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Asymptotic Stability Analysis for Switched Stochastic Nonlinear Systems Using Mode-dependent Uniformly Stable Functions

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Abstract: In this paper, we intend to investigate uniform global asymptotic stability in probability (UGAS-P) for a class of time-varying switched stochastic nonlinear systems. Conventional criteria on stability for switched stochastic systems are based on the negativity of the infinitesimal generator of Lyapunov functions, it is demonstrated that these criteria are conservative. Taking this fact into account, the infinitesimal generator for each active subsystem acting on Lyapunov functions is relaxed to be indefinite with the help of uniformly stable function (USF). Subsequently, improved criteria on asymptotic stability are proposed by applying the weakened condition and mode-dependent average dwell time (MDADT) technique. In addition, numerical examples are presented to verify the effectiveness of the obtained results.

Keywords: Asymptotic stability, mode-dependent average dwell time, switched stochastic nonlinear systems, uniformly stable function.

1. INTRODUCTION

Switched systems consisting of a family of continuous (or discrete) time subsystems orchestrated by a switching rule have been studied extensively due to a wide range of engineering applications and increasingly complexity of engineering systems in recent decades (see, for example, [1–5] and references therein). As a fundamental and challenging issue, stability analysis of switched systems has attracted a lot of attention. Various approaches have been developed to tackle the stability of switched systems under different switching mechanisms, see [6–10] and [5] for a good survey. In particular, uniform asymptotic stability (UAS) for time-varying systems has received more and more attention because of its inherent robustness [11, 12]. Recently, many interesting results have been presented to analyze the UAS of the switched systems, for instance, the extension of LaSalle's invariance principle [13], the limiting systems [14, 15]. However, the existence of switching behaviors and the time-varying characteristic of the systems makes it quite challenging in using LaSalle's invariance principle and guaranteeing the common limiting systems. This motivates us to continue the investigation of this direction.

It is known that switching signals play a crucial role in stability analysis of switched systems, which can be roughly divided into two major types: arbitrary switching and restricted switching. Common Lyapunov function approach is frequently used to develop the stability conditions of switched systems under arbitrary switching. It describes that if a common Lyapunov function has a uniformly negative definite derivative along any subsystem, then the switched system is uniformly global asymptotic stable [8, 9]. In fact, however, it is quite difficult to find such a common Lyapunov function [15, 16]. Thus, considerable attention has been devoted to the stability of switched systems under the restricted switching, such as state-dependent switching [17] or time-controlled switching [18]. Among them, average dwell time (ADT) switching technique introduced by [7] has become a powerful tool to analyze the stability and design controllers of diverse switched systems [19–22]. However, the minimal ADT between consecutive switchings is computed by two common mode-independent parameters for all subsystems, as pointed out in [10], which gives rise to a certain conservativeness. By relaxing the mode-independent parameters to mode-dependent ones, [10] proposed a more flexible and less conservative mode-dependent average dwell time (MDADT) switching scheme. During the most recent years, the MDADT switching scheme has seen widespread applications in different directions of the switched systems, to mention a few, switching stabiliza-

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tion [24], fault detection and control [25], finite-time filtering [26] and the references therein.

Recently, lots of literatures are concentrated on the stability of switched stochastic systems [27–34]. The majority work is dealt with under three types of restricted switching. In the first one, the switching is governed by a Markov process. For instance, exponential stability in the mean square of stochastic delay systems with Markovian switching was studied in [35]. A systematic analysis on the stability of stochastic differential equations with Markovian switching was presented in [36]. Sliding mode control for Markovian jump nonlinear descriptor system and output feedback control for Markovian jump discrete-time switched systems are investigated in [37] and [38] by employing the ADT method. More recently, some significant results for switched neural network modeled by hidden Markov system [39], T-S fuzzy semi-Markov switching systems [40] and Markov switching repeated scalar nonlinear systems [41] have been reported. The second one is time-controlled switching, such as ADT switching. In [42] and [30], the moment stability for switched stochastic systems was investigated by quantitatively characterizing and comparing the convergence of each subsystem. In terms of ADT switching, [31] studied stochastic input-to-state stability for switched stochastic nonlinear systems by using a stochastic comparison principle. Under extended asynchronous switching, asymptotic stability of switched stochastic nonlinear systems and hybrid stochastic retarded systems was developed in [33, 34] by the aid of ADT switching. In the third one, the switching signal is trigged by state-dependent events, which results in that the switching instants are stochastic and there is no information about transition probability of the switching signal. Considering the third switching, a theoretical framework on stability of stochastic nonlinear systems with state-dependent switching was constructed in [32].

It is worth noticing that the results obtained in the above-mentioned literatures are mainly based on the conventional Lyapunov stability theory, that is, time-derivatives of Lyapunov functions or functionals need to be negative definite. However, this condition may be conservative in some sense as pointed out in [43,44]. By introducing uniformly stable functions (USFs) [43, 44], weakened condition (i.e., the time-derivative of Lyapunov function along the trajectory of system can be indefinite) was established to test the asymptotic stability of deterministic time-varying systems. To the best knowledge of the authors, the uniformly global asymptotic stability for non-autonomous switched stochastic nonlinear systems have barely been reported in the existing literature. The considered systems have switching behaviors and time-varying characteristic simultaneously, which makes it quite challenging in using the traditional Lyapunov stability theory. The main contributions can be summarized as follows: 1) The inequality of infinitesimal generator for each active subsystem acting on Lyapunov function are released to be indefinite with the help of the USFs. Compared with the above mentioned methods to deal with the non-autonomous switched stochastic systems, the achieved results on asymptotic stability are weaker, which provides an important theoretical reference. 2) The uniformly stable functions are considered to be mode-dependent, together with the application of the MDADT switching scheme makes the obtained criteria on asymptotic stability more flexible and less conservative.

