of the neural network, an adaptive neural network output-feedback controller is constructed. The controller solves the issue that the states of stochastic nonlinear system cannot be measured, and assures that all signals in the closed-loop system are bounded. Different from the existing adaptive control researches of stochastic nonlinear systems with unmeasured states, the proposed control scheme can guarantee the finite-time stability of the stochastic nonlinear systems. Furthermore, the effectiveness of the proposed control approach is verified by the simulation results.

Keywords: Adaptive neural control, finite-time control, non-strict-feedback form, state observer, stochastic nonlinear systems.

1. INTRODUCTION

In the past few decades, the adaptive control of the nonlinear systems has been paid considerable attention, and some significant results have been published in references [1–6]. The main idea of such control schemes is to model the unknown nonlinear functions by utilizing the function approximation ability of the fuzzy logic systems or neural networks. It should be pointed out that all the control results in [1–6] are applicable to the deterministic systems, but the stability of practical systems is influenced by uncertainties and random disturbances. Because the random differential of stochastic nonlinear systems designed by Lyapunov includes higher order Hessian terms, the control of stochastic nonlinear systems is more difficult than that of deterministic nonlinear systems. Therefore, the study on the control design for the stochastic nonlinear systems has important theoretical and practical significance, and more and more scholars pay attention to it. In particular, for deterministic nonlinear systems [1–6], the control design approaches have been successfully extended to stochastic nonlinear systems [7–10]. Among them, references [7,8] concerned the control design for stochastic nonlinear systems with strict-feedback form, Namadchian *et al*. [9] addressed the issue of adaptive fuzzy control for a kind of stochastic pure-feedback nonlinear systems, and Zhao *et al*. [10] developed the adaptive neural control scheme for a category of stochastic non-strict-feedback nonlinear systems. However, for a class of stochastic nonlinear systems with unmeasurable state variables, the above control strategies [1–10] may not be available.

In practical systems, the state variables are usually unmeasurable or just partly measurable, and some control plans may not be well implemented. The observer can estimate those unmeasurable state variables, it overcomes the difficulties caused by lack of accurate state information. In particular, observer-based neural/fuzzy control has received a lot of attentions in [11–19]. For the nonlinear systems [11,12] whose unknown functions only contain the output term, the issue of adaptive neural/fuzzy output feedback control was solved in [11,12]. Li *et al*. [13] developed the controller design scheme for the nonlinear networked control systems, in which the premise variables are unmeasurable. For a category of nonlinear systems with unknown virtual control coefficients, robust observers were designed in [14,15], and two adaptive control programs based on backstepping were developed in [14,15]. Furthermore, in view of the fact that state variables are unmeasurable and only the system output is available, the adaptive control issue for non-strict-feedback nonlinear systems was addressed in [16]. These design methods of observer-based adaptive fuzzy/neural control in [11–16] have been extended to stochastic nonlinear systems in [17–19]. Despite these developments have been made, it must be acknowledged that these control strategies in [1–19] can only ensure the infinite-time stability of nonlinear systems or stochastic nonlinear sys-

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tems. However, in the practical application, the finite-time stability of the system is more significant than the infinite-time stability.

Recently, the finite-time control has caused extensive attention. Compared with the infinite-time control, the main characteristic of the finite-time control is that the state of the system reaches equilibrium in finite-time and stay there then after. The finite-time control can achieve transient performance rapidly, which is of great significance in the practical application. There are many outstanding investigations on the finite-time stabilization in [20–29]. It must be clarified that literatures [20,21] need to satisfy the linearly parametrization conditions of nonlinearities, references [22,23] assume that the nonlinear terms are completely unknown, and articles [22–24] require the systems to be in a strict-feedback or pure-feedback form. Due to there are obvious differences between the strict-feedback or pure-feedback systems and non-strict-feedback ones, the control methods in [22–24] cannot be employed to non-strict-feedback versions directly. Different from papers [22–24], references [25–27] studied adaptive finite-time control problem of non-strict-feedback nonlinear systems. In addition, the most finite-time control researches [20–27] are based on the fact that the state variables can be measured directly. In reference [28], the finite-time control method for nonlinear systems with unmeasurable state variables was proposed. It should be mentioned that the controllers are designed in [20–28] only for deterministic systems, while deterministic systems ignore the influence of random disturbances. The stochastic disturbances are unavoidable and even bring about the instability of the systems. Hence, the finite-time stability of the stochastic nonlinear systems was investigated in [29]. However, the results in [29] require the state variables to be measurable. So far as is known to authors, no studies have been reported as to the finite-time control for stochastic non-strict-feedback nonlinear systems with unmeasurable state variables due to its complexity, which is an interesting yet challenging issue.

Motivated by the aforementioned findings, this paper investigates the observer-based finite-time neural control problem for stochastic nonlinear systems, where the controlled system is in a non-strict-feedback form and the state variables of system are unmeasurable. To this end, Lemma 6 is applied to solve the difficulties arising from the non-strict-feedback structure, neural networks are introduced to approximate the unknown nonlinear functions, the state observer is employed to estimate the unavailable state variables, and Lemma 5 is used to prove the finite-time stability of stochastic nonlinear systems. And then observer-based adaptive finite-time control scheme is developed for stochastic nonlinear systems. Compared with the existing literatures, the main features of this paper are listed as follows:

- The existing adaptive fuzzy or neural network control strategies in [1–19] can only guarantee the stability of the system in infinite time. In addition, references [20–28] designed the finite-time controllers for deterministic systems. By applying Lemma 5, the proposed control strategy can guarantee the finite-time stability of stochastic nonlinear systems. Therefore, the finite-time investigation in this article is more interesting than the results in [1–28].
- In the studies of finite-time control for deterministic systems [20–27] and stochastic systems [29], the state variables of system need to be known by the designer. In this paper, the unmeasurable state variables make the control strategies in [20–27,29] unavailable. Therefore, this paper forms a novel adaptive output-feedback control strategy by establishing a state observer.
- The proposed control scheme can assure that the state variables are located in a small neighbourhood of the origin in finite-time and that all signals are bounded. As we know, there are no other finite-time control schemes to handle such stochastic nonlinear systems. In addition, the simulation research is given to illustrate the effectiveness of the proposed control scheme.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Preliminaries

This section provides several definitions of stochastic nonlinear systems for the convenience of subsequent stability analyse.

Consider the stochastic nonlinear system as follows:

$$dx = f(x)dt + g(x)dw, \tag{1}$$

where $x \in R^n$ denotes the state variable, w denotes the r-dimension Brownian motion defined on $(\Xi, F, \{F_t\}_{t\geq 0}, P)$ with Ξ, F, $\{F_t\}_{t\geq 0}$ and P represent sample space, $\sigma-$field, filtration, and probability measure respectively. $f : R^n \rightarrow R^n$ and $g : R^n \rightarrow R^{n\times r}$ are continuous and assumed to be Borel measurable about x with $f(0) = g(0) = 0$.

