International Journal of Control, Automation and Systems 19(1) (2021) 20-28
http://dx.doi.org/10.1007/s12555-019-0941-4
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Fault-tolerant Control for Linear System Under Sensor Saturation Constraint

Jun-Wei Zhu, Qiao-Qian Zhou, Jian-Ming Xu*, and Jian-Wei Dong

Abstract: An observer-based fault-tolerant control method is proposed for a linear system with sensor saturation constraint. Considering the linear system with simultaneous actuator faults and sensor faults, the sensor saturation would bring the output measurement error of the system, which would result in the estimation performance degradation. Firstly, the intermediate estimator is modified to estimate the system states and fault signals at the simultaneous time, and the fault-tolerant controller is designed based on the estimation to compensate the effect of actuator faults effectively. Through Lyapunov stability analysis, the sufficient conditions are obtained to ensure the states of closed-loop system to be uniformly ultimately bounded. The effect of sensor saturation error can be suppressed by adjusting some specified parameters directly without introducing any performance index. Finally, the effectiveness and superiority of the proposed method are verified by a simulation example.

Keywords: Actuator faults, fault-tolerant control, intermediate estimator, sensor faults, sensor saturation.

1. INTRODUCTION

Since the 1940s, automatic control system technology has been widely used in practical engineering fields such as robotics, chemical engineering, aerospace, etc. Due to the complexity of production systems and the overload of work, some devices in the system such as sensors and actuators will have malfunctions inevitably. It should be noted that malfunctions will result in system control performance degradation, and even result in major economic losses and casualties. In 2019, an aircraft crash of Ethiopian airlines ET302 happened due to sensor faults. Such examples are not uncommon.

With the increasing demand for stable and reliable operational system, fault detection, and especially fault-tolerant control have received extensive attentions [1–8]. Fault-tolerant control methods can be divided into two categories, passive fault-tolerant control and active fault-tolerant control. The active fault-tolerant control based on various types of observers is an important branch of fault-tolerant control. It uses the observer to estimate the fault signals online and then designs the controller to compensate the faults. So far, the commonly used observers mainly include robust observer, sliding mode observer, adaptive observer, extended state observer and high gain observer. The feature of insensitivity to parameter changes facilitates the application of sliding mode observers, for example, in [9], the augmented sliding mode observer is used for estimating sensor faults, actuator faults and external disturbances simultaneously. However, sliding mode observer needs to get the information of fault and the upper bound of fault derivative, which is difficult to obtain in practical situations. Generally, adaptive observer has fast convergence speed. In [10, 11], adaptive observer-based fault-tolerant control problem of uncertain systems against actuator faults is researched. The advantages of high gain observer are not only can estimate the constant fault parameters, but also can be widely used in various types of nonlinear systems. However, the limitation of high gain observer-based methods is that the system matrix needs to satisfy the restriction of upper triangular structure.

At present, the commonly considered factors in fault-tolerant control generally include actuator faults, sensor faults, disturbances, parameter uncertainty and some problems under network uncertainty conditions, such as time delay, packet dropouts and quantization. In [12], a robust fault-tolerant H_∞ control problem of linear time-invariant systems with actuator faults and perturbation is considered. When it comes to uncertain systems, based on delay replacement, [10] proposes a novel adaptive decentralized fault-tolerant control method for uncertain nonlinear time-delay large scale systems. In addition, a robust adaptive

Manuscript received November 4, 2019; revised March 9, 2020; accepted March 12, 2020. Recommended by Editor Jessie (Ju H.) Park. This work was supported in part by the National Natural Science Foundation of China under Grant 61803334, in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LQ18F030012, in part by the State Scholarship Fund of China Scholarship Council under Scholarship 201908330040.

Jun-Wei Zhu, Qiao-Qian Zhou, Jian-Ming Xu, and Jian-Wei Dong are with the College of Information Engineering, Zhejiang University of Technology, Hangzhou, Zhejiang 310023, China (e-mails: {junweizhu1001, zhouqq, xujm, dongjw}@zjut.edu.cn).
* Corresponding author.

fault-tolerant control is studied for linear system with mismatched parameter uncertainties [11]. On the other hand, in [13], the H_∞ filtering problem for a new class of discrete time networked nonlinear systems with mixed random delays and packet dropouts is investigated. Different from [13], the fault-tolerant control of nonlinear systems with input quantization is solved in [14, 15].

In practical applications, due to physical limitations [16], safety restrictions [17], and aging [18], sensor saturation often occurs in measurement. It should be highlighted that sensor saturation will cause the output measurement error and lead to the estimation performance degradation. Under this limitation, some problems have been studied, such as state estimation problems, filtering problems and output feedback control problems. For example, the output feedback control problem is addressed for linear system [19,20] as well as time-varying nonlinear system [21] subject to sensor saturation. In addition, the problem of robust filtering for discrete-time linear systems subject to sensor saturation is considered and a generalized dynamic filter architecture is proposed [22]. Further, in [23], the H_∞ filtering problem of nonlinear discrete stochastic systems is studied. In [24], the distributed states are estimated by distributed state esimator. However, up to now, the problem of fault-tolerant control for fault-affected systems under sensor saturation has not been considered in the literature.

Motivated by the above reasons, this paper studies the fault-tolerant control problem of linear systems under sensor saturation. Since the sensor saturation error resulting from sensor saturation constraint always leads to estimation performance deterioration, how to cope with the effect of the sensor saturation error is a difficulty. For this reason, the improved intermediate estimator is designed to estimate the system states and fault signals simultaneously, and the fault-tolerant controller is designed based on the estimates to compensate the actuator fault effectively. The main contributions are as follows:.

1) A fault-tolerant control method based on improved intermediate estimator is proposed. Aiming at the sensor saturation constraint problem, a fault-tolerant control method based on improved intermediate estimator is proposed. Most of existing research on sensor saturation are mainly for fault-control problem under network uncertainty and system distrubances. For example, disturbances are considered in [18, 25, 26], time delay is studied in [27, 28]. However, the results have not fully considered sensor saturation.

2) Generally, the robustness against sensor saturation error can be ensured by introducing H_∞ performance index [19, 20]. However, the H_∞ performance index is only used to characterize the worst case, thus the estimation performance is not satisafactory in some situations. Different from it, the estimation performance can be guaranteed by adjusting some specific parameters directly under the proposed method.

This paper is organized as follows: Section 2 presents the mathematical description. Section 3 provides the concrete controller design process, and the stability of the closed-loop system is rigorously analyzed. Then, a simulation example is given in Section 4 before this paper is concluded in Section 5.