Notations: For a vector x, $|x|$ denotes the Euclidean norm, x^{T} denotes its transpose, $\|X\|$ is the 2-norm of a matrix X. $\mathbb{R}^n$ is the real n-dimensional space, $\mathbb{R}^{n\times m}$ is the space of $n\times m$ real matrices and $\mathbb{R}_+$ is the space of all nonnegative real numbers. $\mathrm{tr}\{M\}$ stands for the trace of the square matrix M, i.e., the sum of all elements on the main diagonal line. $\mathcal{C}^0$ denotes the space of functions $V(x,t):\mathbb{R}^n\times\mathbb{R}_+\to\mathbb{R}$ local Lipschitz in x and absolutely continuous in t, $\mathcal{C}^{2,1}$ denotes the space of functions $V(x,t):\mathbb{R}^n\times\mathbb{R}_+\to\mathbb{R}$ twice continuously differentiable in x and once in t, $\mathcal{PC}$ denotes the space of piecewise continuous functions $\mu(t):\mathbb{R}_+\to\mathbb{R}$. $\mathcal{K}$ denotes the set of all functions $\alpha(t):\mathbb{R}_+\to\mathbb{R}_+$, which are continuous, strictly increasing and vanishing at zero, $\mathcal{K}_\infty$ denotes the set of all functions which are of class $\mathcal{K}$ and unbounded, $\mathcal{KL}$ denotes the set of all functions $\beta(s,t):\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}_+$ which is of class $\mathcal{K}$ for each fixed t, and decreases to zero as $t\to\infty$ for each fixed s. For any $a,b\in\mathbb{R}$, define $a\wedge b=\min\{a,b\}$. The symbol $\circ$ stands for the composition operator between two functions.

2. PRELIMINARIES

Consider the following stochastic nonlinear system described by

$$dx(t)=f(x(t),t)dt+g(x(t),t)dw(t),\quad x(t_0)=x_0,\qquad (1)$$

where $x(t)\in\mathbb{R}^n$ is the state, $x(t_0)=x_0$ denotes initial condition, $w(t)$ is a m-dimensional independent standard Wiener process defined on a complete probability space $\left(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq t_0},\mathbb{P}\right)$ with a filtration $\{\mathscr{F}_t\}_{t\geq t_0}$ satisfying the usual conditions, i.e., it is increasing and right continuous while $\mathscr{F}_{t_0}$ contains all $\mathbb{P}$-null sets. The drift filed $f:\mathbb{R}^n\times[t_0,\infty)\to\mathbb{R}^n$ and the diffusion $g:\mathbb{R}^n\times[t_0,\infty)\to\mathbb{R}^{n\times m}$ are piecewise continuous in t, satisfying $|f(0,t)|=\|g(0,t)\|\equiv 0$ and locally Lipschitz in x, namely, for any $R>0$, there exists a constant $L_R\geq 0$ possibly depending on R such that for $\forall x_1,x_2\in U_R, x_1\neq x_2$ and all $t\geq 0$

$$\begin{aligned}&|f(x_2,t)-f(x_1,t)|+\|g(x_2,t)-g(x_1,t)\|\\&\leq L_R|x_2-x_1|.\end{aligned}$$

For system (1), given any $r>0$, define the first exit time

$$\tau_r=\inf\{t\geq t_0,|x(t)|\geq r\}\qquad (2)$$

with special case $\inf\emptyset=\infty$. Obviously, τ_r is increasing so it has limit $\tau_\infty=\lim_{r\to\infty}\tau_r$ which is called the escape (or explosion) time and $\tau_\infty<\infty$ or $\tau_\infty=\infty$.

Lemma 1 [36, Theorem 3.15]**:** Under the locally Lipschitz condition, system (1) has a unique maximal local solution $x(t)=x(t,x_0,t_0)$ on the maximal interval $[t_0,\tau_\infty)$.

The following result shows that the maximal local solution will become a forward complete solution, i.e., $\tau_\infty=\infty$ a.s. (almost surely).

Lemma 2 [46, Lemma 1]**:** Assume that there exist a nonnegative function $V(x,t)\in\mathcal{C}^{2,1}$, constants d and $D\geq 0$ such that for all $t\geq t_0$ and $r>0$

$$\mathbb{E}\left[V\left(x\left(t\wedge\tau_r\right),t\wedge\tau_r\right)\right]\leq De^{d(t-t_0)}, \tag{3}$$

$$\lim_{r\to\infty}\left(\inf_{t\geq t_0,|x|>r}V(x,t)\right)=\infty. \tag{4}$$

Then system (1) has a unique solution $x(t)$ on $[t_0,\infty)$.

