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Fixed-time Output Feedback Consensus of Second-order Multi-agent Systems with Settling Time Estimation

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Abstract: This study discusses fixed-time consensus problem of second-order multi-agent systems with unmeasurable velocity and uncertain disturbance. The proposed control scheme includes two parts: one part is a fixed-time convergent state observer to estimate the unknown velocity while the other part is a fixed-time consensus algorithm based on integral sliding mode. Mathematical proof is given and some stability conditions are derived. Moreover, the settling time depends on the parameters of state observer and consensus algorithm, which can be theoretically estimated offline regardless of initial states. Finally, the proposed control scheme is employed to coordinated control of single-link robotic manipulators and the simulation examples verify the efficiency of the results.

Keywords: Disturbances, fixed-time consensus, integral sliding mode, settling time, unmeasurable velocity.

1. INTRODUCTION

An increasing interest in coordinated control of large scale autonomous agents exists in many applications, such as spacecraft [1], biological systems [2], robots [3] and unmanned aerial vehicles [4], which is to design and implement distributed algorithms such that a group of agents can achieve an agreement via local communication [5]. Consensus problems where agents are specified by different dynamics models have been reported in various reviews [6–9].

A challenging problem is to construct distributed algorithms that agents' states converge to the origin within a finite settling time. Therefore, finite-time consensus problems and estimation of the settling time have become an attractive and popular research area [10, 11]. However, an important point to be noted is that the settling time of finite-time consensus depends on the initial states of the agents [12]. Due to large initial states, the settling time may be sufficiently large and cannot be guaranteed for a predefined convergence time, or cannot be provided if the initial states are unknown [9, 13]. To tackle this problem, researchers have made significant research on fixed-time consensus problem, which converges within the settling time regardless of the initial states [14, 15]. Research results with different agent dynamics were presented in the work of Tian *et al*. [16], Polyakov [17], Khanzadeh and Pourgholi [18], Liu *et al*. [19], etc.

It is worth mentioning that the aforementioned literatures assume that agent's velocity information is available, which is infeasible in some scenarios due to technology constraints, equipment cost or environmental disturbances [20–23]. To solve this problem, state observer is naturally employed. Then, finite-time observers were proposed based on homogeneous theory [24, 25], which cannot estimate the settling time. Thus, Hua *et al*. [22] proposed an observer based on integral sliding mode, which can obtain the relationship between settling time and initial states. It should be noted that the finite-time convergent observer for systems does not hold for systems under fixed-time stability. Especially, it is nontrivial to construct a Lyapunov function and give the stability proof [26]. For a single system, Zhang *et al*. [27] proposed fixed-time convergent state observer and non-singular fast terminal sliding mode controller for a VTVL reusable launch vehicle. Zhang *et al*. [28] proposed fixed-time convergent state observer and fixed-time trajectory tracking controller for a marine surface vessel to track a time-varying reference trajectory. Then, in the work of Francisco *et al*. [29], fixed-time convergent state observer was designed by using implicit Lyapunov function method for perturbed linear control system. For first-order nonlinear systems, Basin, Yu *et al*. [30] proposed fixed-time sliding mode observer by utilizing recursive algorithm formulations. For Brunovsky Systems, Ni *et al*. [31] proposed fixed-time disturbance observer composed of uniform and fixed

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time convergent part. For linear second-order systems, [32, 33] proposed fixed-time convergent state observers and observer-based fixed-time consensus protocols. On the basis of [33], Huang and Jia [34] proposed a fixed-time convergent state observer for second-order nonlinear multi-agent systems. On the basis of [32], [35] proposed fixed-time convergent state observer and consensus protocol for second-order multi-agent systems with uncertain disturbance. For the case where the nonlinear function is known, Zou and Li [36] proposed fixed-time convergent state observer and employed homogeneity property to show that the control algorithm can guarantee the consensus tracking errors converging to the origin in finite time. Note that the above mentioned fixed-time convergent state observers and fixed-time consensus algorithms of second-order multi-agent system were developed by bi-limit homogeneous systems theory. Bi-limit homogeneity can only prove that state observer and consensus algorithm are fixed-time stable, but doesn't give the settling time, which cannot be applied to scenarios with convergence time requirements.

Motivated by the above discussions, we consider the fixed-time consensus problem of second-order multi-agent systems with unmeasurable velocity and uncertain disturbance. The main contributions of this paper can be summarized in both theoretical and practical aspects. In theoretical aspects, we introduce them in three parts: 1) By constructing integral sliding mode variable, a distributed fixed-time consensus algorithm is proposed for second-order multi-agent systems with uncertain disturbance. Compared with the existing distributed algorithm [35], the proposed algorithm has a more concise structure and does not require the control input of the neighbor agents. 2) It is theoretically proved that the settling time depends on parameters of observer and algorithm, which can be estimated offline regardless of initial states. However, the previous results [32–36] only use bi-limit homogeneity to prove that systems are fixed-time stable, but doesn't give the theoretical settling time. 3) The chattering problem is solved by utilizing a saturation function. In practical aspects, we introduce contributions in two parts: 1) A state observer is presented to estimate the unmeasurable velocity in fixed settling time, which is feasible in some scenarios due to technology constraints, equipment cost or environmental disturbances. 2) The proposed control scheme can be applied to scenarios with convergence time requirements and the method can be used to evaluate the convergence time in advance.

The remainder of this paper is organized as follows: Section 2 gives preliminaries and problem formulation. Section 3 describes the state observer and consensus algorithm. Section 4 further discusses the proposed method. Section 5 gives the simulation. The conclusions and future topics are provided in Section 6.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Notations and graph theory

Define $\mathrm{sig}(p)^\gamma = \left[\mathrm{sig}(p_1)^\gamma, \cdots, \mathrm{sig}(p_n)^\gamma\right]^T$, $\mathrm{sig}(p_i)^\gamma = |p_i|^\gamma \mathrm{sgn}(p_i)$, where $\mathrm{sgn}(\bullet)$ denotes the signum function. For a matrix $M = [m_{ij}]_{n\times n}$, $\lambda_{\max}(M), \lambda_{\min}(M)$ denote the maximum and minimum eigenvalues, respectively. I_n denotes the identity matrix. $\|\bullet\|$ denotes the 2-norm. $\otimes$ denotes the Kronecker product. R^n denotes n-dimensional Euclidean space.

The topology of n agents is modeled as an undirected graph $G = \{V, \zeta, A\}$, where $\zeta \subseteq \{(i,j), i,j \in V\}$ is the edge set, $V = \{1, 2, \cdots, n\}$ is a finite set of nodes, and $A = [a_{ij}]_{n\times n}$ is the associated adjacency matrix, where $a_{ii} = 0$, and $a_{ij} = 1$ is the weight if $(j,i) \in \zeta$ or $a_{ij} = 0$, otherwise. The neighbor set of i is defined as $N_i = \{j \in V : a_{ij} = 1\}$. Denote the matrix $D = diag\{d_{11}, d_{22}, \cdots, d_{nn}\}$ with $d_{ii} = \sum_{j=1, j\neq i}^n a_{ij}$. Then, the Laplacian matrix L can be expressed by $L = D - A$ and L is symmetric.

2.2. Some useful definition and lemmas

Consider the following differential equation:

$$\dot{x} = f(t,x), x(0) = x_0, x \in R^n, \tag{1}$$

where $f(t,x) : R_+ \times R^n \to R^n$ is a nonlinear function which may be discontinuous, the solutions of (1) are understood in the sense of Filippov [37]. Suppose that the origin is an equilibrium point of system (1).

Definition 1 [17]**:** The origin of system (1) is said to be a fixed-time stable equilibrium point if it is globally fixed-time stable with bounded convergence time $T(x_0)$.

Lemma 1 [17]: If there exists a continuous radially unbounded function $V : R^n \to R_+ \cup 0$ such that

1) $V(x) = 0 \Rightarrow x = 0$;

2) $\dot{V}(x(t)) \leq -\left(\alpha V^p(x(t)) + \beta V^q(x(t))\right)^k$

for some α, β, p, q, $k > 0$, $pk < 1$, $qk > 1$ and for any solution $x(t)$. Then the origin of system (1) is globally fixed-time stable and the settling time satisfies that $T(x_0) \leq \frac{1}{\alpha^k(1-pk)} + \frac{1}{\beta^k(qk-1)}, \forall x_0 \in R^n$.

If $k = 1$, the origin of system (1) is globally fixed-time stable with settling time T bounded by $T \leq T_{\max} := \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)}$, where $\alpha, \beta > 0$, $0 < p < 1$ and $q > 1$.

