

# Event-triggered Stabilization of Linear Time-delay Systems by Static Output Feedback Control

Xiaoli Wang, Peng Xiang, Wenfeng Hu\* , and Tingwen Huang

**Abstract:** In this paper, we study the stabilization problem for a class of linear systems with a time-varying state delay. An event-triggered static output feedback controller is proposed, such that the sampling frequency and the update time of the controller are both lowered. First, we present a novel event-triggering mechanism depending not only on the output but also on an exponential term, with which less sampling is required and Zeno behavior can be excluded at the same time. Some sufficient conditions are then obtained, under which an exponential convergence can be achieved by means of the comparison principle approach. It is further shown that the parameters design can be easily given if the case reduces to the state feedback control. Moreover, two examples are presented to show the effectiveness of the results.

**Keywords:** Event-triggered control, stabilization problem, static output feedback, time-varying delay.

## 1. INTRODUCTION

As one fundamental issue, stabilization problem has been extensively studied for both linear and nonlinear systems. Many control problems, including the networked control problem and the state estimation problem, can be eventually converted into the stabilization problems (see [1, 2]).

According to the information available for feedback control, the controller can be divided into state feedback and output feedback (see [3–7]). In particular, compared with the state feedback controller, the output feedback controller is more practical, because the system state may not be measurable in many scenarios. From a viewpoint of application, the static output feedback controller is easier for implementation, in contrast with the observer-based output feedback controller [8]. On the other hand, in practice, the time delay is unavoidable when the remote control systems through the Internet are distant from each other in the remote locations, which often leads to instability, oscillation, or other poor performance [9, 10]. In particular, compared with other types of time delays, the state delays are more general and inevitable. Recently, many attentions have been devoted to the stability analysis of the systems with time-varying delays, see [11–13] and the references therein.

In the implementation of the controllers, the information used is usually sampled periodically, which may cause the huge waste of energy. As an alternative method, the event-triggered control strategy can be used to lower the sampling frequency, which is quite important in a resource-limited environment [14]. Hence, the event-triggered control has received considerable attention in recent years and a lot of research results have been reported, such as [15–17, 19–21]. However, it is worth mentioning that the developed controllers in the aforementioned literatures are state dependent, and few event-triggered controllers are static output feedback.

Motivated by this observation, the event-triggered control for various systems based on output feedback has been recently studied, e.g., [22–27]. More specifically, in [22], the distributed control problem for large-scale systems was studied via static output feedback and an event-triggered control scheme was introduced to reduce the communication frequency. Under a distributed self-triggered control framework, the authors in [23] studied the consensus problem for multi-agent systems via local static output feedback or observer-type dynamic feedback. However, in [22–24], the time delays were neglected, which may cause instability or other poor performance. The considered time-varying state delay in this paper makes the approaches in the aforementioned litera-

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tures not applicable anymore. It is noted that many event-triggered control problems for linear multi-agent systems can be eventually converted to the stabilization problem of linear time-delay systems. In particular, in [25–27], the closed-loop system with the event-triggered mechanism was represented by a linear system with an interval time-varying delay, which is similar to the system in the paper. However, the developed techniques are different in the following two aspects. First, the event-triggered mechanisms in [25–27] were based on the sampled data, where the Zeno exclusion problem no longer needs to be considered due to the introduction of the sampler, while the event-triggered mechanism in this paper is based on the continuous signal. Second, different from the stability proof in [25–27] where the Lyapunov-Krasovskii functional approach was utilized and the derived sufficient conditions were given in the form of some linear matrix inequalities (LMIs), this paper adopts the comparison principle approach and the proof by contradiction, and a relatively simple inequality condition is obtained with less dimension and computational complexity. In this case, it is further shown that under some extra conditions, the solutions to such inequality can be guaranteed. Moreover, with our approach, the exponential convergence is achieved with the convergence rate being able to be estimated. Until recently, there have been some results on the event-triggered output-feedback control problem with time delays [28, 29]. However, the authors in [28] were concerned with the stability of the input feed-forward output feedback passive system. In [29], the observer-based dynamic output-feedback approach was applied, which makes the implementation of the controller relatively complicated.

In this paper, by incorporating a novel event-triggering mechanism into a static output feedback controller, we investigate the event-triggered stabilization problem for a class of linear systems subject to a time-varying state delay. In comparison with some existing results, the main contributions of this paper lie in the following four aspects:

- The stabilization problem for a class of linear systems with a time-varying state delay is addressed. Under the obtained sufficient conditions, an exponential convergence can be achieved by means of the comparison principle approach.
- We propose a novel event-triggered control scheme, where both the controller and the triggering mechanism are only based on the measured output without involving any extra dynamic observer. Besides, different from those state-dependent triggering mechanisms in [15–17], an exponential term is introduced in the triggering mechanism. The proposed triggering mechanism is somehow different from the solely time-dependent triggering mechanism [18] in the sense that our triggering mechanism incorporates not only a static output-feedback term but also an exponential term. With the developed event-triggering mechanism, the time interval of two consecutive triggering times can be further prolonged, and Zeno behavior can be strictly excluded.
- In contrast to the event-triggered dynamic output feedback controllers [29, 30], which rely on the observer-based approach, this paper develops the event-triggered static output feedback controller, which is relatively simple for implementation.
- If the states are available for feedback control, the derived results can also be applied to such scenario, under which the solutions always exist, and the event-triggered control law can be easily designed.

The rest of the paper is organized as follows: The problem is formulated in Section 2, and the main results are derived in Section 3. The Zeno behavior exclusion problem is discussed in Section 4, and a numerical example is presented in Section 5 to verify the obtained results. The conclusion is further drawn in Section 6.