Definition 1 [47, 48]**:** The equilibrium $x(t)=0$ of system (1) is said to be globally asymptotically stable in probability (GAS-P) if, for every $\varepsilon>0$, there exists a function $\beta(\cdot,\cdot)\in\mathcal{KL}$ such that

$$\mathbb{P}\{|x(t)|<\beta(|x_0|,\ t-t_0)\}\geq 1-\varepsilon,$$
$$\forall t\geq t_0,\ \ x_0\in\mathbb{R}^n\backslash\{0\}.$$

Definition 2 [49]**:** For a Markov process $x^{s,x_s}(t)$ and $V(x,t)\in\mathcal{C}^0$. The infinitesimal generator $\mathcal{A}$ of $V(x,t)$ with initial state $x(s)=x_s$ is defined by

$$\mathcal{A}V(x(s),s)\triangleq\lim_{h\to 0^+}\frac{\mathbb{E}^{s,x_s}\left[V(x(s+h),s+h)\right]-V(x_s,s)}{h},$$

where $\mathbb{E}^{s,x_s}$ denotes the expectation with respect to the conditional probability measure

$$\mathbb{P}^{s,x_s}\left(x(s+h)\in\mathcal{B}\right)\triangleq\mathbb{P}\left(x(s+h)\in\mathcal{B}|x(s)=x_s\right)$$

for all Borel subsets $\mathcal{B}$ of $\mathbb{R}^n$.

The differential operator $\mathcal{L}$ of $V(x,t)\in\mathcal{C}^{2,1}$ along the system (1) is defined by

$$\mathcal{L}V(x,t)=\frac{\partial V(x,t)}{\partial t}+\frac{\partial V(x,t)}{\partial x}f(x,t)+\frac{1}{2}\mathrm{tr}\left\{g^{\mathrm{T}}(x,t)\frac{\partial^2 V(x,t)}{\partial x^2}g(x,t)\right\}.$$

According to [49], if process $x(t)$ is a solution of system (1), then $\mathcal{L}V(x,t)=\mathcal{A}V(x,t)$.

The following notion and result of uniformly stable functions given by [43] and [44] are recalled.

Definition 3: A function $\mu(t)\in\mathcal{PC}$ is said to be a uniformly stable function if the following linear time-varying system is uniformly globally asymptotically stable:

$$\dot{y}(t)=\mu(t)y(t),\ \ \forall t\geq t_0,$$

where $y(t):[t_0,\infty)\to\mathbb{R}$ is the state variable.

Lemma 3 [43, Lemma 2]**:** $\mu(t)$ is a USF if and only if there exist two constants $c>0$ and $\delta\geq 0$ such that

$$\int_{t_0}^{t}\mu(s)ds\leq -c(t-t_0)+\delta,\ \ \forall t\geq t_0. \tag{5}$$

3. STABILITY ANALYSIS BASED ON UNIFORMLY STABLE FUNCTIONS

In this section, with the help of USFs, we will provide the criteria on stability for non-switched stochastic nonlinear system and switched stochastic nonlinear system, respectively.

3.1. GAS-P for non-switched stochastic nonlinear system

Theorem 1: Consider the stochastic nonlinear system (1). If there exist a function $V(x,t)\in\mathcal{C}^{2,1}$, a USF $\mu(t)\in\mathcal{PC}$, and functions α_1, $\alpha_2\in\mathcal{K}_\infty$ such that

$$\alpha_1(|x|)\leq V(x,t)\leq\alpha_2(|x|), \tag{6}$$

$$\mathcal{L}V(x,t)\leq\mu(t)V(x,t), \tag{7}$$

then system (1) has a unique solution on $[t_0,\infty)$ and the equilibrium $x(t)=0$ is GAS-P.

Proof: According to Lemma 1, for every $x_0\in\mathbb{R}^n$, system (1) has a unique solution $x(t)$ on the maximal interval $[t_0,\tau_\infty)$. Next, we need to show $\tau_\infty=\infty$ a.s..

We let $W(x,t)=V(x,t)e^{-\int_{t_0}^{t}\mu(s)ds}$. Applying Itô's formula along the system (1), $W(x,t)$ has the following stochastic differential

$$dW(x,t)=\left[\mathcal{L}V(x,t)-\mu(t)V(x,t)\right]e^{-\int_{t_0}^{t}\mu(s)ds}dt+\frac{\partial V(x,t)}{\partial x}g(x,t)e^{-\int_{t_0}^{t}\mu(s)ds}dw(t).$$

Taking integrals in $[t_0,t\wedge\tau_r)$ (where τ_r is defined as (2)) on both sides of the above equation, it yields to

$$\begin{aligned}&V\left(x\left(t\wedge\tau_r\right),t\wedge\tau_r\right)\\&=V\left(x_0,t_0\right)e^{\int_{t_0}^{t\wedge\tau_r}\mu(s)ds}\\&\quad+\int_{t_0}^{t\wedge\tau_r}\left[\mathcal{L}V(x,s)-\mu(s)V(x,s)\right]e^{\int_{s}^{t\wedge\tau_r}\mu(\theta)d\theta}ds\\&\quad+\int_{t_0}^{t\wedge\tau_r}\frac{\partial V(x,s)}{\partial x}g(x,s)e^{\int_{s}^{t\wedge\tau_r}\mu(\theta)d\theta}dw(s).\end{aligned}$$