Notations: In this paper, R, R^q and $R^{p\times q}$ mean the sets of integer, q-dimension space and real matrix, respectively; I_q indicates $q\times q$ identity matrix; $diag\{.\}$ represents a diagonal matrix; $\|.\|$ and $|.|$ denote the Euclidean norm and 1-norm of a vector, respectively; and the symbol $*$ within a matrix means the symmetric entry; $\bar{\lambda}(.)$ and $\lambda_{min}(.)$ denote the maximum and minimum eigenvalues of a real symmetric matrix, respectively.

2. PROBLEM DESCRIPTION

2.1. System description

The block diagram of the control system concerned in this paper is shown in Fig. 1, where the actuator faults and sensor faults occur at the same time, and the output information is subject to the sensor saturation constraint before being transmitted to the estimator.

Consider a class of linear systems described by

$$\begin{aligned} x(k+1) &= Ax(k)+B(u(k)+a_u(k)),\\ y(k) &= Cx(k)+Da_y(k), \end{aligned} \tag{1}$$

where $x(k)\in R^n$, $y(k)\in R^p$, $u(k)\in R^m$ represent the system state vector, the measured output vector, the control input, respectively. $a_u(k)\in R^m$ and $a_y(k)\in R^q$ represent actuator faults signal and sensor faults signal, respectively. A,B,C,D are real constant matrices with appropriate dimension. Besides, pair (A,C) is observable. In the sequel, three assumptions are given.

Assumption 1: The unknown faults signals satisfy $\|\delta_u(k)\|\leq\eta_u$, $\|\delta_y(k)\|\leq\eta_y$, where $\|\delta_u(k)\| = \|a_u(k+$

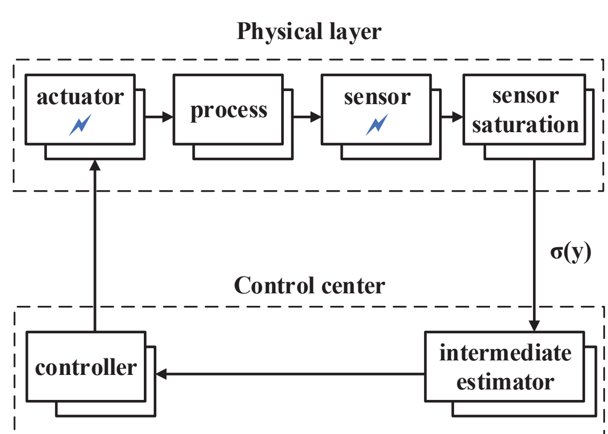

Fig. 1. Structure diagram of control system with sensor saturation.

$1) - a_u(k)\|$, $\|\delta_y(k)\| = \|a_y(k+1) - a_y(k)\|$, $\eta_u \geq 0$, and $\eta_y \geq 0$.

Assumption 2: B and D are of full column rank, i.e., $rank(B) = m$ and $rank(D) = q$.

Assumption 3: For every complex number λ with non-negative real part, $rank \begin{bmatrix} A-\lambda I & B \\ C & 0 \end{bmatrix} = n+m$.

Remark 1: It is reasonable to assume that the fault rate is bounded by Assumption 1. If the rate of change is unbounded, the stability of the estimation error system cannot be guaranteed. Therefore, most of the literatures for fault estimation [29] require Assumption 1.

Remark 2: The condition of full column rank exists commonly in the study of the fault estimation. The basic fault observability requirement (see Asumption 3) is not satisfied without this restriction on B. The pole placement cannot be realized in the intermediate estimator if B is not of full column rank (i.e., B^TB is not positive definite), which can be easily derived by checking formula (6). Besides, if the matrix D is not of full column rank, the observer cannot reconstruct the fault signal to get an accurate estimation. Assumption 2 and Assumption 3 are quite common in the literature on fault estimation [27,29].

2.2. Sensor saturation constraint

The saturation function is defined as $\sigma : R \longrightarrow R$,

$$\sigma(v) = sign(v)\min\{\partial, |v|\}, \tag{2}$$

where $sign$ is the symbol function, ∂ is the saturation level. The measured values received on the estimator side are

$$s(k) = \sigma(Cx(k) + Da_y(k)). \tag{3}$$

Remark 3: The upper bound of sensor saturation is genearal "1" in existing results, while the saturation levels are not "1" in practical control systems. Besides, the parameter ∂ is more general in the sense that it can be unknown.

2.3. Design of the intermediate estimator based fault tolerant control method under saturation constraint

Based on (1)-(3), the system under sensor saturation constraint can be described as

$$\begin{aligned} x(k+1) &= Ax(k) + B(u(k) + a_u(k)), \\ s(k) &= \sigma(Cx(k) + Da_y(k)). \end{aligned} \tag{4}$$

In order to estimate the sensor faults, set the augmented state variable

$$\zeta(k) = \begin{bmatrix} x^T(k) & a_y^T(k) \end{bmatrix}^T. \tag{5}$$

Thus the system (4) can be rewritten as the following augmented system

$$\begin{aligned} \zeta(k+1) &= A_a\zeta(k) + B_a(u(k) + a_u(k)) + Ma_y(k+1), \\ s(k) &= \sigma(C_a\zeta(k)), \end{aligned} \tag{6}$$

where $A_a = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$, $B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $M = \begin{bmatrix} 0 \\ I_q \end{bmatrix}$, $C_a = \begin{bmatrix} C & D \end{bmatrix}$.

Remark 4: Although the intermediate estimator proposed in [29] can estimate the system states and faults, it cannot be applied to the fault-tolerant control under sensor saturation constraint directly. Therefore, the improved intermediate estimator is used to estimate the multiple faults simultaneously under the sensor saturation constraint.

An intermediate variable is introduced as

$$\tau(k) = a_u(k) - \omega B_a^T\zeta(k), \tag{7}$$

where ω is the adjustable parameter. Through adjusting the value of ω can improve the estimation performance. The improved intermediate estimator is proposed as

$$\begin{aligned} \hat{\zeta}(k+1) &= A_a\hat{\zeta}(k) + B_au(k) + B_a\hat{a}_u(k) + M\hat{a}_y(k) \\ &\quad + L[\sigma(y(k)) - C_a\hat{\zeta}(k)], \\ \hat{\tau}(k+1) &= \hat{a}_u(k) - \omega E_a^T[A_a\hat{\zeta}(k) + B_au(k) + B_a\hat{a}_u(k) \\ &\quad + M\hat{a}_y(k)], \\ \hat{a}_u(k) &= \hat{\tau}(k) + \omega B_a^T\hat{\zeta}(k), \end{aligned} \tag{8}$$

where L is the observer gain and needs to be determined. $\hat{x}(k)$, $\hat{\xi}(k)$, $\hat{\tau}(k)$, $\hat{a}_u(k)$, $\hat{a}_y(k)$ are the estimates of $x(k)$, $\xi(k)$, $\tau(k)$, $a_u(k)$, $a_y(k)$, respectively. In order to obtain better fault-tolerant control performance, the control law based on fault estimates is designed as follows:

$$u(k) = -K\hat{x}(k) - \hat{a}_u(k), \tag{9}$$

where K is the state feedback coefficient and needs to be designed. $A - BK$ needs to be Hurwitz matrix. Define the error signals

$$\begin{aligned} &e_\zeta(k) = \zeta(k) - \hat{\zeta}(k),\ e_u(k) = a_u(k) - \hat{a}_u(k), \\ &e_\tau(k) = \tau(k) - \hat{\tau}(k),\ e_s(k) = a_y(k) - \hat{a}_y(k). \end{aligned} \tag{10}$$

