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Separable Recursive Gradient Algorithm for Dynamical Systems Based on the Impulse Response Signals

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Abstract: The identification for process control systems is considered in this paper based on the impulse response signals from the discrete measurements. By taking advantage of impulse signals and through the model parameter decomposition, two dependent identification models are constructed and two identification sub-algorithms are presented based on the nonlinear gradient optimization. In terms of the associated items of the parameters to be estimated between two derived sub-algorithms, a separable recursive gradient parameter estimation method is proposed by designing an interactive and recursive estimation. The performance tests and the comparison experiments are carried out by the simulation examples.

Keywords: Gradient search, impulse signal, parameter estimation, recursive algorithm, system response.

1. INTRODUCTION

The dynamical behavior of systems can be obtained from the measurements of the input and output of systems. Therefore, the selection of excitation signals is an important step in the design of the identification experiments. On the other hand, the identification results can be significantly influenced by the input signals which are applied to the systems to be identified [1–5]. In terms of identification experiments, sinusoidal waves and combinational sine signals, step signals, impulse signals and pseudo-random binary sequences are the most frequently used test signals. The systems to be identified are excited by these excitation signals and will generate the system responses, which contain the important dynamical information. Through the discrete observations of the system responses, we can develop identification methods to construct the models of the systems. Generally, the impulse signal is the simplest and easy to realize in many applications among the excitation signals. Some identification methods are obtained based on special input signal excitations [6–8].

There is an ordinary phenomenon that some models show the features of the combination of linear and nonlinear relations, in which one can separate the system parameters into two parts: a linear part and a nonlinear part [9]. In system identification, the separable technique is adopted to decompose the identification models into several submodels to reduce the complexity [10, 11]. Many identification methods have been proposed in terms of linear systems or nonlinear systems by means of the model decomposition or parameter decomposition [12–14] and can be applied to different fields [15–18].

Many nonlinear optimization can be solved by separable nonlinear least squares method, in which a model can be denoted by a combination of linear and nonlinear functions. Recently, these models are used widely in a variety of applications such as neural networks, time-series analysis, signal analysis and other fields [19]. For the problems of separable nonlinear least squares, the separated parameter sets are dependent on each other. For this problem, the general solution is to optimize all the parameters of the systems regardless of the features of parameters which cannot show the preponderance of the separable methods. Meanwhile, that the parameter decomposition gives rise to a new difficulty is how to solve the associated items among the separated sub-algorithms. It is worth noting that many identification approaches are obtained by experiment techniques [20, 21]. Many parameter estimation methods have been developed for linear systems [22, 23] and nonlinear systems [24, 25] and can be applied to many areas [26–31] for constructing their mathematical models.

The following Section 2 introduces the identification problem and the characteristic of identification models.

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Section 3 derives a recursive gradient sub-algorithm in terms of the the linear parameters. Section 4 presents a recursive gradient sub-algorithm for the nonlinear parameters. Section 5 proposes a separable recursive gradient algorithm by uniting two sub-algorithms. Section 6 puts forward an integral recursive gradient algorithm for comparing the performance between the separable method and the non-separable method. Section 7 provides an example and takes the Monte Carlo experiment and comparison test to illustrate the effectiveness of the proposed separable identification method. Section 8 is the conclusions of this paper.

2. PROBLEM ANALYSIS

The transfer function models are of popularity for describing linear time-invariant systems. In general, the transfer function model for the linear time-invariant system with n order takes the following form:

$$G(s) = g(s,\vartheta) = \frac{C(s)}{D(s)}, \tag{1}$$

where $C(s)$ and $D(s)$ are the numerator polynomial and denominator polynomial, i.e.,

$$C(s) = c_m s^m + c_{m-1}s^{m-1} + \cdots + c_1 s + c_0,$$
$$D(s) = d_n s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0, \quad d_n \neq 0.$$

The notations are illustrated as follows. ϑ is the parameter vector which contains the system parameters to be identified, i.e., d_n, d_{n-1}, $\cdots$, d_0, c_m, c_{m-1}, $\cdots$, c_0. m is the order of the numerator polynomial. n is the order of the denominator polynomial, and $n > m$.

Under the condition that the system only has distinct poles and a unit impulse excitation, the impulse response takes the following mathematical form:

$$y(t) = f(a,b,t) = \sum_{i=1}^{n} b_i \mathrm{e}^{-a_i t} + v(t),$$

where $a := [a_1, a_2, \cdots, a_n] \in \mathbb{R}^n$ and $b := [b_1, b_2, \cdots, b_n]$ are the unknown parameter vectors, t is the time variable and $v(t)$ is the observation noise.

From the above description, we find that even though the system is linear time-invariant, the system response is a highly nonlinear function. Moreover, the unknown parameters a_i and b_i are comprised in the response function. Therefore, one can acquire the information of the system by collecting the discrete measurements from the system response signals. Then, the problem of the parameter estimation is converted into an optimal problem by building a cost function regarding to the unknown parameters.

Here is a special characteristic that $y(t)$ is a linear function about the parameters b_i and $y(t)$ is a nonlinear function about the parameters a_i. This issue motivates us to separate the parameters into two parameter vectors to be identified. One is the linear parameter vector $b := [b_1, b_2, \cdots, b_n]^{\mathrm{T}} \in \mathbb{R}^n$, the other is the nonlinear parameter vector $a := [a_1, a_2, \cdots, a_n]^{\mathrm{T}} \in \mathbb{R}^n$. For the purpose of gaining enough real-time information of the systems, we employ the dynamical data with increasing data length so as to obtain the whole process data and use these data dynamically.