Then taking expectation on both sides of the above equation and from (7), we have

$$\mathbb{E}[V(x(t\wedge\tau_r),t\wedge\tau_r)]\leq\mathbb{E}\left[V(x_0,t_0)e^{\int_{t_0}^{t\wedge\tau_r}\mu(s)ds}\right]. \tag{8}$$

Since $\mu(t)$ is a USF and $t\wedge\tau_r\geq t_0$ a.s., from Lemma 3, there exist constants $c>0$ and $\delta\geq 0$ such that (8) implies

$$\mathbb{E}\left[V\left(x\left(t\wedge\tau_r\right),t\wedge\tau_r\right)\right]\leq\mathbb{E}\left[V\left(x_0,t_0\right)e^{-c(t\wedge\tau_r-t_0)+\delta}\right]$$

$$\leq V(x_0,t_0)e^{\delta} < \infty.$$

By Lemma 2, we can obtain $\tau_\infty = \infty$ a.s.. Hence, combining (6) with (8) leads to

$$\begin{aligned}\mathbb{E}[V(x(t),t)] \leq & V(x_0,t_0)e^{\int_{t_0}^{t}\mu(s)ds} \\ \leq & \alpha_2(|x_0|)e^{-c(t-t_0)+\delta}, \quad \forall t \in [t_0,\infty).\end{aligned}$$

Denoting $\tilde{\beta}(|x_0|,t-t_0) \triangleq \alpha_2(|x_0|)e^{-c(t-t_0)+\delta}$, obviously, $\tilde{\beta}(\cdot,\cdot)$ is a $\mathcal{KL}$ function. Then the above inequality can be rewritten as

$$\mathbb{E}[V(x(t),t)] \leq \tilde{\beta}(|x_0|,t-t_0), \quad \forall t \in [t_0,\infty).$$

Using Chebyshev's inequality and the above inequality, there exist a function $\gamma(\cdot) \in \mathcal{K}_\infty$ and a sufficiently small $\varepsilon = \varepsilon(\gamma)$ such that for all $t \geq t_0$,

$$\begin{aligned}&\mathbb{P}\left\{|x(t)| \geq \alpha_1^{-1}\left(\gamma\left(\tilde{\beta}(|x_0|,t-t_0)\right)\right)\right\} \\ &\leq \mathbb{P}\left\{V(x(t),t) \geq \gamma\left(\tilde{\beta}(|x_0|,t-t_0)\right)\right\} \\ &\leq \frac{\mathbb{E}[V(x(t),t)]}{\gamma\left(\tilde{\beta}(|x_0|,t-t_0)\right)} \leq \varepsilon.\end{aligned}$$

Taking $\beta(\cdot,\cdot) \triangleq \alpha_1^{-1}\circ\gamma\circ\tilde{\beta}(\cdot,\cdot)$, we have $\beta(\cdot,\cdot) \in \mathcal{KL}$ and

$$\mathbb{P}\{|x(t)| < \beta(|x_0|,t-t_0)\} \geq 1-\varepsilon, \ \forall t \in [t_0,\infty).$$

This completes the proof. □

Remark 1: Different from the classical Lyapunov stability theory, the right hand side of inequality (7) is allowed to be indefinite due to the property of USF $\mu(t)$. Therefore, the conditions given in Theorem 1 are less conservative to test the asymptotic stability for a class of time-varying stochastic nonlinear systems.

3.2. GAS-P for switched stochastic nonlinear system with MDADT switching

Consider the following time-varying switched stochastic nonlinear system

$$\begin{aligned}&dx(t) = f_{\sigma(t)}(x(t),t)dt + g_{\sigma(t)}(x(t),t)dw(t), \\ &x(t_0) = x_0,\end{aligned} \tag{9}$$

where $x(t) \in \mathbb{R}^n$ is the state. $\sigma(t) : [t_0,\infty) \to \mathscr{P}$ is called a switching signal and is right hand continuous and piecewise constant in t, $\mathscr{P} = \{1,2,\cdots,M\}$ is a finite index set with M being the number of subsystems. For each $p \in \mathscr{P}$, functions $f_p : \mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}^n$ and $g_p : \mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}^{n\times m}$ are locally Lipschitz in x and piecewise continuous in t, and $|f_p(0,t)| = \|g_p(0,t)\| \equiv 0$. Other explanations are as the same as for system (1).

Remark 2: For switched system (9), only local Lipschitz condition is assumed to the functions f_p and g_p without the linear growth constraint which is different from [45]. However, based on some mild and easily verified conditions, the switched system (9) still has a unique forward complete solution under an arbitrary switching signal as shown in Theorem 2.

For a switching signal $\sigma(t)$, the instants $t_1 < \cdots < t_i < t_{i+1} < \cdots$ represent the switching sequence and the $\sigma(t_i)^{\text{th}}$ subsystem is active when $t \in (t_i,t_{i+1})$. Before presenting the criteria on stability of switched stochastic nonlinear system (9), the following definition of mode-dependent average dwell time (MDADT) is given.

