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Event-triggered Control for Switched Affine Linear Systems

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Abstract: Event-triggered control problem for switched affine linear systems with a state-dependent switching law is addressed in this paper. By constructing a piecewise differential Lyapunov function with time-scheduled matrices, an event-triggered scheme and a switching signal are proposed. The switching signal depends on the state of the trigger instant. A sufficient condition is developed to ensure that the switched affine system exponentially convergences to a small neighborhood of the desired equilibrium point. The proposed result is then generated to a disturbance attenuation performance analysis. The results are presented in the form of linear matrix inequalities (LMIs). Finally, two examples are provided to illustrate the effectiveness of the proposed results.

Keywords: Affine systems, disturbance attenuation performance, event-triggered control, switched systems.

1. INTRODUCTION

A switched system is a hybrid system which has received the attention of scholars due to their applications in many areas, such as physical systems, chemical processes, aircraft control, networked control systems, and so on [1]. Such system consists of a number of subsystems and a logical law between them. The stability analysis and controller design of switched systems are important issues and have been studied by many scholars in the past few decades [2–6]. For example, the literature [2] studied the stability analysis of switched linear systems under switching conditions, and gave the necessary and sufficient conditions for asymptotic stability. A hidden Markov model-based nonfragile state estimator was designed for switched neural network with probabilistic quantized outputs in [3]. A multi-step and multi-class Lyapunov functions method was proposed to investigate the asymptotic stability of discrete-time switched systems with time-varying switching signals in [4]. Static output feedback control of switched systems with quantization was investigated by utilizing a nonhomogeneous sojourn probability approach in [5]. Considering that there exist various disturbances in practical engineering, H_∞ performance or L_2-gain performance analysis and control were also investigated, and many important results were developed in the literature. For instance, combining the multiple Lyapunov function method with the average dwell time approach, sufficient conditions for asymptotically stability with L_2-gain performance were obtained in [6]. By constructing a proper Lyapunov function, sufficient conditions for the existence of nonstationary L_2-L_∞ filter for Markov switching repeated scalar nonlinear systems with randomly occurring nonlinearities was proposed in [7].

As a special class of switched systems, switched affine systems have also received extensive attention. Many practical systems can be described as switched affine systems, such as converters in power electronics, biochemical systems and flight systems [8]. Such systems may have several equilibrium points due to the existence of affine terms, which makes the analysis and controller design of switched affine systems become more difficult. Therefore, many scholars were devoted themselves to the study of switched affine systems, and many important results are available in the literature [9–17]. For example, the global stability of such systems was ensured by determining the appropriate switching signal, and the guaranteed secondary cost was minimized in [8]. By using the Lyapunov-Metzler inequality, a state feedback controller was proposed for switched affine systems to ensure the practical stability of the desired equilibrium point in [10]. A stability condition of discrete-time switched affine systems was established by constructing the time-varying quadratic Lyapunov function with the concavity and convexity function centered on the minimax theory in [13]. Linear matrix inequality (LMI) conditions for the existence of a state feedback controller were proposed to ensure the H_∞ performance of the closed-loop system in

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[14]. A new method for estimating the attraction domain of switched affine systems was developed, and sufficient conditions for the practical asymptotic stability of the error system were obtained in [16].

On the other hand, the sampling data control was proposed as the development of computer technology, microgrid technology and communication network. In recent years, the sampling data control was applied to switched systems [18–20]. As a further improvement of the traditional sampled-data system, the event-triggered control was introduced. In the event-triggered control scheme, the execution controller is generated by designing appropriate event-triggered conditions. Compared with the sampled data scheme, the event-triggered control is a typical aperiodic control, which can significantly reduce the number of executions of control tasks while maintaining satisfactory performance. At present, the event-triggered control of switched systems was investigated in [21–33]. For instant, under the asynchronous switching control scheme, a set of event-triggered finite-time bounded controllers and input-output finite-time stability controllers were designed and sufficient conditions for event-based asynchronous closed-loop systems with finite time bounded and input-output finite time stability were proposed in [21]. An exponential stability condition was developed by designing an event-triggered scheme with dynamic thresholds and a switching signal in [22, 24, 25]. A state-dependent switching law based on the event-triggered scheme was proposed to reduce the conservatism of results in [30]. A hybrid quantization control strategy combining average dwell time and event-triggered condition was proposed to ensure exponential stability of switching system in [31]. In addition, in order to avoid behavior similar to Zeno, the trigger time should be small enough to be considered in [32]. Sufficient conditions are presented to guarantee the H_∞ performance of the closed-loop switched system by using the average dwell time, the piecewise Lyapunov function method and proposing decentralized event-triggered scheme (DETMs) with switching structure in [33]. The average dwell time method was used to establish sufficient conditions for L_2-gain performance of event-triggered switched systems in [34]. However, to the best of our knowledge, there is no result reported on the event-triggered control of switched affine systems, and the main aim of this paper is to shorten such a gap.

In this paper, the event-triggered control of switched affine system with a state-dependent switching law is studied. An event-triggered scheme and a switching signal are proposed by constructing a piecewise differential Lyapunov function with time-scheduled matrices. The switch only occurs at the trigger instant when the switching condition is satisfied. In order to ensure that the switched affine system exponentially convergences to a small neighborhood of the desired equilibrium point, we develop a sufficient condition which has less conservativeness than that in [20], where the sampled data scheme and a Lyapunov function with constant matrices were utilized. Furthermore, we extend the proposed results to a disturbance attenuation performance analysis. The correctness and effectiveness of the proposed results are verified by an example of DC-DC converters and a numerical example.

The remainder of the paper is organized as follows: The problem formulation and preliminaries are given in Section 2. Section 3 proposes the main results on the stability and a disturbance attenuation performance of switched affine systems. In Section 4, an example of DC-DC converter and a numerical example are given to show the effectiveness of the proposed approach. At the end, we give a conclusion for our work in Section 5.

Notations: Throughout this paper, R denotes the set of real numbers. R^n stands for the n-dimension Euclidean space and $R^{n\times n}$ is the $n\times n$ dimension Euclidean space. The superscript "T" stands for matrix transposition. The "I" denotes the identity. For a square symmetric matrix, $P>0$ ($P<0$) indicates that P is positive (negative) definite. I is the identity matrix. $L_2[0,\infty)$represents the space of square summable functions on $[0,\infty)$.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider the following continuous-time switched affine linear system:

$$\dot{x} = A_{\sigma(t)}x + b_{\sigma(t)} + E_{\sigma(t)}\omega(t), \tag{1}$$

$$z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}\omega(t), \tag{2}$$

where $x(t)$, $x(0)=x_0\in R^n$, $\forall t>0$ are the state of the affine system and the initial condition, respectively. $\omega(t)\in L_2[0,\infty)$ is the exogenous disturbance and the $z(t)\in R^q$ is the controlled output. The switching signal $\sigma(t):[0,\infty)\to \underline{N}=\{1, 2, 3, ..., N\}$ is a piecewise constant function with N being the number of subsystems. $A_i\in R^{n\times n}$ and $b_i\in R^n$, $i\in\underline{N}$, are constant matrices.

We give the unit simplex

$$\Lambda = \left\{\lambda = [\lambda_1, \lambda_2, ..., \lambda_{\underline{N}}] \in R^N,\ \lambda_i \geq 0,\ \sum_{i=1}^{N}\lambda_i = 1\right\}, \tag{3}$$

and define the following matrices

$$A(\lambda) = \sum_{i=1}^{N}\lambda_i A_i,\ \ b(\lambda) = \sum_{i=1}^{N}\lambda_i b_i,\ \ \lambda\in\Lambda. \tag{4}$$

The subset of Λ associated to Hurwitz matrices is given as follows:

$$\Lambda_H = \left\{\begin{array}{l}\lambda\in\Lambda : \exists P = P^T > 0, \\ \text{s.t. } A^T(\lambda)P + PA(\lambda) < 0\end{array}\right\}. \tag{5}$$

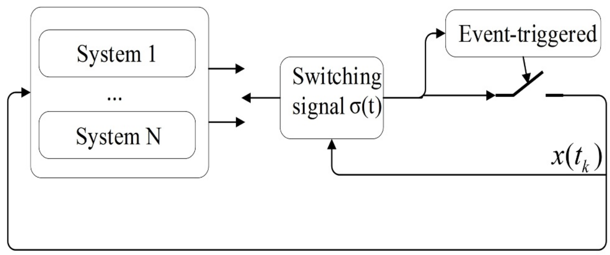

Fig. 1. General scheme of event-triggered switched control systems.

