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Pinning Synchronization of Stochastic T-S Fuzzy Delayed Complex Dynamical Networks with Heterogeneous Impulsive Delays

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Abstract: In this paper, we deal with the exponential pinning synchronization (PS) of stochastic T-S fuzzy delayed complex dynamical networks (FDCDNs) with heterogeneous impulsive delays. Unlike the existing works, a fuzzy memory pinning impulsive control (FMPIC) approach is proposed. In order to conquer the difficulties of studying such a general system, sufficient conditions that depend on the discrete-delay and distributed-delay impulsive effects are obtained by employing the Lyapunov function, inequality techniques and stochastic analysis theory. It is shown that the PS of FDCDNs can be achieved under the designed FMPIC. Numerical simulation on the basis of BA scale-free coupled network is used to illustrate the effectiveness of the theoretical results.

Keywords: Complex dynamical networks, heterogeneous impulsive delays, pinning synchronization, T-S fuzzy system.

1. INTRODUCTION

Complex dynamical networks (CDNs) have been extensively researched over the past few decades because of their potential applications in various fields of science and engineering, such as sensor networks, communication network and linear and nonlinear programming problems [1–4]. Meanwhile, among all of the dynamical behavior CDNs, synchronization has been a fascinating topic since there exist some collective features in individual node naturally. In addition, time delay is very common in natural and man-made systems, and the existence of time delay usually results in poor performance, including divergence, oscillation, or even instability of CDNs [5–9]. Hence, the synchronization of DCDNs have drawn noticeable attention, and a large number of outstanding results have been reported in [10–15] and references cited therein.

It is well known that Takagi-Sugeno (T-S) fuzzy model has good properties in analyzing, synthesizing, and approximating complex dynamical behaviors by a set of IF-THEN rules, which can give local linear representation of the nonlinear system. To this end, many well-known linear system theories can be used to system analysis and synthesis of nonlinear complex systems [16–20]. Moreover, in many real-world and natural processes, the actual signal transmission between subsystems of coupled CDNs is inevitably subject to stochastic perturbation from various uncertainties [21, 22], which may lead to package loss or transmitted signals not fully being detected and received [23]. Naturally, in order to model more realistic CDNs, we should take full account of all the aspects that are influential in studying the CDNs. So far, this issue has recently attracted increasing attention, many literatures regarding the synchronization of T-S fuzzy CDNs (FCDNs) related to stochastic perturbation have been introduced and have left many fruitful results [24–28]. For example, A truck-trailer system is presented to the applicability of the T-S fuzzy systems in [25]. Tang *et al*. [26] studied the T-S fuzzy discrete-time CDNs with stochastic disturbances based on the Lyapunov functional (LF) and some stochastic analysis techniques (SATs) to ensure the mean-square synchronization of the considered system. By employing Kronecker product and SATs, delay-dependent synchronisation criterions of stochastic T-S FCDNs were obtained in [27]. Moreover, because of interval type-2 (IT2) for the system with strong uncertainty and nonlinearity, in order to reduce the conservatism of obtained results [26, 27], on basis of the parallel-distributed compensation (PDC) scheme, the issue of synchronization of IT2 stochastic T-S FCDNs by using fuzzy pinning control was addressed in [28].

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On the other hand, compared with continuous control approach [29], the impulsive control has been validated to CDNs since the impulsive control is activated only at impulsive instants while the continuous control does so at every moment [30]. Based on LF, SATs and a new lemma, Yang *et al*. [31] investigated the synchronization of TS FCDNs with stochastic effects via delayed impulsive distributed control, where the stochastic T-S FCDN was applied to small-world network coupled with time-delayed Lorenz system. Meanwhile, in view of the control cost will be expensive for the application point of view, the idea of controlling a small portion of nodes, e.g., pinning impulsive control (PIC) strategy was proposed by [32, 33]. Accordingly, the investigation of PIC of CDNs is very interesting. Very recently, by using LF together with PIC, pinning impulsive synchronization of a class of CDNs with time-varying delay was solved in [34]. Moreover, a finite-time synchronization condition for CDNs on time scales under PIC was established in [35]. For other recent works, please refer to [36–40] and references cited therein.

It should be noted that the impulsive effects in these published works such as in [31, 38, 40] are only discrete delays, which have been studied by a large number of researchers. However, in real life, such as financial markets, fishery industry and population dynamics, we need to consider distributed-delay dependent impulsive effects. The main difference between discrete-delay and distributed-delay impulses is that the jumps of systems states of distributed-delay depend on the accumulation of the system states over a history time period, while discrete-delay utilizes the states at each impulsive instant or the states at history time. For example, as shown in [41, 42], in order to keep the financial market stable, at some times, new interest rates should be artificially controlled in taking the interest rates during a history time period as a reference. Therefore, the distributed-delay dependent impulse is introduced such that the overall impact of calculation for the new interest rates more accurately. Although distributed-delay dependent impulse is featured with such advantages, the research of FMPIC synchronization of stochastic T-S FDCDNs with heterogeneous impulsive delays is still a difficult task, which is the motivation of this paper.

Motivated by the above discussions, we devote to investigating the issue of PS for stochastic T-S FDCDNs in this paper. The main contributions can be summarized as follows: 1) The heterogeneous time-varying coupling delays and impulsive delays are taken into account. 2) Based on LF, inequality techniques and SATs, sufficient conditions that depend on the discrete-delay and distributed-delay impulsive effects are established. 3) Furthermore, in order to solve the difficulties of studying such general system, an FMPIC approach is designed to guarantee the exponential PS of the considered stochastic T-S FDCDNs. This approach is a more general method than these results [31–33, 36–38, 40] in the previous approach.

Notations: Let I_n denote the $n\times n$ identity matrix. $\Re^n$ denotes n-dimensional Euclidean space, $\Re^{n\times n}$ is the set of all $n\times n$ real matrices. For any symmetric matrices $\mathcal{X}$ and $\mathcal{Y}$, the notation $\mathcal{X}>\mathcal{Y}(\mathcal{X}\geq\mathcal{Y})$ means that the matrix $\mathcal{X}-\mathcal{Y}$ is positive definite (nonnegative). Symbol $\otimes$ stands for Kronecker product and $diag\{\dots\}$ denotes the block diagonal matrix. $\lambda_{min}(\mathcal{X})$ and $\lambda_{max}(\mathcal{X})$ are the minimum and maximum eigenvalues of $\mathcal{X}$. The superscript T denotes matrix or vector transposition. $\|\cdot\|_2$ refers to the induced matrix 2-norm. Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$ be a complete probability space with filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions(i.e., the filtration contains all P-null sets and is right continuous) and $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to the given probability measure P. $\mathcal{N}^{\dagger}$ denotes the set $\{1,2,\dots,N\}$.

