International Journal of Control, Automation and Systems 18(12) (2020) 3015-3022
http://dx.doi.org/10.1007/s12555-019-0643-y
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Trajectory Tracking Control for Nonholonomic Wheeled Mobile Robots with External Disturbances and Parameter Uncertainties

Hui Ye* and **Shuai Wang**

Abstract: Aiming at the trajectory tracking problem of nonholonomic wheeled mobile robots (WMR) with bounded external disturbances and parameter uncertainties, a dual-loop attitude tracking robust controller is proposed for WMR. Firstly, a kinematic controller is designed to generate the virtual velocity based on the kinematic error model. Secondly, the sliding mode control with a modified reaching law is adopted to ensure the actual velocity can converge to the virtual velocity in finite time based on the dynamic model. Finally, the stability of the controller is verified through the Lyapunov function. Numerical simulation shows the robustness and effectiveness of the proposed dual-loop tracking controller.

Keywords: Adaptive sliding mode control, Lyapunov stability, nonholonomic wheeled mobile robot.

1. INTRODUCTION

In recent years, with the continuous prosperity of the world economy, the mobile robot has been increasingly used in different industrial processes. The design of high-performance control method is regarded as a challenging task because the mobile robot is a typical nonholonomic system. In addition, the wheeled mobile robot has unknown internal parameter perturbation and unknown external interference while moving in the real environment. In order to solve these problems, various control methods have been deeply studied, including backstepping approach [1], input-output linearization [2], sliding mode control [3–5] and intelligent control [6–9]. As for unknown nonlinear terms, [10, 11] proposed a homogenous high-gain observer with two novel dynamic gains to estimate the system states. Based on the fact that the linearized system is uniformly completely controllable along the desired reference trajectory, the work [12] develops a linearization-based tracking controller for nonholonomic constrained systems. A sliding mode control method for mobile robots based on two-dimensional polar kinematics is proposed in [3], which eliminates the conditional constraints on the desired linear velocity, angular velocity and the attitude of the mobile robot. Since these studies are based on kinematic models under polar coordinates, there are singularities at the origin of polar coordinates. To solve this problem, a kinematics model in Cartesian coordinates was presented in [13]. In [14], the work has proposed a classical kinematics stability tracking control approach for nonholonomic robot based on Lyapunov method.

There are two main methods to eliminate the uncertainty of lumped disturbance. One method is the disturbance observer technology. The work [15] gives the general framework of perturbed nonlinear systems by using disturbance observer based control technique. A semi-global stability condition of a composite controller composed of a nonlinear controller and a nonlinear disturbance observer is established. The work [16] has proposed a new observer-based tracking strategy for leader follower formation and verified the effectiveness, robustness and applicability in different case studies. A new scheme of designing adaptive virtual speed controller and torque control law is presented for the robot with external disturbances and unknown parameters in [17]. Meanwhile, the disturbance observer is used to estimate the lumped disturbance to realize feedforward compensation. [18] discussed the cascaded time-varying system consisting of two uniformly finite-time stable subsystems.

In actual applications, it is difficult to obtain the upper bound of lumped disturbance precisely. Therefore, a controller is usually designed to suppress lumped disturbances by selecting enough switching gains. In [19], a dynamic adaptive control rule is proposed for nonholonomic mobile robot with unknown dynamic parameters. The adaptive control method of mobile robot is derived by using backstepping technology. Considering the problem of global tracking and stabilization of an internally damped mobile robot with unknown parameters and external disturbance of input torque saturation, the work [20] has pro-

Manuscript received August 9, 2019; revised November 26, 2019 and Februry 7, 2020; accepted February 25, 2020. Recommended by Associate Editor Quoc Chi Nguyen under the direction of Editor Myo Taeg Lim.

Hui Ye is with the School of Science, Jiangsu University of Science and Technology, Zhenjiang, 212000, China (e-mail: yehui@just.edu.cn). Shuai Wang is with the School of Automation, Southeast University, Nanjing, Jiangsu 210096, China (e-mail: swang1115@163.com).
* Corresponding author.

posed a new adaptive scheme. The work [21] designed an adaptive super-torsion algorithm to track the predetermined trajectory which ignores chattering. To overcome this limitation, the terminal sliding mode [22, 23] control approach has been proposed in several control applications. A design scheme of terminal sliding mode control for uncertain dynamic systems with pure feedback was presented in [22]. Based on the fast finite time control algorithm, the work [23] has constructed a fast finite time state feedback controller and a fast finite time observer. In the literature of [24], the output feedback sliding mode control (SMC) problem for discrete-time uncertain nonlinear systems through T-S fuzzy dynamic models was addressed. An adaptive sliding-mode unit vector control approach based on monitoring functions to deal with disturbances of unknown bounds was proposed in [25]. The work [26] proposed a novel time shift approach for actuator fault reconstruction of systems with output time-delay based on a sliding mode observer. Adaptive high-gain stabilizers for a class of linear time-invariant state space systems were presented in [27]. However, there always suffer from such issues as input saturation, input dead-zone, unidirectional input constraints, etc. The work [28] gave the first continuous control solution for pneumatic artificial muscle systems that can simultaneously compensate parametric uncertainties, reject external disturbances, and meet unidirectional constraints. The work [29] proposed adaptive law availably to compensate parameter/structure uncertainties for ship-mounted crane systems.

Neural networks have the ability to approach strongly nonlinear and complex systems when the unmodeled parameter uncertainties and nonparametric uncertainty of the nonholonomic wheeled mobile robot are considered. A nonholonomic kinematics controller for mobile robots and a neural network controller for calculating torque are proposed in [6]. The work [7] developed an adaptive neural sliding mode controller for nonholonomic wheeled mobile robots with model uncertainties and external disturbances. The input torque of the robot can be extended to the pseudo dead zone in [9]. The work [30] has proposed a new adaptive tracking controller based on neural network, where neural network is used to compensate the uncertainty caused by wheel slip and external force in order to achieve the expected tracking performance. The main contributions of this paper are summarized: (i) We design a novel adaptive sliding mode controller based on state feedback by selecting the switching function of WMR; (ii) Without explicit knowledge of parameter uncertainties and disturbances, we also can deal with it by the sliding mode that we designed to estimate the bound of it; (iii) The new adaptive sliding mode controller has the characteristics of simple algorithm, fast response, easy realization and strong anti-interference.