When $b_\sigma = 0$, system (1) with $\omega(t) = 0$ is the switched linear system so that all subsystems have a common equilibrium point $x_e = 0$. Note that, whenever $b_i \neq 0$ the switched system presents several equilibrium points $x_e \in X_e \subset R^n$ belonging to a set of attainable ones defined by

$$X_e = \left\{ x_e \in X_e \left| \begin{array}{l} x_e = -A^{-1}(\lambda) b(\lambda), \\ A(\lambda) \in H, \lambda \in \Lambda \end{array} \right. \right\}. \tag{6}$$

For system (1), the barycentric coordinates λ may not be unique [20], i.e., there exist M vectors $\lambda^j \in \Lambda_e$, $j \in \Im = \{1, 2, ..., M\}$, then $l(\lambda^j) = 0$, $j \in \Im$ where

$$\Lambda_e = \{\lambda \in \Lambda | \exists x_e \in R^n, \text{ s.t. } A(\lambda)x_e = -b(\lambda)\}. \tag{7}$$

In this paper, we will propose an event-triggered control scheme for system (1) and (2). In the event-triggered control scheme, the switching signal is generated by designing appropriate event-triggered conditions. Compared with the sampled data scheme, event-triggered control is a typical aperiodic control, which can significantly reduce the number of executions of control tasks while maintaining satisfactory closed-loop performance.

Fig. 1 shows a scheme of event-triggered switched control systems. It can be seen that the values of the system state are available at trigger instants $0 = t_0 < t_1 < ... < t_k < ...$, with $\lim_{k\to\infty} t_k = \infty$. The value of $\sigma(t)$ only changes at the trigger instant t_k if $\sigma(t_{k+1}) \neq \sigma(t_k)$, otherwise it holds its most recent value. The interval between two trigger moments is no less than T_s, i.e., $t_{k+1} - t_k \geq T_s$. Because the upper bound of the switching frequency is limited to $1/T_s$, this constraint significantly prevents the chattering phenomenon or the Zeno phenomenon of the switching action.

In this paper, since the system state $x(t)$ is only obtained at the trigger instant, the switch only occurs at the trigger instant when the switching condition is satisfied, that is, the switching signal depends on the state of the trigger instant.

The aim of this paper is to find an appropriate trigger condition and a switching signal to enable the system attach the equilibrium point $x_e = -A^{-1}(\lambda)b(\lambda)$ and has a disturbance attenuation performance.

Through the state transition $\xi(t) = x(t) - x_e$, system (1) and (2) can be transformed into the following system:

$$\dot{\xi}(t) = A_{\sigma(t)}\xi(t) + l_{\sigma(t)} + E_{\sigma(t)}\omega(t), \quad \xi_0 = 0, \tag{8}$$

$$z_e(t) = C_{\sigma(t)}\xi(t) + D_{\sigma(t)}\omega(t), \tag{9}$$

where $l_\sigma = A_\sigma x_e + b_\sigma$ for which $\sum_{i\in \underline{N}} \lambda_i l_i = 0$, and $z_e(t) = z(t) - C_{\sigma(t)}x_e$.

In this paper, the trigger time sequence $\{t_k\}_{k\geq 1}$ is generated by

$$t_k = \inf\left\{ t > t_{k-1} + T_s \left| \begin{array}{l} e^T(t)P_j(t)e(t) \\ \geq \eta\xi^T(t)\xi(t) \end{array} \right. \right\}, \tag{10}$$

where $\eta \in (0, 1]$, $P_j(t)$, $j \in \Im$, are positive definite matrices which will be determined later, and

$$e(t) = \xi(t) - \xi(t_{k-1}), \ t \in [t_{k-1}, t_k). \tag{11}$$

Remark 1: Due to the existence of the affine term, the equilibrium point of system (1) with $\omega(t) = 0$ is not zero and may not be unique, which brings some difficulties in the performance analysis and control design. In this paper, system (1) is transformed into system (8) via a state transformation. The event-triggered scheme (10) is constructed to reduce the unnecessary waste of data sampling and communication resources. (11) implies that the interval between two trigger moments is no less than T_s. A smaller T_s may increase the trigger frequency and lead to unnecessary waste of data sampling and communication resources, while a larger T_s may make a bad performance of the system. Thus the selection of T_s should be traded off between the two factors.

The following definition will be used in the later section.

Definition 1: System (8)-(9) is said to be exponentially attracted to a small neighborhood of the equilibrium point x_e with a disturbance attenuation performance, if the following is satisfied:

(i) When $\omega(t) = 0$, system (8)-(9) is said to be exponentially attracted to a small neighborhood of the equilibrium point x_e.

(ii) Under zero initial conditions, the following inequality holds for all non-zero $\omega(t) \in L_2[0, \infty)$

$$\int_0^\infty e^{-ct} z_e(t)^T z_e(t) dt \leq \int_0^\infty \gamma^2 \omega(t)^T \omega(t) dt + d, \tag{12}$$

where $c \geq 0$, $d \geq 0$ and $\gamma > 0$.

Remark 2: Since d is a constant, the inequality (12) means that the output $z_e(t)$ caused by the disturbance $\omega(t)$ will be limited. Thus, (12) can characterize the disturbance attenuation performance to some extent. When $c = 0$ and $d = 0$, (12) becomes

$$\int_0^\infty z_e(t)^T z_e(t) dt \leq \int_0^\infty \gamma^2 \omega(t)^T \omega(t) dt.$$

Then, the system has a standard L_2-gain γ.

3. MAIN RESULTS

3.1. Stability analysis

In this section, we will propose a switching signal for system (1) under the event-triggered scheme by constructing a piecewise differential Lyapunov function with time-scheduled matrices.

For system (1) under this case that the barycentric coordinates λ may not be unique, Lyapunov functions $V_j(\xi) = \xi^T P_j \xi$ are constructed for stability analysis in [20], where $P_j = P_j^T > 0$, $j \in \Im$.

Now we divide the interval $[0, T_s]$ into $\Gamma \in N$ segments described $[\varphi_m, \varphi_{m+1})$, $m \in \mho = \{0, 1, ..., \Gamma-1\}$, which are of equal length $\varphi = T_s/\Gamma$, and $\varphi_0 = 0$, $\varphi_m = m\varphi = mT_s/\Gamma$, $m = 0, 1, ..., \Gamma-1$.

To obtain less conservativeness results, we propose the following piecewise differential functions with time-scheduled matrices:

$$V_j(t,\xi) = \xi^T(t) P_j(t) \xi(t), \quad j \in \Im, \tag{13}$$

where

$$\begin{cases} P_j(t) = (1-\alpha(t))P_{j,m} + \alpha(t)P_{j,m+1}, \\ \alpha(t) = \Gamma(t-t_k)/(t_{k+1}-t_k) - m, \end{cases}$$
$$t \in [t_k + \varphi_m, \ t_k + \varphi_{m+1}),$$

and

$$\begin{cases} P_j(t) = (1-\alpha(t))P_{j,\Gamma-1} + \alpha(t)P_{j,\Gamma}, \\ \alpha(t) = \Gamma(t-t_k)/(t_{k+1}-t_k) - (\Gamma-1), \end{cases}$$
$$t \in [t_k + T_s, t_{k+1}).$$

It is easy to obtain that $P_j(t_k^+) = P_{j,0}$ and $P_j(t_k^-) = P_{j,\Gamma-1}$.

