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Extended Gradient-based Iterative Algorithm for Bilinear State-space Systems with Moving Average Noises by Using the Filtering Technique

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Abstract: This paper develops a filtering-based iterative algorithm for the combined parameter and state estimation problems of bilinear state-space systems, taking account of the moving average noise. In order to deal with the correlated noise and unknown states in the parameter estimation, a filter is chosen to filter the input-output data disturbed by colored noise and a Kalman state observer (KSO) is designed to estimate the states by minimizing the trace of the error covariance matrix. Then, a KSO extended gradient-based iterative (KSO-EGI) algorithm and a filtering based KSO-EGI algorithm are presented to estimate the unknown states and unknown parameters jointly by the iterative estimation idea. The simulation results demonstrate the effectiveness of the proposed algorithms.

Keywords: Bilinear system, data filtering, iterative search, parameter estimation, state estimation.

1. INTRODUCTION

System identification is studying the theory and methods of establishing the mathematical models of systems [1–5]. Due to the vast existence of nonlinear systems in industry, there is increasing interest on the modeling, analysis, synthesis, and control of nonlinear systems [6–8]. Bilinear systems as a special class of nonlinear systems have relatively simple model structures. Despite their simplicity, they are capable of capturing the dynamics of a number of nonlinear systems, such as nuclear fission, heat exchangers and automobile braking systems. They provide a higher degree of the approximation to nonlinear models than traditional linear models. Moreover, a bilinear system can be seen as a linear parameter-varying system with the input signal as the scheduling parameter. In addition, the bilinear systems exhibit makes their structure one of the closest to linear systems. Thus, we can apply several techniques and procedures that have been developed for linear systems to bilinear systems [9].

Bilinear systems have attracted much interest in the control and identification area [10]. There are several approaches to identify bilinear systems. For example, Lopes dos Santos *et al*. presented a subspace identification method for the bilinear systems by regarding the bilinear term as a second-order white noise process based on the Picard decomposition [11]. The subspace methods require a lot of computational cost because the matrix dimensions grow exponentially with the system order increasing [12–14]. Verdult and Verhaegen overcame this difficulty and presented a kernel method depending on the dimension of the kernel matrix [15]. Another way for identifying the bilinear systems is to utilize the maximum likelihood principle, which can operate directly on the time domain data. Li and Liu proposed a maximum likelihood iterative method to identify the parameters of bilinear systems with colored noise [16]. On the basis of the work in [16], Hafezi and Arefi used the input-output representation of bilinear systems and presented a recursive maximum likelihood algorithm for decreasing the computational burden [17]. However, they only developed the parameter estimation for bilinear systems, while the state estimation was not taken into consideration. Thus, this paper studies the combined estimation of the states and parameters for the considered system.

The state-space model is an effective mathematical model to describe the dynamic behaviors of physical systems. Compared with the transfer function models, The state-space models show the relationship between the system states and the input-output variables [18–20]. The Kalman filter is regarded as the optimal filter for linear state-space systems, and its various modifications, such

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as the extended Kalman filter and the unscented Kalman filter are widely applied to estimate the unknown states [21, 22]. Moreover, the estimation and compensation of state-dependent nonlinearity were studied based on the equivalent-input-disturbance approach for compensating the effects of unknown external disturbances [23]. In addition, a linear-extended-state-observer based repetitive control method was presented to enhance the disturbance-rejection performance for the systems with aperiodic uncertainties and disturbances [24]. Bilinear state-space systems are nonlinear but the Kalman filter is only suitable for linear systems, therefore, it cannot be applied to bilinear systems directly.

The bilinear system considered in this paper is disturbed by the moving average noise, and the data filtering technique is introduced to deal with the noise disturbance. Differing from the ways to handle the disturbance in [25, 26], the filtering approach in system identification changes the structure of the noise model, but does not change the relationship between the inputs and outputs. For the unknown states, a Kalman state observer (KSO) is designed by means of replacing the unknown parameters with their estimates and minimizing the trace of the error covariance matrix. Then, a KSO extended gradient-based iterative (KSO-EGI) algorithm is derived for estimating the parameters and states jointly. To deal with the colored noise and improve the estimation accuracy, a filtering based KSO-EGI (F-KSO-EGI) algorithm is proposed to filter the input-output data.

The rest of this paper is organized as follows: Section 2 derives an identification model of the bilinear state-space system. Section 3 proposes a filtering extended gradient-based iterative (F-EGI) algorithm based on the data filtering technique. Section 4 presents an F-KSO-EGI algorithm. A KSO-EGI algorithm is given in Section 5. Section 6 provides an example to demonstrate the effectiveness of the proposed algorithms. Finally, some concluding remarks are given in Section 7.

2. SYSTEM DESCRIPTION AND IDENTIFICATION MODEL

First of all, let us introduce some notation. "$X := A$" stands for "A is defined as X"; the symbol I (I_n) represents an identity matrix of appropriate size ($n\times n$); z denotes a unit forward shift operator like $zx(t) = x(t+1)$ and $z^{-1}x(t) = x(t-1)$; the superscript T symbolizes the vector/matrix transpose; $\hat{\theta}_k$ denotes the estimate of θ at iteration k; $\mathbf{1}_n$ represents an n-dimensional column vector whose entries are all 1; tr$[X]$ denotes the trace of the square matrix X; col$[X]$ represents the vector obtained by arranging the columns of the matrix X in order. Specifically, for the matrix $X = [x_{i,j}] \in \mathbb{R}^{m\times n}$, we have col$[X] = [x_{1,1}, \cdots, x_{m,1}, x_{1,2}, \cdots, x_{m,2}, \cdots, x_{1,n}, \cdots, x_{m,n}]^{\mathrm{T}} \in \mathbb{R}^{mn}$. Some materials use vec$X$ instead of col$[X]$.

