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Stabilization of Enforced Positive Switched Linear Systems with Bounded Controls

Jinjin Liu

Abstract: This paper is concerned with the controller synthesis issue for enforced positive switched linear systems via output feedback. First, the stabilization problem is studied with output feedback under average dwell time switching signal, and the controllers we proposed guarantee stability and positivity of the closed-loop systems. Second, the output feedback stabilization issue is investigated by introducing special form of diagonal matrices, and the constraints on states and control inputs are solved based on limited initial conditions. Then, the derived conditions are described via linear programming, also extending the theoretical findings to constrained output issue. Finally, the simulation results demonstrate the feasibility of the control strategy.

Keywords: Bounded control, linear programming, output feedback, positive system, switched linear systems.

1. INTRODUCTION

As we all know, it is inevitable to encounter situations with constraint in practical control systems. It can be classified into three categories: first, the constraints on the system state; second, the constraints on the control quantity; third, the constraints on the system output. Generally speaking, the system is often subject to a variety of constraints at the same time, and the emergence of these constraints increases the difficulty to controller design and system performance satisfaction [1, 2]. Therefore, it is not only great practical significance but also challenging to study the control technology under constraint conditions.

When the systems states are confined within a "cone" located in the positive orthant rather than in linear spaces, such kind of systems are referred to as positive systems. The positivity constraint is inherent to many real world systems, such as absolute temperature, concentrations of chemical, number of living things, probability in economics [3, 4]. As a special kind of positive systems, switched positive linear systems (SPLSs) contain a finite number of positive subsystems and a rule orchestrating the switching among them. These systems have attracted attention from many researchers because of widespread application value in communication network [5], medical treatment [6], multi-agent system [7] and so on.

From the theoretical perspective, the past decade has witnessed an increasing interesting in stability analysis and controller synthesis of positive systems and switched systems. The stability analysis for positive systems with continuous and discrete system is investigated by Rami *et al*. [8]. Based on linear programming (LP), these papers proposed linear copositive Lyapunov function method which makes full use of the nonnegative nature of positive systems [8, 9]. Since then, a great deal of systematic research has been carried out, the stabilization of positive systems [10, 11], positive delay systems [12, 13], uncertain positive systems [14], positive observer systems [15–17], L1-gain problem [18], impulsive positive systems [19, 20]. For the research on such positive systems, scholars have also made corresponding achievements in fuzzy control [21, 22], fractional order system [23, 24], event-triggered system [25] and so on. Some interesting results on switched systems can be seen in [26–30]. As a special positive system, SPLSs also attract people's research enthusiasm. It is divided into arbitrary switching signal [31], dwell time switching signal [32, 33], average dwell time(ADT) switching signal [34, 35], mode-dependent average dwell time switching signal [36, 37] and state-dependent switching [38, 39].

On the other hand, it is sometimes difficult to obtain the full information of state in engineering applications because of restricting by measurement means of internal variable. Hence, it is necessary to investigate the output feedback stabilization problem. For positive systems, static output feedback stabilization problem was studied

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Jinjin Liu is with the Department of Mathematics and Information Science, Henan University of Economics and Law, Zhengzhou 450046, China (e-mail: jjliu2020@126.com; jjliu@hotmail.com).

in [40] and the controller has one rank gains. By using the singular value decomposition approach, the static output feedback stabilization problem is concerned for systems with interval uncertainties [41]. By means of linear programming approach, state and output feedback controllers for SPLSs with ADT switching are designed [42]. References [43, 44] considered constrained controller synthesis issue of switched linear systems by state feedback. The output-feedback controller synthesis issue for a class of switched linear systems with constrained controls and output is addressed by decomposing control gain matrix [45], nevertheless the rank of gain matrix is not full.

Motivated by above discussion, in this paper, we investigate the stabilization issue of enforced positive switched linear systems with bounded controls under ADT switching scheme. Based on the calculation technique of matrix, the output feedback stabilization issue is investigated by introducing special form of diagonal matrices. The main advantages of the method we proposed include: i) the controllers we establish for stabilization of enforced positive switched linear systems under ADT switching scheme satisfy bounded condition; ii) the rank of controller gain matrix we designed is full, which can make full use of the whole information of the output matrix and overcome the limitation on the rank of controller gain; iii) all the proposed conditions are formulated as linear inequalities, thus the controller parameters can be determined by linear programming, which is powerful for solving higher dimension problems than LMI. The rest of paper is organized as follows: In Section 2, problem formulation and some preliminaries are given. Output feedback stabilization and stabilization with bounded controls are discussed in Sections 3 and 4, respectively. Section 5 solved constrained output problem. Two numerical examples are shown in Section 6. The paper is concluded in Section 7.

Notations: $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ matrices with entries from the field of real numbers $\mathbb{R}$. $\mathbb{R}^n = \mathbb{R}^{n\times 1}$ stands for the set of n dimensional column vectors. Further, a matrix A is nonnegative(negative) if all of its elements are nonnegative and can be denoted by $A \succeq 0 (A \prec 0)$, and a vector λ is nonnegative(negative) if all of its elements are nonnegative and can be denoted by $\lambda \succeq 0 (\lambda \prec 0)$. Second, the main advantage of the method we proposed is the rank of controller gain matrix we designed is full, which can make full use of the whole information of the output matrix and overcome the limitation on the rank of controller gain. On the other hand, the stabilization issue of enforced positive switched linear systems with bounded controls under ADT switching scheme is also our advantage.