Definition 1: Define a differential operator of $V(x,t) \in C^2$ as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\{g^T \frac{\partial^2 V}{\partial x^2} g\}, \tag{2}$$

where Tr denotes a matrix trace.

Definition 2: If there exists a parameter $\varepsilon > 0$ and the settling time $T(\varepsilon, z_0) < \infty$, which make $E(|z(t)|^2) < \varepsilon$ for $\forall t > t_0 + T$, then the stochastic nonlinear system (1) is practical finite-time stable in mean square for $\forall z(t_0) = z_0$.

Lemma 1 [32]**:** For $z_\iota \in R, \iota = 1, \ldots, o, \sigma \in (0,1)$, we have:

$$\Big(\sum_{\iota=1}^{o} |z_\iota|\Big)^\sigma \leq \sum_{\iota=1}^{o} |z_\iota|^\sigma \leq o^{1-\sigma} \Big(\sum_{\iota=1}^{o} |z_\iota|\Big)^\sigma. \tag{3}$$

Lemma 2 [28]**:** If $\dot{\hat{\eta}}(t) = -\nu\hat{\eta}(t) + \upsilon\psi(t)$, then from $\hat{\eta}(t_0) \geq 0$ for $\forall t \geq t_0$, we have $\hat{\eta}(t) \geq 0$ with $\nu > 0, \upsilon > 0$ and $\psi(t) > 0$.

Lemma 3 [33]**:** For $\forall \aleph \in R, \Re \in R$, $s > 0, v > 0, \mu > 0$, we have:

$$|\aleph|^s|\Re|^v \leq \frac{s}{s+v}\mu|\aleph|^{s+v} + \frac{v}{s+v}\mu^{\frac{-s}{v}}|\Re|^{s+v}. \tag{4}$$

Lemma 4 [34]**:** For $\forall(x,y) \in R^2$, the following inequality holds:

$$xy \leq \frac{q^\kappa}{\kappa}|x|^\kappa + \frac{1}{\varsigma q^\varsigma}|y|^\varsigma \tag{5}$$

where $q > 0, \kappa > 1, \varsigma > 1$, and $(\kappa - 1)(\varsigma - 1) = 1$.

Lemma 5 [29]**:** If there exists a function $\rho(x(t)) \in C^2$, three parameters $c, \sigma, d \in (0,1)$, κ_∞ functions τ_1 ,τ_2 for the system $\dot{x}(t) = f(x(t), u)$, such that

$$\begin{cases} \tau_1(\|x(t)\|) \leq \rho(x(t)) \leq \tau_2(\|x(t)\|), \\ \rho(x(t)) - \rho(x(s)) \leq -c\displaystyle\int_s^t \rho^\sigma(x(\vartheta))d\vartheta \\ \qquad\qquad + d(t-s), \forall 0 \leq s \leq t. \end{cases} \tag{6}$$

Then, we can find two parameters $\Delta > 0$ and $\varepsilon > 0$, which meet $\|x(t)\| < \varepsilon$ for $\forall t \geq \Delta$.

2.2. Problem formulation

Think about the stochastic non-strict-feedback nonlinear system as follows:

$$\begin{cases} dx_i = (x_{i+1} + f_i(x))dt + g_i(y)dw, \quad 1 \leq i \leq n-1, \\ dx_n = (u + f_n(x))dt + g_n(y)dw, \\ y = x_1, \end{cases} \tag{7}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ represents the state variable; $u \in R$ and $y \in R$ represent the system input and output respectively; f_i and $g_i (i = 1, 2, \ldots, n)$ represent the unknown nonlinear functions; w is defined by (1).

Remark 1: It can be known from the system (7) that the functions $f_i(\cdot)$ and $g_i(\cdot)$ include the entire state variables. Therefore, the system (7) is in non-strict-feedback form, which is different from the stochastic pure-feedback nonlinear system [9] and the stochastic strict-feedback nonlinear systems [7,17,18,29]. Many physical procedures, such as ball-beam system [30] and hyperchaotic inductor-capacitor oscillation circuit system [31], are in non-strict-feedback form. And in many practical systems, random interference is unavoidable. Hence, many controlled actual systems can be represented as the system (7).

Remark 2: It should be pointed that the existing control methods [1–10,20–27,29] are based on the assumption that the state variables of the system are measurable. Due to the state variables $x_2, \ldots, x_n$ are unmeasurable in (7), the control schemes in [1–10,20–27,29] are unavailable. In addition, the finite-time design scheme [28] is not suitable for the control design of the stochastic nonlinear system with unmeasurable states. Unlike the existing control investigations, the purpose of this article is to develop an observer-based adaptive finite-time control scheme for the stochastic non-strict-feedback nonlinear system (7).

Assumption 1 [19]**:** For the function $g_i(y)$, there exists another function $\bar{g}_i(y)$, such that

$$g_i(y) = y\bar{g}_i(y). \tag{8}$$

Remark 3: The unknown nonlinear function $\bar{g}_i(y)$ will be concentrated in the appropriate unknown function by Assumption 1, which will be compensated by neural networks in this article. This assumption facilitates the subsequent Lyapunov stability analysis.

2.3. Neural networks

In the subsequent control design procedure, radial basis function (RBF) neural networks (NNs) will be employed to handle unknown nonlinear functions. As shown in [27], if the number of nodes ℓ is large enough, then the RBF NN $\xi^{*T}\Phi(x)$ can approach continuous function $f(x)$ on the compact set $\Xi \subset R^p$ with $\forall\varepsilon > 0$ as follows:

$$f(x) = \xi^{*T}\Phi(x) + \delta(x), \forall x \in \Xi \subset R^p, \tag{9}$$

where $\delta(x)$ represents the approximation error, and it meets $|\delta(x)| \leq \varepsilon$. $\xi^* = [\xi_1, \xi_2, ..., \xi_\ell]^T \in R^\ell$ represents the ideal constant weight vector and defined as $\xi^* := \arg\min_{\xi\in R^\ell}\{\sup_{x\in\Xi}|f(x) - \xi^T\Phi(x)|\}$ with $\xi \in R^\ell$ is a weight vector. $\Phi(x) = [\phi_1(x), \phi_2(x), ..., \phi_\ell]^T$ represents the basis function vector, and $\phi_i(x)$ uses the structure of Gaussian function and it can be formulated as follows:

$$\phi_i(x) = \exp[-\frac{(x-r_i)^T(x-r_i)}{\iota_i^2}], \quad i = 1, 2, ..., \ell, \tag{10}$$

where ι_i represents the width of the $\phi_i(x)$ and $r_i = [r_{i1}, r_{i2}, ..., r_{ip}]^T$ denotes the center of the receptive field.