Definition 4 [10]**:** For a switching signal $\sigma(t)$ and any $T \geq t \geq t_0$, let $N_{\sigma p}(T,t)$ be the switching numbers that the p^{th} subsystem is activated over the interval $[t,T]$ and $T_p(T,t)$ denote the total running time of the p^{th} subsystem over the interval $[t,T]$, $p \in \mathscr{P}$. We say that $\sigma(t)$ has a MDADT τ_{ap} if there exist positive numbers N_{0p} (N_{0p} is the mode-dependent chatter bounds here) and τ_{ap} such that

$$N_{\sigma p}(T,t) \leq N_{0p} + \frac{T_p(T,t)}{\tau_{ap}}, \quad \forall T \geq t \geq t_0.$$

Theorem 2: Consider the switched stochastic nonlinear system (9). Suppose that there exist functions $V_p(x,t) \in \mathcal{C}^{2,1}$, class $\mathcal{K}_\infty$ functions $\underline{\alpha}_p$, $\bar{\alpha}_p$, USFs $\mu_p(t) \in \mathcal{PC}$, and positive constants $\eta_p \geq 1$, $c_p > 0$, $\delta_p \geq 0$, $p \in \mathscr{P}$ such that for $\forall p \in \mathscr{P}$ and all $x \in \mathbb{R}^n$, $t \geq t_0$

$$\underline{\alpha}_p(|x|) \leq V_p(x,t) \leq \bar{\alpha}_p(|x|), \tag{10}$$

$$\mathcal{L}V_p(x,t) \leq \mu_p(t)V_p(x,t), \tag{11}$$

and $\sigma(t_i) = p$, $\sigma(t_i^-) = q$ with $\forall p,q \in \mathscr{P}$, $p \neq q$

$$\mathbb{E}[V_p(x(t_i),t_i)] \leq \eta_p\mathbb{E}\left[V_q\left(x(t_i^-),t_i^-\right)\right], \tag{12}$$

then there exists a unique solution to system (9) on $[t_0,\infty)$ and the equilibrium $x(t) = 0$ of system (9) is GAS-P for any switching signal with MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{\ln\eta_p + \delta_p}{c_p}. \tag{13}$$

Proof: From Theorem 1, for each $p \in \mathscr{P}$, the p^{th} subsystem has a unique solution on $[t_0,\infty)$. Thus, under an arbitrary switching signal $\sigma(t)$, the switched stochastic nonlinear system (9) has a unique solution on $[t_0,\infty)$ by a recursive method similar to [50]. The first conclusion is obtained.

For any $T > 0$, let $t_1,\cdots,t_i,t_{i+1},\cdots,t_{N_\sigma}$ be the switching times on interval $[t_0,T]$, where $N_\sigma \triangleq N_\sigma(T,t_0)$ stands for the total switching numbers of the whole switched system over the interval $[t_0,T]$. From Definition 4, let $N_{\sigma p} \triangleq N_{\sigma p}(T,t_0)$ denote the switching numbers that the p^{th} subsystem is activated over the interval $[t_0,T]$. Thus, $N_\sigma = \sum_{p=1}^{M} N_{\sigma p}$. Then for any $t \in [t_i,t_{i+1})$, we set

$$W_{\sigma(t_i)}(x(t),t) \triangleq V_{\sigma(t_i)}(x(t),t)e^{-\int_{t_0}^{t}\mu_{\sigma(t_i)}(s)ds}.$$

Similar to the proof of Theorem 1, for any $t \in [t_i, t_{i+1})$, we can obtain

$$\mathbb{E}\left[V_{\sigma(t_i)}(x(t),t)\right] \le \mathbb{E}\left[V_{\sigma(t_i)}\left(x\left(t_i\right),t_i\right)\right] e^{\int_{t_i}^{t} \mu_{\sigma(t_i)}(s)ds}.$$

