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Improved Function Augmented Sliding Mode Control of Uncertain Nonlinear Systems with Preassigned Settling Time

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Abstract: This paper investigates the robust tracking control of second-order uncertain nonlinear systems by adopting the function augmented sliding mode control approach. An improved version of this approach is proposed such that the exact information of the initial tracking error is not required when generating the desired trajectory of the tracking error. This evidently enlarges the application scope of the function augmented sliding mode control approach. Performance functions are introduced to form the performance envelopes for the sliding mode variables. A robust sliding mode controller is constructed such that the sliding mode variables are confined within their performance envelopes. This further guarantees that the tracking error converges to the given neighbourhood of zero within the preassigned settling time provided that proper control parameters are selected. An application example on the rendezvous control of spacecraft is also employed to illustrate the effectiveness of the proposed control approach.

Keywords: Nonlinear systems, preassigned settling time, robust control, sliding mode control, uncertain systems.

1. INTRODUCTION

Settling time, which characterizes the convergence rate of the system response, is recognized as an important performance specification of control system design. To achieve better performance and stronger robustness, fast convergence is usually pursued in practice. Therefore, finite time control has received considerable attention since it could guarantee that the state of a dynamic system converges to the desired point in finite time [1–5]. Although convergence can be achieved in finite time, the estimation of the settling time in the existing finite time control results explicitly depends on the initial state. This, to some extent, limits their application scope since the initial state may be unknown a priori. To overcome this problem, a strategy named fixed-time control is proposed [6,7]. It has been shown that fixed-time control could guarantee that the settling time is irrelevant to the initial condition. As a result, the research on the fixed-time control has been a hot topic and fruitful results have been obtained [8–10].

An exciting approach which can realize control with the preassigned settling time is called the function augmented sliding mode control approach, which is proposed by Park and Tsuj [11]. Different from the traditional sliding mode control approach where the sliding mode variable is constructed directly based on the tracking error, in the function augmented sliding mode control approach, a desired trajectory of the tracking error which converges to zero in the preassigned settling time is generated, and the sliding mode variable is constructed based on the error between the tracking error and its desired trajectory, then a controller is constructed to make the error between the tracking error and its desired trajectory always zero, which means that the tracking error converges to zero in the preassigned settling time as the desired trajectory converges to zero. The prominent advantage of this approach is that the settling time can be set arbitrarily a priori by adjusting the desired trajectory. Therefore, this approach has attracted considerable attention in recent years. In the theoretical extension aspect, [12] directly extends this result to high-order nonlinear systems; [13] proposes an adaptive function augmented sliding mode control approach for high-order uncertain nonlinear systems; [14] combines the function augmented sliding mode control approach with the fuzzy logic which is used to approximate the unknown nonlinear functions; [15] and [16] integrate the function

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augmented sliding mode control approach with the disturbance observer which is used to estimate the unknown disturbances; and [17] studies the adaptive tracking control of second-order nonlinear systems with nonlinearly parameterized uncertainties and disturbances, as well as multiplicative uncertainty in the control coefficient matrix. In the application research aspect, by adopting the function augmented sliding mode control approach, [18] addresses the control problem of aeroelastic systems; [19] studies the velocity control of a novel narrow vehicle based on the mobile wheeled inverted pendulum; [20] designs a three-dimensional guidance law with impact angle constraints; [21, 22] and [23] consider the integrated translational and rotational motion control of spacecraft; and [24] considers the control of robotic manipulators. More related results can be found in such as [25–27], and references therein.

Although a great progress has been made on the function augmented sliding mode control approach, there is a common problem in all of the existing results: to generate the desired trajectory of the tracking error which plays a key role in the function augmented sliding mode control design, the exact initial value of the tracking error must be known, however, this requirement cannot always be satisfied. For example, in practical applications, it may be required to generate the desired trajectory of the tracking error by using online numerical computation methods, in order to obtain some kind of optimal performance when the initial tracking error is relative large or in order to meet some particular requirement, for example, obstacle avoidance. In this situation, the desired trajectory generally cannot be obtained instantly since the online numerical computation needs a certain amount of time. This means that the desired trajectory cannot be generated by using the real value of the initial tracking error but its estimation. As a result, the existing function augmented sliding mode control results cannot be applied directly any more. In a word, how to design the function augmented sliding mode control algorithm when there is error between the real initial value of the tracking error and its estimation used to generate the desired trajectory is still a meaningful and challenging problem to be solved. This paper focuses on this problem, and an improved function augmented sliding mode control approach is proposed for a class of second order uncertain nonlinear systems. The main contributions of this paper can be summarized as follows. First of all, the exact value of the initial tracking error is not required any more when generating the desired trajectory of the tracking error. Hence, the application scope of the function augmented sliding mode control approach can be significantly enlarged. Secondly, compared with the existing function augmented sliding mode control results, the proposed one can make the control algorithm more simple when the initial values of the tracking error and its derivative are small, since in this case the desired trajectory of the tracking error can be simply set as zero rather than generated by using any other computation methods. Last but not least, the performance functions are introduced, which form the performance envelopes for the sliding mode variables, and the controller is constructed such that the sliding mode variables are confined within the performance envelopes, which could guarantee that the tracking error converges to the given neighbourhood of zero within the preassigned settling time with proper design parameter selection. This is the most significant difference between our result and the existing ones. The proposed control approach is finally applied to the rendezvous control of spacecraft, and simulation results show the effectiveness of our approach.

Throughout the paper, for a vector $\zeta = [\zeta_1, \cdots, \zeta_n]^{\mathrm{T}}$, define $|\zeta| = [|\zeta_1|, \cdots, |\zeta_n|]^{\mathrm{T}}$. For any positive integer n, define $\mathrm{I}_{[1,n]} = \{1, 2, \cdots, n\}$. $[a_{ij}]$ denotes the matrix whose (i, j) element is a_{ij}. $\mathcal{C}^i$, $i \in \{0, 1, 2, \cdots\}$ denotes the set of functions whose i-th derivative is continuous.