Consider a bilinear state-space system disturbed by the moving average noise, whose observability canonical form is given by

$$x(t+1) = Ax(t)+Mx(t)u(t)+bu(t), \tag{1}$$
$$y(t) = cx(t)+w(t), \tag{2}$$

where $x(t) := [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the system input and output variables, and $A \in \mathbb{R}^{n\times n}$, $M \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^{1\times n}$ are the system parameter matrices or vectors,

$$A := \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$b := \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \in \mathbb{R}^n, \quad M := \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix} \in \mathbb{R}^{n\times n}, m_l \in \mathbb{R}^{1\times n},$$

$$c := [1,0,\ldots,0] \in \mathbb{R}^{1\times n},$$

and $w(t)$ is a disturbance vector with a moving average process of the white noise $v(t)$, that is, $w(t) := D(z)v(t)$, $\mathrm{E}[v(t)] = 0$, $\mathrm{E}[v^2(t)] = \sigma^2$, $\mathrm{E}[v(t)v(i)] = 0$ $(t \neq i)$, where $D(z)$ is a scalar polynomial in z^{-1} and its expression is

$$D(z) := 1+d_1z^{-1}+d_2z^{-2}+\cdots+d_nz^{-n}, \quad d_i \in \mathbb{R}.$$

The parameters a_i, b_i, m_{ij}, d_i are to be identified from the observation data $u(t)$ and $y(t)$. Without loss of generality, assume that the dimension n of the system state is known, $y(t) = 0$ and $v(t) = 0$ for $t \leqslant 0$.

From (1) and (2), we have the following relations:

$$x_i(t+1) = x_{i+1}(t)+b_iu(t)+m_ix(t)u(t), \quad i = 1,2,\cdots,n-1, \tag{3}$$
$$x_n(t+1) = -\sum_{i=1}^{n} a_ix_{n-i+1}(t)+b_nu(t)+m_nx(t)u(t). \tag{4}$$

Referring to the method in [27] and multiplying both sides of (3) and (4) by z^{-i} and z^{-n}, respectively, we have

$$x_1(t) = -\sum_{i=1}^{n} a_ix_{n-i+1}(t-n)+\sum_{i=1}^{n} b_iu(t-i) + \sum_{i=1}^{n} m_ix(t-i)u(t-i). \tag{5}$$

Define the parameter vector ϑ and the information vector $\varphi(t)$ as

$$\vartheta := \begin{bmatrix} \theta \\ \theta_v \end{bmatrix} \in \mathbb{R}^{n^2+3n},$$

$$\begin{aligned}
\theta &:= [a^{\mathrm{T}}, b^{\mathrm{T}}, m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n^2+2n},\\
a &:= [a_1, a_2, \cdots, a_n]^{\mathrm{T}} \in \mathbb{R}^n,\\
b &:= [b_1, b_2, \cdots, b_n]^{\mathrm{T}} \in \mathbb{R}^n,\\
m &:= \mathrm{col}[M^{\mathrm{T}}] \in \mathbb{R}^{n^2},\\
\theta_v &:= [d_1, d_2, \cdots, d_n]^{\mathrm{T}} \in \mathbb{R}^n,\\
\varphi(t) &:= \begin{bmatrix} \phi(t) \\ \phi_v(t) \end{bmatrix} \in \mathbb{R}^{n^2+3n},\\
\phi(t) &:= [\phi_a^{\mathrm{T}}(t), \phi_b^{\mathrm{T}}(t), \phi_m^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{n^2+2n},\\
\phi_a(t) &:= [-x_n(t-n), \cdots, -x_1(t-n)]^{\mathrm{T}} \in \mathbb{R}^n,\\
\phi_b(t) &:= [u(t-1), u(t-2), \cdots, u(t-n)]^{\mathrm{T}} \in \mathbb{R}^n,\\
\phi_m(t) &:= [x^{\mathrm{T}}(t-1)u(t-1), x^{\mathrm{T}}(t-2)u(t-2), \cdots,\\
&\qquad x^{\mathrm{T}}(t-n)u(t-n)]^{\mathrm{T}} \in \mathbb{R}^{n^2},\\
\phi_v(t) &:= [v(t-1), v(t-2), \cdots, v(t-n)]^{\mathrm{T}} \in \mathbb{R}^n.
\end{aligned}$$

Inserting (5) into (2) gives

$$\begin{aligned}
y(t) =& \phi_a^{\mathrm{T}}(t)a + \phi_b^{\mathrm{T}}(t)b + \phi_m^{\mathrm{T}}(t)m + w(t)\\
=& \phi^{\mathrm{T}}(t)\theta + \phi_v^{\mathrm{T}}(t)\theta_v + v(t)\\
=& \varphi^{\mathrm{T}}(t)\vartheta + v(t). \qquad (6)
\end{aligned}$$

Equation (6) is the identification model of the bilinear state-space system in (1) and (2). In the information vector $\varphi(t)$, there are unknown states $x(t-i)$ and the unknown noise $v(t-i)$, which makes it difficult to identify the parameter vector ϑ containing the parameters a_i, b_i, $m_{i,j}$ and d_i. The objective of the next section is to use the iterative identification idea and the filtering technique to derive the parameter estimation algorithm, which can reduce the impact of colored noise to the parameter estimates.

3. THE F-EGI ALGORITHM

The data filtering technique was used to eliminate noises, and to reduce the noise-to-signal ratio in signal processing. It has been employed to handle the parameter estimation problem of systems with colored noises.

In this section, a filter $L(z)$ is introduced to deal with the colored noise. For the bilinear system in (1) and (2), if the input-output data of the system are filtered through $L(z) := \frac{1}{D(z)}$, the identification model in (6) can be transformed into two submodels. Then, we derive an F-EGI algorithm to identify each submodel, respectively. This method can reduce the influence of the colored noise to the parameter estimates and improve the identification accuracy.