2. ROBOT MODEL AND PROBLEM FORMULATION

There are one front castor wheel and two driving wheels in the WMR (Fig. 1). The castor wheel prevents the robot from tipping over as it moves on a horizontal plane. The two driving wheels are independently driven by two actuators(e.g. DC motors) for the motion and orientation. Two wheels have the same radius denoted by r and two driving wheels are separated by $2b$. The robot is in the fixed coordinate system $\{X, O, Y\}$ and θ is the angle between the direction of motion of the robot and the X axis, which denotes the orientation of the robot frame with respect to the Cartesian frame. The trajectory tracking problem of a nonholonomic mobile robot can be reduced to that the robot reaches and tracks a given reference trajectory from the initial position in the inertial coordinate system.

Via Euler-Lagrangian formulation, the robot dynamics can be obtained as [6]

$$M(q)\ddot{q}+C(q,\dot{q})+F(\dot{q})+\tau_d=B(q)\tau-A(q)^T\lambda, \quad (1)$$

where $q=[x,y,\theta]^T$ is the robot posture. $\tau=[\tau_1,\tau_2]^T$ is the control torque, $M(q)$ is a symmetric positive definite inertia matrix, $C(q,\dot{q})$ is the centripetal Coriolis matrix, $B(q)$ is the input transformation matrix, $A(q)$ is the matrix associated with nonholonomic constraints, τ_d denotes the unknown disturbance, $F(\dot{q})$ denotes the surface friction while λ is the Lagrange multiplier of constraint forces.

For the sake of simplicity, define the variables as [5]:

$$M(q)=\begin{bmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I\end{bmatrix},\ A^T(q)=\begin{bmatrix} -\sin\theta\\ \cos\theta\\ 0\end{bmatrix},$$

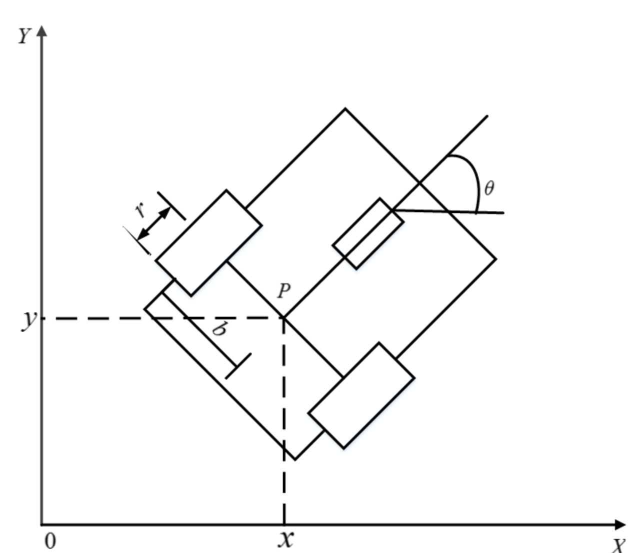

Fig. 1. Motion model of wheeled mobile robot.

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ b & -b \end{bmatrix}. \tag{2}$$

When its motion satisfies the condition of pure rolling without slipping, the robot motion has the following speed constraint

$$-\dot{x}\sin\theta + \dot{y}\cos\theta = 0. \tag{3}$$

The kinematic model of the nonholonomic WMR under the situation of pure rolling and non-slipping is given as [14]

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = J\mu, \ \ t \geq 0, \tag{4}$$

where $\mu = [\upsilon, \omega]^T$ are the linear and angular velocities of the mobile robot.

Let $[x_r, y_r, \theta_r]^T$ represent the desired posture of the WMR, and its motion must also satisfy nonholonomic constraints. The reference trajectory is described by the following equation:

$$\dot{q}_r = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon_r \\ \omega_r \end{bmatrix}, \ \ t \geq 0, \tag{5}$$

where $[\upsilon_r, \omega_r]^T$ denotes the desired linear and angular velocities of the WMR.

According to the geometric relationship, the posture tracking error of the mobile robot is defined as [20]

$$q_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \tag{6}$$

Then, by a direct calculation, the derivative of the posture tracking error can be expressed as [17]

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e\omega + \upsilon_r\cos\theta_e - \upsilon \\ -x_e\omega + \upsilon_r\sin\theta_e \\ \omega_r - \omega \end{bmatrix}. \tag{7}$$

Under the new state variable $[x_e, y_e, \theta_e]^T$, the trajectory tracking problem of nonholonomic mobile robot (4) is transformed into the stabilization problem of tracking error model (7). In other words, the controller which stabilizes the nonlinear error is designed to make the tracking error tend to zero.

Under the new state variable $[x_e, y_e, \theta_e]^T$, the trajectory tracking problem of nonholonomic mobile robot (4) is transformed into the stabilization problem of tracking error model (7). In other words, the controller which stabilizes the nonlinear error is designed to make the tracking error tend to zero.

Taking time derivatives of (4) results in

$$\ddot{q} = \dot{J}(\theta)\mu(t) + J(\theta)\dot{\mu}(t). \tag{8}$$

By substituting (8) into (1), it yields

$$M(q)J(q)\dot{\mu} + M(q)\dot{J}(q)\mu + \tau_d = B(q)\tau - A(q)^T\lambda. \tag{9}$$

Multiplying both sides by $J(q)^T$, because $J(q)^T A(q)^T = 0$, $J(q)^T M(q)\dot{J}(q)\mu = 0$, it yields

$$\bar{M}\dot{\mu} + \bar{\tau}_d = \bar{B}\tau, \tag{10}$$

where $\bar{M} = J^T MJ$, $\bar{\tau}_d = J^T\tau_d$, $\bar{B} = J^T B$.