Let the initial sampled moment be t_1. Then, the measurements from the initial moment t_1 to the current moment t_k are described as $y(t_1)$, $y(t_2)$, $\cdots$, $y(t_k)$. With the increasing of t_k over time, the length of the collected data also increases. Based on the dynamical observations with the increasing data length, define the objective function

$$J(a,b) := \frac{1}{2}\sum_{j=1}^{k} [y(t_j) - f(a,b,t_j)]^2.$$

From $J(a,b)$, it can be seen that the criterion function with incremental data length continuously absorbs new observed data. Therefore, more data can be involved in the recursive computation. Based on the separated parameter vectors a and b, the objective function is separated into two sub-objective functions:

$$J(a) := \frac{1}{2}\sum_{j=1}^{k} [y(t_j) - f(a,b,t_j)]^2,$$
$$J(b) := \frac{1}{2}\sum_{j=1}^{k} [y(t_j) - \varphi_b(a,t_j)b]^2,$$

where $\varphi_b(a,t_j) = [\mathrm{e}^{-a_1 t_j}, \mathrm{e}^{-a_2 t_j}, \cdots, \mathrm{e}^{-a_n t_j}]^{\mathrm{T}} \in \mathbb{R}^n$. As a result, two sub-algorithms can be derived by optimizing the above two separated criterion functions $J(a)$ and $J(b)$ separately. The deriving process is described in the following section.

3. THE RECURSIVE GRADIENT SUB-ALGORITHM FOR LINEAR PARAMETERS

Let us make a hypothesis that the nonlinear parameter vector a is known and the linear parameter vector b is unknown. Under this hypothesis, the aim of identification is to obtain the parameter estimate of the linear parameter vector b.

Define the information vector

$$\varphi_b(a,t_k) = [\mathrm{e}^{-a_1 t_k}, \mathrm{e}^{-a_2 t_k}, \cdots, \mathrm{e}^{-a_n t_k}]^{\mathrm{T}} \in \mathbb{R}^n.$$

Define the error between the observed output and model output:

$$v(a,b,t_k) := y(t_k) - \sum_{i=1}^{n} b_i \mathrm{e}^{-a_i t_k} \in \mathbb{R}.$$

Then, $v(a,b,t_k)$ can be expressed as

$$v(a,b,t_k) = y(t_k) - \varphi_b^{\mathrm{T}}(a,t_k)b.$$

Finding the search direction by taking the first-order derivative of the objective function $J(b)$ gives the gradient vector:

$$\mathrm{grad}[J(b)] := \frac{\partial J(b)}{\partial b} = -\sum_{j=1}^{k} \varphi_b(a,t_j)v(a,b,t_j) \in \mathbb{R}^n.$$

Because the measurements are stacked with the time increasing, we define the stacked output vector $Y(t_k)$ and the stacked information matrix $\Phi_b(a,t_k)$ as

$$Y(t_k) := \begin{bmatrix} y(t_1) \\ y(t_2) \\ \vdots \\ y(t_k) \end{bmatrix} \in \mathbb{R}^k,$$

$$\Phi_b(a,t_k) := \begin{bmatrix} \varphi_b^{\mathrm{T}}(a,t_1) \\ \varphi_b^{\mathrm{T}}(a,t_2) \\ \vdots \\ \varphi_b^{\mathrm{T}}(a,t_k) \end{bmatrix} \in \mathbb{R}^{k\times n}.$$

As a result, the gradient vector $\mathrm{grad}[J(b)]$ becomes

$$\begin{aligned} \mathrm{grad}[J(b)] =& -\Phi_b^{\mathrm{T}}(a,t_k)[Y(t_k) - \Phi_b(a,t_k)b] \\ =& -[\Phi_b^{\mathrm{T}}(a,t_k)Y(t_k) - \Phi_b^{\mathrm{T}}(a,t_k)\Phi_b(a,t_k)b] \\ =& -[\xi_b(t_k) - R_b(t_k)b], \end{aligned}$$

where

$$\begin{aligned} \xi_b(t_k) :=& \Phi_b^{\mathrm{T}}(a,t_k)Y(t_k) \\ =& \xi_b(t_{k-1}) + \varphi_b(a,t_k)y(t_k) \in \mathbb{R}^n, \\ R_b(t_k) :=& \Phi_b^{\mathrm{T}}(a,t_k)\Phi_b(a,t_k) \\ =& R_b(t_{k-1}) + \varphi_b(a,t_k)\varphi_b^{\mathrm{T}}(a,t_k) \in \mathbb{R}^{n\times n}. \end{aligned}$$

The estimate of the linear parameter vector b at t_k is denoted by $\hat{b}(t_k) := [\hat{b}_1(t_k), \hat{b}_2(t_k), \cdots, \hat{b}_n(t_k)]^{\mathrm{T}}$. By means of the negative gradient search, the recursive gradient sub-algorithm for identifying the linear parameter vector b is developed as follows:

$$\begin{aligned} \hat{b}(t_k) &= \hat{b}(t_{k-1}) - \frac{1}{r_b(t_k)}\mathrm{grad}[J(\hat{b}(t_{k-1}))] \\ &= \hat{b}(t_{k-1}) + \frac{1}{r_b(t_k)}[\xi_b(t_k) - R_b(t_k)\hat{b}(t_{k-1})], \quad (2) \\ r_b(t_k) &= r_b(t_{k-1}) + \|\varphi_b(a,t_k)\|^2, \quad (3) \\ \xi_b(t_k) &= \xi_b(t_{k-1}) + \varphi_b(a,t_k)y(t_k), \quad (4) \\ R_b(t_k) &= R_b(t_{k-1}) + \varphi_b(a,t_k)\varphi_b^{\mathrm{T}}(a,t_k), \quad (5) \\ \varphi_b(a,t_k) &= [\mathrm{e}^{-a_1t_k}, \mathrm{e}^{-a_2t_k}, \cdots, \mathrm{e}^{-a_nt_k}]^{\mathrm{T}}, \quad (6) \\ \hat{b}(t_k) &= [\hat{b}_1(t_k), \hat{b}_2(t_k), \cdots, \hat{b}_n(t_k)]^{\mathrm{T}}. \quad (7) \end{aligned}$$