**Notations:** Throughout this paper,  $\mathbb{R}$  is the set of real numbers,  $\mathbb{R}^n$  denotes the  $n$  dimensional Euclidean space,  $\mathbb{R}^{n \times m}$  is the real matrices set with dimension  $n \times m$ , and  $\mathbb{C}^{n \times n}$  is the complex matrices set with dimension  $n \times n$ . The notation  $X \leq Y$  (respectively,  $X < Y$ ), where  $X$  and  $Y$  are symmetric matrices, means that the matrix  $X - Y$  is negative semidefinite (respectively, negative definite). For vector  $x \in \mathbb{R}^n$ ,  $\|x\| = \sqrt{x^T x}$ . For matrix  $A \in \mathbb{R}^{n \times n}$ ,  $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$ ,  $A^H$  denotes the conjugate transpose of  $A$ ,  $\lambda_{\max}(A)$  and  $\lambda_{\min}(A)$  represents the largest and minimum eigenvalue of matrix  $A$ , respectively.  $I_n$  denotes the  $n$  dimensional unit matrix.

## 2. PROBLEM FORMULATION

Consider the following linear system with the time-varying state delay

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Cx(t - \tau(t)), \\ y(t) &= Dx(t), \end{aligned} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the system state,  $u(t) \in \mathbb{R}^l$  is the control input,  $y(t) \in \mathbb{R}^m$  is the measured output and  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times l}$ ,  $C \in \mathbb{R}^{m \times n}$ ,  $D \in \mathbb{R}^{m \times n}$  are constant system matrices. Besides,  $\tau(t)$  describes the unknown time-varying delay.

We will consider a static output feedback controller of the following form

$$u(t) = Ky(t_i), \quad t \in [t_i, t_{i+1}), \quad (2)$$

where  $K \in \mathbb{R}^{l \times m}$  is the gain matrix to be designed, and the time instants  $t_i (i \in N)$  are considered as the execution times when the plant's output is sampled and they satisfy  $t_0 < t_1 < \dots < t_i < \dots$ .

To determine  $t_i$ , we consider the following triggering mechanism that based on the output information,

$$t_{i+1} = \inf\{t > t_i | e^T(t)e(t) \geq \alpha y^T(t)y(t) + \beta_1 e^{-\beta_2 t}\}, \quad (3)$$

where  $\alpha$ ,  $\beta_1$  and  $\beta_2$  are design parameters, and  $e(t)$  is the measurement error defined as

$$e(t) = y(t_i) - y(t), \quad t \in [t_i, t_{i+1}). \quad (4)$$

With (2) and (4), the closed-loop system is then written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BK(e(t) + Dx(t)) + Cx(t - \tau(t)) \\ &= (A + BKD)x(t) + Cx(t - \tau(t)) + BKe(t). \end{aligned} \quad (5)$$

**Definition 1:** Consider the linear time-delay system (1). Design a static output feedback controller of form (2) with the event-triggering mechanism (3), such that the closed-loop system (5) is exponentially stable. We say that the event-triggered stabilization of system (1) is achieved by static output feedback control.

### 3. MAIN RESULTS

Before obtaining the main results, we need the following assumptions and lemmas:

**Assumption 1:**  $(A, B)$  is stabilizable.

**Assumption 2:** The unknown time-varying delay satisfies  $0 < \tau(t) < \tau$ , where  $\tau$  is a positive constant.

**Lemma 1** (Lemma 4.6.11 of [31]): If  $P \in \mathbb{R}^{n \times n}$  is a symmetric and positive definite matrix and  $Q \in \mathbb{R}^{n \times n}$  is a symmetric matrix, then there must exist a nonsingular matrix  $T \in \mathbb{C}^{n \times n}$  such that

$$T^H P T = I_n, \quad T^H Q T = \Lambda,$$

where  $\Lambda$  is a diagonal matrix which can be denoted as  $\Lambda = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n]$ .

**Lemma 2:** If  $P \in \mathbb{R}^{n \times n}$  is a symmetric and positive definite matrix and  $Q \in \mathbb{R}^{n \times n}$  is a symmetric matrix, then

$$\lambda_{\min}(P^{-1}Q)x^T P x \leq x^T Q x \leq \lambda_{\max}(P^{-1}Q)x^T P x.$$

**Proof:** Based on Lemma 1, let  $T = [\theta_1, \dots, \theta_n]$  where  $\theta_i$  represents the  $i$ th column vector of matrix  $T$ . From Lemma 1, we have  $QT = (T^H)^{-1}\Lambda = P T \Lambda = P[\lambda_1 \theta_1, \dots, \lambda_n \theta_n]$ . Thus,  $Q\theta_i = \lambda_i P \theta_i$ . Since  $P$  is nonsingular, the equation can be converted to  $P^{-1}Q\theta_i = \lambda_i \theta_i$ , which implies that  $\lambda_i$  is the eigenvalue of  $P^{-1}Q$  associated with eigenvector  $\theta_i$ . Obviously,  $\lambda_{\min}(P^{-1}Q) \leq \lambda_i \leq \lambda_{\max}(P^{-1}Q)$ . Note that  $Q = (T^{-1})^H \Lambda T^{-1}$  and  $(T^{-1})^H \lambda_{\min}(P^{-1}Q) T^{-1} \leq (T^{-1})^H \Lambda T^{-1} \leq (T^{-1})^H \lambda_{\max}(P^{-1}Q) T^{-1}$ . Then, we can get

$$(T^{-1})^H \lambda_{\min}(P^{-1}Q) T^{-1} \leq Q$$

$$\leq (T^{-1})^H \lambda_{\max}(P^{-1}Q) T^{-1}. \quad (6)$$

By noting that  $P = (T^{-1})^H T^{-1}$ , left multiplying (6) by  $x^T$  and right multiplying (6) by  $x$  yields that

$$\begin{aligned} \lambda_{\min}(P^{-1}Q)x^T P x &\leq x^T Q x \\ &\leq \lambda_{\max}(P^{-1}Q)x^T P x. \end{aligned} \quad \square$$

Then, we are ready to present the main result of this paper, which can be summarized as the following theorem.