Assuming that the parameters of the mobile robot are uncertain, from (10) we can get

$$(\bar{M} + \Delta\bar{M})\dot{\mu} + \bar{\tau}_d = \bar{B}\tau, \tag{11}$$

where $\Delta\bar{M} = J^T \Delta MJ$ system parameter perturbation.

From (11), one has

$$\begin{aligned} \dot{\mu} &= \bar{M}^{-1}\bar{B}\tau - \bar{M}^{-1}(\Delta\bar{M}\dot{\mu} + \bar{\tau}_d) \\ &= G\tau - d(q,\dot{q},\ddot{q}), \end{aligned} \tag{12}$$

where $d(q,\dot{q},\ddot{q}) = \bar{M}^{-1}(\Delta\bar{M}\dot{\mu} + \tau_d)$ represents lumped disturbance including the friction, external interference and parameter disturbance.

3. SLIDING MODE TRACKING CONTROLLER DESIGN

Our control objective is to design torque controller τ for the WMR to make the real trajectory track the desired one. To achieve this objective, some assumptions are made.

Assumption 1: The reference linear and angular velocities υ_r, ω_r and their first-order derivatives $\dot{\upsilon}_r$, $\dot{\omega}_r$ are bounded and continuous.

Assumption 2: The lumped disturbance $d(q,\dot{q},\ddot{q})$ is bounded [31]

$$\|d(q,\dot{q},\ddot{q})\| < \xi_0 + \xi_1\|q\| + \xi_2\|\dot{q}\|^2, \tag{13}$$

where ξ_0, ξ_1, ξ_2 are unknown constants.

Remark 1: According to the actual motion of the mobile robot, the above assumptions are reasonable shown as [32, 33]. In addition, because d is limited by external interference, its exact value does not need to be known.

Lemma 1 [34]**:** Consider nonlinear systems:

$$\dot{x}(t) = f(t,x), \ x(0) = x_0, \ \ x \in R^n, \tag{14}$$

where $f(t,x)$ is a continuous function, if there exists a C^1 positive definite and proper function $V : R^n \to R$, and real number $k > 0$ and $\sigma \in (0,1)$ such that $\dot{V}(x) + kV^\sigma$ is negative semi-definite. Then, the origin of system (14) is a global finite-time stable equilibrium.

Choosing the virtual feedback $\bar{\theta}_e = \theta_e - \delta$ with $\delta = -\arctan(v_r y_e)$, the kinematic controller for tracking the reference trajectory is designed as

$$\begin{cases} \upsilon_c = \upsilon_r \cos\theta_e + \dfrac{\partial\delta}{\partial y_e}\sin(\dfrac{\bar{\theta}_e}{2})\omega_c + k_1 x_e, \\ \omega_c = 2y_e v_r \cos(\delta + \dfrac{\bar{\theta}_e}{2}) - \dfrac{\partial\delta}{\partial\upsilon_r}\dot{\upsilon}_r - \dfrac{\partial\delta}{\partial y_e}\upsilon_r\sin\theta_e \\ \qquad + k_2\sin(\dfrac{\bar{\theta}_e}{2}) + \omega_r, \end{cases} \tag{15}$$

where υ_c and ω_c are the linear and angular velocities, respectively. k_1, k_2 are positive constants.

Without considering the parameter uncertainties and external disturbance, (12) becomes

$$\dot{\mu} = \bar{M}^{-1}\bar{B}\tau = G\tau. \tag{16}$$

Define the velocity tracking error as

$$\mu_e = [\upsilon_e, \omega_e]^T = \mu_c - \mu. \tag{17}$$

Next, we choose the sliding mode surface as

$$\begin{aligned} s =& \mu_e(t) - \mu_e(0) + \beta_1\int_0^t sig^{\varepsilon_1}(\mu_e)dt \\ &+ \beta_2\int_0^t sig^{\varepsilon_2}(\mu_e)dt, \end{aligned} \tag{18}$$

where $s = [s_1, s_2]^T$, $\beta_i > 0$, $0 < \varepsilon_i < 1$, $i = 1, 2$ are constants. $sig^{\varepsilon_i}(\mu_e) = [|\upsilon_e|^{\varepsilon_i} sign(\upsilon_e), |\omega_e|^{\varepsilon_i} sign(\omega_e)]^T$.

Substituting (16), (17) into (18), one has

$$\begin{aligned} \dot{s} &= \dot{\mu}_e + \beta_1 sig^{\varepsilon_1}(\mu_e) + \beta_2 sig^{\varepsilon_2}(\mu_e) \\ &= (\dot{\mu}_c - G\tau) + \beta_1 sig^{\varepsilon_1}(\mu_e) + \beta_2 sig^{\varepsilon_2}(\mu_e). \end{aligned} \tag{19}$$

According to (19), letting $\dot{s} = 0$, we can obtain the nominal control law τ_{eq}:

$$\tau_{eq} = G^{-1}\big(\dot{\mu}_c + \beta_1 sig^{\varepsilon_1}(\mu_e) + \beta_2 sig^{\varepsilon_2}(\mu_e)\big). \tag{20}$$

Then, with considering the parameter uncertainties and disturbance, the discontinuous control law τ_{sw} is introduced

$$\begin{aligned} \tau_{sw} =& G^{-1}\big(\sigma_1\|s\|^{\eta_1} sgn(s) + \sigma_2\|s\|^{\eta_2} s \\ &+ \hat{d}(q,\dot{q},\ddot{q})sgn(s)\big), \end{aligned} \tag{21}$$

where $sgn(s) = [sign(s_1), sign(s_2)]^T$, $\sigma_i > 0$, $0 < \eta_i < 1$ $(i = 1, 2)$. $\hat{d}(q,\dot{q},\ddot{q}) = \hat{\xi}_0 + \hat{\xi}_1\|q\| + \hat{\xi}_2\|\dot{q}\|^2$ is the estimate of $d(q,\dot{q},\ddot{q})$, whose adaptive rate is chosen as [31]

$$\dot{\hat{\xi}}_0 = \|s\|, \quad \dot{\hat{\xi}}_1 = \|s\|\|q\|, \quad \dot{\hat{\xi}}_2 = \|s\|\|\dot{q}\|^2. \tag{22}$$

