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Robust Fractional-order PID Tuning Method for a Plant with an Uncertain Parameter

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Abstract: The robust design of fractional-order proportional-integral-differential (FOPID) controllers for controlled plants with uncertainty is a popular research topic. The well-studied "flat phase" condition is effective for the gain variation but not for variations in other parameters. This paper addresses the problem of tuning a robust FOPID controller for a plant with a known structure and an uncertain parameter (a coefficient or order in the plant transfer function). The method is based on preserving the phase margin of the open-loop system when the plant parameter varies around the nominal value. First, the partial derivatives of the gain crossover frequency with respect to the plant parameters are calculated. Then, the partial derivatives of the phase margin with respect to the plant parameters are obtained as the robust performance indexes. In addition, the equations needed to compute FOPID parameters that meet the specifications in the frequency domain are obtained and used as nonlinear constraints. Finally, the FOPID parameters can be obtained by optimizing the robust performance indexes under these constraints. Simulation experiments are carried out on examples with different types of uncertain parameters to verify the effectiveness of the tuning method. The results show that the requirements are fulfilled and that the system with the proposed FOPID controller is stable and robust to variations in the uncertain parameters. Comparisons clearly show that the controllers designed by the proposed method provide relatively robust performance.

Keywords: FOPID controller, phase margin, plant parameter, robustness.

1. INTRODUCTION

Research and applications in the fields of fractional calculus and nonintegral calculus have been increasingly favored and popularized by researchers over the past two decades, mainly because fractional calculus has been shown to play a prominent role in broad and abundant fields of science and engineering. To date, this tool has been successfully and widely applied in various fields, such as control systems [1–11] image processing [12], thermal systems [13], signal processing [14], and electrochemistry [15]. In the abovementioned applications, the performance of fractional-order controllers based on fractional calculus has been found to be outstanding, particularly when compared with the classic integer-order controller. In fact, fractional-order controllers can offer more possibilities for improving system performance than integer-order controllers [16].

According to multiple reports, industrial control is still dominated by the PID controller despite the rapid development of control theory and technology [17–19]. This can be attributed to its simple structure, strong robustness, reliable performance and easy implementation and manipulation in hardware. However, the general increase in the complexity of modern engineering platforms has led to a gradual increase in stringency in the selection of controllers, which has motivated the re-engineering of the PID framework. To preserve the characteristics of the classic integer-order PID (IOPID) controller and take advantage of fractional-order control, the FOPID controller was first proposed by Podlubny [20]. As a generalization of the IOPID controller, its advantage is that in addition to the three gains, the tunable parameters also include differential and integral orders; thus, the controller offers more freedom than IOPID controllers. Because of the increase in the number of tunable parameters, the FOPID controller can be demonstrated to be significantly superior to its predecessors in terms of its dynamic and steady-state performance.

Inevitably, considerable uncertainty arises in systems as a result of inaccurate modeling or environmental changes [21]. Moreover, intricate and diverse uncertainties are

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difficult to identify and eliminate completely. Therefore, much emphasis has been placed on robust control strategies, and various robust tuning rules for FOPID controllers have been explored and described. Usually, a plant with uncertainty has a known structure, but the parameters are unknown [22, 23]. To obtain robustness to variations in these parameters, tuning FOPID controllers in the frequency domain is one of the most widely used options because of its intuitiveness and convenience in operation. To impart robustness against gain variations, the "flat phase" condition is regarded as an important part of the design in many reports [24–28]. In [29], high-frequency noise and output disturbances are rejected by specifying a sensitivity function and complementary sensitivity function. In [30], a simple technique based on a Bode ideal transfer function for tuning fractional-order PI controllers is recommended. In addition to the gain, variations in the time constants of the model of a plant are unavoidable. For this reason, robust tuning strategies for time constant uncertainty have been discussed in the literature. In [31], from the derivatives of the plant's phase and modulus with respect to the time constant and the crossover frequency, equations are established to determine the robustness of the time constant. In addition, robust rules against large uncertainty in the time constant of a system are available in [32]. However, these robust tuning methods do not work for other uncertain parameters of the plant.

Most existing works use the "flat phase" condition to design robust FOPID controllers, and the obtained controllers are robust to gain variations but not to variations in other plant parameters. This paper focuses on the robust FOPID tuning method for fractional plants with an uncertain coefficient or order. To obtain robustness, an effective approach should be to preserve the phase margin of the control system when the plant parameter varies. However, it is difficult to determine the behavior of the phase margin when the parameter is disturbed. The deviations in the parameter will cause variations in the gain crossover frequency, which is closely related to the phase margin. To address these problems, we construct an implicit function consisting of the gain crossover frequency and plant parameters. Then, the partial derivatives of the phase margin with respect to all the plant parameters can be obtained analytically. These partial derivatives are used as robust performance indexes in the tuning of FOPID controllers. The main contributions of our work are summarized as follows:

1) The controlled plant is fractional, and the uncertain parameters are not limited to the gain but include the coefficients and orders. These features show that the proposed method has universal applicability.