Lemma 2 [10]**:** For any nonnegative real numbers $x_1, x_2, \cdots, x_n$, the following inequality holds: $\left(\sum_{i=1}^n x_i\right)^p \leq \sum_{i=1}^n x_i^p \leq n^{1-p}\left(\sum_{i=1}^n x_i\right)^p$, where $p \in (0,1]$.

Lemma 3 [19]**:** For any nonnegative real numbers x_1, x_2, $\cdots$, x_n, the following inequality holds: $\left(\sum_{i=1}^n x_i\right)^q \geq \sum_{i=1}^n x_i^q \geq n^{1-q}\left(\sum_{i=1}^n x_i\right)^q$, where $q > 1$.

2.3. Problem formulation

Considering a multi-agent system composed of n agents (called follower) and a leader agent (labeled as 0). The dynamics of ith agent is described by

$$\begin{cases}\dot{r}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t)+\Delta_i(t),\end{cases}\tag{2}$$

where $r_i, u_i \in R^m$ denote the position and control input, respectively, $v_i \in R^m$ denotes the velocity, which is unmeasurable, $\Delta_i(t) \in R^m$ denotes the uncertain disturbance, which is bounded by a known constant.

The dynamics of the leader is described as

$$\begin{cases}\dot{r}_0(t)=v_0(t),\\ \dot{v}_0(t)=u_0(t)\end{cases}\tag{3}$$

where $r_0,\ v_0,\ u_0 \in R^m$ denote the position, velocity and control input of the leader, respectively.

Definition 2: The fixed-time consensus is achieved if there exists a fixed-time T such that $\lim\limits_{t\to T}\|r_i(t)-r_0(t)\|=0$, $\lim\limits_{t\to T}\|v_i(t)-v_0(t)\|=0$ and $\|r_i(t)-r_0(t)\|=0$, $\|v_i(t)-v_0(t)\|=0$ when $t\geq T$ $(i=1,2,\cdots,n)$. The settling time T is fixed and bounded, i.e., for any initial states, $\exists T_{\max}>0$ such that $T\leq T_{\max}$.

Assumption 1: For the multi-agent system (2) and (3), the undirected graph G is connected and there is at least one agent that can directly interact with the leader.

Assumption 2: There exists a positive constant Δ such that $\|\Delta_i(t)\|\leq\Delta$, $i=1,\ 2,\ \cdots,\ n$. The disturbance Δ_i is continuously differentiable and $\left\|\dot{\Delta}_i(t)\right\|\leq\Delta'$, where Δ' denotes a positive constant.

Lemma 4 [38]**:** Define diagonal matrix $H=diag(a_{10}, a_{20},\cdots,a_{N0})\geq 0$ with $a_{i0}>0$ if the ith agent is connected to the leader. Under Assumption 1, the symmetric matrix $B=L+H$ is positive definite.

Proof: Let $\lambda_1,\cdots,\lambda_n$ be the eigenvalues of Laplacian matrix L in the increasing order, then $\lambda_1=0$, $\lambda_j>0$, $2\leq j\leq n$. Denote n eigenvectors of L by ς_i, $i=1,\ 2,\ \cdots,\ n$ with $\varsigma_1=\mathbf{1}$, an eigenvector of L corresponding to $\lambda_1=0$. Then any nonzero vector $z\in R^n$ can be expressed by $z=\sum\limits_{i=1}^{n}c_i\varsigma_i$ for some constants c_i, $i=1,\ 2,\ \cdots,\ n$. Moreover, $H\neq\mathbf{0}$. Without loss of generality, we assume $a_{j0}>0$ for some j, and it is obvious $\varsigma_1^T H\varsigma_1\geq a_{j0}$. Therefore, in either the case when $c_i=0$, $i=2,\ \cdots,\ n$ $(c_1\neq 0)$ or the case when $c_i\neq 0$ for some $i\geq 2$, we always have $z^TBz=z^TLz+z^THz\geq\sum\limits_{i=2}^{n}\lambda_i c_i^2\varsigma_i^T\varsigma_i+z^THz>0$, for $z\neq\mathbf{0}$, which implies the conclusion. □

3. MAIN RESULTS

In this section, we first propose a fixed-time state observer to estimate velocity; subsequently, we present distributed algorithms for multi-agent systems with disturbance and estimate the settling time.

For the considered multi-agent system (2) with unknown velocity and uncertain disturbance, the fixed-time state observer is constructed to estimate velocity, which is specified as

$$\begin{aligned}\dot{\eta}_{ri}&=\eta_{vi}-c_1\sigma sig^{\alpha_1}(\eta_{ri}-r_i)\\&\quad-c_1(1-\sigma)sig^{\beta_1}(\eta_{ri}-r_i),\\ \dot{\eta}_{vi}&=u_i(t)+\eta_{\Delta i}-c_2\sigma sig^{\alpha_2}(\eta_{ri}-r_i)\\&\quad-c_2(1-\sigma)sig^{\beta_2}(\eta_{ri}-r_i),\\ \dot{\eta}_{\Delta i}&=-c_3\sigma sig^{\alpha_3}(\eta_{ri}-r_i)\\&\quad-c_3(1-\sigma)sig^{\beta_3}(\eta_{ri}-r_i),\end{aligned}\tag{4}$$

where $\eta_{ri},\ \eta_{vi},\ \eta_{\Delta i}\in R^m$ are the estimates of r_i, v_i, Δ_i, respectively, c_1, c_2, c_3 are positive constants, $\alpha_k=(k+1)\alpha-k$, $\beta_k=(k+1)\beta-k$, $\alpha\in(1-\varepsilon,1)$, $\beta\in(1,1+\varepsilon)$, $k=1,\ 2,\ 3$, $\varepsilon>0$ is sufficiently small, the gains c_1, c_2, c_3 are assigned such that the matrix $M=\begin{bmatrix}-c_1&1&0\\-c_2&0&1\\-c_3&0&0\end{bmatrix}$ is Hurwitz, $\sigma(t)$ is a switching function such that $\sigma(t)=\begin{cases}0,t\in[0,T_u],\\1,t\in(T_u,\infty),\end{cases}$ T_u is switching time.

Then, we can obtain that the observer (4) can estimate the velocity within

$$T_1\leq\frac{4\lambda_{\max}(\Upsilon)}{(1-\alpha)\lambda_{\min}(\mathrm{Z})}\left(\frac{(\beta-1)\lambda_{\min}(\mathrm{Z})T_u}{4\lambda_{\max}(\Upsilon)}\right)^{-\alpha}+T_u,\tag{5}$$

where Υ is a positive definite matrix satisfying $M^T\Upsilon+\Upsilon M=-\mathrm{Z}$, Z is a positive definite matrix.

The design of this observer is based on the results provided in [31, 39]. The proof can refer to the proof process for Brunovsky Systems [31], and is hence omitted here. T_u is typically selected through estimation error. For $t\leq T_u$, $\sigma=0$ and observer (4) becomes $\begin{cases}\dot{\eta}_{ri}=\eta_{vi}-c_1sig^{\beta_1}(\eta_{ri}-r_i),\\ \dot{\eta}_{vi}=u_i(t)+\eta_{\Delta i}-c_2sig^{\beta_2}(\eta_{ri}-r_i),\\ \dot{\eta}_{\Delta i}=-c_3sig^{\beta_3}(\eta_{ri}-r_i).\end{cases}$ Define the estimation errors $\varphi_{ri}=\eta_{ri}-r_i$, $\varphi_{vi}=\eta_{vi}-v_i$, $\varphi_{\Delta i}=\eta_{\Delta i}-\Delta_i$ and choose a Lyapunov function $V(\beta,\varphi)=\Xi^T\Upsilon\Xi$, where $\varphi=(\varphi_1,\varphi_2,\varphi_3)^T$ and $\Xi=\left(\varphi_1,\varphi_2^{\frac{1}{\beta_1}},\varphi_3^{\frac{1}{\beta_2}}\right)^T$. When $t=T_u$, Lyapunov function $V(\beta,\varphi)$ satisfies $V(T_u)<\left(\frac{\beta-1}{4}\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}T_u\right)^{\frac{2}{1-\beta}}$, which indicates the relationship between T_u and the evaluation error.

Remark 1: For linear second-order multi-agent systems, [32, 33] proposed fixed-time convergent state observers. Then, [35] proposed fixed-time convergent observer for multi-agent systems with disturbance. All these results only used bi-limit homogeneity to prove that the observer is fixed-time stable, but didn't give the settling time. Therefore, fixed-time convergent observer which can estimate the settling time is provided in this paper.