Therefore, the torque control law τ is designed as

$$\begin{aligned} \tau = & G^{-1}\big(\beta_1 sig^{\varepsilon_1}(\mu_e) + \beta_2 sig^{\varepsilon_2}(\mu_e) + \sigma_1\|s\|^{\eta_1} sgn(s) \\ &+ \sigma_2\|s\|^{\eta_2} s + \dot{\mu}_c + \hat{d}(q,\dot{q},\ddot{q})\cdot sgn(s)\big). \end{aligned} \tag{23}$$

4. STABILITY ANALYSIS

In what follows, we will prove the stability of the system in two parts: Firstly, we will prove the stability of the dynamic model of the inner loop system, then the stability of the kinematic model of the outer loop system will be proved. The block diagram of the closed-loop system is shown in Fig. 2.

Theorem 1: In the presence of parametric uncertainties and external disturbances, the designed adaptive law (22) and the torque controller (23) for system (1) can make the tracking error of WMR can converge to zero in finite time.

Proof: Choose the Lyapunov function as

$$V_1 = \frac{1}{2}(s^T s + \tilde{\xi}_0^2 + \tilde{\xi}_1^2 + \tilde{\xi}_2^2), \tag{24}$$

where $\tilde{\xi}_i = \hat{\xi}_i - \xi_i$, $i = 0, 1, 2$.

The time derivative of (24) becomes

$$\begin{aligned} \dot{V}_1 =& s^T\dot{s} + (\hat{\xi}_0 - \xi_0)\dot{\hat{\xi}}_0 + (\hat{\xi}_1 - \xi_1)\dot{\hat{\xi}}_1 + (\hat{\xi}_2 - \xi_2)\dot{\hat{\xi}}_2 \\ =& s^T\big(-\sigma_1\|s\|^{\eta_1} sgn(s) - \sigma_2\|s\|^{\eta_2} s - \hat{d}(q,\dot{q},\ddot{q})sgn(s) \end{aligned}$$

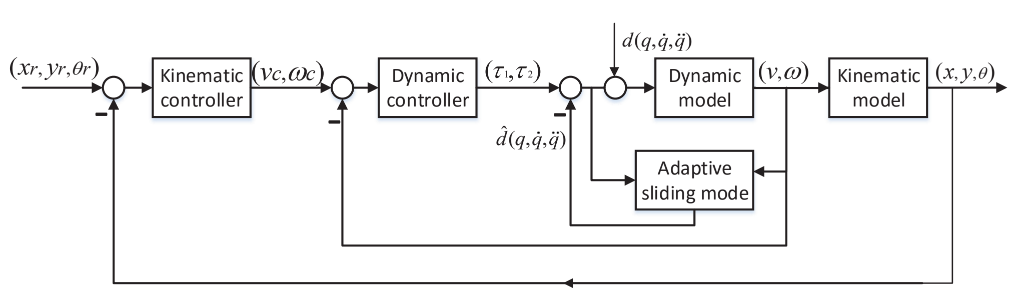

Fig. 2. The block diagram of the closed-loop system.

$$
\begin{aligned}
&+d(q,\dot{q},\ddot{q})\big)+(\hat{\xi}_0-\xi_0)\dot{\hat{\xi}}_0+(\hat{\xi}_1-\xi_1)\dot{\hat{\xi}}_1\\
&+(\hat{\xi}_2-\xi_2)\dot{\hat{\xi}}_2\\
\leq&-\sigma_1\|s\|^{\eta_1+1}-\sigma_2\|s\|^{\eta_2+2}-s^T\|\hat{d}(q,\dot{q},\ddot{q})\|sgn(s)\\
&+s^T\|d(q,\dot{q},\ddot{q})\|+(\xi_0+\xi_1\|q\|+\xi_2\|\dot{q}\|^2)\|s\|\\
&-(\xi_0+\xi_1\|q\|+\xi_2\|\dot{q}\|^2)\|s\|+(\hat{\xi}_0-\xi_0)\|s\|\\
&+(\hat{\xi}_1-\xi_1)\|s\|\|q\|+(\hat{\xi}_2-\xi_2)\|s\|\|\dot{q}\|^2\\
\leq&\|d(q,\dot{q},\ddot{q})\|\|s\|-(\hat{\xi}_0+\hat{\xi}_1\|q\|+\hat{\xi}_2\|\dot{q}\|^2)\|s\|\\
&+(\xi_0+\xi_1\|q\|+\xi_2\|\dot{q}\|^2)\|s\|-(\xi_0+\xi_1\|q\|\\
&+\xi_2\|\dot{q}\|^2)\|s\|+(\hat{\xi}_0-\xi_0)\|s\|+(\hat{\xi}_1-\xi_1)\|s\|\|q\|\\
&+(\hat{\xi}_2-\xi_2)\|s\|\|\dot{q}\|^2\leq-\kappa\|s\|,
\end{aligned}
\tag{25}
$$

where $\kappa=(\xi_0+\xi_1\|q\|+\xi_2\|\dot{q}\|^2)-\|d(q,\dot{q},\ddot{q})\|>0$. According to Assumption (2), torque control law (23) and adaptive laws (22), one has $\tilde{\xi}_i$ and s are bounded.