By (7)-(10), the closed-loop system dynamics can be given as follows:

$$\begin{aligned} \zeta(k+1) &= C_u\zeta(k) + C_be_\zeta(k) + B_ae_\tau(k) + M\Delta_y(k), \\ e_\zeta(k+1) &= (A_u - LC_a)e_\zeta(k) + B_ae_\tau(k) + M\Delta_y(k) \\ &\quad - L\sigma(C_a\zeta(k)) + LC_a\zeta(k), \\ e_\tau(k+1) &= B_be_\tau(k) + B_ue_\zeta(k) + \Delta_u(k) \\ &\quad - wB_a^TM\Delta_y(k), \end{aligned} \tag{11}$$

where

$$\begin{aligned} B_b &= I - wB_a^TB_a, \\ B_u &= wB_a^T(I - A_a - MM^T - wB_aB_a^T), \\ A_u &= A_a + MM^T + wB_aB_a^T, \end{aligned}$$

$$\Delta_u(k) = a_u(k+1) - a_u(k),$$
$$k_1 = [K \ \ 0], \ \ \Delta_y(k) = a_y(k+1) - a_y(k),$$
$$e_s(k) = Me_\zeta(k),$$
$$C_b = B_a k_1 + wB_a B_a^T, \ \ C_u = A_a - B_a k_1 + MM^T.$$

This paper focuses on solving the problem of fault-tolerant control of linear systems under sensor saturation constraint. At first, the improved intermediate estimator is designed to estimate the states of system and fault signals. Based on the estimates, the fault-tolerant controller is designed to ensure the states of the closed-loop system be uniformly ultimately bounded.

3. STABILITY ANALYSIS FOR THE CLOSED-LOOP SYSTEM AND THE ALGORITHM

In this section, according to Lyapunov stability theory, the stability of the closed-loop system (11) is proved, and the procedure for the improved intermediate estimator-based fault-tolerant control strategy is given.

3.1. Stability analysis for the closed-loop system

Theorem 1: Suppose Assumptions 1 and 2 hold, given adjustable parameters $\omega > 0$, $\varepsilon > 0$, $\varepsilon_1 > 0$; if the matrices $P_1 \in R^{(n+q)\times(n+q)} > 0$, $P_2 \in R^{(n+q)\times(n+q)} > 0$, $P_3 \in R^{m\times m} > 0$ satisfy

$$\Pi < 0, \tag{12}$$

where

$$\Sigma_{11} = C_u^T P_1 C_u - \varepsilon_1 C_a^T \Lambda C_a - P_1,$$
$$\Sigma_{12} = C_u^T P_1 C_b + C_a^T H A_u,$$
$$\Sigma_{34} = -B_a^T H, \ \ \Sigma_{38} = B_a^T P_1 M, \ \ \Sigma_{4,12} = H^T M,$$
$$\Sigma_{5,5} = -P_2, \ \ \Sigma_{33} = B_a^T P_1 B_a + B_a^T P_2 B_a + B_b^T P_3 B_b - P_3,$$
$$\Sigma_{44} = \varepsilon I - \varepsilon_1 I, \ \ \Sigma_{24} = -A_u^T H, \ \Sigma_{25} = -C_a^T H^T,$$
$$\Sigma_{45} = -H^T, \ \ \Sigma_{15} = C_a^T H^T, \ \ \Sigma_{19} = C_a^T H^T M,$$
$$\Sigma_{2,17} = {A_u}^T P_2 M, \ \ \Sigma_{2,15} = wB_u^T P_3^T B_a^T M,$$
$$\Sigma_{3,11} = B_a^T P_2 M, \ \ \Sigma_{2,10} = C_a^T H^T M,$$
$$\Sigma_{3,14} = \omega B_b^T P_3^T B_a^T M, \ \ \Sigma_{13} = C_u^T P_1 B_a + C_a^T H B_a,$$
$$\Sigma_{3,16} = B_b^T P_3^T, \ \ \Sigma_{27} = C_b^T P_1 M,$$
$$\Sigma_{22} = C_b^T P_1 C_b + A_u^T P_2 A_u - A_u^T H C_a - C_a^T H^T A_u + B_u^T P_3 B_u - P_2,$$
$$\Sigma_{2,13} = B_u^T P_3, \ \ \Sigma_{14} = \frac{\varepsilon_1}{2} C_a^T (I+\Lambda), \ \ \Sigma_{16} = C_u^T P_1 M,$$
$$\Sigma_{6,6} = \Sigma_{9,9} = \Sigma_{10,10} = \Sigma_{11,11} = \Sigma_{12,12} = \Sigma_{13,13} = \Sigma_{14,14} = \Sigma_{7,7} = \Sigma_{8,8} = \Sigma_{15,15} = \Sigma_{16,16} = \Sigma_{17,17} = -\varepsilon I,$$
$$\Sigma_{23} = C_b^T P_1 B_a + A_u^T P_2 B_a - C_a^T H^T B_a + B_u^T P_3 B_b,$$
$$\Sigma_{14} = \frac{\varepsilon_1}{2} C_a^T (I+\Lambda),$$

then the states of the closed-loop system (11) are uniformly ultimately bounded. Besides, from inequality (12), the gain of the improved intermediate estimator can be obtained as $L = P_2^{-1} H$.

Proof: Define the Lyapunov function as follows:

$$V(k) = \zeta(k)^T P_1 \zeta(k) + e_\zeta^T(k) P_2 e_\zeta(k) + e_\tau^T(k) P_3 e_\tau(k).$$

Above all, derive the Lyapunov function $V(k)$ and calculate the three components. Substituting (11) into $V(k)$ yields the first component as

$$\begin{aligned}
&\zeta^T(k+1) P_1 \zeta(k+1)\\
&= 2e_\zeta^T(k) C_b^T P_1 M \Delta_y(k) + \Delta_y^T(k) M^T P_1 M \Delta_y(k)\\
&\quad + 2\zeta^T(k) C_u^T P_1 C_b e_\zeta(k) + 2e_\zeta^T(k) C_b^T P_1 B_a e_\tau(k)\\
&\quad + 2e_\tau^T(k) B_a^T P_1 M \Delta_y(k) + \zeta^T(k) C_u^T P_1 C_u \zeta(k)\\
&\quad + 2\zeta^T(k) C_u^T P_1 B_a e_\tau(k) + e_\zeta^T(k) C_b^T P_1 C_b e_\zeta(k)\\
&\quad + e_\tau^T(k) B_a^T P_1 B_a e_\tau(k) + 2\zeta^T(k) C_u^T P_1 M \Delta_y(k).
\end{aligned}$$