The above equation together with (12) yields that

$$\begin{aligned}
&\mathbb{E}[V_{\sigma(t_i)}(x(t_{i+1}^-),t_{i+1}^-)] \\
&\le \mathbb{E}[V_{\sigma(t_i)}(x(t_i),t_i)]e^{\int_{t_i}^{t_{i+1}} \mu_{\sigma(t_i)}(s)ds} \\
&\le \eta_{\sigma(t_i)}\mathbb{E}[V_{\sigma(t_{i-1})}(x(t_i^-),t_i^-)]e^{\int_{t_i}^{t_{i+1}} \mu_{\sigma(t_i)}(s)ds} \\
&\le \eta_{\sigma(t_i)}\mathbb{E}[V_{\sigma(t_{i-1})}(x(t_{i-1}),t_{i-1})]e^{\int_{t_{i-1}}^{t_i} \mu_{\sigma(t_{i-1})}(s)ds} \\
&\quad \times e^{\int_{t_i}^{t_{i+1}} \mu_{\sigma(t_i)}(s)ds} \\
&\le \eta_{\sigma(t_i)}\eta_{\sigma(t_{i-1})}\mathbb{E}[V_{\sigma(t_{i-2})}(x(t_{i-1}^-),t_{i-1}^-)] \\
&\quad \times e^{\int_{t_{i-1}}^{t_i} \mu_{\sigma(t_{i-1})}(s)ds+\int_{t_i}^{t_{i+1}} \mu_{\sigma(t_i)}(s)ds} \\
&\le \cdots \le \prod_{k=0}^{i-1} \eta_{\sigma(t_{k+1})} e^{\sum_{k=0}^{i}\int_{t_k}^{t_{k+1}} \mu_{\sigma(t_k)}(s)ds} V_{\sigma(t_0)}(x_0,t_0).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathbb{E}\left[V_{\sigma(T^-)}\left(x\left(T\right),T\right)\right] \\
&\le \mathbb{E}\left[V_{\sigma(t_{N_\sigma})}\left(x\left(t_{N_\sigma}\right),t_{N_\sigma}\right)\right] e^{\int_{t_{N_\sigma}}^{T} \mu_{\sigma(t_{N_\sigma})}(s)ds} \\
&\le \prod_{k=0}^{N_\sigma-1} \eta_{\sigma(t_{k+1})} e^{\sum_{k=0}^{N_\sigma-1}\int_{t_k}^{t_{k+1}} \mu_{\sigma(t_k)}(s)ds+\int_{t_{N_\sigma}}^{T} \mu_{\sigma(t_{N_\sigma})}(s)ds} \\
&\quad \times V_{\sigma(t_0)}\left(x_0,t_0\right) \\
&\le \prod_{p=1}^{M} \eta_p^{N_{\sigma p}} e^{\sum_{p=1}^{M}\sum_{l\in\psi(p)}\int_{t_l}^{t_{l+1}} \mu_p(s)ds+\int_{t_{N_\sigma}}^{T} \mu_{\sigma(t_{N_\sigma})}(s)ds} \\
&\quad \times V_{\sigma(t_0)}\left(x_0,t_0\right),
\end{aligned} \tag{14}$$

where $\psi(p)$ denotes the set of l satisfying $\sigma(t_l) = p$, $t_l \in \{t_0, t_1, \cdots, t_i, t_{i+1}, \cdots, t_{N_\sigma-1}\}$. Since, for each $p \in \mathscr{P}$, $\mu_p(t)$ is a USF, from Lemma 3, there exist constants $c_p > 0$ and $\delta_p \ge 0$ such that

$$\int_{\tau_1}^{\tau_2} \mu_p(s)ds \le -c_p(\tau_2-\tau_1)+\delta_p, \quad \forall \tau_2 \ge \tau_1 \ge t_0. \tag{15}$$

Combining (14) with (15) gives

$$\begin{aligned}
&\mathbb{E}\left[V_{\sigma(T^-)}\left(x\left(T\right),T\right)\right] \\
&\le \prod_{p=1}^{M} \eta_p^{N_{\sigma p}} e^{\sum_{p=1}^{M}\sum_{l\in\psi(p)}[-c_p(t_{l+1}-t_l)+\delta_p]-c_{\sigma(t_{N_\sigma})}(T-t_{N_\sigma})+\delta_{\sigma(t_{N_\sigma})}} \\
&\quad \times V_{\sigma(t_0)}\left(x_0,t_0\right) \\
&\le e^{\sum_{p=1}^{M}(N_{0p}\ln\eta_p+N_{\sigma p}\delta_p)} e^{\sum_{p=1}^{M}\left(\frac{\ln\eta_p}{\tau_{ap}}-c_p\right)T_p} V_{\sigma(t_0)}\left(x_0,t_0\right) \\
&\le e^{\sum_{p=1}^{M}N_{0p}(\ln\eta_p+\delta_p)} e^{\sum_{p=1}^{M}\left(\frac{\ln\eta_p+\delta_p}{\tau_{ap}}-c_p\right)T_p} V_{\sigma(t_0)}\left(x_0,t_0\right).
\end{aligned}$$

Hence, if there exist constants τ_{ap} satisfying (13), we have

$$\begin{aligned}
&\mathbb{E}[V_{\sigma(T^-)}(x(T),T)] \\
&\le e^{\sum_{p=1}^{M}N_{0p}(\ln\eta_p+\delta_p)+\max_{p\in\mathscr{P}}(\frac{\ln\eta_p+\delta_p}{\tau_{ap}}-c_p)(T-t_0)} V_{\sigma(t_0)}(x_0,t_0).
\end{aligned} \tag{16}$$

By denoting $\lambda \triangleq -\max_{p\in\mathscr{P}}\left(\frac{\ln\eta_p+\delta_p}{\tau_{ap}}-c_p\right) \ge 0$, from (10) and (16), there exists a class $\mathcal{KL}$ function

$$\tilde{\beta}(|x_0|,T-t_0) \triangleq e^{\sum_{p=1}^{M}N_{0p}(\ln\eta_p+\delta_p)-\lambda(T-t_0)} \bar{\alpha}_{\sigma(t_0)}(|x_0|)$$

such that

$$\mathbb{E}[V_{\sigma(T^-)}(x(T),T)] \le \tilde{\beta}(|x_0|,T-t_0), \quad \forall T \in [t_0,\infty).$$

In consequence, for any $\varepsilon > 0$, there exists a function $\beta(\cdot,\cdot) \in \mathcal{KL}$ such that

$$\mathbb{P}\{|x(T)| < \beta(|x_0|,T-t_0)\} \ge 1-\varepsilon, \quad \forall T \in [t_0,\infty).$$

This allows us to conclude. □

Remark 3: Compared with the restriction $\int_{t_0}^{\infty}\phi^+(s)ds < \infty$ ($\phi^+(t)$ is the positive parts of indefinite function $\phi(t)$) in [45], the USFs $\mu_p(t)$, $p \in \mathscr{P}$ may be unbounded and the integral of $\mu_p^+(t)$ may be infinite, i.e., $\int_{t_0}^{\infty}\mu_p^+(s)ds = \infty$, such as $\mu_1(t)$ and $\mu_2(t)$ in Example 2. Therefore, the criteria on global asymptotic stability proposed in this paper are weaker.