Choose the Lyapunov function as $V_0=\frac{1}{2}s^Ts$. According to (25), one has

$$
\begin{aligned}
\dot{V}_0=&\dot{V}_1+(\xi_0-\hat{\xi}_0)\dot{\hat{\xi}}_0+(\xi_1-\hat{\xi}_1)\dot{\hat{\xi}}_1+(\xi_2-\hat{\xi}_2)\dot{\hat{\xi}}_2\\
\leq&-\kappa\|s\|+(\xi_0-\hat{\xi}_0)\dot{\hat{\xi}}_0+(\xi_1-\hat{\xi}_1)\dot{\hat{\xi}}_1\\
&+(\xi_2-\hat{\xi}_2)\dot{\hat{\xi}}_2.
\end{aligned}
\tag{26}
$$

In what follows, we will discuss the accessibility of the sliding mode s in two cases:

Case 1: If $\hat{\xi}_i(0)<\xi_i, i=0,1,2$, then it is obviously that $\dot{\hat{\xi}}_i\geq 0$. Firstly, assuming a finite time $T_1\geq 0$ that makes all $\hat{\xi}_i(t)\geq\xi_i, t\geq T_1$, which can conclude that $\dot{V}_0(t)<-\kappa\|s\|$. Otherwise, there will be a finite time T_2 makes at least one of its adaptive estimates satisfy $\dot{\hat{\xi}}_i=0$ and $\hat{\xi}_i(t)<\xi_i, t\geq T_2$. In general, suppose $\dot{\hat{\xi}}_2(t)=0$ and $\hat{\xi}_2<\xi_2, t\geq T_2$, we can get that $\dot{V}_0(t)\leq-\kappa\|s\|+(\xi_0-\hat{\xi}_0)\dot{\hat{\xi}}_0+(\xi_1-\hat{\xi}_1)\dot{\hat{\xi}}_1+(\xi_2-\hat{\xi}_2)\dot{\hat{\xi}}_2, t\geq T_2$. As for $\hat{\xi}_0,\hat{\xi}_1$, we can get the same conclusion. Therefore, there will always be a limited time to make $\dot{V}_0<-\kappa\|s\|$.

Case 2: While at least one $\hat{\xi}_i(0)$ satisfy $\hat{\xi}_i(0)\geq\xi_1$, we can get $(\xi_i-\hat{\xi}_i)\dot{\hat{\xi}}_i\leq 0$. As for $\hat{\xi}_j(0)<\xi_j, j\neq i$, the analysis process is the same as Case1, thus, s will stable in limited time.

Based on the above analysis, it is not hard to find that for any initial condition $\big(s(0),\hat{\xi}_i(0)\big)$, there always be a positive constant κ, such that

$$
\dot{V}_0(t)\leq-\kappa\|s\|\leq-\sqrt{2}\kappa V_0^{\frac{1}{2}}.
\tag{27}
$$

By Lemma 1, it can be obtained that the sliding mode s will reach zero in a finite time. □

Theorem 2: For system (4), the posture of tracking error (7) is globally asymptotically converge to zero if the control law (15) is applied.

Proof: The Lyapunov function candidate is chosen as

$$
V_2=\frac{1}{2}x_e^2+\frac{1}{2}y_e^2+2(1-\cos\frac{\bar{\theta}_e}{2}).
\tag{28}
$$

The time derivative of V_2 is

$$
\begin{aligned}
\dot{V}_2=&x_e\dot{x}_e+y_e\dot{y}_e+\dot{\bar{\theta}}_e\sin\frac{\bar{\theta}_e}{2}\\
=&x_e(\omega y_e-\upsilon+\upsilon_r\cos\theta_e)+y_e(-\omega x_e+\upsilon_r\sin\theta_e)\\
&+\sin(\frac{\bar{\theta}_e}{2})\big(\omega_r-\omega-\frac{\partial\delta}{\partial\upsilon_r}\dot{\upsilon}_r\\
&-\frac{\partial\delta}{\partial y_e}(-\omega x_e+\upsilon_r\sin\theta_e)\big)\\
=&x_e(-\upsilon+\upsilon_r\cos\theta_e+\frac{\partial\delta}{\partial y_e}\omega\sin(\frac{\bar{\theta}_e}{2}))+y_e\upsilon_r\sin\delta\\
&+2y_e\upsilon_r\sin(\frac{\bar{\theta}_e}{2})\cos(\frac{\bar{\theta}_e}{2}+\delta)+\sin(\frac{\bar{\theta}_e}{2})(\omega_r\\
&-\omega-\frac{\partial\delta}{\partial\upsilon_r}\dot{\upsilon}_r-\frac{\partial\delta}{\partial y_e}\upsilon_r\sin\theta_e).
\end{aligned}
\tag{29}
$$

Putting (15) into (29) yields

$$
\begin{aligned}
\dot{V}_2=&x_e(-\upsilon_r\cos\theta_e-\frac{\partial\delta}{\partial y_e}\omega\sin(\frac{\bar{\theta}_e}{2})-k_1x_e+\upsilon_r\cos\theta_e\\
&+\frac{\partial\delta}{\partial y_e}\omega\sin\frac{\bar{\theta}_e}{2})+2y_e\upsilon_r\sin\frac{\bar{\theta}_e}{2}\cos(\frac{\bar{\theta}_e}{2}+\delta)\\
&+y_e\upsilon_r\sin\delta+\sin\frac{\bar{\theta}_e}{2}\big(\omega_r-2y_e\upsilon_r\cos(\delta+\frac{\bar{\theta}_e}{2})\\
&+\frac{\partial\delta}{\partial\upsilon_r}\dot{\upsilon}_r+\frac{\partial\delta}{\partial y_e}\upsilon_r\sin\theta_e-k_2\sin(\frac{\bar{\theta}_e}{2})-\omega_r\\
&-\frac{\partial\delta}{\partial\upsilon_r}\dot{\upsilon}_r-\frac{\partial\delta}{\partial y_e}\upsilon_r\sin\theta_e\big)\\
=&-k_1x_e^2-y_e\upsilon_r\sin\big(\arctan(y_e\upsilon_r)\big)-k_2\sin^2\frac{\bar{\theta}_e}{2}.
\end{aligned}
\tag{30}
$$