The second component can be expanded as follows:

$$\begin{aligned}
&e_\zeta^T(k+1) P_2 e_\zeta(k+1)\\
&= 2\zeta^T(k) C_a^T L^T P_2 (A_u - LC_a) e_\zeta(k)\\
&\quad + 2\zeta^T(k) C_a^T L^T P_2 B_a e_\tau(k)\\
&\quad - 2\zeta^T(k) C_a^T L^T P_2 L \sigma(C_a \zeta(k))\\
&\quad + 2\zeta^T(k) C_a^T L^T P_2 M \Delta_y(k)\\
&\quad + 2e_\zeta^T(k) C_a^T L^T P_2 L \sigma(C_a \zeta(k))\\
&\quad + \Delta_y^T(k) M^T P_2 M \Delta_y(k)\\
&\quad + 2e_\zeta^T(k) (A_u - LC_a)^T P_2 B_a e_\tau(k)\\
&\quad + \sigma(C_a \zeta(k))^T L^T P_2 L \sigma(C_a \zeta(k))\\
&\quad + e_\tau^T(k) B_a^T P_2 B_a e_\tau(k) - 2e_\zeta^T(k) C_a^T L^T P_2 M \Delta_y(k)\\
&\quad - 2e_\zeta^T(k) A_u^T P_2 L \sigma(C_a \zeta(k)) + 2e_\tau^T(k) B_a^T P_2 M \Delta_y(k)\\
&\quad - 2\sigma^T(C_a \zeta(k)) L^T P_2 M \Delta_y(k)\\
&\quad + 2e_\zeta^T(k) A_u^T P_2 M \Delta_y(k) - 2e_\tau^T(k) B_a^T P_2 L \sigma(C_a \zeta(k))\\
&\quad + e_\zeta^T(k) (A_u - LC_a)^T P_2 (A_u - LC_a) e_\zeta(k)\\
&\quad + \zeta^T(k) C_a^T L^T P_2 L C_a \zeta(k).
\end{aligned}$$

The third component can be expanded as follows:

$$\begin{aligned}
&e_\tau^T(k+1) P_3 e_\tau(k+1)\\
&= \omega^2 \Delta_y^T(k) M^T B_a P_3 B_a^T M \Delta_y(k)\\
&\quad - 2\omega \Delta_u^T(k) P_3 B_a^T M \Delta_y(k) + e_\zeta^T(k) B_u^T P_3 B_u e_\zeta(k)\\
&\quad + e_\tau^T(k) B_b^T P_3 B_b e_\tau(k) + 2e_\tau^T(k) B_b^T P_3 \Delta_u(k)\\
&\quad + \Delta_u^T(k) P_3 \Delta_u(k) - 2\omega e_\zeta^T(k) B_u^T P_3 B_a^T M \Delta_y(k)\\
&\quad - 2\omega e_\tau^T(k) B_b^T P_3 B_a^T M \Delta_y(k) + 2e_\zeta^T(k) B_u^T P_3 B_b e_\tau(k)\\
&\quad + 2e_\zeta^T(k) B_u^T P_3 \Delta_u(k).
\end{aligned}$$

According to Assumption 1, the following inequalities always hold:

$$\begin{aligned}
&\Delta_u^T(k)P_3\Delta_u(k) \le \bar{\lambda}(P_3)\eta_u^2,\\
&\Delta_y^T(k)M^TP_1M\Delta_y(k) \le \bar{\lambda}\left(M^TP_1M\right)\eta_y^2,\\
&\Delta_y^T(k)M^TP_2M\Delta_y(k) \le \bar{\lambda}(M^TP_2M)\eta_y^2,\\
&\omega^2\Delta_y^T(k)M^TB_aP_3B_a^TM\Delta_y(k)\\
&\le \bar{\lambda}(\omega^2M^TB_aP_3B_a^TM)\eta_y^2,\\
&2e_\zeta^T(k)B_u^TP_3\Delta_u(k) \le \varepsilon e_\zeta^T(k)B_u^TP_3P_3^TB_ue_\zeta(k)+\frac{\eta_u^2}{\varepsilon},\\
&2e_\tau^T(k)B_b^TP_3\Delta_u(k) \le \varepsilon e_\tau^T(k)B_b^TP_3P_3^TB_be_\tau(k)+\frac{\eta_u^2}{\varepsilon},\\
&2e_\tau^T(k)B_a^TP_2M\Delta_y(k)\\
&\le \varepsilon e_\tau^T(k)B_a^TP_2MM^TP_2^TB_ae_\tau(k)+\frac{\eta_y^2}{\varepsilon},\\
&2\zeta^T(k)C_u^TP_1M\Delta_y(k)\\
&\le \varepsilon\zeta^T(k)C_u^TP_1MM^TP_1^TC_u\zeta(k)+\frac{\eta_y^2}{\varepsilon},\\
&2\zeta(k)C_a^TL^TP_2M\Delta_y(k)\\
&\le \varepsilon\zeta^T(k)C_a^TH^TMM^THC_a\zeta(k)+\frac{\eta_y^2}{\varepsilon},\\
&2e_\zeta^T(k)C_b^TP_1M\Delta_y(k)\\
&\le \varepsilon e_\zeta^T(k)C_b^TP_1MM^TP_1^TC_be_\zeta(k)+\frac{\eta_y^2}{\varepsilon},\\
&2e_\zeta^T(k)A_u^TP_2M\Delta_y(k)\\
&\le \varepsilon e_\zeta^T(k)A_u^TP_2MM^TP_2^TA_ue_\zeta(k)+\frac{\eta_y^2}{\varepsilon},\\
&-2{e_\zeta}^TC_a^TL^TP_2M\Delta_y\\
&\le \varepsilon e_\zeta^T(k)C_a^TL^TP_2MM^TP_2^TLC_ae_\zeta(k)+\frac{\eta_y^2}{\varepsilon},\\
&-2\omega e_\zeta^T(k)B_u^TP_3B_a^TM\Delta_y(k)\\
&\le \varepsilon\omega^2e_\zeta^T(k)B_u^TP_3B_a^TMM^TB_aP_3B_ue_\zeta(k)+\frac{\eta_y^2}{\varepsilon},\\
&-2\omega e_\tau^T(k)B_b^TP_3B_a^TM\Delta_y(k)\\
&\le \varepsilon\omega^2e_\tau^T(k)B_b^TP_3B_a^TMM^TB_aP_3B_be_\tau(k)+\frac{\eta_y^2}{\varepsilon},\\
&2e_\tau^T(k)B_a^TP_1M\Delta_y(k)\\
&\le \varepsilon e_\tau^T(k)B_a^TP_1MM^TP_1^TB_ae_\tau(k)+\frac{\eta_y^2}{\varepsilon},\\
&-2\omega\Delta_u^T(k)P_3B_a^TM\Delta_y(k)\\
&\le \varepsilon\bar{\lambda}(\omega^2P_3B_a^TMM^TB_aP_3)\eta_y^2+\frac{\eta_u^2}{\varepsilon},\\
&-2\sigma(C_a\zeta(k))^TL^TP_2M\Delta_y\\
&\le \varepsilon\sigma(C_a\zeta(k))^TL^TP_2MM^TP_2L\sigma(C_a\zeta(k))+\frac{\eta_y^2}{\varepsilon}.
\end{aligned}$$

