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Output Feedback Regulation of a Class of Lower Triangular Nonlinear Systems with Arbitrary Unknown Measurement Sensitivity

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Abstract: In this paper, a regulation problem for a class of lower triangular nonlinear systems under unknown measurement sensitivity by output feedback is considered. The distinguished feature is that the unknown measurement sensitivity is only required to be positive and bounded. The analysis is carried out to show the relation between the gain selection of an output feedback controller and the bound of the measurement sensitivity. Then, the adaptive gain-scalings of the controller are utilized to dominate the unknown growth rate of the nonlinearity.

Keywords: Lower triangular nonlinear system, measurement sensitivity, output feedback controller.

1. INTRODUCTION

The global regulation problems by output feedback for nonlinear systems coped with uncertain nonlinearities have been studied for the past decades. For a class of nonlinear systems with lower triangular nonlinearities, various control schemes with observers that have on-line adaptive gains are constructed in [1–4] under some conditions such as unknown linear growth rate, bounding functions depending on output feedback rate, and unknown control direction. In [5], both high-gain and low-gain output feedback controllers are developed and engaged to systems depending on the nonlinearity types. Then, in [6], a switching control scheme is developed for systems whose nonlinearity types are not known a priori. All aforementioned results share a common fact that their control schemes are developed based on the assumption of 'clean' feedback circumstances. That is, the measured feedback through sensors is assumed to be so accurate.

Recently, the stabilization or regulation problems of a class of nonlinear system under the measurement noise or sensitivity have attracted much attention [7–16], because some discrepancy between the real system state values and measured feedback values via sensors can occur in practice [13, 17, 18]. The study in [10] considers a case of the measurement noise where the error due to the measurement noise causes the increasing gain which can deteriorate the state estimation error. In [9], the observer with adaptive law is designed to deal with the output measurement noise in the form of $y = x_1 + s(t)$ where $s(t)$ is the noise.

The so-called measurement sensitivity considers a different feedback distortion case such as $\theta_i(t)x_i$ where x_i denote system state in convention and $\theta_i(t)$ denote some bounded positive functions. Then, in [8], an output feedback controller with dual-domination technique is proposed to obtain the system stabilization under $y = \theta(t)x_1$ in which $\theta(t)$ is not necessarily a differentiable function. However, as addressed in [8], the allowed bound of $\theta(t)$ is limited to some small ranges such as $\theta(t) \in [1-\theta^*, 1+\theta^*]$ where θ^* is somewhat small. Moreover, in [8], the size of θ^* tends to be reduced significantly with respect to the increase of the system dimension.

In this paper, we consider the output feedback regulation problem for systems with lower triangular nonlinearity under the same measurement sensitivity as set in [8]. However, two new distinguished features are that (i) $\theta(t)$ is only required to be positive and bounded, that is, the allowed bound of $\theta(t)$ is much enlarged such as $0 < \theta(t) < +\infty$, so the major shortcoming of [8] - the shrink of the allowed range of $\theta(t)$ with respect to the system dimension size does not occur; (ii) the growth rate of nonlinearity is unknown such that the previous fixed controller gains approach cannot be used. In order to solve our control problem, two main approaches are (i) for the given bound of $\theta(t)$, the fixed gains of the output feedback controller are determined based on the Lyapunov inequality analysis motivated by [19]; (ii) two gain-scalings are designed and coupled with an output feedback controller to deal with the unknown growth rate.

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2. SYSTEM FORMULATION AND PROBLEM STATEMENT

Consider the following system:

$$
\begin{aligned}
\dot{x}_i &= x_{i+1} + \delta_i(t,x,u), \quad i = 1,\cdots,n-1,\\
\dot{x}_n &= u + \delta_n(t,x,u),\\
y &= \theta(t)x_1,
\end{aligned}
\tag{1}
$$

where $x = [x_1,\cdots,x_n]^T \in \mathcal{R}^n$, $u \in \mathcal{R}$, and $y \in \mathcal{R}$ are the states, input, output, respectively. The $\delta_i(t,x,u) : \mathcal{R}^+ \times \mathcal{R}^n \times \mathcal{R} \to \mathcal{R}$, $i = 1, \cdots, n$, are continuous functions. The following conditions are imposed on the measurement sensitivity and lower triangular nonlinearity.

Assumption 1: The measurement sensitivity $\theta(t)$ is an uncertain, continuous, and bounded function and there exist positive constants $0 < \theta_l \le 1$ and $1 \le \theta_u < \infty$ such that

$$
\theta_l \le \theta(t) \le \theta_u, \tag{2}
$$

for all $t \ge 0$.

Assumption 2: There exists an unknown constant $c \ge 0$ such that

$$
|\delta_i(t,x,u)| \le c\sum_{j=1}^{i}|x_j|, \; i = 1,\cdots,n, \tag{3}
$$

for all t, x, u.

Here, we formally state our control problem.

Problem Statement 1: Globally regulate the system (1) under Assumptions 1 and 2 by an output feedback controller.

To show the generality of our control problem, we take the following example directly from [8]:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + d_1(t)\sin x_1,\\
\dot{x}_2 &= u + d_2(t)\ln(1+x_1^2),\\
y &= \theta(t)x_1.
\end{aligned}
$$

When there is no sensitivity feedback issue, that is, the feedback is ideal, $\theta(t)$ naturally becomes 1. So, without loss of generality, we can also express $\theta(t)$ as $\theta(t) = 1 + \delta\theta(t)$ as well and $\delta\theta(t)$ denotes the uncertain measurement sensitivity part. Then, in [8], their results obtain the allowed range of $\delta\theta(t)$ as $-0.4383 \le \delta\theta(t) \le +0.4383$ or $(-48.83\% \le \delta\theta(t) \le +48.83\%)$ for system dimension $n = 2$. Note that this allowed range of $\delta\theta(t)$ significantly decreases as the system dimension n increases. Moreover, in [8], the upper bounds of $d_1(t)$ and $d_2(t)$ are required to be known. On the other hand, as already stated, these restrictions are removed in our case. That is, now, $\theta(t)$ is virtually arbitrary as long as it is positive and finite. In other words, our condition is $-1 < \delta\theta(t) < +\infty$ regardless of the system dimension. Notably, $\theta(t)$ is not necessarily differentiable in our case as well. Moreover, the upper bounds of $d_1(t)$ and $d_2(t)$ are not known. Thus, our generalized features are well clear over [8].

To the best of our knowledge, there have been no results dealing with our control problem yet. Our approach to the considered problem is summarized as follows: First, our proposed output feedback controller has fixed gains and two adaptive gain-scaling factors. Then, for any given bound of $\theta(t)$ satisfying Assumption 1, the fixed gains are determined based on the analysis using Lyapunov inequality technique developed by [19]. Next, two adaptive gain-scaling factors are designed to tackle the unknown growth rate of nonlinearity. Combining these methods together, the system regulation will be shown using Lyapunov stability analysis.