2. PROBLEM FORMULATION

Consider the following switched linear systems

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \\ y(t) = C_{\sigma(t)}x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^s$ are system state, input and output respectively. $\sigma(t) : [0,\infty) \to \mathbb{P} = \{1,2,\dots,N\}$ is switching signal, and N is the number of the subsystems. When $\sigma(t) = p$, p-th subsystem is active, $A_p = [a_{pij}] \in \mathbb{R}^{n\times n}$, $B_p = [b_{pij}] \in \mathbb{R}^{n\times m}$ and $C_p = [c_{pij}] \in \mathbb{R}^{s\times n}$ are subsystem matrices. The purpose of this paper is to design constrained output feedback controller

$$u(t) = G_p y(t), \tag{2}$$

subject to the corresponding closed-loop system with initial condition $x_0 \succeq 0$,

$$\dot{x}(t) = A_{cp}x(t) \tag{3}$$

is positive and asymptotically stable under ADT switching signal. Here $G_p \in \mathbb{R}^{m\times s}$ is output feedback gain matrix and A_{cp} is given by

$$A_{cp} = A_p + B_p G_p C_p, \quad \forall p \in \mathbb{P}. \tag{4}$$

Next, some definitions and lemmas are listed for our further study.

Definition 1 [4]**:** The continuous-time linear system

$$\dot{x}(t) = Ax(t) \tag{5}$$

is said to be positive if for the nonnegative initial condition, the corresponding trajectory of system $x(t) \succeq 0$ for all $t \geq 0$, where $A \in \mathbb{R}^{n\times n}$.

Lemma 1 [4]**:** System (5) is positive if and only if A is a Metzler matrix, i.e. all the off-diagonal entries of the matrix A are nonnegative.

Remark 1: As the switching signal is right continuous at switching instants, the positivity condition of linear time-invariant system can be easily extended to switched system. Clearly, the closed-loop system (3) is positive if system matrix A_{cp} is Metzler for each $p \in \mathbb{P}$.

Definition 2 [13]**:** Let $\sigma(t)$ be a switching signal and $N_\sigma(t_1,t_2)$ be the switching number of $\sigma(t)$ in time interval $[t_1,t_2]$. If there exist two constants $N_0 \geq 0$ and $\tau^* > 0$ such that

$$N_\sigma(t_1,t_2) \leq N_0 + (t_2 - t_1)/\tau_\alpha,$$

then τ_α is an ADT of the switching signal $\sigma(t)$ and N_0 is the chatter bound.

Lemma 2 [37]**:** If exist vector $\lambda_p \succ 0$ and constant $\gamma > 0$ such that $(A_p + \gamma I)\lambda_p \prec 0$, a SPLS $\dot{x}(t) = A_p x(t)$ is asymptotically stable under ADT switching signal $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{\lambda_p}{\lambda_q}$ for $(p,q) \in \mathbb{P}\times\mathbb{P}$.

Lemma 3 [40]**:** Consider a SPLS $\dot{x}(t) = A_p x(t)$, if exist vector $\lambda_p \succ 0$ and constant $\gamma > 0$ such that $(A_p + \gamma I)\lambda_p \prec 0$ set up for $0 \preceq x_0 \preceq \lambda_p$, then the state trajectory of SPLS satisfies that $0 \preceq x(t) \preceq \lambda_p$ for $p \in \mathbb{P}$.

3. OUTPUT FEEDBACK STABILIZATION

In this section, we consider output feedback stabilization of switched linear systems. Unlike the other issues, we request that the closed-loop system not only asymptotically stable and positive under ADT switching signal.

Theorem 1: Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0, Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ satisfying the following linear programming inequalities

$$a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} \geq 0, \ \ \forall 1 \leq i \neq j \leq n, \quad (6)$$

$$\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} + \gamma d_p < 0, \quad \forall 1 \leq i \leq n, \quad (7)$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = Z_p D_p^{-1}. \quad (8)$$

Proof: First, using the notations of (4), one can obtain

$$a_{cpij} = a_{pij} + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}g_{pkl}c_{plj}, \ \ \forall 1 \leq i, j \leq n. \quad (9)$$

Multiplying both sides of (9) by d_p, it follows that

$$a_{cpij}d_p = a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}g_{pkl}c_{plj}d_p.$$

From (8), $z_{pkj} = g_{pkj}d_p$ holds. Owing to the fact that multiplication of numbers satisfies the commutative property, then

$$a_{cpij}d_p = a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj}.$$

Utilising (6), $a_{cpij}d_p \geq 0$ sets up for $1 \leq i \neq j \leq n$. Thus, the formula $a_{cpij} \geq 0$ is true for $1 \leq i \neq j \leq n$ due to $d_p > 0$. This implies A_{cp} is Metzler matrix, so the closed-loop (3) system is positive by Lemma 1. Next, the following part focuses on the proof of asymptotically stable. Define the vector $d_{vp} = [d_p, d_p, \ldots, d_p]^T \in \mathbb{R}^n$ and based on the calculation technique of matrix, it yields

$$[A_{cp}d_{vp}]_i = \sum_{j=1}^{n}[(a_{pij} + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}g_{pkl}c_{plj})d_p] = \sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj}.$$

Then, we can get $(A_{cp} + \gamma I)d_{vp} = (A_p + B_pG_pC_p + \gamma I)d_{vp} \prec 0$ from (7). Hence the closed-loop system is asymptotically stable by Lemma 2. This completes the proof. □

Remark 2: The output feedback stabilization problem we think about is to find controller subject to the closed-loop system positive and asymptotically stable, even if the open-loop system is not positive at all. In other words, this issue can be interpreted as enforcing the system to be positive. Under this circumstance, the system is called enforced positive.

4. STABILIZATION WITH BOUNDED CONTROLS

In the following, we divide this section into three kinds of condition and establish output feedback controllers which are limited to the prescribed bounds.

4.1. Sign-restricted controls

In this subsection, the stabilization issue for switched linear systems is investigated with nonnegative or negative controls. First we focus on the condition of $u(t) \succeq 0$.