2. PROBLEM FORMULATION

Consider the following second-order vector nonlinear system

$$\ddot{x} = f(x, \dot{x}) + (I + \Delta) G(x, \dot{x})u + \delta(x, \dot{x}, t), \tag{1}$$

where $x \in \mathcal{R}^n$ and $\dot{x} \in \mathcal{R}^n$ are the states, $u \in \mathcal{R}^n$ is the input, $f(x, \dot{x})$ is an available $\mathcal{C}^0$ vector function, $G(x, \dot{x})$ is an available $\mathcal{C}^0$ invertible matrix function, Δ denotes the multiplicative uncertainty, of which the elements satisfy that

$$|\Delta_{ij}| \le \bar{\Delta}_{ij},\ i = 1, \cdots, n;\ j = 1, \cdots, n,$$

and $\|\bar{\Delta}\| < 1$ where $\bar{\Delta} = [\bar{\Delta}_{ij}]$, and $\delta(x, \dot{x}, t)$ is a $\mathcal{C}^0$ vector function, denoting the lumped uncertain term including internal uncertainties and external disturbances and satisfying that

$$|\delta_i(x, \dot{x}, t)| \le d_i,\ i = 1, \cdots, n, \tag{2}$$

where $d_i = d_i(x, \dot{x}, t) \ge 0$ are available $\mathcal{C}^0$ functions.

The control design objective of this paper is stated as follows: Given a reference signal x_d, a settling time $T_f > 0$ and admissible tracking error bounds ε_i, $i = 1, \cdots, n$, where T_f and ε_i are preassigned and independent from one another, design a proper control law for system (1) such that the states of the resulted closed-loop system are bounded and satisfy that the tracking errors $|x_i - x_{id}| \le \varepsilon_i$ $(i = 1, 2, \cdots, n)$ when $t \ge T_f$.

In this paper, the given reference signal is assumed to satisfy the following condition.

Assumption 1: The reference signal x_d satisfies that x_d, $\dot{x}_d$ and $\ddot{x}_d$ are all bounded.

3. ROBUST SLIDING MODE CONTROL DESIGN

Define the tracking error as

$$e = x - x_d, \tag{3}$$

and

$$z = e - \eta, \tag{4}$$

where $\eta : \Re_{\geq 0} \to \Re^n$ denotes the desired trajectory of the tracking error $e(t)$. That is, $z(t)$ denotes the error between the tracking error $e(t)$ and its desired trajectory $\eta(t)$. The desired trajectory of the tracking error $\eta(t)$ is defined by the user. In the traditional function augmented sliding mode control approach, $\eta(t)$ is determined according to the following conditions:

1) $\eta(t)$ is a piecewise $\mathcal{C}^2$ differentiable function defined on $[0,\infty)$;
2) $\eta(t)$, $\dot{\eta}(t)$ and $\ddot{\eta}(t)$ are all bounded;
3) $\eta(0) = e(0)$ and $\dot{\eta}(0) = \dot{e}(0)$;
4) $\eta(t) = 0$ and $\dot{\eta}(t) = 0$ when $t \geq T_f$.

According to the above four conditions, the function $\eta(t)$ can be constructed based on all kinds of function interpolation methods, viewing $t = 0$ and $t = T_f$ as the interpolation nodes. For example, in [11], $\eta_i(t)$, $t \in [0, T_f]$, $i = 1, 2, \cdots, n$ are constructed as cubic polynomials. As shown in [17], this may lead to large undershoot-like phenomenon or large overshoot-like phenomenon. To overcome these problems, in [17], $\eta_i(t)$, $t \in [0, T_f]$ are constructed as piece-wise cubic polynomials. However, when $\|e(0)\|$ is relative large, $\eta(t)$ may need to be generated by using online numerical methods instead of explicit formulas, in order to obtain some kind of optimal performance or meet some particular requirement. In this case, $\eta(t)$ generally cannot be generated instantly since the numerical computation needs a certain amount of time. Hence, the initial values $\eta(0)$ and $\dot{\eta}(0)$ used to generate $\eta(t)$ cannot respectively be the real values $e(0)$ and $\dot{e}(0)$ but their estimate values $\hat{e}(0)$ and $\hat{\dot{e}}(0)$. As a result, the third condition used to construct $\eta(t)$ should be replaced by the following condition.

3') $\eta(0) = \hat{e}(0)$ and $\dot{\eta}(0) = \hat{\dot{e}}(0)$, where $\hat{e}(0)$ and $\hat{\dot{e}}(0)$ are respectively the estimate values of $e(0)$ and $\dot{e}(0)$.

Generally, $\hat{e}(0)$ and $\hat{\dot{e}}(0)$ are close to $e(0)$ and $\dot{e}(0)$, respectively. Therefore, although $z(0) = e(0) - \eta(0)$ and $\dot{z}(0) = \dot{e}(0) - \dot{\eta}(0)$ are not zero, it is reasonable to suppose that they satisfy the following condition.

Assumption 2: The initial errors $z_i(0)$ and $\dot{z}_i(0)$, $i = 1, 2, \cdots, n$, satisfy that

$$|z_i(0)| \leq r_i,$$
$$|\dot{z}_i(0)| \leq r_i',$$

where r_i and r_i' are known constants.

Another interesting case is that, when $\|e(0)\|$ and $\|\dot{e}(0)\|$ are small, it is in fact only needed to set $\eta(t) \equiv 0$ rather than to generate $\eta(t)$ by using any other computation methods which would make the control algorithm complicated. In this case, it is also reasonable to assume that $z_i(0)$ and $\dot{z}_i(0)$ satisfy Assumption 2, since in this situation

$$z_i(0) = e_i(0) - \eta_i(0) = e_i(0),$$
$$\dot{z}_i(0) = \dot{e}_i(0) - \dot{\eta}_i(0) = \dot{e}_i(0).$$

Define the sliding mode variable as

$$s = \dot{z} + Cz, \tag{5}$$

where $C = \mathrm{diag}(c_1, \cdots, c_n)$ with $c_i > 0$ being design parameters.