3. MAIN RESULTS

Lemma 1: Suppose that Assumption 1 holds. Let $A_L(\theta(t))$ be $n \times n$ matrices as

$$
A_L(\theta(t)) = \begin{bmatrix} -l_1\theta(t) & 1 & 0 & \cdots & 0\\ -l_2\theta(t) & 0 & 1 & \cdots & 0\\ \vdots & \vdots & & \ddots & \vdots\\ -l_{n-1}\theta(t) & 0 & 0 & \cdots & 1\\ -l_n\theta(t) & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{4}
$$

Select l_i, $i = 1, \cdots, n$ as

$$
\begin{aligned}
l_1 &= b_2 + \frac{1}{2} + l_0,\\
l_i &= b_i l_{i-1} - b_i\prod_{k=2}^{i} b_k + \prod_{k=2}^{i+1} b_k, \quad i = 2,\cdots,n,
\end{aligned}
\tag{5}
$$

where l_0 is to be presented later on and

$$
b_i = b_{i+1} + \frac{i}{2} + 1 + \bar{b}_i, \tag{6}
$$

where $b_{n+1} = 0$, $\bar{b}_n = 0$, and, $i = 2, \cdots, n-1$,

$$
\bar{b}_i = \frac{1}{2}\sum_{m=i+1}^{n-1}\left(\bar{b}_m + 1 + \frac{m}{2}\right)^2 \prod_{k=i+1}^{m} b_k^2 + \frac{1}{2}b_n^2\prod_{k=i+1}^{n} b_k^2, \tag{7}
$$

with $b_{n+1} = 0$.

Then, there exists a positive constants l_0^* such that, for $l_0 \ge l_0^*$, we have the following inequality as

$$
A_L(\theta(t))^T P_L + P_L A_L(\theta(t)) \le -\min\{l_0\theta_l, 1\}I, \tag{8}
$$

where I is an $n \times n$ identity matrix and $P_L = P_1^T P_1$ is a positive definite matrix with

$$
P_1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0\\ -b_2 & 1 & 0 & \cdots & 0\\ 0 & -b_3 & 1 & \cdots & 0\\ & & & \vdots & \\ 0 & \cdots & 0 & -b_n & 1 \end{bmatrix}. \tag{9}
$$

Proof: Proof is in Appendix A.1. □

We introduce an output feedback controller as follows:

$$u = \sum_{i=1}^{n} \left(k_i \gamma(t)^{n-i+1} \varepsilon(t)^{n-i+1}\right) \hat{x}_i, \tag{10}$$

$$\begin{aligned}\dot{\hat{x}}_i &= \hat{x}_{i+1} + l_i \gamma(t)^i (y - \hat{x}_1), \ \ i = 1, \cdots, n-1, \\ \dot{\hat{x}}_n &= u + l_n \gamma(t)^n (y - \hat{x}_1),\end{aligned} \tag{11}$$

where k_i and l_i, $i = 1, \cdots, n$ are selected as follows:

- Select k_i, $i = 1, \cdots, n$ as $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1 = 0$ becomes a Hurwitz polynomial.
- Select l_i, $i = 1, \cdots, n$, as described in (5) with $l_0 \geq l_0^*$ calculated from given $\theta(t)$ in Assumption 1.

The dynamic gains $\gamma(t)$ and $\varepsilon(t)$ are designed as

$$\dot{\gamma}(t) = \frac{|y| + \sum_{i=1}^{n} |\hat{x}_i|}{1 + |y| + \sum_{i=1}^{n} |\hat{x}_i|} \left(\frac{\varepsilon(t)^{n-1} + 1}{\gamma(t)^{n-1} \varepsilon(t)^{n-1}} \right), \tag{12}$$

$$\dot{\varepsilon}(t) = \frac{|y| + \sum_{i=1}^{n} |\hat{x}_i|}{1 + |y| + \sum_{i=1}^{n} |\hat{x}_i|} \left(\frac{1}{\gamma(t)^{n-1} \varepsilon(t)^{n-1}} \right), \tag{13}$$

with $\gamma(0) = \varepsilon(0) = 1$.

Lemma 2: Suppose that Assumptions 1-2 hold. Let the controller (10)-(11) with (12)-(13) be applied to the system (1). If $\gamma(t)$ converges to a finite positive constant, then all states of the system (1) are globally regulated and $\varepsilon(t)$ converges to a finite positive constant.

Proof: Proof is in Appendix A.2. □

Lemma 3: Suppose that Assumptions 1-2 hold. Let the controller (10)-(11) with (12)-(13) be applied to the system (1). If $\varepsilon(t)$ converges to a finite positive constant, then all states of the system (1) are globally regulated and $\gamma(t)$ converges to a finite positive constant.

Proof: Proof is in Appendix A.3. □

Theorem 1: Suppose that Assumptions 1-2 hold. Then, with the controller (10)-(11) with (12)-(13), all states of the closed system (1) are globally regulated and $\gamma(t)$ and $\varepsilon(t)$ converge to finite positive constants.

Proof: Define $e = [e_1, \cdots, e_n]^T$ and $z = [z_1, \cdots, z_n]^T$ where, for $i = 1, \cdots, n$,

$$e_i = \gamma(t)^{-(i-1)} (x_i - \hat{x}_i), \tag{14}$$

$$z_i = \gamma(t)^{-(i-1)} \varepsilon(t)^{-(i-1)} \hat{x}_i. \tag{15}$$

Note that, from (1), (11), (14), (15), for $i = 1, \cdots, n$, we obtain

$$\begin{aligned}\dot{e} =& \gamma(t) A_L(\theta(t)) e + (1 - \theta(t)) \gamma(t) L z_1 \\ &+ E_{\gamma(t)} \delta(t, x, u) + \gamma(t)^{-1} \dot{\gamma}(t) D_1 e,\end{aligned} \tag{16}$$

where $E_{\gamma(t)} = \mathrm{diag}[1, \gamma(t)^{-1}, \cdots, \gamma(t)^{-(n-1)}]$, $D_1 = \mathrm{diag}[0, -1, \cdots, -(n-1)]$, $L = [l_1, \cdots, l_n]^T$, $\delta(t,x,u) = [\delta_1(t,x,u), \cdots, \delta_n(t,x,u)]^T$.