Theorem 2: Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0, Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ such that

$$a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} \geq 0, \ \ \forall 1 \leq i \neq j \leq n, \quad (10)$$

$$\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} + \gamma d_p < 0, \quad \forall 1 \leq i \leq n, \quad (11)$$

$$\sum_{l=1}^{s} z_{pil}c_{plj} \geq 0, \ \ \forall 1 \leq i \leq m, 1 \leq j \leq n, \quad (12)$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = Z_p D_p^{-1}.$$

Proof: Combining (10) and (11), the closed-loop system (3) is positive and asymptotically stable following the same reasoning as Theorem 1. From (12), it is easy to see that $Z_pC_p = G_pC_pd_p \succeq 0$. Recalling the fact that $d_p > 0$, it follows $G_pC_p \succeq 0$. Thus, we have $u(t) = G_py(t) = G_pC_px(t) \succeq 0$. □

Remark 3: We are obligated to act on the system by only nonnegative inputs since practical factors for instance, the control law can be pressure, heating or voltage, etc. Next we consider the $u(t) \preceq 0$ condition of output feedback stabilization for switched linear systems under ADT switching.

Theorem 3: Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0, Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ subject to

$$\begin{aligned}
&a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} \geq 0, \ \ \forall 1 \leq i \neq j \leq n,\\
&\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} + \gamma d_p < 0,\\
&\quad \forall 1 \leq i \leq n,\\
&\sum_{l=1}^{s} z_{pil}c_{plj} \leq 0, \ \ \forall 1 \leq i \leq m, 1 \leq j \leq n,
\end{aligned} \tag{13}$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = Z_p D_p^{-1}.$$

Proof: The closed-loop system (3) is positive and asymptotically stable following the same reasoning as Theorem 1. From (13), it is easy to see that $Z_pC_p = G_pC_pd_p \preceq 0$. Recalling the fact that $d_p > 0$, it follows $G_pC_p \preceq 0$. Thus, we have $u(t) = G_py(t) = G_pC_px(t) \preceq 0$.□

4.2. Bounded controls

The following part is concerned with bounded controls issues. First, we consider the trajectory of closed-loop system with initial condition $0 \preceq x_0 \preceq d_{vp}$ is positive and the input is limited to be nonnegative and bounded by a predetermined $\bar{u} \succ 0$, where $d_{vp} = [d_p, d_p, \ldots, d_p]^T \in \mathbb{R}^n$.

Theorem 4: Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0, Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ such that

$$\begin{aligned}
&a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} \geq 0, \ \ \forall 1 \leq i \neq j \leq n,\\
&\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} + \gamma d_p < 0,\\
&\quad \forall 1 \leq i \leq n,\\
&\sum_{l=1}^{s} z_{pil}c_{plj} \geq 0, \ \ \forall 1 \leq i \leq m, 1 \leq j \leq n,\\
&\sum_{j=1}^{n}\sum_{l=1}^{s} z_{pil}c_{plj} \leq \bar{u}_i, \ \ \forall 1 \leq i \leq m,
\end{aligned} \tag{14}$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = Z_p D_p^{-1}.$$

Proof: We can get the closed-loop system is positive and asymptotically stable from Theorem 1. Further, $u(t) \succeq 0$ holds by Theorem 2. For initial condition $0 \preceq x_0 \preceq d_{vp}$, the trajectory satisfies $0 \preceq x(t) \preceq d_{vp}$ according to Lemma 3. Thus, it follows that

$$u(t) = G_py(t) = G_pC_px(t) \preceq G_pC_pd_{vp}.$$

Take $e = [1, 1 \ldots, 1]^T \in \mathbb{R}^s$, we have $d_{vp} = d_pe$. Then the above can be turned into

$$G_pC_pd_{vp} = G_pd_pC_pe = Z_pC_pe.$$

Therefore , bounded control $u(t) \preceq \bar{u}$ can obtain by the inequality (14). In conclusion, the resulting closed-loop system is positive with ADT scheme and control input meet the constraint $0 \preceq u(t) \preceq \bar{u}$. □

Next, the constraint satisfaction of nonpositive control $-\tilde{u} \preceq u(t) \preceq 0$ is studied by enforced positive systems for fixed $\tilde{u} \succ 0$.

Theorem 5: Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0, Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ subject to

$$\begin{aligned}
&a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} \geq 0, \ \ \forall 1 \leq i \neq j \leq n,\\
&\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} + \gamma d_p < 0,\\
&\quad \forall 1 \leq i \leq n,\\
&\sum_{l=1}^{s} z_{pil}c_{plj} \leq 0, \ \ \forall 1 \leq i \leq m, 1 \leq j \leq n,\\
&-\tilde{u}_i \preceq \sum_{j=1}^{n}\sum_{l=1}^{s} z_{pil}c_{plj}, \ \ \forall 1 \leq i \leq m,
\end{aligned} \tag{15}$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = Z_p D_p^{-1}.$$

Proof: We can get the closed-loop system is positive and asymptotically stable from Theorem 1. Further, $u(t) \preceq 0$ holds by Theorem 3. For initial condition $0 \preceq x_0 \preceq d_{vp}$, the trajectory satisfies $0 \preceq x(t) \preceq d_{vp}$ according to Lemma 3. Similar to the proof of Theorem 4, we have

$$\begin{aligned}
u(k) =& G_py(k) = G_pC_px(k)\\
\succeq& G_pC_pd_{vp} = G_pd_pC_pe = Z_pC_pe.
\end{aligned}$$

Therefore, bounded control $u(t) \succeq -\tilde{u}$ can obtain by the inequality (15). In conclusion, the resulting closed-loop system is positive with ADT scheme and control input meet the constraint $-\tilde{u} \preceq u(t) \preceq 0$. □

Finally in this part, bounded control issue can be formulated as a problem of finding admissible output feedback control law $-\tilde{u} \preceq u(t) \preceq \bar{u}$ to stabilize switched linear system.