Remark 1: The recursive sub-algorithm in (2)–(7) only can be used for estimating the linear parameter vector b when the nonlinear parameter vector a is known. If the nonlinear parameter vector a is unknown, the recursive sub-algorithm (2)–(7) is in vain because there contains the unknown nonlinear parameter vector a in the information vector $\varphi_b(a,t_k)$.

4. THE RECURSIVE GRADIENT SUB-ALGORITHM FOR NONLINEAR PARAMETER VECTOR

Based on the separable parameters, here derives the recursive gradient sub-algorithm for the nonlinear parameter vector a. Suppose that the linear parameter vector b is known and the nonlinear parameter vector a is unknown.

Taking the first-order derivative of the separated objective function $J(a)$ obtains the search direction, i.e., the gradient of $J(a)$:

$$\begin{aligned} \mathrm{grad}[J(a)] :=& \frac{\partial J(a)}{\partial a} \\ =& \left[\frac{\partial J(a)}{\partial a_1}, \frac{\partial J(a)}{\partial a_2}, \cdots, \frac{\partial J(a)}{\partial a_n}\right]^{\mathrm{T}} \in \mathbb{R}^n, \\ \frac{\partial J(a)}{\partial a_l} =& \sum_{j=1}^{k} b_l t_j \mathrm{e}^{-a_l t_j} v(a,b,t_j), \quad l = 1,2,\cdots,n, \\ v(a,b,t_j) =& y(t_k) - \sum_{i=1}^{n} b_i \mathrm{e}^{-a_i t_j}. \end{aligned}$$

The information vector is given by

$$\begin{aligned} \varphi_a(a,b,t_k) :=& [b_1 t_k \mathrm{e}^{-a_1 t_k}, b_2 t_k \mathrm{e}^{-a_2 t_k}, \cdots, b_n t_k \mathrm{e}^{-a_n t_k}]^{\mathrm{T}} \\ & \in \mathbb{R}^n. \end{aligned}$$

The stacked information matrix is given by

$$\Phi_a(a,b,t_k) := \begin{bmatrix} \varphi_a^{\mathrm{T}}(a,b,t_1) \\ \varphi_a^{\mathrm{T}}(a,b,t_2) \\ \vdots \\ \varphi_a^{\mathrm{T}}(a,b,t_k) \end{bmatrix} \in \mathbb{R}^{k\times n}.$$

Define the model output at t_k as $f(a,b,t_k) := \sum_{i=1}^{n} b_i \mathrm{e}^{-a_i t_k} \in \mathbb{R}$. The stacked model output vector is defined as

$$F(a,b,t_k) := \begin{bmatrix} f(a,b,t_1) \\ f(a,b,t_2) \\ \vdots \\ f(a,b,t_k) \end{bmatrix} \in \mathbb{R}^k.$$

As a result, the gradient vector $\mathrm{grad}[J(a)]$ is expressed as

$$\begin{aligned} \mathrm{grad}[J(a)] =& \Phi_a^{\mathrm{T}}(a,b,t_k)[Y(t_k) - F(a,b,t_k)] \\ =& \Phi_a^{\mathrm{T}}(a,b,t_k)Y(t_k) - \Phi_a^{\mathrm{T}}(a,b,t_k)F(a,b,t_k). \end{aligned}$$

Define the following recursive relationships:

$$\xi_a(a,b,t_k) := \Phi_a^{\mathrm{T}}(a,b,t_k)Y(t_k)$$

$$
\begin{aligned}
&= \Phi_a^{\mathrm{T}}(a,b,t_{k-1})Y(t_{k-1}) \\
&\quad + \varphi_a(a,b,t_k)y(t_k) \in \mathbb{R}^n, \\
\zeta_a(a,b,t_k) &:= \Phi_a^{\mathrm{T}}(a,b,t_k)F(a,b,t_k) \\
&= \Phi_a^{\mathrm{T}}(a,b,t_{k-1})F(a,b,t_{k-1}) \\
&\quad + \varphi_a(a,b,t_k)f(t_k) \in \mathbb{R}^n.
\end{aligned}
$$