3.1. The filtered identification model

Define the filtered state $\bar{x}(t)$, the filtered input $\bar{u}(t)$ and the filtered output $\bar{y}(t)$ as

$$\bar{x}(t) := \frac{x(t)}{D(z)} = x(t) + [1-D(z)]\bar{x}(t),$$

$$\bar{u}(t) := \frac{u(t)}{D(z)} = u(t) + [1-D(z)]\bar{u}(t),$$

$$\bar{y}(t) := \frac{y(t)}{D(z)} = y(t) + [1-D(z)]\bar{y}(t).$$

Multiplying both sides of (1) and (2) by the filter $L(z)$ gives

$$\bar{x}(t+1) = A\bar{x}(t) + M\bar{x}(t)u(t) + b\bar{u}(t), \qquad (7)$$

$$\bar{y}(t) = c\bar{x}(t) + v(t). \qquad (8)$$

From (7) and (8), we can get

$$\begin{aligned}
\bar{x}_1(t) =& -\sum_{i=1}^{n} a_i \bar{x}_{n-i+1}(t-n) + \sum_{i=1}^{n} b_i \bar{u}(t-i)\\
&+ \sum_{i=1}^{n} m_i \bar{x}(t-i)u(t-i). \qquad (9)
\end{aligned}$$

Define the filtered information vector

$$\begin{aligned}
\bar{\phi}(t) :=& [-\bar{x}_n(t-n), -\bar{x}_{n-1}(t-n), \cdots, -\bar{x}_1(t-n),\\
&\bar{u}(t-1), \bar{u}(t-2), \cdots, \bar{u}(t-n),\\
&\bar{x}^{\mathrm{T}}(t-1)u(t-1), \cdots, \bar{x}^{\mathrm{T}}(t-n)u(t-n)]^{\mathrm{T}}.
\end{aligned}$$

Inserting (9) into (8) yields

$$\begin{aligned}
\bar{y}(t) =& \bar{x}_1(t) + v(t)\\
=& \bar{\phi}^{\mathrm{T}}(t)\theta + v(t). \qquad (10)
\end{aligned}$$

The colored noise $w(t)$ is expressed as

$$w(t) = \phi_v^{\mathrm{T}}(t)\theta_v + v(t). \qquad (11)$$

Equations (10) and (11) are the two new submodels of (6). However, the polynomial $D(z)$ is unknown, then the filtered input $\bar{u}(t)$ and output $\bar{y}(t)$, the filtered state vector $\bar{x}(t)$ and the colored noise $w(t)$ are unknown. The filtered information vector $\bar{\phi}(t)$ contains these unknown items. In the next subsection, we apply the iterative identification idea to solve this difficulty. When a batch of measured data are collected, we use all these data to update the parameter estimates at each iteration [28, 29]. The gradient search, is applied to the parameter estimation and signal processing. Then, an F-EGI algorithm is derived in the following.

3.2. The derivation of the F-EGI algorithm

On the basis of the two models in (10) and (11), we define two quadratic criterion functions:

$$J_1(\theta) := \frac{1}{2}\sum_{t=1}^{p}[\bar{y}(t) - \bar{\phi}^{\mathrm{T}}(t)\theta]^2,$$

$$J_2(\theta_v) := \frac{1}{2}\sum_{t=1}^{p}[w(t) - \phi_v^{\mathrm{T}}(t)\theta_v]^2.$$

Let $k = 1, 2, 3, \ldots$ be an iterative variable, $\mu_k \geqslant 0$ be the iterative step-size. $\hat{\theta}_k$ and $\hat{\theta}_{v,k}$ be the parameter estimates of θ and θ_v at iteration k. Using the negative gradient search and minimizing $J_1(\theta)$ and $J_2(\theta_v)$ get

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \mu_{1,k}\sum_{t=1}^{p}\bar{\phi}(t)[\bar{y}(t) - \bar{\phi}^{\mathrm{T}}(t)\hat{\theta}_{k-1}], \qquad (12)$$

$$\hat{\theta}_{v,k}=\hat{\theta}_{v,k-1}+\mu_{2,k}\sum_{t=1}^{p}\phi_v(t)[w(t)-\phi_v^{\mathrm{T}}(t)\hat{\theta}_{v,k-1}]. \quad (13)$$

Due to the unknown items $\bar{y}(t)$, $\bar{\phi}(t)$, $w(t)$ and $\phi_v(t)$ in (12) and (13), the parameter estimates $\hat{\theta}_k$ and $\hat{\theta}_{v,k}$ cannot be worked out. To cope with this problem, we use $\hat{v}_{k-1}(t-i)$ to construct the estimate of $\phi_v(t)$ as

$$\hat{\phi}_{v,k}(t):=[\hat{v}_{k-1}(t-1),\cdots,\hat{v}_{k-1}(t-n)]^{\mathrm{T}}\in\mathbb{R}^n. \quad (14)$$

Then, using the estimate $\hat{\theta}_{v,k}:=[\hat{d}_{1,k},\hat{d}_{2,k},\cdots,\hat{d}_{n,k}]^{\mathrm{T}}\in\mathbb{R}^n$ to construct the estimate of $D(z)$ gives

$$\hat{D}_k(z):=1+\hat{d}_{1,k}z^{-1}+\hat{d}_{2,k}z^{-2}+\cdots+\hat{d}_{n,k}z^{-n}.$$

The estimates $\hat{\bar{x}}_k(t)$, $\hat{\bar{u}}_k(t)$ and $\hat{\bar{y}}_k(t)$ of $\bar{x}(t)$, $\bar{u}(t)$ and $\bar{y}(t)$ are

$$\begin{aligned}\hat{\bar{x}}_k(t)&:=\hat{x}_k(t)+[1-\hat{D}_k(z)]\hat{\bar{x}}_k(t)\\&=\hat{x}_k(t)-\sum_{i=1}^{n}\hat{d}_{i,k}\hat{\bar{x}}_k(t-i), \quad (15)\\ \hat{\bar{u}}_k(t)&:=u(t)+[1-\hat{D}_k(z)]\hat{\bar{u}}_k(t)\\&=u(t)-\sum_{i=1}^{n}\hat{d}_{i,k}\hat{\bar{u}}_k(t-i), \quad (16)\\ \hat{\bar{y}}_k(t)&:=y(t)+[1-\hat{D}_k(z)]\hat{\bar{y}}_k(t)\\&=y(t)-\sum_{i=1}^{n}\hat{d}_{i,k}\hat{\bar{y}}_k(t-i). \quad (17)\end{aligned}$$