2) The expressions of the robust performance indexes for all the plant parameters are developed. They are extensions of the "flat phase" condition.

3) New equations for FOPID parameters to satisfy the required phase margin and gain crossover frequency are formulated to reduce the computational burden.

The rest of this paper is organized as follows: In Section 2, the descriptions of fractional-order systems and the gain robustness condition are addressed. The novel robust design method for the FOPID controller is presented in Section 3. In Section 4, simulation examples are carried out to verify the proposed method. The conclusions of this work are drawn in Section 5.

2. PRELIMINARIES

2.1. Definition of fractional-order systems

A fractional-order system is based on fractional calculus. At present, several mathematical expressions of fractional differentiation have been derived through different approaches [33]. Here, the Riemann-Liouville (RL) expression is given as

$$ {}_{t_0}D_t^p f(t) = \frac{1}{\Gamma(n-p)} \frac{d^n}{dt^n} \int_{t_0}^{t} (t-\tau)^{n-p-1} f(\tau)\, d\tau, \tag{1}$$

where n is an integer, $n-1<p<n$, t_0 and t are the limits, $\Gamma(\cdot)$ represents the gamma function, and p is the order of the differentiation.

The transfer function of a linear fractional-order system can be given as

$$ P_{fo}(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\gamma_n} + a_{n-1} s^{\gamma_{n-1}} + \cdots + a_0 s^{\gamma_0}} e^{-Ls}, \tag{2}$$

where L is the time delay; a_i ($i = 0, 1, ..., n$) and b_k ($k = 0, 1, ..., m$) are coefficients; γ_i and β_k are real numbers, where $0 \le \gamma_0 \le \gamma_1 \le \cdots \le \gamma_n$ and $0 \le \beta_0 \le \beta_1 \le \cdots \le \beta_m$.

In this paper, we consider the following controlled plant:

$$ P(s) = \frac{k}{a_n s^{\gamma_n} + a_{n-1} s^{\gamma_{n-1}} + \cdots + a_0 s^{\gamma_0}} e^{-Ls}, \tag{3}$$

where k is the gain. The coefficients and fractional orders of $P(s)$ are arranged well to ensure a proper plant.

The standard transfer function of a FOPID controller is as follows:

$$ C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu}, \tag{4}$$

where k_p, k_i, and k_d represent the gains of the proportion, fractional integration and fractional differentiation components, respectively. The parameters λ and μ are the orders of the fractional calculus; $0 < \lambda$ and $\mu < 2$.

2.2. Robust tuning method for systems with gain variation

Consider the feedback control system shown in Fig. 1. The open-loop transfer function of the system model containing the FOPID controller is

$$ G(s) = C(s)P(s). \tag{5}$$

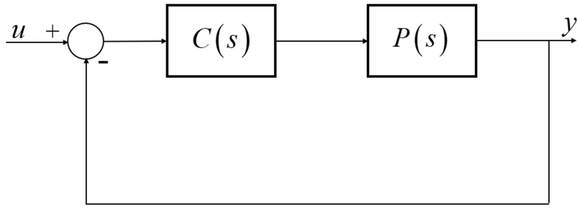

Fig. 1. Feedback control system.

To obtain FOPID controllers that are robust to variation in the gain of the system, a tuning method has been widely reported [5,29,34–36] that contains three primary specifications in the frequency domain:

(i) Gain crossover frequency specification:

$$|G(j\omega_c)| = 1, \tag{6}$$

where ω_c is the specified gain crossover frequency.

(ii) Phase margin specification:

$$\arg(G(j\omega_c)) = -\pi + \phi_{PM}. \tag{7}$$

As an important measure of robustness, an appropriate phase margin ϕ_{PM} is always taken into account in controller design.

(iii) Robustness to gain variations ("flat phase" condition):

$$\left.\frac{d(\arg(G(j\omega)))}{d\omega}\right|_{\omega=\omega_c} = 0. \tag{8}$$

The phase of the open-loop transfer function is guaranteed to be flat near the gain crossover frequency under this condition; that is, the phase remains constant within a certain interval around ω_c.