Under Assumption 2, it is easy to obtain that

$$|s_i(0)| \leq |\dot{z}_i(0)| + c_i |z_i(0)| \leq r_i' + c_i r_i, \ i = 1, 2, \cdots, n.$$

Furthermore, one has the following result.

Theorem 1: View (5) as a dynamical system with $z(t)$ being the state and $s(t)$ the input. If there exist time varying functions

$$\rho_i(t) = (\rho_{i0} - \rho_{i\infty}) \exp(-c_i t) + \rho_{i\infty}, \ i = 1, 2, \cdots, n \tag{6}$$

with $\rho_{i0} \geq r_i' + c_i r_i$, $0 < \rho_{i\infty} < \rho_{i0}$ such that $|s_i(t)| \leq \rho_i(t)$, $\forall i \in I_{[1,n]}$, $\forall t \geq 0$, then

$$|z_i(t)| < \frac{\rho_{i0}}{c_i}, \quad |\dot{z}_i(t)| < 2\rho_{i0}, \ \forall i \in I_{[1,n]}, \ \forall t \geq 0;$$

in addition, if the design parameters satisfy that, $\forall i \in I_{[1,n]}$,

$$\frac{\rho_{i0} - c_i r_i}{c_i (\rho_{i0} - \rho_{i\infty})} \leq T_f, \tag{7}$$

and

$$\begin{aligned} &r_i \exp(-c_i T_f) + (\rho_{i0} - \rho_{i\infty}) \exp(-c_i T_f) T_f \\ &\quad + \frac{\rho_{i\infty}}{c_i} (1 - \exp(-c_i T_f)) \leq \varepsilon_i, \end{aligned} \tag{8}$$

then

$$|z_i(t)| \leq \varepsilon_i, \ \forall i \in I_{[1,n]}, \ \forall t \geq T_f.$$

Proof: From (5), one has that, $\forall i \in \mathrm{I}_{[1,n]}$,

$$\dot{z}_i = -c_i z_i + s_i, t \geq 0.$$

Its solution is

$$z_i(t) = \exp(-c_i t) z_i(0) + \int_0^t \exp[-c_i(t-\tau)] s_i(\tau) \mathrm{d}\tau.$$

Hence,

$$\begin{aligned} |z_i(t)| &= \left| \exp(-c_i t) z_i(0) + \int_0^t \exp[-c_i(t-\tau)] s_i(\tau) \mathrm{d}\tau \right| \\ &\leq \exp(-c_i t) |z_i(0)| + \int_0^t \exp[-c_i(t-\tau)] |s_i(\tau)| \mathrm{d}\tau \end{aligned}$$

$$\begin{aligned}\leq & r_i \exp(-c_i t) \\ & + \int_0^t \exp[-c_i(t-\tau)] \\ & \times [(\rho_{i0}-\rho_{i\infty})\exp(-c_i\tau)+\rho_{i\infty}]\mathrm{d}\tau \\ = & r_i \exp(-c_i t) + (\rho_{i0}-\rho_{i\infty})\exp(-c_i t)t \\ & + \frac{\rho_{i\infty}}{c_i}(1-\exp(-c_i t)).\end{aligned}$$

Define

$$\begin{aligned}\sigma(t) = & r_i \exp(-c_i t) + (\rho_{i0}-\rho_{i\infty})\exp(-c_i t)t \\ & + \frac{\rho_{i\infty}}{c_i}(1-\exp(-c_i t)),\end{aligned}$$

of which the derivative satisfies that

$$\dot{\sigma}(t) = [-c_i r_i - c_i(\rho_{i0}-\rho_{i\infty})t + \rho_{i0}]\exp(-c_i t).$$

From $\dot{\sigma}(t)=0$, it can be obtained that

$$t = t_{st} \triangleq \frac{\rho_{i0}-c_i r_i}{c_i(\rho_{i0}-\rho_{i\infty})}.$$

Clearly, when $t<t_{st}$, $\dot{\sigma}(t)>0$ and when $t>t_{st}$, $\dot{\sigma}(t)<0$. Hence, it can be obtained that, at $t=t_{st}$, $\sigma(t)$ has its maximum value

$$\begin{aligned}\sigma(t_{st}) = & r_i \exp(-c_i t_{st}) + (\rho_{i0}-\rho_{i\infty})\exp(-c_i t_{st})t_{st} \\ & + \frac{\rho_{i\infty}}{c_i}(1-\exp(-c_i t_{st})) \\ = & \frac{\rho_{i0}-\rho_{i\infty}}{c_i}\exp(-c_i t_{st}) + \frac{\rho_{i\infty}}{c_i} \\ < & \frac{\rho_{i0}}{c_i},\end{aligned}$$

which implies that, $\forall t \geq 0$,

$$|z_i(t)| \leq \sigma(t) \leq \sigma(t_{st}) < \frac{\rho_{i0}}{c_i},$$

and

$$\begin{aligned}|\dot{z}_i(t)| &= |-c_i z_i + s_i| \leq c_i|z_i| + |s_i| \\ &< c_i\frac{\rho_{i0}}{c_i} + \rho_{i0} = 2\rho_{i0}.\end{aligned}$$

Since $\sigma(t)$ is strictly decreasing when $t \geq t_{st}$, one has that, if $T_f \geq t_{st}$, then when $t \geq T_f$,

$$\begin{aligned}\sigma(t) \leq \sigma(T_f) = & r_i\exp(-c_i T_f) + (\rho_{i0}-\rho_{i\infty})\exp(-c_i T_f)T_f \\ & + \frac{\rho_{i\infty}}{c_i}(1-\exp(-c_i T_f)).\end{aligned}$$

Therefore, if the design parameters are selected such that conditions (7) and (8) are satisfied, then one has that

$$|z_i(t)| \leq \sigma(t) \leq \sigma(T_f) \leq \varepsilon_i,\ \forall t \geq T_f.$$

This completes the proof. □

Remark 1: The time varying function

$$\rho_i(t) = (\rho_{i0}-\rho_{i\infty})\exp(-c_i t) + \rho_{i\infty}$$

is called the performance function, and the region between $-\rho_i(t)$ and $\rho_i(t)$ is called the performance envelop for $s_i(t)$.