Replacing $\bar{x}(t)$ and $\bar{u}(t)$ in the information vector $\bar{\phi}(t)$ with their estimates $\hat{\bar{x}}_k(t)$ and $\hat{\bar{u}}_k(t)$ obtains

$$\begin{aligned}\hat{\bar{\phi}}_k(t):=&[-\hat{\bar{x}}_{n,k-1}(t-n),-\hat{\bar{x}}_{n-1,k-1}(t-n),\cdots,\\&-\hat{\bar{x}}_{1,k-1}(t-n),\hat{\bar{u}}_{k-1}(t-1),\hat{\bar{u}}_{k-1}(t-2),\cdots,\\&\hat{\bar{u}}_{k-1}(t-n),\hat{\bar{x}}_{k-1}^{\mathrm{T}}(t-1)u(t-1),\\&\hat{\bar{x}}_{k-1}^{\mathrm{T}}(t-2)u(t-2),\cdots,\\&\hat{\bar{x}}_{k-1}^{\mathrm{T}}(t-n)u(t-n)]^{\mathrm{T}}\in\mathbb{R}^{n^2+2n}. \quad (18)\end{aligned}$$

Then, the estimate of $w(t)$ can be calculated by

$$\hat{w}_k(t)=y(t)-\hat{\phi}_k^{\mathrm{T}}(t)\hat{\theta}_{k-1}, \quad (19)$$

where the estimate $\hat{\phi}_k(t)$ of $\phi(t)$ is defined as

$$\begin{aligned}\hat{\phi}_k(t):=&[-\hat{x}_{n,k-1}(t-n),-\hat{x}_{n-1,k-1}(t-n),\cdots,\\&-\hat{x}_{1,k-1}(t-n),u(t-1),u(t-2),\cdots,\\&u(t-n),\hat{x}_{k-1}^{\mathrm{T}}(t-1)u(t-1),\\&\hat{x}_{k-1}^{\mathrm{T}}(t-2)u(t-2),\cdots,\\&\hat{x}_{k-1}^{\mathrm{T}}(t-n)u(t-n)]^{\mathrm{T}}\in\mathbb{R}^{n^2}. \quad (20)\end{aligned}$$

After that, $\hat{v}_k(t)$ can be calculated by

$$\hat{v}_k(t)=\hat{w}_k(t)-\hat{\phi}_{v,k}^{\mathrm{T}}(t)\hat{\theta}_{v,k-1}. \quad (21)$$

Replacing $\bar{y}(t)$, $\bar{\phi}(t)$, $w(t)$ and $\phi_v(t)$ in (12) and (13) with $\hat{\bar{y}}_k(t)$, $\hat{\bar{\phi}}_k(t)$, $\hat{w}_k(t)$ and $\hat{\phi}_k(t)$, we can obtain

$$\hat{\theta}_k=\hat{\theta}_{k-1}+\mu_{1,k}\sum_{t=1}^{p}\hat{\bar{\phi}}_k(t)[\hat{\bar{y}}_k(t)-\hat{\bar{\phi}}_k^{\mathrm{T}}(t)\hat{\theta}_{k-1}], \quad (22)$$

$$\mu_{1,k}\leqslant 2\lambda_{\max}^{-1}\left[\sum_{t=1}^{p}\hat{\bar{\phi}}_k(t)\hat{\bar{\phi}}_k^{\mathrm{T}}(t)\right], \quad (23)$$

$$\hat{\theta}_{v,k}=\hat{\theta}_{v,k-1}+\mu_{2,k}\sum_{t=1}^{p}\hat{\phi}_{v,k}(t)[\hat{w}_k(t)-\hat{\phi}_{v,k}^{\mathrm{T}}(t)\hat{\theta}_{v,k-1}], \quad (24)$$

$$\mu_{2,k}\leqslant 2\lambda_{\max}^{-1}\left[\sum_{t=1}^{p}\hat{\phi}_{v,k}(t)\hat{\phi}_{v,k}^{\mathrm{T}}(t)\right], \quad (25)$$

where $\lambda_{\max}[X]$ is the maximum eigenvalue of the real symmetric matrix. Equations (14)–(25) form the F-EGI algorithm for estimating the parameters of the bilinear system.

Remark 1: In the following, we give a brief discussion about how to choose the iterative step-sizes in this algorithm. Equations (22) and (24) can be seen as the discrete-time systems of $\hat{\theta}_k$ and $\hat{\theta}_{v,k}$. In order to ensure the convergence of $\hat{\theta}_k$ and $\hat{\theta}_{v,k}$, it is required that all the eigenvalues of matrices $[I_{n^2+2n}-\mu_{1,k}\sum_{t=1}^{p}\hat{\bar{\phi}}_k(t)\hat{\bar{\phi}}_k^{\mathrm{T}}(t)]$ and $[I_n-\mu_{2,k}\sum_{t=1}^{p}\hat{\phi}_{v,k}(t)\hat{\phi}_{v,k}^{\mathrm{T}}(t)]$ are inside the unit circle, and there is no duplicate eigenvalue on the unit circle. Thus, $\mu_{1,k}$ and $\mu_{2,k}$ in (23) and (25) are the conservative choices of the iterative step-sizes.

Based on the parameter estimates obtained by the filtering extended gradient-based iterative algorithm, we will design a state observer to estimate the state vector in the next section.

4. THE F-KSO-EGI ALGORITHM

In the previous section, it is assumed that the states are known to derive the F-EGI algorithm. In fact, the state vector $x(t)$ is unknown and a bilinear state observer is designed to estimate the state vector based on the Kalman filtering principle. Then, the parameters and states are estimated by the F-KSO-EGI algorithm.