Remark 4: Since USFs are mode-dependent, the MDADT switching scheme can guarantee that the obtained results are less conservative than that by using ADT switching scheme. Moreover, the application of the USFs makes the conditions of UGAS feasible for nonautonomous switched stochastic systems by comparing with the existing literature on autonomous the switched systems with MDADT switching [10, 24, 25].

4. NUMERICAL EXAMPLES

The obtained stability criteria can be used as the design conditions of a set of stabilizing controllers for nonautonomous stochastic nonlinear systems with MDADT switching. In this section, numerical examples are presented to study the stabilization problem. Based on these conditions, the stabilizing controllers can be designed to achieve the stabilization of the systems, which demonstrates the potential and validity of the results obtained.

Example 1: Consider a non-switched stochastic nonlinear system as follows:

$$dx = f(x,t)dt + g(x,t)dw, \tag{17}$$

where $x = (x_1,x_2)^{\mathrm{T}}$, $w = (w_1,w_2)^{\mathrm{T}}$, w_1 and w_2 are two mutually independent scalar standard Wiener processes,

$$f(x,t) = \begin{pmatrix} -\frac{1}{4}x_1^3+\left(\frac{t\cos t}{2(1+t)}-\frac{1}{4}\right)x_1-2x_2\cos t \\ \left(\frac{t\cos t}{2(1+t)}-\frac{3}{4}\right)x_2+2x_1\cos t \end{pmatrix},$$

$$g(x,t)=\begin{pmatrix}\frac{x_1^2}{2\sqrt{1+x_2^2}} & -\frac{1}{\sqrt{2}}x_2\sin t\\ \frac{1}{\sqrt{2}}x_2\sin t & \frac{x_1^2}{2\sqrt{1+x_2^2}}\end{pmatrix}.$$

Let $V(x)=\frac{1}{2}\left(x_1^2+x_2^2\right)$. Then, we have

$$\mathcal{L}V(x)\leq\mu(t)V(x),$$

where $\mu(t)=\frac{t\cos t}{1+t}-\frac{1}{2}$. We calculate that

$$\begin{aligned}\int_s^t\mu(\theta)d\theta&=-\frac{1}{2}(t-s)+\sin t-\sin s\\&\quad-\frac{\sin t}{1+t}+\frac{\sin s}{1+s}-\int_s^t\frac{\sin\theta}{(1+\theta)^2}d\theta\\&\leq-\frac{1}{2}(t-s)+5,\ \ \forall t\geq s\geq 0.\end{aligned}$$

Thus, $\mu(t)$ is a USF. The conditions of Theorem 1 are satisfied, then the zero solution $x(t)=0$ of system (17) is GAS-P.

Example 2: Consider the following switched stochastic nonlinear control system.

$$dx=f_\sigma(x,u_\sigma,t)dt+g_\sigma(x,u_\sigma,t)dw, \tag{18}$$

where the state is $x=(x_1,x_2)^{\mathrm{T}}\in\mathbb{R}^2$, the switching signal is $\sigma:[t_0,\infty)\to\{1,2\}$, $u_p=(u_{p1},u_{p2})^{\mathrm{T}}\in\mathbb{R}^2$ $(p=1,2)$ are control inputs, $w=(w_1,w_2)^{\mathrm{T}}$, w_1 and w_2 are two mutually independent scalar standard Wiener processes,

$$\begin{aligned}f_1(x,u_1,t)&=\begin{pmatrix}-\frac{1}{4}x_1^3+\frac{t\cos t^2}{4}x_1-2x_2\cos t+u_{11}\\ \frac{t\cos t^2}{4}x_2+x_1\cos t+u_{12}\end{pmatrix},\\ f_2(x,u_2,t)&=\begin{pmatrix}\frac{t\cos t^2}{2}x_1+2|\cos t|x_2+u_{21}\\ \frac{t\cos t^2}{2}x_2-2|\cos t|x_1+u_{22}\end{pmatrix},\\ g_1(x,u_1,t)&=\begin{pmatrix}\frac{x_1^2}{2\sqrt{1+x_2^2}} & -\frac{1}{\sqrt{2}}x_2\sin t\\ \frac{1}{2}x_2\sin t & \frac{x_1^2}{2\sqrt{2(1+x_2^2)}}\end{pmatrix},\\ g_2(x,u_2,t)&=\begin{pmatrix}\frac{x_1}{\sqrt{1+x_2^2}} & 0\\ 0 & \frac{x_2}{\sqrt{2}}\end{pmatrix}.\end{aligned}$$