Remark 2: According to Theorem 1, if a control law is designed such that $\forall i \in I_{[1,n]}$, $s_i(t)$ is confined within the performance envelop formed by $\rho_i(t)$, that is, $|s_i(t)| \leq \rho_i(t), \forall t \geq 0$, and if the design parameters are chosen such that conditions (7) and (8) are satisfied for $i=1, 2, \cdots, n$, then one can obtain that $|z_i(t)| \leq \varepsilon_i$, $\forall i \in I_{[1,n]}$, $\forall t \geq T_f$. This further implies that, $\forall i \in I_{[1,n]}$,

$$|x_i(t)-x_{id}(t)| = |e_i(t)| = |z_i(t)+\eta_i(t)| \leq \varepsilon_i,$$

when $t \geq T_f$ since $\eta_i(t)=0$ when $t \geq T_f$, that is, the control objective can be achieved.

Considering that for any T_f, ε_i, r_i and r_i', one can select proper parameters c_i, ρ_{i0} and $\rho_{i\infty}$ such that conditions (7) and (8) are satisfied. Hence, to achieve the control objective, the remained main task is to design a control law such that $|s_i(t)| \leq \rho_i(t)$, $\forall i \in I_{[1,n]}$, $\forall t \geq 0$.

From (5), one has that

$$\begin{aligned}\dot{s} = & \ddot{z} + C\dot{z} \\ = & \ddot{e} - \ddot{\eta} + C(\dot{e}-\dot{\eta}) \\ = & \ddot{x} - \ddot{x}_d - \ddot{\eta} + C(\dot{x}-\dot{x}_d-\dot{\eta}) \\ = & f(x,\dot{x}) + (I+\Delta)G(x,\dot{x})u + \delta(x,\dot{x},t) - w,\end{aligned} \tag{9}$$

where

$$w = \ddot{x}_d + \ddot{\eta} - C(\dot{x}-\dot{x}_d-\dot{\eta}). \tag{10}$$

Define $\xi = \begin{bmatrix}\xi_1 & \cdots & \xi_n\end{bmatrix}^{\mathrm{T}}$ with

$$\xi_i = \frac{s_i}{\rho_i}, \tag{11}$$

then one has that

$$\begin{aligned}\dot{\xi} = & M_1(\dot{s}-M_2 s) \\ = & M_1[f(x,\dot{x}) + (I+\Delta)G(x,\dot{x})u + \delta(x,\dot{x},t) - w - M_2 s],\end{aligned}$$

where

$$M_1 = \begin{bmatrix}\frac{1}{\rho_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\rho_n}\end{bmatrix},$$

$$M_2 = \begin{bmatrix}\frac{\dot{\rho}_1}{\rho_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\dot{\rho}_n}{\rho_n}\end{bmatrix}.$$

Construct the robust sliding mode control law as follows:

$$u = G^{-1}(x,\dot{x})\left[-f(x,\dot{x})+w+M_2 s-\mathrm{SAT}(\xi)\chi\right], \tag{12}$$

where

$$\mathrm{SAT}(\xi)=\begin{bmatrix} \mathrm{sat}(\xi_1) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathrm{sat}(\xi_n) \end{bmatrix}$$

with

$$\mathrm{sat}(\xi_i)=\begin{cases} 1, & \xi_i>1, \\ \xi_i, & |\xi_i|\le 1, \quad i=1,2,\cdots,n, \\ -1, & \xi_i<-1, \end{cases}$$

and

$$\chi = (I_n-\bar{\Delta})^{-1}[\bar{\Delta}|f(x,\dot{x})-w-M_2 s|+d+k], \tag{13}$$

where $d=\begin{bmatrix} d_1 & \cdots & d_n\end{bmatrix}^{\mathrm{T}}$, $k=\begin{bmatrix} k_1 & \cdots & k_n\end{bmatrix}^{\mathrm{T}}$ with $k_i>0$ being design parameters.

For the stability of the resulted closed-loop system, one has the following theorem.

Theorem 2: System (1) with control law (12) under Assumptions 1 and 2 is such that x and $\dot{x}$ are bounded, and for the preassigned settling time $T_f>0$ and admissible tracking error bounds $\varepsilon_i>0$, the tracking errors satisfy that $|x_i-x_{id}|\le\varepsilon_i$ ($i=1,2,\cdots,n$) when $t\ge T_f$, if conditions (7) and (8) are satisfied for all $i\in I_{[1,n]}$.

Proof: Substituting (12) into (9) yields that

$$\begin{aligned}\dot{\xi}(t)=&M_1\big[f(x,\dot{x})\\ &+(I+\Delta)(-f(x,\dot{x})+w+M_2 s-\mathrm{SAT}(\xi)\chi)\\ &+\delta(x,\dot{x},t)-w-M_2 s\big]\\ =&M_1\big[-(I+\Delta)\mathrm{SAT}(\xi)\chi+\Delta(-f(x,\dot{x})+w+M_2 s)\\ &+\delta(x,\dot{x},t)\big].\end{aligned}$$

Consider the i-th ($\forall i\in \mathrm{I}_{[1,n]}$) subsystem

$$\begin{aligned}\dot{\xi}_i=&-\frac{1}{\rho_i}(1+\Delta_{ii})\mathrm{sat}(\xi_i)\chi_i-\frac{1}{\rho_i}\sum_{j=1,j\neq i}^{n}\Delta_{ij}\mathrm{sat}(\xi_j)\chi_j\\ &-\frac{1}{\rho_i}\sum_{j=1}^{n}\Delta_{ij}\left(f_j(x,\dot{x})-w_j-\frac{\dot{\rho}_j}{\rho_j}s_j\right)+\frac{1}{\rho_i}\delta_i(x,\dot{x},t).\end{aligned}$$