First, we consider the case when the leader's control input is zero, i.e., $u_0 = 0_m$. Based on the fixed-time state observer (4), an integral sliding mode variable $S_i(t)$ is designed as

$$S_i(t) = \eta_{vi}(t) - \eta_{vi}(0) - \int_0^t u_{i1}(t)d\tau, \tag{6}$$

where $S_i = [s_{i1},\ s_{i2},\ \cdots,\ s_{im}]^T$, $u_{i1}(t) = -\psi_i(t) - sig(\psi_i(t))^p - sig(\psi_i(t))^{\frac{1}{p}}$, $\psi_i(t) = k_1 \sum_{j=1}^{n} a_{ij}(r_i(t) - r_j(t)) + k_2 \sum_{j=1}^{n} a_{ij}(\eta_{vi}(t) - \eta_{vj}(t)) + k_1 a_{i0}(r_i(t) - r_0(t)) + k_2 a_{i0}(\eta_{vi}(t) - v_0(t))$, $k_1, k_2 > 0$, $p \in (0.5, 1)$.

The fixed-time consensus algorithm based on integral sliding mode is proposed as

$$u_i(t) = u_{i1}(t) - \gamma_1 \operatorname{sgn}(S_i) - \gamma_2 sig(S_i)^q, \tag{7}$$

where $\gamma_1 > \Delta$, γ_2 is a positive constant, $q \in (1, 2)$.

Remark 2: From (10), q is used to adjust the settling time. In applications, when $m > 2$, in the case that other parameters remain unchanged, increasing q will result in longer settling time. When $m = 1$ or 2, increasing q will result in shorter settling time. For the convenience of the readers, we stipulate that q is selected in the interval $(1, 2)$. Meanwhile, we can guarantee the settling time by selecting the parameters γ_1, γ_2.

The structure diagram of control process is given in Fig. 1.

Remark 3: From Fig. 1, the leader broadcasts its states and then the followers who can interact with the leader receive the leader's information. For each follower agent, its sensor can receive the states of its neighbors broadcast.

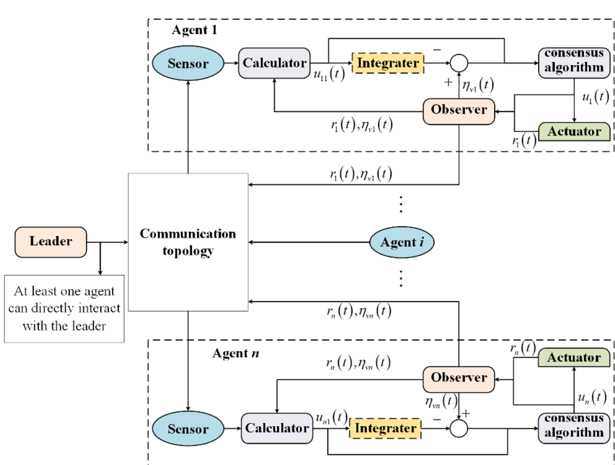

Fig. 1. Structure diagram of control processes.

Meanwhile, the calculator calculates the variable u_{i1} by using the received information and the observer's output. Then, each agent calculate its control input according to the variable u_{i1} and a integrator. Since only the position of the agent is measurable, the observer is introduced to measure the velocity. Each agent broadcasts its position and the observer's output.

Lemma 5: Suppose that Assumption 1 and Assumption 2 hold. The multi-agent system (2) and the leader (3) under consensus algorithm (7) do not escape in finite time interval $[0, T_1]$ if the parameters are selected as $\gamma_1 > \Delta$, $\gamma_2 > 0$, $k_2 > \sqrt{k_1 \lambda_{\max}(B^{-1})}$, that is, the system states are bounded.

Proof: See Appendix A. □

Theorem 1: Consider the multi-agent system (2) and the leader (3) with $u_0 = 0_m$. Suppose that Assumption 1 and Assumption 2 hold. With the fixed-time observer (4), the distributed consensus algorithm (7) can achieve fixed-time leader-following consensus if the parameters are selected as $\gamma_1 > \Delta$, $\gamma_2 > 0$, $k_2 > \sqrt{k_1 \lambda_{\max}(B^{-1})}$.

Furthermore, the settling time is upper bounded by

$$\begin{aligned} T \leq& T_1 + T_2 + T_3 \\ =& \frac{4\lambda_{\max}(\Upsilon)}{(1-\alpha)\lambda_{\min}(Z)} \left(\frac{(\beta-1)\lambda_{\min}(Z) T_u}{4\lambda_{\max}(\Upsilon)} \right)^{-\alpha} + T_u \\ &+ \frac{\sqrt{2}}{\gamma_1 - \Delta} + \frac{1}{\gamma_2 (q-1) m^{(1-q)/2} 2^{(q-1)/2}} \\ &+ \frac{2}{(1-p)\sqrt{\delta^2 \xi / \lambda}^{p+1}} \\ &+ \frac{2p}{(1-p)\sqrt{nm}^{1-\frac{1}{p}} \sqrt{\delta^2 \xi / \lambda}^{\frac{1}{p}+1}}, \end{aligned}$$

where T_1 is defined in (5), T_2 is defined in (10), T_3 is defined in (19).

Proof: From (5), the observer (4) can estimate the velocity within fixed-time upper bounded by $t \leq T_1$, i.e., $\eta_{ri} = r_i, \eta_{vi} = v_i$ when $t > T_1$. The next proof will be divided into two steps: (1) it will be shown that sliding mode surface $S_i(t)$ is reached within fixed time; (2) we will prove that the system (2) is globally stable within fixed time.

Step 1: Choose the Lyapunov function as

$$V_1(t) = \frac{1}{2} S_i^T S_i. \tag{8}$$

Taking the derivative of $V_1(t)$ yields

$$\begin{aligned} \dot{V}_1(t) =& S_i^T \dot{S}_i = S_i^T (u_i(t) + \Delta_i(t) - u_{i1}(t)) \\ =& S_i^T (\Delta_i(t) - \gamma_1 \operatorname{sgn}(S_i) - \gamma_2 sig(S_i)^q) \end{aligned}$$

$$
\begin{aligned}
&\le \Delta\|S_i\| - \gamma_1 \sum_{j=1}^{m} |s_{ij}| - \gamma_2 \sum_{j=1}^{m} |s_{ij}|^{q+1} \\
&\le -(\gamma_1-\Delta)\|S_i\| - \gamma_2 m^{(1-q)/2}\|S_i\|^{q+1} \\
&= -\sqrt{2}(\gamma_1-\Delta)V_1^{\frac{1}{2}}(t) - \gamma_2 m^{\frac{1-q}{2}} 2^{\frac{q+1}{2}} V_1^{\frac{q+1}{2}}(t),
\end{aligned} \tag{9}
$$

where $\gamma_1-\Delta>0$. Then, from Lemma 1, each sliding mode $S_i(t)$ will converge to origin within

$$
T_2 \le \frac{\sqrt{2}}{\gamma_1-\Delta} + \frac{1}{\gamma_2(q-1)m^{(1-q)/2}2^{(q-1)/2}}, \tag{10}
$$

Step 2: According to (6), after all $S_i(t)$ converge to origin, i.e., $S_i(t)=\dot{S}_i(t)=0_m$, sliding mode manifold is maintained. It follows that $\dot{\eta}_{vi}=\dot{v}_i=u_{i1}(t)$.

Define tracking errors $\varepsilon_{ri}(t)=r_i(t)-r_0(t)$, $\varepsilon_{vi}(t)=v_i(t)-v_0(t)$ and their dynamics are

$$
\begin{cases}
\dot{\varepsilon}_{ri}(t)=\varepsilon_{vi}(t),\\
\dot{\varepsilon}_{vi}(t)=u_{i1}(t).
\end{cases} \tag{11}
$$

Furthermore, set $\varsigma_i=\sum\limits_{j=1}^{n} a_{ij}(\varepsilon_{ri}-\varepsilon_{rj})+a_{i0}\varepsilon_{ri}$, $\kappa_i=\sum\limits_{j=1}^{n} a_{ij}(\varepsilon_{vi}-\varepsilon_{vj})+a_{i0}\varepsilon_{vi}$. Then, we have

$$
\begin{cases}
\dot{\varsigma}=\kappa,\\
\dot{\kappa}=-B\psi(t)-Bsig(\psi(t))^p-Bsig(\psi(t))^{\frac{1}{p}},\\
\psi(t)=k_1\varsigma+k_2\kappa,
\end{cases} \tag{12}
$$

where $\varsigma=[\varsigma_1^T,\varsigma_2^T,\cdots,\varsigma_n^T]^T$, $\kappa=[\kappa_1^T,\kappa_2^T,\cdots,\kappa_n^T]^T$, $B=L+H$.