3. A NEW ROBUST TUNING METHOD

3.1. Frequency response of the control system

By substituting $s = j\omega$ into the controlled plant $P(s)$ described in (3), we have

$$P(j\omega) = \frac{ke^{-jL\omega}}{A_1(\omega) + jA_2(\omega)}, \tag{9}$$

where

$$A_1(\omega) = \sum_{i=0}^{n} a_i \omega^{\gamma_i} \cos\frac{\pi}{2}\gamma_i, \tag{10}$$

$$A_2(\omega) = \sum_{i=0}^{n} a_i \omega^{\gamma_i} \sin\frac{\pi}{2}\gamma_i. \tag{11}$$

The well-known equation $j = e^{j0.5\pi}$ has been applied in the above calculation. Similarly, the FOPID controllers in (4) can be described as

$$C(j\omega) = C_1(\omega) + jC_2(\omega), \tag{12}$$

where

$$C_1(\omega) = k_p + k_i\omega^{-\lambda}\cos\frac{\pi}{2}\lambda + k_d\omega^{\mu}\cos\frac{\pi}{2}\mu, \tag{13}$$

$$C_2(\omega) = -k_i\omega^{-\lambda}\sin\frac{\pi}{2}\lambda + k_d\omega^{\mu}\sin\frac{\pi}{2}\mu. \tag{14}$$

From (5), the transfer function of the open-loop system $G(j\omega)$ is given as

$$G(j\omega) = P(j\omega)C(j\omega) = \frac{ke^{-jL\omega}}{A_1(\omega) + jA_2(\omega)}\left(C_1(\omega) + jC_2(\omega)\right). \tag{15}$$

For convenience in the following sections, the variables A_1, A_2, C_1 and C_2 denote $A_1(\omega_c)$, $A_2(\omega_c)$, $C_1(\omega_c)$ and $C_2(\omega_c)$, respectively.

3.2. Robust performance indexes

To obtain a FOPID controller that is robust to the plant parameters, all the plant parameters k, a_i, γ_i in (3) are considered variables. From the specification in (6), the parameters of the plant and FOPID controller can be related to the gain crossover frequency. This is a complex implicit relationship that can be described as follows:

$$|G(j\omega_c)| = \left|\frac{k}{A_1 + jA_2}\right| |C_1 + jC_2| = 1. \tag{16}$$

We define an implicit function F, which (16) shows is equivalent to:

$$F = k^2\left(C_1^2 + C_2^2\right) - \left(A_1^2 + A_2^2\right) = 0. \tag{17}$$

Assume that the specified nominal gain crossover frequency is ω_{0c}. When subjected to environmental or internal physical changes, the system parameters are perturbed. As a result, the parameter ω_{0c} changes to satisfy (17). Therefore, the relationship of ω_c to a_i, γ_i, and k can be described by an implicit function through (17). From this function, we can calculate the partial derivative of ω_c with respect to each parameter as follows:

$$\frac{\partial\omega_c}{\partial a_i} = -\frac{F_{a_i}}{F_{\omega_c}}, \tag{18}$$

$$\frac{\partial\omega_c}{\partial\gamma_i} = -\frac{F_{\gamma_i}}{F_{\omega_c}}, \tag{19}$$

$$\frac{\partial\omega_c}{\partial k} = -\frac{F_k}{F_{\omega_c}}, \tag{20}$$

where F_{ω_c}, F_{a_i}, F_{γ_i} and F_k denote the partial derivatives of F with respect to ω_c, a_i, γ_i and k, respectively.

Equations (18)-(20) represent the rate of change in the gain crossover frequency caused by parameter variation. Now, we focus on the phase of the open-loop transfer function. From (15), the phase at ω_c can be expressed as

$$\phi_M = \arg(G(j\omega_c))$$

$$=\arg(C_1+jC_2)-\arg(A_1+jA_2)-L\omega_c \\ =-\pi+\phi_{PM}. \quad (21)$$

Since ω_c is a variable related to the plant parameters, the phase ϕ_M can be considered to be dependent on only the plant parameters when the controller is specified. Assuming that a_i, γ_i, and k undergo slight disturbances Δa_i, $\Delta\gamma_i$, and Δk, respectively, the corresponding phase variation $\Delta\phi_M$ can be expressed as

$$\Delta\phi_M=\sum_{i=0}^{n}\frac{\partial\phi_M}{\partial a_i}\Delta a_i+\sum_{i=0}^{n}\frac{\partial\phi_M}{\partial\gamma_i}\Delta\gamma_i+\frac{\partial\phi_M}{\partial k}\Delta k, \quad (22)$$

where the partial derivatives represent the slope of the phase with respect to the parameters, reflecting the sensitivity of the phase with respect to these parameter variations. Regarding ω_c as a function of the plant parameters determined by (17), the partial derivatives of ϕ_M with respect to these parameters can be determined. After using some mathematical manipulations, we obtain