2. PRELIMINARIES

Consider the following stochastic T-S FDCDNs with r rules:

Plant Rule s: If $z_1(t)$ is $\mathscr{M}_{s1}$, $z_2(t)$ is $\mathscr{M}_{s2}$, ..., $z_p(t)$ is $\mathscr{M}_{sp}$,

then

$$\begin{cases} dx_i(t)=[\mathcal{A}_s x_i(t)+\mathcal{B}_s x_i(t-\tau_1(t))+c_1\sum_{j=1}^{N}c_{ij}^{s}x_j(t) \\ \qquad +c_2\sum_{j=1}^{N}d_{ij}^{s}x_j(t-\tau_2(t))]dt \\ \qquad +\sigma_s(x_i(t),x_i(t-\tau_3(t)))dw(t),\ \ i\in\mathcal{N}^{\dagger}, \\ x_i(t)=\phi_i(t),\ \ \forall t\in[-\tau^*,0], \end{cases} \tag{1}$$

$$\begin{cases} dy_i(t)=[\mathcal{A}_s y_i(t)+\mathcal{B}_s y_i(t-\tau_1(t))+c_1\sum_{j=1}^{N}c_{ij}^{s}y_j(t) \\ \qquad +c_2\sum_{j=1}^{N}d_{ij}^{s}y_j(t-\tau_2(t))+u_i(t)]dt \\ \qquad +\sigma_s(y_i(t),y_i(t-\tau_3(t)))dw(t),\ \ i\in\mathcal{N}^{\dagger}, \\ y_i(t)=\psi_i(t),\ \ \forall t\in[-\tau^*,0], \end{cases} \tag{2}$$

where $z_\ell(t)$ denotes the premise variable and $\mathscr{M}_{s\ell}$ is the fuzzy set, $\ell=1,\ 2,\ \dots,\ p$, $s=1,\ 2,\ \dots,\ r$, r is a positive integer. $x_i(t)=(x_{i1}(t),\ x_{i2}(t),\ \dots,\ x_{in}(t))^T\in\Re^n$ and $y_i(t)=(y_{i1}(t),\ y_{i2}(t),\ \dots,\ y_{in}(t))^T\in\Re^n$ stand for the master state vector and the slave state variable of the ith node, respectively. $\tau_\vartheta(t)$ is the time-varying delay satisfying $\underline{\tau}_\vartheta\leq\tau_\vartheta(t)\leq\overline{\tau}_\vartheta$, $max\{\overline{\tau}_\vartheta\}=\tau^*$, in which τ^*, $\underline{\tau}_\vartheta$, $\overline{\tau}_\vartheta$ ($\vartheta=1,\ 2,\ 3$) are the positive scalars. c_1 and c_2 are coupling strengths. Matrices $\mathcal{A}_s\in\Re^{n\times n}$, $\mathcal{B}_s\in\Re^{n\times n}$ are known real matrices, $\mathcal{C}_s=(c_{ij}^s)_{N\times N}$ and $\mathcal{D}_s=(d_{ij}^s)_{N\times N}$ with the sum of each row being zero are the non-delayed and delayed outer couplings of the whole networks. $u_i(t)$ is a control input to be designed later. $\sigma_s(\cdot,\cdot):\Re^n\times\Re^n\to\Re^{n\times m}$ is a Borel function. $w(t)=(w_1(t),\ w_2(t),\ \dots,\ w_m(t))^T$ is an m-

dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ satisfying $\mathbf{E}\{dw(t)\} = 0$, $\mathbf{E}\{dw(t)^2\} = dt$. $\phi_i(tT)$ and $\psi_i(t)$ are the initial conditions.

By utilizing the singleton fuzzifier, product inference, and the center average defuzzifier, the final stochastic T-S fuzzy systems (1) and (2) are described as

$$\begin{cases} dx_i(t) = \sum_{s=1}^{r} h_s(z(t))[\mathcal{A}_s x_i(t) + \mathcal{B}_s x_i(t-\tau_1(t)) \\ \qquad + c_1 \sum_{j=1}^{N} c_{ij}^s x_j(t) + c_2 \sum_{j=1}^{N} d_{ij}^s x_j(t-\tau_2(t))]dt \\ \qquad + \sum_{s=1}^{r} h_s(z(t))\sigma_s(x_i(t), x_i(t-\tau_3(t)))dw(t) \\ dy_i(t) = \sum_{s=1}^{r} h_s(z(t))[\mathcal{A}_s y_i(t) + \mathcal{B}_s y_i(t-\tau_1(t)) \\ \qquad + c_1 \sum_{j=1}^{N} c_{ij}^s y_j(t) \\ \qquad + c_2 \sum_{j=1}^{N} d_{ij}^s y_j(t-\tau_2(t)) + u_i(t)]dt \\ \qquad + \sum_{s=1}^{r} h_s(z(t))\sigma_s(y_i(t), y_i(t-\tau_3(t)))dw(t), \end{cases} \tag{3}$$

where $h_s(z(t))$ stands for the normalized membership function satisfying $0 \leq h_s(z(t)) \leq 1$ $(s = 1, 2, \ldots, r)$ and $\sum_{s=1}^{r} h_s(z(t)) = 1$.

Let $e_i(t) = y_i(t) - x_i(t)$, $i \in \mathcal{N}^\dagger$ as the synchronization error, then we have

$$\begin{cases} de_i(t) = \sum_{s=1}^{r} h_s(z(t))[\mathcal{A}_s e_i(t) + \mathcal{B}_s e_i(t-\tau_1(t)) \\ \qquad + c_1 \sum_{j=1}^{N} c_{ij}^s e_j(t) \\ \qquad + c_2 \sum_{j=1}^{N} d_{ij}^s e_j(t-\tau_2(t)) + u_i(t)]dt \\ \qquad + \sum_{s=1}^{r} h_s(z(t))\overline{\sigma}_s(e_i(t), e_i(t-\tau_3(t)))dw(t), \\ e_i(t) = \varphi_i(t), \quad \forall t \in [-\tau^*, 0], \end{cases} \tag{4}$$

where $\overline{\sigma}_s(e_i(t), e_i(t-\tau_3(t))) = \sigma_s(y_i(t), y_i(t-\tau_3(t))) - \sigma_s(x_i(t), x_i(t-\tau_3(t)))$ and $\varphi_i(t) = \psi_i(t) - \phi_i(t)$.

In this paper, the following assumptions, definitions and lemmas are useful in deriving our main results.

Assumption 1: There exist nonnegative constants ρ_{1s}, ρ_{2s} such that

$$\begin{aligned} &trace\{[\sigma_s(x,y) - \sigma_s(\bar{x},\bar{y})]^T[\sigma_s(x,y) - \sigma_s(\bar{x},\bar{y})]\} \\ &\leq \rho_{1s}(x-\bar{x})^T(x-\bar{x}) + \rho_{2s}(y-\bar{y})^T(y-\bar{y}) \end{aligned} \tag{5}$$

holds for all $s = 1, 2, \ldots, r$, and any $x, \bar{x}, y, \bar{y} \in \Re^n$.