Consider the Lyapunov function as

$$
V_2(t)=\frac{1}{2}\begin{pmatrix}\varsigma\\ \kappa\end{pmatrix}^T\left(\begin{pmatrix}2k_1k_2I_n & k_1B^{-1}\\ k_1B^{-1} & k_2B^{-1}\end{pmatrix}\otimes I_m\right)\begin{pmatrix}\varsigma\\ \kappa\end{pmatrix}, \tag{13}
$$

where I_n, I_m are unit matrix of order n,m, respectively. From Lemma 4, the symmetric matrix $B=L+H$ is positive definite. Clearly, if $k_2>\sqrt{k_1\lambda_{\max}(B^{-1})}$, we have $\begin{pmatrix}2k_1k_2I_n & k_1B^{-1}\\ k_1B^{-1} & k_2B^{-1}\end{pmatrix}>0$. Thus, $V_2(t)\ge 0$ and $V_2(t)=0$ if and only if $\|\varsigma\|=0$, $\|\kappa\|=0$. The derivative of $V_2(t)$ versus time is

$$
\begin{aligned}
\dot{V}_2(t) =& 2k_1k_2\varsigma^T\kappa + k_1\kappa^T\left(B^{-1}\otimes I_m\right)\kappa \\
&+\left(k_1\varsigma^T+k_2\kappa^T\right)\left(B^{-1}\otimes I_m\right)\dot{\kappa} \\
\le& -k_1^2\varsigma^T\varsigma - \left(k_2^2-k_1\lambda_{\max}\left(B^{-1}\right)\right)\kappa^T\kappa \\
&-\sum_{i=1}^{n}\sum_{j=1}^{m}|\psi_{ij}|^{p+1} - \sum_{i=1}^{n}\sum_{j=1}^{m}|\psi_{ij}|^{\frac{1}{p}+1} \\
\le& -k_1^2\varsigma^T\varsigma - \left(k_2^2-k_1\lambda_{\max}\left(B^{-1}\right)\right)\kappa^T\kappa \\
&-\|\psi\|^{p+1} - \sqrt{nm}^{1-\frac{1}{p}}\|\psi\|^{\frac{1}{p}+1}.
\end{aligned} \tag{14}
$$

With the condition $k_2>\sqrt{k_1\lambda_{\max}(B^{-1})}$, one can obtain that $\dot{V}_2(t)<0$. The state $\left(\varsigma^T,\kappa^T\right)^T$ will asymptotically converge to $\left(0_{mn}^T,0_{mn}^T\right)^T$, which implies that $r_i(t)-r_0(t)=0_m$, $v_i(t)-v_0(t)=0_m$.

Next, we will show that $\left(\varsigma^T,\kappa^T\right)^T$ converge to $\left(0_{mn}^T,0_{mn}^T\right)^T$ in fixed-time.

First, we give the following analysis. If $\psi_i(t)=0_m$, for all $i=1,2,\cdots,n$, then $\varsigma_i=0_m$, $\kappa_i=0_m$. Suppose that some $\varsigma_i\neq 0_m$ or $\kappa_i\neq 0_m$. If $\psi_i(t)=0_m$, it follows that $\psi_i(t)=k_1\kappa_i+k_2\dot{\kappa}_i=0_m$, which implies $\dot{\kappa}_i=-\frac{k_1}{k_2}\kappa_i$, $\dot{\kappa}=-diag\left\{\frac{k_1}{k_2},\frac{k_1}{k_2},\cdots,\frac{k_1}{k_2}\right\}\kappa$. According to (12), we obtain that $\dot{\kappa}=-B\psi(t)-Bsig(\psi(t))^p-Bsig(\psi(t))^{\frac{1}{p}}=0_{nm}$. We further get $\kappa_i=0_m$, $\varsigma_i=-\frac{k_2}{k_1}\kappa_i=0_m$, which contradicts that some $\varsigma_i\neq 0_m$ or $\kappa_i\neq 0_m$ is not 0_m. Therefore, we can conclude that if $\psi_i(t)=0_m$, for all $i=1,2,\cdots,n$, then $\varsigma_i=0_m$, $\kappa_i=0_m$.

Before consensus is achieved, it is obvious that at least one of $\varsigma_i(t)$ and $\kappa_i(t)$ is not 0_m, $\psi^T(t)\psi(t)>0$. Define $\varepsilon_i(t)=\left(k_1\varsigma_i^T,k_2\kappa_i^T\right)^T$, then we have $\psi_i=\begin{pmatrix}1 & 1\end{pmatrix}\otimes I_m\varepsilon_i(t)$. Before consensus is achieved, it follows from Lemma 6 [40] that

$$
\psi^T\psi \ge \xi\sum_{i=1}^{n}\left(k_1^2\|\varsigma_i\|^2+k_2^2\|\kappa_i\|^2\right), \tag{15}
$$

where $\xi=\min_{\frac{\varepsilon}{\|\varepsilon\|}}\left(\frac{\varepsilon}{\|\varepsilon\|}\right)^T\begin{pmatrix}1&1\\1&1\end{pmatrix}\otimes I_{mn}\left(\frac{\varepsilon}{\|\varepsilon\|}\right)>0$, $\varepsilon=\left[\varepsilon_1^T,\varepsilon_2^T,,\varepsilon_n^T\right]^T$.

$$
\begin{aligned}
\frac{\|\psi(t)\|^{p+1}}{V_2(t)^{\frac{p+1}{2}}} &\ge \frac{\|\psi(t)\|^{p+1}}{\lambda^{\frac{p+1}{2}}\|\varsigma^T\varsigma+\kappa^T\kappa\|^{\frac{p+1}{2}}} \\
&\ge \frac{\left\|\xi\delta^2\sum\limits_{i=1}^{n}\left(\|\varsigma_i\|^2+\|\kappa_i\|^2\right)\right\|^{\frac{p+1}{2}}}{\lambda^{\frac{p+1}{2}}\|\varsigma^T\varsigma+\kappa^T\kappa\|^{\frac{p+1}{2}}} = \sqrt{\delta^2\xi/\lambda}^{p+1},
\end{aligned} \tag{16}
$$

where $\delta=\min\{k_1,k_2\}$ and λ is the maximum eigenvalue of the matrix $\frac{1}{2}\begin{pmatrix}2k_1k_2I_n & k_1B^{-1}\\ k_1B^{-1} & k_2B^{-1}\end{pmatrix}$.

Similarly, we get

$$
\begin{aligned}
\frac{\sqrt{nm}^{1-\frac{1}{p}}\|\psi\|^{\frac{1}{p}+1}}{V_2(t)^{\frac{1}{2p}+\frac{1}{2}}} &\ge \frac{\sqrt{nm}^{1-\frac{1}{p}}\|\psi\|^{\frac{1}{p}+1}}{\lambda^{\frac{1}{2p}+\frac{1}{2}}\|\varsigma^T\varsigma+\kappa^T\kappa\|^{\frac{1}{2p}+\frac{1}{2}}} \\
&\ge \frac{\sqrt{nm}^{1-\frac{1}{p}}\left\|\xi\delta^2\sum\limits_{i=1}^{n}\left(\|\varsigma_i\|^2+\|\kappa_i\|^2\right)\right\|^{\frac{1}{2p}+\frac{1}{2}}}{\lambda^{\frac{1}{2p}+\frac{1}{2}}\|\varsigma^T\varsigma+\kappa^T\kappa\|^{\frac{p+1}{2}}} \\
&= \sqrt{nm}^{1-\frac{1}{p}}\sqrt{\delta^2\xi/\lambda}^{\frac{1}{p}+1}.
\end{aligned} \tag{17}
$$

Thus, submitting (16), (17) into (14) can yield

$$
\dot{V}_2(t)+\sqrt{\delta^2\xi\lambda}^{p+1}V_2(t)^{\frac{p+1}{2}}
$$

$$+\sqrt{nm}^{1-\frac{1}{p}}\sqrt{\delta^2\xi\lambda}^{\frac{1}{p}+1}V_2(t)^{\frac{1}{2p}+\frac{1}{2}} \leq 0. \tag{18}$$

It follows from Lemma 1 that $\left(\varsigma^T,\kappa^T\right)^T$ converges to $\left(0_{mn}^T,0_{mn}^T\right)^T$ within the settling time

$$T_3 \leq \frac{2}{(1-p)\sqrt{\delta^2\xi/\lambda}^{p+1}} + \frac{2p}{(1-p)\sqrt{nm}^{1-\frac{1}{p}}\sqrt{\delta^2\xi/\lambda}^{\frac{1}{p}+1}}. \tag{19}$$

Then, we get $r_1=\cdots=r_n=r_0$, $v_1=\cdots=v_n=v_0$. Based on the analysis above, we can conclude that the proposed observer and integral sliding mode control algorithm can solve fixed-time consensus problem of multi-agent system (2) and leader (3) with $u_0=0_m$. From (5), (10) and (19), the settling time is upper bounded by $T\leq T_1+T_2+T_3$. This is the end of proof. □

Remark 4: The proposed consensus algorithm (7) not only can solve the fixed-time consensus problem with unmeasurable velocity, but also can estimate the settling time, while previous results [32–35] only used bi-limit homogeneity to prove that multi-agent systems are fixed-time stable. In practical applications, these control methods cannot be applied to scenarios with convergence time requirements. From (5), (10), (19), it can be seen that the settling time depends on observer parameters, controller parameters, which can be selected properly to satisfy strict settling-time requirements.

