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Leaderless Consensus of Non-linear Mixed delay Multi-agent Systems with Random Packet Losses via Sampled-data Control

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Abstract: This paper inspects the consensus problem of nonlinear mixed delay multi-agent systems with random packet losses through the sampled-data control using the undirected graph without any specified leader for the other following agents. The probabilistic time varying delay is taken in the control input delay that Bernoulli distributed white sequence is engaged to formulate the random packet losses between the agents. The consensus problem can be changed over into a stabilization problem by using the Laplacian matrix which can be obtained by undirected graph. By framing a Lyapunov-Krasovskii functional with triple integral terms and implementation of the property of Kronecker product together with some well known matrix inequality techniques, a mean square consensus for mixed delay multi-agent system can be achieved. Terminally, two numerical examples are provided to illuminate the advantages of the suggested techniques.

Keywords: Consensus, Kronecker product, multi-agent systems (MASs), probabilistic time delay.

1. INTRODUCTION

For the past few years, multi-agent systems has became an interesting topic among the researchers due to its huge probable applications in lots of fields listed as unmanned aerial vehicles formation control, scheduling of automated highway systems, world wide web, robotics, fish school, complex networks, rendezvous and so on [1–3]. Especially, the consensus problem is the very basic concept in the study of MASs. All interconnected agents can reach a common goal by distributed controller is the consensus. Many researchers discussed about the two types of consensus i.e., leader following and leaderless consensus due to its immense applications such as cooperative of a group of agents in [4, 5]. During the early phase, literatures have interested to investigate the consensus problem of MASs in the class of first order and then their attention turned in to the second-order consensus problem of MASs and then they are attracted by consensus problem for higher-order MASs. It is realized that, practically speaking, nonlinearities are unavoidable in genuine frameworks. Thus,the nonlinearity issue, these days, has been one of the noteworthy research points when investigating the consensus of multi-agent systems. Various intriguing and productive outcomes have been accounted for on the accord issues for the nonlinear multi-agent systems [6].

Time delay made large impact on numerous practical systems, it may create some complex dynamic behaviors such as divergence, oscillation and instability of systems. Discrete time delay [7,8], continuous time delay [9], fractional- order time delay [10] are already discussed by many researchers in both networks and systems. The class of multi-agent systems with time varying delay will reach the mean square consensus are investigated in [11]. Although, time-varying delays are frequently exist in a random form in some of the practical system. Besides, its some probabilistic features of system with delay such as Bernoulli and Poisson distribution, can generally acquired by well known statistical methods [12]. By virtue of these true attitude, researchers have inserted the notion of probabilistic delays in the time delay systems [13,14]. The stability analysis of mixed delayed neural networks are generally discussed when compare to the consensus of mixed delay multi-agent system. Li [15] concerned the leader following consensus of multi-agent system with mixed delay via adaptive pinning intermittent control.

A control system manages, commands, directs, or regulates the behavior of other devices or systems using control loops. Control systems are found in abundance in all sectors of industry such as: quality control of manu-

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factured products, computer control, power systems, intelligent systems, traffic system. The sampled-data control has developed frequently due to its applications in computer hardware. When compare to continuous control, the sampled-data control is a more forceful and effective method. In [16], the impact of controller gain fluctuation and communication delay, a novel sampled-data control scheme with variable sampling interval is designed for each agent is well examined. The issue of quantized sampled-data fuzzy control for chaotic systems with variable sampling is concerned in [17]. [18] explained the synchronization problem for Lur'e systems with uncertainty parameters and variable sampling control by employing an augment LKF and the FMB inequality approach. Researchers in [19] developed some novel stabilization criteria for uncertainty fuzzy system with time-varying delay and disturbance under memory sampled data control.

In numerous genuine circumstance, the loss of control packet will happens due to the actuator faults, communication disturbance, crowding and so on. As continual appearance of control packet loss is inevitable, it may leads to instability of the system. So it is essential to introduce the impact of control packet loss. The two types of packet losses are deterministic packet loss, random packet loss. In [20] consensus of multi-agent system with probabilistic time delays and packet losses using sampled-data control has been investigated. This paper explains how to use the Bernoulli white sequence to describe random packet loss among agents and switched system to denote the packet dropouts in a deterministic method. The sampled data consensus of multi-agent system with packet losses are examined in [21]. In random packet loss random indicates the packet loss doesn't occur at a regular interval or time of day [22, 23].

Motivated by above facts, this paper discuss the leaderless consensus of mixed delayed multi-agent system with probabilistic time delay and random packet loss using sampled-data control. This paper mainly discuss the consensus of nonlinear MASs with uncertain parameters through sampled-data control scheme by constructing the suitable Lyapunov-Krasovskii functional.

The main aspects of this paper is listed below:

- The mixed delayed multi agent systems with Random packet losses is examined by using suitable Lyapunov-Krasovskii functional with Kronecker product. The special case of this paper is that the system is modeled as multi-agent system with nonlinear dynamics, discrete time delay and distributed time delays.
- Consensus will obtain without considering any specific leader to lead the other agents. The communication between the agents are represented through the undirected graph.
- Laplacian matrices are used to convert the consensus problem. Newly improved techniques are used to obtain the mean square consensus.
- Numerical examples are presented to show the validity of the results.

Notations: Throughout this paper, the following notations are used. $\mathbb{R}^n$ represents the n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. The symmetric terms in a symmetric matrix is expressed as $*$. A^T represents the transpose of A and A^{-1} denotes the inverse of A. I is the identity matrix with appropriate dimension. $X>0$ means that the matrix X is real symmetric positive definite with appropriate dimension and $diag\{a,b,\ldots,z\}$ indicates the block-diagonal matrix with $a,b,\ldots,z$ in the diagonal entries. $A\otimes B$ denotes the Kronecker product of matrices A and B.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Let us consider a multi-agent system contains N number of agents and the interaction between the agents are represented by the weighted undirected graph $\mathcal{G}=(\mathcal{V}, \mathcal{E}, \mathcal{A})$ with order N, where $\mathcal{V}=\{v_1, v_2, \ldots, v_N\}$ denotes the set of all agents, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ represents the edges set, and $\mathcal{A}=[a_{ij}]_{N\times N}$ is the adjacency matrix with $a_{ij}>0$ if $(v_i,v_j)\in\mathcal{E}$ and $a_{ij}=0$ otherwise. The neighbour set of v_i can be represented as $\mathcal{N}_i=\{j:(v_i,v_j)\in\mathcal{E}\}$. Let $\mathcal{D}=diag\{deg(1),deg(2),\ldots,deg(i)\}$ be the degree matrix of the undirected graph $\mathcal{G}$ with entries $deg(i)=\sum_{j\in\mathcal{N}_i}a_{ij}$. Then the Laplacian matrix of $\mathcal{G}$ can be expressed as $\mathcal{L}=(l_{ij})_{N\times N}$ is defined as $l_{ii}=-\sum_{j=1,\ j\neq i}l_{ij}$, $l_{ij}=-a_{ij}$, $i\neq j$.