Definition 1: The stochastic T-S fuzzy system (4) is said to be mean square exponentially stable if there exist a pair of positive constants $M \geq 1$ and α such that

$$\mathbf{E}(\|e(t)\|^2) \leq M \sup_{-\tau^* \leq s \leq 0} \mathbf{E}(\|\varphi(s)\|^2)e^{-\alpha(t-t_0)}, \quad t \geq t_0. \tag{6}$$

Definition 2: If system (4) satisfies the following condition:

$$\lim_{t\to\infty} \|e_i(t)\| = \lim_{t\to\infty} \|y_i(t) - x_i(t)\| = 0, \quad i \in \mathcal{N}^\dagger. \tag{7}$$

then the T-S FCDNs (3) are said to achieve synchronization.

Lemma 1: If $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ are real matrices with appropriate dimensions, then there exists scalars $\delta > 0$, $\varepsilon > 0$ such that the following inequality holds

$$\begin{aligned} &(\mathcal{X}+\mathcal{Y}+\mathcal{Z})^T(\mathcal{X}+\mathcal{Y}+\mathcal{Z}) \\ &\leq (1+\delta)\mathcal{X}^T\mathcal{X} + (1+\delta^{-1})(1+\varepsilon)\mathcal{Y}^T\mathcal{Y} \\ &\quad + (1+\delta^{-1})(1+\varepsilon^{-1})\mathcal{Z}^T\mathcal{Z}. \end{aligned} \tag{8}$$

Proof: The inequality (8) can be derived straightforwardly based on [lemma 2.1, 38] $\mathcal{X}^T\mathcal{Y} + \mathcal{Y}^T\mathcal{X} \leq \delta\mathcal{X}^T\mathcal{X} + \delta^{-1}\mathcal{Y}^T\mathcal{Y}$. Hence, the proof is omitted here.

Lemma 2 [31]: Consider the following impulsive differential inequalities:

$$\begin{cases} D^+V(t) \leq \alpha V(t) + \alpha_1\{V(t)\}_{\overline{\tau}_1} + \alpha_2\{V(t)\}_{\overline{\tau}_2} \\ \qquad + \cdots + \alpha_{\aleph}\{V(t)\}_{\overline{\tau}_{\aleph}}, \quad t \neq t_k, \quad t \geq 0, \\ V(t_k^+) \leq \beta_k V(t_k^-) + \beta_k^1\{V(t_k^-)\}_{\overline{\tau}_1} + \beta_k^2\{V(t_k^-)\}_{\overline{\tau}_2} \\ \qquad + \cdots + \beta_k^{\aleph}\{V(t_k^-)\}_{\overline{\tau}_{\aleph}}, \; k \in \mathbb{Z}^+, \\ V(t) = \varphi(t), \; t \in [t_0 - \overline{\tau}, t_0], \end{cases}$$

where $\alpha \in \Re$, $\alpha_i \geq 0$, $\beta_k \geq 0$, $\beta_k^i \geq 0$, $\overline{\tau}_i \geq 0$ $(i = 1, 2, \ldots, \aleph)$ and the function $V(t) \in PC(\mathbb{R}, \mathbb{R}^+)$, $\{V(t)\}_{\overline{\tau}_i} = \sup_{t-\overline{\tau}_i \leq s \leq t} V(s)$, $\{V(t_k)\}_{\overline{\tau}_i} = \sup_{t_k-\overline{\tau}_i \leq s < t_k} V(s)$. Suppose that

$$\beta_k + \sum_{i=1}^{\aleph} \beta_k^i < 1,$$

$$\alpha + \frac{\sum_{i=1}^{\aleph} \alpha_i}{\beta_k + \sum_{j=1}^{\aleph} \beta_k^j} + \frac{ln(\beta_k + \sum_{j=1}^{\aleph} \beta_k^j)}{t_{k+1} - t_k} < 0,$$

then exist $M > 1$ and $\lambda > 0$ such that

$$V(t) \leq \sup_{t_0 - \overline{\tau} \leq s \leq t_0} V(s) M e^{-\lambda(t-t_0)}, \; t \geq 0,$$

where $\overline{\tau} = max\{\overline{\tau}_i, \; i = 1, 2, \ldots, \aleph\}$.

3. MAIN RESULTS

3.1. Synchronization with FMPIC

In the FMPIC scheme, we assume that the fuzzy impulsive controllers share the same premise parts as in (3). That is to say, the impulsive controller for rule ϑ is designed by

Plant Rule ϑ: If $z_1(t)$ is $\mathcal{M}_{\vartheta 1}$, $z_2(t)$ is $\mathcal{M}_{\vartheta 2}$, $\dots$, $z_p(t)$ is $\mathcal{M}_{\vartheta p}$,

then

$$u_i(t)=\begin{cases}\sum_{k=1}^{\infty}[d_{1,k}^{\vartheta}e_i(t_k^-)+d_{2,k}^{\vartheta}e_i(t_k-\zeta_1(t_k))\\ \quad +d_{3,k}^{\vartheta}\int_{t_k-\zeta_2(t_k)}^{t_k}e_i(s)ds]\delta(t-t_k),\\ \qquad\qquad i\in\mathcal{D}_k^{\dagger}\text{ and }\sharp\mathcal{D}_k^{\dagger}=\partial_k,\\ 0,\qquad\qquad i\notin\mathcal{D}_k^{\dagger},\end{cases}\tag{9}$$

where $d_{1,k}^{\vartheta}$, $d_{2,k}^{\vartheta}$ and $d_{3,k}^{\vartheta}$ denote the impulsive control gains to be designed, $\delta(\cdot)$ is the Dirac delta function. $\zeta_i(t)$ is the impulsive delay of controller, which satisfies $\underline{\zeta}_i\le\zeta_i(t)\le\overline{\zeta}_i$ $(i=1,2)$, $\tau=\max\{\tau^*,\overline{\zeta}_j\}$ $(j=1,2)$. In addition, we assume that the impulsive time instant is generated by using a zero-order hold (ZOH) function with a sequence of hold times $0=t_0<t_1<t_2<\cdots<t_k<\cdots$, $t_k\to\infty$ as $k\to\infty$. Moreover, ∂_k denotes the number of nodes to be controlled at impulsive instant t_k. The set $\mathcal{D}_k^{\dagger}$ is defined as follows: we can reorder the vector states $e_1(t_k)$, $e_2(t_k)$, $\dots$, $e_N(t_k)$ such that $\|e_{\ell 1}(t_k)\|\ge\|e_{\ell 2}(t_k)\|\ge\dots\ge\|e_{\ell\partial_k}(t_k)\|\ge\|e_{\ell(\partial_k+1)}(t_k)\|\ge\dots\ge\|e_{\ell N}(t_k)\|$ at the impulsive instant t_k, then we have $\mathcal{D}_k^{\dagger}=\{\ell 1,\ell 2,\dots,\ell\partial_k\}$ and $\sharp\mathcal{D}_k^{\dagger}=\partial_k$.