Give the state $x(t)$ an initial value, and use the parameter estimates obtained by the F-EGI algorithm in (14)–(25) to construct the estimates $\hat{A}_k$, $\hat{M}_k$ and $\hat{b}_k$ of the system parameter matrices/vector A, M and b. By means of the idea of the Kalman filtering, we design the state observer of the filtered system in (7) and (8) in the following:

$$\begin{aligned}\hat{x}_k(t)=&\hat{\bar{x}}_k(t)+\hat{d}_{1,k}\hat{\bar{x}}_k(t-1)+\hat{d}_{2,k}\hat{\bar{x}}_k(t-2)+\cdots\\&+\hat{d}_{n,k}\hat{\bar{x}}_k(t-n), \quad (26)\\ \hat{\bar{x}}_k(t+1)=&\hat{A}_k\hat{\bar{x}}_k(t)+\hat{M}_k\hat{\bar{x}}_k(t)u(t)+\hat{b}_k\hat{\bar{u}}_k(t)\\&+L_k(t)[\hat{\bar{y}}_k(t)-c\hat{\bar{x}}_k(t)], \quad (27)\\ =&[\hat{A}_k+\hat{M}_ku(t)-L_k(t)c]\hat{\bar{x}}_k(t)+\hat{b}_ku(t)\\&+L_k(t)\hat{\bar{y}}_k(t), \quad (28)\end{aligned}$$

$$L_k(t) = \frac{[\hat{A}_k + \hat{M}_k u(t)]P_k(t)c^{\mathrm{T}}}{cP_k(t)c^{\mathrm{T}} + \sigma^2}, \tag{29}$$

$$P_k(t+1) = [\hat{A}_k + \hat{M}_k u(t)]P_k(t)[\hat{A}_k + \hat{M}_k u(t)]^{\mathrm{T}} - L_k(t)cP_k(t)[\hat{A}_k + \hat{M}_k u(t)]^{\mathrm{T}}, \tag{30}$$

$$\hat{A}_k = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -\hat{a}_{n,k} & -\hat{a}_{n-1,k} & -\hat{a}_{n-2,k} & \cdots & -\hat{a}_{1,k} \end{bmatrix}, \tag{31}$$

$$\hat{M}_k = \begin{bmatrix} \hat{m}_{1,k} \\ \hat{m}_{2,k} \\ \vdots \\ \hat{m}_{n,k} \end{bmatrix}, \quad \hat{b}_k = \begin{bmatrix} \hat{b}_{1,k} \\ \hat{b}_{2,k} \\ \vdots \\ \hat{b}_{n,k} \end{bmatrix}, \tag{32}$$

where $\hat{x}_k(t) := [\hat{x}_{1,k}(t), \hat{x}_{2,k}(t), \cdots, \hat{x}_{n,k}(t)]^{\mathrm{T}} \in \mathbb{R}^n$, $L_k(t) \in \mathbb{R}^n$ is the optimal gain vector, and $P_k(t+1) \in \mathbb{R}^{n\times n}$ is the state estimation error covariance matrix, σ^2 is the noise variance.

Remark 2: Assume that $\mathcal{D} \in \mathbb{R}^n$ is a closed set which includes $x(t)$, that is, $\eta \in \mathcal{D}$ means that $\|\eta\|$ is bounded. If $\hat{x}_k(t) \in \mathcal{D}$, then $\hat{x}_k(t)$ is the state estimate at the iteration k and current time t; if $\hat{x}_k(t) \notin \mathcal{D}$, then let $\hat{x}_k(t) := \hat{x}_{k-1}(t)$. The choice of the gain vector $L_k(t)$ is such that $[\hat{A}_k + \hat{M}_k u(t) - L_k(t)c]$ in (28) has its eigenvalues inside the unit circle or its eigenvalues most close to the unit circle.

Remark 3: The following gives the derivation of the state estimation algorithm. According to the design idea of the observer, suppose that the state observer is in the form of (27). The state estimation error is defined as $e_k(t) := x(t) - \hat{x}_k(t)$. Define the state estimation error covariance matrix $P_k(t) := \mathrm{E}[e_k(t)e_k^{\mathrm{T}}(t)]$. The aim is to choose an optimal gain vector $L_k(t)$ to minimize the sum of the squares of the estimation errors. The sum is defined by

$$\begin{aligned} J_e(t) :=& \mathrm{E}[(x_1(t) - \hat{x}_{1,k}(t))^2] + \mathrm{E}[(x_2(t) - \hat{x}_{2,k}(t))^2] \\ &+ \cdots + \mathrm{E}[(x_n(t) - \hat{x}_{n,k}(t))^2] \\ =& \mathrm{E}\{\mathrm{tr}[e_k(t)e_k^{\mathrm{T}}(t)]\} = \mathrm{tr}[P_k(t)], \end{aligned}$$

where $\mathrm{tr}[P_k(t)]$ denotes the trace of the state error covariance matrix $P_k(t)$. The above derivation means that our goal is to minimize $\mathrm{tr}[P_k(t)]$. Therefore, $L_k(t)$ in (29) is the optimal gain vector which makes the state estimation error covariance matrix minimum.

Equations (14)–(25) and (26)–(32) form the F-KSO-EGI algorithm. The steps involved in the F-KSO-EGI algorithm for bilinear systems are listed as follows:

1) To initialize: Give the data length p, the parameter estimation accuracy ε. Let $\hat{\theta}_0 = \mathbf{1}_{n^2+2n}/p_0$, $\hat{\theta}_{v,0} = \mathbf{1}_n/p_0$, $\hat{x}_0(t) = \hat{\hat{x}}_0(t) = \mathbf{1}_n/p_0$, $\hat{\hat{y}}_0(t) = \hat{\hat{u}}_0(t) = 1/p_0$, $\hat{w}_0(t) = \hat{v}_0(t) = 1/p_0$, $p_0 = 10^6$, $t = 1,2,\cdots,p$.
2) Let $k = 1$, collect the input-output data $u(t)$ and $y(t)$, $t = 1,2,\cdots,p$.
3) Form the information vectors $\hat{\phi}_k(t)$ and $\hat{\phi}_{v,k}(t)$ using (20) and (14). Compute $\mu_{2,k}$, $\hat{w}_k(t)$ and $\hat{v}_k(t)$ by (25), (19), and (21), $t = 1,2,\cdots,p$.
4) Update the parameter estimate $\hat{\theta}_{v,k}$ by (24).
5) Compute $\hat{\hat{y}}_k(t)$ and $\hat{\hat{u}}_k(t)$ using (17) and (16), form $\hat{\hat{\phi}}_k(t)$ by (18), $t = 1,2,\cdots,p$. Compute $\mu_{1,k}$ using (23).
6) Update the parameter estimate $\hat{\theta}_k$ by (22).
7) Construct $\hat{A}_k$, $\hat{M}_k$ and $\hat{b}_k$ using (31) and (32).
8) Let $t = 1$, set $\hat{x}_k(1) = \mathbf{1}_n/p_0$, $P_k(1) = I_n$.
9) Compute the gain vector $L_k(t)$ and the error covariance matrix $P_k(t+1)$ by (29) and (30).
10) Compute $\hat{\hat{x}}_k(t+1)$ using (27) and the system state estimation vector $\hat{x}_k(t)$ using (26).
11) If $t \leqslant p-1$, increase t by 1 and go to Step 9; otherwise, go to the next step.
12) If $\|\hat{\theta}_k - \hat{\theta}_{k-1}\| + \|\hat{\theta}_{v,k} - \hat{\theta}_{v,k-1}\| > \varepsilon$, then, increase k by 1 and go to Step 3; otherwise, obtain the parameter estimates $\hat{\theta}_k$ and $\hat{\theta}_{v,k}$, terminate this process.

Remark 4: The proposed algorithm offers a connection between the identification problem and the state estimation problem. One can indeed use the system identification as a technique to find the desired observer gain, and an optimal observer gain is identified simultaneously with the system model.

5. THE KSO-EGI ESTIMATION ALGORITHM

To show the superiority of the F-KSO-EGI algorithm, we generalize the KSO-EGI algorithm for identifying the parameter vector ϑ and the state vector $x(t)$ of bilinear systems with moving average noise.

Consider p data from $t = 1$ to $t = p$. Define the stacked output vector $Y(p)$ and the stacked information matrix $\Phi(p)$ as

$$Y(p) := \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(p) \end{bmatrix} \in \mathbb{R}^p, \quad \Phi(p) := \begin{bmatrix} \varphi^{\mathrm{T}}(1) \\ \varphi^{\mathrm{T}}(2) \\ \vdots \\ \varphi^{\mathrm{T}}(p) \end{bmatrix} \in \mathbb{R}^{p\times(n^2+3n)}.$$

Define a quadratic cost function

$$J_3(\vartheta) := \frac{1}{2}\|Y(p) - \Phi(p)\vartheta\|^2.$$

Let $\mu_k \geqslant 0$ be the iterative step-size. $\hat{\vartheta}_k$ represents the estimate of ϑ at iteration k. Using the negative gradient search and minimizing $J_3(\vartheta)$ get

$$\begin{aligned} \hat{\vartheta}_k =& \hat{\vartheta}_{k-1} - \mu_k \mathrm{grad}[J_3(\hat{\vartheta}_{k-1})] \\ =& \hat{\vartheta}_{k-1} + \mu_k \Phi^{\mathrm{T}}(p)[Y(p) - \Phi(p)\hat{\vartheta}_{k-1}]. \end{aligned} \tag{33}$$

For the reason that the information vector $\varphi(t)$ in $\Phi(p)$ contains the unknown state vector $x(t-i)$ and the noise

$v(t-i)$, the parameter estimate $\hat{\vartheta}_k$ cannot be calculated by (33) directly. The scheme is to replace $x(t-i)$ and $v(t-i)$ with their estimates $\hat{x}_{k-1}(t-i)$ and $\hat{v}_{k-1}(t-i)$ at iteration $k-1$. Referring the derivation in the F-EGI algorithm, the following yields the EGI algorithm to estimate the parameter vector ϑ for the bilinear system:

$$\hat{\vartheta}_k = \hat{\vartheta}_{k-1} + \mu_k \hat{\Phi}_k^{\mathrm{T}}(p)[Y(p) - \hat{\Phi}_k(p)\hat{\vartheta}_{k-1}], \tag{34}$$

$$\mu_k \leqslant 2\lambda_{\max}^{-1}[\hat{\Phi}_k^{\mathrm{T}}(p)\hat{\Phi}_k(p)], \tag{35}$$

$$Y(p) = [y(1), y(2), \cdots, y(p)]^{\mathrm{T}}, \tag{36}$$

$$\hat{\Phi}_k(p) = [\hat{\varphi}_k(1), \hat{\varphi}_k(2), \cdots, \hat{\varphi}_k(p)]^{\mathrm{T}}, \tag{37}$$

$$\hat{\varphi}_k(t) = [\hat{\phi}_{a,k}^{\mathrm{T}}(t), \phi_b^{\mathrm{T}}(t), \hat{\phi}_{m,k}^{\mathrm{T}}(t), \hat{\phi}_{v,k}^{\mathrm{T}}(t)]^{\mathrm{T}}, \tag{38}$$

$$\hat{\phi}_{a,k}(t) = [-\hat{x}_{n,k-1}(t-n), \cdots, -\hat{x}_{1,k-1}(t-n)]^{\mathrm{T}}, \tag{39}$$

$$\phi_b(t) = [u(t-1), u(t-2), \cdots, u(t-n)]^{\mathrm{T}}, \tag{40}$$

$$\begin{aligned}\hat{\phi}_{m,k}(t) = [&\hat{x}_{k-1}^{\mathrm{T}}(t-1)u(t-1), \cdots, \\ &\hat{x}_{k-1}^{\mathrm{T}}(t-n)u(t-n)]^{\mathrm{T}},\end{aligned} \tag{41}$$

$$\hat{\phi}_{v,k}(t) = [\hat{v}_{k-1}(t-1), \cdots, \hat{v}_{k-1}(t-n)]^{\mathrm{T}}, \tag{42}$$

$$\hat{v}_k(t) = y(t) - \hat{\varphi}_k^{\mathrm{T}}(t)\hat{\vartheta}_k, \tag{43}$$

$$\begin{aligned}\hat{\vartheta}_k = [&\hat{a}_{1,k}, \hat{a}_{2,k}, \cdots, \hat{a}_{n,k}, \hat{b}_{1,k}, \hat{b}_{2,k}, \cdots, \hat{b}_{n,k}, \\ &\hat{m}_{1,k}, \hat{m}_{2,k}, \cdots, \hat{m}_{n,k}, \hat{d}_{1,k}, \hat{d}_{2,k}, \cdots, \hat{d}_{n,k}]^{\mathrm{T}}.\end{aligned} \tag{44}$$