Thus, the gradient vector $\mathrm{grad}[J(a)]$ can be expressed as

$$
\mathrm{grad}[J(a)] = \xi_a(a,b,t_k) - \zeta_a(a,b,t_k).
$$

Denote $\hat{a}(t_k) = [\hat{a}_1(t_k), \hat{a}_2(t_k), \cdots, \hat{a}_n(t_k)]^{\mathrm{T}} \in \mathbb{R}^n$, where $\hat{a}(t_k)$ is the recursive estimate of the nonlinear parameter vector a at t_k. According to the theory of the negative gradient search and optimizing the objective function $J(a)$, the recursive gradient sub-algorithm for estimating the nonlinear parameter vector a is summarized as follows:

$$
\begin{aligned}
\hat{a}(t_k) &= \hat{a}(t_{k-1}) - \frac{1}{r_a(t_k)}\mathrm{grad}[J(\hat{a}(t_{k-1}),b)] \\
&= \hat{a}(t_{k-1}) - \frac{1}{r_a(t_k)}[\hat{\xi}_a(t_k) - \hat{\zeta}_a(t_k)], \qquad (8) \\
r_a(t_k) &= r_a(t_{k-1}) + \|\varphi_a(\hat{a}(t_{k-1}),b,t_k)\|^2, \qquad (9) \\
\hat{\xi}_a(t_k) &= \xi_a(\hat{a}(t_{k-1}),b,t_k) \\
&= \hat{\xi}_a(t_{k-1}) + \varphi_a(\hat{a}(t_{k-1}),b,t_k)y(t_k), \qquad (10) \\
\hat{\zeta}_a(t_k) &= \zeta_a(\hat{a}(t_{k-1}),b,t_k) \\
&= \hat{\zeta}_a(t_{k-1}) + \varphi_a(\hat{a}(t_{k-1}),b,t_k)\hat{f}_a(t_k), \qquad (11) \\
\hat{\varphi}_a(t_k) &= \varphi_a(\hat{a}(t_{k-1}),b,t_k) \\
&= [b_1 t_k \mathrm{e}^{-\hat{a}_1(t_{k-1})t_k}, \cdots, b_n t_k \mathrm{e}^{-\hat{a}_n(t_{k-1})t_k}]^{\mathrm{T}}, \qquad (12) \\
\hat{f}_a(t_k) &= f(\hat{a}(t_{k-1}),b,t_k) = \sum_{i=1}^{n} b_i \mathrm{e}^{\hat{a}_i(t_{k-1})t_k}, \qquad (13) \\
\hat{a}(t_k) &= [\hat{a}_1(t_k), \hat{a}_2(t_k), \cdots, \hat{a}_n(t_k)]^{\mathrm{T}}. \qquad (14)
\end{aligned}
$$

Remark 2: From the presented recursive gradient sub-algorithm in (8)–(14), we notice that the algorithm only can be used for determining the estimate of the nonlinear parameter vector a in the condition that the linear parameter vector b is known. Because the unknown parameters b exist in the information vector $\hat{\varphi}_a(t_k)$, the recursive gradient sub-algorithm in (8)–(13) cannot be realized. This is the same problem as the sub-algorithm in (2)–(7).

5. THE SEPARABLE RECURSIVE GRADIENT ALGORITHM

In order to solve the problem which the sub-algorithms are unavailable, we combine the sub-algorithm in (2)–(7) for estimating the linear parameter vector b and the sub-algorithm in (8)–(14) for estimating the nonlinear parameter vector a. The key difficulty of the unavailable sub-algorithms is that there are the associated unknown terms between the sub-algorithms. Thus the following measures are taken:

1) use $\hat{a}(t_{k-1})$ to replace a in (2)–(7);
2) use $\hat{b}(t_{k-1})$ to replace b in (8)–(14);
3) estimate the separable parameter vectors a and b by interactive estimation technique. Then we obtain the separable recursive gradient (SRG) algorithm as follows:

$$
\begin{aligned}
\hat{a}(t_k) &= \hat{a}(t_{k-1}) - \frac{1}{r_a(t_k)}\mathrm{grad}[J(\hat{a}(t_{k-1})] \\
&= \hat{a}(t_{k-1}) - \frac{1}{r_a(t_k)}[\hat{\xi}_a(t_k) - \hat{\zeta}_a(t_k)], \qquad (15) \\
r_a(t_k) &= r_a(t_{k-1}) + \|\hat{\varphi}_a(t_k)\|^2, \qquad (16) \\
\hat{\xi}_a(t_k) &= \xi_a(\hat{a}(t_{k-1}),\hat{b}(t_{k-1}),t_k) \\
&= \hat{\xi}_a(t_{k-1}) + \hat{\varphi}_a(t_k)y(t_k), \qquad (17) \\
\hat{\zeta}_a(t_k) &= \zeta_a(\hat{a}(t_{k-1}),\hat{b}(t_{k-1}),t_k) \\
&= \hat{\zeta}_a(t_{k-1}) + \hat{\varphi}_a(t_k)\hat{f}(t_k), \qquad (18) \\
\hat{\varphi}_a(t_k) &= \varphi_a(\hat{a}(t_{k-1}),\hat{b}(t_{k-1}),t_k) \\
&= [\hat{b}_1(t_{k-1})t_k \mathrm{e}^{-\hat{a}_1(t_{k-1})t_k}, \hat{b}_2(t_{k-1})t_k \mathrm{e}^{-\hat{a}_2(t_{k-1})t_k}, \cdots, \\
&\quad \hat{b}_n(t_{k-1})t_k \mathrm{e}^{-\hat{a}_n(t_{k-1})t_k}]^{\mathrm{T}}, \qquad (19) \\
\hat{f}(t_k) &= f(\hat{a}(t_{k-1}),\hat{b}(t_{k-1}),t_k) \\
&= \sum_{i=1}^{n} \hat{b}_i(t_{k-1})\mathrm{e}^{\hat{a}_i(t_{k-1})t_k}, \qquad (20) \\
\hat{b}(t_k) &= \hat{b}(t_{k-1}) - \frac{1}{r_b(t_k)}\mathrm{grad}[J(\hat{b}(t_{k-1}))] \\
&= \hat{b}(t_{k-1}) + \frac{1}{r_b(t_k)}[\hat{\xi}_b(t_k) - R_b(t_k)\hat{b}(t_{k-1})], \qquad (21) \\
r_b(t_k) &= r_b(t_{k-1}) + \|\varphi_b(\hat{a}(t_{k-1}),t_k)\|^2, \qquad (22) \\
\hat{\xi}_b(t_k) &= \hat{\xi}_b(t_{k-1}) + \varphi_b(\hat{a}(t_{k-1}),t_k)y(t_k), \qquad (23) \\
R_b(t_k) &= R_b(t_{k-1}) + \varphi_b(\hat{a}(t_{k-1}),t_k)\varphi_b^{\mathrm{T}}(\hat{a}(t_{k-1}),t_k), \qquad (24) \\
\hat{\varphi}_b(t_k) &= \varphi_b(\hat{a}(t_{k-1}),t_k) \\
&= [\mathrm{e}^{-\hat{a}_1(t_{k-1})t_k}, \mathrm{e}^{-\hat{a}_2(t_{k-1})t_k}, \cdots, \mathrm{e}^{-\hat{a}_n(t_{k-1})t_k}]^{\mathrm{T}}, \qquad (25) \\
\hat{a}(t_k) &= [\hat{a}_1(t_k), \hat{a}_2(t_k), \cdots, \hat{a}_n(t_k)]^{\mathrm{T}}, \qquad (26) \\
\hat{b}(t_k) &= [\hat{b}_1(t_k), \hat{b}_2(t_k), \cdots, \hat{b}_n(t_k)]^{\mathrm{T}}. \qquad (27)
\end{aligned}
$$