**Theorem 1:** Under Assumptions 1 and 2, consider the linear time-delay system (1) with the control law (2). Suppose  $B$  has full row rank. If there exists a symmetric and positive definite matrix  $P \in \mathbb{R}^{n \times n}$ , a matrix  $K \in \mathbb{R}^{l \times m}$  and two constants  $\rho > 0$ ,  $\mu > \eta = \frac{1}{\lambda_{\min}(P)}$ , such that the following inequality is satisfied,

$$\begin{aligned} PA + A^T P + PBKD + (PBKD)^T + PCC^T P \\ + \frac{PBK K^T B^T P}{\rho} \leq -\mu P, \end{aligned} \quad (7)$$

then the closed-loop system is exponentially stable with the properly designed event-triggering mechanism (3). The parameters are chosen as follows,  $\alpha = \frac{\gamma \lambda_{\min}(P)}{\rho \|D\|^2}$ ,  $\beta_2 > \mu - \gamma$  and  $0 < \beta_1 \leq \frac{\beta_2 + \gamma - \mu}{\rho}$ , where  $0 < \gamma < \mu - \eta$ . Furthermore, the convergence rate  $\sigma$  is determined by the solution of  $\sigma + \eta e^{\sigma \tau} = \mu - \gamma$ .

**Proof:** Consider the following Lyapunov function

$$V(t) = x^T(t) P x(t).$$

Then, the time derivative of the Lyapunov function along the trajectory of (5) is

$$\begin{aligned} \dot{V}(t) &= 2x^T(t) P \dot{x}(t) \\ &= 2x^T(t) P [(A + BKD)x(t) + Cx(t - \tau(t)) \\ &\quad + BKe(t)]. \end{aligned} \quad (8)$$

By using some matrices inequality, one can derive that

$$\begin{aligned} &2x^T(t) P C x(t - \tau) \\ &\leq x^T(t) P C C^T P x(t) + x^T(t - \tau(t)) x(t - \tau(t)) \\ &\leq x^T(t) P C C^T P x(t) \\ &\quad + \frac{1}{\lambda_{\min}(P)} x^T(t - \tau(t)) P x(t - \tau(t)), \end{aligned} \quad (9)$$

$$\begin{aligned} &2x^T(t) P B K e(t) \\ &\leq \frac{x^T(t) P B K K^T B^T P x(t)}{\rho} + \rho e(t)^T e(t), \end{aligned} \quad (10)$$

where  $\rho > 0$  is a scalar. On the other hand, it follows from the event-triggering condition (3) and Lemma 2 that

$$e^T(t)e(t) \leq \alpha y^T(t)y(t) + \beta_1 e^{-\beta_2 t}$$

$$\begin{aligned}
&= \alpha x^T(t) D^T D x(t) + \beta_1 e^{-\beta_2 t} \\
&\leq \frac{\alpha \|D\|^2}{\lambda_{\min}(P)} x^T(t) P x(t) + \beta_1 e^{-\beta_2 t}. \quad (11)
\end{aligned}$$

Denoting  $\eta = \frac{1}{\lambda_{\min}(P)}$ ,  $\gamma = \frac{\rho \alpha \|D\|^2}{\lambda_{\min}(P)}$  and taking (9), (10), (11) into (8) yields

$$\begin{aligned}
\dot{V}(t) = & x^T(t) \left[ PA + A^T P + PBKD + (PBKD)^T \right. \\
& \left. + PCC^T P + \frac{PBKK^T B^T P}{\rho} \right] x(t) + \gamma x^T(t) P x(t) \\
& + \eta x^T(t - \tau(t)) P x(t - \tau(t)) + \rho \beta_1 e^{-\beta_2 t}. \quad (12)
\end{aligned}$$

It follows from (7) that

$$\dot{V}(t) \leq (-\mu + \gamma)V(t) + \eta V(t - \tau(t)) + \rho \beta_1 e^{-\beta_2 t}.$$

By using the Comparison Lemma [32], we can get the inequality as follows:

$$\begin{aligned}
V(t) \leq & M e^{(-\mu + \gamma)(t - t_0)} \\
& + \int_{t_0}^t e^{(-\mu + \gamma)(t - s)} [\eta V(s - \tau(s)) + \rho \beta_1 e^{-\beta_2 s}] ds, \quad (13)
\end{aligned}$$

where  $M = \sup_{t_0 - \tau < t < t_0} V(t)$ . To complete the proof, the mathematical induction approach will be used. First, we will consider the following two cases:  $t \in [t_0 - \tau, t_0]$  and  $t \in (t_0, +\infty)$ .

1) If  $t \in [t_0 - \tau, t_0]$ , we can easily get

$$V(t) \leq M < (M + 1)e^{-\sigma(t - t_0)},$$

where  $\sigma > 0$  is a constant.

2) If  $t \in (t_0, +\infty)$ , the proof is completed by contradiction. Suppose that  $V(t) < (M + 1)e^{-\sigma(t - t_0)}$  does not hold for some  $t'$ , that is, there exists  $t^* \in (t_0, t']$  such that  $t^* = \inf\{t > t_0 | V(t) \geq (M + 1)e^{-\sigma(t - t_0)}\}$ . Then, we have

$$V(t^*) = (M + 1)e^{-\sigma(t^* - t_0)}, \quad (14)$$

and

$$V(t) < (M + 1)e^{-\sigma(t - t_0)},$$

for  $t_0 < t < t^*$ . Then, according to (13), one has

$$\begin{aligned}
V(t^*) \leq & M e^{(-\mu + \gamma)(t^* - t_0)} \\
& + \int_{t_0}^{t^*} e^{(-\mu + \gamma)(t^* - s)} [\eta V(s - \tau(s)) + \rho \beta_1 e^{-\beta_2 s}] ds \\
\leq & e^{(-\mu + \gamma)(t^* - t_0)} \left\{ M + \int_{t_0}^{t^*} e^{(\mu - \gamma)(s - t_0)} [\rho \beta_1 e^{-\beta_2 s} \right. \\
& \left. + \eta(M + 1)e^{-\sigma(s - \tau(s) - t_0)}] ds \right\} \\
\leq & e^{(-\mu + \gamma)(t^* - t_0)} \left\{ M + \int_{t_0}^{t^*} \rho \beta_1 e^{(\mu - \gamma - \beta_2)s - (\mu - \gamma)t_0} ds \right. \\
& \left. + (M + 1)\eta e^{\sigma\tau} \int_{t_0}^{t^*} e^{(\mu - \gamma - \sigma)(s - t_0)} ds \right\}. \quad (15)
\end{aligned}$$