For $\xi(t_k) \in R^n$, the quadratic form of the minimum value is represented by the index set

$$J_{\min}(\xi(t_k)) = \left\{ \begin{matrix} j \in \Im \,|\, \xi^T(t_k) P_{j,0}\, \xi(t_k) \\ \leq \xi^T(t_k) P_{r,\Gamma-1} \xi(t_k), \forall r \in \Im \end{matrix} \right\}. \tag{14}$$

Then, a switching signal is proposed as follows:

$$\sigma(t) = \sigma((\xi(t_k)), J^*(\xi(t_k))), \ \ t \in [t_k, t_{k+1}), \tag{15}$$

where

$$\sigma((\xi(t_k)), J^*(\xi(t_k)))$$
$$\in \arg \min_{(i,j) \in N \times J_{\min}(\xi(t_k))} \xi^T(t_k) P_{j,0}(A_i \xi(t_k) + l_i).$$

The switching signal in (15) means that the switch only occurs at the trigger moment when the switching condition is satisfied, and the value of the switching signal remains the same during the interval between two adjacent trigger moments. Therefore, the proposed switching signal satisfies that the interval between two adjacent switching instants is no less than T_s.

By (13), (14) and (15), a Lyapunov function candidate is constructed for system (7)

$$V(t,\xi) = \xi^T(t) P_{j^*}(t) \xi(t), \tag{16}$$

where j^* indicates the active index in the Lyapunov function $V(t,\xi)$.

Remark 3: By the definitions of $P_j(t)$ and $\alpha(t)$, the form of $P_j(t)$ on the interval $[t_k + T_s, t_{k+1})$ is the same with the one on the interval $[t_k + \varphi_{\Gamma-1}, t_k + \varphi_\Gamma)$, which guarantees the continuity of $P_j(t)$ at the instant $t_k + T_s$. Actually, $P_j(t)$ is a piecewise differentiable function, and this leads to that the Lyapunov function (16) is piecewise differentiable. (14) is introduced to ensure that the Lyapunov function (16) is non-increasing at the trigger instant t_k.

Before proposing the main results, we give the following lemma.

Lemma 1: For system (8) with $\omega(t) = 0$, if there exist a piecewise differentiable function $V(t,\xi)$ in the form of (16) and a continuous function $w(t)$ with $w(t_k) = 0$ and $w(t) > 0$, $\forall t \in (t_k, t_{k+1})$, such that

$$V(t_k^+, \xi(t_k)) \leq V(t_k^-, \xi(t_k)), \tag{17}$$

$$\dot{V}(t,\xi) + \dot{w}(t) + 2\varsigma(V(t,\xi) + w(t)) < \beta T_s,$$
$$\forall t \in (t_k, t_{k+1}), \tag{18}$$

where ς and β are positive constants, then we have

$$\begin{aligned} |\xi(t)|^2 \leq & eig_{\max_{j\in\Im, m\in\mho}}(P_{j,m}) |\xi(0)|^2 e^{-2\varsigma T_s} / eig_{\min_{j\in\Im, m\in\mho}}(P_{j,m}) \\ & + \beta T_s / 2\varsigma eig_{\min_{j\in\Im, m\in\mho}}(P_{j,m}). \end{aligned} \tag{19}$$

Proof: Following the proof line of Theorem 2 in [20], this lemma can be obtained. □

Remark 4: The condition (17) in Lemma 1 implies that the function $V(t,\xi)$ is non-increasing at the trigger instant t_k. The function $w(t)$ is introduced to ensure the decrease of the function $V(t,\xi)$ at the triggered instant. Actually, the condition (18) guarantees that $V(\xi(t_{k+1})) < V(\xi(t_{k+1})) + w(t_{k+1}^-) < V(\xi(t_k))$ whenever $\xi(t)$ is outside the attractive ellipsoid given by

$$\phi^{T_s} := \left\{ x \in R^n : \min\left\{ eig_{\min_{j\in\Im, m\in\mho}}(P_{j,m}) \right\} |x - x_e|^2 \leq \frac{\beta T_s}{2\varsigma} \right\}.$$

Equation (19) means that $\xi(t)$ exponentially converges to the attractive ellipsoid. When $T_s \to 0$, the ellipsoid becomes the equilibrium point and the system is exponentially stable.

The main result on the stabilization of system (1) is presented by following theorem.

Theorem 1: Consider system (1) with $\omega(t) = 0$ and $\lambda^j \in \Lambda_e$, $j \in \Im$, for give scalar parameters $\varsigma > 0$, $\mu_{j,r} > 0$, $r \in \Im$, $\beta > 0$, if there exist matrices $Q_i > 0$, $P_{j,m} > 0$, $i \in \underline{N}$, $m \in \mho$, such that the following matrix inequalities

$$\begin{bmatrix} \Xi_{j,m} + T_s A_i^T Q_i A_i & T_s A_i^T Q_i l_i + \Psi_{j,m} \\ * & T_s l_i^T Q_i l_i - \beta T_s I \end{bmatrix} \leq 0, \tag{20}$$

$$\begin{bmatrix} \Xi_{j,m+1}+T_sA_i^TQ_iA_i & T_sA_i^TQ_il_i+\Psi_{j,m+1} \\ * & T_sl_i^TQ_il_i-\beta T_sI \end{bmatrix} \leq 0, \quad (21)$$

$$\begin{bmatrix} \Xi_{j,m} & \Psi_{j,m} & -T_s\Theta_{j,0} \\ * & -\beta T_sI & T_sl_i^TP_{j,0} \\ * & * & T_s^2(\Theta_{j,0}-Q_ie^{-2\varsigma T_s}/T_s) \end{bmatrix} \leq 0, \quad (22)$$

$$\begin{bmatrix} \Xi_{j,m+1} & \Psi_{j,m+1} & -T_s\Theta_{j,0} \\ * & -\beta T_sI & T_sl_i^TP_{j,0} \\ * & * & T_s^2(\Theta_{j,0}-Q_ie^{-2\varsigma T_s}/T_s) \end{bmatrix} \leq 0, \quad (23)$$

then under the event-triggered scheme (10) and the switching signal (15), the solution $x(t)$ is exponentially attracted to a small neighborhood of the equilibrium point x_e, where the neighborhood is given by the following ball

$$\phi^{T_s} := \left\{ \begin{array}{l} x \in R^n : \\ \min\left\{ eig_{\min\limits_{j\in\Im,m\in\mho}}(P_{j,m})\right\}|x-x_e|^2 \leq \dfrac{\beta T_s}{2\varsigma} \end{array} \right\}, \quad (24)$$

where

$$\begin{aligned} \Xi_{j,m} &= (A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i) \\ &\quad +(P_{j,m+1}-P_{j,m})\Gamma/T_s+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}) \\ &\quad +A_i^TP_{j,m}+P_{j,m}A_i+2\varsigma P_{j,m}, \\ \Xi_{j,m+1} &= (A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i) \\ &\quad +(P_{j,m+1}-P_{j,m})\Gamma/T_s+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}) \\ &\quad +A_i^TP_{j,m+1}+P_{j,m+1}A_i+2\varsigma P_{j,m+1}, \\ \Psi_{j,m} &= P_{j,m}l_i-P_{j,0}l_i, \\ \Psi_{j,m+1} &= P_{j,m+1}l_i-P_{j,0}l_i, \\ \Theta_{j,0} &= (A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i) \\ &\quad +\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}). \end{aligned}$$

Proof: Choose the following piecewise differentiable function

$$\bar{V}(t) = V(t)+w(t),$$

where

$$w(t) = (t_{k+1}-t)\int_{t_k}^{t} e^{2\varsigma(s-t)}\dot{\xi}^T(s)Q_i\dot{\xi}(s)ds,$$

with Q_i, $i\in \underline{N}$, are positive definite symmetry matrices. It is easy to obtain $w(t_k)=0$, $w(t)\geq 0$, $\forall t\in[t_k, t_{k+1})$.