Define

$$V_i=\frac{1}{2}\xi_i^2.$$

Bear the inequalities on the absolute value in mind, it is easy to conclude that the time derivative of V_i satisfies the following inequality

$$\begin{aligned}\dot{V}_i=&\xi_i\dot{\xi}_i\\ =&-\frac{1}{\rho_i}(1+\Delta_{ii})\xi_i\mathrm{sat}(\xi_i)\chi_i-\frac{1}{\rho_i}\xi_i\sum_{j=1,j\neq i}^{n}\Delta_{ij}\mathrm{sat}(\xi_j)\chi_j\\ &-\frac{1}{\rho_i}\xi_i\sum_{j=1}^{n}\Delta_{ij}\left(f_j(x,\dot{x})-w_j-\frac{\dot{\rho}_j}{\rho_j}s_j\right)+\frac{1}{\rho_i}\xi_i\delta_i(x,\dot{x},t)\\ \le&-\frac{1}{\rho_i}(1-\bar{\Delta}_{ii})|\xi_i|\chi_i+\frac{1}{\rho_i}|\xi_i|\sum_{j=1,j\neq i}^{n}\bar{\Delta}_{ij}\chi_j\\ &+\frac{1}{\rho_i}|\xi_i|\sum_{j=1}^{n}\bar{\Delta}_{ij}\left|f_j(x,\dot{x})-w_j-\frac{\dot{\rho}_j}{\rho_j}s_j\right|+\frac{1}{\rho_i}|\xi_i|d_i,\end{aligned}$$

when $|\xi_i|\ge 1$, where the well-known triangle inequality is used repeatedly. Considering that the derivation of this inequality is tedious but not difficult, the details are omitted here. Since condition (13) is equivalent to

$$\begin{aligned}\left(1-\bar{\Delta}_{ii}\right)\chi_i=&\sum_{j=1,j\neq i}^{n}\bar{\Delta}_{ij}\chi_j+d_i+k_i\\ &+\sum_{j=1}^{n}\bar{\Delta}_{ij}\left|f_j(x,\dot{x})-w_j-\frac{\dot{\rho}_j}{\rho_j}s_j\right|,\end{aligned}$$

one has that

$$\dot{V}_i\le-\frac{k_i}{\rho_i}|\xi_i|<0,$$

when $|\xi_i|\ge 1$. This implies that

$$|\xi_i(t)|\le 1,\ \ \forall t\ge 0,$$

since

$$|\xi_i(0)|=\left|\frac{s_i(0)}{\rho_i(0)}\right|\le\frac{r_i'+c_i r_i}{\rho_{i0}}\le 1.$$

Hence, $\left|\frac{s_i(t)}{\rho_i(t)}\right|\le 1$, $\forall t\ge 0$, or equivalently, $|s_i(t)|\le\rho_i(t)$, $\forall t\ge 0$.

According to Theorem 1, one can conclude that

$$\begin{aligned}&|z_i(t)|<\frac{\rho_{i0}}{c_i},\\ &|\dot{z}_i(t)|<2\rho_{i0},\forall t\ge 0,\end{aligned}$$

or equivalently,

$$\begin{aligned}&|x_i-(x_{id}+\eta_i)|<\frac{\rho_{i0}}{c_i},\\ &|\dot{x}_i-(\dot{x}_{id}+\dot{\eta}_i)|<2\rho_{i0}.\end{aligned}$$

Therefore, it is easy to obtain that

$$\begin{aligned}|x_i|=&|x_i-(x_{id}+\eta_i)+(x_{id}+\eta_i)|\\ \le&|x_{id}+\eta_i|+|x_i-(x_{id}+\eta_i)|\\ <&|x_{id}|+|\eta_i|+\frac{\rho_{i0}}{c_i},\end{aligned}$$

and

$$|\dot{x}_i|=|\dot{x}_i-(\dot{x}_{id}+\dot{\eta}_i)+(\dot{x}_{id}+\dot{\eta}_i)|$$

$$\leq |\dot{x}_{id}+\dot{\eta}_i|+|\dot{x}_i-(\dot{x}_{id}+\dot{\eta}_i)|$$
$$< |\dot{x}_{id}|+|\dot{\eta}_i|+2\rho_{i0}.$$

According to Assumption 1 and the conditions on η, one has that x_{id}, $\dot{x}_{id}$, η_i and $\dot{\eta}_i$ are all bounded. Hence, x_i and $\dot{x}_i$ are bounded.

In addition, as shown in Remark 2, if conditions (7) and (8) are satisfied, then one has that $|x_i(t)-x_{id}(t)|\leq\varepsilon_i$ when $t\geq T_f$. This completes the proof. □

4. APPLICATION ON THE RENDEZVOUS CONTROL OF SPACECRAFT

In this section, the obtained results are applied to the rendezvous control of spacecraft. If the target orbit is circular, then the motion of the chaser relative to the target in the local-vertical-local-horizontal reference frame centered on the target is governed by (please see [28])

$$\ddot{x}=f(x,\dot{x})+u+a_d, \tag{14}$$

where

$$x=\begin{bmatrix}x_r\\y_r\\z_r\end{bmatrix},$$

$$f(x,\dot{x})=\begin{bmatrix}n_0^2x_r+2n_0\dot{y}_r-\dfrac{\mu_g(x_r+R)}{[y_r^2+z_r^2+(x_r+R)^2]^{\frac{3}{2}}}+\dfrac{\mu_g}{R^2}\\ -2n_0\dot{x}_r+n_0^2y_r-\dfrac{\mu_g y_r}{[y_r^2+z_r^2+(x_r+R)^2]^{\frac{3}{2}}}\\ -\dfrac{\mu_g z_r}{[y_r^2+z_r^2+(x_r+R)^2]^{\frac{3}{2}}}\end{bmatrix},$$

with $x=[x_r\ y_r\ z_r]^{\mathrm{T}}$ being the relative position of the chaser with respect to the target, R the scalar radius of the target from the center of the Earth, n_0 the orbital angular rate of the target, μ_g the gravitational parameter, $u=a_c=[a_{c1}\ a_{c2}\ a_{c3}]^{\mathrm{T}}$ the control acceleration produced by the propeller, and $a_d=[a_{d1}\ a_{d2}\ a_{d3}]^{\mathrm{T}}$ the disturbance acceleration. a_d is assumed to be bounded, that is, $|a_{di}|\leq d_i$, where d_i are known positive constants.