$$\frac{\partial\phi_M}{\partial a_i}=XF_{a_i}-Y_1(i), \quad (23)$$

$$\frac{\partial\phi_M}{\partial\gamma_i}=XF_{\gamma_i}-a_i(\ln\omega_c)Y_1(i)-Y_2(i), \quad (24)$$

$$\frac{\partial\phi_M}{\partial k}=XF_k, \quad (25)$$

where the variables X, $Y_1(i)$ and $Y_2(i)$ are

$$X=\left(-\frac{1}{F_{\omega_c}}\right)(-X_A+X_C-L), \quad (26)$$

$$X_A=\frac{\sum_{i=0}^{n-1}\sum_{j=i+1}^{n}a_ia_j\omega_c^{\gamma_i+\gamma_j-1}(\gamma_i-\gamma_j)\sin\frac{\pi}{2}(\gamma_i-\gamma_j)}{\sum_{i=0}^{n}(a_i\omega_c^{\gamma_i})^2+2\sum_{0\le i<j\le n}a_ia_j\omega_c^{\gamma_i+\gamma_j}\cos\frac{\pi}{2}(\gamma_i-\gamma_j)}, \quad (27)$$

$$X_C=\frac{C_1(\partial C_2/\partial\omega_c)-C_2(\partial C_1/\partial\omega_c)}{C_1^2+C_2^2}, \quad (28)$$

$$Y_1(i)=\frac{\omega_c^{\gamma_i}\sum_{j=0}^{n}a_j\omega_c^{\gamma_j}\sin\frac{\pi}{2}(\gamma_i-\gamma_j)}{\sum_{i=0}^{n}\left(a_i\omega_c^{\gamma_i}\right)^2+2\sum_{0\le i<j\le n}a_ia_j\omega_c^{\gamma_i+\gamma_j}\cos\frac{\pi}{2}(\gamma_i-\gamma_j)}, \quad (29)$$

$$Y_2(i)=\frac{\frac{\pi}{2}a_i\omega_c^{\gamma_i}\sum_{j=0}^{n}a_j\omega_c^{\gamma_j}\cos\frac{\pi}{2}(\gamma_i-\gamma_j)}{\sum_{i=0}^{n}\left(a_i\omega_c^{\gamma_i}\right)^2+2\sum_{0\le i<j\le n}a_ia_j\omega_c^{\gamma_i+\gamma_j}\cos\frac{\pi}{2}(\gamma_i-\gamma_j)}. \quad (30)$$

To preserve the phase margin when a plant parameter p ($p\in\{a_i,\gamma_i,k\}$) changes, the partial derivative of ϕ_M with respect to p should be kept as close to zero as possible. Therefore, the robust performance index for the parameter p can be expressed as

$$R(p)=\left|\frac{\partial\phi_M}{\partial p}\right|. \quad (31)$$

Remark 1: All the partial derivatives of ϕ_M with respect to p contain the parameters of the FOPID controller. That is, the value of $R(p)$ can be tuned by the FOPID parameters. It should be noted that the robust performance index is invalid for the time delay. Considering that ω_c is independent of L, the partial derivative of ϕ_M with respect to L can be computed as

$$\frac{\partial\phi_M}{\partial L}=-\omega_c. \quad (32)$$

Hence, the value of $\partial\phi_M/\partial L$ cannot be changed by the FOPID controller after specifying the gain crossover frequency.

Remark 2: The robust performance indexes for different parameters are not independent of each other. From (23)-(25), one can easily obtain the following equations:

$$\frac{\partial\phi_M}{\partial a_i}=\frac{\partial\phi_M}{\partial k}\frac{F_{a_i}}{F_k}-Y_1(i), \quad (33)$$

$$\left(\frac{\partial\phi_M}{\partial a_i}+Y_1(i)\right)F_{a_j}=\left(\frac{\partial\phi_M}{\partial a_j}+Y_1(j)\right)F_{a_i}, \quad (34)$$

$$\frac{\partial\phi_M}{\partial\gamma_i}=\frac{\partial\phi_M}{\partial k}\frac{F_{\gamma_i}}{F_k}-a_i(\ln\omega_c)Y_1(i)-Y_2(i), \quad (35)$$

$$\left(\frac{\partial\phi_M}{\partial\gamma_i}+a_i(\ln\omega_c)Y_1(i)+Y_2(i)\right)F_{\gamma_j} \\ =\left(\frac{\partial\phi_M}{\partial\gamma_j}+a_j(\ln\omega_c)Y_1(j)+Y_2(j)\right)F_{\gamma_i}, \quad (36)$$

$$\left(\frac{\partial\phi_M}{\partial a_i}+Y_1(i)\right)F_{\gamma_j} \\ =\left(\frac{\partial\phi_M}{\partial\gamma_j}+a_j(\ln\omega_c)Y_1(j)+Y_2(j)\right)F_{a_i}, \quad (37)$$

where $0\le i,\ j\le n$. Equations (33)-(37) indicate that for different plant parameters p_1 and p_2, if $R(p_1)=0$, then $R(p_2)$ may not be zero. In some cases, a smaller value of $R(p_1)$ results in a large value of $R(p_2)$. This means that the robustness indexes for different parameters restrict each other.