Now let's assumed at time t_k the i-th system is activated, i.e., $\sigma(\xi(t_k))=i$.

When $t\in[t_k+\varphi_m, t_k+\varphi_{m+1})$, we have

$$\begin{aligned} \dot{V}(t,\xi) &= \dot{\xi}^T(t)P_j(t)\xi(t)+\xi^T(t)P_j(t)\dot{\xi}(t) \\ &\quad +\xi^T(t)\dot{P}_j(t)\xi(t) \\ &= 2l_i^TP_j(t)\xi(t)+\xi^T(t)(A_i^TP_j(t)+P_j(t)A_i \\ &\quad +\Gamma(P_{j,m+1}-P_{j,m})/(t_{k+1}-t_k))\xi(t) \\ &\leq 2l_i^TP_j(t)\xi(t)+\xi^T(t)(A_i^TP_j(t)+P_j(t)A_i \\ &\quad +\Gamma(P_{j,m+1}-P_{j,m})/T_s)\xi(t). \end{aligned} \quad (25)$$

When $t\in[t_k+T_s, t_{k+1})$, we have

$$\begin{aligned} &\dot{V}(t,\xi) \\ &= \dot{\xi}^T(t)P_j(t)\xi(t)+\xi^T(t)P_j(t)\dot{\xi}(t)+\xi^T(t)\dot{P}_j(t)\xi(t) \\ &= 2l_i^TP_j(t)\xi(t)+\xi^T(t)(A_i^TP_j(t)+P_j(t)A_i \\ &\quad +\Gamma(P_{j,\Gamma}-P_{j,\Gamma-1})/(t_{k+1}-t_k))\xi(t) \\ &\leq 2l_i^TP_j(t)\xi(t)+\xi^T(t)(A_i^TP_j(t)+P_j(t)A_i \\ &\quad +\Gamma(P_{j,\Gamma}-P_{j,\Gamma-1})/T_s)\xi(t). \end{aligned} \quad (26)$$

Set

$$\theta(t) = e(t)\tau^{-1}(t) = (\xi(t)-\xi(t_k))\tau^{-1}(t), \quad (27)$$

with $\tau(t)=t-t_k$ for all $t\in[t_k, t_{k+1})$.

According to the Jensen inequality, we get the following inequality

$$\begin{aligned} \dot{w}(t)+2\varsigma w(t) \leq &(T_s-\tau(t))\left[A_i\xi(t)+l_i\right]^TQ_i\left[A_i\xi(t)+l_i\right] \\ &-\tau(t)\theta^T(t)Q_i\theta(t)e^{-2\varsigma T_s}. \end{aligned} \quad (28)$$

From (25) and (28), we obtain, $\forall\, t\in[t_k+\varphi_m, t_k+\varphi_{m+1})$,

$$\begin{aligned} &\dot{V}(\xi(t))+\dot{w}(t)+2\varsigma(V(\xi(t))+w(t))-\beta T_s \\ &= \xi^T(t)((1-\alpha(t))\Omega_{j,m}+\alpha(t)\Omega_{j,m+1} \\ &\quad +(T_s-\tau(t))A_i^TQ_iA_i)\xi(t) \\ &\quad +2\xi^T(t)(((1-\alpha(t))P_{j,m}+\alpha(t)P_{j,m+1})l_i \\ &\quad +(T_s-\tau(t))A_i^TQ_il_i)+(T_s-\tau(t))l_i^TQ_il_i-\beta T_sI \\ &\quad -\tau(t)\theta^T(t)Q_i\theta(t)e^{-2\varsigma T_s}, \end{aligned} \quad (29)$$

where

$$\begin{aligned} \Omega_{j,m} &= A_i^TP_{j,m}+P_{j,m}A_i+(P_{j,m+1}-P_{j,m})\Gamma/T_s+2\varsigma P_{j,m}, \\ \Omega_{j,m+1} &= A_i^TP_{j,m+1}+P_{j,m+1}A_i+2\varsigma P_{j,m+1} \\ &\quad +(P_{j,m+1}-P_{j,m})\Gamma/T_s. \end{aligned}$$

From (14) and (15), we have

$$\begin{aligned} &2\xi^T(t_k)P_{j,0}(A_f\xi(t_k)+l_f) \\ &\quad -2\xi^T(t_k)P_{j,0}(A_i\xi(t_k)+l_i)\geq 0, \ \ j\in\Im, \end{aligned} \quad (30)$$

$$\xi^T(t_k)(P_{j,0}-P_{r,\Gamma-1})\xi(t_k)\leq 0, \ \ \forall r\in\Im. \quad (31)$$

It is easy to obtain from (27) that

$$\xi(t_k) = \xi(t)-\tau(t)\theta(t). \quad (32)$$

For simplicity, we denote $\tau=\tau(t)$ and $\theta=\theta(t)$. Multiplying the left and right sides of (30) by λ^j yields

$$2\xi^T(t_k)P_{j,0}(A(\lambda^j)-A_i)\xi(t_k)-2\xi^T(t_k)P_{j,0}l_i\geq 0. \quad (33)$$

Substituting (32) into (31) leads to

$$\begin{aligned}
&(\xi(t)-\tau\theta)^T\left[\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0})\right](\xi(t)-\tau\theta)\\
&=\xi(t)^T(\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\xi(t)\\
&\quad-2\tau\xi(t)^T(\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\theta\\
&\quad+\tau^2\theta^T(\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\theta\geq 0. \qquad (34)
\end{aligned}$$

From (33) and (34), we have

$$\begin{aligned}
\phi(t)=&\xi^T(t)((A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i)\\
&+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\xi(t)\\
&-2\tau\xi^T(t)(A(\lambda^j)-A_i)^TP_{j,0}\\
&+P_{j,0}(A(\lambda^j)-A_i)+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\theta(t)\\
&+\tau^2\theta^T(t)((A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i)\\
&+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\theta(t)\\
&+2\tau l_i^TP_{j,0}\theta(t)-2\xi^T(t)P_{j,0}l_i\geq 0. \qquad (35)
\end{aligned}$$

Combining (29) and (35) leads to, $\forall\ t\in[t_k+\varphi_m,\ t_k+\varphi_{m+1})$,

$$\begin{aligned}
&\dot{V}_j(\xi(t))+\dot{w}(t)+2\varsigma(V_j(\xi(t))+w(t))-\beta T_s+\phi(t)\\
&=\xi^T(t)(A_i^TP_j(t)+P_j(t)A_i+\frac{\Gamma}{T_s}(P_{j,m+1}-P_{j,m})\\
&\quad+2\varsigma P_{j,m}+(A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i)\\
&\quad+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0})+(T_s-\tau(t))A_i^TQ_iA_i)\xi(t)\\
&\quad-2\tau\xi^T(t)(A(\lambda^j)-A_i)^TP_{j,0}+P_{j,0}(A(\lambda^j)-A_i)\\
&\quad+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\theta(t)+2\tau l_i^TP_{j,0}\theta(t)\\
&\quad+2\xi^T(t)(T_s-\tau(t))A_i^TQ_il_i+P_j(t)l_i-P_{j,0}l_i)\\
&\quad+(T_s-\tau(t)))l_i^TQ_il_i-\beta T_sI\\
&\quad-\tau(t)\theta^T(t)Q_i\theta(t)e^{-2\varsigma T_s}+\tau^2\theta^T(t)((A(\lambda^j)-A_i)^TP_{j,0}\\
&\quad+P_{j,0}(A(\lambda^j)-A_i)+\sum_{r=1}^{M}\mu_{j,r}(P_{r,\Gamma-1}-P_{j,0}))\theta(t), \qquad (36)
\end{aligned}$$