Based on the Lyapunov stability theorem, it can be concluded that $\lim_{t\to\infty}x_e=0$, $\lim_{t\to\infty}y_e=0$, $\lim_{t\to\infty}\bar{\theta}_e=0$.□

Remark 2: Due to the noise and switch delay, $\|\hat{\xi}_i\|$ maybe rise all the time. Motivated by [35], using the dead zone method, the adaptive rate can be adjusted as

$$
\dot{\hat{\xi}}_0=\begin{cases}\|s\|, & \text{if } \|s\|\geq\iota,\\ 0, & \text{if } \|s\|<\iota,\end{cases}
\tag{31}
$$

$$
\dot{\hat{\xi}}_1=\begin{cases}\|s\|\|q\|, & \text{if } \|s\|\geq\iota,\\ 0, & \text{if } \|s\|<\iota,\end{cases}
\tag{32}
$$

$$
\dot{\hat{\xi}}_2=\begin{cases}\|s\|\|\dot{q}\|^2, & \text{if } \|s\|\geq\iota,\\ 0, & \text{if } \|s\|<\iota,\end{cases}
\tag{33}
$$

where ι is a small positive constant.

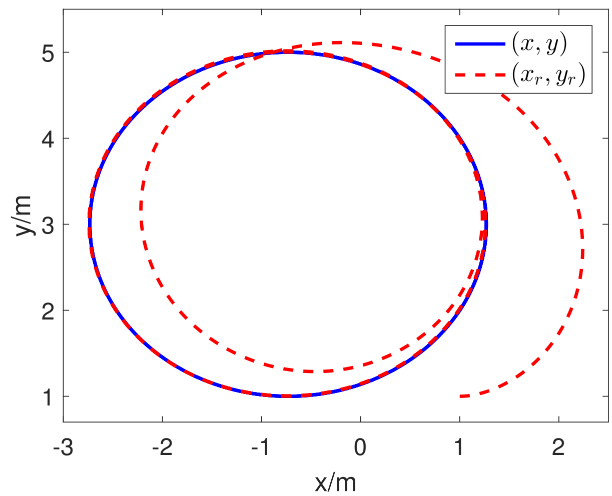

Fig. 3. Circular track tracking.

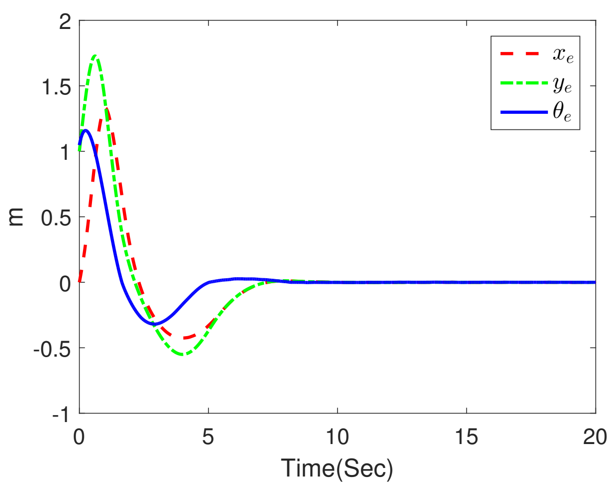

Fig. 4. Circular trajectory tracking error.

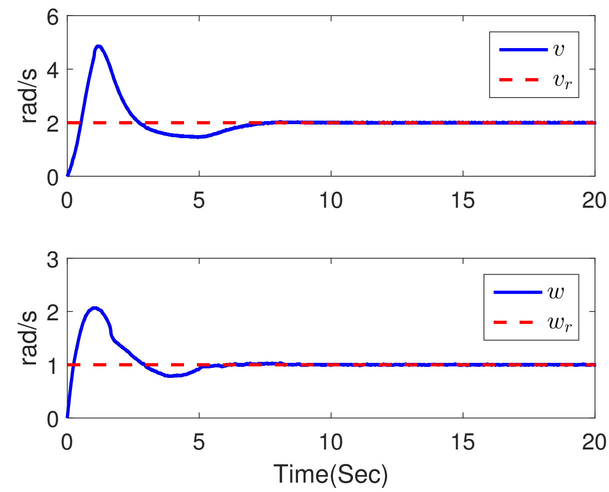

Fig. 5. Linear and angular velocity.

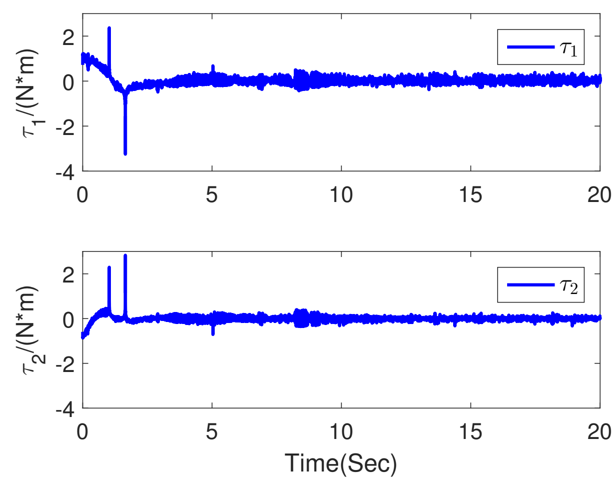

Fig. 6. Adaptive control torque.

5. SIMULATION RESULT

This section illustrates the performance of the proposed controller in the presence of uncertainties by using Matlab Simulink. The physical parameters of robots are set as: $b = 0.25$ m, $I = 2.5$ kg$\cdot$m^2, $m = 4$ kg. The initial position is set as $(x(0), y(0), \theta(0)) = (1, 1, \frac{\pi}{6})$, and the initial position and orientation of the reference input is $(x_r(0), y_r(0), \theta_r(0)) = (1, 2, \frac{\pi}{3})$. The reference velocities are set as $v_r = 2$ m/s, $\omega_r = 1$ rad/s. The parameters in the controller (15) are chosen as $k_1 = k_2 = 1$. The controller (23) parameters have been tuned to the following values: $\beta_1 = 5$, $\beta_2 = 1$, $\sigma_1 = 2$, $\sigma_2 = 1$, $\varepsilon_1 = \frac{4}{3}$, $\varepsilon_2 = \frac{1}{2}$, $\eta_1 = 1.5$, $\eta_2 = 1.1$.