Then, the final FMPIC is represented by

$$\begin{aligned}u_i(t_k)=&\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))(d_{1,k}^{\vartheta}e_i(t_k^-)+d_{2,k}^{\vartheta}e_i(t_k-\zeta_1(t_k))\\ &+d_{3,k}^{\vartheta}\int_{t_k-\zeta_2(t_k)}^{t_k}e_i(s)ds),\quad i\in\mathcal{D}_k^{\dagger}.\end{aligned}\tag{10}$$

Thus, we yield the following error system under FMPIC (10),

$$\begin{cases}de_i(t)=\sum_{s=1}^{r}h_s(z(t))[\mathcal{A}_se_i(t)+\mathcal{B}_se_i(t-\tau_1(t))\\ \qquad +c_1\sum_{j=1}^{N}c_{ij}^se_j(t)+c_2\sum_{j=1}^{N}d_{ij}^se_j(t-\tau_2(t))]dt\\ \qquad +\sum_{s=1}^{r}h_s(z(t))\overline{\sigma}_s(e_i(t),e_i(t-\tau_3(t)))dw(t),\\ \qquad t\ne t_k,\\ \Delta e_i(t_k)=\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))[d_{1,k}^{\vartheta}e_i(t_k^-)+d_{2,k}^{\vartheta}e_i(t_k-\zeta_1(t_k))\\ \qquad +d_{3,k}^{\vartheta}\int_{t_k-\zeta_2(t_k)}^{t_k}e_i(s)ds],\ i\in\mathcal{D}_k^{\dagger},\\ e_i(t)=\varphi_i(t),\ \forall t\in[-\tau,0].\end{cases}\tag{11}$$

Now, with the help of FMPIC and LF approach, Theorems 1-2 can be derived.

Theorem 1: Suppose that Assumption 1 holds, if there exist positive scalars κ_1, κ_2, $\underline{\delta}$, $\overline{\delta}$ such that the following inequalities hold for all $\vartheta=1,2,\dots,r$:

$$\begin{aligned}&\overline{q}_k=1+\frac{\partial_k}{N}(q_k-1)+(1+\underline{\delta}^{-1})(1+\overline{\delta})\overline{d}_{2,k}^2\\ &\quad+(1+\underline{\delta}^{-1})(1+\overline{\delta}^{-1})\overline{\zeta}_2^2\overline{d}_{3,k}^2<1,\\ &p+\frac{\kappa_1^{-1}\overline{b}+c_2\kappa_2^{-1}\underline{d}+\overline{\rho}_2}{\overline{q}_k}+\frac{\ln\overline{q}_k}{t_{k+1}-t_k}<0,\end{aligned}\tag{12}$$

where $q_k=(1+\underline{\delta})\overline{d}_{1,k}^2$, $\overline{d}_{1,k}=\max_{\vartheta}\{|1+d_{1,k}^{\vartheta}|\}$, $\overline{d}_{2,k}=\max_{\vartheta}\{|d_{2,k}^{\vartheta}|\}$, $\overline{d}_{3,k}=\max_{\vartheta}\{|d_{3,k}^{\vartheta}|\}$, $p=\overline{a}+\kappa_1+2c_1\overline{c}+c_2\kappa_2\overline{d}+\overline{\rho}_1$. Then, the stochastic T-S FCDNs (3) can achieve synchronization under the FMPIC (10).

Proof: Consider the following LF

$$V(t)=\sum_{i=1}^{N}e_i^T(t)e_i(t).\tag{13}$$

First of all, when $t=t_k$, we have

$$\begin{aligned}V(t_k)=&\sum_{i=1}^{N}e_i^T(t_k)e_i(t_k)\\ =&\sum_{i\in\mathcal{D}_k^{\dagger}}e_i^T(t_k)e_i(t_k)+\sum_{i\notin\mathcal{D}_k^{\dagger}}e_i^T(t_k)e_i(t_k)\\ =&\sum_{i\in\mathcal{D}_k^{\dagger}}\Big\{\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))[(1+d_{1,k}^{\vartheta})e_i(t_k^-)\\ &+d_{2,k}^{\vartheta}e_i(t_k-\zeta_1(t_k))+d_{3,k}^{\vartheta}\int_{t_k-\zeta_2(t_k)}^{t_k}e_i(s)ds]^T\Big\}\\ &\times\Big\{\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))[(1+d_{1,k}^{\vartheta})e_i(t_k^-)\\ &+d_{2,k}^{\vartheta}e_i(t_k-\zeta_1(t_k))+d_{3,k}^{\vartheta}\int_{t_k-\zeta_2(t_k)}^{t_k}e_i(s)ds]\Big\}\\ &+\sum_{i\notin\mathcal{D}_k^{\dagger}}e_i^T(t_k^-)e_i(t_k^-).\end{aligned}\tag{14}$$

Let $X_{i,1}^{\vartheta}=\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))(1+d_{1,k}^{\vartheta})e_i(t_k^-)$, $X_{i,2}^{\vartheta}=\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))d_{2,k}^{\vartheta}e_i(t_k-\zeta_1(t_k))$, $X_{i,3}^{\vartheta}=\sum_{\vartheta=1}^{r}h_{\vartheta}(z(t_k))d_{3,k}^{\vartheta}\int_{t_k-\zeta_2(t_k)}^{t_k}e_i(s)ds$.

By using Lemma 1,

$$\begin{aligned}&\sum_{i\in\mathcal{D}_k^{\dagger}}(X_{i,1}^{\vartheta}+X_{i,2}^{\vartheta}+X_{i,3}^{\vartheta})^T(X_{i,1}^{\vartheta}+X_{i,2}^{\vartheta}+X_{i,3}^{\vartheta})\\ &\le(1+\underline{\delta})\sum_{i\in\mathcal{D}_k^{\dagger}}\{X_{i,1}^{\vartheta}\}^T\{X_{i,1}^{\vartheta}\}\\ &\quad+(1+\underline{\delta}^{-1})(1+\overline{\delta})\sum_{i\in\mathcal{D}_k^{\dagger}}\{X_{i,2}^{\vartheta}\}^T\{X_{i,2}^{\vartheta}\}\\ &\quad+(1+\underline{\delta}^{-1})(1+\overline{\delta}^{-1})\sum_{i\in\mathcal{D}_k^{\dagger}}\{X_{i,3}^{\vartheta}\}^T\{X_{i,3}^{\ell}\},\end{aligned}\tag{15}$$