Also, note that, from (10), (11), (14), (15), we have

$$\begin{aligned}\dot{z} =& \gamma(t) \varepsilon(t) A_K z + \theta(t) \gamma(t) E_{\varepsilon(t)} L e_1 \\ &+ (\theta(t) - 1) \gamma(t) E_{\varepsilon(t)} L z_1 \\ &+ \gamma(t)^{-1} \dot{\gamma}(t) D_1 z + \varepsilon(t)^{-1} \dot{\varepsilon}(t) D_1 z,\end{aligned} \tag{17}$$

where $A_K = A + BK$ with the Brunovsky canonical pair (A, B) and $K = [k_1, \cdots, k_n]$, and $E_{\varepsilon(t)} = \mathrm{diag}[1, \varepsilon(t)^{-1}, \cdots, \varepsilon(t)^{-(n-1)}]$.

Define a Lyapunov function $V_1(e) = e^T P_L e$. Then, taking a time-derivative of $V_1(e)$ along the trajectory of (16), we obtain

$$\begin{aligned}\dot{V}_1(e) \leq& \gamma(t) e^T \Big(A_L(\theta(t))^T P_L + P_L A_L(\theta(t)) \Big) e \\ &+ 2\gamma(t) |1 - \theta(t)| \|P_L\| \|L\| \|e\| |z_1| \\ &+ 2 \|P_L\| \|e\| \|E_{\gamma(t)} \delta(t, x, u)\|_1 \\ &+ 2\gamma(t)^{-1} \dot{\gamma}(t) \|P_L\| \|D_1\| \|e\|^2.\end{aligned} \tag{18}$$

By using (14)-(15), we have

$$\begin{aligned}&\|E_{\gamma(t)} \delta(t, x, u)\|_1 \\ &\leq c \sum_{i=1}^{n} \gamma(t)^{-(i-1)} \Big(\gamma(t)^{(i-1)} |e_i| + \gamma(t)^{(i-1)} \varepsilon(t)^{(i-1)} |z_i| \Big) \\ &\leq c \sum_{i=1}^{n} \Big(|e_i| + \varepsilon(t)^{(i-1)} |z_i| \Big) \\ &\leq cn(n+1) \Big(\|e\| + \varepsilon(t)^{(n-1)} \|z\| \Big).\end{aligned} \tag{19}$$

By substituting (8) and (19) into (18) and using $|z_1| \leq \|z\|$, we obtain

$$\begin{aligned}\dot{V}_1(e) \leq& -\theta_M \gamma(t) e^T e + c_1 |1 - \theta(t)| \|e\| \|z\| + c_1 \|e\|^2 \\ &+ c_1 \varepsilon(t)^{n-1} \|e\| \|z\| + c_1 \gamma(t)^{-1} \dot{\gamma}(t) \|e\|^2,\end{aligned} \tag{20}$$

where $\theta_M = \min\{l_0 \theta_l, 1\}$, $c_1 = 2\max\{\|P_L\| \|L\|, cn(n+1)\|P_L\|, \|P_L\| \|D_1\|\}$.

Next, set the Lyapunov function as $V_2(z) = z^T P_K z$ where P_K is from the Lyapunov equation $A_K^T P_K + P_K A_K = -I$. Then, taking a time-derivative of $V_2(z)$ along the trajectory of (17), we obtain

$$\begin{aligned}\dot{V}_2(z) \leq& -\gamma(t) \varepsilon(t) \|z\|^2 + 2\theta(t) \gamma(t) \|P_K\| \|z\| \|E_{\varepsilon(t)} L e_1\| \\ &+ 2|\theta(t) - 1| \gamma(t) \|P_K\| \|z\| \|E_{\varepsilon(t)} L z_1\| \\ &+ 2\Big(\gamma(t)^{-1} \dot{\gamma}(t) + \varepsilon(t)^{-1} \dot{\varepsilon}(t) \Big) \|P_K D_1\| \|z\|^2.\end{aligned} \tag{21}$$

From $\varepsilon(t) \geq 1$, $|e_1| \leq \|e\|$, and $|z_1| \leq \|z\|$, we have $\|E_{\varepsilon(t)}Le_1\| \leq \|L\|\|e\|$ and $\|E_{\varepsilon(t)}Lz_1\| \leq \|L\|\|z\|$. Using these inequalities and (21), we obtain

$$\begin{aligned}\dot{V}_2(z) \leq & -\gamma(t)\varepsilon(t)\|z\|^2 \\ & + c_2\theta(t)\|z\|\|e\| + c_2|\theta(t)-1|\|z\|^2 \\ & + c_2\Big(\gamma(t)^{-1}\dot{\gamma}(t) + \varepsilon(t)^{-1}\dot{\varepsilon}(t)\Big)\|z\|^2, \end{aligned} \tag{22}$$

where $c_2 = 2\max\{\|P_K\|\|L\|, \|P_K\|\|D_1\|\}$.

Note that, from (12)-(13), using $\gamma(t) \geq 1$, $\varepsilon(t) \geq 1$, we have the following properties as

$$\dot{\gamma}(t) \leq 2, \;\; \dot{\varepsilon}(t) \leq 1, \tag{23}$$

$$\dot{\gamma}(t) = \dot{\varepsilon}(t)(\varepsilon(t)^{n-1}+1) \geq \varepsilon(t)^{n-1}\dot{\varepsilon}(t). \tag{24}$$

From (24), we have $\gamma(t) - 1 \geq \frac{1}{n}(\varepsilon(t)^n - 1)$. Using this inequality, we arrive at

$$\gamma(t) \geq \frac{1}{n}\varepsilon(t)^n. \tag{25}$$

Using (25), the fourth term in the right-hand side of (20) is bounded as

$$c_1\varepsilon(t)^{n-1}\|e\|\|z\| \leq c_1 n\gamma(t)\|e\|\|z\|. \tag{26}$$

Using (23), we have

$$\gamma(t)^{-1}\dot{\gamma}(t) \leq 2\gamma(t)^{-1}, \quad \varepsilon(t)^{-1}\dot{\varepsilon}(t) \leq \varepsilon(t)^{-1}. \tag{27}$$

Now, we set a composite Lyapunov function $V(e,z) = V_1(e) + V_2(z)$ for $t \in [0, T_f)$ where T_f denotes an arbitrary large finite time. Then, taking a time-derivative of $V(e,z)$ with (20) and (22), (26), (27), we have

$$\begin{aligned} & \dot{V}(e,z) \\ & \leq -\frac{\theta_M}{2}\gamma(t)\|e\|^2 - \frac{1}{2}\gamma(t)\varepsilon(t)\|z\|^2 \\ & \quad - \left(\frac{\theta_M}{4}\gamma(t) - c_1 - 2c_1\gamma(t)^{-1}\right)\|e\|^2 \\ & \quad - \left(\frac{1}{4}\gamma(t)\varepsilon(t) - c_2|\theta(t)-1| - \frac{2c_2}{\gamma(t)} - \frac{c_2}{\varepsilon(t)}\right)\|z\|^2 \\ & \quad - \begin{bmatrix} e \\ z \end{bmatrix}^T \begin{bmatrix} \frac{\theta_M}{4}\gamma(t) & \Pi(t) \\ \Pi(t) & \frac{1}{4}\gamma(t)\varepsilon(t) \end{bmatrix} \begin{bmatrix} e \\ z \end{bmatrix}, \end{aligned} \tag{28}$$

where

$$\Pi(t) = -\frac{1}{2}\Big(c_1|1-\theta(t)| + c_1 n\gamma(t) + c_2\theta(t)\Big)].$$