Theorem 6: Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0, Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$, $W_p = [w_{pij}] \in \mathbb{R}^{m\times s}$ subject to

$$a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}(z_{pkl} - w_{pkl})c_{plj} \geq 0,$$
$$\forall 1 \leq i \neq j \leq n,$$
$$\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}(z_{pkl} - w_{pkl})c_{plj} + \gamma d_p < 0,$$
$$\forall 1 \leq i \leq n,$$
$$\sum_{l=1}^{s} z_{pil}c_{plj} \geq 0, \;\; \forall 1 \leq i \leq m, 1 \leq j \leq n, \tag{16}$$
$$\sum_{k=1}^{s} w_{pil}c_{plj} \geq 0, \;\; \forall 1 \leq i \leq m, 1 \leq j \leq n, \tag{17}$$
$$\sum_{j=1}^{n}\sum_{l=1}^{s} z_{pil}c_{plj} \leq \bar{u}_i, \;\; \forall 1 \leq i \leq m, \tag{18}$$
$$\sum_{j=1}^{n}\sum_{l=1}^{s} w_{pil}c_{plj} \leq \tilde{u}_i, \;\; \forall 1 \leq i \leq m, \tag{19}$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = (Z_p - W_p)D_p^{-1}.$$

Proof: Similarly, we know the closed-loop system is positive and asymptotically stable from Theorem 1. For initial condition $0 \preceq x_0 \preceq d_{vp}$, the trajectory satisfies $0 \preceq x(t) \preceq d_{vp}$ according to Lemma 3. Recalling vector e and together with (16-17), then

$$\begin{aligned}0 &\preceq Z_pD_p^{-1}y(t) = Z_pD_p^{-1}C_px(t)\\ &\preceq Z_pD_p^{-1}C_pd_{vp} = Z_pC_pe,\\ 0 &\preceq W_pD_p^{-1}y(t) = W_pD_p^{-1}C_px(t)\\ &\preceq W_pD_p^{-1}C_pd_{vp} = W_pC_pe.\end{aligned}$$

Using the properties of the inequality, the above one becomes

$$-W_pC_pe \preceq -W_pD_p^{-1}y(t) \preceq 0.$$

Combining (18) and (19), one has

$$-\tilde{u} \preceq u(t)G_py(t) = (Z_p - W_p)D_p^{-1}y(t) \preceq \bar{u}.$$

In the end, bounded output feedback controllers such that the closed-loop system (3) is not only positive but also asymptotically stable under ADT switching signal was determined. This completes the proof. □

5. CONSTRAINED OUTPUT

If $C_p \succeq 0$, this section is dedicated to find output feedback controller such that the resultant closed-loop system is positive and asymptotically stable via ADT switching signals rule. Also the output ensures the given bound $0 \preceq y(t) \preceq \bar{y}$, where $\bar{y} \succ 0$.

Theorem 7: Assume that $C_p \succeq 0$. Consider system (1), if exist a constant $\gamma > 0$ and matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s} \succ 0$, $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ such that

$$a_{pij}d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} \geq 0, \;\; \forall 1 \leq i \neq j \leq n, \tag{20}$$
$$\sum_{j=1}^{n} a_{pij}d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}z_{pkl}c_{plj} + \gamma d_p < 0,$$
$$\forall 1 \leq i \leq n, \tag{21}$$
$$\sum_{j=1}^{n} c_{pij}d_p \leq \bar{y}_i, \;\; \forall 1 \leq i \leq s, \tag{22}$$

then the closed-loop system (3) is positive and asymptotically stable under ADT switching strategy $\tau_\alpha > \tau_\alpha^* = \frac{\ln\mu}{\gamma}$, where $\mu = \max \frac{d_p}{d_q}, \forall (p,q) \in \mathbb{P}\times\mathbb{P}$. Moreover, the admissible output feedback gain is given by

$$G_p = Z_pD_p^{-1}.$$

Proof: From (20)-(21), the closed-loop system (3) is positive and asymptotically stable following the same reason as Theorem 1. Noting Lemma 3 and $C_p \succeq 0$, then we have $y(t) = C_px(t) \preceq C_pd_{vp}$. According to (22) and $x(t) \succeq 0$, it yields that $0 \preceq y(t) \preceq \bar{y}$. □

6. NUMERICAL EXAMPLES

Consider system (1) with parameters as follows:

$$A_1 = \begin{pmatrix} -2 & -0.8 \\ -0.5 & -1.2 \end{pmatrix}, \; B_1 = \begin{pmatrix} 1.8 & -1.2 \\ -0.6 & 1 \end{pmatrix},$$
$$C_1 = \begin{pmatrix} 1 & 0.8 \\ 0.5 & 1 \end{pmatrix}; \; C_2 = \begin{pmatrix} 0.6 & 1 \\ 1 & 0.5 \end{pmatrix},$$
$$A_2 = \begin{pmatrix} -1.2 & -0.5 \\ 0.5 & -1 \end{pmatrix}, \; B_2 = \begin{pmatrix} -2.2 & 0.8 \\ -0.8 & -1.8 \end{pmatrix}.$$