i.e.,

$$\begin{aligned}
&\dot{V}(\xi(t))+\dot{w}(t)+2\varsigma(V(\xi(t))+w(t))-\beta T_s+\phi(t)\\
&=z^T(\Psi_i^1(\tau)+\Psi_i^2(\tau))z,\quad t\in[t_k,t_k+T_s),
\end{aligned}$$

where $z=[\xi^T\ 1\ \theta^T]^T$ and

$$\Psi_i^1(\tau)=\begin{bmatrix}\Psi_i^{11}(\lambda,\tau) & \Psi_i^{12}(\tau) & -\tau\Theta_{j,0}\\ * & \Psi_i^{13}(\tau) & \tau l_i^TP_{j,0}\\ * & * & \Psi_i^{14}(T_s,\tau)\end{bmatrix}, \qquad (37)$$

$$\Psi_i^2(\tau)=\begin{bmatrix}\Psi_i^{21}(\lambda,\tau) & \Psi_i^{22}(\tau) & -\tau\Theta_{j,0}\\ * & \Psi_i^{13}(\tau) & \tau l_i^TP_{j,0}\\ * & * & \Psi_i^{14}(T_s,\tau)\end{bmatrix}, \qquad (38)$$

where

$$\begin{aligned}
&\Psi_i^{11}(\lambda,\tau)=\Xi_{j,m}+(T_s-\tau(t))A_i^TQ_iA_i,\\
&\Psi_i^{12}(\tau)=(T_s-\tau(t))A_i^TQ_il_i+\Psi_{j,m},\\
&\Psi_i^{22}(\tau)=(T_s-\tau(t))A_i^TQ_il_i+\Psi_{j,m+1},\\
&\Psi_i^{13}(\tau)=(T_s-\tau(t))l_i^TQ_il_i-\beta T_sI,\\
&\Psi_i^{14}(T_s,\tau)=-\tau Q_ie^{-2\varsigma T_s}+\tau^2\Theta_{j,0},\\
&\Psi_i^{21}(\lambda,\tau)=\Xi_{j,m+1}+(T_s-\tau(t))A_i^TQ_iA_i.
\end{aligned}$$

It follows from (20)-(23) that

$$\begin{aligned}
&\Psi_i^1(0)\leq 0,\ \ \Psi_i^2(0)\leq 0,\\
&\Psi_i^1(T_s)\leq 0,\ \ \Psi_i^2(T_s)\leq 0.
\end{aligned}$$

Since $\Psi_i(\tau)\in co\{\Psi_i(0),\Psi_i(T_s)\}$,
where

$$\Psi_i(\tau)=\Psi_i^1(\tau)+\Psi_i^2(\tau),\ \ \forall\tau\in[0,T_s],$$

we obtain

$$z^T\Psi_i(0)z\leq 0, \qquad (39)$$
$$z^T\Psi_i(T_s)z\leq 0. \qquad (40)$$

From (37)-(41), we obtain, $\forall\ t\in[t_k+\varphi_m,\ t_k+\varphi_{m+1})$,

$$\dot{V}_j(\xi(t))+\dot{w}(t)+2\varsigma(V_j(\xi(t))+w(t))-\beta T_s+\phi(t)\leq 0.$$

Since $\phi(t)\geq 0$, when $\forall\ t\in[t_k+\varphi_m,\ t_k+\varphi_{m+1})$, we get

$$\dot{V}_j(\xi(t))+\dot{w}(t)+2\varsigma(V_j(\xi(t))+w(t))\leq\beta T_s. \qquad (41)$$

Similarly, it can be obtained that (41) holds for $t\in[t_k+T_s,\ t_{k+1})$. Thus (41) is satisfied for $t\in[t_k,\ t_{k+1})$.

From (17), we have

$$\begin{aligned}
V(\xi(t_k^+),t_k^+)=&\xi^T(t_k^+)P_j(t_k^+)\xi(t_k^+)\\
&\leq\xi^T(t_k^-)P_j(t_k^-)\xi(t_k^-)=V(\xi(t_k^-),t_k^-). \qquad (42)
\end{aligned}$$

By Lemma 1, we get

$$\begin{aligned}
|\xi(t)|^2\leq& eig_{\max\limits_{j\in\Im,m\in\mho}}(P_{j,m})\,|\xi(0)|^2e^{-2\varsigma T_s}/eig_{\min\limits_{j\in\Im,m\in\mho}}(P_{j,m})\\
&+\beta T_s/2\varsigma eig_{\min\limits_{j\in\Im,m\in\mho}}(P_j(t)),
\end{aligned}$$

i.e., the solution $x(t)$ is exponentially attracted to a small neighborhood of the equilibrium point x_e, where the neighborhood is given by the following ball

$$\phi^{T_s} := \left\{ \begin{array}{l} x \in R^n : \\ \min\left\{ eig_{\substack{\min \\ j\in\Im, m\in\mho}} (P_{j,m}) \right\} |x - x_e|^2 \le \dfrac{\beta T_s}{2\varsigma} \end{array} \right\}. \quad (43)$$

□

Remark 5: It should be noted that [20] gives the stabilization result for switched affine systems by using a sampled data control scheme and a switched Lyapunov functions with constant matrices. The proposed result in Theorem 1 is more general than that in [20] since the event-triggered control scheme and the piecewise differential Lyapunov function (16) with time-scheduled matrices is adopted in this paper. When $P_{j,m} = P_j$, $m = 0, 1, ..., \Gamma - 1$, and regardless of the event-triggered condition, Theorem 1 reduces to Theorem 2 in [20].

Since the switched piecewise differential Lyapunov function is used in this paper, $A(\lambda^j)$ is not required to be Hurwitz. When there is only one j, we have the following corollary.

Corollary 1: Consider system (1) with $\omega(t) = 0$ and $\lambda^j \in \Lambda_e$, $j = 1$, for give scalar parameters $\varsigma > 0$, $\beta > 0$, if there exist matrices $Q_i > 0$, $P_m > 0$, $i \in \underline{N}$, $m \in \mho$, such that matrix inequalities hold

$$\begin{bmatrix} \Xi_m + T_s A_i^T Q_i A_i & T_s A_i^T Q_i l_i + \Psi_m \\ * & T_s l_i^T Q_i l_i - \beta T_s I \end{bmatrix} \le 0, \quad (44)$$

$$\begin{bmatrix} \Xi_{m+1} + T_s A_i^T Q_i A_i & T_s A_i^T Q_i l_i + \Psi_{m+1} \\ * & T_s l_i^T Q_i l_i - \beta T_s I \end{bmatrix} \le 0, \quad (45)$$

$$\begin{bmatrix} \Xi_m & \Psi_m & -T_s\Theta_0 \\ * & -\beta T_s I & T_s l_i^T P_0 \\ * & * & T_s^2(\Theta_0 - Q_i e^{-2\varsigma T_s}/T_s) \end{bmatrix} \le 0, \quad (46)$$

$$\begin{bmatrix} \Xi_{m+1} & \Psi_{m+1} & -T_s\Theta_0 \\ * & -\beta T_s I & T_s l_i^T P_0 \\ * & * & T_s^2(\Theta_0 - Q_i e^{-2\varsigma T_s}/T_s) \end{bmatrix} \le 0, \quad (47)$$

$$\begin{aligned} \Xi_m &= (A(\lambda) - A_i)^T P_0 + P_0(A(\lambda) - A_i) \\ &\quad + (P_{m+1} - P_m)\Gamma/T_s + A_i^T P_m + P_m A_i + 2\varsigma P_m, \\ \Xi_{m+1} &= (A(\lambda) - A_i)^T P_0 + P_0(A(\lambda) - A_i) + 2\varsigma P_{m+1} \\ &\quad + (P_{m+1} - P_m)\Gamma/T_s + A_i^T P_{m+1} + P_{m+1} A_i, \\ \Psi_m &= P_m l_i - P_0 l_i, \\ \Psi_{m+1} &= P_{m+1} l_i - P_0 l_i, \\ \Theta_0 &= (A(\lambda) - A_i)^T P_0 + P_0(A(\lambda) - A_i), \end{aligned}$$

then under the event-triggered scheme (10) and the switching signal (15), the solution $x(t)$ is exponentially attracted to a small neighborhood of the equilibrium point x_e, where the neighborhood is given by the following ball

$$\phi^{T_s} := \left\{ \xi \in R^n : \xi^T P_m \xi \le \frac{\beta T_s}{2\varsigma} \right\}. \quad (48)$$

3.2. Disturbance attenuation performance analysis

In this subsection, we will investigate the disturbance attenuation performance of system (1) and (2).