Similarly, minimizing the trace of the error covariance matrix and referring to the state estimation algorithm in (26)–(32), we can summarize the Kalman state observer:

$$\begin{aligned}\hat{x}_k(t+1) = &\hat{A}_k\hat{x}_k(t) + \hat{M}_k\hat{x}_k(t)u(t) + \hat{b}_k u(t) \\ &+ L_k(t)[y(t) - c\hat{x}_k(t) - \hat{\phi}_{v,k}^{\mathrm{T}}(t)\hat{\theta}_{v,k}],\end{aligned} \tag{45}$$

$$L_k(t) = \frac{[\hat{A}_k + \hat{M}_k u(t)]P_k(t)c^{\mathrm{T}}}{\sigma^2 + cP_k(t)c^{\mathrm{T}}}, \tag{46}$$

$$\begin{aligned}P_k(t+1) = &[\hat{A}_k + \hat{M}_k u(t)]P_k(t)[\hat{A}_k + \hat{M}_k u(t)]^{\mathrm{T}} \\ &- L_k(t)cP_k(t)[\hat{A}_k + \hat{M}_k u(t)]^{\mathrm{T}},\end{aligned} \tag{47}$$

$$\hat{A}_k = \begin{bmatrix} \mathbf{0} & I_{n-1} \\ -\hat{a}_{n,k} & -\hat{a}_k \end{bmatrix}, \tag{48}$$

$$\hat{a}_k = [\hat{a}_{n-1,k}, \hat{a}_{n-2,k}, \cdots, \hat{a}_{1,k}], \tag{49}$$

$$\hat{M}_k = [\hat{m}_{1,k}^{\mathrm{T}}, \hat{m}_{2,k}^{\mathrm{T}}, \cdots, \hat{m}_{n,k}^{\mathrm{T}}]^{\mathrm{T}}, \tag{50}$$

$$\hat{b}_k = [\hat{b}_{1,k}, \hat{b}_{2,k}, \cdots, \hat{b}_{n,k}]^{\mathrm{T}}. \tag{51}$$

Equations (34)–(44) and (45)–(51) form the KSO-EGI algorithm for estimating the states and parameters simultaneously. The proposed algorithm in this paper can combine other estimation algorithms [30–34] to study the parameter estimation issues for linear and nonlinear systems [35–40] and can be applied to other control and schedule areas such as the information processing and transportation communication systems [41–47].

Remark 5: From the above steps, we can see that for each iterative calculation of $\hat{\vartheta}_k$, the state estimate $\hat{x}_k(t)$ is recursively calculated p times from $t=1$ to $t=p$.

6. EXAMPLE

Consider the following bilinear system:

$$\begin{aligned}x(t+1) = &\begin{bmatrix} 0 & 1 \\ 0.31 & 0.32 \end{bmatrix} x(t) + \begin{bmatrix} 0.18 \\ 0.14 \end{bmatrix} u(t) \\ &+ \begin{bmatrix} 0.18 & 0.10 \\ 0.08 & 0.07 \end{bmatrix} x(t)u(t),\end{aligned}$$

$$\begin{aligned}y(t) &= [1,\ 0]x(t) + w(t), \\ w(t) &= D(z)v(t) \\ &= v(t) + 0.80v(t-1) + 0.27v(t-2).\end{aligned}$$

The parameter vector to be identified is

$$\begin{aligned}\vartheta = &[a_1, a_2, m_{11}, m_{12}, m_{21}, m_{22}, b_1, b_2, d_1, d_2]^{\mathrm{T}} \\ = &[-0.32, -0.31, 0.18, 0.10, 0.08, 0.07, 0.18, 0.14, \\ &0.80, 0.27]^{\mathrm{T}}.\end{aligned}$$

In simulation, the input $\{u(t)\}$ is taken as a random binary sequence generated by the Matlab function u=idinput([datalength,1], 'rbs', [0,0.25], [-0.7,0.7]), and $\{v(t)\}$ as a white noise sequence with zero mean and variance $\sigma^2 = 0.20^2$, $\{w(t)\}$ is the moving average process of the white noise $\{v(t)\}$. Take the data length $p = 1000$ and let the maximum iterative number $k_{\max} = 50$. The output is generated by simulating the model with the input signal. The simulated input-output data are recorded and shown in Fig. 1 (Samples 1 to 300).

Apply the F-KSO-EGI algorithm in (14)–(25) and (26)–(32) to estimate the state vector $x(t)$ and parameter vector ϑ of this bilinear system. The states $x_1(t)$ and $x_2(t)$, their estimates $\hat{x}_{1,k}(t)$ and $\hat{x}_{1,k}(t)$ against t are shown in Figs. 2 and 3. From Figs. 2 and 3, it can be found that the state estimates approach their true values with t increasing.

Taking the noise variances $\sigma^2 = 0.15^2$ and $\sigma^2 = 0.20^2$, respectively, we apply the F-KSO-EGI algorithm to estimate this bilinear system. The parameter estimates and their estimation errors $\delta := \|\hat{\vartheta}_k - \vartheta\|/\|\vartheta\| \times 100\%$ against k of the F-KSO-EGI algorithm with different noise variances are shown in Fig. 4. From the parameter estimation error curves in Fig. 4, it can be concluded that the

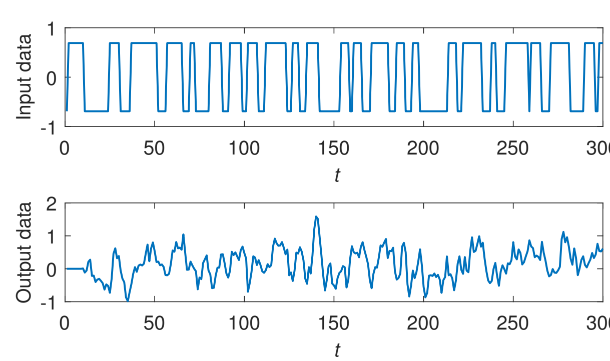

Fig. 1. The input and output signals against t.