Lemma 6 [27]**:** Let $\bar{x}_p = [x_1, \cdots, x_p]^T$, and $\Phi(\bar{x}_p) = [\phi_1(\bar{x}_p), \cdots, \phi_\ell(\bar{x}_p)]^T$ is the basis function vector of the RBF NN. For $\forall p \geq k$, the following relationship can be obtained:

$$||\Phi(\bar{x}_p)||^2 \leq ||\Phi(\bar{x}_k)||^2. \tag{11}$$

Remark 4: Similar to the finite-time research [27], lemma 6 provides an effective feature of RBF NN. By applying this lemma in the subsequent stability analysis, the adaptive backstepping design approach can be easily extended to the non-strict-feedback system (7).

3. OUTPUT-FEEDBACK CONTROLLER DESIGN BASED ON FINITE-TIME

3.1. The design of state observer

In many practical applications, the state variables of the system are not available for measurement. Therefore, it is essential to design a control strategy for stochastic nonlinear systems with unmeasurable state variables. In this article, the following state observer is designed to evaluate the unmeasurable state variables:

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} - l_i\hat{x}_1, \ \ 1 \le i \le n-1, \\ \dot{\hat{x}}_n = u - l_n\hat{x}_1, \end{cases} \tag{12}$$

where $\hat{x}_i$ denotes the estimate of x_i, l_i denotes the design parameter vector.

Define $e = x - \hat{x}$ is the observer error with $e = [e_1, e_2, \ldots, e_n]^T$, $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$. Combining the stochastic non-strict-feedback nonlinear system (7) and the state observer (12), the following relationship can be obtained:

$$de = (Ae + F(x))dt + g(y)dw, \tag{13}$$

where $F(x) = [f_1(x) + l_1x_1, f_2(x) + l_2x_1, \ldots, f_n(x) + l_nx_1]^T$, $g(y) = [g_1(y), g_2(y), \ldots, g_n(y)]^T$, and A denotes the strict Hurwitz matrix as follows:

$$A = \begin{bmatrix} -l_1 & & \\ \vdots & I_{n-1} & \\ -l_n & \ldots & 0 \end{bmatrix}.$$

For a given parameter $\omega > 0$, there exists a matrix $\Gamma > 0$, the following relationship can be obtained:

$$A^T\Gamma + \Gamma A = -\omega I. \tag{14}$$

From (7), (12), (13) and (14), the interconnected systems are obtained as follows:

$$\begin{cases} de = (Ae + F(x))dt + g(y)dw, \\ dy = dx_1 = (x_2 + f_1(x))dt + g_1(y)dw \\ \quad = (e + \hat{x}_2 + f_1(x))dt + g_1(y)dw, \\ \dot{\hat{x}}_i = \hat{x}_{i+1} - l_i\hat{x}_1, \\ \dot{\hat{x}}_n = u - l_n\hat{x}_1. \end{cases} \tag{15}$$

Remark 5: We designed a state observer (12) to estimate the unmeasurable state variables of the system. The results of observer-based adaptive control [11–19] can only ensure the infinite-time stability of systems. The observer-based finite-time control method [28] is only suitable for deterministic systems. So far, there has not been any results to be reported on observer-based finite-time adaptive neural control for stochastic nonlinear systems with non-strict-feedback structure.

3.2. Finite-time controller design

Define $\hat{\eta}$ is the estimate of η, and $\eta = \max\{l\|\xi_0^*\|^2, \|\xi_i^*\|^2,\ i = 1, \ldots, n\}$, l denotes the number of the neural network nodes and ξ_i^* denotes the ideal weight vector, $\tilde{\eta} = \eta - \hat{\eta}$. Then, we define the following coordinate transformation:

$$z_1 = y, \ \ z_i = \hat{x}_i - \alpha_{i-1}, \ \ i = 2, \ldots, n, \tag{16}$$

where α_i denotes the virtual controller that

$$\alpha_i = -k_i z_i^{4\sigma-3} - \frac{1}{2a_i^2} z_i^3 \hat{\eta} \Phi_i^T(X_i)\Phi_i(X_i). \tag{17}$$

In the representation above, $\Phi_i(X_i)$ denotes the basis function vector with $X_i = (\hat{x}_1, \ldots, \hat{x}_i, \hat{\eta}, y)^T$, $k_i > 0, a_i > 0$ and $\sigma = \frac{2\flat-1}{2\flat+1}$ ($\flat > 2, \flat \in N$) are design parameters, and the adaptive law is obtained as follows:

$$\dot{\hat{\eta}} = \sum_{i=1}^{n} \frac{q}{2a_i^2} z_i^6 \Phi_i^T(X_i)\Phi_i(X_i) - \gamma\hat{\eta}, \hat{\eta}(t_0) \ge 0. \tag{18}$$

where $q > 0, \gamma > 0$ denote the design parameters.

The controller can be expressed as follows:

$$u = -k_n z_n^{4\sigma-3} - \frac{1}{2a_n^2} z_n^3 \hat{\eta} \Phi_n^T(X_n)\Phi_n(X_n), \tag{19}$$

where $X_n = (\hat{x}_1, \ldots, \hat{x}_n, \hat{\eta}, y)^T$, $k_n > 0$ denotes the design parameter.

Theorem 1: Consider the stochastic non-strict-feedback nonlinear system (7). Suppose that Assumption 1 is valid, the virtual controller (17), the parameter adaptive law (18), and the controller (19) are selected. Then, we can conclude that the closed-loop system is actually finite-time stable in mean square.

Proof: For the stochastic non-strict-feedback nonlinear system (7), by employing It$\hat{o}$ formula, we have:

$$dz_1 = (x_2 + f_1(x))dt + g_1(y)dw, \tag{20}$$

$$dz_i = (\hat{x}_{i+1} - l_i\hat{x}_1 - L\alpha_{i-1})dt - \frac{\partial \alpha_{i-1}}{\partial x_1} g_1(y)dw, \tag{21}$$

$$dz_n = (u - l_n\hat{x}_1 - L\alpha_{n-1})dt - \frac{\partial \alpha_{n-1}}{\partial x_1} g_1(y)dw, \tag{22}$$

where

$$\begin{aligned} L\alpha_{i-1} = & \frac{\partial \alpha_{i-1}}{\partial x_1}(x_2 + f_1(x)) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\eta}_j} \dot{\hat{\eta}}_j \\ & + \frac{1}{2} \frac{\partial^2 \alpha_{i-1}}{\partial x_1^2} g_1^T(y) g_1(y) \\ & + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j} \left(\hat{x}_{j+1} - l_j\hat{x}_1 \right). \end{aligned} \tag{23}$$

Consider the following stochastic Lyapunov function:

$$V = e^T\Gamma e + \sum_{i=1}^{n} \frac{1}{4} z_i^4 + \frac{1}{2q}\tilde{\eta}^2, \tag{24}$$

where $\tilde{\eta} = (\tilde{\eta}_1, \ldots, \tilde{\eta}_n)^T$.