The simulation results are shown in the figures. Fig. 3 shows the effect of tracking a circular trajectory. Fig. 4 indicates the circular trajectory tracking error, where x_e, y_e, θ_e converge to zero asymptotically. Fig. 5 shows that the actual linear velocity and angular velocity of the mobile robot has been successfully tracked with the desired linear velocity. The adaptive tracking torques under additional disturbance torque in Fig. 6. Fig. 7 shows the disturbance. It meets the control requirements of trajectory tracking for wheeled mobile robots. Fig. 8 shows the adaptive gains, it can be seen that the adaptive estimation $\hat{\xi}_0, \hat{\xi}_1, \hat{\xi}_2$ can reach a stable state in a finite time.

6. CONCLUSION

In this paper, aiming at the trajectory tracking control problem of WMR, a new adaptive sliding mode controller is designed to deal with external disturbances and uncertainties of system parameters. The stability of the proposed controller is analyzed by Lyapunov theory, and the robustness and effectiveness of the proposed controller are verified by numerical simulations. In the future work, we will discuss the problem of input saturation and time-delay for WMR.

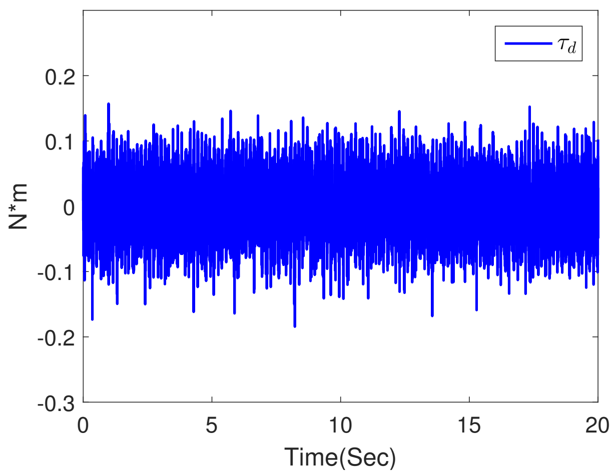

Fig. 7. Disturbance.

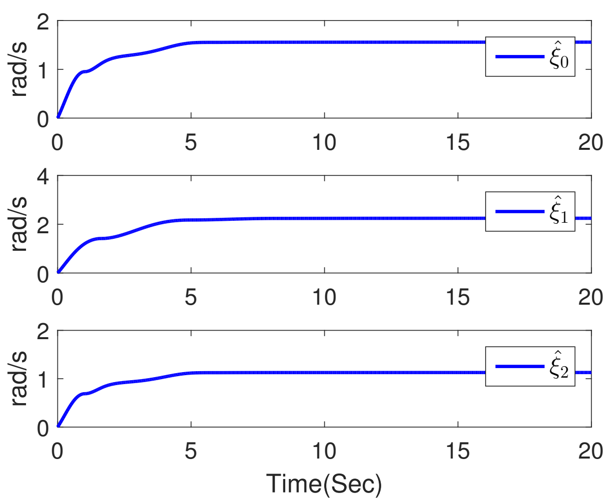

Fig. 8. Adaptive gains.

REFERENCES

[1] Z. Jiang and H. Nijmeijer, "Tracking control of mobile robots: a case study in backstepping," *Automatica*, vol. 33, no. 7, pp. 1393-1399, February, 1997.

[2] D. Kim and T. Oh, "Tracking control of a two-wheeled mobile robot using input-output linearization," *Control Engineering Practice*, vol. 7, no. 3, pp. 369-373, July, 1999.

[3] D. Chwa, "Sliding-mode control of nonholonomic wheeled mobile robots in polar coordinates," *IEEE Trans. on Control Systems Technology*, vol. 12, no. 4, pp. 637-644, July, 2004.

[4] O. Mofid and S. Mobayen, "Adaptive sliding mode control for finite-time stability of quad-rotor UAVs with parametric uncertainties," *ISA Trans.*, vol. 72, pp. 1-14, January, 2018.

[5] J. Zhai and Z. Song, "Adaptive sliding mode trajectory tracking control for wheeled mobile robots," *International Journal of Control*, vol. 92, no. 10, pp. 2255-2262, October, 2019.

[6] R. Fierro and F. Lewis, "Control of a nonholonomic mobile robot using neural networks," *IEEE Trans. on Neural Networks*, vol. 9, no. 4, pp. 589-600, April, 1998.

[7] B. Park, S. Yoo, J. Park, Y. Choi, "Adaptive neural sliding mode control of nonholonomic wheeled mobile robots with model uncertainty," *IEEE Trans. on Control Systems Technology*, vol. 17, no. 1, pp. 207-214, January, 2009.

[8] H. Du, J. Zhai, M. Chen, and W. Zhu, "Robustness analysis of a continuous higher order finite-time control system under sampled-data control," *IEEE Trans. on Automatic Control*, vol. 64, no. 6, pp. 2488-2494, June, 2019.

[9] S. Li, L. Ding, H. Gao, C. Chen, Z. Liu, and Z. Deng, "Adaptive neural network tracking control-based reinforcement learning for wheeled mobile robots with skidding and slipping," *Neurocomputing*, vol. 283, pp. 20-30, December, 2017.

[10] J. Y. Zhai and H. R. Karimi, "Universal adaptive control for uncertain nonlinear systems via output feedback," *Information Sciences*, vol. 500, pp. 140-155, October, 2019.

[11] J. Y. Zhai and H. R. Karimi, "Global output feedback control for a class of nonlinear systems with unknown homogenous growth condition," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 7, pp. 2082-2095, May, 2019.