Since $\gamma(t)$ and $\varepsilon(t)$ are strictly increasing and $0 < \theta(t) < \infty$, the terms multiplied by $\|e\|^2$ and $\|z\|^2$ in the second and the third lines of (28) are positive and the matrix in the last line of (28) is positive definite if there exist positive constants $\bar{\gamma}$, $\bar{\varepsilon}$ such that $\gamma(t) \geq \bar{\gamma}$ and $\varepsilon(t) \geq \bar{\varepsilon}$ satisfying

$$\frac{\theta_M}{4}\gamma(t) - 3c_1 > 0, \tag{29}$$

$$\frac{1}{4}\gamma(t)\varepsilon(t) - c_2\bar{\theta} - 3c_2 > 0, \tag{30}$$

$$\begin{aligned} & \frac{\theta_M}{16}\gamma(t)^2\varepsilon(t) - \Pi(t)^2 \\ & \geq \frac{\theta_M}{16}\gamma(t)^2\varepsilon(t) - \frac{1}{4}\Big(c_1 n\gamma(t) + c_1\bar{\theta} + c_2\theta_u\Big)^2 > 0, \end{aligned} \tag{31}$$

where $\bar{\theta} = \max\{1-\theta_l, \theta_u - 1\}$.

Because $\gamma(t)$ and $\varepsilon(t)$ are nondecreasing, we consider two cases: (i) $\gamma(t) \leq \bar{\gamma}$ or $\varepsilon(t) \leq \bar{\varepsilon}$ and (ii) $\gamma(t) \geq \bar{\gamma}$ and $\varepsilon(t) \geq \bar{\varepsilon}$. In the case (i), the global regulation of the system (1) is achieved by Lemmas 2 and 3. Then, we only need to consider the case (ii).

Now, we suppose that $\lim_{t \to T_f} \gamma(t) \to \infty$ and $\lim_{t \to T_f} \varepsilon(t) \to \infty$. Then, from (28)-(31), and $\gamma(t) > 1$, $\varepsilon(t) > 1$, for $t \in [t_1^*, T_f)$ where t_1^* is a particular time such that the case (ii) is satisfied for $t \in [t_1^*, T_f)$, we have

$$\begin{aligned}\dot{V}(e,z) & \leq -\frac{1}{2}\Big(\theta_M\gamma(t)\|e\|^2 + \gamma(t)\varepsilon(t)\|z\|^2\Big) \\ & \leq -c_3\Big(\|e\|^2 + \|z\|^2\Big), \end{aligned} \tag{32}$$

where $c_3 = \frac{1}{2}\min\{1, \theta_M\}$.

Note that

$$m(\|e\|^2 + \|z\|^2) \leq V(e,z) \leq M(\|e\|^2 + \|z\|^2), \tag{33}$$

where $m = \min\{\lambda_{\min}(P_K), \lambda_{\min}(P_L)\}$ and $M = \max\{\lambda_{\max}(P_K), \lambda_{\max}(P_L)\}$.

From (32)-(33), we have, for $t \in [t_1^*, T_f)$,

$$V(e,z) \leq V(e(t_1^*), z(t_1^*))e^{-\frac{c_3}{M}(t-t_1^*)}. \tag{34}$$

Then, using (13) and (34), we arrive at

$$\begin{aligned} \varepsilon(t) - 1 & = \int_0^t \frac{|y| + \sum_{i=1}^n |\hat{x}_i|}{1 + |y| + \sum_{i=1}^n |\hat{x}_i|}\gamma(t)^{-(n-1)}\varepsilon(t)^{-(n-1)}dt \\ & \leq \int_0^t \left(|y| + \sum_{i=1}^n \gamma(t)^{-(i-1)}\varepsilon(t)^{-(i-1)}|\hat{x}_i|\right)dt \\ & \leq \int_0^t \left(\theta_u(|e_1| + |z_1|) + \sum_{i=1}^n |z_i|\right)dt \\ & \leq (\theta_u + n^2 + n)\int_0^t \sqrt{\|e\|^2 + \|z\|^2}dt \\ & \leq c_4\int_0^t \sqrt{V(e(t_1^*), z(t_1^*))}e^{-\frac{c_3}{2M}(t-t_1^*)}dt < \infty, \end{aligned} \tag{35}$$

where $c_4 = (\theta_u + n^2 + n)/\sqrt{m}$.

Since $\varepsilon(t)$ is bounded as shown in (35), it is contradictory to the supposition of $\lim_{t\to T_f}\varepsilon(t)\to\infty$. Therefore, we conclude that $\varepsilon(t)$ is bounded on $t\in[0,T_f)$. Then, from Lemma 3, the global regulation and the convergence of $\gamma(t)$ are achieved. So, the global system regulation is followed. □

Remark 1: The contributions are summarized as follows: (i) In [8], the allowable sensitivity error is $\theta(t)\in[1-\bar{\theta},1+\bar{\theta}]$ where $0\le\bar{\theta}<1$. Our proposed controller by Lemma 1 is designed for the desired bounds of the sensitivity so that the bounds of the sensitivity can be much relaxed as shown in (2) of Assumption 1. (ii) Our proposed controller has dynamic gains $\varepsilon(t)$ and $\gamma(t)$ unlike the static gains in [8]. The roles of $\gamma(t)$ and $\varepsilon(t)$ are to treat the effect of the nonlinearity under Assumption 2 and the measurement sensitivity, respectively. These features are clearly distinguished from the results of [8].

4. EXAMPLE

(i) Illustrative example: We revisit the example in Section 2. We set $\theta(t)=3+2.7\sin t$, $d_1(t)=1+\cos t$ and $d_2(t)=2-\sin 20t$. Thus, the bound of $\theta(t)$ is $\theta(t)\in[0.3,5.7]$ or in an expression of $\theta(t)=1+\delta\theta(t)$, $-0.7\le\delta\theta(t)\le 4.7$ or $(-70\%\le\delta\theta(t)\le+470\%)$ which is well over the allowed bound suggested by [8]. Remind that the bounds of $d_1(t)$ and $d_2(t)$ are not needed in our controller design. Thus, this clearly shows the generality of our result over [8]. For fixed gains of the controller, we set $k_1=-4$, $k_2=-4$. From the proof of Lemma 1 and the bounds of $\theta(t)$, choose $l_0=149$ from (A.9) by calculating with $b_2=2$ in (6) and $\rho_1=3/2$, $\rho_2=4$ in (A.5). Thus, we set $l_1=151.5$,$l_2=299$, and the initial condition of the controller as $u(0)=0$ with $\hat{x}_1(0)=0$ and $\hat{x}_2(0)=0$. Coupled with two gain-scaling factors, the system with the initial condition $x_1(0)=1$ and $x_2(0)=-1$ is regulated as shown in Fig. 1.