By employing (7), (15) and (16), the time derivative of V can be obtained as follows:

$$\begin{aligned}
LV =& 2e^T\Gamma(Ae+F(x)) + Tr[g^T(y)\Gamma g(y)] \\
&+ z_1^3(x_2+f_1(x)) + \frac{3}{2}z_1^2 g_1^T(y)g_1(y) \\
&+ \sum_{i=2}^{n-1} z_i^3\Big[\hat{x}_{i+1} - l_i\hat{x}_1 - \frac{\partial\alpha_{i-1}}{\partial x_1}(x_2+f_1(x)) \\
&- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}_j}\dot{\hat{\eta}}_j - \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}g_1^T(y)g_1(y) \\
&- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} - l_j\hat{x}_1)\Big] \\
&+ \frac{3}{2}\sum_{i=2}^{n-1} z_i^2\|\frac{\partial\alpha_{i-1}}{\partial x_1}g_1(y)\|^2 \\
&+ z_n^3\Big[u - l_n\hat{x}_1 - \frac{\partial\alpha_{n-1}}{\partial x_1}(x_2+f_1(x)) \\
&- \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\eta}_j}\dot{\hat{\eta}}_j - \frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}g_1^T(y)g_1(y) \\
&- \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} - l_j\hat{x}_1)\Big] + \frac{3}{2}z_n^2\|\frac{\partial\alpha_{n-1}}{\partial x_1}g_1(y)\|^2 \\
&- \frac{\tilde{\eta}\dot{\hat{\eta}}}{q} \\
=& e^T(A^T\Gamma + \Gamma A)e + 2e^T\Gamma F(x) + Tr[g^T(y)\Gamma g(y)] \\
&+ z_1^3\Big(e_2 + z_2 + \alpha_1 + f_1(x)\Big) + \frac{3}{2}z_1^2 g_1^T(y)g_1(y) \\
&+ \sum_{i=2}^{n-1} z_i^3\Big[z_{i+1} + \alpha_i - l_i\hat{x}_1 \\
&- \frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2 + e_2 + f_1(x)) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}_j}\dot{\hat{\eta}}_j \\
&- \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}g_1^T(y)g_1(y) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} - l_j\hat{x}_1)\Big] \\
&+ \frac{3}{2}\sum_{i=2}^{n-1} z_i^2\|\frac{\partial\alpha_{i-1}}{\partial x_1}g_1(y)\|^2 + z_n^3\Big[u - l_n\hat{x}_1 \\
&- \frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2 + e_2 + f_1(x)) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\eta}_j}\dot{\hat{\eta}}_j \\
&- \frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}g_1^T(y)g_1(y) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} - l_j\hat{x}_1)\Big] \\
&+ \frac{3}{2}z_n^2\|\frac{\partial\alpha_{n-1}}{\partial x_1}g_1(y)\|^2 - \frac{\tilde{\eta}\dot{\hat{\eta}}}{q}. \qquad (25)
\end{aligned}$$

Define $F(x) = (F_1(x),\ F_2(x),\ \ldots,\ F_n(x))^T$. Since the function $F_i(x) = f_i(x) + l_i x_1$ ($i = 1, 2, \ldots, n$) is unknown, for $\forall\varepsilon_{i0} > 0$, there exists a RBF NN $\xi_{i0}^{*T}\Phi_{i0}(X_0)$ such that

$$F_i(x) = \xi_{i0}^{*T}\Phi_{i0}(X_0) + \delta_{i0}(X_0),\ \ |\delta_{i0}(X_0)| \le \varepsilon_{i0}, \quad (26)$$

where $\xi_{i0}^{*T} \in R^l$, $\Phi_{i0} = \exp[-\frac{(X_0-\iota_{0m})^T(X_0-\iota_{0m})}{\iota^2}]$, $m = 1, 2, ...,$ l, $X_0 = x \in \Omega_{X_0}$, Ω_{X_0} is a pre-defined compact set, and the state trajectory can be propagated through this compact set, $\iota_{0m} = [\iota_{0m1}, \cdots, \iota_{0mn}]^T$ represents the center of the receptive field, and $\iota > 0$ denotes the width of the Gaussian function, $\delta_{i0}(X_0)$ represents the approximation error.

In view of (26), we have:

$$F(x) = \xi_0^{*T}\Phi_0(X_0) + \delta_0(X_0),\ \|\delta_0(X_0)\| \le \varepsilon_0, \quad (27)$$

where

$$\begin{aligned}
&\xi_0^* = (\xi_{10}^*, \ldots, \xi_{n0}^*), \\
&\delta_0(X_0) = (\delta_{10}(X_0), \ldots, \delta_{n0}(X_0))^T, \\
&\varepsilon_0 = \sqrt{\varepsilon_{10}^2 + \ldots + \varepsilon_{n0}^2}. \qquad (28)
\end{aligned}$$

With the definition of Φ_{i0} and η, we can get $\Phi_0^T\Phi_0 = \sum_{i=1}^{l}\Phi_{i0}^2 \le l$ and $\|\xi_0^*\|^2 \le \eta/l$ respectively.

Therefore, $2e^T\Gamma F$ is expressed as:

$$\begin{aligned}
2e^T\Gamma F(x) &= 2e^T\Gamma[\xi_0^{*T}\Phi_0(X_0) + \delta_0(X_0)] \\
&\le 2\|e\|^2 + \|\Gamma\|^2\eta + \|\Gamma\|^2\varepsilon_0^2. \qquad (29)
\end{aligned}$$

In view of Lemma 4, the following inequalities hold:

$$z_1^3 e_2 \le \frac{3\sqrt[3]{4}}{4}z_1^4 + \frac{1}{4}\|e\|^4 \le \frac{3}{2}z_1^4 + \frac{1}{4}\|e\|^4, \quad (30)$$

$$z_i^3 z_{i+1} \le \frac{3}{4}z_i^4 + \frac{1}{4}z_{i+1}^4, \quad (31)$$

$$Tr[g^T(y)\Gamma g(y)] \le \frac{1}{2}\|\Gamma\|^2 + \frac{1}{2}y^4\sum_{i=1}^{n}\|\bar{g}_i(y)\|^4, \quad (32)$$

$$-z_i^3\frac{\partial\alpha_{i-1}}{\partial x_1}e_2 \le \|e\|^2 + \frac{1}{4}(\frac{\partial\alpha_{i-1}}{\partial x_1})^2 z_i^6, \quad (33)$$

$$\frac{3}{2}z_i^2\|\frac{\partial\alpha_{i-1}}{\partial x_1}g_1(y)\|^2 \le \frac{3}{4}b_i^{-2}z_i^4\|\frac{\partial\alpha_{i-1}}{\partial x_1}g_1(y)\|^4 + \frac{3}{4}b_i^2, \quad (34)$$

$$\frac{3}{2}z_1^2 g_1^T(y)g_1(y) = \frac{3}{2}z_1^4\|\bar{g}_1(y)\|^2. \quad (35)$$