Now, consider a nonlinear mixed delayed multi-agent system in the following form

$$\begin{aligned}\dot{x}_i(t)=&A_ax_i(t)+A_bf(x_i(t))+A_cf(x_i(t-h(t)))\\&+A_d\int_{t-d(t)}^{t}f(x_i(s))ds+Bu_i(t),\\&i=1,2,\ldots,N,\end{aligned}\tag{1}$$

where N is the number of agents, n is the number of states of agent i, $x_i(t)=[x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T\in\mathbb{R}^n$ denotes the state vector of the agent i, $u_i(t)=[u_{i1}(t), u_{i2}(t), \ldots, u_{in}(t)]^T\in\mathbb{R}^n$ is the control input acting on agent i and $f(x_i(t))=[f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t))]^T$ is a nonlinear vector function to describe the time-varying nonlinear dynamics of agent i; A_a, A_b, A_c, A_d, B are real constant matrices with appropriate dimensions. $h(t)$, $d(t)$ are probabilistic time varying delay and distributed delay respectively. $d(t)>0$ it satisfies $0\leq d(t)\leq d$ where d is the constant.

The sampled-data controller contains random packet loss can be represented as, $u_i(t)=\alpha(t_k)K\sum_{j=1}^{N}a_{ij}(x_j(t_k)-x_i(t_k))$, $t_k\leq t<t_{k+1}$, $i=1, 2, \ldots, N$, where the stochastic variable $\alpha(t_k)$ is a Bernoulli-distributed white sequence with $prob\{\alpha(t_k)=1\}=\mathbb{E}\{\alpha(t_k)\}=\overline{\alpha}$, $prob\{\alpha(t_k)=0\}=\mathbb{E}\{\alpha(t_k)\}=1-\overline{\alpha}$, in which $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation, $0\leq\overline{\alpha}\leq 1$. $K\in\mathbb{R}^{n\times n}$ is the

feedback matrix to be determined, a_{ij} is (i, j)-th entry of the adjacency matrix of communication topology $\mathcal{G}$, t_k is the updating instant time satisfying $0 = t_0 < t_1 < .... < t_k < .... < lim_{k\to\infty} t_k = +\infty$. The sampling interval is defined as $t_{k+1} - t_k = \eta_k \leq \eta$ for any integer $k \geq 0$, where $\eta > 0$ represents the largest sampling interval. Define $x_i(t_k) = x_i(t - \eta(t))$ with $\eta(t) = t - t_k$, $0 \leq \eta(t) \leq \eta$ for $t \neq t_k$. Then, eqn.(2) can be expressed as $u_i(t) = \alpha(t - \eta(t))K\sum_{j=1}^{N} a_{ij}(x_j(t-\eta(t)) - x_i(t-\eta(t)))$, $t_k \leq t < t_{k+1}$, $i = 1, 2, ..., N$.

Then, system (1) can be expressed as

$$\dot{x}_i(t) = A_a x_i(t) + A_b f(x_i(t)) + A_c f(x_i(t-h(t))) + A_d \int_{t-d(t)}^{t} f(x_i(s))ds + \alpha(t-\eta(t))BK\sum_{j=1}^{N} a_{ij} (x_j(t-\eta(t)) - x_i(t-\eta(t))). \tag{2}$$

For our convenience, we denote the subsequent vector, $x(t) = [x_1^T(t)\ x_2^T(t)\ ...\ x_N^T(t)]^T$, $f(x(t)) = [f^T(x_1(t))\ f^T(x_2(t))\ ...\ f^T(x_N(t))]^T$, $f(x(t-h(t))) = [f^T(x_1(t-h(t)))\ f^T(x_2(t-h(t)))\ ...\ f^T(x_N(t-h(t)))]^T$.

Then by using the Kronecker product property, (4) can be written in the following compact form:

$$\begin{aligned}\dot{x}(t) =& (I_N \otimes A_a)x(t) + (I_N \otimes A_b)f(x(t)) \\ &+ (I_N \otimes A_c)f(x(t-h(t))) \\ &+ (I_N \otimes A_d)\int_{t-d(t)}^{t} f(x(s))ds \\ &- \alpha(t-\eta(t))(\mathcal{L}\otimes B)Kx(t-\eta(t)).\end{aligned} \tag{3}$$

For further derivations, the below mentioned assumptions are utilized in this paper.

Assumption 1 [26]**:** For any $j \in \{1, 2, ..., n\}$, $f_j(0) = 0$, and there exist constants Λ_j^- and Λ_j^+ such that

$$\Lambda_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq \Lambda_j^+,\ \ \forall\ \alpha_1 \neq \alpha_2.$$

Assumption 2 [14]**:** The probability distribution of the time-varying delay $h(t)$ is defined by $prob\{h(t) \in [0, h_1)\} = q_0$, $prob\{h(t) \in [h_1, h_2]\} = 1 - q_0$, where $0 \leq q_0 \leq 1$ is a constant. Then, the stochastic variable $q(t)$ can be defined as

$$q(t) = \begin{cases} 1, & \text{for } h(t) \in [0, h_1), \\ 0, & \text{for } h(t) \in [h_1, h_2]. \end{cases}$$

$q(t)$ is a Bernoulli distributed sequence with $prob\{q(t) = 1\} = prob\{0 \leq h(t) < h_1\} = \mathbb{E}\{q(t)\} = q_0$, $prob\{q(t) = 0\} = prob\{h_1 \leq h(t) \leq h_2\} = 1 - \mathbb{E}\{q(t)\} = 1 - q_0$, where $\mathbb{E}\{q(t)\}$ is the mathematical expectation of $q(t)$. It implies $\mathbb{E}\{q(t) - q_0\} = 0$, $\mathbb{E}\{(q(t) - q_0)^2\} = q_0(1 - q_0)$.