As can be seen from the definition of the saturation function, the nonlinear function σ satisfies: $[\sigma(v_i)-a_iv_i][\sigma(v_i)-v_i]\le 0$, $(|v_i|\le \frac{\partial}{a_i})$, where a_i is a positive scalar and $0<a_i<1$ is satisfied. Order $\Lambda=\mathrm{diag}\{a_1, a_2, ..., a_m\}$, it has $[\sigma(C_a\zeta(k))-\Lambda C_a\zeta(k)]^T[\sigma(C_a\zeta(k))-\Lambda C_a\zeta(k)]\le 0$.

Based on the above analysis, the upper bound of the difference $\Delta V(k)$ can be obtained

$$\begin{aligned}
\Delta V(k) \le& V(k+1)-V(k)-\varepsilon_1[\sigma(C_a\zeta(k))-\Lambda C_a\zeta(k)]^T\\
&\times[\sigma(C_a\zeta(k))-C_a\zeta(k)]\\
=&\varepsilon_1\sigma(C_a\zeta(k))^TC_a\zeta(k)-V(k)\\
&+\varepsilon_1\zeta(k)^TC_a^T\Lambda^T\sigma(C_a\zeta(k))\\
&-\varepsilon_1\sigma(C_a\zeta(k))^T\sigma(C_a\zeta(k))\\
&-\varepsilon_1\zeta(k)^TC_a^T\Lambda^TC_a\zeta(k)+V(k+1).
\end{aligned}$$

The derivative of energy function $V(k)$ can be written as

$$\Delta V(k)\le \alpha^T\cdot\Phi\cdot\alpha+\beta, \tag{13}$$

where

$$\begin{aligned}
&\alpha=\left[\zeta^T(k)\ \ e_\zeta^T(k)\ \ e_\tau^T(k)\ \ \sigma^T(C_a\zeta(k))\right]^T,\\
&\Phi=\begin{bmatrix}\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14}\\ * & \Psi_{22} & \Psi_{23} & -(A_u-LC_a)^TH\\ * & * & \Psi_{33} & -B_a^TH\\ * & * & * & \Psi_{44}\end{bmatrix},\\
&\Psi_{11}=C_u^TP_1C_u+\varepsilon C_u^TP_1MM^TP_1^TC_u+C_a^TH^TP_2{}^{-1}HC_a\\
&\qquad+\varepsilon C_a^TH^TMM^THC_a-\varepsilon_1C_a^T\Lambda C_a-P_1,\\
&\Psi_{12}=C_u^TP_1C_b+C_a^TH^T(A_u-LC_a),\\
&\Psi_{13}=C_u^TP_1B_a+C_a^TH^TB_a,\\
&\Psi_{14}=\frac{\varepsilon_1}{2}C_a^T(I+\Lambda)-C_a^TH^TL,\\
&\Psi_{22}=C_a^TH^TP_2{}^{-1}HC_a+C_b^TP_1C_b+A_u^TP_2A_u+B_u^TP_3B_u\\
&\qquad-C_a^TH^TA_u-A_u^THC_a-P_2+\varepsilon A_u^TP_2MM^TP_2{}^TA_u\\
&\qquad+\varepsilon C_b^TP_1MM^TP_1{}^TC_b+\varepsilon C_a^TH^TMM^THC_a\\
&\qquad+\varepsilon\omega^2B_u^TP_3B_a^TMM^TB_aP_3{}^TB_u+\varepsilon B_u^TP_3P_3{}^TB_u,\\
&\Psi_{23}=C_b^TP_1B_a+A_u^TP_2B_a-C_a^TH^TB_a+B_u^TP_3B_b,\\
&\Psi_{33}=B_a^TP_1B_a+B_a^TP_2B_a+\varepsilon\omega^2B_b^TP_3B_a^TMM^TB_aP_3B_b\\
&\qquad-P_3+\varepsilon B_a^TP_1MM^TP_1{}^TB_a+\varepsilon B_a^TP_2MM^TP_2{}^TB_a\\
&\qquad+\varepsilon B_b^TP_3P_3{}^TB_b+B_b^TP_3B_b,\\
&\Psi_{44}=\varepsilon I-\varepsilon_1I+H^TP_2{}^{-1}H+\varepsilon H^TMM^TH,\\
&\beta=\bar{\lambda}\left(\omega^2M^TB_aP_3B_a^TM\right)\eta_y^2+\bar{\lambda}(M^TP_2M)\eta_y^2\\
&\qquad+\varepsilon\bar{\lambda}(\omega^2P_3B_a^TMM^TB_aP_3)\eta_y^2+\bar{\lambda}\left(M^TP_1M\right)\eta_y^2\\
&\qquad+\bar{\lambda}(P_3)\eta_u^2+3\frac{\eta_u^2}{\varepsilon}+10\frac{\eta_y^2}{\varepsilon}.
\end{aligned}$$

According to the Shur complement theory, the available inequality (12) can be gotten from Φ. If $\Phi<0$, then

$$\Delta V(k)\le\bar{\lambda}(\Phi)\left(\|\zeta(k)\|^2+\|e_\zeta(k)\|^2+\|e_\tau(k)\|^2\right)+\beta.$$

Combining (12), it shows

$$\begin{aligned} V(k) \leq & \bar{\lambda}(P_1)\|\zeta(k)\|^2 + \bar{\lambda}(P_2)\|e_\zeta(k)\|^2 \\ & + \bar{\lambda}(P_3)\|e_\tau(k)\|^2 \\ \leq & \max\left[\bar{\lambda}(P_1), \bar{\lambda}(P_2), \bar{\lambda}(P_3)\right] (\|\zeta(k)\|^2 \\ & + \|e_\zeta(k)\|^2 + \|e_\tau(k)\|^2). \end{aligned}$$

Hence inequality (13) can be expressed as $\Delta V(k) \leq -\kappa V(k) + \beta$, where

$$\kappa = \frac{-\overline{\lambda}(\Phi)}{\max\left[\overline{\lambda}(P_1), \overline{\lambda}(P_2), \overline{\lambda}(P_3)\right]} > 0.$$