Substituting (14) and (29)-(35) into (25). It follows that

$$\begin{aligned}
LV \le& -e^T\omega e + 2\|e\|^2 + \|\Gamma\|^2\eta + \|\Gamma\|^2\varepsilon_0^2 + \frac{1}{2}\|\Gamma\|^2 \\
&+ \frac{1}{2}y^4\sum_{i=1}^{n}\|\bar{g}_i(y)\|^4 + \frac{3}{4}z_1^4 + \frac{1}{4}\|e\|^2 + \frac{3}{4}z_1^4 \\
&+ \frac{1}{4}z_2^4 + z_1^3\Big(\alpha_1 + f_1(x)\Big) + \frac{3}{2}z_1^4\|\bar{g}_1(y)\|^2 \\
&+ \sum_{i=2}^{n-1}(\frac{3}{4}z_i^4 + \frac{1}{4}z_{i+1}^4) + \sum_{i=2}^{n-1} z_i^3\Big[\alpha_i - l_i\hat{x}_1 \\
&- \frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2 + f_1(x)) - \frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}} \\
&- \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}g_1^T(y)g_1(y) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} - l_j\hat{x}_1)\Big]
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=2}^{n}\Big[\|e\|^2+\frac{1}{4}(\frac{\partial\alpha_{i-1}}{\partial x_1})^2 z_i^6\Big]-\frac{\tilde{\eta}\dot{\hat{\eta}}}{q}\\
&+\sum_{i=2}^{n}\Big[\frac{3}{4}b_i^{-2}z_i^4\|\frac{\partial\alpha_{i-1}}{\partial x_1}g_1(y)\|^4+\frac{3}{4}b_i^2\Big]\\
&+z_n^3\Big[u-l_n\hat{x}_1-\frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2+f_1(x))\\
&-\frac{\partial\alpha_{n-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}-\frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}g_1^T(y)g_1(y)\\
&-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1}-l_j\hat{x}_1)\Big]\\
=&-\omega\|e\|^2+\sigma_0+\frac{5}{4}\|e\|^2+z_1^3\Big(\alpha_1+\bar{f}_1(x)\Big)\\
&+\sum_{i=2}^{n-1}z_i^3\Big[\bar{f}_i+\alpha_i+\psi_i(X_i)-\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}\Big]\\
&+\sum_{i=1}^{n}\|e\|^2-\sum_{i=1}^{n}\frac{3}{4}z_i^4+\sum_{i=2}^{n}\frac{3}{4}b_i^2\\
&+z_n^3\Big[\bar{f}_n+u-\frac{\partial\alpha_{n-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}+\psi_n(X_n)\Big]-\frac{\tilde{\eta}\dot{\hat{\eta}}}{q}\\
=&-(\omega-(n+\frac{5}{4}))\|e\|^2+\sigma_0\\
&+z_1^3\Big(\alpha_1+\bar{f}_1(x)-\frac{3}{4}z_1\Big)\\
&+\sum_{i=2}^{n-1}z_i^3\Big[\bar{f}_i+\alpha_i+\psi_i(X_i)-\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}-\frac{3}{4}z_i\Big]\\
&+z_n^3\Big[\bar{f}_n+u-\frac{\partial\alpha_{n-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}+\psi_n(X_n)-\frac{3}{4}z_n\Big]\\
&+\sum_{i=2}^{n}\frac{3}{4}b_i^2-\frac{\tilde{\eta}\dot{\hat{\eta}}}{q},
\end{aligned}
\tag{36}
$$

where

$$
\sigma_0=\|\Gamma\|^2\eta+\|\Gamma\|^2\varepsilon_0^2+\frac{1}{2}\|\Gamma\|^2, \tag{37}
$$

$$
\bar{f}_1=f_1(x)+\frac{1}{2}z_1\sum_{i=1}^{n}\|\bar{g}_i(y)\|^4+\frac{3}{2}z_1\|\bar{g}_1(y)\|^2+\frac{9}{4}z_1, \tag{38}
$$

$$
\begin{aligned}
\bar{f}_i=&-l_i\hat{x}_1-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1}-l_j\hat{x}_1)+\frac{7}{4}z_i-\psi_i(X_i)\\
&-\frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2+f_1(x))-\frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}g_1^T(y)g_1(y)\\
&+\frac{1}{4}\Big(\frac{\partial\alpha_{i-1}}{\partial x_1}\Big)^2z_i^3+\frac{3}{4}b_i^{-2}z_i\|\frac{\partial\alpha_{i-1}}{\partial x_1}g_1(y)\|^4,\\
&(2\le i\le n-1),
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
\bar{f}_n=&-l_n\hat{x}_1-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1}-l_j\hat{x}_1)+z_n-\psi_n(X_n)\\
&-\frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2+f_1(x))-\frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}g_1^T(y)g_1(y)\\
&+\frac{1}{4}\Big(\frac{\partial\alpha_{n-1}}{\partial x_1}\Big)^2z_n^3+\frac{3}{4}b_n^{-2}z_n\|\frac{\partial\alpha_{n-1}}{\partial x_1}g_1(y)\|^4,
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
\psi_i(X_i)=&-c_0\hat{\eta}\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}-\sum_{m=2}^{i}z_i\frac{r}{2a_m^2}|z_m\frac{\partial\alpha_{m-1}}{\partial\hat{\eta}_m}|\\
&+\sum_{m=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\frac{q}{2a_m^2}z_m^2\xi_m^T\xi_m,\ 2\le i\le n.
\end{aligned}
\tag{41}
$$

Note $\bar{f}_1$ contains the unknown function $f_1(x)$, the RBF NN $\xi_1^{*T}\Phi_1(\zeta_1)$ is used to approximate $\bar{f}_1$ with $\zeta_1=(x_1,x_2,\ldots,x_n,y)^T$. So, we have

$$
\bar{f}_1=\xi_1^{*T}\Phi_1(\zeta_1)+\delta_1(\zeta_1), \tag{42}
$$

where $\delta_1(\zeta_1)$ represents the approximation error.

Remark 6: It is noteworthy that $\bar{f}_1$ includes all the state variables $x_1,x_2,\ldots,x_n$. Due to $x_2,\ldots,x_n$ are immeasurable, $\xi_1^{*T}\Phi_1(\zeta_1)$ cannot be directly employed to the control process. Therefore, Lemma 6 and (12) will be applied to handle this difficulty in the subsequent produce.