Choose $V_1=\frac{1}{2}x_1^2+x_2^2$ and $V_2=\frac{1}{2}\left(x_1^2+x_2^2\right)$. Based on the MDADT switching scheme, the controller is designed

$$u_1=\left(-\frac{1}{4}x_1,\ -\frac{1}{2}x_2\right)^{\mathrm{T}},\ \ u_2=\left(-x_1,\ -\frac{3}{4}x_2\right)^{\mathrm{T}}$$

such that

$$\mathcal{L}V_p(x)\leq\mu_p(t)V_p(x),\ \ p=1,2$$

with $\mu_1(t)=\frac{1}{2}t\cos t^2-\frac{1}{2}$ and $\mu_2(t)=t\cos t^2-1$. Since for $\forall t\geq s\geq 0$, we have $\int_s^t\mu_1(\theta)d\theta\leq-\frac{1}{2}(t-s)+\frac{1}{2}$ and $\int_s^t\mu_2(\theta)d\theta\leq-(t-s)+1$, $\mu_1(t)$ and $\mu_2(t)$ are two USFs.

Table 1. USFs and admissible switching signals under two different switching schemes.

Switching schemes	MDADT	ADT
USFs	$\mu_1(t)=\frac{1}{2}t\cos t^2-\frac{1}{2}$, $\mu_2(t)=t\cos t^2-1$	$\mu(t)=\frac{1}{2}t\cos t^2-\frac{1}{2}$
Switching signals	$\tau_{a1}^*=2.4$, $\tau_{a2}^*=1$ ($\eta_1=2$, $\eta_2=1$, $c_1=\frac{1}{2}$, $\delta_1=\frac{1}{2}$, $c_2=1$, $\delta_2=1$)	$\tau_a^*=2.4$ ($\eta_1=\eta_2=2$, $c=\frac{1}{2}$, $\delta=\frac{1}{2}$)

For comparison, the controllers designed by applying ADT switching scheme are given as follows:

$$\begin{aligned}u_1&=\left(-\frac{1}{4}x_1,\ -\frac{1}{2}x_2\right)^{\mathrm{T}},\\ u_2&=\left(-\left(\frac{t\cos t}{4}+\frac{3}{4}\right)x_1,\ -\left(\frac{t\cos t}{4}+\frac{1}{2}\right)x_2\right)^{\mathrm{T}}.\end{aligned}$$

Then we have $\mathcal{L}V_p(x)\leq\mu(t)V_p(x)$, $p=1,2$, where $\mu(t)=\frac{1}{2}t\cos t^2-\frac{1}{2}$. The USFs and admissible switching signals under the two switching schemes are listed in Table 1. From Table 1, we can see that ADT switching is a special case of MDADT switching. According to Theorem 2, the zero solution $x(t)=0$ of switched system (18) is GAS-P under the both switching schemes of MDADT and ADT.

By choosing the same initial condition $x_0=(1,-1)^{\mathrm{T}}$, the corresponding state responses of the closed-loop system are shown in Fig. 1 and Fig. 2, respectively. It can

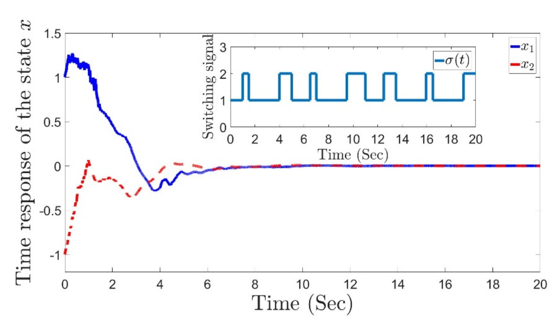

Fig. 1. Time response of the state under MDADT switching signal σ with $\tau_{a1}=2.5$ and $\tau_{a2}=1$.

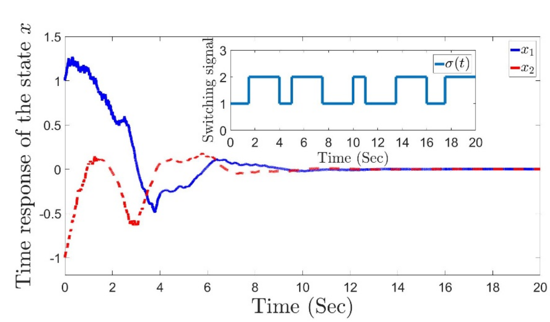

Fig. 2. Time response of the state under ADT switching signal σ with $\tau_a=2.5$.

be seen that the fluctuation of the state trajectories under the MDADT switching scheme is smaller than that under the ADT switching scheme. Thus, as demonstrated in [10], the MDADT switching scheme can guarantee that the proposed results of this paper are less conservative.

5. CONCLUSIONS

The global asymptotic stability in probability for the time-varying switched stochastic nonlinear systems is investigated in this paper. By virtue of the properties of uniformly stable functions, the right hand side of infinitesimal generator for each active subsystem acting on Lyapunov function is relaxed to be indefinite. Then improved criteria on asymptotic stability for the switched stochastic nonlinear systems are derived. Moreover, MDADT switching scheme is applied to guarantee that the obtained results are less conservative. Finally, numerical examples are given to illustrate the effectiveness of the proposed results.

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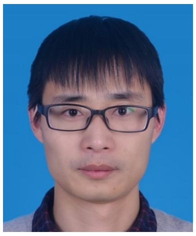

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