Theorem 2: Consider system (1)-(2) and $\lambda^j \in \Lambda_e$, $j \in \Im = \{1, 2, ..., M\}$, for give scalar parameters $\varsigma > 0$ and $\beta > 0$, if there exist matrices $Q_i > 0$, $P_{j,m} > 0$, $i \in \underline{N}$, $m \in \mho$, such that the following matrix inequalities hold

$$\begin{bmatrix} \begin{array}{c}\Upsilon_{j,m} + C_i^T C_i \\ + T_s A_i^T Q_i A\end{array} & T_s A_i^T Q_i l_i & \begin{array}{c}P_{j,m} E_i + C_i^T D_i \\ + T_s A_i^T Q_i E_i\end{array} \\ * & T_s l_i^T Q_i l_i - \beta T_s I & T_s l_i^T Q_i E_i \\ * & * & \begin{array}{c}D_i^T D_i - \gamma^2 I \\ + T_s E_i^T Q_i E_i\end{array} \end{bmatrix} \le 0, \quad (49)$$

$$\begin{bmatrix} \begin{array}{c}\Upsilon_{j,m+1} + C_i^T C_i \\ + T_s A_i^T Q_i A\end{array} & T_s A_i^T Q_i l_i & \begin{array}{c}P_{j,m+1} E_i + C_i^T D_i \\ + T_s A_i^T Q_i E_i\end{array} \\ * & T_s l_i^T Q_i l_i - \beta T_s I & T_s l_i^T Q_i E_i \\ * & * & \begin{array}{c}D_i^T D_i - \gamma^2 I \\ + T_s E_i^T Q_i E_i\end{array} \end{bmatrix} \le 0, \quad (50)$$

$$\begin{bmatrix} \Upsilon_{j,m} + C_i^T C_i & 0 & * & P_{j,m} E_i + C_i^T D_i \\ * & -\beta T_s & * & * \\ * & * & -T_s Q_i e^{-2\varsigma T_s} & * \\ * & * & * & D_i^T D_i - \gamma^2 I \end{bmatrix} \le 0, \quad (51)$$

$$\begin{bmatrix} \Upsilon_{j,m+1} + C_i^T C_i & 0 & * & P_{j,m+1} E_i + C_i^T D_i \\ * & -\beta T_s & * & * \\ * & * & -T_s Q_i e^{-2\varsigma T_s} & * \\ * & * & * & D_i^T D_i - \gamma^2 I \end{bmatrix} \le 0, \quad (52)$$

where

$$\begin{aligned} \Upsilon_{j,m} &= A^T(\lambda^j) P_{j,m} + P_{j,m} A(\lambda^j) \\ &\quad + (P_{j,m+1} - P_{j,m})\Gamma/T_s + 2\varsigma P_{j,m}, \\ \Upsilon_{j,m+1} &= A^T(\lambda^j) P_{j,m+1} + P_{j,m+1} A(\lambda^j) \\ &\quad + (P_{j,m+1} - P_{j,m})\Gamma/T_s + 2\varsigma P_{j,m+1}. \end{aligned}$$

then system (1) is exponentially attracted to a small neighborhood of the equilibrium point x_e with a disturbance attenuation performance under the trigger scheme (10) and the switching signal (15).

Proof: Equations (49)-(52) imply that (20)-(23) are satisfied. By Theorem 1, system (1) with $\omega(t) = 0$ is exponentially attracted to a small neighborhood of the equilibrium point x_e.

Next, we will focus on the disturbance attenuation performance. For any non-zero $\omega(t) \in L_2[0, \infty)$ and zero initial conditions, we construct the indicator function as follows:

$$J(t_{k+1}^-)$$

$$= \int_{t_k}^{t_{k+1}} \left[(\dot{\bar{V}}(\xi(t)) + 2\varsigma\bar{V}(\xi(t)) - \beta T_s) + z_e(t)^T z_e(t) - \gamma^2 \omega(t)^T \omega(t)\right] dt, \tag{53}$$

where $t \in [t_k, t_{k+1})$, $\bar{V}(\xi(0)) = 0$ under zero initial conditions.
From (15), we get

$$\begin{aligned}&\min_{i\in\underline{N}}(2\xi(t)^T P_j(t)(A_i\xi(t) + l_i))\\&= \min_{i\in\underline{N}}(\xi(t)^T(A_i^T P_j(t) + P_j(t)A_i)\xi(t) + 2\xi(t)^T P_j(t) l_i),\end{aligned}$$

which implies

$$\begin{aligned}&\xi(t)^T(A_i^T P_j(t) + P_j(t)A_i)\xi(t) + 2\xi(t)^T P_j(t) l_i\\&\le \sum_{j=1}^{M} \lambda^j(\xi(t)^T(A_i^T P_j(t) + P_j(t)A_i)\xi(t)\\&\quad + 2\xi(t)^T P_j(t) l_i)\\&= \xi(t)^T(A^T(\lambda^j)P_j(t) + P_j(t)A(\lambda^j))\xi(t).\end{aligned} \tag{54}$$

From (8)-(9), (25) and (28)-(29), we have

$$\begin{aligned}&\dot{\bar{V}}(\xi(t)) + z_e(t)^T z_e(t) - \gamma^2 \omega(t)^T \omega(t)\\&= G^T[\Delta_i^1(\tau) + \Delta_i^2(\tau)]G,\end{aligned}$$

where $G = [\xi^T(t)\ 1\ \theta^T(t)\ \omega^T(t)]^T$, $\tau = \tau(t)$ and

$$\Delta_i^1(\tau) = \begin{bmatrix} \Delta_i^{11}(\tau) & \Delta_i^{12}(\tau) & * & \Delta_i^{14}(\tau) \\ * & \Delta_i^{22}(\tau) & * & \Delta_i^{24}(\tau) \\ * & * & \Delta_i^{33}(\tau) & * \\ * & * & * & \Delta_i^{44}(\tau) \end{bmatrix}, \tag{55}$$

$$\Delta_i^2(\tau) = \begin{bmatrix} \Delta_i^{211}(\tau) & \Delta_i^{12}(\tau) & * & \Delta_i^{214}(\tau) \\ * & \Delta_i^{22}(\tau) & * & \Delta_i^{24}(\tau) \\ * & * & \Delta_i^{33}(\tau) & * \\ * & * & * & \Delta_i^{44}(\tau) \end{bmatrix}, \tag{56}$$

$$\begin{aligned}&\Delta_i^{11}(\tau) = \Upsilon_{j,m} + C_i^T C_i + (T_s - \tau(t))A_i^T Q_i A,\\&\Delta_i^{12}(\tau) = (T_s - \tau(t))A_i^T Q_i l_i,\\&\Delta_i^{22}(\tau) = (T_s - \tau(t))l_i^T Q_i l_i - \beta T_s I,\\&\Delta_i^{33}(\tau) = -\tau(t)Q_i e^{-2\varsigma T_s},\ \ \Delta_i^{24}(\tau) = (T_s - \tau(t))l_i^T Q_i E_i,\\&\Delta_i^{14}(\tau) = P_{j,m}E_i + C_i^T D_i + (T_s - \tau(t))A_i^T Q_i E_i,\\&\Delta_i^{44}(\tau) = D_i^T D_i - \gamma^2 I + (T_s - \tau(t))E_i^T Q_i E,\\&\Delta_i^{211}(\tau) = \Upsilon_{j,m+1} + C_i^T C_i + (T_s - \tau(t))A_i^T Q_i A,\\&\Delta_i^{214}(\tau) = P_{j,m+1}E_i + C_i^T D_i + (T_s - \tau(t))A_i^T Q_i E_i.\end{aligned}$$