Remark 3: It can be shown from (17) that $F_k\neq 0$. Note that

$$\left.\frac{d(\arg(G(j\omega)))}{d\omega}\right|_{\omega=\omega_c}=-X_A+X_C-L, \quad (38)$$

with $R(k)=0$, will lead to the "flat phase" condition (8). Therefore, the robust performance indexes given in (31) can be regarded as the promotion and generalization of the gain robustness condition.

3.3. Specifications in the frequency domain

From the specifications given in (6) and (7), we have the following theorem:

Theorem 1: For a plant described in (3), the parameters of the FOPID controller that ensure the desired phase margin φ_{PM} and gain crossover frequency ω_c can be determined by the following equations:

$$k_p = -\frac{k_d\omega_c^{\mu}\sin\frac{\pi}{2}(\lambda+\mu)}{\sin\frac{\pi}{2}\lambda} - \frac{\sqrt{A_1^2+A_2^2}\sin\left(\frac{\pi}{2}\lambda+\theta\right)}{k\sin\frac{\pi}{2}\lambda}, \tag{39}$$

$$k_i = \frac{k_d\omega_c^{\lambda+\mu}\sin\frac{\pi}{2}\mu}{\sin\frac{\pi}{2}\lambda} + \frac{\omega_c^{\lambda}\sqrt{A_1^2+A_2^2}\sin\theta}{k\sin\frac{\pi}{2}\lambda}, \tag{40}$$

where

$$\theta = \varphi_{PM} + L\omega_c + \angle(A_1 + jA_2). \tag{41}$$

Proof: Substituting (39) and (40) into (12), $C(j\omega_c)$ can be expressed as

$$\begin{aligned} &C(j\omega_c) \\ &= k_d\left((j\omega_c)^{\mu} - \frac{\omega^{\mu}\sin\frac{\pi}{2}(\lambda+\mu)}{\sin\frac{\pi}{2}\lambda} + \frac{\omega^{\lambda+\mu}\sin\frac{\pi}{2}\mu}{(j\omega_c)^{\lambda}\sin\frac{\pi}{2}\lambda}\right) \\ &\quad + \frac{\sqrt{A_1^2+A_2^2}\sin\theta}{j^{\lambda}k\sin\frac{\pi}{2}\lambda} - \frac{\sqrt{A_1^2+A_2^2}\sin\left(\frac{\pi}{2}\lambda+\theta\right)}{k\sin\frac{\pi}{2}\lambda}. \end{aligned} \tag{42}$$

We also have the following equations:

$$\sin\left(\frac{\pi}{2}\lambda+\theta\right) = \sin\frac{\pi}{2}\lambda\cos\theta + \cos\frac{\pi}{2}\lambda\sin\theta, \tag{43}$$

$$\cos\left(\frac{\pi}{2}\lambda+\theta\right) = \cos\frac{\pi}{2}\lambda\cos\theta - \sin\frac{\pi}{2}\lambda\sin\theta, \tag{44}$$

which allow (42) to be rewritten as

$$C(j\omega_c) = -e^{j\theta}\frac{\sqrt{A_1^2+A_2^2}}{k}. \tag{45}$$

Then, we have

$$|G(j\omega_c)| = \left|-e^{j\theta}\frac{\sqrt{A_1^2+A_2^2}}{k}\right| \times \left|k\frac{A_1-jA_2}{A_1^2+A_2^2}e^{-jL\omega_c}\right| = 1, \tag{46}$$

$$\angle G(j\omega_c) = -\pi + \varphi_{PM}. \tag{47}$$

It can be observed from (46) and (47) that the specifications in (6) and (7) are fully met. This completes the proof. □

Remark 4: Equations (39) and (40) provide direct calculations of the FOPID parameters to meet the specifications of the phase margin and gain crossover frequency. They simplify the complex equations given in (6) and (7).

3.4. Tuning procedure for the FOPID controller

To improve and perfect the tuning of the FOPID controller, high-frequency noise attenuation and output disturbance rejection [29] should be considered. Then, the robust tuning method for a system with an undetermined parameter p can be formulated as follows:

1) Specify the gain crossover frequency ω_c and the desired phase margin ϕ_{PM}; then, we obtain (39) and (40).