Define $\Omega = \left\{ \begin{bmatrix} \zeta(k) \\ e_\zeta(k) \\ e_\tau(k) \end{bmatrix} \middle| \begin{array}{c} \lambda_{\min}(P_1)\|\zeta(k)\|^2 + \\ \lambda_{\min}(P_2)\|e_\zeta(k)\|^2 + \\ \lambda_{\min}(P_3)\|e_\tau(k)\|^2 \leq \frac{\beta}{\kappa} \end{array} \right\}$ and denote $\overline{\Omega}$ as its complement. It can be obtained that $\Delta V(k) < 0$ if $\begin{bmatrix} \zeta^T(k) & e_\zeta^T(k) & e_\tau^T(k) \end{bmatrix}^T \in \overline{\Omega}$. Thus the system states are uniformly ultimately bounded if $\Pi < 0$. This completes the proof. □

Remark 5: There are results on fault estimation and fault-tolerant control with network uncertainty, such as the time-delay [10] and packet dropouts [5]. Under the sensor saturation constraint, these problems have been considered, such as distributed states estimation [24], robust filtering [22], output feedback controller design [19], and output feedback H_∞ controller design. However, the tolerant-control under the sensor saturation constraint has not been studied fully.

Remark 6: The parameter ω can be adjusted to improve the speed and accuracy of fault estimation. The traditional methods generally process the sensor saturation error by introducing the performance index H_∞ to change the quantification problem into the robust control problem. In this paper, the effect of sensor saturation can be suppressed by adjusting feedback gain K and ω.

3.2. Procedure for the algorithm

The flow chart of the improved intermediate estimator-based fault-tolerant control method is shown in Fig. 2.

In Fig. 2, $\|e_y(k)\| = \|\sigma(y(k)) - C_a\hat{\zeta}(k)\|$, where $\sigma(y(k))$ is the measurement value of sensor, and $C_a\hat{\zeta}(k)$ is the estimate of output. There are mainly 5 steps:

1) Collect the data when the system is running;

2) Set the adjustable parameter and feedback gain;

3) Calculate the observer gain by (12);

4) Using the improved intermediate estimator to estimate the system states, actuator faults and sensor faults simultaneously. If the estimation performance does not satisfy the judgment condition, re-adjust the adjustment parameter ω and recalculate the observer gain;

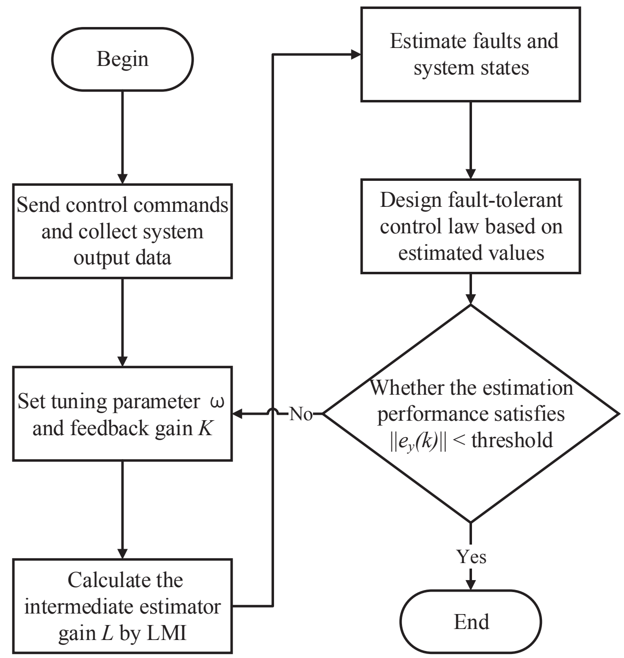

Fig. 2. Fault-tolerant control flow chart.

5) Using the estimated value to design a fault-tolerant control law that compensates the effect of actuator faults effectively.

Remark 7: The parameter selection is a nontrivial problem. Generally, taking a bigger ω can improve the convergence speed, enhance the accuracy of the estimation, and get better estimation performance.

Remark 8: Since the actual fault estimation error is not available in the implementation of fault estimation scheme, hence, the threshold is chosen as an index for evaluating the estimation performance. More importantly, the threshold provides a reference for parameter selection, therefore, it enhances the reliability of the proposed method.

Remark 9: The main differences between improved intermediate estimator and nominal intermediate estimator are as follows: 1) The stability conditions under sensor saturation constraint are quite different from those of nominal intermediate estimator. 2) To enhance the reliability of the estimation scheme, the output error $\|e_y(k)\|$ is used as an index for parameter selection in the proposed method.

4. SIMULATION EXAMPLE

In this section, the effectiveness and merits of the proposed improved intermediate estimator are verified by a simulation example. The dynamic model of the system is given by

$$x(k+1) = Ax(k) + B(u(k) + a_u(k)),$$

Table 1. Sensor faults and actuator faults.

t/s	[0, 31)	[31, 62)	[62, 80)	[80, 120)
$a_{u1}(t)$	3.47sin(0.4t)			
$a_{u2}(t)$	0			
$a_y(t)$	0	6sin(0.2t)	0	0.5

$$y(k) = Cx(k) + Da_y(k), \tag{14}$$

where $A = \begin{bmatrix} 0.9743 & -3.265 \\ 0.00389 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.04202 & 0 \\ 0 & -0.01 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$. In the simulation, set the sensor saturation level as 4.164. Besides, the system actuator faults and sensor faults are created as Table 1. Two types of fault signals are considered, i.e., constant and sinusoidal.

Design the compensation fault-tolerant control law as $u(k) = -K\hat{x}(k) - \hat{a}_u(k)$, where $K = \begin{bmatrix} 22.7069 & -74.9063 \\ -38.9971 & -91.5298 \end{bmatrix}$. Taking $\varepsilon_1 = 100$, $\varepsilon = 1$ and setting $\omega = 800$, while it is found that the estimated error exceeds the threshold (0.1), hence adjusting $\omega = 890$. According to inequality (12), the gain of improved intermediate estimator can be gotten as

$$L = \begin{bmatrix} 0.0036 & 0.0887 \\ 0.0007 & 0.0456 \\ 0.1658 & -0.2783 \end{bmatrix}.$$

To verify the superiority of the proposed method, a comparative simulation study of nominal intermediate estimator is performed. When $\omega = 20$, the gain of the nominal intermediate estimator is obtained as

$$L_1 = \begin{bmatrix} 0.8612 & -0.7287 \\ -0.0609 & 0.6271 \\ 0.0266 & -0.2735 \end{bmatrix}.$$

Set $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. The ideal outputs and actual outputs with sensor saturation are shown in Fig. 3, where y, s are ideal outputs and actual outputs with sensor saturation, respectively. The effect of the actuator faults and sensor faults cause output 1 to exceed the sensor saturation.