In accordance with the above algorithm in (15)–(27), we summarize the computational steps for computing the parameter estimation vectors $\hat{a}(t_k)$ and $\hat{b}(t_k)$ of the system which is studied in this work.

1) Initiation: Let $k = 1$, set $\hat{a}(t_0) = [\hat{a}_1(t_0), \hat{a}_2(t_0), \cdots, \hat{a}_n(t_0)]^{\mathrm{T}}$, $\hat{b}(t_0) = [\hat{b}_1(t_0), \hat{b}_2(t_0), \cdots, \hat{b}_n(t_0)]^{\mathrm{T}}$, $r_a(t_0) = 1$, $r_b(t_0) = 1$, $\hat{\xi}_a(t_0) = \mathbf{0}$, $\hat{\zeta}_a(t_0) = \mathbf{0}$, $\hat{\xi}_b(t_0) = \mathbf{0}$, $R_b(t_0) = \mathbf{0}$, and a recursive length $k_{\max}$.

2) Collect the impulse response data $y(t_k)$.

3) Calculate and construct the information vector $\hat{\varphi}_a(t_k)$ via (19), calculate the model output $\hat{f}(t_k)$ via (20) and calculate and construct the information vector $\hat{\varphi}_b(t_k)$ via (25).

4) Calculate $r_a(t_k)$ via (16), calculate vector $\hat{\xi}_a(t_k)$ via (17), calculate vector $\hat{\zeta}_a(t_k)$ via (18) and refresh the parameter estimation vector $\hat{a}(t_k)$ via (15).

5) Calculate $r_b(t_k)$ via (22), calculate vector $\hat{\xi}_b(t_k)$ via (23), calculate the matrix $R_b(t_k)$ via (24) and refresh the parameter estimation vector $\hat{b}(t_k)$ via (21).

6) Acquire the nonlinear parameter estimate $\hat{a}_i(t_k)$ from the vector $\hat{a}(t_k)$ in (26) and the linear parameter estimate $\hat{b}_i(t_k)$ from vector $\hat{b}(t_k)$ in (27), $i = 1, 2, \cdots, n$.

7) If recursion k satisfies $k < k_{\max}$, then $k := k+1$ and go to Step 2); otherwise obtain the parameter estimation vectors $\hat{a}(t_k)$ and $\hat{b}(t_k)$ and terminate the computational process.

6. INTEGRAL RECURSIVE GRADIENT ALGORITHM

For comparing the performance of the proposed separable algorithm, the integral recursive gradient algorithm is provided directly, in which the parameters to be identified are not separated. The recursive gradient (RG) algorithm is as follows:

$$\begin{aligned}\hat{\theta}(t_k) &= \hat{\theta}(t_{k-1}) + \frac{1}{r(t_k)} \\ &\quad \times [\xi(\hat{\theta}(t_{k-1}),t_k) - \zeta(\hat{\theta}(t_{k-1}),t_k)] \\ &= \hat{\theta}(t_{k-1}) + \frac{1}{r(t_k)}[\hat{\xi}(t_k) - \hat{\zeta}(t_k)], \end{aligned} \tag{28}$$

$$\begin{aligned} r(t_k) &= \mathrm{tr}[\Phi^{\mathrm{T}}(\hat{\theta}(t_{k-1}),t_k)\Phi(\hat{\theta}(t_{k-1}),t_k)] \\ &= r(t_{k-1}) + \|\varphi(\hat{\theta}(t_{k-1}),t_k)\|^2 \\ &= r(t_{k-1}) + \|\hat{\varphi}(t_k)\|^2, \quad r(t_0) = 1, \end{aligned} \tag{29}$$

$$\begin{aligned} \hat{\xi}(t_k) &:= \xi(\hat{\theta}(t_{k-1}),t_k) \\ &= \xi(\hat{\theta}(t_{k-1}),t_{k-1}) + \varphi(\hat{\theta}(t_{k-1}),t_k)y(t_k) \\ &= \hat{\xi}(t_{k-1}) + \hat{\varphi}(t_k)y(t_k), \end{aligned} \tag{30}$$