The approximated error $\delta_1(\zeta_1)$ satisfies $|\delta_1(\zeta_1)|\le\varepsilon_1$. Then, for $\forall\varepsilon_1>0$ and $\forall a_1>0$, according to lemma 4 and Lemma 6, we obtain

$$
\begin{aligned}
z_1^3\bar{f}_1&=z_1^3(\xi_1^{*T}\Phi_1(\zeta_1)+\delta_1(\zeta_1))\\
&\le|z_1^3|(||\xi_1^*||||\Phi_1(X_1)||+\varepsilon_1)\\
&\le\frac{1}{2a_1^2}z_1^6\eta\Phi_1^T(X_1)\Phi_1(X_1)+\frac{1}{2}a_1+\frac{3}{4}z_1^4+\frac{1}{4}\varepsilon_1^4,
\end{aligned}
\tag{43}
$$

where $X_1=y$, $\eta=\max\{\|\xi_i^*\|^2, i=1,2,\ldots,n\}$.

Similarly, by applying the RBF NN $\xi_i^{*T}\Phi_i(\zeta_i)$ to approximate the unknown nonlinear function $\bar{f}_i$ and adopting Young's inequality, we have

$$
\begin{aligned}
z_i^3\bar{f}_i=&z_i^3(\xi_i^{*T}\Phi_i(\zeta_i)+\delta_i(\zeta_i))\\
\le&|z_i|^3(\|\xi_i^*\|\|\Phi_i(X_i)\|+\varepsilon_i)\\
\le&\frac{1}{2a_i^2}z_i^6\eta\Phi_i^T(X_i)\Phi_i(X_i)+\frac{1}{2}a_i^2+\frac{3}{4}z_i^4+\frac{1}{4}\varepsilon_i^4,\\
&|\delta_i(\zeta_i)|\le\varepsilon_i,
\end{aligned}
\tag{44}
$$

where $2\le i\le n$, $\eta=\max\{\|\xi_i^*\|^2, i=1,2,\ldots,n\}$, $X_i=(\hat{x}_1,\ldots,\hat{x}_i,\hat{\eta},y)^T\in\Omega_{X_i}\subset R^{i+2}$, and $a_i>0$ is a design parameter.

Substituting (17)-(19) and (43)-(44) into (36). It follows that

$$
\begin{aligned}
LV\le&-(\omega-(n+\frac{5}{4}))\|e\|^2-\sum_{i=1}^{n}k_iz_i^{4\sigma}+\sigma_0\\
&-\frac{1}{q}\tilde{\eta}\Big(\dot{\hat{\eta}}-\sum_{i=1}^{n}\frac{q}{2a_i^2}z_i^6\Phi_i^T\Phi_i\Big)+\sum_{i=1}^{n}(\frac{1}{2}a_i^2+\frac{1}{4}\varepsilon_i^4)\\
&+\sum_{i=2}^{n}z_i^3\Big(\psi_i(X_i)-\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}\Big)+\sum_{i=2}^{n}\frac{3}{4}b_i^2\\
\le&-(\omega-(n+\frac{5}{4}))\|e\|^2-\sum_{i=1}^{n}k_iz_i^{4\sigma}+\sigma_1+\frac{\gamma}{q}\tilde{\eta}\hat{\eta}
\end{aligned}
$$

$$+\sum_{i=2}^{n} z_i^3\Big(\psi_i(X_i)-\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}\Big), \tag{45}$$

where $\sigma_1=\sigma_0+\sum_{i=1}^{n}(\frac{1}{2}a_i^2+\frac{1}{4}\varepsilon_i^4)+\sum_{i=2}^{n}\frac{3}{4}b_i^2$.

In view of the reference [3], we have

$$\sum_{i=2}^{n} z_i^3\Big(\psi_i(X_i)-\frac{\partial\alpha_{i-1}}{\partial\hat{\eta}}\dot{\hat{\eta}}\Big)\le 0. \tag{46}$$

In addition, by using Lemma 4, we have

$$\tilde{\eta}\hat{\eta}=\tilde{\eta}(\eta-\tilde{\eta})\le -\frac{1}{2}\tilde{\eta}^2+\frac{1}{2}\eta^2. \tag{47}$$

Then, (45) can be analyzed as

$$\begin{aligned}LV\le&-(\omega-(n+\frac{5}{4}))\|e\|^2-\sum_{i=1}^{n}k_iz_i^{4\sigma}-\frac{\gamma}{2q}\tilde{\eta}^2+\sigma_1\\&+\frac{\gamma}{2q}\eta^2\\ \le&-\tau e^T\Gamma e-\sum_{i=1}^{n}k_iz_i^{4\sigma}-\frac{\gamma}{2q}\tilde{\eta}^2+\sigma_1+\frac{\gamma}{2q}\eta^2.\end{aligned} \tag{48}$$

where $\tau=\frac{\omega-(n+\frac{5}{4})}{\lambda_{max}(\Gamma)}$.

Adopting Lemma 3, let $\aleph=e^T\Gamma e$, $\Re=1$, $s=\sigma$, $v=1-\sigma$, and $\mu=\frac{1}{\sigma}$, we obtained that

$$(e^T\Gamma e)^\sigma\le(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+e^T\Gamma e. \tag{49}$$

And then, let $\aleph=\frac{1}{2q}\tilde{\eta}^2$, $\Re=1$, and $s=\sigma$, $v=1-\sigma$, $\mu=\frac{1}{\sigma}$, we also obtained

$$\Big(\frac{1}{2q}\tilde{\eta}^2\Big)^\sigma\le(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+\frac{1}{2q}\tilde{\eta}^2. \tag{50}$$

With the results of (49)-(50), the inequality (48) is computed as

$$\begin{aligned}LV\le&-\tau(e^T\Gamma e)^\sigma-\sum_{i=1}^{n}k_iz_i^{4\sigma}-\gamma\Big(\frac{1}{2q}\tilde{\eta}^2\Big)^\sigma+\sigma_1\\&+\frac{\gamma}{2q}\eta^2+\tau(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+\gamma(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}\\ \le&-c(e^T\Gamma e)^\sigma-c\sum_{i=1}^{n}(\frac{1}{4}z_i^4)^\sigma-c\Big(\frac{1}{2q}\tilde{\eta}^2\Big)^\sigma+d,\end{aligned} \tag{51}$$

where $c=\min\{\tau,4^\sigma k_i,\gamma,i=1,2,\dots,n\}$ and $d=\sigma_1+\tau(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+\gamma(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}+\frac{\gamma}{2q}\eta^2$. By employing Lemma 1, (51) can be analyzed as

$$\begin{aligned}LV(Z_n(t))\le&-c(e^T\Gamma e)^\sigma-c\Big(\sum_{i=1}^{n}\frac{1}{4}z_i^4\Big)^\sigma\\&-c\Big(\frac{1}{2q}\tilde{\eta}^2\Big)^\sigma+d\\ \le&-cV^\sigma(Z_n(t))+d.\end{aligned} \tag{52}$$

Remark 7: Unlike the investigations in [1–28], Lemma 5 will be adopted to analyze the finite-time stability of stochastic nonlinear system (7), and the presented control scheme is based on the finite-time stability criterion "$LV\le -cV^\sigma+d$".