Example 1: In this part, we are interesting in finding constrained output feedback controller $0 \preceq u(t) \preceq \bar{u}$ subject to the corresponding closed-loop system with initial condition $x_0 \succeq 0$ is positive and asymptotically stable under ADT switching strategy. Choose $\bar{u} = [6, 8]^T$. Using the conditions given by Theorem 4 and linear programming algorithm in MATLAB, we can obtain $D_1 = diag\{6.8574, 6.8574\}$, $D_2 = diag\{9.2086, 9.2086\}$ and matrices

$$Z_1 = \begin{pmatrix} -2.6754 & 6.5333 \\ 9.4164 & -7.1827 \end{pmatrix},$$

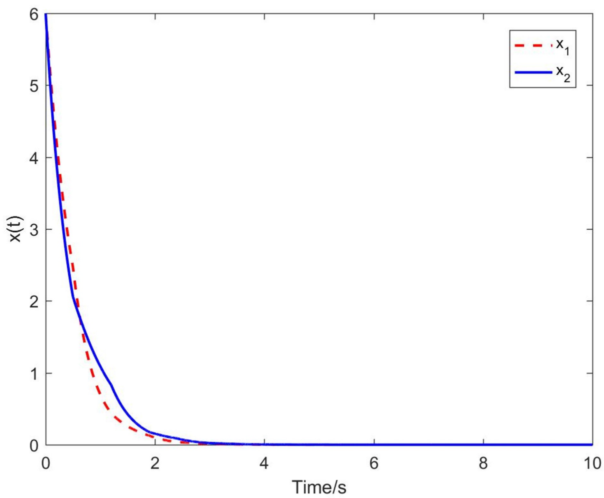

Fig. 1. Simulation of closed-loop system state trajectories (Example 1).

$$Z_2 = \begin{pmatrix} -1.4277 & 3.4822 \\ 9.2367 & -5.0638 \end{pmatrix}.$$

By means of (25), the gain matrix is calculated by

$$G_1 = \begin{pmatrix} -0.3901 & 0.9527 \\ 1.3732 & -1.0474 \end{pmatrix},$$
$$G_2 = \begin{pmatrix} -0.1550 & 0.3781 \\ 1.0031 & -0.5499 \end{pmatrix}.$$

It can be seen that the gain matrix G_1 and G_2 are nonsingular. Combining with the computation of (7), we can get the closed-loop system matrices

$$A_{c1} = \begin{pmatrix} -2.8641 & 0.2918 \\ 0.2977 & -1.5333 \end{pmatrix},$$
$$A_{c2} = \begin{pmatrix} -1.7857 & 0.0076 \\ 0.1784 & -2.3378 \end{pmatrix}.$$

Obviously, A_{c1} and A_{c2} are Metzler matrices, so the close-loop system is positive. By computing, one has $\mu = \frac{d_2}{d_1} = 1.3429$. Take $\gamma = 0.5$, we can obtain $\tau_\alpha^* = 0.5897$. The state trajectories of the closed-loop system with initial $x_0 = (6,6)^T$ are given in Fig. 1. Fig. 2 plots the control input, which illustrates that the controls meet the fixed bound $0 \preceq u(k) \preceq [6,8]^T$ when the initial value is limit to $0 \preceq x_0 \preceq d_{v1} = (6.8574, 6.8574)^T$. The switching signal $\sigma(t)$ with ADT is shown in Fig. 3.

Example 2: Here, our goal is to design bounded output feedback controller $-\tilde{u} \preceq u(t) \preceq 0$ such that the corresponding closed-loop system with initial condition $x_0 \succeq 0$ is positive and asymptotically stable under ADT switching strategy. Let $\tilde{u} = [3,5]^T$. According to the conditions given in theorem 5 and linear programming algorithm in MATLAB, we can obtain $D_1 = diag\{1.5499, 1.5499\}$, $D_2 =$

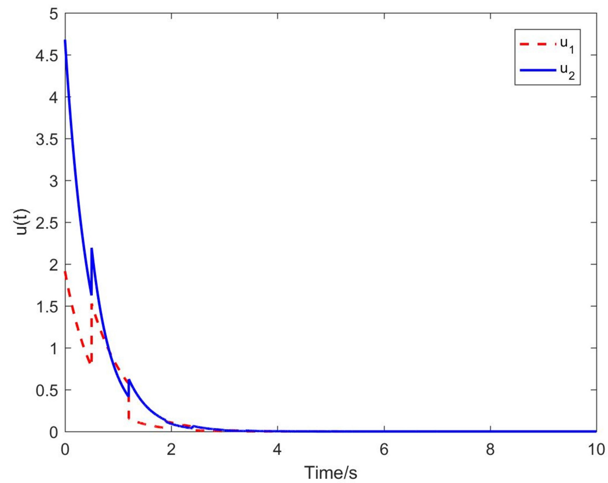

Fig. 2. Simulation of control input (Example 1).

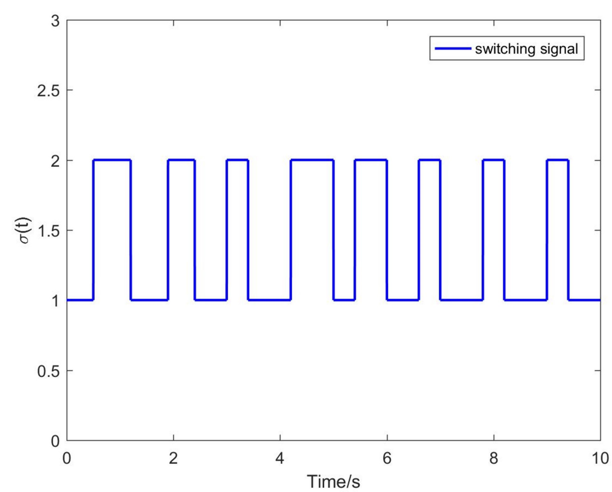

Fig. 3. Simulation of switching signal (Example 1).