2) Specify the high-frequency noise attenuation:

$$\begin{aligned} &\left|T(j\omega) = \frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)}\right| \le h_1, \ \forall\omega \ge \omega_t, \\ &\to T(j\omega_t) = h_1, \end{aligned} \tag{48}$$

where h_1 is an appropriate constant.

3) Specify the output disturbance rejection

$$\begin{aligned} &\left|S(j\omega) = \frac{1}{1+C(j\omega)P(j\omega)}\right| \le h_2, \ \forall\omega \le \omega_s, \\ &\to S(j\omega_s) = h_2, \end{aligned} \tag{49}$$

where h_2 is a desired value.

4) Determine the p-robust performance index $R(p)$.

For a plant with variations in the parameter p, $R(p)$ can be taken as the objective function to tune the FOPID controller with the other four specifications.

5) Perform steady-state error cancellation: The fractional integrator $s^{-\lambda}$ is as efficient as an integer-order integrator. Therefore, this condition can always be fulfilled [22].

The above tuning method includes four nonlinear constraints and one objective function (31). Therefore, the five parameters (k_p, k_i, k_d, λ, and μ) of the FOPID controller can be obtained based on the solution of the nonlinear optimization problem. It should be noted that for a parameter p, the minimum value of $R(p)$ may not be zero under the other four specifications. In addition, determining the appropriate specifications is important because inappropriate specifications can lead to system instability.

4. EXAMPLES

The proposed tuning method is tested for two examples. The types of variable parameters involved in each example are different.

Example 1: A liquid level system is modeled by a first-order system with a time delay [29]

$$P_1(s) = \frac{3.13}{Ts+1}e^{-50s}, \tag{50}$$

where T is a time constant that either undergoes a perturbation or cannot be accurately measured. The nominal value is estimated to be $T_0 = 433.33$ s. We aim to design a FOPID controller with robustness to T. The design specifications are

- gain crossover frequency $\omega_{0c} = 0.008$ rad/s;
- phase margin $\phi_{0PM} = 60°$;
- output disturbance rejection:

$$|S(j\omega)| \leq 0.1, \ \ \forall \omega \leq \omega_s = 0.001 \text{ rad/s};$$

- high-frequency noise attenuation:

$$|T(j\omega)| \leq 0.1, \ \ \forall \omega \geq \omega_t = 10 \text{ rad/s}.$$

From (23), a performance index $R(T)$ that is robust to the time constant T can be obtained. Using the proposed tuning method, the transfer function of the robust FOPID controller can be expressed as

$$C_3 = 1.0379 + \frac{0.0044}{s^{0.9642}} + 6.0349s^{1.4069}. \tag{51}$$

As a comparison, applying the tuning method in [29] and the transfer function of a FOPID controller with gain robustness produces

$$C_4 = 0.6152 + \frac{0.01}{s^{0.8968}} + 4.3867s^{0.4773}. \tag{52}$$

Fig. 2 shows the step responses of the closed-loop system with C_3. It can be seen that in the time-constant variation range [303.33, 563.33] (±30% variations from the nominal value of 433.33), the overshoot does not change much, and the closed-loop system remains stable. Fig. 3 shows the step responses of the closed-loop system with controller C_4, which has a larger variation in overshoot in the variation range. Table 1 presents a summary of the results corresponding to Figs. 2 and 3. It can be seen that the average variation in overshoot of the system with controller C_3 is 1.625%, while that of the system with controller C_4 is 4.05%. In Fig. 4, the variations in the phase margin as T varies are shown. The system using the proposed FOPID controller has an almost constant phase margin near the nominal value of 60°, which demonstrates its greater robustness to variations in T. Fig. 5 shows that the system with C_4 exhibits significant instability as T changes to 100, but the system with C_3 remains stable. This finding shows that the robustness performance of the proposed FOPID controller is better.

Example 2: Consider a fractional-order plant with uncertain order described by the following transfer function:

$$P_2(s) = \frac{1}{2s^{\gamma} + 1}, \tag{53}$$

where the order γ of the plant cannot be measured precisely but has a nominal value of 1.4. The design specifications are given as

- gain crossover frequency $\omega_{0c} = 0.8$ rad/s;
- phase margin $\phi_{0PM} = 70°$;

Table 1. Summary of the results corresponding to Figs. 2 and 3.