With Itô formula, for $0\le s<t$, we obtained that

$$\begin{aligned}EV(Z_n(t))&=EV(Z_n(s))+E\int_s^t LV(Z_n(s))ds\\&=EV(Z_n(s))+\int_s^t E[LV(Z_n(s))]ds.\end{aligned} \tag{53}$$

With (52) and Jessen's inquality, one has

$$\begin{aligned}E[LV(Z_n(s))]&\le -cE[V^\sigma(Z_n(s)]+d\\&\le -c[EV(Z_n(s))]^\sigma+d.\end{aligned} \tag{54}$$

According to the inequality (54), the equality (53) is computed as

$$\begin{aligned}EV(Z_n(t))\le&EV(Z_n(s))\\&+\int_s^t\{-c[EV(Z_n(s))]^\sigma+d\}ds.\end{aligned} \tag{55}$$

Then, we have

$$\begin{aligned}EV(Z_n(t))-EV(Z_n(s))\le&-c\int_s^t[EV(Z_n(s))]^\sigma ds\\&+d(t-s).\end{aligned} \tag{56}$$

Let $\rho(t)=EV(Z_n(t))$, from [29], for $\forall t\ge\Delta$, there is a settling time

$$\Delta=\frac{1}{(1-\sigma)\beta c}\Big[(EV(Z_n(0)))^{1-\sigma}-\Big(\frac{d}{(1-\beta)c}\Big)^{\frac{(1-\sigma)}{\sigma}}\Big], \tag{57}$$

which makes $EV(Z_n(t))\le\varepsilon$ with $\varepsilon=4(\frac{d}{(1-\beta)c})^{1/4\sigma}$.

From the definition of $V(Z_n(t))$, one has

$$E\Big(\sum_{j=1}^{n}z_j^4\Big)\le 4E[V(Z_n(t))]\le 4\varepsilon,\ \ t\ge\Delta. \tag{58}$$

From the nature of mathematical expectation, we have

$$\Big[E\big(z_j^2\big)\Big]^2\le E\Big(z_j^4\Big)\le E\Big(\sum_{j=1}^{n}z_j^4\Big)\le 4\varepsilon,\ \ t\ge\Delta. \tag{59}$$

Therefore

$$E(z_j^2)\le 2\sqrt{\varepsilon},\ \ t\ge\Delta. \tag{60}$$

Similarly, the following inequality is also obtained:

$$E(\eta_j^2)\le 2q_{\max}\varepsilon,\ t\ge\Delta, \tag{61}$$

where $q_{\max}=\max\{q_j,\ 1\le j\le n\}$.

From (60) and (61), we proved that the closed system is practical finite-time stable in mean square. □

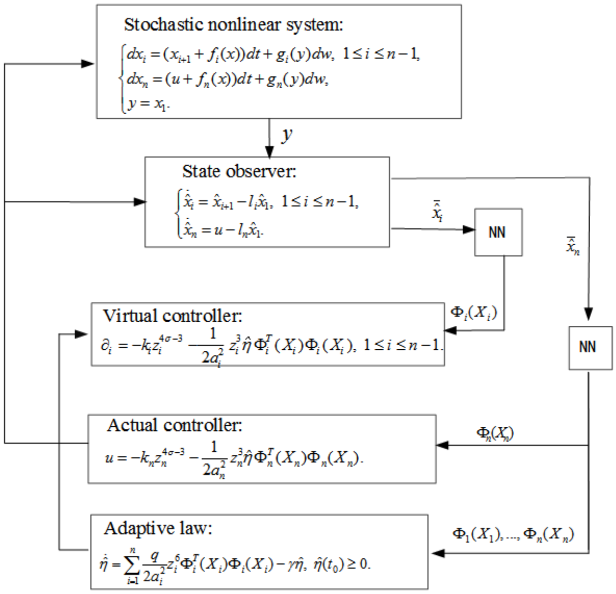

Fig. 1. Block diagram of control system.

The design process of the controller can be seen from the block diagram shown in Fig. 1.

Remark 8: Note that Δ depends on the initial value of $z(0)$. Under this condition, the system is said to be finite-time stable. If Δ can be independent of $z(0)$, the system is called fixed-time stable. The fixed-time stability of system may be a research direction in the future

4. SIMULATION EXAMPLE

In this part, we will show the significance of the presented control strategy by a numerical example. Consider the following stochastic nonlinear system in the non-strict-feedback form

$$\begin{cases} dx_1 = (x_2 + x_1x_2\sin(x_3) + \dfrac{x_1^2}{1+x_1^2})dt \\ \qquad +0.5x_1\cos(x_1)dw, \\ dx_2 = (x_3 - 10\sin(x_1) - x_2 + x_2x_3^2)dt \\ \qquad +0.1\sin(x_1^2)dw, \\ dx_3 = (u - x_2^2\sin(x_1) - 8x_3)dt + 0.1x_1\sin(2x_1)dw, \\ y = x_1, \end{cases} \tag{62}$$

where x_1, x_2 and x_3 represent the state variables (x_2 and x_3 are unmeasurable), u and y denote the system input and output respectively, and w is defined by (1). Due to the state variables of the stochastic nonlinear system (62) are unmeasurable, the existing finite-time control schemes for stochastic nonlinear system cannot be employed to the system (62).

To ensure the stability of the system (62) in finite-time, the state observer (12), the virtual control signal (17), the adaptive laws (18), and the actually controller (19) are applied to (62). The design parameters are chosen as $a_1 = 0.65$, $a_2 = 0.7$, $a_3 = 0.6$, $l_1 = 31$, $l_2 = 275$, $l_3 = 794$, $k_1 = 2$, $k_2 = 2$, $k_3 = 2.6$ $q = 6$, $\gamma = 2.5$, $\sigma = \frac{199}{201}$. Furthermore, the RBF NN $\xi_1^{*T}\Phi_1(\zeta_1)$ contains 7^3 nodes, the centers spaced evenly in the interval $[-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5]$ with widths being equal to 2. The RBF NN $\xi_2^{*T}\Phi_2(\zeta_2)$ includes 7^6 nodes with centers spaced evenly in the interval $[-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5]$ with widths being equal to 2. The RBF NN $\xi_3^{*T}\Phi_3(X_3)$ contains 7^7 nodes with centers spaced evenly in the interval $[-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5]$ with widths being equal to 2. The initial conditions are given as $[x_1(0), x_2(0), x_3(0), \hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0), \hat{\eta}(0)]^T = [-0.3, -0.2, 0.1, 0.1, 0.1, 0, 0.1, 0, 0]^T$.