The control objective is stated as follows: Given a settling time $T_f=100$ s, and admissible tracking error bounds $\varepsilon_i=0.05$ m, $i=1,2,3$, design a proper control law for system (14) such that x_r converges to a small neighborhood of x_{rf} which denotes the desired final value of x_r when completing the rendezvous task, y_r and z_r converge to a small neighborhood of zero, and when $t\geq T_f$, $|x_r-x_{rf}|\leq\varepsilon_1$, $|y_r|\leq\varepsilon_2$ and $|z_r|\leq\varepsilon_3$.

Theorem 2 is employed to design the control law. The sliding mode variable is defined as

$$s=\dot{z}+Cz,$$

where $C=\mathrm{diag}(c_1,c_2,c_3)$ with $c_i>0$, and

$$z=e-\eta,$$

where

$$e=x-\begin{bmatrix}x_{rf}\\0\\0\end{bmatrix}.$$

Suppose that the chaser is not near to the target and some online trajectory planning scheme is needed to generate $\eta:\Re_{\geq 0}\to\Re^3$ based on the four conditions 1), 2), 3') and 4) stated in Section 3.

When the real values $x(0)$ and $\dot{x}(0)$ are obtained, one can determine $r_i\geq|z_i(0)|=|e_i(0)-\eta_i(0)|$ and $r_i'\geq|\dot{z}_i(0)|=|\dot{e}_i(0)-\dot{\eta}_i(0)|$, $i=1,\,2,\,3$. Further, one can obtain the performance functions

$$\rho_i(t)=(\rho_{i0}-\rho_{i\infty})\exp(-c_it)+\rho_{i\infty},\ i=1,2,3$$

with $\rho_{i0}\geq r_i'+c_ir_i$, $0<\rho_{i\infty}<\rho_{i0}$ and define

$$\xi_i=\frac{s_i}{\rho_i},\ i=1,2,3.$$

Then, the control law can be obtained as follows:

$$u=-f(x,\dot{x})+w+M_2s-\mathrm{SAT}(\xi)(d+k),$$

where

$$\xi=[\ \xi_1\quad \xi_2\quad \xi_3\ ]^{\mathrm{T}},$$
$$w=\ddot{\eta}-C(\dot{x}-\dot{\eta}),$$
$$M_2=\begin{bmatrix}\frac{\dot{\rho}_1}{\rho_1}&0&0\\0&\frac{\dot{\rho}_2}{\rho_2}&0\\0&0&\frac{\dot{\rho}_3}{\rho_3}\end{bmatrix},$$
$$\mathrm{SAT}(\xi)=\begin{bmatrix}\mathrm{sat}(\xi_1)&0&0\\0&\mathrm{sat}(\xi_2)&0\\0&0&\mathrm{sat}(\xi_3)\end{bmatrix},$$
$$d=[\ d_1\quad d_2\quad d_3\ ]^{\mathrm{T}},$$
$$k=[\ k_1\quad k_2\quad k_3\ ]^{\mathrm{T}},$$

with $k_i>0$. The design parameters are selected such that conditions (7) and (8) are satisfied for $i=1,\,2,\,3$.

For numerical simulation, the system parameters are set as follows: $R=42241$ km, $n_0=7.2722\times10^{-5}$ rad/s, and $\mu_g=3.986\times10^{14}$ m^3/s^2. The upper bound of $|a_{di}|$ are set as $d_i=0.1$ m/s^2, $i=1,\,2,\,3$. x_{rf} is set to be 2 m. The estimate values of the initial states are $\hat{x}(0)=[\ 50\quad 0\quad 0\ ]^{\mathrm{T}}$ m and $\hat{\dot{x}}(0)=[0\ 0\ 0]^{\mathrm{T}}$m/s, but their real values are $x(0)=[\ 49\quad 1\quad -0.5\ ]^{\mathrm{T}}$m and $\dot{x}(0)=[\ -0.1\quad 0.1\quad -0.1\ ]^{\mathrm{T}}$ m/s. For simplicity, suppose the desired trajectory η is generated as follows:

$$\eta_i(t)=\begin{cases}a_{i0}+a_{i1}t+a_{i2}t^2+a_{i3}t^3, & \text{if } 0\leq t\leq T_f,\\ 0, & \text{if } t>T_f,\end{cases} \tag{15}$$

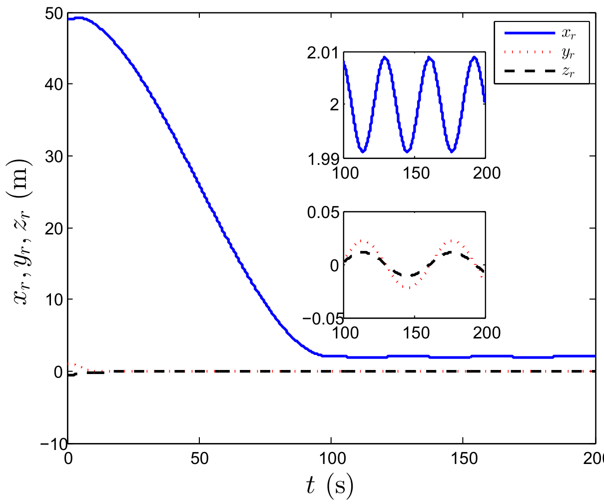

Fig. 1. Curves of x_r, y_r and z_r.