Furthermore, it follows from  $\mu - \gamma > 0$  that

$$0 < e^{(\mu - \gamma - \beta_2)s - (\mu - \gamma)t_0} < e^{(\mu - \gamma - \beta_2)s}.$$

Then (15) can be rewritten as

$$\begin{aligned}
V(t^*) &< e^{(-\mu + \gamma)(t^* - t_0)} \{M + (M + 1)[e^{(\mu - \gamma - \sigma)(t^* - t_0)} - 1] \\
& \quad + \int_{t_0}^{t^*} \rho \beta_1 e^{(\mu - \gamma - \beta_2)s} ds\} \\
&= e^{(-\mu + \gamma)(t^* - t_0)} \{ (M + 1)e^{(\mu - \gamma - \sigma)(t^* - t_0)} - 1 \\
& \quad + \int_{t_0}^{t^*} \rho \beta_1 e^{-(\mu + \gamma + \beta_2)s} ds \},
\end{aligned}$$

where the equation  $\sigma + \eta e^{\sigma\tau} = \mu - \gamma$  is used. According to the fact that the exponential distribution function satisfies  $\int_0^\infty \sigma e^{-\sigma s} ds \leq 1$ , ( $\sigma > 0$ ), we can get  $\int_{t_0}^{t^*} \rho \beta_1 e^{-(\mu + \gamma + \beta_2)s} ds \leq 1$  if  $0 < \rho \beta_1 \leq -\mu + \gamma + \beta_2$ . Then

$$\begin{aligned}
V(t^*) &< e^{(-\mu + \gamma)(t^* - t_0)} \{ (M + 1)e^{(\mu - \gamma - \sigma)(t^* - t_0)} \} \\
&= (M + 1)e^{-\sigma(t^* - t_0)},
\end{aligned}$$

which is contradictory with (14) and therefore we have  $V(t) < (M + 1)e^{-\sigma(t - t_0)}$  for all  $t \geq t_0 - \tau$ . Then, it is concluded that the considered closed-loop system is exponentially stable.

In the end, we will discuss the existence of  $\sigma$ . Define  $f(\sigma) = \sigma + \eta e^{\sigma\tau} - \mu + \gamma$ . Obviously,  $f(\sigma)$  is a monotone increasing function since  $\dot{f}(\sigma) = 1 + \eta \tau e^{\sigma\tau} > 0$ . Then, as  $f(0) = \gamma + \eta - \mu < 0$  and  $\lim_{\sigma \rightarrow \infty} f(\sigma) = \infty$ , it follows from the Zero Point Theorem [33] that a positive solution  $\sigma > 0$  to  $\sigma + \eta e^{\sigma\tau} = \mu - \gamma$  always exists.  $\square$

It is noted that in some existing results on event-triggered output feedback control (see in [26, 28]), the conventional triggering mechanisms are usually given as follows:

$$t_{i+1} = \inf\{t > t_i | e^T(t) e(t) \geq \alpha y^T(t) y(t)\}. \quad (16)$$

Next, the following proposition shows that, under the same initial conditions, the next triggering time determined by (3) is larger than that given by (16).

**Proposition 1:** Let  $t_{i+1}^1$  be given by the rule (3) and  $t_{i+1}^2$  be given by the rule (16), then  $t_{i+1}^1 \geq t_{i+1}^2$ .

**Proof:** Suppose that  $t_{i+1}^1 < t_{i+1}^2$ . Then, from rule (16), we can get

$$e^T(t_{i+1}^1) e(t_{i+1}^1) < \alpha y^T(t_{i+1}^1) y(t_{i+1}^1). \quad (17)$$

Since  $t_{i+1}^1$  is the next triggering time determined by (3), we have

$$\begin{aligned}
e^T(t_{i+1}^1) e(t_{i+1}^1) &\geq \alpha y^T(t_{i+1}^1) y(t_{i+1}^1) + \beta_1 e^{-\beta_2 t_{i+1}^1} \\
&> \alpha y^T(t_{i+1}^1) y(t_{i+1}^1),
\end{aligned}$$

which contradicts (17). Therefore,  $t_{i+1}^1 \geq t_{i+1}^2$ .

Given the previous discussions, it can be concluded that the introduction of the addition exponential term has two advantages. First, with the help of the exponential term, we would get a longer triggering time interval in comparison with the conventional output dependent triggering condition (16), while an exponential convergence is achieved. Second, the exponential term plays an important role in the Zeno exclusion proof, as shown in the proof of Theorem 2.

**Remark 1:** It is noted that Theorem 1 is a sufficient condition for the solvability of the stabilization problem of (1) with (2). The parameters of the controller and triggering mechanism are given at the same time. Besides, it can be easily seen that the decay rate  $\sigma$  is proportional to  $\beta_2$  while it is inversely proportional to the parameters  $\alpha$  and  $\beta_1$ . Since  $\sigma$  is the positive solution to equation  $\sigma + \eta e^{\sigma\tau} = \mu - \gamma$ , the larger  $\tau$  is, the smaller  $\sigma$  will be. Therefore, if the time delay increases,  $\alpha$  and  $\beta_1$  will increase, while  $\beta_2$  will decrease.

To explicitly determine above parameters, the following algorithm is developed.

**Algorithm 1:**

- i) Choose a relatively large positive  $\mu > 0$ ;
- ii) Solve the following LMI to get  $P > 0$ ,  $W$ , and  $\rho > 0$ ,

$$\begin{bmatrix} PA + A^T P + WD + D^T W^T + \mu P & PC & W \\ C^T P & -I_n & 0 \\ W^T & 0 & -\rho \end{bmatrix} < 0,$$

if  $\mu \leq \frac{1}{\lambda_{\min}(P)}$ , go back to i, otherwise go to iii);

- iii) Solve  $W = PBK$  to get gain matrix  $K$ ;
- iv) Choose  $0 < \gamma < \mu - \eta$ ;
- v) Choose  $\alpha = \frac{\gamma \lambda_{\min}(P)}{\rho \|D\|^2}$ ,  $\beta_2 > \mu - \gamma$ , and  $0 < \beta_1 \leq \frac{\beta_2 + \gamma - \mu}{\rho}$ .