[12] G. Walsh, D. Tilbury, S. Sastry, R. Murray, and J. Laumond, "Stabilization of trajectories for systems with nonholonomic constraints," *IEEE Trans. on Automatic Control*, vol. 39, no. 1, pp. 216-222, January, 1994.

[13] S. Lee, Y. Cho, M. Hwang, B. You, and S. Oh, "A stable target-tracking control for unicycle mobile robots," *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 1822-1827, March, 2000.

[14] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for a non-holonomic mobile robot," *Proceedings of IEEE/RSJ International Workshop on Intelligent Robots and Systems*, pp. 1236-1241, November, 1991.

[15] W. Chen, "Disturbance observer based control for nonlinear systems," *IEEE/ASME Trans. on Mechatronics*, vol. 9, no. 4, pp. 706-710, April, 2004.

[16] M. Hassana, E. Aljuwaiser, and R. Badr, "A new online observer-based controller for leader-follower formation of multiple nonholonomic mobile robots," *Journal of the Franklin Institute*, vol. 355, no. 5, pp. 2436-2472, February, 2018.

[17] D. Huang, J. Zhai, W. Ai, and S. Fei, "Disturbance observer-based robust control for trajectory tracking of wheeled mobile robots," *Neurocomputing*, vol. 198, pp. 74-79, March, 2016.

[18] S. Li and Y. Tian, "Finite-time stability of cascaded time-varying systems," *International Journal of Control*, vol. 80, no. 4, pp. 646-657, April, 2007.

[19] F. Pourboghrat and M. Karlsson, "Adaptive control of dynamic mobile robots with nonholonomic constraints," *Computers and Electrical Engineering*, vol. 28, no. 4, pp. 241-253, April, 2002.

[20] J. Huang, C. Wen, W. Wang, and Z. Jiang, "Adaptive stabilization and tracking control of a nonholonomic mobile robot with input saturation and disturbance," *System and Control Letters*, vol. 62, no. 3, pp. 234-241, January, 2013.

[21] I. Matraji, A. Durra, A. Haryono, K. Wahedi, and M. Khousa, "Trajectory tracking control of skid-steered mobile robot based on adaptive second order sliding mode control," *Control Engineering Practice*, vol. 72, pp. 167-176, March, 2018.

[22] Y. Wu, X. Yu, and Z. Man, "Terminal sliding mode control design for uncertain dynamic systems," *System and Control Letters*, vol. 34, no. 5, pp. 281-287, March, 1998.

[23] D. Wu, Y. Cheng, H. Du, W. Zhu, and M. Zhu, "Finite-time output feedback tracking control for a nonholonomic wheeled mobile robot," *Aerospace Science and Technology*, vol. 78, pp. 574-579, July, 2018.

[24] W. Ji, J. Qiu, and H. Karimi, "Fuzzy-model-based output feedback sliding mode control for discrete-time uncertain nonlinear systems," *IEEE Trans. on Fuzzy Systems*, 2019. DOI: 10.1109/TFUZZ.2019.2917127

[25] L. Hsu, T. Oliveira, J. Cunha, and L. Yan, "Adaptive unit vector control of multivariable systems using monitoring functions," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 3, pp. 583-600, February, 2019.

[26] P. Pinto, R. Oliveira, and L. Hsu, "Sliding mode observer for fault reconstruction of time-delay and sampled-output systems - a Time Shift Approach," *Automatica*, vol. 106, pp. 390-400, May, 2019.

[27] A. Ilchmann and D. Owens, "Adaptive stabilization with exponential decay," *Systems and Control Letters*, vol. 14, no. 5, pp. 437-443, January, 1990.

[28] N. Sun, D. Liang, Y. Wu, Y. Chen, Y. Qin, and Y. Fang, "Adaptive control for pneumatic artificial muscle systems with parametric uncertainties and unidirectional input constraints," *IEEE Trans. on Industrial Informatics*, vol. 16, no. 2, pp. 969-979, February, 2020.

[29] T. Yang, N. Sun, H. Chen, and Y. Fang, "Neural network-based adaptive antiswing control of an underactuated ship-mounted crane with roll motions and input dead zones," *IEEE Trans. on Neural Networks and Learning System*, vol. 31, no. 3, pp. 901-914, March, 2020.

[30] N. Hoang and H. Kang, "Neural network-based adaptive tracking control of mobile robots in the presence of wheel slip and external disturbance force," *Neurocomputing*, vol. 188, pp. 12-22, February, 2016.

[31] J. Moreno, D. Negrete, V. Torres-González, and L. Fridman, "Adaptive continuous twisting algorithm," *International Journal of Control*, vol. 89, no. 9, pp. 1798-1806, July, 2016.

[32] M. Boukattaya, N. Mezghani, and T. Damak, "Adaptive nonsingular fast terminal sliding mode control for the tracking problem of uncertain dynamical system," *ISA Trans.*, vol. 77, pp. 1-19, June, 2018.

[33] S. Yi and J. Zhai, "Adaptive second-order fast nonsingular terminal sliding mode control for robotic manipulators," *ISA Trans.*, vol. 90, pp. 41-51, July, 2019.

[34] S. Bhat and D. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control and Optimization*, vol. 38, no. 3, pp. 751-766, March, 2000.

[35] S. Mondal and C. Mahanta, "Adaptive second order terminal sliding mode controller for robotic manipulators," *Journal of the Franklin Institute*, vol. 351, no. 4, pp. 2356-2377, April, 2014.

Hui Ye received her M.S. degree from College of Mathematics and Information Science, Jiangsu University, Zhenjiang, China. She is an associate professor at School of Science, Jiangsu University of Science and Technology. Her current research interests include robot control, stability analysis and fault tolerant control of switched systems.

Shuai Wang received her B.S. degree at Measurement and Control Technology and Instruments, University of Electronic Science and Technology of China, in 2017. She is currently pursuing her M.S. degree at the School of Automation, Southeast University. Her research interests include robot control and nonlinear systems.