(ii) Application example: Consider the LLC resonant circuit system in [6] with the output function $y=\theta(t)i_{L_1}$.

$$\begin{aligned}\dot{i}_{L_1}&=-\frac{v_c}{L_1}-\frac{R_a}{L_1}(i_{L_2}-0.5\sin v_c),\\ \dot{v}_c&=\frac{i_{L_2}}{C}-\frac{0.5\sin v_c}{C},\\ \dot{i}_{L_2}&=-\frac{R_b}{L_2}i_{L_2}+\frac{u}{L_2},\\ y&=\theta(t)(i_{L_1}+2v_c),\end{aligned}\tag{36}$$

where $L_1=L_2=1mH$, $C=2nF$, $R_a=1k\Omega$, and $R_bk\Omega$ is an unknown variable resistance. The measurement sensitivity range is $0.8\le\theta(t)\le1.2$.

By the transformation $x_1=i_{L_1}+2v_c$, $x_2=-v_c$, $x_3=-0.5(i_{L_2}-0.5\sin v_c)$ and $u=-2u_1$, we have

$$\begin{aligned}&\dot{x}_1=x_2,\ \dot{x}_2=x_3,\\ &\dot{x}_3=u_1+R_b(x_3-0.25\sin x_2)-0.125x_3\cos x_2,\\ &y=\theta(t)x_1.\end{aligned}\tag{37}$$

The system (37) satisfies Assumption 2 with $n=3$. We set $k_1=-8$, $k_2=-12$, $k_3=-6$ by using the Hurwitz polynomial $s^3+k_3s^2+k_2s+k_1=0$. From (6), $b_2=12$, $b_3=2$ are calculated and we have $\rho_1=12.5$, $\rho_2=120$, $\rho_3=48$ in (A.5) by using b_2 and b_3. From the proof of Lemma 1 and the bounds of $\theta(t)$ with $\theta_l=0.8$ and $\theta_u=1.2$, we choose $l_0=900$. Thus, we set the fixed gain as $l_1=912.5$, $l_2=10830$, $l_3=21612$ in (5) and the initial condition of the controller as $u(0)=0$ with $\hat{x}_1(0)=0$, $\hat{x}_2(0)=0$, $\hat{x}_3(0)=0$. The system with the initial condition $i_{L_1}(0)=1$, $v_c(0)=0$, $i_{L_2}(0)=2$ is globally regulated as shown in Fig. 2.

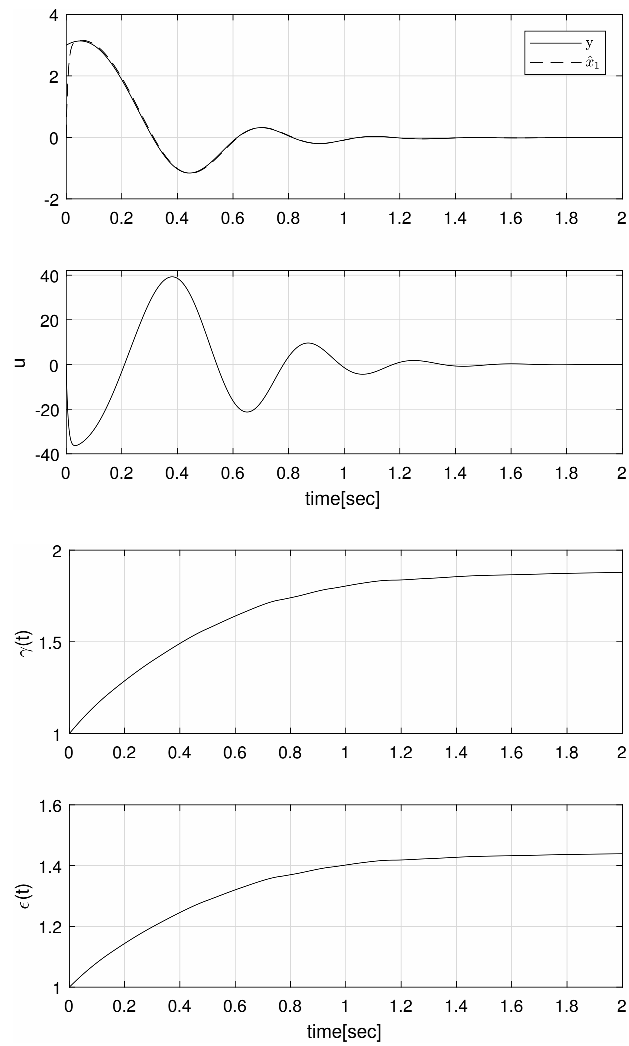

Fig. 1. Trajectories of the system/observer states, gain-scaling factors, output, and input for (i) Illustrative example.

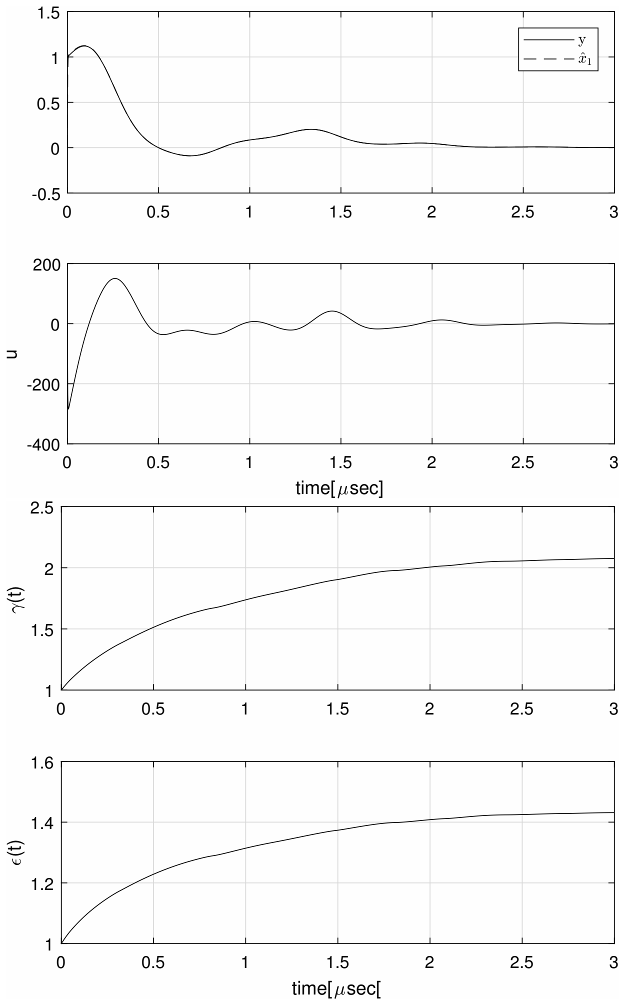

Fig. 2. Trajectories of the system output/observer and input for (ii) Application example.