**Remark 2:** Since  $P$  is a positive definite matrix, we have  $BK = P^{-1}W$ . Let  $K = [k_1, k_2, \dots, k_m]$  and  $Z = P^{-1}W = [z_1, z_2, \dots, z_m]$  where  $k_i$  is a  $l$  dimensional column vector and  $z_i$  is a  $n$  dimensional column vector for any  $i = 1, 2, \dots, m$ . Then, the matrix equality  $W = PBK$  can be converted to the following  $m$  linear equations,

$$\begin{cases} Bk_1 = z_1, \\ Bk_2 = z_2, \\ \vdots \\ Bk_m = z_m. \end{cases}$$

Since  $B$  is full row rank, the above  $m$  linear equations with respect to  $k_i$  have unique solutions.

It is worth mentioning that the problem considered in the paper includes the state feedback control problem as a special case. In particular, by letting  $m = n$ ,  $D = I_n$ , the system reduces to

$$\dot{x}(t) = Ax(t) + Bu(t) + Cx(t - \tau(t)),$$

$$y(t) = x(t). \quad (18)$$

which implies that the state of the system is available for feedback control. In this case, the results in Theorem 1 can reduce to the state feedback controller. Before deriving the main result, we need the following lemma.

**Lemma 3:** Under Assumptions 1 and 2, for any sufficiently large  $\bar{\mu}$ , there always exists  $K \in \mathbb{R}^{l \times n}$  and a positive definite matrix  $P \in \mathbb{R}^{n \times n}$ , such that

$$PA + A^T P + PBK + K^T B^T P + \bar{\mu} P P = 0. \quad (19)$$

**Proof:** Under Assumption 1, it can be concluded that there always exists a matrix  $K$  such that  $(A + BK)$  is Hurwitz, and then so is  $(A + BK)^T$ . It follows from the Lyapunov's Theorem that, for any  $-\bar{\mu}I_n$ , the following equation

$$Q(A + BK)^T + (A + BK)Q = -\bar{\mu}I_n \quad (20)$$

has a unique solution  $Q > 0$ . Left multiply and right multiply  $Q^{-1}$  on both two sides yields that

$$(A + BK)^T Q^{-1} + Q^{-1}(A + BK) = -\bar{\mu}Q^{-1}Q^{-1}, \quad (21)$$

which is equivalent to (19) by letting  $P = Q^{-1}$ .  $\square$

Then, the results are summarized as Corollary 1.

**Corollary 1:** Consider the linear time-delay system (18) with the control law  $u(t) = Kx(t_i)$ ,  $t \in [t_i, t_{i+1})$ . Under Assumption 1, choose proper  $K$  such that  $(A + BK)$  is Hurwitz, and select sufficiently large  $\bar{\mu} > 0$  and  $\rho > 0$  such that  $\bar{\mu} - \|C\|^2 - \frac{\|BK\|^2}{\rho} > \frac{1}{\lambda_{\min}^2(P)}$ , where  $K$  and  $P > 0$  are the solutions to (19). Then, the closed-loop system is exponentially stable with some properly designed event-triggering mechanism (3). More specifically, let  $\mu = \lambda_{\min}(P)(\bar{\mu} - \|C\|^2 - \frac{\|BK\|^2}{\rho})$ ,  $\alpha = \frac{\gamma \lambda_{\min}(P)}{\rho}$ ,  $\beta_2 > \mu - \gamma$  and  $0 < \beta_1 \leq \frac{\beta_2 + \gamma - \mu}{\rho}$ , where  $0 < \gamma < \mu - \frac{1}{\lambda_{\min}(P)}$ .

**Proof:** Under Assumption 1, there always exists  $K$  such that  $(A + BK)$  is Hurwitz, and it further follows from Lemma 3 that there always exists a positive definite matrix  $P$  to equation (19). By letting  $D = I_n$ , it follows from (19) that

$$\begin{aligned} & PA + A^T P + PBKD + (PBKD)^T + PCC^T P \\ & + \frac{PBKK^T B^T P}{\rho} \\ & = -P(\bar{\mu}I_n - CC^T - \frac{BKK^T B^T}{\rho})P \\ & \leq -\lambda_{\min}(P)(\bar{\mu} - \|C\|^2 - \frac{\|BK\|^2}{\rho})P \\ & = -\mu P, \end{aligned} \quad (22)$$

which implies that inequality (7) in Theorem 1 is satisfied. Some other parameters can be obtained from Theorem 1. The proof is thus completed.  $\square$

**Remark 3:** It is noted that the event-triggered control in Corollary 1 reduces to the state feedback control law if the state is available. In this case, by choosing proper  $K$ , there always exists a solution to (7). Moreover, how to choose the optimal parameters is another interesting problem to be addressed in the future.

**Remark 4:** As we mentioned before, the derived results can also be applied to the multi-agent systems with general linear dynamics. In particular, if  $\dot{x}_i = Ax_i + Bu_i$  and  $u_i = K \sum_{j=1}^N (\hat{x}_j - \hat{x}_i)$  where  $x_i$  and  $\hat{x}_i$  represent the state of agent  $i$  and the state at the triggering time, respectively. In this case, the closed-loop system in compact form is  $\dot{x} = (I_n \otimes A - L \otimes BK)x + (-L \otimes BK)e$ , and the cooperative control of such multi-agent systems can be converted to the stabilization problem of the closed-loop system.

#### 4. ZENO BEHAVIOR EXCLUSION

Zeno behavior is an abnormal behavior caused by the triggering execution, which means there appears accumulation of infinite number of triggering instants in a finite time. Thus, it is necessary to exclude Zeno behavior such that the feasibility of this proposed triggering mechanism can be guaranteed.

**Definition 2:** If there exists a positive constant  $T_0$  such that  $\lim_{i \rightarrow \infty} t_i = T_0$ , then it implies that Zeno behavior appears.

**Theorem 2:** Under Assumptions 1 and 2, consider the linear time-delay system (1) and the output feedback controller (2). With the event-triggering mechanism (3), Zeno behavior is excluded.