where

$$\{X_{i,1}^{\vartheta}\}^T\{X_{i,1}^{\vartheta}\}$$

$$
\begin{aligned}
&= \sum_{\vartheta=1}^{r}\sum_{\ell=1}^{r} h_\vartheta(z(t_k))h_\ell(z(t_k))(1+d_{1,k}^{\vartheta})(1+d_{1,k}^{\ell}) \\
&\quad \times e_i^T(t_k^-)e_i(t_k^-) \\
&\le \sum_{\vartheta=1}^{r}\sum_{\ell=1}^{r} h_\vartheta(z(t_k))h_\ell(z(t_k))[(1+d_{1,k}^{\vartheta})^2/2 \\
&\quad +(1+d_{1,k}^{\ell})^2/2]e_i^T(t_k^-)e_i(t_k^-) \\
&= \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))(1+d_{1,k}^{\vartheta})^2 e_i^T(t_k^-)e_i(t_k^-),
\end{aligned}
$$

$$
\begin{aligned}
&\{X_{i,2}^{\vartheta}\}^T\{X_{i,2}^{\vartheta}\} \\
&= \sum_{\vartheta=1}^{r}\sum_{\ell=1}^{r} h_\vartheta(z(t_k))h_\ell(z(t_k))d_{2,k}^{\vartheta}d_{2,k}^{\ell}e_i^T(t_k-\zeta_1(t_k)) \\
&\quad \times e_i(t_k-\zeta_1(t_k)) \\
&\le \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))(d_{2,k}^{\vartheta})^2 e_i^T(t_k-\zeta_1(t_k))e_i(t_k-\zeta_1(t_k)),
\end{aligned}
$$

$$
\begin{aligned}
&\{X_{i,3}^{\vartheta}\}^T\{X_{i,3}^{\vartheta}\} \\
&= \sum_{\vartheta=1}^{r}\sum_{\ell=1}^{r} h_\vartheta(z(t_k))h_\ell(z(t_k))d_{3,k}^{\vartheta}d_{3,k}^{\ell}\int_{t_k-\zeta_2(t_k)}^{t_k} e_i^T(s)ds \\
&\quad \times \int_{t_k-\zeta_2(t_k)}^{t_k} e_i(s)ds \\
&\le \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))(d_{3,k}^{\vartheta})^2\int_{t_k-\zeta_2(t_k)}^{t_k} e_i^T(s)ds \\
&\quad \int_{t_k-\zeta_2(t_k)}^{t_k} e_i(s)ds \\
&\le \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))\bar{\zeta}_2(d_{3,k}^{\vartheta})^2\int_{t_k-\zeta_2(t_k)}^{t_k} e_i^T(s)e_i(s)ds.
\end{aligned}
$$

Additionally, since

$$
\frac{1}{N-\partial_k}\sum_{i\notin\mathcal{D}_k^{\dagger}} e_i^T(t_k^-)e_i(t_k^-) \le \frac{1}{N}\sum_{i=1}^{N} e_i^T(t_k^-)e_i(t_k^-).
$$

Then, we have

$$
\begin{aligned}
V(t_k) \le& \sum_{i\in\mathcal{D}_k^{\dagger}}\Big[(1+\underline{\delta})\overline{d}_{1,k}^2 e_i^T(t_k^-)e_i(t_k^-) \\
&+(1+\underline{\delta}^{-1})(1+\overline{\delta})\overline{d}_{2,k}^2 e_i^T(t_k-\zeta_1(t_k)) \\
&\times e_i(t_k-\zeta_1(t_k))+(1+\underline{\delta}^{-1})(1+\overline{\delta}^{-1})\overline{\zeta}_2\overline{d}_{3,k}^2 \\
&\times\int_{t_k-\zeta_2(t)}^{t_k} e_i^T(s)e_i(s)ds\Big]+\sum_{i\notin\mathcal{D}_k^{\dagger}} e_i^T(t_k^-)e_i(t_k^-) \\
\le&[1+\frac{\partial_k}{N}(q_k-1)]\sum_{i=1}^{N} e_i^T(t_k^-)e_i(t_k^-) \\
&+(1+\underline{\delta}^{-1})(1+\overline{\delta})\overline{d}_{2,k}^2\sum_{i=1}^{N} e_i^T(t_k-\zeta_1(t_k)) \\
&\times e_i(t_k-\zeta_1(t_k))+(1+\underline{\delta}^{-1})(1+\overline{\delta}^{-1})\overline{\zeta}_2\overline{d}_{3,k}^2 \\
&\times\sum_{i=1}^{N}\int_{t_k-\zeta_2(t_k)}^{t_k} e_i^T(s)e_i(s)ds \\
\le&[1+\frac{\partial_k}{N}(q_k-1)]V(t_k^-)+(1+\underline{\delta}^{-1})(1+\overline{\delta}) \\
&\times\overline{d}_{2,k}^2\{V(t_k^-)\}_{\overline{\zeta}_1}+(1+\underline{\delta}^{-1})(1+\overline{\delta}^{-1})\overline{\zeta}_2^2 \\
&\times\overline{d}_{3,k}^2\{V(t_k^-)\}_{\overline{\zeta}_2},
\end{aligned}
\tag{16}
$$

where $q_k=(1+\underline{\delta})\overline{d}_{1,k}^2$, $\overline{d}_{1,k}=\underset{\vartheta}{max}\{|1+d_{1,k}^{\vartheta}|\}$, $\overline{d}_{2,k}=\underset{\vartheta}{max}\{|d_{2,k}^{\vartheta}|\}$, $\overline{d}_{3,k}=\underset{\vartheta}{max}\{|d_{3,k}^{\vartheta}|\}$.

Taking mathematical expectations of both sides for the (17), one has

$$
\begin{aligned}
\mathbf{E}\{V(t_k)\}\le&[1+\frac{\partial_k}{N}(q_k-1)]\mathbf{E}\{V(t_k^-)\} \\
&+(1+\underline{\delta}^{-1})(1+\overline{\delta})\overline{d}_{2,k}^2\mathbf{E}\{V(t_k)\}_{\overline{\zeta}_1} \\
&+(1+\underline{\delta}^{-1})(1+\overline{\delta}^{-1})\overline{\zeta}_2^2\overline{d}_{3,k}^2\mathbf{E}\{V(t_k^-)\}_{\overline{\zeta}_2}.
\end{aligned}
\tag{17}
$$

On the other hand, when $t\in[t_{k-1},t_k)$, the weak infinitesimal operator $\mathscr{L}$ of the stochastic process along the evolution of $V(t)$ yields

$$
\begin{aligned}
&\mathscr{L}V(t) \\
&=2\sum_{s=1}^{r} h_s(z(t))[\sum_{i=1}^{N} e_i^T(t)(\mathcal{A}_s e_i(t) \\
&\quad +\mathcal{B}_s e_i(t-\tau_1(t)))+c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)c_{ij}^s e_j(t) \\
&\quad +c_2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)d_{ij}^s e_j(t-\tau_2(t))] \\
&\quad +\sum_{i=1}^{N}\frac{1}{2}trace(\sum_{s=1}^{r} h_s(z(t))\overline{\sigma}_s^T(e_i(t),e_i(t-\tau_3(t))) \\
&\quad \times 2I\sum_{s=1}^{r} h_s(z(t))\overline{\sigma}_s(e_i(t),e_i(t-\tau_3(t)))).
\end{aligned}
\tag{18}
$$