Next, we consider the case where the leader's control input is not zero. Since only the followers that have direct communication link to leader can receive the leader's state, fixed-time observer is constructed to estimate the leader's state.

$$\dot{\sigma}_i = \frac{\sum_{j=0}^{n} a_{ij}\dot{\sigma}_j}{\sum_{j=0}^{n} a_{ij}} - \frac{d_1}{\sum_{j=0}^{n} a_{ij}} sig\left(\sum_{j=0}^{n} a_{ij}(\sigma_i-\sigma_j)\right)^{\alpha_0} - \frac{d_2}{\sum_{j=0}^{n} a_{ij}} sig\left(\sum_{j=0}^{n} a_{ij}(\sigma_i-\sigma_j)\right)^{1/\alpha_0}, \tag{20}$$

where d_1, d_2 denote observer gains and d_1, $d_2>0$, $\alpha_0\in(0.5,1)$, σ_i $(i=1,2,\cdots,n)$ are the estimate of ith agent, $\sigma_0=r_0$.

Then, the integral sliding mode variable (6) and consensus algorithm (7) will be modified to

$$S_i(t)=\eta_{vi}(t)-\eta_{vi}(0)-\int_0^t(u_{i0}(t)+u_{i1}(t))d\tau, \tag{21}$$

$$u_i(t)=u_{i0}(t)+u_{i1}(t)-\gamma_1\operatorname{sgn}(S_i)-\gamma_2 sig(S_i)^q, \tag{22}$$

where $u_{i0}(t)=\ddot{\sigma}_i$.

Theorem 2: Consider the multi-agent system (2) and the leader (3) with $u_0\neq 0_m$. Suppose that Assumption 1 and Assumption 2 hold. With the fixed-time observer (4) (20), the distributed consensus algorithm (22) can achieve fixed-time leader-following consensus if the parameters are selected as $\gamma_1>\Delta$, $\gamma_2>0$, $k_2>\sqrt{k_1\lambda_{\max}(B^{-1})}$.

Furthermore, the settling time is upper bounded by $T\leq T_1+T_2+T_3+T_4$, where T_1 is defined in (5), T_2 is defined in (10), T_3 is defined in (19), T_4 is defined in (27).

Proof: First, we prove that the observer (20) can estimate the leader's states in fixed-time.

Set $\omega_i=\sum_{j=0}^{n}a_{ij}(\sigma_i-\sigma_j)=\sum_{j=1}^{n}a_{ij}(\tilde{\sigma}_i-\tilde{\sigma}_j)+a_{i0}\tilde{\sigma}_i$, $\tilde{\sigma}_i=\sigma_i-\sigma_0$, $\omega=\left(\omega_1^T,\omega_2^T,\cdots,\omega_n^T\right)^T$, $\tilde{\sigma}=\left(\tilde{\sigma}_1^T,\tilde{\sigma}_2^T,\cdots,\tilde{\sigma}_n^T\right)^T$, it follows that $\omega=(B\otimes I_m)\tilde{\sigma}$. Then we rewrite (20) as

$$\dot{\omega}_i=-d_1 sig(\omega_i)^{\alpha_0}-d_2 sig(\omega_i)^{1/\alpha_0}. \tag{23}$$

Consider the Lyapunov function as

$$V_1(t)=\frac{1}{2}\omega^T\omega. \tag{24}$$

Taking the derivative of $V(t)$ yields

$$\begin{aligned}\dot{V}_1(t) &= -d_1\sum_{i=1}^{n}\omega_i^T sig(\omega_i)^{\alpha_0}-d_2\sum_{i=1}^{n}\omega_i^T sig(\omega_i)^{1/\alpha_0}\\ &= -d_1\sum_{i=1}^{n}\sum_{j=1}^{m}|\omega_{ij}|^{\alpha_0+1}-d_2\sum_{i=1}^{n}\sum_{j=1}^{m}|\omega_{ij}|^{1/\alpha_0+1}\\ &\leq -d_1\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\omega_{ij}^2\right)^{\frac{\alpha_0+1}{2}}\\ &\quad -d_2(nm)^{\frac{1-1/\alpha_0}{2}}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\omega_{ij}^2\right)^{\frac{1/\alpha_0+1}{2}}\\ &= -d_1\|\omega\|^{\alpha_0+1}-d_2(nm)^{\frac{1-1/\alpha_0}{2}}\|\omega\|^{1/\alpha_0+1}\\ &= -d_1 2^{\frac{\alpha_0+1}{2}}V_1^{\frac{\alpha_0+1}{2}}(t)\\ &\quad -d_2(nm)^{\frac{1-1/\alpha_0}{2}}2^{\frac{1/\alpha_0+1}{2}}V_1^{\frac{1/\alpha_0+1}{2}}(t).\end{aligned} \tag{25}$$

Define $\xi=\sqrt{2V(t)}$, it follows that

$$\dot{\xi}\leq -d_1\xi^{\alpha_0}-d_2(nm)^{\frac{1-1/\alpha_0}{2}}\xi^{1/\alpha_0}. \tag{26}$$

From Lemma 1, ω will converge to the origin within fixed-time upper bounded by

$$T_4\leq\frac{1}{d_1(1-\alpha_0)}+\frac{\alpha_0}{d_2(1-\alpha_0)\sqrt{nm}^{1-1/\alpha_0}}. \tag{27}$$

It follows from Lemma 4 that $\tilde{\sigma}_i=\sigma_i-\sigma_0=0_m$ and $\sigma_1=\cdots=\sigma_n=\sigma_0$ within T_4.

Then, similar to the proof process of Theorem 1, it is easy to obtain that Theorem 2 holds and the proof is omitted here. This is the end of proof. □

Remark 5: The observer (20) can estimate the leader's states within T_4. The controller (22) involves differential term, which results in difficulties in control implementation. A uniform robust exact differentiator [41] can accurately calculate the derivative of σ_i, i.e., $\ddot{\sigma}_1 = \cdots = \ddot{\sigma}_n = u_0$.

Remark 6: Comparing Theorems 1 and 2 shows that when the leader's control input is not zero, which not only affects the controller design process, but also increases the settling time due to the estimation of the leader's state. Obviously, the case when the leader's control input is 0 is a special case where the control input is not 0. The controller whose leader's control input is not 0 is more complicated than the controller with $u_0 = 0_m$.

Considering the absence of disturbance for multi-agent system (2), we have the following corollary.

Corollary 1: Consider the multi-agent system (2) and the leader (3) in the absence of uncertain disturbance. Suppose that Assumption 1 holds. If the state observer and consensus algorithm are designed as

$$\begin{aligned}\dot{\eta}_{ri} &= \eta_{vi} - c_1\sigma sig^{\alpha_1}(\eta_{ri} - r_i) \\ &\quad - c_1(1-\sigma)\, sig^{\beta_1}(\eta_{ri} - r_i), \\ \dot{\eta}_{vi} &= u_i(t) - c_2\sigma sig^{\alpha_2}(\eta_{ri} - r_{ii}) \\ &\quad - c_2(1-\sigma)\, sig^{\beta_2}(\eta_{ri} - r_i), \qquad (28)\\ u_i(t) &= u_{i0}(t) + u_{i1}(t), \qquad (29)\end{aligned}$$

then leader-following consensus can be achieved in fixed time.

Proof: The proof is similar to Theorems 1 and is omitted here. This is the end of proof. □

It is worth noting that the sign function is used in the control algorithm (22), which may lead to chattering in practical applications. To tackle this problem, the following saturation function is defined as

$$sat(x) = \begin{cases} \dfrac{x}{\omega}, & |x| < \omega, \\ sgn(x), & |x| \geq \omega, \end{cases} \qquad (30)$$

where ω denotes a very small positive constant, $sat(X) = (sat(x_1), sat(x_2), \cdots, sat(x_n))^T$, $X = (x_1, x_2, \cdots, x_n)^T$.