**Proof:** Firstly, suppose that Zeno behavior appears, then it follows from Definition 2 that there exists a positive constant  $T_0$  such that  $\lim_{i \rightarrow \infty} t_i = T_0$ . From the definition of the limit, for any  $\varepsilon > 0$ , there exists  $N(\varepsilon)$  such that if  $i > N(\varepsilon)$ ,  $t_i \in [T_0 - \varepsilon, T_0]$ , which implies  $t_{N(\varepsilon)+1} - t_{N(\varepsilon)} \leq \varepsilon$ . Then, for  $t \in [t_{N(\varepsilon)}, t_{N(\varepsilon)+1})$ , computing the upper right-hand Dini derivative of  $\|e(t)\|$  yields

$$D^+ \|e(t)\| \leq \|\dot{e}(t)\|. \quad (23)$$

It follows from (4) and (5) that

$$\begin{aligned} \dot{e}(t) &= -\dot{y}(t) = -D\dot{x}(t) \\ &= -D[(A + BKD)x(t) + Cx(t - \tau(t)) + BK e(t)]. \end{aligned} \quad (24)$$

Then substituting (24) into (23) leads

$$\begin{aligned} D^+ \|e(t)\| &\leq \|\dot{e}(t)\| \\ &\leq \|D\|[\|A + BKD\| \|x(t)\| + \|C\| \|x(t - \tau(t))\| \\ &\quad + \|BK\| \|e(t)\|]. \end{aligned} \quad (25)$$

We further have

$$\|x(t)\| \leq \sqrt{\frac{x^T P x}{\lambda_{\min}(P)}} < \sqrt{\frac{(M+1)e^{-\sigma(t-t_0)}}{\lambda_{\min}(P)}}$$

$$< \sqrt{\frac{(M+1)}{\lambda_{\min}(P)}},$$

and

$$\begin{aligned} \|e(t)\| &= \sqrt{e(t)^T e(t)} \leq \sqrt{\alpha y(t)^T y(t) + \beta_1 e^{-\beta_2 t}} \\ &< \sqrt{\alpha x^T D^T D x} + \sqrt{\beta_1} \\ &\leq \sqrt{\alpha \lambda_{\max}(P^{-1} D^T D) V(t)} + \sqrt{\beta_1} \\ &< \sqrt{\alpha \lambda_{\max}(P^{-1} D^T D) (M+1)} + \sqrt{\beta_1}. \end{aligned}$$

Then, (25) can be rewritten as

$$\begin{aligned} D^+ \|e(t)\| &< \|D\|[\|A + BKD\| + \|C\|] \sqrt{\frac{(M+1)}{\lambda_{\min}(P)}} \\ &\quad + \|BK\| \sqrt{\alpha \lambda_{\max}(P^{-1} D^T D) (M+1)} \\ &\quad + \|BK\| \sqrt{\beta_1}. \end{aligned}$$

Denoting

$$\begin{aligned} \xi &= \|D\|[\|A + BKD\| + \|C\|] \sqrt{\frac{(M+1)}{\lambda_{\min}(P)}} \\ &\quad + \|BK\| \sqrt{\alpha \lambda_{\max}(P^{-1} D^T D) (M+1)} + \|BK\| \sqrt{\beta_1}. \end{aligned}$$

Obviously,  $\xi > 0$ . Then

$$D^+ \|e(t)\| < \xi.$$

Finally, by using Comparison Lemma, we have

$$\|e(t)\| < \xi(t - t_{N(\varepsilon)}).$$

Considering  $t = t_{N(\varepsilon)+1}^-$ , we can get

$$\|e(t_{N(\varepsilon)+1}^-)\| < \xi(t_{N(\varepsilon)+1}^- - t_{N(\varepsilon)}),$$

that is,

$$t_{N(\varepsilon)+1}^- - t_{N(\varepsilon)} > \frac{\|e(t_{N(\varepsilon)+1}^-)\|}{\xi}. \quad (26)$$

Besides, it follows from (3) that the next event will not be triggered before  $\|e(t)\| = \sqrt{\alpha \|y(t)\|^2 + \beta_1 e^{-\beta_2 t}}$ , which implies

$$\|e(t_{N(\varepsilon)+1}^-)\| \geq \sqrt{\beta_1} e^{-\frac{\beta_2}{2} t_{N(\varepsilon)+1}^-}. \quad (27)$$

Define  $\varepsilon = \frac{\sqrt{\beta_1} e^{-\frac{\beta_2}{2} T_0}}{\xi}$ , and take (27) into (26). we can finally obtain

$$t_{N(\varepsilon)+1} - t_{N(\varepsilon)} > \frac{\|e(t_{N(\varepsilon)+1}^-)\|}{\xi} \geq \frac{\sqrt{\beta_1} e^{-\frac{\beta_2}{2} T_0}}{\xi} = \varepsilon,$$

which contradicts the fact that  $t_{N(\varepsilon)+1} - t_{N(\varepsilon)} \leq \varepsilon$  for any  $\varepsilon > 0$ . Thus Zeno behavior can be excluded.  $\square$

## 5. EXAMPLES

### 5.1. Example 1: Reactor train with delayed recycle

In this section, we present a two stage chemical reactor with recycle as an example to illustrate the usefulness of our results to some practical system, which was first considered in [10].

As shown in Fig. 1, we consider the irreversible, first order, isothermal reaction which occurs in the two stage reactor system. To maintain constant reactor temperature, the composition of product streams from the two reactors  $c_1, c_2$  need to be controlled via manipulating the feed compositions to the two reactors, namely,  $c_{1f}, c_{2f}$ . In some ideal cases, we can ignore the process disturbance caused by extra feed streams, namely  $F_d = 0$  and  $c_d = 0$ . The flow rates to the reactor system are fixed and only the compositions vary. In this case, the state delay is usually inevitable due to the transportation lag in the recycle stream.