$$\begin{aligned} \hat{\zeta}(t_k) &:= \zeta(\hat{\theta}(t_{k-1}),t_k) \\ &= \zeta(\hat{\theta}(t_{k-1}),t_{k-1}) + \varphi(\hat{\theta}(t_{k-1}),t_k)f(\hat{\theta}(t_{k-1}),t_k) \\ &= \hat{\zeta}(t_{k-1}) + \hat{\varphi}(t_k)\hat{f}(t_k), \end{aligned} \tag{31}$$

$$\begin{aligned} \hat{\varphi}(t_k) &:= \varphi(\hat{\theta}(t_{k-1}),t_k) \\ &= [-\hat{b}_1(t_{k-1})t_k\mathrm{e}^{-\hat{a}_1(t_{k-1})t_k}, \mathrm{e}^{-\hat{a}_1(t_{k-1})t_k}, \\ &\quad -\hat{b}_2(t_{k-1})t_k\mathrm{e}^{-\hat{a}_2(t_{k-1})t_k}, \mathrm{e}^{-\hat{a}_2(t_{k-1})t_k}, \cdots, \\ &\quad -\hat{b}_n(t_{k-1})t_k\mathrm{e}^{-\hat{a}_n(t_{k-1})t_k}, \mathrm{e}^{-\hat{a}_n(t_{k-1})t_k}]^{\mathrm{T}}, \end{aligned} \tag{32}$$

$$\begin{aligned} \hat{f}(t_k) &= f(\hat{\theta}(t_{k-1}),t_k) \\ &= \sum_{i=1}^{n} \hat{b}_i(t_{k-1})\mathrm{e}^{-\hat{a}_i(t_{k-1})t_k}, \end{aligned} \tag{33}$$

$$\hat{\theta}(t_k) = [\hat{a}_1(t_k), \cdots, \hat{a}_n(t_k), \cdots, \hat{b}_1(t_k), \cdots, \hat{b}_n(t_k)]^{\mathrm{T}}. \tag{34}$$

In accordance with the algorithm (28)–(34), we summarize the computational steps for estimating the parameter estimation vector $\hat{\theta}(t_k)$ as follows.

1) Initiation: Let $k = 1$, set $\hat{\theta}(t_0) = [\hat{a}_1(t_0), \hat{a}_2(t_0), \cdots, \hat{a}_n(t_0), \hat{b}_1(t_0), \hat{b}_2(t_0), \cdots, \hat{b}_n(t_0)]^{\mathrm{T}}$, $r(t_0) = 1$, $\hat{\xi}(t_0) = \mathbf{0}$, $\hat{\zeta}(t_0) = \mathbf{0}$, and set the recursive step $k_{\max}$.

2) Collect the impulse response data $y(t_k)$.

3) Calculate and construct the information vector $\hat{\varphi}(t_k)$ via (32), and calculate the model output $\hat{f}(t_k)$ via (33).

4) Calculate $r(t_k)$ via (29), calculate vector $\hat{\xi}(t_k)$ via (30), and calculate vector $\hat{\zeta}(t_k)$ via (31)

5) Refresh the parameter estimation vector $\hat{\theta}(t_k)$ via (28).

6) If recursion k satisfies $k < k_{\max}$, then $k := k+1$ and go to Step (2); otherwise obtain the parameter estimates $\hat{\theta}(t_k)$ from (34) and terminate the computational process.

The methods proposed in this paper can combine some mathematical tools [32–37] and strategies [38–41] to study the parameter estimation problems of different systems with colored noises and can be applied to other engineering systems [42–46] such as information systems [47–50] and networked systems [51–56] and so on.

7. ILLUSTRATED EXAMPLE

Consider the following nonlinear function

$$y(t) = b_1\mathrm{e}^{-a_1 t} + b_2\mathrm{e}^{-a_2 t} + b_3\mathrm{e}^{-a_3 t},$$

where the true parameters of the system are $a_1 = 5.4$, $a_2 = 4.3$, $a_3 = 3.2$, $b_1 = 1$, $b_2 = 2$ and $b_3 = 1.8$. Its Laplace transform gives the transfer function $G(s) = \frac{1}{s+5.4} + \frac{2}{s+4.3} + \frac{1.8}{s+3.2}$. $y(t)$ is the unit impulse response of $G(s)$. In this section, the following simulation experiments are taken and the simulation results are analyzed.

7.1. Monte Carlo experiment

In this subsection, the Monte Carlo simulation tests using the measured data under different circumstance are taken to test the effectiveness of the proposed separable identification method. The experimental conditions are set as follows: 1) the noise variance is $\sigma^2 = 0.10^2$; 2) the number of the Monte Carlo simulation tests is 20; 3) the recursive step is $k_{\max} = 200$, $k_{\max} = 500$, $k_{\max} = 1000$ and $k_{\max} = 1500$, respectively; 4) the estimation error is computed according to the following equation:

$$\delta(t_k) := \frac{\sqrt{\|\hat{a}(t_k) - a\|^2 + \|\hat{b}(t_k) - b\|^2}}{\|a\| + \|b\|}.$$

The parameter estimates obtained by the Monte Carlo tests are shown in Table 1, where SRG means the separable recursive gradient method. The estimation errors under different recursive steps for 20 Monte Carlo tests are drawn in Fig. 1.