In view of Assumption 1 and the condition $\sum_{s=1}^{r} h_s(z(t))=1$,

$$
\begin{aligned}
&\frac{1}{2}trace(\sum_{s=1}^{r} h_s(z(t))\overline{\sigma}_s^T(e_i(t),e_i(t-\tau_3(t)))2I \\
&\quad \times\sum_{s=1}^{r} h_s(z(t))\overline{\sigma}_s(e_i(t),e_i(t-\tau_3(t)))) \\
&=\frac{1}{2}trace(\sum_{s=1}^{r}\sum_{\vartheta=1}^{r} h_s(z(t))h_\vartheta(z(t))2 \\
&\quad \times\overline{\sigma}_s^T(e_i(t),e_i(t-\tau_3(t)))\overline{\sigma}_\vartheta(e_i(t),e_i(t-\tau_3(t)))) \\
&\le\frac{1}{2}trace(\sum_{s=1}^{r}\sum_{\vartheta=1}^{r} h_s(z(t))h_\vartheta(z(t)) \\
&\quad \times[\overline{\sigma}_s^T(e_i(t),e_i(t-\tau_3(t)))\overline{\sigma}_s(e_i(t),e_i(t-\tau_3(t))) \\
&\quad +\overline{\sigma}_\vartheta^T(e_i(t),e_i(t-\tau_3(t)))\overline{\sigma}_\vartheta(e_i(t),e_i(t-\tau_3(t)))]) \\
&=\sum_{s=1}^{r} h_s(z(t))trace(\bar{\sigma}_s^T(e_i(t),e_i(t-\tau_3(t)))
\end{aligned}
$$

$$\begin{aligned}&\times \bar{\sigma}_s(e_i(t),e_i(t-\tau_3(t))))\\ &\leq \sum_{s=1}^{r} h_s(z(t))[\rho_{1s}e_i^T(t)e_i(t)\\ &\quad+\rho_{2s}e_i^T(t-\tau_3(t))e_i(t-\tau_3(t))]. \end{aligned} \tag{19}$$

In addition, it is calculated that

$$\begin{aligned}&2\sum_{i=1}^{N} e_i^T(t)\mathcal{B}_s e_i(t-\tau_1(t))\\ &\leq \sum_{i=1}^{N}\kappa_1 e_i^T(t)e_i(t)\\ &\quad+\kappa_1^{-1}\sum_{i=1}^{N} e_i^T(t-\tau_1(t))\mathcal{B}_s^T\mathcal{B}_s e_i^T(t-\tau_1(t))\\ &\leq \sum_{i=1}^{N}\kappa_1 e_i^T(t)e_i(t)\\ &\quad+\kappa_1^{-1}\overline{b}\sum_{i=1}^{N} e_i^T(t-\tau_1(t))e_i^T(t-\tau_1(t)), \end{aligned} \tag{20}$$

$$\begin{aligned}&2c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)c_{ij}^s e_j(t)\\ &=2c_1\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}^s\sum_{\ell=1}^{n} e_{i\ell}^T(t)e_{j\ell}(t)\\ &=2c_1\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{n} e_{i\ell}^T(t)c_{ij}^s e_{j\ell}(t)\\ &=2c_1\sum_{\ell=1}^{n}(e^{\ell}(t))^T\mathcal{C}_s e^{\ell}(t)\\ &\leq 2c_1\overline{c}\sum_{i=1}^{N} e_i^T(t)e_i(t), \end{aligned} \tag{21}$$

and

$$\begin{aligned}&2c_2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)d_{ij}^s e_j(t-\tau_2(t))\\ &\leq 2c_2\sum_{i=1}^{N}\sum_{j=1}^{N}\|e_i^T(t)\||d_{ij}^s|\|e_j(t-\tau_2(t))\|\\ &\leq c_2\sum_{i=1}^{N}\sum_{j=1}^{N}|d_{ij}^s|(\kappa_2 e_i^T(t)e_i(t)\\ &\quad+\kappa_2^{-1}e_j^T(t-\tau_2(t))e_j^T(t-\tau_2(t)))\\ &\leq c_2\kappa_2\overline{d}\sum_{i=1}^{N} e_i^T(t)e_i(t)\\ &\quad+c_2\kappa_2^{-1}\underline{d}\sum_{i=1}^{N} e_i^T(t-\tau_2(t))e_i^T(t-\tau_2(t)), \end{aligned} \tag{22}$$

where $\overline{b}=\max_s\|\mathcal{B}_s\|^2$, $\overline{c}=\max_s\|\mathcal{C}_s\|$, $\overline{d}=\max_{s,i}\sum_{j=1}^{N}|d_{ij}^s|$, $\underline{d}=\max_{s,j}\sum_{i=1}^{N}|d_{ij}^s|$.

Substituting (20)-(23) into (19), we have

$$\begin{aligned}\mathscr{L}V(t)\leq&2\sum_{s=1}^{r} h_s(z(t))[\sum_{i=1}^{N} e_i^T(t)(\mathcal{A}_s e_i(t)\\ &+\mathcal{B}_s e_i(t-\tau_1(t)))+c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)c_{ij}^s e_j(t)\\ &+c_2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)d_{ij}^s e_j(t-\tau_2(t))]\\ &+\sum_{s=1}^{r} h_s(z(t))[\sum_{i=1}^{N}\overline{\rho}_1 e_i^T(t)e_i(t)\\ &+\sum_{i=1}^{N}\overline{\rho}_2 e_i^T(t-\tau_3(t))e_i(t-\tau_3(t))]\\ \leq&\sum_{s=1}^{r} h_s(z(t))[(\overline{a}+\kappa_1+2c_1\overline{c}+c_2\kappa_2\overline{d}+\overline{\rho}_1)\\ &\times\sum_{i=1}^{N} e_i^T(t)e_i(t)\\ &+\kappa_1^{-1}\overline{b}\sum_{i=1}^{N} e_i^T(t-\tau_1(t))e_i(t-\tau_1(t))\\ &+c_2\kappa_2^{-1}\underline{d}\sum_{i=1}^{N} e_i^T(t-\tau_2(t))e_i^T(t-\tau_2(t))\\ &+\overline{\rho}_2\sum_{i=1}^{N} e_i^T(t-\tau_3(t))e_i(t-\tau_3(t))]\\ \leq&pV(t)+\kappa_1^{-1}\overline{b}\{V(t)\}_{\overline{\tau}_1}+c_2\kappa_2^{-1}\underline{d}\{V(t)\}_{\overline{\tau}_2}\\ &+\overline{\rho}_2\{V(t)\}_{\overline{\tau}_3}, \end{aligned} \tag{23}$$

where $p=\overline{a}+\kappa_1+2c_1\overline{c}+c_2\kappa_2\overline{d}+\overline{\rho}_1$, $\overline{a}=\max_s\|\mathcal{A}_s\|$, $\overline{\rho}_1=\max_s\rho_{1s}$, $\overline{\rho}_2=\max_s\rho_{2s}$.