Estimator is used to estimate both states and faults. Figs. 4 and 5 exhibit the estimation performance on actuator faults and sensor faults, respectively. It can be seen that the improved intermediate estimator can track the actuator faults and the sensor faults well. At the same time, the esimation performance of nominal intermediate estimator is not accurate enough due to sensor saturation effect. As can be seen in Fig. 5, in the vicinity of 40 seconds and 55 seconds, neither method can track the sensor faults signals accurately.

The state response curve under fault-tolerant control is shown in Fig. 6. In the closed-loop system, the state response 1 is stable between −0.05 and 0.05, and the state response 2 is stable between −0.04 and 0.04. Therefore, it can be concluded that the proposed method can obtain ideal results and fault-tolerant control performance.

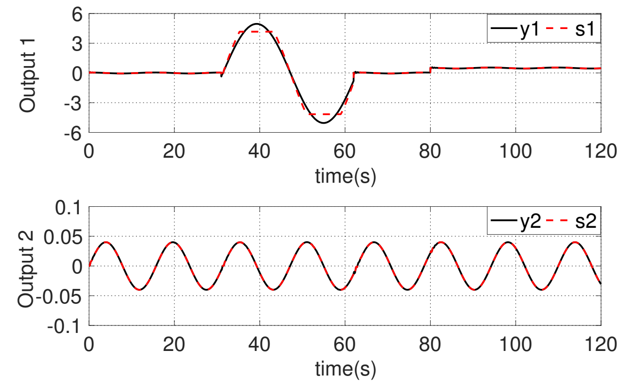

Fig. 3. Ideal outputs and actual outputs with sensor saturation.

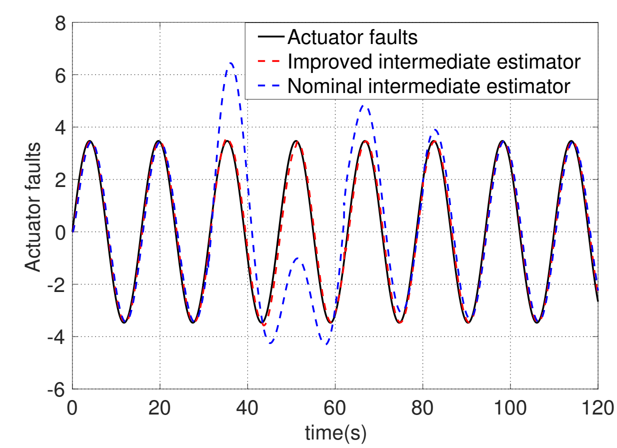

Fig. 4. Actuator faults and estimation.

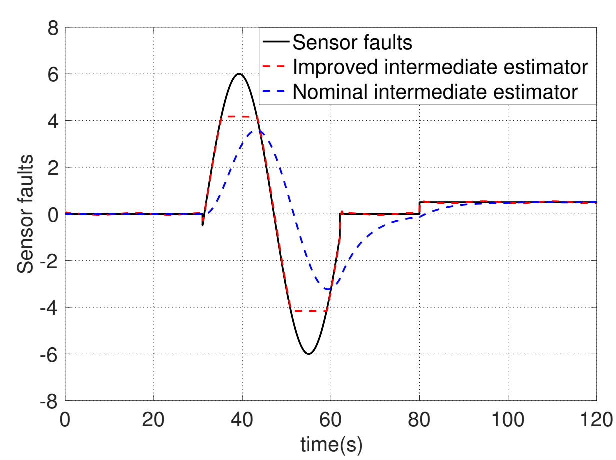

Fig. 5. Sensor faults and estimation.

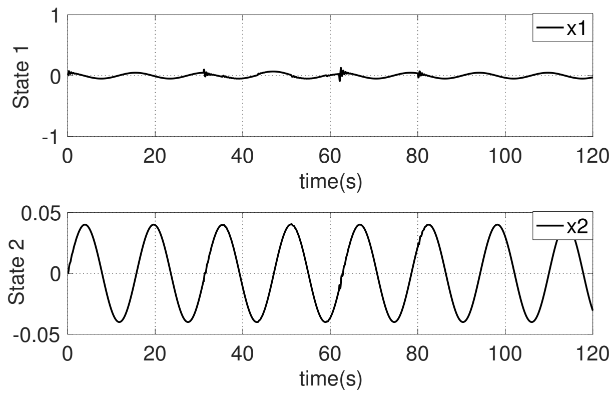

Fig. 6. The response curve of system states.

5. CONCLUSION

Aiming at the linear system under sensor saturation constraint, the fault-tolerant control method based on improved intermediate estimator is designed. Considering the actuator faults and sensor faults, the fault-tolerant controller is designed according to the estimates of states and fault signals. The simulation results show that the proposed method can maintain good fault identification performance under sensor saturation constraint and ensure satisfactory fault-tolerant control performance. In future work, the problem will be consided in the framework of multi-agent systems.

REFERENCES

[1] Z. Gao, Z. Zhou, G. Jiang, M. Qian, and J. Lin, "Active fault tolerant control scheme for satellite attitude systems: multiple actuator faults case," *International Journal of Control, Automation and Systems*, vol. 16, no. 4, pp. 1794-1804, August 2018.

[2] F. Chen, D.Lu, and X. Li, "Robust observer based fault-tolerant control for one-sided Lipschitz Markovian jump systems with general uncertain transition rates," *International Journal of Control, Automation and Systems*, vol. 17, no. 7, pp. 1614-1625, July 2019.

[3] L. B. Wu, J. H. Park, and N. N. Zhao, "Robust adaptive fault-tolerant tracking control for nonaffine stochastic nonlinear systems with full-state constraints," *IEEE Trans. on Cybernetics*, 2019. DOI: 10.1109/TCYB.2019.2940296

[4] J. H. Park, H. Shen, X. H. Chang, and T. H. Lee, *Recent Advances in Control and Filtering of Dynamic Systems with Constrained Signals*, Springer, Switzerland, 2019.

[5] G. H. Yang and D. Ye, "Distributed fixed-time consensus tracking control of uncertain nonlinear multiagent systems: a prioritized strategy," *IEEE Trans. on Cybernetics*, vol. 50, no. 6, pp. 2627-2638, 2020.

[6] Y. Long, J. H. Park, and D. Ye, "Finite frequency fault detection for a class of nonhomogeneous Markov jump systems with nonlinearities and sensor failures," *Nonlinear Dynamics*, vol. 96, no. 1, pp. 285-299, January 2019.

[7] X. Wang and G. H. Yang, "Fault-tolerant consensus tracking control for linear multiagent systems under switching directed network," *IEEE Trans. on Cybernetics*, vol. 50, no. 5, pp. 1921-1930, 2020.