It follows from (50)-(53) that $\Delta_i^1(0) \le 0, \Delta_i^2(0) \le 0$, $\Delta_i^1(T_s) \le 0$ and $\Delta_i^2(T_s) \le 0$, which implies $\Delta_i^1(\tau) + \Delta_i^2(\tau) \le 0$.
Then we have

$$\begin{aligned}&\dot{\bar{V}}(\xi(t)) + 2\varsigma\bar{V}(\xi(t) + z_e(t)^T z_e(t) - \gamma^2 \omega(t)^T \omega(t)\\&\quad - \beta T_s \le 0.\end{aligned} \tag{57}$$

Multiplying both sides of (58) by $e^{-(\rho-2\varsigma)t}$, with ρ being a positive constant satisfying $\rho > 2\varsigma$, we have

$$\begin{aligned}&e^{-(\rho-2\varsigma)t}(\dot{\bar{V}}(\xi(t)) + 2\varsigma\bar{V}(\xi(t)) - \beta T_s\\&\quad + z_e(t)^T z_e(t) - \gamma^2 \omega(t)^T \omega(t)) \le 0.\end{aligned} \tag{58}$$

It follows that

$$\begin{aligned}&\int_0^\infty e^{-(\rho-2\varsigma)t} z_e(t)^T z_e(t) dt\\&\le \int_0^\infty e^{-(\rho-2\varsigma)t}\gamma^2 \omega(t)^T \omega(t) dt\\&\quad - \sum_{k=0}^{\infty} \int_{t_k^+}^{t_{k+1}^-} e^{2\varsigma t}(\dot{\bar{V}}(\xi(t)) + 2\varsigma\bar{V}(\xi(t)))dt\\&\quad + \int_0^\infty e^{-(\rho-2\varsigma)t}\beta T_s dt\\&\le \int_0^\infty e^{-(\rho-2\varsigma)t}\gamma^2 \omega(t)^T \omega(t) dt + \int_0^\infty e^{-(\rho-2\varsigma)t}\beta T_s dt\\&\quad - \sum_{k=0}^{\infty}(e^{2\varsigma t_{k+1}}\bar{V}(\xi(t_{k+1})) - e^{2\varsigma t_k}\bar{V}(\xi(t_k^+))).\end{aligned} \tag{59}$$

Noting from (14) that

$$\bar{V}(\xi(t_{k+1}^+)) \le \bar{V}(\xi(t_{k+1}^-)),\ \ k = 1, 2...,$$

we have

$$\begin{aligned}&\int_0^\infty e^{-(\rho-2\varsigma)t} z_e(t)^T z_e(t) dt\\&\le \int_0^\infty e^{-(\rho-2\varsigma)t}\gamma^2 \omega(t)^T \omega(t) dt\\&\quad + \int_0^\infty e^{-(\rho-2\varsigma)t}\beta T_s dt\\&\quad - \sum_{k=0}^{\infty}(e^{2\varsigma t_{k+1}}\bar{V}(\xi(t_{k+1}^-)) - e^{2\varsigma t_k}\bar{V}(\xi(t_k^+)))\\&\quad - \sum_{k=0}^{\infty} e^{2\varsigma t_{k+1}}(\bar{V}(\xi(t_{k+1}^+)) - \bar{V}(\xi(t_{k+1}^-)))\\&\le \int_0^\infty e^{-(\rho-2\varsigma)t}\gamma^2 \omega(t)^T \omega(t) dt + \int_0^\infty e^{-(\rho-2\varsigma)t}\beta T_s dt\\&\quad + \bar{V}(\xi(t_0)) - \bar{V}(\xi(\infty)).\end{aligned} \tag{60}$$

Since $\bar{V}(\xi(t_0)) = 0$ and $\bar{V}(\xi(\infty)) > 0$, we obtain from (61) that

$$\begin{aligned}&\int_0^\infty e^{-(\rho-2\varsigma)t} z_e(t)^T z_e(t) dt\\&\le \int_0^\infty e^{-(\rho-2\varsigma)t}\gamma^2 \omega(t)^T \omega(t) dt + \beta T_s/(\rho - 2\varsigma).\end{aligned} \tag{61}$$

It follows that

$$\int_0^\infty e^{-ct} z_e(t)^T z_e(t) dt \le \int_0^\infty \gamma^2 \omega(t)^T \omega(t) dt + d, \tag{62}$$

where $c = \rho - 2\varsigma$ and $d = \frac{\beta T_s}{c}$.
This completes the proof. □

Remark 6: In Theorem 2, we present the result on the disturbance attenuation performance characterized by (62) for system (1) and (2) under the event-triggered scheme. When there is no event-triggered mechanism, i.e., $T_s \to 0$, (57) becomes

$$\dot{V}(\xi(t)) + 2\varsigma\bar{V}(\xi(t) + z_e(t)^T z_e(t) - \gamma^2\omega(t)^T\omega(t) \le 0,$$

which leads to

$$\int_0^\infty z_e(t)^T z_e(t)dt \le \int_0^\infty \gamma^2\omega(t)^T\omega(t)dt.$$

Then the system has a standard L_2-gain γ.

Remark 7: In this paper, we divide the interval $[0, T_s]$ into Γ segments. A larger Γ is favorable to reduce the conservativeness of the results, but will increase the number of variables and inequalities in Theorems 2, which may increase the computational complexity. Thus the selection of Γ should be traded off between the two factors.

4. NUMERICAL EXAMPLES

In this section, we will give two examples to show the effectiveness of the proposed results.

Example 1: Consider an example used in [20] with matrices as follows:

$$A_1 = A_3 = \begin{bmatrix} -3 & -6 & 3 \\ 2 & 2 & -3 \\ 1 & 0 & -2 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1 & 3 & 3 \\ -1 & -3 & -3 \\ 0 & 0 & -2 \end{bmatrix},$$
$$b_1 = -b_3 = \begin{bmatrix} -35 & 0 & 0 \end{bmatrix}^T, \; b_2 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T.$$

In order to show the advantage of the proposed method, we give the same parameters with those in [20], i.e.,

$$\lambda^1 = \lambda^4 = \begin{bmatrix} 0.5 & 0 & 0.5 \end{bmatrix}^T,$$
$$\lambda^2 = \lambda^3 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T,$$
$$\beta = 2.5 \times 10^3, \; \varsigma = 0.5 \times 10^{-3}.$$

By Theorem 1, we find that inequalities (20)-(23) are feasible for any trigger intervals with $T_s \le 0.7$ s, while the conditions of Theorem 2 in [20] are found to be feasible only for $T_{\max} \le 3.2 \times 10^{-2}$ s. This demonstrates that the proposed method in this paper is less conservative than that presented in [20].