Then, based on (24)

$$\begin{aligned}\mathbf{E}\{\mathscr{L}V(t)\}\leq&p\mathbf{E}\{V(t)\}+\kappa_1^{-1}\overline{b}\mathbf{E}\{V(t)\}_{\overline{\tau}_1}\\ &+c_2\kappa_2^{-1}\underline{d}\mathbf{E}\{V(t)\}_{\overline{\tau}_2}+\overline{\rho}_2\mathbf{E}\{V(t)\}_{\overline{\tau}_3}. \end{aligned} \tag{24}$$

Therefore, according to (18) and (25), all the conditions of Lemma 2 are satisfied. This completes the proof. □

Remark 1: Compared with early results studied by [31, 38, 40], the impulsive effects in these references are only discrete delays. However, in real life, such as financial markets, fishery industry and population dynamics, we need to consider distributed-delay dependent impulsive effects, in which the jumps of systems states of distributed-delay depend on the accumulation of the system states over a history time period. Therefore, it is important to consider the heterogeneous impulsive delays effects in T-S FDCDNs.

Remark 2: When $d_{2,k}^{\vartheta}=0$, $d_{3,k}^{\vartheta}=0$ in controller (9), the FMPIC approach for the synchronization of stochastic T-S FDCDNs is reduced to the conventional fuzzy impulsive control (FIC) method studied in [32, 34, 36] and the references therein.

In what follows, a result is presented in the case of free delayed impulsive effects.

3.2. Synchronization with FPIC

Now, under the fuzzy pinning impulsive control (FPIC), we can get the following error system:

$$\begin{cases} de_i(t) = \sum_{s=1}^{r} h_s(z(t))[\mathcal{A}_s e_i(t) + \mathcal{B}_s e_i(t-\tau_1(t)) \\ \quad + c_1 \sum_{j=1}^{N} c_{ij}^s e_j(t) + c_2 \sum_{j=1}^{N} d_{ij}^s e_j(t-\tau_2(t))]dt \\ \quad + \sum_{s=1}^{r} h_s(z(t)) \overline{\sigma}_s(e_i(t), e_i(t-\tau_3(t)))dw(t), \\ \quad t \neq t_k, \\ \Delta e_i(t_k) = \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k)) d_{1,k}^{\vartheta} e_i(t_k^-),\ i \in \mathcal{D}_k^{\dagger}, \\ e_i(t) = \varphi_i(t),\ \ \forall t \in [-\tau, 0]. \end{cases} \tag{25}$$

Theorem 2: Suppose that Assumption 1 holds, if there exist positive scalars κ_1, κ_2, such that the following inequalities hold for all $\vartheta = 1, 2, \ldots, r$:

$$\widetilde{q}_k = 1 + \frac{\partial_k}{N}(q_k - 1) < 1, \tag{26}$$

$$p + \frac{\kappa_1^{-1}\overline{b} + c_2\kappa_2^{-1}\underline{d} + \overline{\rho}_2}{\widetilde{q}_k} + \frac{ln\widetilde{q}_k}{t_{k+1} - t_k} < 0, \tag{27}$$

where $p = \overline{a} + \kappa_1 + 2c_1\overline{c} + c_2\kappa_2\overline{d} + \overline{\rho}_1$. Then, the stochastic T-S FCDNs (3) can achieve synchronization under the FPIC.

Proof: Consider the same LF as in (14) for error system (26).

As analysed in Theorem 1, when $t = t_k$, one has

$$\begin{aligned} V(t_k) &= \sum_{i=1}^{N} e_i^T(t_k)e_i(t_k) = \sum_{i\in\mathcal{D}_k^{\dagger}} e_i^T(t_k)e_i(t_k) \\ &\quad + \sum_{i\notin\mathcal{D}_k^{\dagger}} e_i^T(t_k)e_i(t_k) \\ &= \sum_{i\in\mathcal{D}_k^{\dagger}} \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))(1+d_{1,k}^{\vartheta})e_i^T(t_k^-) \\ &\quad \times \sum_{\ell=1}^{r} h_\ell(z(t_k))(1+d_{1,k}^{\ell})e_i(t_k^-) \\ &\quad + \sum_{i\notin\mathcal{D}_k^{\dagger}} e_i^T(t_k^-)e_i(t_k^-) \\ &\leq \sum_{i\in\mathcal{D}_k^{\dagger}} \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))(1+d_{1,k}^{\vartheta})^2 e_i^T(t_k^-)e_i(t_k^-) \\ &\quad + \sum_{i\notin\mathcal{D}_k^{\dagger}} e_i^T(t_k^-)e_i(t_k^-) \\ &= \sum_{\vartheta=1}^{r} h_\vartheta(z(t_k))(1+d_{1,k}^{\vartheta})^2 \sum_{i=1}^{N} e_i^T(t_k^-)e_i(t_k^-) \\ &\quad + (1-\overline{d}_{1,k}^2) \sum_{i\notin\mathcal{D}_k^{\dagger}} e_i^T(t_k^-)e_i(t_k^-) \end{aligned}$$

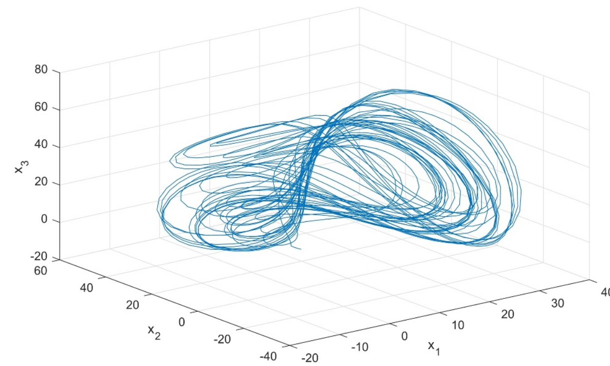

Fig. 1. Chaotic behavior of the Lorenz system (30).

$$\leq [1 + \frac{\partial_k}{N}(q_k - 1)]V(t_k^-), \tag{28}$$

where $q_k = \overline{d}_{1,k}^2$, $\overline{d}_{1,k} = \max_{\vartheta}\{|1 + d_{1,k}^{\vartheta}|\}$.

Next, with the same lines as in proof of the Theorem 1, we have Theorem 2 immediately. This concludes the proof.□

Remark 3: With the help of the LF, inequality techniques and SATs, the PS criteria of stochastic T-S FDCDNs are presented. Different from the existing works in the literature such as [33, 36, 37], the DCDNs with heterogeneous impulsive delays of this paper are composed of stochastic disturbance and T-S fuzzy model, which make the addressed model general and practical.