Then, the integral sliding mode controller which can eliminate chattering is designed as

$$S_i(t) = \eta_{vi}(t) - \eta_{vi}(0) - \int_0^t (u_{i0}(t) + u_{i1}(t))\, d\tau, \quad (31)$$

$$\begin{aligned}u_i(t) &= u_{i0}(t) + u_{i1}(t) - \Delta sat(S_i) \\ &\quad - \gamma_1 sig(S_i)^{q_1} - \gamma_2 sig(S_i)^{q_2}, \qquad (32)\end{aligned}$$

where $u_{i0}(t) = \ddot{\sigma}_i$, $\gamma_1 > \Delta$, $\gamma_2 > 0$, $q_1 \in (0.5, 1)$, $q_2 \in (1, 2)$.

Theorem 3: Consider the multi-agent system (2) and the leader (3) with $u_0 \neq 0_m$. Suppose that Assumption 1 and Assumption 2 hold. With the fixed-time convergent state observers (4) (20), the distributed consensus algorithm (32) can make the multi-agent system fixed-time stable with a bounded error $\|S_i\| = (\omega/4)^{\frac{1}{q_1+1}}$ if the parameters are selected as $\gamma_1 > \Delta$, $\gamma_2 > 0$, $k_2 > \sqrt{k_1 \lambda_{\max}(B^{-1})}$.

Furthermore, the settling time is upper bounded by $T \leq T_1 + T_2 + T_3 + T_4$, where T_1 is defined in (5), T_2 is defined in (38), T_3 is defined in (19), T_4 is defined in (27).

Proof: First, we prove that the sliding mode surface $S_i(t)$ is reached with a bounded error. Choose the Lyapunov function as

$$V_1(t) = \frac{1}{2} S_i^T S_i. \qquad (33)$$

Taking the derivative of $V_1(t)$ yields

$$\begin{aligned}\dot{V}_1(t) =& S_i^T \dot{S}_i = S_i^T (u_i(t) + \Delta_i(t) - u_{i0}(t) + u_{i1}(t)) \\ =& S_i^T(\Delta_i(t) - \Delta sat(S_i) - \gamma_1 sig(S_i)^{q_1} - \gamma_2 sig(S_i)^{q_2}) \\ \leq& \Delta\left(\|S_i\| - S_i^T sat(S_i)\right) - \gamma_1 \sum_{j=1}^m |s_{ij}|^{q_1+1} \\ &- \gamma_2 \sum_{j=1}^m |s_{ij}|^{q_2+1}. \qquad (34)\end{aligned}$$

When $|s_{ij}(t)| \geq \omega$, it follows that $sat(s_{ij}(t)) = \mathrm{sgn}(s_{ij}(t))$ and $S_i^T sat(S_i) = \sum_{j=1}^m |s_{ij}|$. Then, we have

$$\begin{aligned}\dot{V}_1(t) \leq& \Delta\left(\|S_i\| - \sum_{j=1}^m |s_{ij}|\right) - \gamma_1 \sum_{j=1}^m |s_i|^{q_1+1} \\ &- \gamma_2 \sum_{j=1}^m |s_i|^{q_2+1} \\ \leq& -\gamma_1 \sum_{j=1}^m |s_i|^{q_1+1} - \gamma_2 \sum_{j=1}^m |s_i|^{q_2+1} \\ \leq& -\gamma_1 \|S_i\|^{q_1+1} - \gamma_2 m^{(1-q_2)/2} \|S_i\|^{q_2+1} \\ =& -\gamma_1 2^{\frac{q_1+1}{2}} V_1^{\frac{q_1+1}{2}}(t) - \gamma_2 m^{\frac{1-q_2}{2}} 2^{\frac{q_2+1}{2}} V_1^{\frac{q_2+1}{2}}(t). \qquad (35)\end{aligned}$$

From Lemma 1, it is easy to obtain that $s_{ij}(t)$ converges to the region $|s_{ij}(t)| < \omega$ within $t_1 \leq \frac{1}{\gamma_1(1-q_1)2^{(q_1-1)/2}} + \frac{1}{\gamma_2(q_2-1)m^{(1-q_2)/2}2^{(q_2-1)/2}}$.

Then, we have

$$\begin{aligned}\dot{V}_1(t) \leq& \Delta\left(\|S_i\| - \frac{\|S_i\|^2}{\omega}\right) - \gamma_1 \sum_{j=1}^m |s_i|^{q_1+1} \\ &- \gamma_2 \sum_{j=1}^m |s_i|^{q_2+1}\end{aligned}$$

$$\begin{aligned}&\leq \Delta\left(\|S_i\|-\frac{\|S_i\|^2}{\omega}\right)-\gamma_1\|S_i\|^{q_1+1}\\&\quad-\gamma_2 m^{\frac{1-q_2}{2}}\|S_i\|^{q_2+1}\\&=-\Delta\left(\frac{\omega}{4}-\|S_i\|+\frac{\|S_i\|^2}{\omega}\right)\\&\quad+\left(\frac{\omega\Delta}{4}-\Delta\|S_i\|^{q_1+1}\right)-(\gamma_1-\Delta)\|S_i\|^{q_1+1}\\&\quad-\gamma_2 m^{\frac{1-q_2}{2}}\|S_i\|^{q_2+1},\end{aligned}\tag{36}$$

where $\gamma_1-\Delta>0$. Obviously, $-\Delta\left(\frac{\omega}{4}-\|S_i\|+\frac{\|S_i\|^2}{\omega}\right)\leq 0$ and we choose $\frac{\omega}{4}=\|S_i\|^{q_1+1}$, it follows that

$$\begin{aligned}\dot{V}_1(t)&\leq-(\gamma_1-\Delta)\|S_i\|^{q_1+1}-\gamma_2 m^{\frac{1-q_2}{2}}\|S_i\|^{q_2+1}\\&=-(\gamma_1-\Delta)2^{\frac{q_1+1}{2}}V_1^{\frac{q_1+1}{2}}(t)\\&\quad-\gamma_2 m^{\frac{1-q_2}{2}}2^{\frac{q_2+1}{2}}V_1^{\frac{q_2+1}{2}}(t).\end{aligned}\tag{37}$$

Then, from Lemma 1, each sliding mode $S_i(t)$ will converge to region $\|S_i\|=(\omega/4)^{\frac{1}{q_1+1}}$ within $t_2\leq\frac{1}{(\gamma_1-\Delta)(1-q_1)2^{(q_1-1)/2}}+\frac{1}{\gamma_2(q_2-1)m^{(1-q_2)/2}2^{(q_2-1)/2}}$.

Based on the analysis above, we can obtain that each sliding mode $S_i(t)$ will converge to the region $\|S_i\|=(\omega/4)^{\frac{1}{q_1+1}}$ with settling time

$$T_2=t_1+t_2.\tag{38}$$

The next proof process is similar to Theorem 1 and is omitted here. This is the end of proof. □

4. FURTHER DISCUSSION

In this section, the results of this paper are further compared with existing results and the novelty of the results is demonstrated.

Most of the existing consensus protocols can achieve consensus by using both the position and velocity information. However, in some cases, the agents are not equipped with the velocity measurement devices or the velocity information cannot be measured precisely due to the technology constraints or the environment disturbances. The existing results on fixed-time output feedback consensus, for example, see [32–36], can only prove fixed-time stable without settling time estimation. In this paper, uncertain disturbance is considered and the proposed control scheme can achieve fixed-time consensus with settling time estimation.

The consensus algorithms of second-order multi-agent system [32–36] were developed by bi-limit homogeneous systems theory. Bi-limit homogeneity can only prove that state observer and consensus algorithm are fixed-time stable, but doesn't give the settling time, which cannot be applied to scenarios with convergence time requirements. Among these literatures, only [35] investigated fixed-time output feedback consensus with uncertain disturbance and proposed an algorithm $u_i(t)=-(l_{ii}+b_i)^{-1}\left(\sum_{j=1,j\neq i}^{j=M}l_{ij}u_j+u_i'\right)$, where u_j denotes jth agent's control input. One can see clearly that each agent requires its neighbors' control input values to calculate its own control input value by using this controller. On one hand, each agent must continuously obtain its neighbors' control inputs, which will increase the communication burden in the network. On the other hand, information of controller updates for each agent is regarded as private and may be inaccessible to other agents. This will make it impossible to carry out the control scheme in some practical applications. This paper presents a novel method to tackle the settling time estimation problem, which does not require obtaining the control input of neighbors.