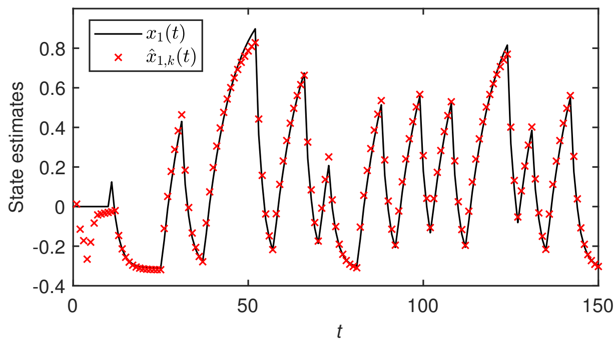

Fig. 2. State $x_1(t)$ and its estimate $\hat{x}_{1,k}(t)$ against t for the F-KSO-EGI algorithm.

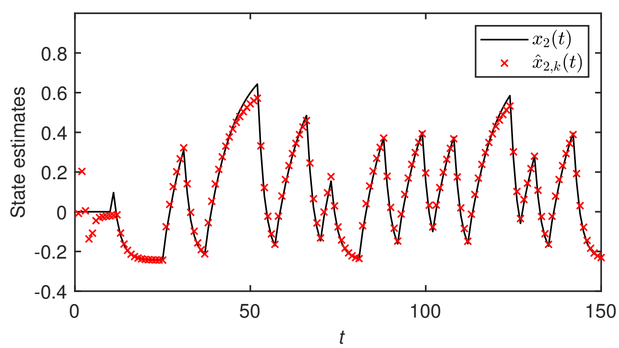

Fig. 3. State $x_2(t)$ and its estimate $\hat{x}_{2,k}(t)$ against t for the F-KSO-EGI algorithm.

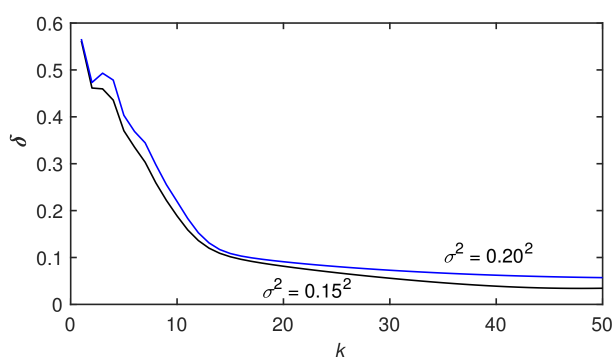

Fig. 4. The F-KSO-EGI estimation errors δ against k with different variances.

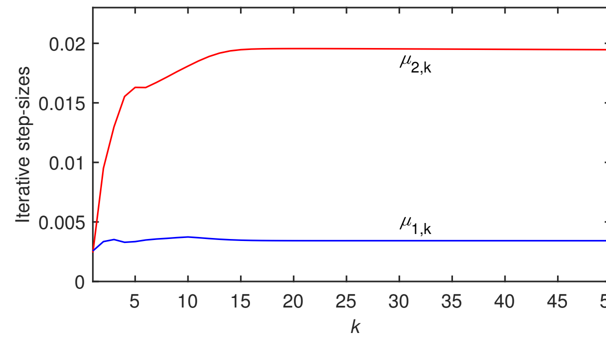

Fig. 5. The iterative step-sizes $\mu_{1,k}$ and $\mu_{2,k}$ against k for the F-KSO-EGI algorithm.

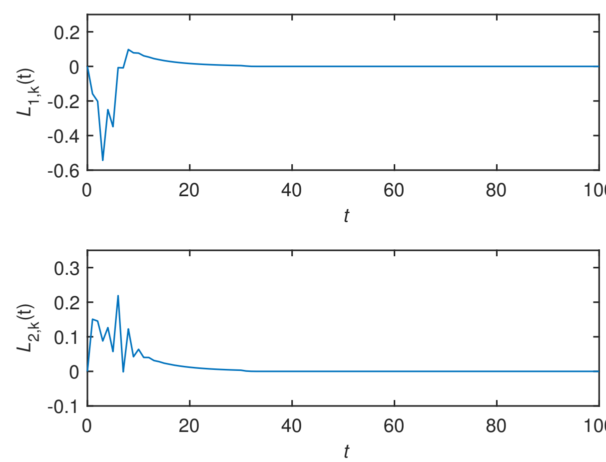

Fig. 6. The observer gain $L_k(t)$ against t for the F-KSO-EGI algorithm $(k = 30)$.

smaller noise variance results in more accurate parameter estimates.

In Fig. 5, the iterative step-sizes $\mu_{1,k}$ and $\mu_{2,k}$ in the F-KSO-EGI algorithm against k are given to show their behaviors. It is obvious that the iterative step-sizes $\mu_{1,k}$ and $\mu_{2,k}$ gradually become stable for large k, so the parameter estimation algorithm is effective. Furthermore, the behavior of the observer gain $L_k(t) = [L_{1,k}(t), L_{2,k}(t)]^{\mathrm{T}} \in \mathbb{R}^2$ in the F-KSO-EGI algorithm is shown in Fig. 6 when the iterative number $k = 30$.

7. CONCLUSIONS

In this paper, a state estimator is designed to estimate the system states, and an F-KSO-EGI algorithm and a KSO-EGI algorithm are proposed to realize the combined estimation of parameters and states for bilinear systems with the moving average noise. Compared with the KSO-EGI algorithm, the F-KSO-EGI algorithm has smaller estimation errors by employing the data filtering technique. The filtering-based iterative methods proposed in this paper can combine other methods and strategies to study the parameter estimation problems of different systems with colored noises [48–54] and can be applied to other fields [55–61] such as information processing and communication networked systems and so on.

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