$diag\{10.5541, 10.5541\}$ and

$$Z_1 = \begin{pmatrix} -3.5059 & 2.5327 \\ 2.4073 & -5.4269 \end{pmatrix},$$
$$Z_2 = \begin{pmatrix} -3.3789 & 1.7867 \\ 0.1792 & -0.5548 \end{pmatrix}.$$

Then we have

$$G_1 = \begin{pmatrix} -2.2620 & 1.6341 \\ 1.5532 & -3.5015 \end{pmatrix},$$
$$G_2 = \begin{pmatrix} -0.3202 & 0.1693 \\ 0.0170 & -0.0526 \end{pmatrix}.$$

It is easy to verify that the ranks of gain matrices are full. By (7), the closed-loop system matrices are

$$A_{c1} = \begin{pmatrix} -4.3639 & 1.5948 \\ 0.1694 & -3.3536 \end{pmatrix},$$
$$A_{c2} = \begin{pmatrix} -1.1837 & 0.0107 \\ 0.5945 & -0.7948 \end{pmatrix}.$$

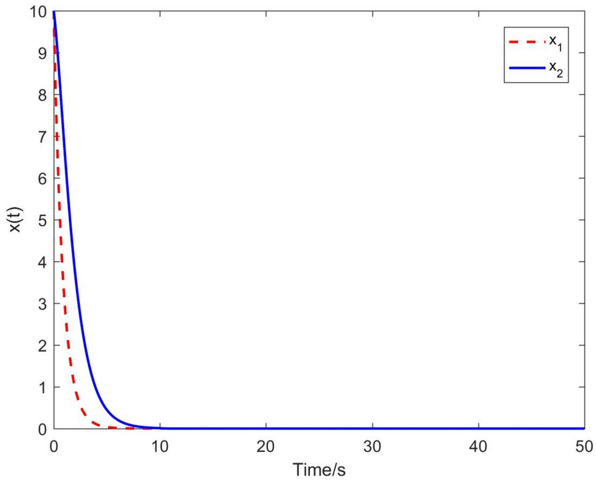

Fig. 4. Simulation of closed-loop system state trajectories (Example 2).

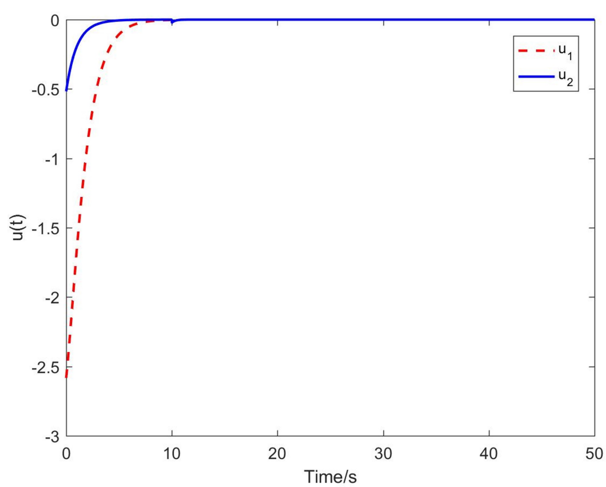

Fig. 5. Simulation of control input (Example 2).

It is obvious that A_{c1} and A_{c2} are Metzler matrices, namely the closed-loop system is positive. By computing $\mu = \frac{d_2}{d_1} = 6.8095$, and seleting $\gamma = 0.2$, which yields $\tau_\alpha^* = 9.5916$. The simulation results are depicted in Figs. 4-6 with initial condition $x_0 = (10,10)^T$ under ADT switching. Fig. 4 shows the state trajectories of closed-loop system. Fig. 5 plots the control input which limited to bound $-[3,5]^T \preceq u(t) \preceq 0$ when initial condition meets $0 \preceq x_0 \preceq d_{v1}$. The switching signal $\sigma(t)$ with ADT is given in Fig. 6.

Example 3: In this part, our goal is to design bounded output feedback controller $-\tilde{u} \preceq u(t) \preceq \bar{u}$ such that the corresponding closed-loop system with initial condition $x_0 \succeq 0$ is positive and asymptotically stable under ADT switching strategy. Choose $\tilde{u} = [5,10]^T$ and $\bar{u} = [6,8]^T$. According to the conditions given in theorem 6 and linear programming algorithm in MATLAB, we can obtain $D_1 = diag\{8.1678, 8.1678\}$, $D_2 = diag\{19.7475, 19.7475\}$ and

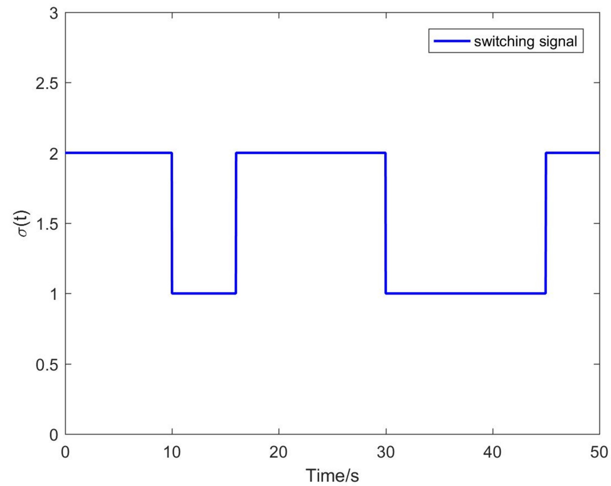

Fig. 6. Simulation of switching signal (Example 2).