Bi-limit homogeneous systems theory only needs to prove that the system is globally asymptotically stable and homogeneous in the bi-limit. The controller design and proof method is relatively simple [32–36]. In order to prove stability and theoretically give a time estimation, we construct a Lyapunov function and skillfully derive the relationship between it and its derivative.

Theoretically, the proposed control scheme can achieve fixed-time output feedback consensus with settling time estimation. Practically, the proposed control scheme is more suitable for scenarios with convergence time requirements than those in [32–36].

5. SIMULATION RESULTS

To verify the validity of main results, we consider consensus problem of single-link robotic manipulators [15, 42]. Consider a group of single-link robotic manipulators, which are linked as shown in Fig. 2. The single-link robotic manipulator consists of a rigid link coupled through a gear train to a DC motor. The dynamics of

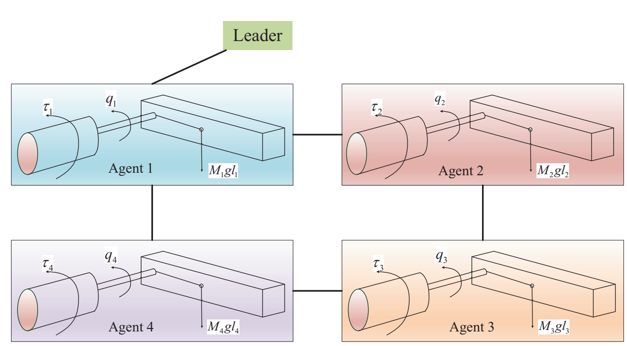

Fig. 2. The interaction graph.

single-link robotic manipulator is described by

$$J_i\ddot{q}_i + B_i\dot{q}_i + M_i g l_i \sin(q_i) = \tau_i, \tag{39}$$

where the states q_i and $\dot{q}_i$ are angle and angular velocity of the ith manipulator, J_i is the total rotational inertia of the link and the motor, B_i is the damping coefficient, M_i denotes the total mass of the link, g is the gravitational acceleration, l_i is the distance from the joint axis to the link center of mass for ith manipulator.

To achieve fixed-time consensus, the controller for robotic manipulator is constructed as

$$\tau_i = u_{in}^i + u_{out}^i = B_i\dot{q}_i + M_i g l_i \sin(q_i) + J_i(u_i + \Delta_i), \tag{40}$$

where the control of arm can be decomposed into the internal control u_{in}^i and external control u_{out}^i, i.e., $u_{in}^i = B_i\dot{q}_i + M_i g l_i \sin(q_i)$ and $u_{out}^i = J_i(u_i + \Delta_i)$. The internal control is designed by using the robotic manipulator's own output and the external control is generated by the external controller. With the controller (31), the manipulators' dynamics can be rewritten as

$$\ddot{q}_i = u_i. \tag{41}$$

Set $r_i = q_i$, $v_i = \dot{q}_i$, the manipulators' dynamics becomes multi-agent system (2) with uncertain disturbance. Next, two simulation examples are provided to verify the validity of fixed-time consensus algorithm.

In the first example, we verify the fixed-time consensus of multi-agent systems with $u_0 = 0$. Assume that $\Delta_i(t) = 0.1\cos(t)$. We design the controller (7) with $k_1 = 5$, $k_2 = 5$, $p = 0.8$, $q = 1.2$, $\gamma_1 = \gamma_2 = 1$ and the state observer (4) with $c_1 = c_2 = c_3 = 150$, $\alpha = 0.8$, $\beta = 1.2$, $T_u = 0.1$ s. The initial conditions for the single-link robotic manipulators are chosen as $r(0) = [\pi/11,\ -3\pi/7,\ 2\pi/5,\ 3\pi/10]^T$ rad, $v(0) = [0.12, -0.14, 0.25, -0.13]^T$ rad/s. The initial states of leader is selected as $[r_0(0),\ v_0(0)] = [0,\ 0.2]^T$. By using algorithm (7), we implement the simulations and plot the state trajectories and observers' output in Fig. 3 and Fig. 4, respectively.

It can be seen from Fig. 3 that the position and velocity of all followers converge rapidly from their respective initial states to leader's states with convergence time 5.96s, which verifies the validity of Theorem 1. From Fig. 4(a) and Fig. 4(c), we can obtain that each state observer rapidly estimates agent's position and the estimated errors rapidly converge to zero with convergence time 0.545s. From Fig. 4(b) and Fig. 4(d), we can obtain that each observer estimates agent's velocity with a longer convergence time 5.38s, which satisfies (5) and is mainly due to disturbance. From Fig. 3(a) and Fig. 3(c), when $t > 0.545s$, we get $\eta_{ri} = r_i$, further $\dot{\eta}_{\Delta i} = -c_3\sigma sig^{\alpha_3}(\eta_{ri} - r_i) - c_3(1-\sigma)sig^{\beta_3}(\eta_{ri} - r_i) = 0$. Note that $\Delta_i(t) = 0.1\cos(t)$ and it is impossible to achieve

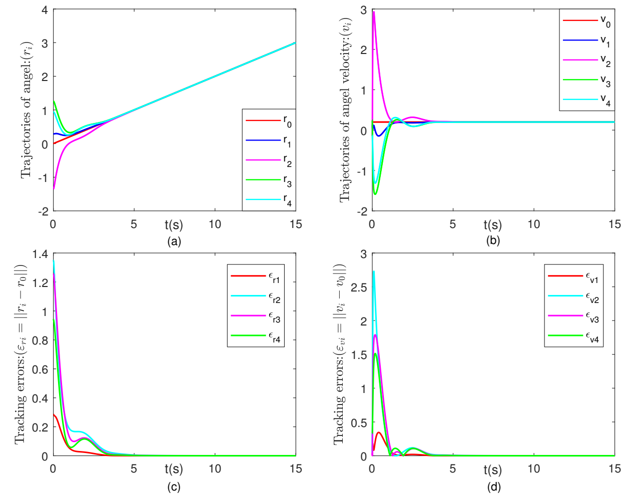

Fig. 3. Trajectories of system states and tracking errors: (a) r_i; (b) v_i; (c) $\varepsilon_{ri} = \|r_i - r_0\|$; (d) $\varepsilon_{vi} = \|v_i - v_0\|$.

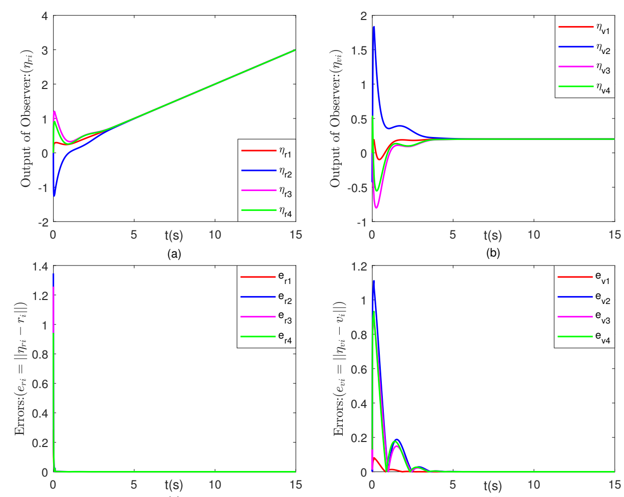

Fig. 4. Output of state observer and observer errors: (a) η_{ri}; (b) η_{vi}; (c) $e_{ri} = \|\eta_{ri} - r_i\|$; (d) $e_{vi} = \|\eta_{vi} - v_i\|$.

the identity $\dot{\eta}_{\Delta i} = 0$, which causes the observer spend extra time adjusting the estimation.

In the second example, the initial states and the parameters of controller and observer are selected to be the same as in the first example. The leader's control input is set to $u_0 = -0.2\sin(t)$ and $[r_0(0), v_0(0)] = [0, 0.1]$. The observer for leader is selected as $d_1 = d_2 = 50$, $\alpha_0 = 0.8$. By using algorithm (22), we implement the simulations and the results are shown in Fig. 5, Fig. 6 and Fig. 7, respectively.