Example 2: Consider a DC-DC converter shown in Fig. 2. The model of the DC-DC converter has the form

$$\dot{x}(t) = \bar{A}_{\sigma(t)}x(t) + \bar{b}_{\sigma(t)} + E_{\sigma(t)}\omega(t),$$

where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $\sigma(t) = \{1, 2\}$, x_1 is the inductor current and x_2 is the capacitor voltage,

$$\bar{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix}, \; \bar{b}_1 = \begin{bmatrix} U/L \\ 0 \end{bmatrix},$$
$$\bar{A}_2 = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix}, \; \bar{b}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

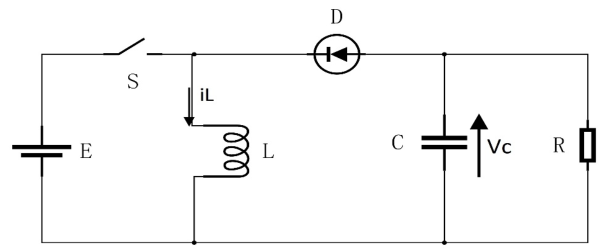

Fig. 2. Buck-boost converter.

$$E_1 = \begin{bmatrix} 1/L & 0 \end{bmatrix}^T, \; E_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T.$$

Let the capacitor voltage be the performance output, i.e., $z = x_2$, $C_1 = C_2 = [0\ 1]$ and $D_1 = D_2 = [0\ 0]$.

Choose $U = 12$ V, $R = 35$ Ω, $L = 100$ mH, and $C = 450$ μF. It is easy to verify that $A(\lambda)$ is Hurwitz for $\lambda^1 = [0.6\ 0.4]^T$ and $x_e = [1.29\ 18]^T$.

In order to deal with large numerical values and avoid ill conditioned matrix inequalities, we use the time scale change $t = \vartheta t'$ with $\vartheta = 10^4$, then the model of the DC-DC converter can be described as system (8) with $A_i = \vartheta^{-1}\bar{A}_i$ and $l_i = \vartheta^{-1}(\bar{A}_i x_r + \bar{b}_i)$.

For the numerical test, we choose $M = 1$, $\beta = 2.0 \times 10^{-1}$, $\varsigma = 280$, $T_s = 0.0005$ s and $\Gamma = 4$, then solving the matrix inequalities in Theorem 2 yields

$$P_{11} = \begin{bmatrix} 1.8727 & -5.4 \times 10^{-4} \\ -5.4 \times 10^{-4} & 2.1792 \end{bmatrix},$$
$$P_{12} = \begin{bmatrix} 1.7329 & -5.3 \times 10^{-4} \\ -5.3 \times 10^{-4} & 2.0184 \end{bmatrix},$$
$$P_{13} = \begin{bmatrix} 1.6029 & -5.1 \times 10^{-4} \\ -5.1 \times 10^{-4} & 1.8686 \end{bmatrix},$$
$$P_{14} = \begin{bmatrix} 1.4818 & -4.9 \times 10^{-4} \\ -4.9 \times 10^{-4} & 1.7293 \end{bmatrix},$$
$$P_{15} = \begin{bmatrix} 1.3691 & -4.8 \times 10^{-4} \\ -4.8 \times 10^{-4} & 1.5995 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 739.82 & 0.0057 \\ 0.0057 & 806.06 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 806.063 & 5.2 \times 10^{-7} \\ 5.2 \times 10^{-7} & 806.064 \end{bmatrix},$$
$$\gamma = 10.03.$$

By applying the proposed method in this paper, the event-triggered condition is designed as follows:

$$e^T(t)P(t)e(t) \ge 0.01\xi^T(t)\xi(t),$$

where

$$t_{k+1} = \inf\left\{t > t_k + T_s \mid e^T(t)P(t)e(t) \ge 0.01\xi^T(t)\xi(t)\right\}.$$

Since

$$\arg\min \xi(t_k)^T P(t_k)(\vartheta\bar{A}_i\xi(t_k) + \vartheta l_i)$$

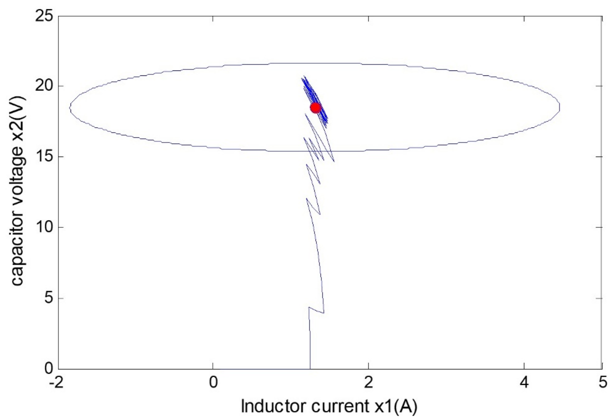

Fig. 3. The evolution of system state and the attraction ball (24).

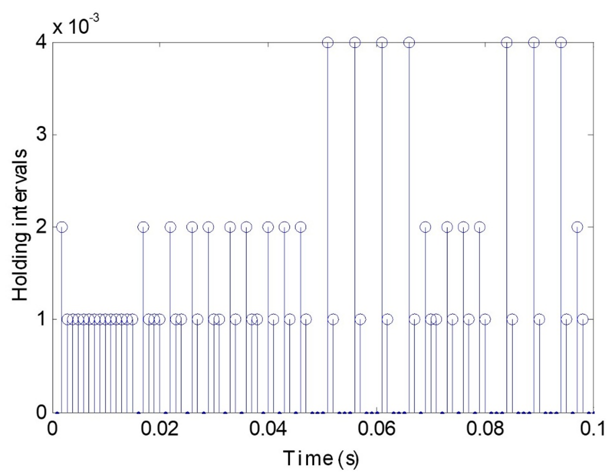

Fig. 4. The triggered instants of the event-triggered mechanism.

$$= \arg\min \xi(t_k)^T P(t_k)(A_i\xi(t_k) + \vartheta l_i),$$

the switching signal can be constructed as follows:

$$\sigma(\xi(t_k)) \in \arg\min_{i=1,2} \xi^T(t_k)P_0(A_i\xi(t_k) + l_i).$$

Fig. 3 shows the evolution of system state and the attraction ball (24). The event-triggered instants are shown in Fig. 4. The switching signal of the system is shown in Fig. 5. The initial value is $x = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ and the disturbance is

$$\begin{cases} \omega(t) = \sin(960\pi t), & 0 \leq t \leq 0.06, \\ \omega(t) = 0 & t > 0.06. \end{cases}$$

It can be seen from Fig. 3 that the inductor current and the capacitor voltage are convergent to a small neighborhood of the equilibrium point x_e, which demonstrates the correctness of the results.

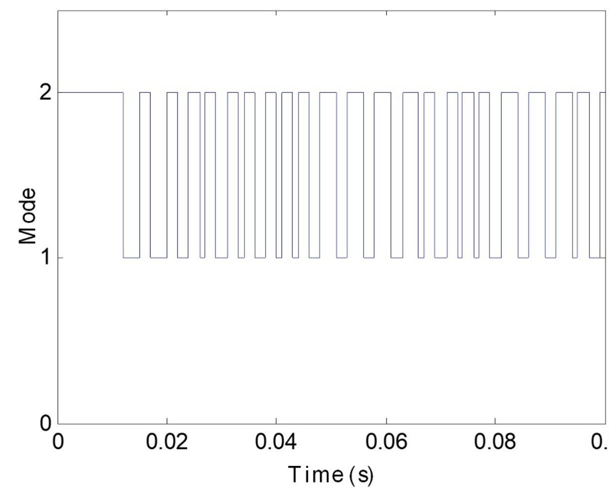

Fig. 5. The evolution of the switching signal.

5. CONCLUSION

In this paper, the event-triggered control problem for switched affine linear systems has been studied. A stability criterion is proposed to ensure that the switched affine system exponentially converges to a small neighborhood of the desired equilibrium point. An event triggering scheme and a state-dependent switching signal are proposed by constructing a piecewise differential Lyapunov function with time-scheduled matrices. We also present the result on the disturbance attenuation performance analysis. In future work, we will focus on the controller design and robust control for the considered systems. We will also extend the proposed results to switched affine nonlinear systems.

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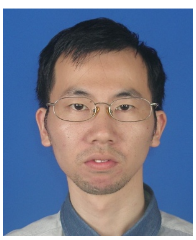

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