$$Z_1 = \begin{pmatrix} -1.5522 & 4.7916 \\ 9.0570 & -6.6216 \end{pmatrix},$$
$$W_1 = \begin{pmatrix} 4.5566 & -3.0390 \\ -4.3445 & 10.3046 \end{pmatrix},$$
$$Z_2 = \begin{pmatrix} -1.5967 & 4.6338 \\ 7.5856 & -3.2431 \end{pmatrix},$$
$$W_2 = \begin{pmatrix} 4.4682 & -2.1315 \\ -2.1870 & 4.7679 \end{pmatrix}.$$

Then we have

$$G_1 = \begin{pmatrix} -0.7479 & 0.9587 \\ 1.6408 & -2.0723 \end{pmatrix},$$
$$G_2 = \begin{pmatrix} -0.3071 & 0.3426 \\ 0.4949 & -0.4057 \end{pmatrix}.$$

It is easy to verify that the ranks of gain matrices are full. By (7), the closed-loop system matrices are

$$A_{c1} = \begin{pmatrix} -3.2089 & 0.7603 \\ 0.2658 & -2.1759 \end{pmatrix},$$
$$A_{c2} = \begin{pmatrix} -1.6353 & 0.0325 \\ 0.5691 & -1.4170 \end{pmatrix}.$$

It is obvious that A_{c1} and A_{c2} are Metzler matrices, namely the closed-loop system is positive. By computing $\mu = \frac{d_2}{d_1} = 2.4177$, and seleting $\gamma = 0.6$, which yields $\tau_\alpha^* = 1.7656$. The simulation results are depicted in Figs. 7-9 with initial condition $x_0 = (8,8)^T$ under ADT switching. Fig. 7 shows the state trajectories of closed-loop system. Fig. 8 plots the control input which limited to bound $-[5,10]^T \preceq u(t) \preceq [6,8]^T$ when initial condition meets $0 \preceq x_0 \preceq d_{v1}$. The switching signal $\sigma(t)$ with ADT is given in Fig. 9.

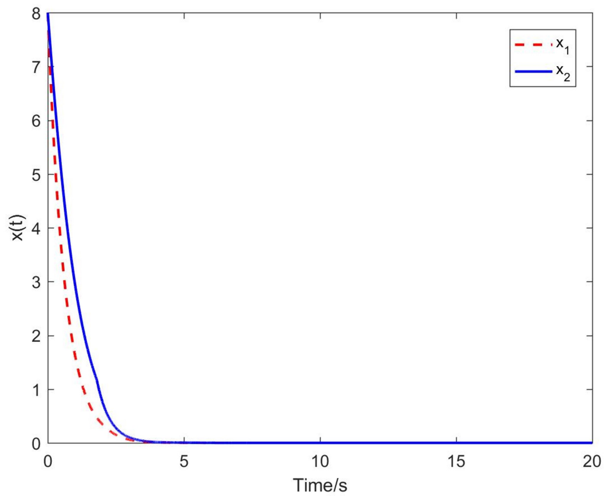

Fig. 7. Simulation of closed-loop system state trajectories (Example 3).

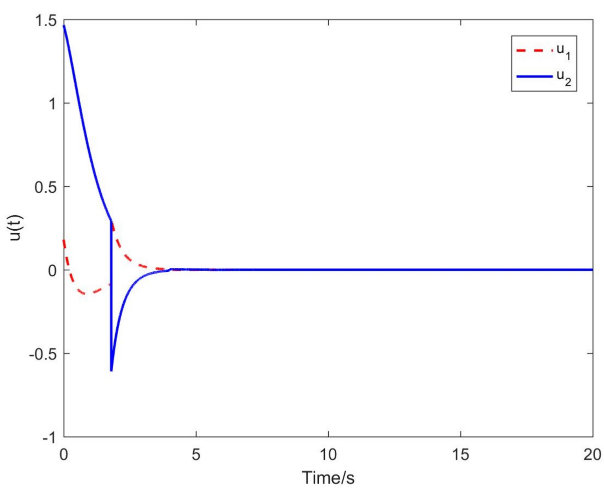

Fig. 8. Simulation of control input (Example 3).

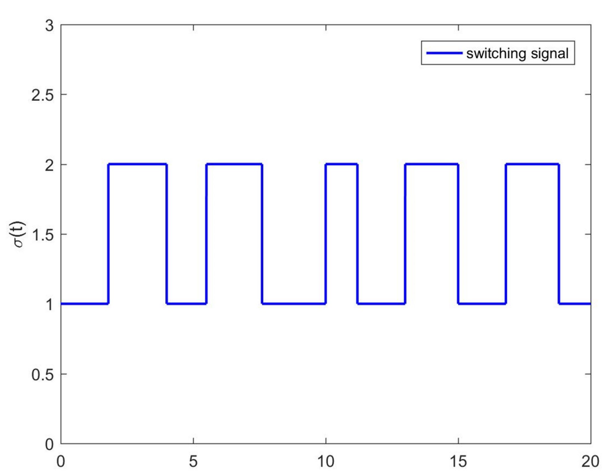

Fig. 9. Simulation of switching signal (Example 3).