Now, we introduce time varying delays $h_1(t)$ and $h_2(t)$ such that

$$h(t) = \begin{cases} h_1(t), & h(t) \in [0, h_1), \\ h_2(t), & h(t) \in [h_1, h_2], \end{cases}$$

where $\dot{h_1}(t) \leq \mu_1 < 1$, $\dot{h_2}(t) \leq \mu_2 < 1$, μ_1 and μ_2 are constants.

The closed-loop system (5) with probabilistic time varying delay can be represented as

$$\begin{aligned}\dot{x}(t) =& (I_N \otimes A_a)x(t) + (I_N \otimes A_b)f(x(t)) + q(t)(I_N \otimes A_c) \\ &\times f(x(t-h_1(t))) + (1-q(t))(I_N \otimes A_c) \\ &\times f(x(t-h_2(t))) + (I_N \otimes A_d)\int_{t-d(t)}^{t} f(x(s))ds \\ &+ (\bar{\alpha} - \alpha(t-\eta(t)) - \bar{\alpha})(\mathcal{L}\otimes B)Kx(t-\eta(t)).\end{aligned} \tag{4}$$

Further, it can be equivalently rewritten as

$$\begin{aligned}\dot{x}(t) =& (I_N \otimes A_a)x(t) + (I_N \otimes A_b)f(x(t)) + q_0(I_N \otimes A_c) \\ &\times f(x(t-h_1(t))) + (1-q_0)(I_N \otimes A_c)f(x(t \\ &-h_2(t))) + (q(t)-q_0)(I_N \otimes A_c)[f(x(t-h_1(t)) \\ &-f(x(t-h_2(t)))] + (I_N \otimes A_d)\int_{t-d(t)}^{t} f(x(s))ds \\ &+ (\bar{\alpha} - \alpha(t-\eta(t)) - \bar{\alpha})(\mathcal{L}\otimes B)Kx(t-\eta(t)).\end{aligned} \tag{5}$$

Definition 1 [14]**:** The consensus of system is said to be achieved asymptotically in the sense of mean-square if, for each agent $i \in \{1, 2, ..., N\}$, there is a local state feedback u_i of $x_i : j \in N_i$ such that the closed loop system satisfies $\lim_{t\to\infty} \mathbb{E}\{\|x_i(t) - x_j(t)\|^2\} = 0$.

Lemma 1 [24]**:** For any constant matrix $M \in \mathbb{R}^{n\times n}$, $M^T = M > 0$, scalars α and β with $\alpha > \beta$ and vector $x : [\beta, \alpha] \to \mathbb{R}^n$, such that the following integrations are well defined, then

$$\begin{aligned}&-(\alpha-\beta)\int_{\beta}^{\alpha} x^T(s)Mx(s)ds \\ &\leq -\left(\int_{\beta}^{\alpha} x(s)ds\right)^T M \left(\int_{\beta}^{\alpha} x(s)ds\right), \\ &-\frac{(\alpha-\beta)^2}{2}\int_{\beta}^{\alpha}\int_{u}^{\alpha} x^T(s)Mx(s)dsdu \\ &\leq -\left(\int_{\beta}^{\alpha}\int_{u}^{\alpha} x(s)dsdu\right)^T M \left(\int_{\beta}^{\alpha}\int_{u}^{\alpha} x(s)dsdu\right).\end{aligned}$$

Lemma 2 [25]**:** For any constant positive matrix $V \in \mathbb{R}^{n\times n}$, scalar $0 \leq \tau(t) \leq \tau$, and vector function $\dot{x}(t) : [-\tau, 0] \to \mathbb{R}^n$, it holds that

$$\begin{aligned}&-\tau\int_{t-\tau}^{t} \dot{x}^T(s)V\dot{x}(s)ds \\ &\leq \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau) \end{bmatrix}^T \begin{bmatrix} -V & V & 0 \\ * & -2V & V \\ * & * & -V \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau) \end{bmatrix}.\end{aligned} \tag{6}$$

Lemma 3 [5]**:** For any given X, $Y \in \mathbb{R}^n$, matrices $F > 0$, Γ and Σ have appropriate dimensions, one has $-2X^T\Gamma\Sigma Y \leq X^T\Gamma F\Gamma^T X + Y^T\Sigma^T F^{-1}\Sigma Y$.

3. MAIN RESULTS

In this section, the consensus criteria for the closed loop system (7) can be achieved in the form of LMIs using the sampled-data controller.

Theorem 1: For any known positive scalars $0 < h_1 < h_2$, μ_1, μ_2, q_0, the consensus of system (7) can be reached, if there exist positive definite symmetric matrices $P > 0$, $Q_i > 0$, $R_i > 0$, $S_i > 0$, $T_j > 0$, $(i = 1, ..., 4)$, $(j = 1, 2)$ and positive diagonal matrices M, $\mathbf{N}_1$, O, any matrix W with appropriate dimensions such that the following inequality hold:

$$[\breve{\Psi}]_{15\times 15} < 0, \tag{7}$$

where

$$\begin{aligned}
\breve{\Psi}_{11} &= (I_N\otimes Q_1)+(I_N\otimes Q_2)+(I_N\otimes Q_3)+(I_N\otimes Q_4)\\
&\quad +(I_N\otimes R_3)-(1-\mu_1)(I_N\otimes S_2)-(1-\mu_2)(I_N\otimes S_3)\\
&\quad -(I_N\otimes S_4)-h_1^2(I_N\otimes T_1)-\Lambda_1(I_N\otimes M)+(I_N\otimes W)\\
&\quad \times(I_N\otimes A_a)+(I_N\otimes A_a)^T(I_N\otimes W)^T+\bar{\alpha}(\mathcal{L}\mathcal{L}^T\otimes BB^T)X,\\
\breve{\Psi}_{12} &= (I_N\otimes P)-(I_N\otimes W)+(I_N\otimes W)(I_N\otimes A_a),\\
\breve{\Psi}_{13} &= (1-\mu_1)(I_N\otimes S_2), \breve{\Psi}_{14} = (1-\mu_2)(I_N\otimes S_3),\\
\breve{\Psi}_{17} &= (I_N\otimes S_4),\\
\breve{\Psi}_{19} &= \Lambda_2(I_N\otimes M)+(I_N\otimes W)(I_N\otimes A_b),\\
\breve{\Psi}_{1,10} &= q_0(I_N\otimes W)(I_N\otimes A_c),\\
\breve{\Psi}_{1,11} &= (1-q_0)(I_N\otimes W)(I_N\otimes A_c),\\
\breve{\Psi}_{1,12} &= h_1(I_N\otimes T_1), \quad \breve{\Psi}_{1,15} = (I_N\otimes W)(I_N\otimes A_d),\\
\breve{\Psi}_{22} &= h_2^2(I_N\otimes S_1)+h_1^2(I_N\otimes S_2)+h_2^2(I_N\otimes S_3)+\eta^2(I_N\otimes S_4)\\
&\quad -(I_N\otimes W)-(I_N\otimes W)^T+\bar{\alpha}(\mathcal{L}\mathcal{L}^T\otimes BB^T)X\\
&\quad +\frac{h_1^4}{4}(I_N\otimes T_1)+\frac{h_{21}^4}{4}(I_N\otimes T_2),\\
\breve{\Psi}_{29} &= (I_N\otimes W)(I_N\otimes A_b),\\
\breve{\Psi}_{2,10} &= q_0(I_N\otimes W)(I_N\otimes A_c),\\
\breve{\Psi}_{2,15} &= (I_N\otimes W)(I_N\otimes A_d), \quad \breve{\Psi}_{3,10} = \Lambda_2(I_N\otimes \mathbf{N}_1),\\
\breve{\Psi}_{33} &= -(1-\mu_1)(I_N\otimes Q_1)-(1-\mu_1)(I_N\otimes S_2)\\
&\quad -\Lambda_1(I_N\otimes \mathbf{N}_1),\\
\breve{\Psi}_{2,11} &= (1-q_0)(I_N\otimes W)(I_N\otimes A_c),\\
\breve{\Psi}_{44} &= -(1-\mu_2)(I_N\otimes Q_3)-(1-\mu_2)(I_N\otimes S_3)\\
&\quad -\Lambda_1(I_N\otimes O),\\
\breve{\Psi}_{4,11} &= \Lambda_2(I_N\otimes O),\\
\breve{\Psi}_{55} &= -(I_N\otimes Q_2)-h_{21}^2(I_N\otimes T_2),\\
\breve{\Psi}_{5,13} &= h_{21}(I_N\otimes T_2),\\
\breve{\Psi}_{66} &= -(I_N\otimes Q_4), \breve{\Psi}_{77} = -2(I_N\otimes S_4)+2\bar{\alpha},\\
\breve{\Psi}_{78} &= (I_N\otimes S_4), \quad \breve{\Psi}_{88} = -(I_N\otimes R_3)-(I_N\otimes S_4),\\
\breve{\Psi}_{99} &= (I_N\otimes R_1)+(I_N\otimes R_2)-(I_N\otimes M)\\
&\quad +d^2(I_N\otimes R_4),\\
\breve{\Psi}_{10,10} &= -(1-\mu_1)(I_N\otimes R_1)-(I_N\otimes \mathbf{N}_1),\\
\breve{\Psi}_{11,11} &= -(I_N\otimes O)-(1-\mu_2)(I_N\otimes R_2),\\
\breve{\Psi}_{12,12} &= -(I_N\otimes T_1), \quad \breve{\Psi}_{13,13} = -(I_N\otimes T_2),\\
\breve{\Psi}_{14,14} &= -(I_N\otimes S_1), \quad \breve{\Psi}_{15,15} = -(I_N\otimes R_4),
\end{aligned}$$

and other terms are zero. Moreover the control gain matrix is given as $K = W^{-1}X$.

Proof: We construct Lyapunov-Krasovskii functional as follows:

$$\begin{aligned}
V(t,x_t) =& V_1(t,x_t)+V_2(t,x_t)+V_3(t,x_t)+V_4(t,x_t)\\
&+V_5(t,x_t),
\end{aligned}$$

where

$$\begin{aligned}
V_1(t,x_t) &= x^T(t)(I_N\otimes P)x(t),\\
V_2(t,x_t) &= \int_{t-h_1(t)}^{t} x^T(s)(I_N\otimes Q_1)x(s)ds\\
&\quad +\int_{t-h_1}^{t} x^T(s)(I_N\otimes Q_2)x(s)ds\\
&\quad +\int_{t-h_2(t)}^{t} x^T(s)(I_N\otimes Q_3)x(s)ds\\
&\quad +\int_{t-h_2}^{t} x^T(s)(I_N\otimes Q_4)x(s)ds,\\
V_3(t,x_t) &= \int_{t-h_1(t)}^{t} f^T(x(s))(I_N\otimes R_1)f(x(s))ds\\
&\quad +\int_{t-h_2(t)}^{t} f^T(x(s))(I_N\otimes R_2)f(x(s))ds\\
&\quad +\int_{t-\eta}^{t} x^T(s)(I_N\otimes R_3)x(s)ds\\
&\quad +d\int_{-d(t)}^{0}\int_{t+\beta}^{t} f^T(x(s))(I_N\otimes R_4)f(x(s))ds,\\
V_4(t,x_t) &= h_2\int_{-h_2}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)(I_N\otimes S_1)\dot{x}(s)dsd\beta\\
&\quad +h_1\int_{-h_1(t)}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)(I_N\otimes S_2)\dot{x}(s)dsd\beta\\
&\quad +h_2\int_{-h_2(t)}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)(I_N\otimes S_3)\dot{x}(s)dsd\beta\\
&\quad +\eta\int_{-\eta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)(I_N\otimes S_4)\dot{x}(s)dsd\beta,\\
V_5(t,x_t) &= \frac{h_1^2}{2}\int_{t-h_1}^{t}\int_{\gamma}^{t}\int_{\beta}^{t} \dot{x}^T(s)(I_N\otimes T_1)\dot{x}(s)dsd\beta d\gamma\\
&\quad +\frac{h_{21}^2}{2}\int_{t-h_2}^{t-h_1}\int_{\gamma}^{t-h_1}\int_{\beta}^{t-h_1} \dot{x}^T(s)(I_N\otimes T_2)\dot{x}(s)dsd\beta d\gamma,
\end{aligned}$$