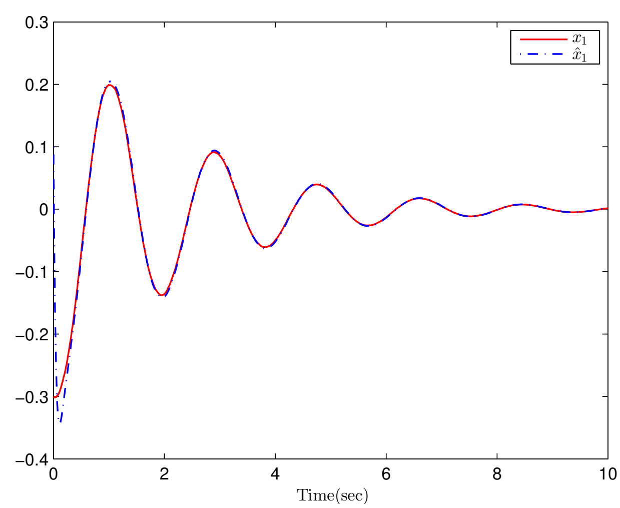

Fig. 2. Responses of x_1 and $\hat{x}_1$.

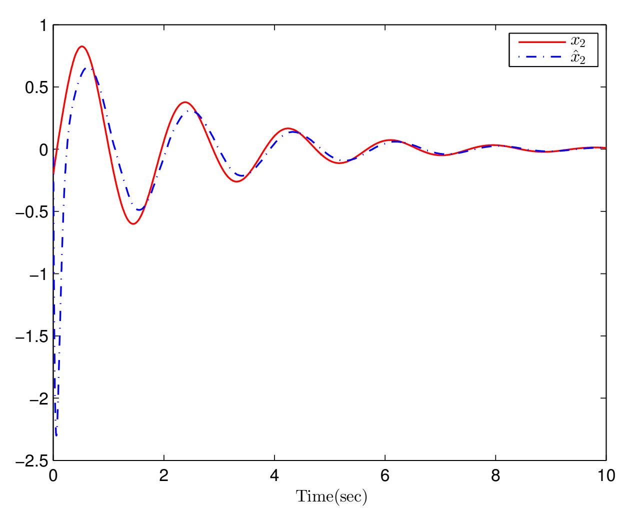

Fig. 3. Responses of x_2 and $\hat{x}_2$.

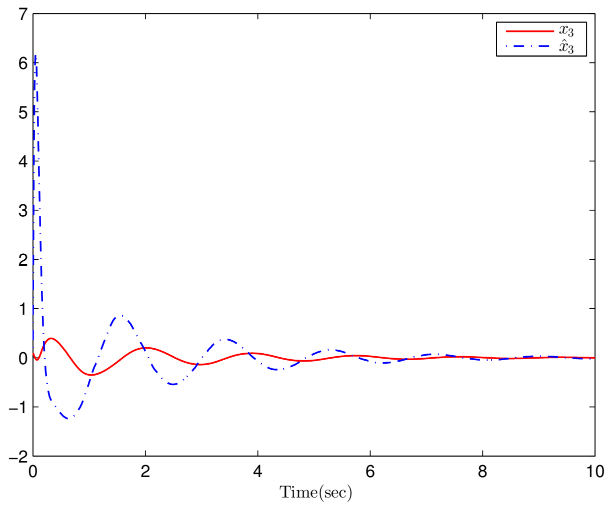

Fig. 4. Responses of x_3 and $\hat{x}_3$.

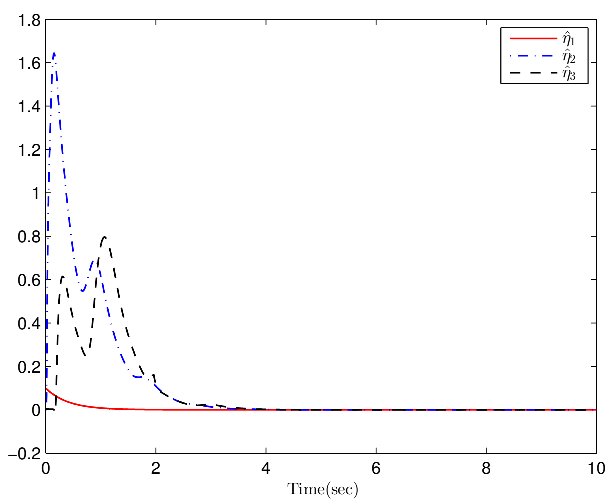

Fig. 5. Responses of $\hat{\eta}_1$, $\hat{\eta}_2$ and $\hat{\eta}_3$.

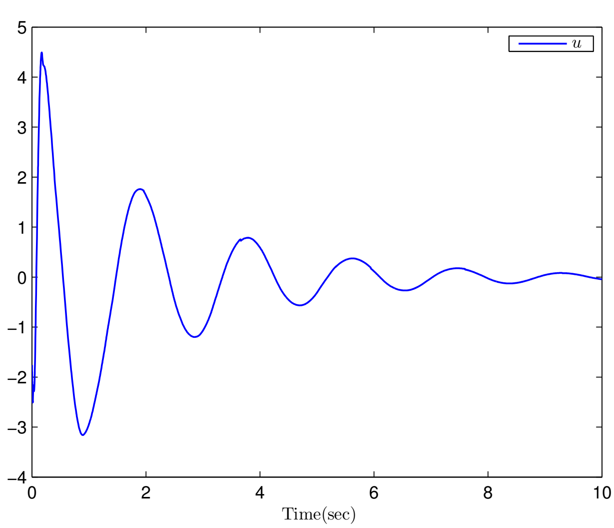

Fig. 6. Response of u.

The simulation results are demonstrated by Figs.2 to 6. The responses of x_1 and $\hat{x}_1$, x_2 and $\hat{x}_2$, x_3 and $\hat{x}_3$ are shown in Figs. 2-4, respectively. It can be seen that the designed observer can estimate the state variables of the system effectively. The adaptive parameter curves and the control input curve under the action of the finite-time adaptive controller are plotted in Figs. 5 and 6, respectively. From Figs. 2-6, we can obtain that although the state variables of the system (62) are unmeasurable, the boundedness of all the signals can be assured in finite-time under the presented control strategy.

5. CONCLUSION

In this paper, the observer-based finite-time neural control scheme has been developed for a category of stochastic nonlinear systems with non-strict-feedback structure. In the process of control design, by introducing RBF NNs, the unknown nonlinear functions have been approximated, and the backstepping design method has been extended from stochastic strict-feedback nonlinear systems to non-strict-feedback ones. With the designed observer, the unmeasurable state variables have been approximated usefully, which lays the foundation for the design of the control program under unmeasurable state variables. In addition, the presented control strategy ensures that all the signals in the closed-loop systems are bounded. By adopting Lemma 5, the finite-time stability of the stochastic nonlinear system has been proved. Finally, simulation results have been rendered to demonstrate the validity and effectiveness of the presented control scheme in this article.

Time-delays and quantization often occur in many stochastic nonlinear systems, which may cause instability and degrade system performance. Therefore, how to achieve fixed-time stability and take into account the time-delay and quantization phenomena on the basis of this paper. This may be considered a possible research topic in the future.

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