Example 4: Here, our goal is to design bounded output feedback controller $0 \preceq y(t) \preceq \bar{y}$ such that the corresponding closed-loop system with initial condition $x_0 \succeq 0$ is positive and asymptotically stable under ADT switching strategy. Choose $\bar{y} = [5,2]^T$. According to the conditions given in Theorem 7 and linear programming algorithm in MATLAB, we can obtain $D_1 = diag\{0.6595, 0.6595\}$, $D_2 = diag\{0.6930, 0.6930\}$ and

$$Z_1 = \begin{pmatrix} -121.6878 & 61.6104 \\ 65.9189 & -165.2996 \end{pmatrix},$$

$$Z_2 = \begin{pmatrix} -36.4831 & 67.1879 \\ 143.7723 & -139.8337 \end{pmatrix}.$$

Then we have

$$G_1 = \begin{pmatrix} -184.5152 & 93.4199 \\ 99.9528 & -250.6438 \end{pmatrix},$$

$$G_2 = \begin{pmatrix} -52.6452 & 96.9522 \\ 207.4636 & -201.7802 \end{pmatrix}.$$

It is easy to verify that the ranks of gain matrices are full. By (7), the closed-loop system matrices are

$$A_{c1} = \begin{pmatrix} -219.6067 & 106.4717 \\ 56.8141 & -139.3662 \end{pmatrix},$$

$$A_{c2} = \begin{pmatrix} -206.8449 & 93.9307 \\ 87.3516 & -189.4971 \end{pmatrix}.$$

It is obvious that A_{c1} and A_{c2} are Metzler matrices, namely the closed-loop system is positive. By computing $\mu = \frac{d_2}{d_1} = 1.0508$, and seleting $\gamma = 0.6$, which yields $\tau_\alpha^* = 0.1239$. The simulation results are depicted in Figs. 10-12 with initial condition $x_0 = (0.6, 0.6)^T$ under ADT switching. Fig. 10 shows the state trajectories of closed-loop system. Fig. 11 plots the output which limited to bound $0 \preceq y(t) \preceq [5,2]^T$ when initial condition meets $0 \preceq x_0 \preceq d_{v1}$. The switching signal $\sigma(t)$ with ADT is given in Fig. 12.

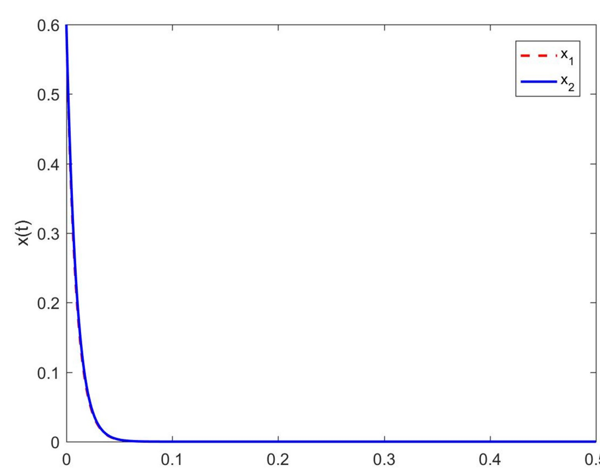

Fig. 10. Simulation of closed-loop system state trajectories (Example 4).

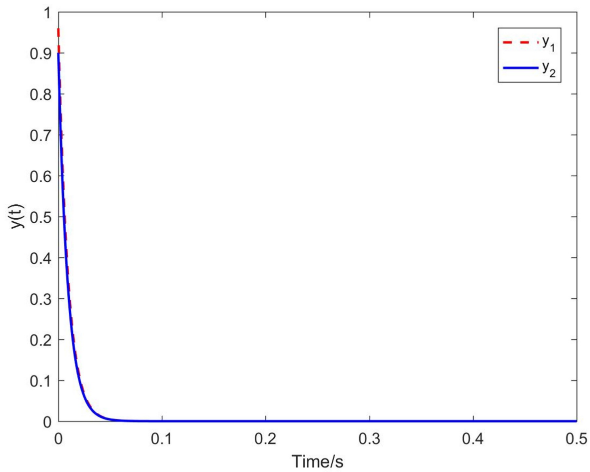

Fig. 11. Simulation of control input (Example 4).

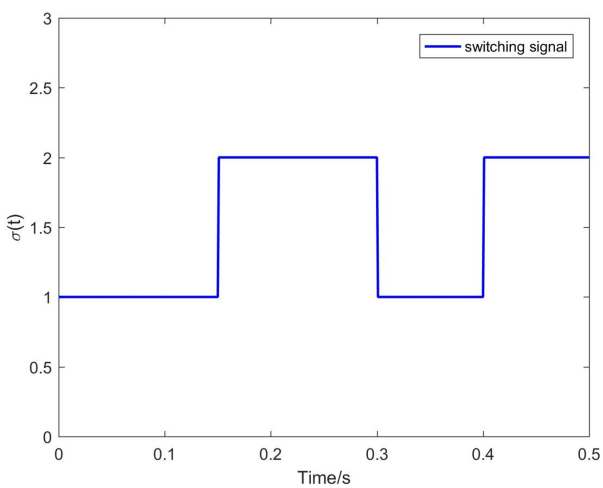

Fig. 12. Simulation of switching signal (Example 4).

7. CONCLUSION

This article investigates the stabilization issue of enforced positive switched linear systems with bounded controls under ADT switching scheme. First, we focus on designing output feedback controller which guaranteeing the closed-loop system positive and asymptotically stable, although the open-loop system is not positive at all. Second, based on the properties of special form of diagonal matrix, the sufficient conditions for the existence of bounded controllers are obtained. Third, the obtained results have been extended to the situation of constrained output. Finally, two examples are given to demonstrate the effectiveness of the proposed approach. All the proposed conditions are formulated in terms of linear inequalities, thus the controller parameters can be determined by linear programming. It is worth noting that the rank of controller gain matrix we designed is full, which overcoming the limitation on the rank of controller gain. Following the approach in this paper, further work may refer to delay problems and uncertain systems, reducing conservatism of the current results.

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Jinjin Liu received her M.S. degree from the College of Mathematics and Information Science, Henan Normal University, in 2011, and a Ph.D. degree from the School of Automation, Southeast University, in 2015. In 2015, she joined the School of Mathematics and Information Science, Henan University of Economics and Law, Zhengzhou. In 2018, she was a Visiting Scholar with the School of Control Science and Engineering, Zhejiang University. Her research interests include control synthesis of positive systems and optimal control of switched systems.