[8] D. Ye and T. Zhang, "Summation detector for false data-injection attack in cyber-physical systems," *IEEE Trans. on Cybernetics*, vol. 50, no. 6, pp. 2338-2345, 2020.

[9] H. Y. Li, H. J. Gao, P. Shi, and X. D. Zhao, "Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach," *Automatica*,vol. 50, no. 7, pp. 1825-1834, July 2014.

[10] T. Guo and Y. J. Liang, "Adaptive decentralized fault-tolerant control for uncertain nonlinear time-delay Large scale systems," *Acta Automatica Sinica*, vol. 43, no. 3, pp. 486-492, March 2017.

[11] L. B. Wu, G. H. Yang, and D. Ye, "Robust adaptive fault-tolerant control for linear systems with actuator failures and mismatched parameter uncertainties," *IET Control Theory and Applications*, vol. 8, no. 6, pp. 441-449, April 2014.

[12] X. Z. Jin, G. H. Yang, X. H. Chang, and W. W. Che, "Robust fault-tolerant H_∞ control with adaptive compensation," *Acta Automatica Sinica*, vol. 39, no. 1, pp. 31-42, January 2013.

[13] R. Yang, P. Shi, and G. P. Liu, "Filtering for discrete-time networked nonlinear systems with mixed random delays and packet dropouts," *IEEE Trans. on Automatic Control*, vol. 56, no.11, pp. 2655-2660, November 2011.

[14] H. Ma, Q. Zhou, L. Bai, and H. Liang, "Observer-based adaptive fuzzy fault-tolerant control for stochastic nonstrict-feedback nonlinear systems with input quantization," *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 2, pp. 287-298, February 2019.

[15] C. C. Wang and G. H. Yang, "Prescribed performance adaptive fault-tolerant tracking control for nonlinear time-delay systems with input quantization and unknown control directions," *Neurocomputing*, vol. 311, pp. 333-343, October 2018.

[16] X. L. Wang and G. H. Yang, "Fault detection for T-S fuzzy systems with past output measurements," *Fuzzy Sets and Systems*, vol. 365, pp. 98-115, May 2019.

[17] J. You, S. Yin, and H. R. Karimi, "Filtering for discrete fuzzy stochastic time-delay systems with sensor saturation," *Mathematical Problems in Engineering*, vol. 2013, pp. 1-10, March 2013.

[18] D. R. Ding, Z. D. Wang, B. Shen, and H. S. Shu, "H_∞ state estimation for discrete-time complex networks with randomly occurring sensor saturations and randomly varying sensor delays," *IEEE Trans. on Neural Networks and Learning Systems*, vol. 23, no. 5, pp. 725-736, May 2012.

[19] Y. Y. Cao, Z. L. Lin, and B. M. Chen, "An output feedback H_∞ controller design for linear systems subject to sensor nonlinearities," *IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications*, vol. 50, no. 7, pp. 914-921, August 2003.

[20] Z. Zuo, J. Wang, and L. Huang, "Output feedback H_∞ controller design for linear discrete-time systems with sensor nonlinearities," *IEE Proceedings-Control Theory and Applications*, vol. 152, no. 1, pp. 19-26, February 2005.

[21] Z. D. Wang, D. W. C. Ho, H. L. Dong, and H. J. Gao, "Robust H_∞ finite-horizon control for a class of stochastic nonlinear time-varying systems subject to sensor and actuator saturations," *IEEE Trans. on Automatic Control*, vol. 55, no. 7, pp. 1716-1722, August 2010.

[22] Y. Xiao, Y. Cao, and Z. Lin, "Robust filtering for discrete-time systems with saturation and its application to transmultiplexers," *IEEE Trans. on Signal Processing*, vol. 52, no. 5, pp. 1266-1277, June 2004.

[23] B. Shen, Z. D. Wang, H. S. Shu, and G. L. Wei, "H_∞ filtering for nonlinear discrete-time stochastic systems with randomly varying sensor delays," *Automatica*, vol. 45, no. 4, pp. 1032-1037, April 2009.

[24] J. L. Liang and W. X. Zheng. "Distributed state estimation for sensor networks with randomly occurring sensor saturations," *Proceedings of the IEEE International Symposium on Circuits and Systems*, pp. 2305-2308, 2014.

[25] J. W. Zhu and G. H. Yang, "Fault-tolerant control for linear systems with multiple faultds and disturbances based on augmented intermediate estimator," *IET Control Theory and Applications*, vol. 11, no. 2, pp. 164-172, October 2017.

[26] L. Sun, and G. Sun, "Robust adaptive saturated fault-tolerant control of autonomous rendezvous with mismatched disturbances," *International Journal of Control, Automation and Systems*, vol. 17, no. 11, pp. 2703-2713, November 2019.

[27] J. W. Zhu and G. H. Yang, "Fault accommodation for linear systems with time-varying delay," *International Journal of Systems Science*, vol. 48, no. 2, pp. 316-323, May 2017.

[28] L. Chen, X. Li, W. Xiao, P. Li, and Q. Zhou, "Fault-tolerant Control for uncertain vehicle active steering systems with time-delay and actuator fault," *International Journal of Control, Automation and Systems*, vol. 17, no. 9, pp. 2234-2241, September 2019.

[29] J. W. Zhu, G. H. Yang, H. Wang, and F. L. Wang, "Fault estimation for a class of nonlinear systems based on intermediate estimator," *IEEE Trans. on Automatic Control*, vol. 61, no. 9, pp. 2518-2524, January 2016.

Jun-Wei Zhu received his B.S. degree in Control Theory and Engineering from Northeastern University, China, in 2008, his M.S. degree in Control Theory and Engineering from Shenyang University, China, in 2011, and his Ph.D. degree in Control Theory and Engineering from Northeastern University, China, in 2016. Currently, he is a special-termed Associate Professor at the College of Information Engineering, Zhejiang University of Technology. He is also a visiting professor of the Institute for Automatic Control and Complex Systems (AKS), University of Duisburg-Essen, Germany, from September 2019 to September 2020. His research interests include cyber-physical systems, fault diagnosis and fault tolerant control.

Qiao-Qian Zhou received her B.Eng. degree in electrical engineering and automation from Ningbo University of Technology, Ningbo, China, in 2016. She is currently pursuing a Ph.D. degree with the College of Information Engineering, Zhejiang University of Technology, Hangzhou, China.

Jian-Ming Xu received his B.S. degree in engineering from Nanchang University in 1998, and his M.S. and Ph.D. degrees from Zhejiang University of Technology, Hangzhou, China. He is currently a Professor in the College of Information Engineering, Zhejiang University of Technology, China. His current research interests include robot control and planning, fault detection.

Jian-Wei Dong received his B.Eng. degree in automation from Suzhou University, Suzhou, China, in 2016. He is currently pursuing a Ph.D. degree with the College of Information Engineering, Zhejiang University of Technology, Hangzhou, China.