A material balance on the reactor train yields

$$V_1 \frac{dc_1}{dt} = F_1 c_{1f} + R c_2(t - \tau) + F_d c_d - (F_1 + R + F_d) c_1 - V_1 k_1 c_1, \quad (28)$$

$$V_2 \frac{dc_2}{dt} = (F_1 + R + F_d - F_{p1}) c_1 + F_2 c_{2f} - (F_{p2} + R) c_2 - V_2 k_2 c_2. \quad (29)$$

where the second product stream,  $F_{p2}$  is given by  $F_{p2} = F_1 + F_d - F_{p1} + F_2$ .

Define the following variables

$$\begin{aligned} \theta_1 &= \frac{V_1}{F_1 + R + F_d}, & \theta_2 &= \frac{V_2}{F_{p2} + R}, \\ \lambda_R &= \frac{R}{F_1 + R + F_d}, & \mu &= \frac{F_{p2} - F_2 + R}{F_{p2} + R}, \\ \lambda_d &= \frac{F_d}{F_1 + R + F_d}, \end{aligned}$$

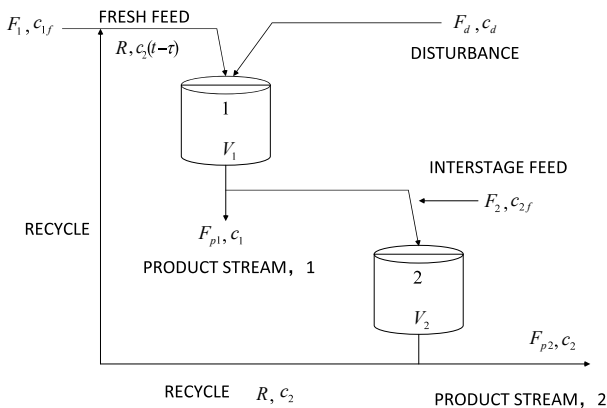


Fig. 1. Two stage chemical reactor train with delayed recycle.

and make the following transformations:  $u_1 = c_{1f} - c_{1fs}$ ,  $u_2 = c_{2f} - c_{2fs}$ ,  $x_1 = c_1 - c_{1s}$ , and  $x_2 = c_2 - c_{2s}$ , where  $c_{1fs}$ ,  $c_{2fs}$ ,  $c_{1s}$ , and  $c_{2s}$  denote steady-state values. Let  $x = [x_1, x_2]^T$  and  $u = [u_1, u_2]^T$ , by using vector-matrix notation, systems (28) and (29) become

$$\dot{x} = Ax(t) + Bu(t) + Cx(t - \tau), \quad (30)$$

$$y = Dx(t), \quad (31)$$

where  $y$  is the measured output and

$$A = \begin{pmatrix} -\frac{1+k_1\theta_1}{\theta_1} & 0 \\ \frac{\mu}{\theta_2} & -\frac{1+k_2\theta_2}{\theta_2} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1-\lambda_R-\lambda_d}{\theta_1} & 0 \\ 0 & \frac{1-\mu}{\theta_2} \end{pmatrix}, \\ C = \begin{pmatrix} 0 & \lambda_R \\ 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

It is noted that in (31), it is assumed that  $x$  can be measured directly, and thus  $D = I$ . In this case, the stabilization of (30) is to design proper control input  $u_1$  and  $u_2$ , such that the  $x = [0, 0]^T$  is asymptotically stable, which implies that  $c_1 \rightarrow c_{1s}$  and  $c_2 \rightarrow c_{2s}$  as  $t \rightarrow \infty$ . Then, we can determine the feed compositions for the two reactors as follows:  $c_{1f} = u_1 + c_{1fs}$ ,  $c_{2f} = u_2 + c_{2fs}$ .

Then we use the example that first used in [34] with ignoring any disturbances, where  $A = \begin{bmatrix} -2 & 0 \\ 0.5 & -2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}$ ,  $C = \begin{bmatrix} 0 & 0.5 \\ 0 & 0 \end{bmatrix}$ , and  $\tau = 1$ .

It is noted that the system is a special case of (1) with  $\tau(t) \equiv 1$  and  $D = I$ . In this case, we can design the event-triggered controller according to Corollary 1. The parameters are as follows,  $K = \begin{bmatrix} -0.7148 & -1.4297 \\ -0.4693 & -0.9387 \end{bmatrix}$ ,  $\alpha = 0.1268$ ,  $\beta_1 = 0.021$ , and  $\beta_2 = 2$ . The time response of the system state with the proposed event-triggered controller is shown in Fig. 2. The simulation result shows the effectiveness of the results.

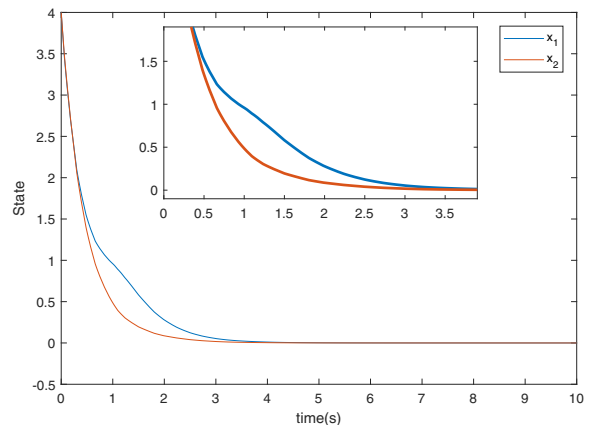


Fig. 2. Time response of the system state with the proposed controller.

## 5.2. Example 2: Numerical model with time-varying delay

In this section, we consider a 2-dimensional example of the model (1) with

$$A = \begin{bmatrix} 0 & 4 \\ 1 & -3 \end{bmatrix}, B = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5 & 0 \\ 1 & -1 \end{bmatrix}, D = \begin{bmatrix} 0.5 & 1 \end{bmatrix}.$$

The time-varying delay is  $\tau(t) = \frac{e^t}{1+e^t}$ , and the parameter is selected as  $\mu = 4$ . Then, according to Theorem 1, we can get the following parameters that satisfy the inequality (7),

$$P = \begin{bmatrix} 0.6013 & -0.2431 \\ -0.2431 & 1.6100 \end{bmatrix}, \rho = 9.4650, \gamma = 2.2.$$

Further more, we can obtain the design parameters as follows

$$K = \begin{bmatrix} 1.7593 \\ -1.5517 \end{bmatrix}, \alpha = 0.1268, \beta_1 = 0.021, \beta_2 = 2.$$

With the proposed event-triggered controller, simulation results are presented with the arbitrarily chosen initial states  $x_0 = [4, 4]^T$ . In particular, the time response of the system state with the proposed event-triggered controller is shown in Fig. 3. It can be easily observed from Fig. 3 that the state of the linear time-delay system indeed converges to 0 eventually.