with $h_{21} = h_2 - h_1$. Then by calculating the derivative of $V(t,x_t)$ along the trajectories of system, we get

$$\begin{aligned}
LV_1(t,x_t) &= 2x^T(t)(I_N\otimes P)\dot{x}(t),\\
LV_2(t,x_t) &\le x^T(t)[(I_N\otimes Q_1)+(I_N\otimes Q_2)+(I_N\otimes Q_3)\\
&\quad +(I_N\otimes Q_4)]x(t)-(1-\mu_1)x^T(t-h_1(t))
\end{aligned}$$

$$\times (I_N\otimes Q_1)x(t-h_1(t))-x^T(t-h_1)$$
$$\times (I_N\otimes Q_2)x(t-h_1)-(1-\mu_2)x^T$$
$$\times (t-h_2(t))(I_N\otimes Q_3)x(t-h_2(t))$$
$$-x^T(t-h_2)(I_N\otimes Q_4)x(t-h_2),$$

$$\begin{aligned}LV_3(t,x_t) = {} & f^T(x(t))[(I_N\otimes R_1)+(I_N\otimes R_2)]f(x(t))\\ & -(1-\mu_1)f^T(x(t-h_1(t)))(I_N\otimes R_1)\\ & \times f(x(t-h_1(t)))-(1-\mu_2)f^T(x(t-h_2(t)))\\ & \times(I_N\otimes R_2)f(x(t-h_2(t)))+x^T(t)(I_N\otimes R_3)\\ & \times x(t)-x(t-\eta)(I_N\otimes R_3)x(t-\eta)\\ & +d^2f^T(x(t))(I_N\otimes R_4)f(x(t))\\ & -d\int_{t-d(t)}^{t}f^T(x(s))(I_N\otimes R_4)f(x(s))ds.\end{aligned}$$

By using Lemma 1, the above integral terms will become

$$\begin{aligned}& -d\int_{t-d(t)}^{t}f^T(x(s))(I_N\otimes R_4)f(x(s))ds\\ & \le \Big(\int_{t-d(t)}^{t}f(x(s))ds\Big)^T(I_N\otimes R_4)\Big(\int_{t-d(t)}^{t}f(x(s))ds\Big),\end{aligned}$$

$$\begin{aligned}LV_4(t,x_t) = {} & h_1^2\dot{x}^T(t)[h_2^2(I_N\otimes S_1)\\ & +h_2^2(I_N\otimes S_2)+h_2^2(I_N\otimes S_3)\\ & +\eta^2(I_N\otimes S_4)]\dot{x}(t)\\ & -h_2\int_{t-h_2}^{t}\dot{x}^T(s)(I_N\otimes S_1)\dot{x}(s)ds\\ & -(1-\mu_1)h_1\int_{t-h_1(t)}^{t}\dot{x}^T(s)(I_N\otimes S_2)\dot{x}(s)ds\\ & -(1-\mu_2)h_2\int_{t-h_2(t)}^{t}\dot{x}^T(s)(I_N\otimes S_3)\dot{x}(s)ds\\ & -\eta\int_{t-\eta}^{t}\dot{x}^T(s)(I_N\otimes S_4)\dot{x}(s)ds.\end{aligned}$$

By using Lemma 1, in the above integral term,

$$\begin{aligned}& -h_2\int_{t-h_2}^{t}\dot{x}^T(s)(I_N\otimes S_1)\dot{x}(s)ds\\ & \le -\left(\int_{t-h_2}^{t}\dot{x}(s)ds\right)(I_N\otimes S_1)\left(\int_{t-h_2}^{t}\dot{x}(s)\right)ds. \qquad (8)\end{aligned}$$

By using Lemma 1, and then simplifying the terms we get

$$\begin{aligned}& -(1-\mu_1)h_1\int_{t-h_1(t)}^{t}\dot{x}^T(s)(I_N\otimes S_2)\dot{x}(s)ds\\ & \le -(1-\mu_1)[x(t)-x(t-h_1(t))]^T(I_N\otimes S_2)\\ & \quad \times[x(t)-x(t-h_1(t))], \qquad (9)\\ & -(1-\mu_2)h_2\int_{t-h_2(t)}^{t}\dot{x}^T(s)(I_N\otimes S_3)\dot{x}(s)ds\\ & \le -(1-\mu_2)[x(t)-x(t-h_2(t))]^T(I_N\otimes S_3)\\ & \quad \times[x(t)-x(t-h_2(t))]. \qquad (10)\end{aligned}$$

By using Lemma 2, we get

$$\begin{aligned}& \eta\int_{t-\eta}^{t}\dot{x}^T(s)(I_N\otimes S_4)\dot{x}(s)ds\\ & \le \begin{bmatrix}x(t)\\ x(t-\eta(t))\\ x(t-\eta)\end{bmatrix}^T\begin{bmatrix}-(I_N\otimes S_4) & (I_N\otimes S_4) & 0\\ * & -2(I_N\otimes S_4) & (I_N\otimes S_4)\\ * & * & -(I_N\otimes S_4)\end{bmatrix}\\ & \quad\times\begin{bmatrix}x(t)\\ x(t-\eta(t))\\ x(t-\eta)\end{bmatrix},\end{aligned}$$

$$\begin{aligned}LV_5(t,x_t) = {} & \frac{h_1^4}{4}\dot{x}^T(t)(I_N\otimes T_1)\dot{x}(t)\\ & +\frac{h_{21}^4}{4}\dot{x}^T(t-h_1)(I_N\otimes T_2)\dot{x}(t-h_1)\\ & -\frac{h_1^2}{2}\int_{t-h_1}^{t}\int_{\beta}^{t}\dot{x}^T(s)(I_N\otimes T_1)\dot{x}(s)dsd\beta\\ & -\frac{h_{21}^2}{2}\int_{t-h_2}^{t-h_1}\int_{\beta}^{t-h_1}\dot{x}^T(s)(I_N\otimes T_2)\dot{x}(s)dsd\beta.\end{aligned}$$