Time constant T	Overshoot (%) in Fig. 2	Overshoot (%) in Fig. 3
303.33	7.8	6.3
368.33	6.8	9.6
433.33	8.1	12.5
498.33	9.7	15.0
563.33	11.4	17.1

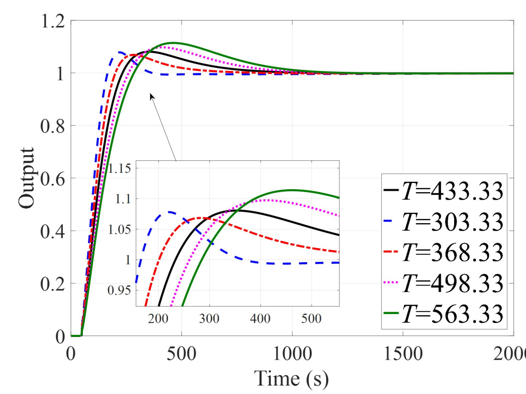

Fig. 2. Step responses of the closed-loop system with the FOPID C_3.

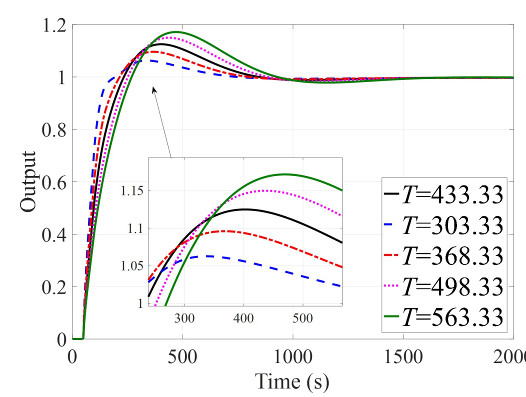

Fig. 3. Step responses of the closed-loop system with the FOPID C_4.

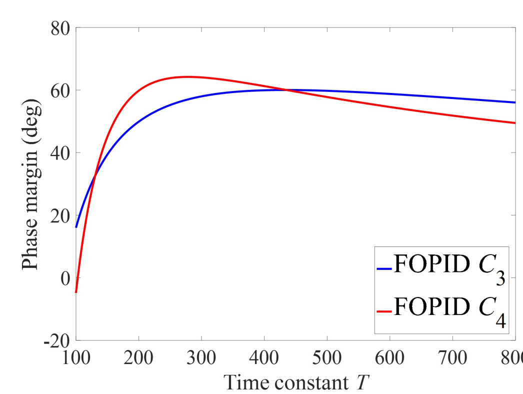

Fig. 4. Curves of the phase margin with respect to T.

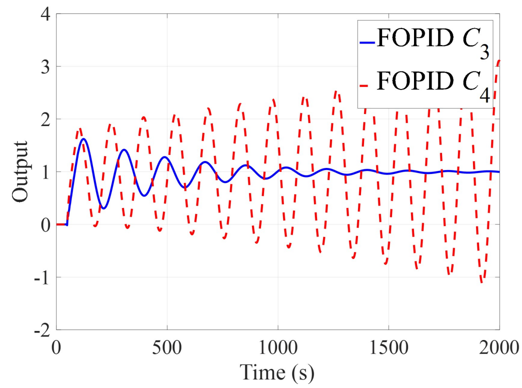

Fig. 5. Step responses of the closed-loop systems with $T = 100$.

- output disturbance rejection:

$$|S(j\omega)| \leq 0.0001, \ \ \forall \omega \leq \omega_s = 0.001 \text{ rad/s};$$

- high-frequency noise attenuation

$$|T(j\omega)| \leq 0.1, \ \ \forall \omega \geq \omega_t = 100 \text{ rad/s}.$$

The robust performance index $R(\gamma)$ can be obtained from (24). Using the proposed tuning method, the robust FOPID controller is given by

$$C_5 = 1.8716 + \frac{1.2366}{s^{1.3026}} + 1.1782s^{1.0342}. \quad (54)$$

Similarly, by applying the tuning method in [29], the FOPID controller is obtained as

$$C_6 = 1.6951 + \frac{0.6539}{s^{1.3948}} + 0.2971s^{1.3381}. \quad (55)$$

When the plant order changes from 1 to 1.8, the closed-loop system with C_5 remains stable, and the overshoot of the step response changes slightly, as shown in Fig. 6. In contrast, the performance of the system with C_6 changes dramatically with order variation, as shown in Fig. 7. The corresponding results are given in Table 2. It can be seen that the average variation in overshoot of the system with controller C_5 is 2.775%, while that of the system with controller C_6 is 8.25%. Fig. 8 shows the mapping of the plant order γ and phase margin. It is clear that with the change in γ, the variation in the phase margin is less in the system with C_5. Fig. 9 shows the step responses of the closed-loop systems with $\gamma = 1.9$. It can be seen that the proposed FOPID controller C_5 provides superior performance compared with the system with C_6, which undergoes a long period of oscillation.