5. CONCLUSIONS

We consider the global regulation problem for lower triangular nonlinear systems under uncertain measurement sensitivity and unknown growth rate of nonlinearity. One distinguished feature is that the measurement sensitivity is only required to be positive and finite and the allowed bound of measurement sensitivity is much enlarged and the growth rate of nonlinearity is not required to be known a priori. By using the simulation example, proposed method is demonstrated. As a future topic, our method can be extended to output feedback regulation for nonlinear system with sampled measurements [11, 14].

APPENDIX A

A.1. Proof of Lemma 1

Consider a system $\dot{\eta} = A_L(\theta(t))\eta$ with $\eta = [\eta_1, \eta_2, \cdots, \eta_n]^T$. Using a transformation $\xi = P_1\eta$ where $\xi = [\xi_1, \xi_2, \cdots, \xi_n]^T$, we have

$$\xi_1 = \eta_1,\ \xi_i = -b_i\eta_{i-1} + \eta_i,\ \ i = 2\cdots, n. \tag{A.1}$$

Then, its inverse transformation is

$$\eta_1 = \xi_1,\ \ \eta_i = \xi_i + \sum_{j=1}^{i-1}\xi_j \prod_{k=j+1}^{i} b_k,\ \ i = 2, \cdots, n. \tag{A.2}$$

Using (A.1) and (A.2), we have the time-derivative of ξ_i, $i = 1, \cdots, n$, as

$$\begin{aligned}
\dot{\xi}_1 &= (b_2 - l_1\theta(t))\xi_1 + \xi_2,\\
\dot{\xi}_i &= \left(b_i l_{i-1}\theta(t) - l_i\theta(t) - b_i\prod_{k=2}^{i} b_k + \prod_{k=2}^{i+1} b_k\right)\xi_1\\
&\quad + (b_{i+1} - b_i)\xi_i + \xi_{i+1}\\
&\quad + \sum_{j=2}^{i-1}\xi_j\left(\prod_{k=j+1}^{i+1} b_k - b_i\prod_{k=j+1}^{i} b_k\right),\\
&\quad i = 2, \cdots, n,
\end{aligned} \tag{A.3}$$

where $\xi_{n+1} = 0$.

By substituting l_i, $i = 2, \cdots, n$, in (5) into (A.3), we have

$$\begin{aligned}
\dot{\xi}_1 &= (b_2 - l_1\theta(t))\xi_1 + \xi_2,\\
\dot{\xi}_i &= \rho_i(\theta(t) - 1)\xi_1 + (b_{i+1} - b_i)\xi_i + \xi_{i+1}\\
&\quad + \sum_{j=2}^{i-1}\xi_j\left(\prod_{k=j+1}^{i} b_k(b_{i+1} - b_i)\right),\ \ i = 2, \cdots, n,
\end{aligned} \tag{A.4}$$

where with $b_{n+1} = 0$,

$$\begin{aligned}
\rho_1 &= b_2 + \frac{1}{2},\\
\rho_i &= b_i\prod_{k=2}^{i} b_k - \prod_{k=2}^{i+1} b_k,\ \ i = 2, \cdots, n.
\end{aligned} \tag{A.5}$$

Using (A.4), l_1 in (5), and $\xi_1\xi_2 \le \frac{1}{2}(\xi_1^2 + \xi_2^2)$, note that

$$\xi_1\dot{\xi}_1 \le \Big(\rho_1(1 - \theta(t)) - l_0\theta(t)\Big)\xi_1^2 + \frac{1}{2}\xi_2^2, \tag{A.6}$$

where ρ_1 is defined in (A.5).

Similar to (A.6), we obtain, $i = 2, \cdots, n$,

$$\begin{aligned}
\xi_i\dot{\xi}_i \le{} & \rho_i(\theta(t) - 1)\xi_1\xi_i + \left(b_{i+1} - b_i + \frac{i-1}{2}\right)\xi_i^2\\
& + \frac{1}{2}\xi_{i+1}^2 + \frac{1}{2}\sum_{j=2}^{i-1}\xi_j^2(b_{i+1} - b_i)^2\prod_{k=j+1}^{i} b_k^2.
\end{aligned} \tag{A.7}$$

Using (A.6)-(A.7) and b_i of (6), we have

$$\begin{aligned}&\sum_{i=1}^{n}\xi_i\dot{\xi}_i\\&\le\Big(\rho_1(1-\theta(t))-l_0\theta(t)\Big)\xi_1^2\\&\quad+\big(\theta(t)-1\big)\sum_{i=2}^{n}\rho_i\xi_1\xi_i-\sum_{i=2}^{n}\xi_i^2-\sum_{i=2}^{n-1}\bar{b}_i\xi_i^2\\&\quad+\frac{1}{2}\sum_{i=2}^{n-1}\xi_i^2\left(\sum_{m=i+1}^{n-1}\left(\left(\bar{b}_m+1+\frac{m}{2}\right)^2\prod_{k=i+1}^{m}b_k^2\right)\right)\\&\quad+\frac{1}{2}b_n^2\sum_{i=2}^{n-1}\xi_i^2\prod_{k=i+1}^{n}b_k^2.\end{aligned}\tag{A.8}$$

Note that if l_0 is selected as $l_0\ge l_0^*$ such that l_0^* satisfy $\rho_1(1-\theta(t))-\frac{l_0^*\theta(t)}{2}\le 0$ and the following inequality as for $i=2,\cdots,n$,

$$\begin{aligned}&\frac{2}{n^2}\left(\frac{l_0^*\theta(t)}{2}-\rho_1(1-\theta(t))\right)-(1-\theta(t))^2\rho_i^2\\&\ge 0,\end{aligned}\tag{A.9}$$

for all t, then, we have $\Big(\rho_1(1-\theta(t))-l_0\theta(t)\Big)\xi_1^2-\big(\theta(t)-1\big)\sum_{i=2}^{n}\rho_i\xi_1\xi_i+\frac{1}{2}\sum_{i=2}^{n}\xi_i^2\ge 0$.