By using Lemma 1 in the above integral terms and simplifying, then we get

$$\begin{aligned}& -\frac{h_1^2}{2}\int_{t-h_1}^{t}\int_{\beta}^{t}\dot{x}^T(s)(I_N\otimes T_1)\dot{x}(s)dsd\beta\\ & \le -\left[h_1x(t)-\int_{t-h_1}^{t}x(s)ds\right]^T(I_N\otimes T_1)\left[h_1x(t)-\int_{t-h_1}^{t}x(s)ds\right],\\ & -\frac{h_{21}^2}{2}\int_{t-h_2}^{t-h_1}\int_{\beta}^{t-h_1}\dot{x}(s)(I_N\otimes T_2)\dot{x}(s)dsd\beta\\ & \le -\left[h_{21}x(t-h_1)-\int_{t-h_2}^{t-h_1}x(s)ds\right]^T(I_N\otimes T_2)\\ & \quad\times\left[h_{21}x(t-h_1)-\int_{t-h_2}^{t-h_1}x(s)ds\right].\end{aligned}$$

For positive diagonal matrices $M,\ \mathbf{N}_1,\ O$, from Assumption 1, we have

$$0\le\begin{bmatrix}x(t)\\ f(x(t))\end{bmatrix}^T\begin{bmatrix}-\Lambda_1(I_N\otimes M) & \Lambda_2(I_N\otimes M)\\ * & -(I_N\otimes M)\end{bmatrix}\begin{bmatrix}x(t)\\ f(x(t))\end{bmatrix},$$

$$\begin{aligned}0\le{} & \begin{bmatrix}x(t-h_1(t))\\ f(x(t-h_1(t)))\end{bmatrix}^T\begin{bmatrix}-\Lambda_1(I_N\otimes \mathbf{N}_1) & \Lambda_2(I_N\otimes \mathbf{N}_1)\\ * & -(I_N\otimes \mathbf{N}_1)\end{bmatrix}\\ & \times\begin{bmatrix}x(t-h_1(t))\\ f(x(t-h_1(t)))\end{bmatrix},\end{aligned}$$

$$\begin{aligned}0\le{} & \begin{bmatrix}x(t-h_2(t))\\ f(x(t-h_2(t)))\end{bmatrix}^T\begin{bmatrix}-\Lambda_1(I_N\otimes O) & \Lambda_2(I_N\otimes O)\\ * & -(I_N\otimes O)\end{bmatrix}\\ & \times\begin{bmatrix}x(t-h_2(t))\\ f(x(t-h_2(t)))\end{bmatrix}.\end{aligned}$$

For appropriated positive matrix $(I_N\otimes W)$, the following equation hold:

$$\begin{aligned}\mathbb{E}\Big\{ & 2[x^T(t)(I_N\otimes W)+\dot{x}(t)(I_N\otimes W)]\\ & \times\Big[-\dot{x}(t)+(I_N\otimes A_a)x(t)+(I_N\otimes A_b)f(x(t))\\ & +q_0(I_N\otimes A_c)f(x(t-h_1(t)))+(1-q_0)(I_N\otimes A_c)\\ & \times f(x(t-h_2(t)))+(q(t)-q_0)(I_N\otimes A_c)\end{aligned}$$

$$
\begin{aligned}
&\times [f(x(t-h_1(t))-f(x(t-h_2(t)))]+(I_N\otimes A_d)\\
&\times \int_{t-d(t)}^{t} f(x(s))ds)+(\bar{\alpha}-\alpha(t-\eta(t)-\bar{\alpha})\\
&\times (\mathcal{L}\otimes BK)x(t-\eta(t))\Big]\Big]\Big\}=0. \qquad (11)
\end{aligned}
$$

From the above equation we get

$$
\begin{aligned}
&-2x^T(t)(I_N\otimes W)\dot{x}(t)+2x^T(t)(I_N\otimes W)(I_N\otimes A_a)x(t)\\
&+2\dot{x}(t)(I_N\otimes W)(I_N\otimes A_b)f(x(t))\\
&+2x^T(t)q_0(I_N\otimes W)(I_N\otimes A_c)f(x(t-h_1(t)))\\
&+x^T(t)(1-q_0)(I_N\otimes W)(I_N\otimes A_c)f(x(t-h_2(t)))\\
&+x^T(t)(\bar{\alpha}-\bar{\alpha}(t-\eta(t))-\bar{\alpha})(I_N\otimes W)(\mathcal{L}\otimes B)K\\
&\times x(t-\eta(t))-2\dot{x}^T(t)(I_N\otimes W)\dot{x}(t)\\
&+2\dot{x}^T(t)(I_N\otimes W)(I_N\otimes A_a)x(t)\\
&+2\dot{x}^T(t)(I_N\otimes W)(I_N\otimes A_b)f(x(t))\\
&+2\dot{x}(t)q_0(I_N\otimes W)(I_N\otimes A_c)f(x(t-h_1(t)))\\
&+2\dot{x}^T(t)(1-q_0)(I_N\otimes W)(I_N\otimes A_c)f(x(t-h_2(t)))\\
&+2x^T(t)(I_N\otimes W)(I_N\otimes A_c)\int_{t-d(t)}^{t} f(x(s))ds\\
&+2\dot{x}(t)(I_N\otimes W)(I_N\otimes A_d)\int_{t-d(t)}^{t} f(x(s))ds\\
&+2x^T(t)(\bar{\alpha}-\alpha(t-\eta(t))-\bar{\alpha})(I_N\otimes W)(\mathcal{L}\otimes B)\\
&\times Kx(t-\eta(t))+2\dot{x}^T(t)(\bar{\alpha}-\alpha(t-\eta(t))-\bar{\alpha})\\
&\times (I_N\otimes W)(\mathcal{L}\otimes BK)x(t-\eta(t))=0. \qquad (12)
\end{aligned}
$$

By using Lemma 3,

$$
\begin{aligned}
&-2\bar{\alpha}x^T(t)(I_N\otimes W)(\mathcal{L}\otimes B)Kx(t-\eta(t))\\
&\leq \bar{\alpha}x^T(t)(\mathcal{L}\mathcal{L}^T\otimes BB^T)(I_N\otimes W)(I_N\otimes W)^T KK^T x(t)\\
&\quad+\bar{\alpha}x^T(t-\eta(t))x(t-\eta(t)), \qquad (13)\\
&-2\bar{\alpha}\dot{x}^T(t)(I_N\otimes W)(\mathcal{L}\otimes B)Kx(t-\eta(t))\\
&\leq \bar{\alpha}\dot{x}^T(t)(\mathcal{L}\mathcal{L}^T\otimes BB^T)(I_N\otimes W)(I_N\otimes W)^T KK^T \dot{x}(t)\\
&\quad+\bar{\alpha}x^T(t-\eta(t))x(t-\eta(t)). \qquad (14)
\end{aligned}
$$