It can be seen that with relatively small  $\beta_1$  and relatively large  $\beta_2$ , the exponential term in the event-triggering mechanism, i.e.,  $\beta_1 e^{-\beta_2 t}$  is quite small. To show the effect of the exponential term, we carry out the simulation by using the event-triggering mechanism without the exponential term (case 2, in this case,  $\beta_1 = \beta_2 = 0$ ), and the time response of the system state is presented in Fig. 4. It follows that a similar convergence rate is achieved, which implies that the exponential term has little impact on the convergence performance. Moreover, time response of the system state with continuous controller is presented in Fig. 5, which shows that with the proposed event-triggered control law, the convergence rate of the system remains similar as that with continuous controller, while sampling frequency can be reduced. Besides, the triggering time instants determined by the event-triggering mechanism with or without the exponential term are recorded in Fig. 6, respectively. It is noted that in Fig. 6,  $T_1$  is associated with the case with the exponential term while  $T_2$  is associated with the other case. It can be clearly observed that comparing with  $T_1$ ,  $T_2$  has more triggering times and some triggering time intervals even tend to be 0.

## 6. CONCLUSION

In this paper, the stabilization problem has been investigated for a class of linear time-delay systems by proposing

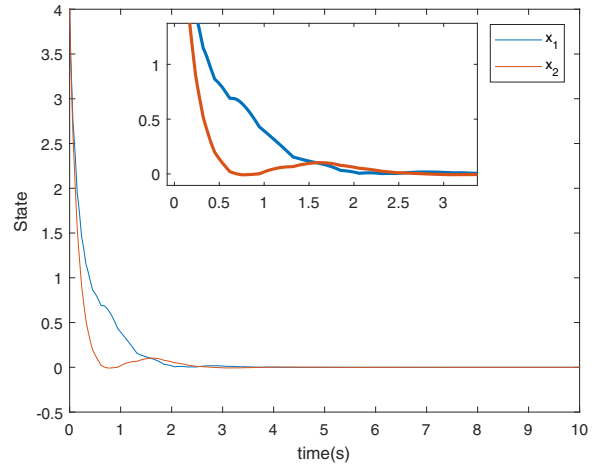


Fig. 3. Time response of the system state with the proposed controller.

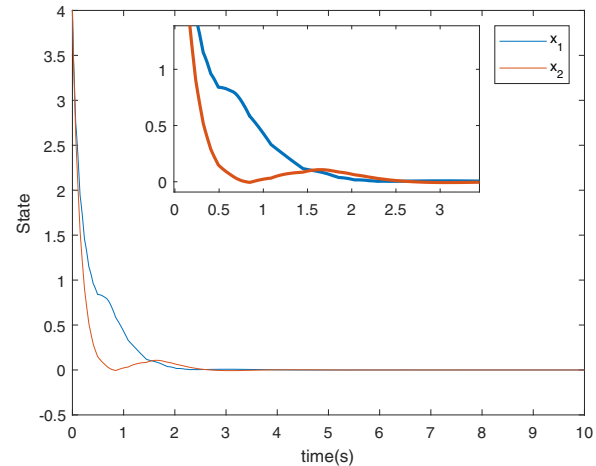


Fig. 4. Time response of the system state in case 2.

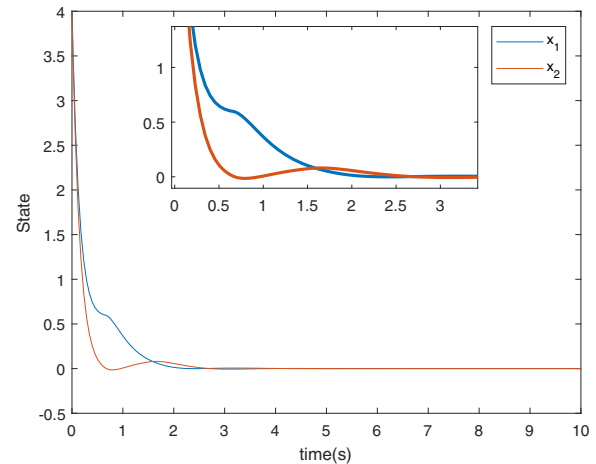


Fig. 5. Time response of the system state without event-triggered controller.



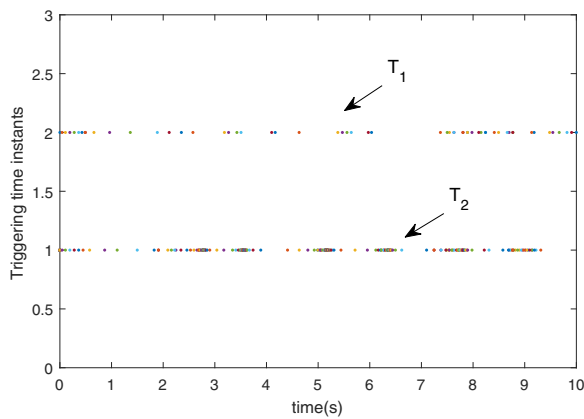


Fig. 6. Event-triggering time instants in the two cases.

an event-triggered output feedback control law. A novel event-triggering mechanism has been proposed to reduce the communication burden, and sufficient conditions have been obtained to ensure that the state of the controlled system exponentially converges to zero. Meanwhile, Zeno behavior has also been strictly excluded. Finally, a simulation has been given to show the effectiveness of the sufficient conditions. Future work will focus on the performance analysis of event/self-triggered control framework and its applications to the cooperative control of multi-agent systems.

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