where $i = 1, 2, 3$,

$$a_{i0} = \hat{e}_i(0), a_{i1} = \dot{\hat{e}}_i(0),$$
$$a_{i2} = -3(\hat{e}_i(0)/T_f^2) - 2(\dot{\hat{e}}_i(0)/T_f),$$
$$a_{i3} = 2(\hat{e}_i(0)/T_f^3) + \dot{\hat{e}}_i(0)/T_f^2.$$

It is easy to check that η satisfies the four conditions (1), (2), (3') and (4). The parameters are set as follows: $C = \text{diag}(0.2, 0.2, 0.2)$, $r_i = 1$, $r_i' = 0.1$, $k_i = 0.1$, $\rho_{i0} = 0.5$, $\rho_{i\infty} = 0.01$. It is easy to check that these parameters satisfy conditions (7) and (8) for $i = 1, 2, 3$. And the disturbance acceleration is set as

$$a_d = \begin{bmatrix} 0.05\cos(0.2t) \\ -0.1\sin(0.1t) \\ -0.05\sin(0.1t) \end{bmatrix} \text{ m/s}^2.$$

The simulation results are given in Fig. 1 to Fig. 4. Fig. 1 shows the relative position variables of the chaser with respect to the target. Clearly, x_r converges to a small neighborhood of x_{rf}, y_r and z_r both converge to a small neighborhood of zero, and when $t \geq T_f = 100$s, $|x_r - x_{rf}| < 0.05$m, $|y_r| < 0.05$m and $|z_r| < 0.05$m. This means that the control objective is achieved. Fig. 2 shows the change rate of the relative position variables. Fig. 3 shows the curves of the control accelerations. The magnitude of every control signal is reasonable. And Fig. 4 shows the curves of the sliding mode variables. One can see that $|s_i(t)| < \rho_i(t), \forall t \geq 0$, that is, every $s_i(t)$ is confined within the performance envelope formed by $\rho_i(t)$, this coincides with the obtained theoretical result. In a word, the presented simulation results show the correctness and effectiveness of the proposed control method.

To further illustrate the robustness of the obtained control law, let us consider the situation when the measured signals x_r, y_r, z_r, $\dot{x}_r$, $\dot{y}_r$ and $\dot{z}_r$ are contaminated by measurement noises. The measurement noises imposed on the

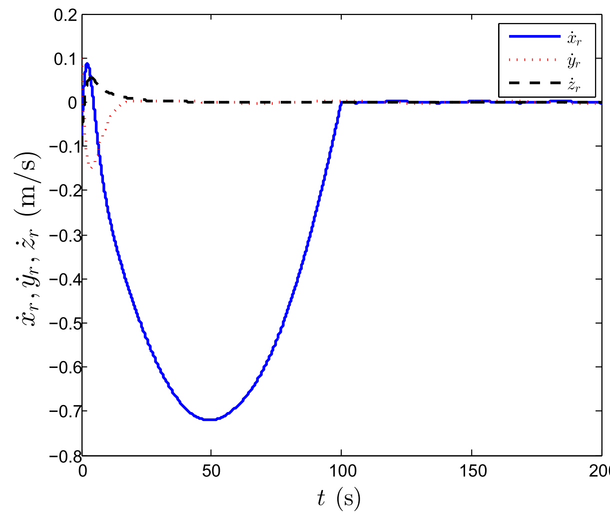

Fig. 2. Curves of $\dot{x}_r$, $\dot{y}_r$ and $\dot{z}_r$.

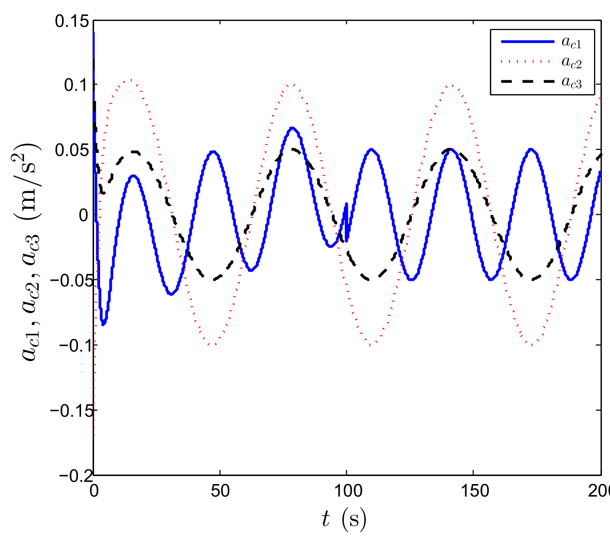

Fig. 3. Curves of a_{c1}, a_{c2} and a_{c3}.

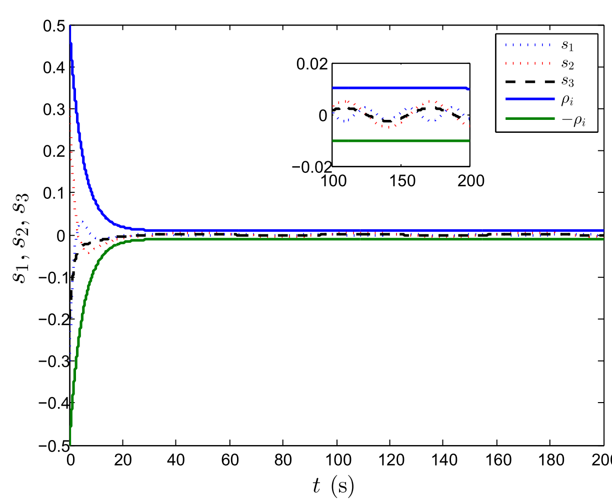

Fig. 4. Curves of s_1, s_2 and s_3.