From (11)-(14) and taking expectation, we can obtain that

$$
\mathbb{E}\{LV(t,x_t)\}\leq \mathbb{E}\{\zeta^T(t)[\breve{\Psi}]_{15\times 15}\zeta(t)\}, \qquad (15)
$$

where $\zeta^T(t)=\big[x^T(t)\ \ \dot{x}^T(t)\ \ x^T(t-h_1(t))\ \ x^T(t-h_2(t))\ \ x^T(t-h_1)\ \ x^T(t-h_2)\ \ x^T(t-\eta(t))\ \ x^T(t-\eta)\ \ f^T(x(t))\ \ f^T(x(t-h_1(t)))\ \ f^T(x(t-h_2(t)))\ \int_{t-h_1}^{t} x^T(s)ds\ \int_{t-h_2}^{t-h_1} x^T(s)ds\ \int_{t-h_2}^{t}\dot{x}^T(s)ds\ \int_{t-d(t)}^{t} f^T(x(s))ds\big]$.

From (9), it is noticed that $\mathbb{E}\{LV(t,x_t)\}<0$ which clearly implies that $\mathbb{E}\{\|x_i(t)-x_j(t)\|^2\}\to 0\ as\ t\to\infty$. Therefore, it concludes that the closed loop system (7) is mean square asymptotic stable. Then by Definition 1, the MASs (1) will reach the consensus in the mean square using the sampled data control. This completes the proof. □

Remark 1: If there is no distributed delay in MASs (5) then it will became

$$
\begin{aligned}
\dot{x}(t)=&(I_N\otimes A_a)x(t)+(I_N\otimes A_b)f(x(t))\\
&+q_0(I_N\otimes A_c)f(x(t-h_1(t)))\\
&+(1-q_0)(I_N\otimes A_c)f(x(t-h_2(t)))\\
&+(q(t)-q_0)(I_N\otimes A_c)\\
&\times [f(x(t-h_1(t))-f(x(t-h_2(t)))]ds\\
&+(\bar{\alpha}-\alpha(t-\eta(t))-\bar{\alpha})(\mathcal{L}\otimes BK)x(t-\eta(t)).
\end{aligned} \qquad (16)
$$

Corollary 1: For any known positive scalars $0<h_1<h_2$, μ_1, μ_2, q_0, the consensus of system (31) can be reached, if there exist positive definite symmetric matrices $P>0$, $Q_i>0$, $R_j>0$, $S_i>0$, $T_k>0$, $(i=1,...,4)$, $(j=1,2,3)$, $(k=1,2)$ and positive diagonal matrices M,$\mathbf{N}_1$, O, any matrix W with appropriate dimensions such that the following inequality hold:

$$
\Omega_1=\begin{bmatrix}
\breve{\Psi}_{11} & \breve{\Psi}_{12} & \breve{\Psi}_{13} & \breve{\Psi}_{14} & 0 & 0 & \breve{\Psi}_{17} & 0 & \breve{\Psi}_{19} & \breve{\Psi}_{1,10} & \breve{\Psi}_{1,11} & \breve{\Psi}_{1,12} & 0 & 0\\
* & \breve{\Psi}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & \breve{\Psi}_{29} & \breve{\Psi}_{2,10} & \breve{\Psi}_{2,11} & 0 & 0 & 0\\
* & * & \breve{\Psi}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & \breve{\Psi}_{3,10} & 0 & 0 & 0 & 0\\
* & * & * & \breve{\Psi}_{44} & 0 & \breve{\Psi}_{46} & 0 & 0 & 0 & 0 & \breve{\Psi}_{4,11} & 0 & 0 & 0\\
* & * & * & * & \breve{\Psi}_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \breve{\Psi}_{5,13} & 0\\
* & * & * & * & * & \breve{\Psi}_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & \breve{\Psi}_{77} & \breve{\Psi}_{78} & 0 & 0 & 0 & 0 & 0 & \\
* & * & * & * & * & * & * & \breve{\Psi}_{88} & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & \Omega_{199} & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & \breve{\Psi}_{10,10} & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & \breve{\Psi}_{11,11} & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & * & \breve{\Psi}_{12,12} & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & * & * & \breve{\Psi}_{13,13} & 0\\
* & * & * & * & * & * & * & * & * & * & * & * & * & \breve{\Psi}_{14,14}
\end{bmatrix}<0, \qquad (17)
$$

where $\Omega_{199}=(I_N\otimes R_1)+(I_N\otimes R_2)-(I_N\otimes M)$ and all other terms are already mentioned in Theorem 1 Moreover the control gain matrix is $K=W^{-1}X$.

Proof: As same as in Theorem 1. □

Remark 2: Sampled-data control is an increasingly substantial technique for nodes to convey in network environment, which settles the weakness of continuous-time control. Theorem 1 guarantees the consensus of non-linear multi-agent system. In the vast majority of the writing, the consistency issue for MASs has been examined in [20, 21, 23] by using sampled-data control. Be that as it may, in actuality, applications, the sampling intervals may meddle by the outside unsettling influence. By utilizing the algorithm suggested in Theorem 1, assures the consensus of multi-agent system via sampled-data control under undirected graph with random packet loss. In the presence of random packet losses, the control gain matrix is designed.

Remark 3: When compare to the existing results in [16] random packet loss is considered in this paper under undirected graph along with probabilistic time varying delay. The consensus of non-linear multi-agent system in (7) via sampled- data control with random packet loss illuminate the interplay among the sampling interval η, the probability of packet losses $\bar{\alpha}$, and the control gain matrix K.

4. NUMERICAL EXAMPLES

In this section, two examples are given to illustrate the effectiveness of the proposed method.
Let us consider a three agents i.e., $N = 3$ and each agents are interconnected with one another through undirected graph.