From (A.8), l_0 is calculated from (A.9), and $\bar{b}_i$ in (7), we obtain

$$\begin{aligned}\sum_{i=1}^{n}\xi_i\dot{\xi}_i&\le-\frac{l_0\theta(t)}{2}\xi_1^2-\frac{1}{2}\sum_{i=2}^{n}\xi_i^2\\&\le-\frac{1}{2}\min\{l_0\theta_l,1\}\sum_{i=1}^{n}\xi_i^2.\end{aligned}\tag{A.10}$$

Thus, from $\frac{1}{2}\xi^T\xi=\frac{1}{2}\sum_{i=1}^{n}\xi_i^2$, $\xi=P_1\eta$, and $\dot{\eta}=A_L(\theta(t))\eta$, note that $\sum_{i=1}^{n}\xi_i\dot{\xi}_i=\frac{1}{2}\frac{d(\xi^T\xi)}{dt}=\frac{1}{2}\Big(\dot{\eta}^TP_1^TP_1\eta+\eta^TP_1^TP_1\dot{\eta}\Big)=\frac{1}{2}\Big(\eta^TA_L(\theta(t))^TP_L\eta+\eta^TP_LA_L(\theta(t))\eta\Big)$. Together with this inequality and (A.10), the inequality (8) is followed.

A.2. Proof of Lemma 2

From (12) and (13),

$$\gamma(t)-1=\frac{1}{n}\varepsilon(t)^n+\varepsilon(t)-\frac{n+1}{n}.\tag{A.11}$$

From (A.11) and the convergence of $\gamma(t)$, it is easy to obtain that there exist constants $\bar{\gamma}$ and $\bar{\varepsilon}$ such that $\gamma(t)<\bar{\gamma}$ and $\varepsilon(t)<\bar{\varepsilon}$. Then, since $\varepsilon(t)$ is nondecreasing, the convergence of $\varepsilon(t)$ is achieved. Also, by integrating (13) and using the upper bounds of $\gamma(t)$ and $\varepsilon(t)$, we have

$$\infty>\varepsilon(t)-1>\frac{1}{\bar{\gamma}^{n-1}\bar{\varepsilon}^{n-1}}\int_0^t\frac{|y|+\sum_{i=1}^{n}|\hat{x}_i|}{1+|y|+\sum_{i=1}^{n}|\hat{x}_i|}dt.$$

Then, by Barbalat's Lemma [20], we have

$$|\theta(t)x_1|+\|\hat{x}\|_1\to 0\ \text{ as }\ t\to\infty.\tag{A.12}$$

Now, define $e=[e_1,\cdots,e_n]^T$ and $z=[z_1\cdots,z_n]^T$ where, for $i=1,\cdots,n$,

$$e_i=\gamma_*^{-(i-1)}(x_i-\hat{x}_i),\tag{A.13}$$
$$z_i=\gamma_*^{-(i-1)}\varepsilon_*^{-(i-1)}\hat{x}_i,\tag{A.14}$$

where $\varepsilon_*\ge\bar{\varepsilon}$ and

$$\gamma_*\ge\max\{8cn(n+1)\|P_L\|/\theta_M,\bar{\gamma}\}.\tag{A.15}$$

Using the transformations (A.13) and (A.14), we have

$$\begin{aligned}\dot{e}=&\gamma_*A_L(\theta(t))e+\theta(t)\Big(\gamma_*I-\gamma(t)E_{\gamma_*}E_{\gamma(t)}^{-1}\Big)Le_1\\&+\gamma(t)(1-\theta(t))E_{\gamma_*}Lz_1+E_{\gamma_*}\delta(t,x,u),\end{aligned}\tag{A.16}$$

where $E_{\gamma_*}=\text{diag}[1,\ \gamma_*^{-1},\ \cdots,\ \gamma_*^{-(n-1)}]$, $E_{\gamma(t)}=\text{diag}[1,\gamma(t)^{-1},\cdots,\gamma(t)^{-(n-1)}]$, where $L=[l_1,\cdots,l_n]^T$.

Define a Lyapunov function $V_1(e)=e^TP_Le$. Then, taking a time-derivative of $V_1(e)$ along the trajectory of (A.16), we have

$$\begin{aligned}\dot{V}_1(e)\le&\gamma_*e^T\Big(A_L(\theta(t))^TP_L+P_LA_L(\theta(t))\Big)e\\&+2\gamma_*|\theta(t)|\|P_L\|\|L\|\|e\||e_1|\\&+2\gamma(t)|\theta(t)|\|P_L\|\|E_{\gamma_*}E_{\gamma(t)}^{-1}L\|\|e\||e_1|\\&+2\gamma(t)|1-\theta(t)|\|P_L\|\|E_{\gamma_*}L\|\|e\||z_1|\\&+2\|P_L\|\|e\|\|E_{\gamma_*}\delta(t,x,u)\|_1.\end{aligned}\tag{A.17}$$

Using $\|E_{\gamma_*}E_{\gamma(t)}^{-1}L\|=\sqrt{\sum_{i=1}^{n}\left(\frac{\gamma(t)}{\gamma_*}\right)^{2(i-1)}|l_i|^2}$ and $\gamma(t)\le\gamma_*$, the term $2\gamma(t)|\theta(t)|\|P_L\|\|E_{\gamma_*}E_{\gamma(t)}^{-1}L\|\|e\||e_1|$ in (A.17) is upper-bounded as

$$\begin{aligned}&2\gamma(t)|\theta(t)|\|P_L\|\|E_{\gamma_*}E_{\gamma(t)}^{-1}L\|\|e\||e_1|\\&\le 2\bar{\gamma}\theta_u\|P_L\|\|L\|\|e\||e_1|.\end{aligned}\tag{A.18}$$

Also, using $\|E_{\gamma_*}L\|\le\|L\|$, the term $2\gamma(t)|1-\theta(t)|\|P_L\|\|E_{\gamma_*}L\|\|e\||z_1|$ in (A.17) is upper-bounded as

$$\begin{aligned}&2\gamma(t)|1-\theta(t)|\|P_L\|\|E_{\gamma_*}L\|\|e\||z_1|\\&\le 2\bar{\gamma}\max\{1-\theta_l,\theta_u-1\}\|P_L\|\|L\|\|e\||z_1|.\end{aligned}\tag{A.19}$$

By using (A.13)-(A.14), the last term in (A.17) is upper-bounded as

$$\begin{aligned}&\|E_{\gamma_*}\delta(t,x,u)\|_1\\&\le c\sum_{i=1}^{n}\gamma_*^{-(i-1)}\Big(\gamma_*^{(i-1)}|e_i|+\gamma_*^{(i-1)}\varepsilon_*^{(i-1)}|z_i|\Big)\\&\le c\sum_{i=1}^{n}\Big(|e_i|+\varepsilon_*^{(i-1)}|z_i|\Big)\\&\le cn(n+1)\Big(\|e\|+\varepsilon_*^{(n-1)}\|z\|\Big).\end{aligned}\tag{A.20}$$