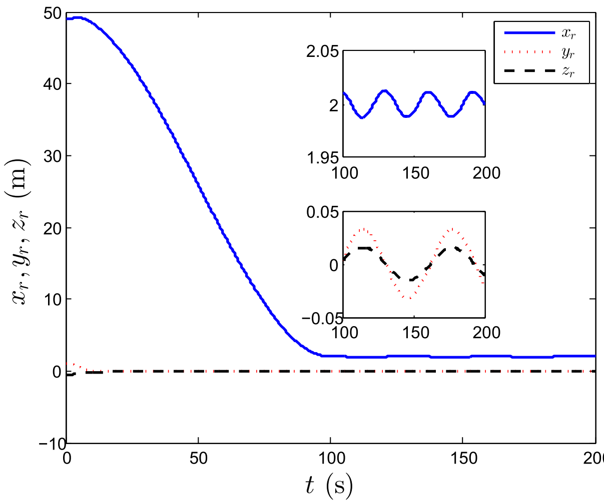

Fig. 5. Curves of x_r, y_r and z_r when considering measurement noises.

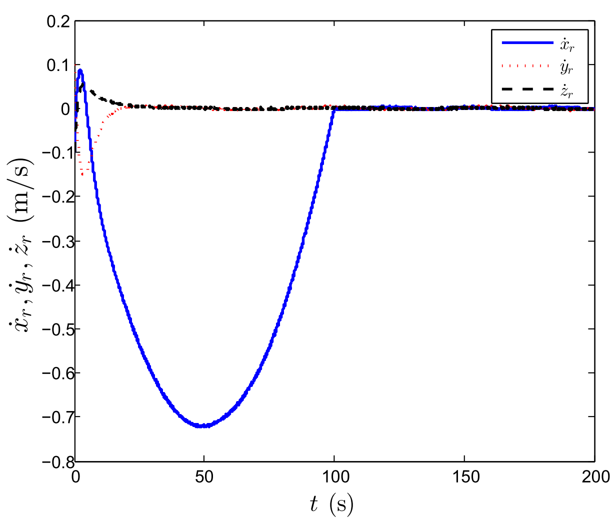

Fig. 6. Curves of $\dot{x}_r$, $\dot{y}_r$ and $\dot{z}_r$ when considering measurement noises.

feedback signals x_r, y_r and z_r are set to be zero mean value and standard deviation 0.02 m; and the measurement noises imposed on the feedback signals $\dot{x}_r$, $\dot{y}_r$ and $\dot{z}_r$ are set to be zero mean value and standard deviation 0.005 m/s.

The simulation results are given in Fig. 5 to Fig. 7, which respectively show the relative position variables of the chaser with respect to the target, the change rate of the relative position variables and the curves of the control accelerations. Fig. 5 and Fig. 6 are similar to Fig. 1 and Fig. 2, respectively. It can also be seen that when $t \geq T_f = 100$s, $|x_r - x_{rf}| < 0.05$m, $|y_r| < 0.05$m and $|z_r| < 0.05$m. This means that the obtained control law possesses good robustness property against the measurement noises. Fig. 7 is different from Fig. 4, but it presents a normal phenomenon caused by the measurement noises which are not further dealt with by utilizing filters here.

Remark 3: In [17], the rendezvous control problem of spacecraft is also considered by using the adaptive function augmented sliding mode control approach. Like all the other results on the function augmented sliding mode control approach, the exact initial values of the relative position variables are required there to generate the function $\eta(t)$. If the exact initial values of the relative position variables are unknown when generating $\eta(t)$, just like the situation considered in this example, the function augmented sliding mode control methods obtained in [17] as well as the other references are not applicable. So our results evidently enlarge the application scope of the function augmented sliding mode control approach.

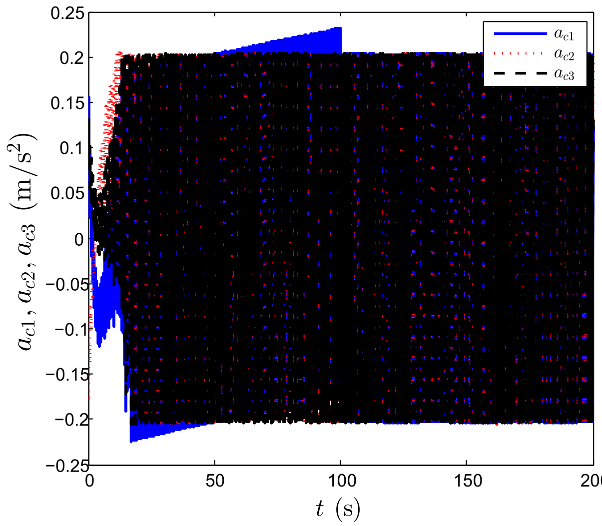

Fig. 7. Curves of a_{c1}, a_{c2} and a_{c3} when considering measurement noises.

5. CONCLUSION

This paper proposes an improved function augmented sliding mode control approach for the tracking control of second-order nonlinear systems with uncertainties including nonlinearly parameterized uncertainties, external disturbances, and multiplicative uncertainty in the control coefficient matrix. Different from the existing results, the proposed approach does not need the exact information of the initial tracking error to generate the desired trajectory of the tracking error, thus evidently enlarges the application scope of the function augmented sliding mode control approach. By introducing performance functions, performance envelopes for the sliding mode variables are formed. Then a robust sliding mode controller is constructed such that the sliding mode variables are confined within their performance envelopes, and this could further guarantee that the tracking error converges to the given neighbourhood of zero within the given settling time provided that proper control parameters are selected. An application example on the rendezvous control of spacecraft shows the correctness and effectiveness of the proposed method. In this paper, all the states are assumed to be available, but in the practical applications, only the output variables can be measured. In this case, how to design the output feedback control law is of great value and will be studied in the near future.

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