Example 1: Consider mixed delay nonlinear multi-agent system (7), with parameters as $\mu_1 = 0.2$, $\mu_2 = 0.7$, $h_1 = 0.3$, $h_2 = 1$, $d = 0.30$, $\bar{\alpha} = 0.5$, $q_0 = 0.6$ and $A_a = \begin{bmatrix} -2.5 & 0.1 \\ 0.3 & 0.05 \end{bmatrix}$, $A_b = \begin{bmatrix} 2 & -0.1 \\ -3 & 1.5 \end{bmatrix}$, $A_c = \begin{bmatrix} 2.5 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_d = \begin{bmatrix} -0.8 & 0.9 \\ 0.9 & -0.8 \end{bmatrix}$, $B = \begin{bmatrix} -0.6 & 0.7 \\ 0.7 & -0.6 \end{bmatrix}$, $\Lambda_1 = 0$, $\Lambda_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$.

The Laplacian matrix is $\mathcal{L} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$, and $f(x(t)) = \tanh(x(t))$. By solving the LMIs in Theorem 1, we obtain the control gain matrix as $K = W^{-1}X$.

$$K = \begin{bmatrix} -0.1659 & -0.5939 \\ 0.1892 & 0.6712 \end{bmatrix}.$$

Example 2: Consider the multi-agent system without distributed delay in (31), with parameters $\mu_1 = 0.3$, $\mu_2 = 0.6$, $h_1 = 0.3$, $h_2 = 1$, $\eta = 0.6$, $d = 0.30$, $\bar{\alpha} = 0.4$, $q_0 = 0.5$ and $A_a = \begin{bmatrix} -2.54 & 0.19 \\ 0.5 & 0.08 \end{bmatrix}$, $A_b = \begin{bmatrix} 3 & -0.1 \\ -3 & 1.5 \end{bmatrix}$, $A_c = \begin{bmatrix} 2.5 & 0 \\ 0 & 0.3 \end{bmatrix}$, $B = \begin{bmatrix} -0.6 & 0.7 \\ 0.7 & -0.6 \end{bmatrix}$, $\Lambda_1 = 0$, $\Lambda_2 = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix}$. By solving the LMIs in Corollary 1 with the same Laplacian matrix, the control gain matrix is obtained as

$$K = \begin{bmatrix} -0.4624 & -1.7199 \\ 0.8245 & 2.9753 \end{bmatrix}.$$

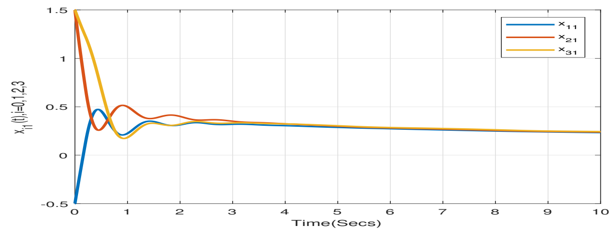

Fig. 1. State trajectory of the system in Example 1.

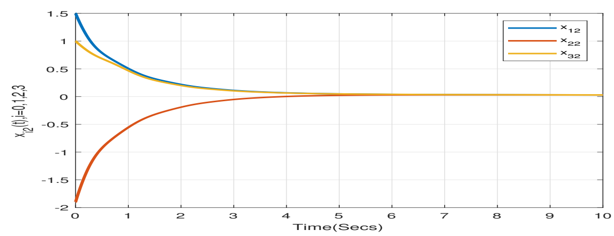

Fig. 2. State trajectory of the system in Example 1.

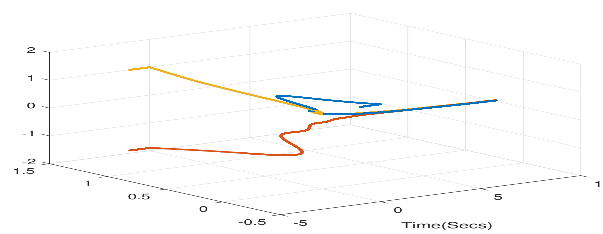

Fig. 3. State trajectory of the system in Example 1.

5. CONCLUSION

In this paper, consensus of non-linear mixed delay multi agent systems with probabilistic time delay and random packet losses through the sampled-data control have been studied. By utilizing a suitable Lyapunov-Krasovskii functional with some integral inequality approach, sufficient conditions are obtained through LMIs which can be easily checked numerically by using the effective and well known LMI tool box in MATLAB and it guarantees the closed-loop multi-agent system to reach mean square consensus and also get the value of controller gain matrix. Furthermore, two numerical examples are given to show the validity of the obtained results. The state trajectories of Examples 1 and 2 are given in Figs. 1-6.

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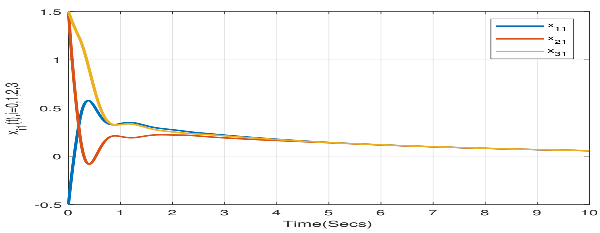

Fig. 4. State trajectory of the system in Example 2.

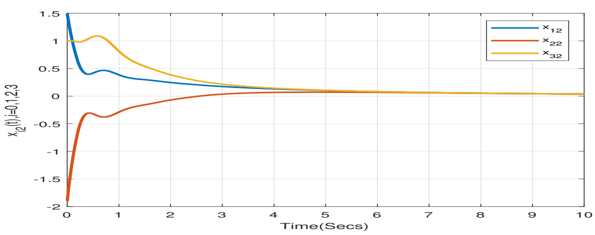

Fig. 5. State trajectory of the system in Example 2.

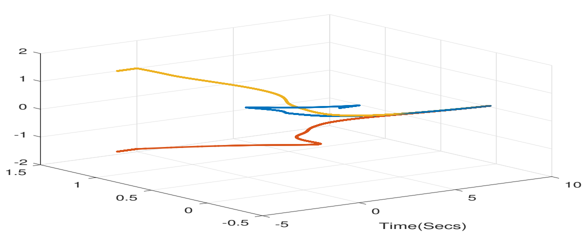

Fig. 6. State trajectory of the system in Example 2.

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