5. CONCLUSION

This study presents a robust tuning method for FOPID controllers based on maintaining the phase margin of an open-loop system. The implicit function of the gain crossover frequency and the plant parameters are constructed, and the partial derivatives of the phase margin with respect to all the plant parameters are obtained as the robust performance indexes. These performance indexes maintain the insensitivity of the phase margin to parameter changes; thus, the robust performance of the system can be ensured. In addition, the formulas for calculating

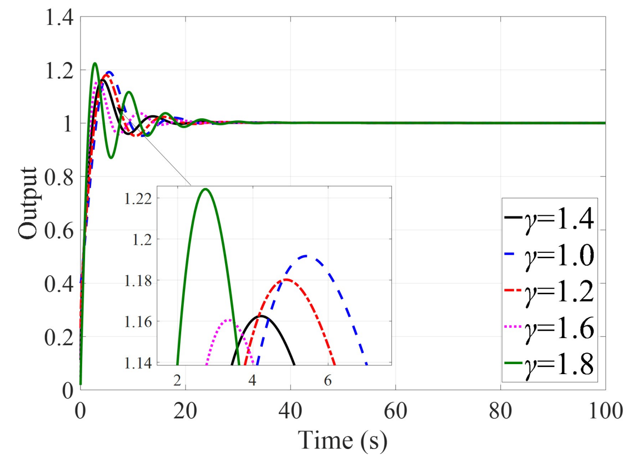

Fig. 6. Step responses of the closed-loop system with FOPID C_5.

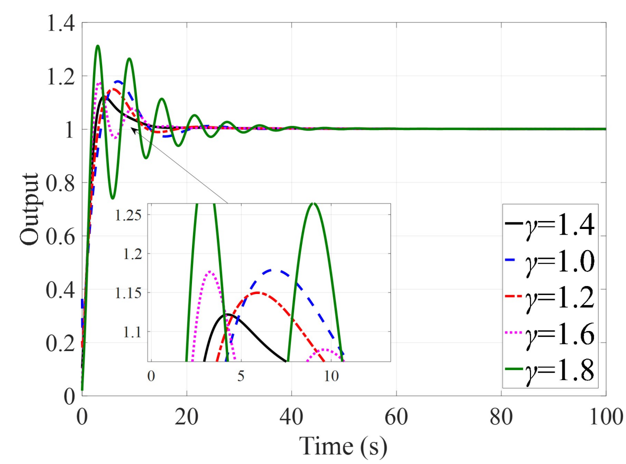

Fig. 7. Step responses of the closed-loop system with the FOPID C_6.

Table 2. Summary of the results corresponding to Figs. 6 and 7.

Fractional order γ	Overshoot (%) in Fig. 6	Overshoot (%) in Fig. 7
1	19.2	17.9
1.2	18.0	15.0
1.4	16.2	12.2
1.6	16.1	17.7
1.8	22.4	31.2

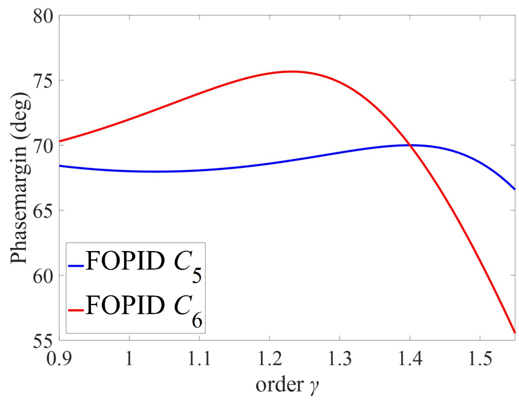

Fig. 8. Curves of the phase margin with respect to γ.

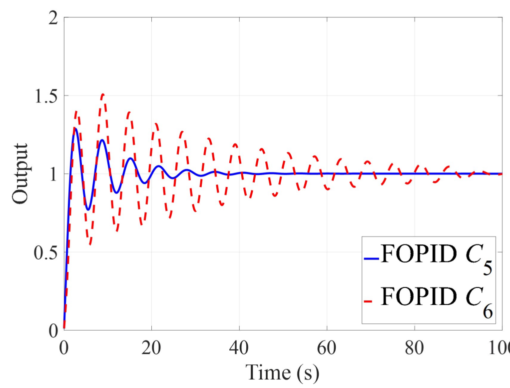

Fig. 9. Step responses of the closed-loop systems with $\gamma = 1.9$.

the parameters of the FOPID controller that meet the specified phase margin and gain crossover frequency are presented and proved. The simulation results clearly show the effectiveness of the proposed tuning method for the FOPID controller. The specifications of the gain crossover frequency, phase margin, complementary sensitivity function and sensitivity function are met. Compared with previous results, the phase margin and overshoot change less when the specified uncertain parameter varies around its nominal value. In our future work, the proposed method will be applied to other fractional-order controllers.

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