4. NUMERICAL SIMULATION

In this section, a numerical example is given to illustrate the effectiveness of the derived results.

Consider the delayed Lorenz system as the node dynamics, which is described by

$$\begin{cases} \dot{x}_1(t) = -10x_1(t) + 10x_2(t-\tau_1(t)), \\ \dot{x}_2(t) = 28x_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - (8/3)x_3(t-\tau_1(t)), \end{cases} \tag{29}$$

where $\tau_1(t) = 1/6$. Fig. 1 depicts the chaotic behavior of the delayed Lorenz system with the initial value $(x_1(t), x_2(t), x_3(t))^T = (1, 0.7, 0.5)^T$. Now, the system (30) can be represented by a T-S fuzzy model:

Rule 1: IF $x_1(t)$ is $h_1(x_1(t))$, THEN

$$\dot{x}(t) = \mathcal{A}_1 x(t) + \mathcal{B}_1 x(t-\tau_1(t)).$$

Rule 2: IF $x_1(t)$ is $h_2(x_1(t))$, THEN

$$\dot{x}(t) = \mathcal{A}_2 x(t) + \mathcal{B}_2 x(t-\tau_1(t)),$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$. The membership functions for rules 1 and 2 are given as $h_1(x_1(t)) = (1 + x_1(t)/d)/2$, $h_2(x_1(t)) = 1 - h_1(x_1(t))$ with $d = 20$, and

$$\mathcal{A}_1 = \begin{bmatrix} -10 & 0 & 0 \\ 28 & -1 & -d \\ 0 & d & 0 \end{bmatrix},\ \mathcal{A}_2 = \begin{bmatrix} -10 & 0 & 0 \\ 28 & -1 & -d \\ 0 & -d & 0 \end{bmatrix},$$

$$\mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} 0 & 10 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -8/3 \end{bmatrix}.$$

Next, consider the following T-S fuzzy error system with $N = 30$ under FMPIC (10),

$$\begin{cases} de_i(t) = \sum_{s=1}^{2} h_s(x_1(t))[\mathcal{A}_s e_i(t) + \mathcal{B}_s e_i(t-\tau_1(t)) \\ \qquad + c_1 \sum_{j=1}^{30} c_{ij}^s e_j(t) + c_2 \sum_{j=1}^{30} d_{ij}^s e_j(t-\tau_2(t))]dt \\ \qquad + \sum_{s=1}^{2} h_s(x_1(t)) \overline{\sigma}_s(e_i(t), e_i(t-\tau_3(t)))dw(t), \\ \qquad t \neq t_k, \\ \Delta e_i(t_k) = \sum_{\vartheta=1}^{2} h_\vartheta(x_1(t_k)) \Big[d_{1,k}^\vartheta e_i(t_k^-) \\ \qquad + d_{2,k}^\vartheta e_i(t_k - \zeta_1(t_k)) \\ \qquad + d_{3,k}^\vartheta \int_{t_k-\zeta_2(t_k)}^{t_k} e_i(s)ds \Big], \;\; i \in \mathcal{D}_k^\dagger, \\ e_i(t) = \varphi_i(t), \;\; \forall t \in [-\tau, 0]. \end{cases} \tag{30}$$

where $c_1 = c_2 = 0.01$, $\overline{\sigma}_s(e_i(t), e_i(t-\tau_3(t))) = 0.1 * diag(e_{i1}(t), e_{i2}(t), e_{i3}(t-\tau_3(t)))$, $\tau_2(t) = 0.015$, $\tau_3(t) = 0.01$, $\zeta_1(t) = 0.5 * |\sin(t)|$, $\zeta_2(t) = 0.1 * \tanh(t)$. Thus we have $\bar{\zeta}_2 = 0.1$ and $\rho_{1s} = \rho_{2s} = 0.01$. In addition, in the example, the delayed and non-delayed topological structure is given as a BA scale-free network with $m_0 = 3$, $m = 3$ and $N = 30$. When we consider the other parameters $\overline{\delta} = 0.5$, $\underline{\delta} = 1$, $\kappa_1 = 15$, $\kappa_2 = 1.5$, the number of pinning node $\partial_k = 16$ and the length of the interval $t_{k+1} - t_k = 0.01 (k \in \mathbb{Z}^+)$, by using the Theorem 1, we can compute the impulsive control gains $\bar{d}_{1,k} = \bar{d}_{2,k} = \bar{d}_{3,k} = 0.1 (k \in \mathbb{Z}^+)$ which satisfy the conditions (12)-(13).

Specially, when we choose $d_{1,k}^1 = -1.1$, $d_{1,k}^2 = -0.9$, $d_{2,k}^1 = 0.1$, $d_{2,k}^2 = 0.05$, $d_{3,k}^1 = -0.1$, $d_{3,k}^2 = -0.08$. Under the random initial condition, the corresponding simulation results can be seen in Fig. 2, which show the trajectories of the synchronization errors $e_{i1}(t)$, $e_{i2}(t)$, and $e_{i3}(t)$, $i = 1, 2, \ldots, 30$. It is verified that the FDCDNs (31) can achieve synchronization.

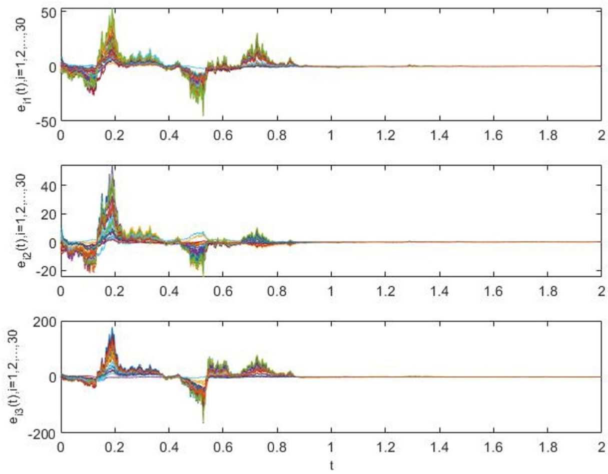

Fig. 2. Trajectories of the synchronization errors $e_{i1}(t)$, $e_{i2}(t)$ and $e_{i3}(t)$ ($i = 1, 2, ..., 30$) under FMPIC.

5. CONCLUSION

This paper considered a general class of stochastic T-S FDCDNs with heterogeneous impulsive delays effects. Different from the existing works, we proposed an FMPIC approach. In order to overcome the difficulties of studying such a general system, sufficient conditions that depend on the discrete-delay and distributed-delay impulsive effects were obtained by employing the LF, inequality techniques and SATs. It is shown that the PS of FDCDNs can be achieved by using the designed FMPIC. Finally, numerical simulation on basis of BA scale-free coupled network is used to show the effectiveness of the theoretical results.

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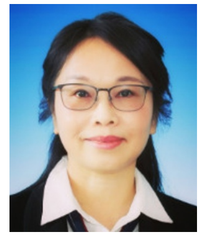

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