Then, by substituting (A.18)-(A.20) into (A.17), the inequality (A.17) is obtained as

$$\dot{V}_1 \leq -\frac{\theta_M \gamma_*}{2}\|e\|^2 - \left(\frac{\gamma_* \theta_M}{4} - 2cn(n+1)\|P_L\|\right)\|e\|^2 - \left(\frac{\gamma_* \theta_M}{4}\|e\| - d_1|e_1| - d_2|z_1| - d_3\|z\|\right)\|e\|, \tag{A.21}$$

where $d_1 = 2\theta_u(\gamma_*\|P_L\|\|L\| + \bar{\gamma}\|P_L\|\|L\|)$, $d_2 = 2\bar{\gamma}\max\{1-\theta_l,\ \theta_u - 1\}\|P_L\|\|L\|$, $d_3 = 2cn(n+1)\varepsilon_*^{n-1}\|P_L\|\|L\|$.

Using γ_* in (A.15), we have $\dot{V}_1 \leq -\frac{\theta_M \gamma_*}{2}\|e\|^2 - (\frac{\gamma_* \theta_M}{4}\|e\| - d_1|e_1| - d_2|z_1| - d_3\|z\|)\|e\|$. Thus, $\dot{V}_1$ is negative if $\|e\| \geq \frac{d_1|e_1| + d_2|z_1| + d_3\|z\|}{0.25\gamma_* \theta_M}$. Then, we have that $\|e\|$ is ultimately bounded by $d_1|e_1| + d_2|z_1| + d_3\|z\|$ from (A.21) [20]. Then, from (A.12), it is obvious that the ultimate bound of $\|e\|$ becomes to zero as $t \to \infty$. Thus, we have that $\|e\| \to 0$ and $\|\hat{x}\| \to 0$ as $t \to \infty$. Therefore, the global regulation is achieved.

A.3. Proof of Lemma 3

Using (A.11) in the proof of Lemma 2 and the convergence of $\varepsilon(t)$, it is obvious that $\gamma(t)$ and $\varepsilon(t)$ have finite positive upper bounds for all t. Since $\gamma(t)$ is nondecreasing, the upper bound of $\gamma(t)$ guarantees the convergence of $\gamma(t)$. Therefore, by using Lemma 2, the global regulation is achieved.

REFERENCES

[1] H. Lei and W. Lin, "Universal adaptive control of nonlinear systems with unknown growth rate by output feedback," *Automatica*, vol. 42, no. 10, pp. 1783-1789, 2006.

[2] H. Lei and W. Lin, "Adaptive regulation of uncertain nonlinear systems by output feedback: A universal control approach," *Systems and Control Letters*, vol. 56, no. 7-8, pp. 529-537, 2007.

[3] L. Liu and J. Huang, "Global robust output regulation of lower triangular systems with unknown control direction," *Automatica*, vol. 44, no. 5, pp. 1278-1284, 2008.

[4] L. Praly, "Asymptotic stabilization via output feedback for lower triangular systems with output dependent incremental rate," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 1103-1108, 2003.

[5] H.-L. Choi and J.-T. Lim, "Global exponential stabilization of a class of nonlinear systems by output feedback," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 255-257, 2005.

[6] M.-S. Koo, H.-L. Choi, and J.-T. Lim, "Global regulation of a class of uncertain nonlinear systems by switching adaptive controller," *IEEE Transactions on Automatic Control*, vol. 55, no. 12, pp. 2822-2827, 2010.

[7] T. Ahmed-Ali, V. Van Assche, J. Massieu, and P. Dorleans, "Continuous-discrete observer for state affine systems with sampled and delayed measurements," *IEEE Transactions on Automatic Control,* vol. 58, no. 4, pp. 1085-1091, 2013.

[8] C.-C. Chen, C. Qian, Z.-Y. Sun, and Y.-W. Liang, "Global output feedback stabilization of a class of nonlinear systems with unknown measurements sensitivity," *IEEE Transactions on Automatic Control*, vol. 63, no. 7, pp. 2212-2217, 2018.

[9] R. G. Sanfelice and L. Praly, "On the performance of high-gain observers with gain adaptation under measurement noise," *Automatica*, vol. 47, no. 10, pp. 2163-2176, 2011.

[10] L. K. Vasiljevic and H. K. Khalil, "Differentiation with high-gain observers the presence of measurement noise," *Proceedings of the 45th IEEE Conference on Desition and Control*, Sandiego, CA, December 13-15, 2006.

[11] H. Wang and Q. Zhu, "Global stabilization of stochastic nonlinear systems via C^1 and C^∞ controllers," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5800-5887, 2017

[12] J. Weston and M. Malisoff, "Sequential predictors under time-varying feedback and measurement delays and sampling," *IEEE Transactions on Automatic Control*, vol. 64, no. 7, pp. 2991- 2996, 2019.

[13] C. Zhang, C. Qian, S. Li, and H. Du, "Global robust stabilization via sampled-data output feedback for nonlinear systems with uncertain measurement and control gains," *Asian Journal of Control*, vol. 17, no. 3, pp. 868-878, 2015.

[14] D. Zhang and Y. Shen, "Global output feedback sampled-data stabilization for upper-triangular nonlinear systems with improved maximum allowable transmission delay," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 2, pp. 212-235, 2017

[15] J.-X. Zhang and G.-H. Yang, "Global finite-time output stabilization of nonlinear systems with unknown measurement sensitivity," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 2, pp. 5158-5172, 2018.

[16] D. Zhang, Y. Shen, and X. Xia, "Globally uniformly ultimately bounded observer design for a class of nonlinear systems with sampled and delayed measurements," *Kybernetika*, vol. 52, no. 3, pp. 441-460, 2016.

[17] D. S. Bernstein, "Sensor performance specification," *IEEE Control System Magazine,* vol. 21, no. 4, pp. 9-18, 2001.

[18] P. Singla, K. Subbarao, and J. L. Junkins, "Adaptive output feedback control for spacecraft rendezvous and docking under measurement uncertainty," *Journal of Guidance, Control, and Dynamics*, vol. 29, no. 4, pp. 892-902, 2006.

[19] P. Krishnamurthy, F. Khorrami, and Z. P. Jiang, "Global output feedback tracking for nonlinear systems in generalized output-feedback canonical form," *IEEE Transactions on Automatic Control*, vol. 40, no. 9, pp. 814-819, 2002.

[20] H. K. Khalil, *Nonlinear Systems*, 3rd ed., Prentice Hall, Upper Saddle Riber, NJ07458, 2002.

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