7.2. Performance comparison

The performance comparison with other algorithms is an important way to illustrate the features of presented methods. In this subsection, the performance comparison

Table 1. The SRG estimates and their estimation errors by the 20 Monte Carlo tests.

k	200	500	1000	1500	True values
a_1	5.19738 ± 0.20447	5.21363 ± 0.18877	5.24370 ± 0.16165	5.27319 ± 0.13560	5.40000
a_2	4.01297 ± 0.29542	4.07954 ± 0.23498	4.20557 ± 0.12641	4.33246 ± 0.06724	4.30000
a_3	3.00986 ± 0.19044	3.01192 ± 0.18882	3.01595 ± 0.18559	3.02015 ± 0.18178	3.20000
b_1	0.98662 ± 0.07630	1.04011 ± 0.09422	1.07623 ± 0.20569	1.03866 ± 0.20443	1.00000
b_2	2.02968 ± 0.11315	2.09499 ± 0.15179	2.14158 ± 0.25357	2.10367 ± 0.26399	2.00000
b_3	1.87035 ± 0.21571	1.86617 ± 0.30278	1.87574 ± 0.35708	1.91011 ± 0.39457	1.80000

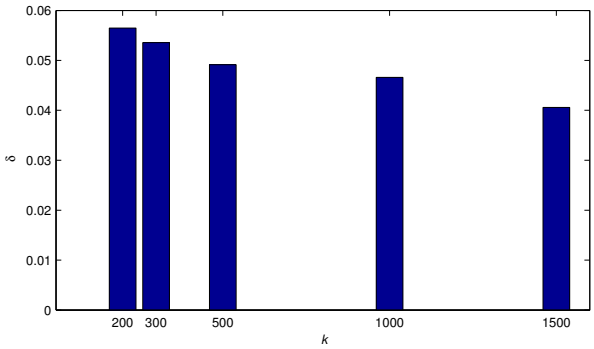

Fig. 1. The estimation errors under different recursive steps.

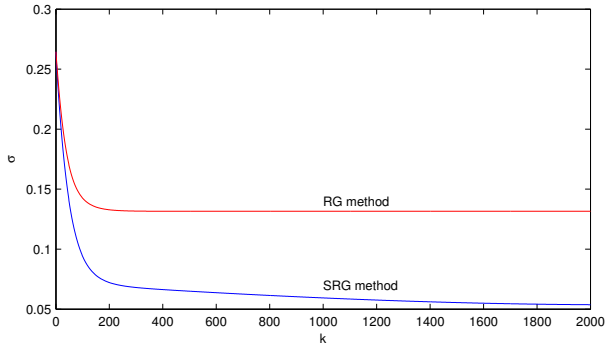

Fig. 2. The SRG and RG estimation errors versus k (SNR= 1.04).

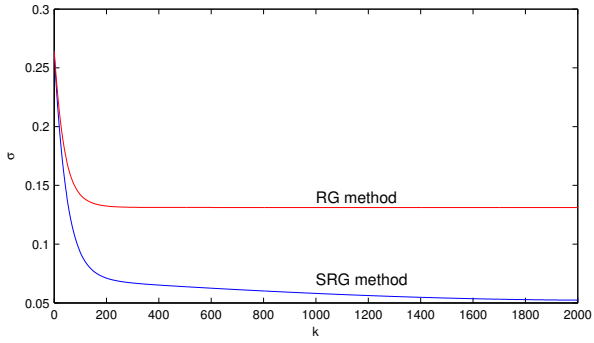

Fig. 3. The SRG and RG estimated errors versus k (SNR= 26.17).

experiment is implemented between the separable recursive gradient algorithm and the recursive gradient algorithm without separation. Firstly, we use the separable recursive gradient algorithm and the recursive gradient algorithm without separation to identify the parameters of the system in this example. Furthermore, different signal-to-noise ratio (SNR) is considered because the noisy observations are involved in this example, where the SNR is defined as the ratio between the variance of signal and the variance of noise. Here, the experimental conditions are set as follows: 1) the sample period is 0.1 s; 2) the recursive step is $k_{\max} = 2000$; 3) the estimation error is computed according to the following equation:

$$\delta(t_k) := \frac{\sqrt{\|\hat{a}(t_k) - a\|^2 + \|\hat{b}(t_k) - b\|^2}}{\|a\| + \|b\|}.$$

Under the noisy measurement scenario, the performance of the proposed separable recursive gradient algorithm and the recursive gradient algorithm without separation are verified based on different SNRs. In the simulation, the SNR is set as 26.17 and 1.04. The estimated values of the system parameters and their estimation errors on different SNRs are displayed in Tables 2–3. The parameter estimation errors versus k obtained via the separable recursive gradient method are shown in Figs. 2 and 3, respectively. The parameter estimates obtained by these two methods under different SNRs and the original system parameters are illustrated in Figs. 4 and 5, respectively.

In Fig. 4, the parameter estimate of a_1 is shown in (a), the parameter estimate of a_2 is shown in (b), the parameter estimate of a_3 is shown in (c), the parameter estimate of b_1 is shown in (d), the parameter estimate of b_2 is shown in (e) and the parameter estimate of b_3 is shown in (f). Moreover, the mark "$\circ$" denotes the original value, the mark "$*$" denotes the estimated value obtained by the recursive gradient (RG) method without separation and the mark "$\triangle$" denotes the estimated value obtained by the separable recursive gradient (SRG) method.

In Fig. 5, the parameter estimate of a_1 is shown in (a), the parameter estimate of a_2 is shown in (b), the parameter estimate of a_3 is shown in (c), the parameter estimate of b_1 is shown in (d), the parameter estimate of b_2 is shown in (e) and the parameter estimate of b_3 is shown in (f). More-

Table 2. The SRG and the RG estimates and their estimation errors (SNR= 1.04).

Method	k	a_1	a_2	a_3	b_1	b_2	b_3	$\delta(\%)$
SRG	100	5.18814	3.97352	3.00857	0.83499	1.70324	1.26882	9.41002
	200	5.19229	3.99105	3.00913	0.88594	1.80900	1.45368	7.21713
	500	5.20472	4.04362	3.01084	0.89871	1.83158	1.48244	6.49668
	1000	5.22544	4.13125	3.01367	0.90558	1.83896	1.47662	5.93689
	2000	5.26688	4.30651	3.01934	0.91899	1.85333	1.46500	5.37002
RG	100	5.09758	3.59083	2.99620	0.78662	1.60216	1.09031	14.25280
	200	5.08594	3.54165	2.99461	0.82521	1.68325	1.23474	13.28037
	500	5.08416	3.53414	2.99437	0.83110	1.69562	1.25677	13.15755
	1000	5.08416	3.53413	2.99436	0.83111	1.69565	1.25682	13.15727
	2000	5.08416	3.53413	2.99436	0.83111	1.69565	1.25682	13.15727
True values		5.40000	4.30000	3.20000	1.00000	2.00000	1.80000	

Table 3. The SRG and the RG estimates and their estimation errors (SNR= 26.17).

Method	k	a_1	a_2	a_3	b_1	b_2	b_3	$\delta(\%)$
SRG	100	5.18818	3.97365	3.00857	0.83762	1.70867	1.27815	9.29701
	200	5.19235	3.99131	3.00914	0.88905	1.81536	1.46431	7.10602
	500	5.20488	4.04428	3.01086	0.90222	1.83850	1.49298	6.37959
	1000	5.22575	4.13257	3.01371	0.90963	1.84659	1.48662	5.80708
	2000	5.26751	4.30914	3.01943	0.92410	1.86234	1.47395	5.23553
RG	100	5.09708	3.58877	2.99613	0.78827	1.60561	1.09636	14.20637
	200	5.08537	3.53931	2.99454	0.82709	1.68716	1.24159	13.24110
	500	5.08359	3.53177	2.99429	0.83301	1.69960	1.26375	13.11990
	1000	5.08359	3.53175	2.99429	0.83302	1.69963	1.26380	13.11962
	2000	5.08359	3.53175	2.99429	0.83302	1.69963	1.26380	13.11962
True values		5.40000	4.30000	3.20000	1.00000	2.00000	1.80000	

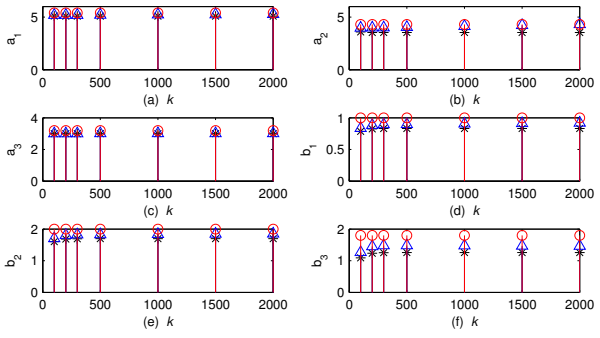

Fig. 4. The SRG and RG estimated parameters (SNR= 1.04).

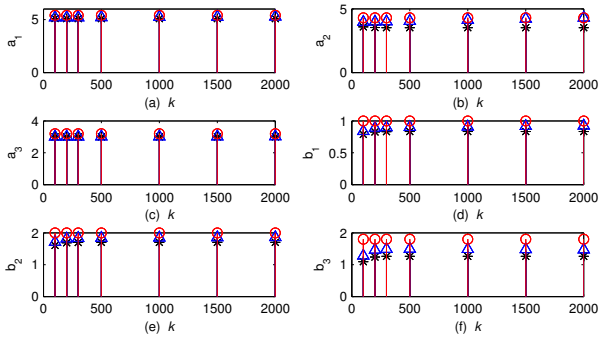

Fig. 5. The SRG and RG estimated parameters (SNR= 26.17).

over, the mark "$\circ$" denotes the original value, the mark "$*$" denotes the estimated value obtained by the recursive gradient (RG) method without separation and the mark "$\triangle$" denotes the estimated value obtained by the separable recursive gradient (SRG) method.

8. CONCLUSIONS

This paper studies the the identification of the transfer functions of process control systems for identifying the industrial process. For the purpose of obtaining higher estimation accuracy and on-line identification, a strategy of using dynamical data with increasing length is developed and applied to derive the identification method based on system response signals. Because the system response function is highly nonlinear, we analyze the relation between the system response function and the system parameters to be identified, and design a separable identification algorithm by separating the whole parameters of the systems into two parameter vectors. On the basis of the

separated parameter vectors and the gradient optimization, two separated identification models are built and two sub-algorithms are derived by combining them together and reciprocal estimation. Finally, the separable recursive gradient method is proposed for the process control systems. The numerical simulation results concerning the performance test and comparison show that the proposed separable recursive gradient method is practically feasible, which can be adopted for industrial applications. The proposed separable recursive gradient algorithm for dynamical systems based on the impulse response signals can combine and other estimation methods [57–59] to explore new identification methods and can be applied to other fields [60–64] such as information processing and engineering systems [65–71] and so on.

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