It can be seen from Fig. 5 that the position and velocity of all followers converge rapidly from their respective initial states to leader's states with convergence time 6.075s, which verifies the validity of Theorem 2. From Fig. 6, we can obtain that each state observer estimates agent's states with convergence time 5.225s. From Fig. 7, we can ob-

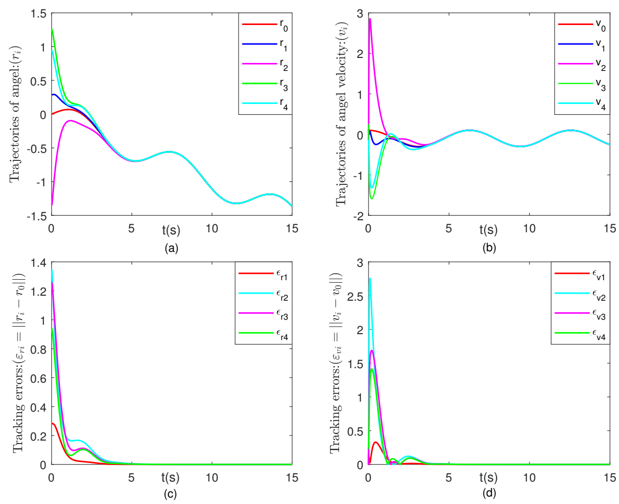

Fig. 5. Trajectories of system states and tracking errors: (a) r_i; (b) v_i; (c) $\varepsilon_{ri} = \|r_i - r_0\|$; (d) $\varepsilon_{vi} = \|v_i - v_0\|$.

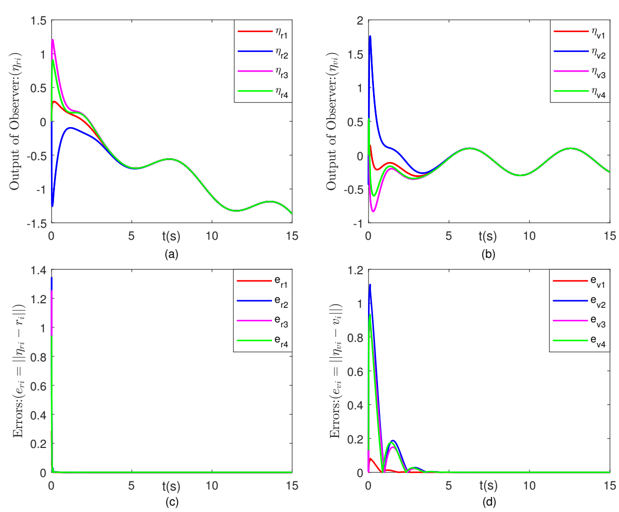

Fig. 6. Output of state observer and observer errors: (a) η_{ri}; (b) η_{vi}; (c) $e_{ri} = \|\eta_{ri} - r_i\|$; (d) $e_{vi} = \|\eta_{vi} - v_i\|$.

tain that each agent can estimate the leader's states within 0.195s.

Then, we conducted a simulation to verify that the proposed algorithm (32) can eliminate the chattering phenomenon. The initial states and the parameters of controller and observer are selected to be the same as in the second example. The saturation parameter is selected as $\omega = 0.005$. By using algorithm (32), we implement the simulations and the results are shown in Fig. 8, and Fig. 9, respectively. Fig. 8 shows that fixed-time leader-follower consensus is achieved and each state observer estimates agent's states in fixed time. It can be seen from Fig. 9 that when the consensus is achieved, the control inputs of all followers are in a smooth state, and no chattering phenomenon occurs.

The above simulation examples verify that the proposed

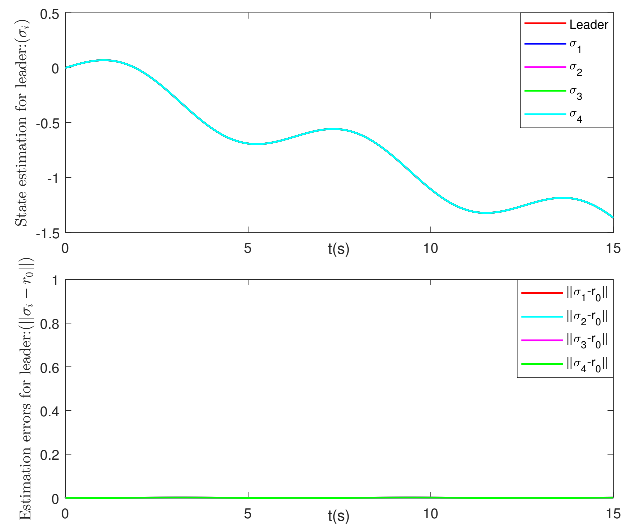

Fig. 7. Estimation for leader.

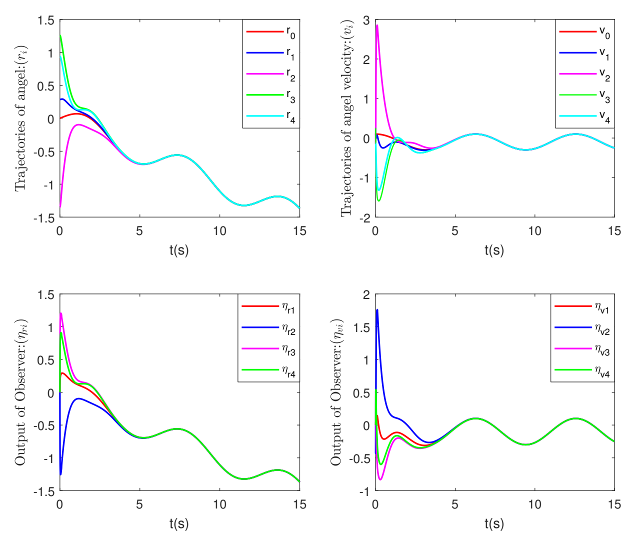

Fig. 8. Trajectories of system states and output of state observer by using controller (32): (a) r_i; (b) v_i; (c) η_{ri}; (d) η_{vi}.

control scheme can achieve fixed-time consensus with unmeasurable velocity and uncertain disturbance. Meanwhile, the proposed algorithm (32) solves the chattering problem caused by the integral sliding mode.

In order to show the advantages of the results in this paper, the simulation comparison with [35] is carried out.

$$u_i(t) = -(l_{ii} + b_i)^{-1} \left(\sum_{j=1, j\neq i}^{j=M} l_{ij} u_j + u_i{}' \right), \tag{42}$$

$$\begin{aligned} u_i{}'(t) = &-\hat{\Delta}_i' - k_{i,1} \left(sign(e_{i,1})^{\alpha_{i,1}} + sign(e_{i,1})^{\alpha'_{i,1}} + e_{i,1} \right) \\ &- k_{i,2} \left(sign(\hat{e}_{i,2})^{\alpha_{i,2}} + sign(e_{i,2})^{\alpha'_{i,2}} + e_{i,2} \right). \end{aligned} \tag{43}$$

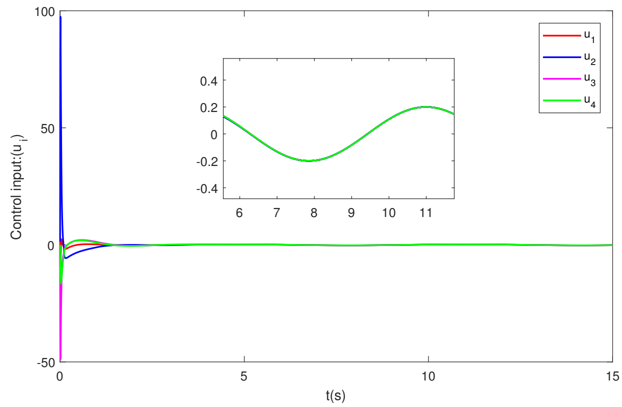

Fig. 9. Control input of each manipulator by using controller (32).

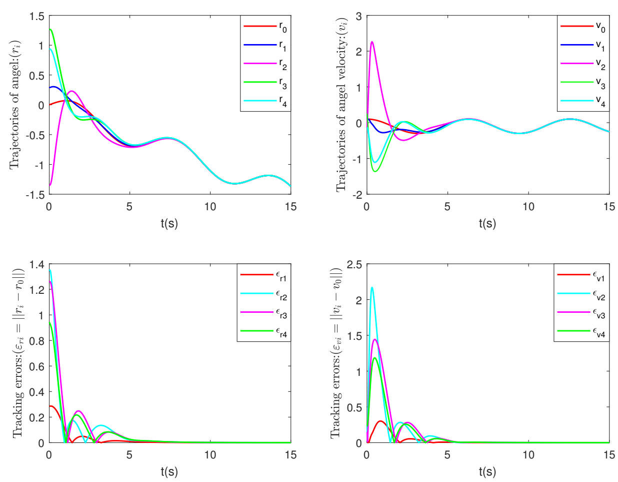

Fig. 10. Trajectories of system states and tracking errors by using controller (42): (a) r_i; (b) v_i; (c) $\varepsilon_{ri} = \|r_i - r_0\|$; (d) $\varepsilon_{vi} = \|v_i - v_0\|$.

The initial states are the same as in the second example. The coupling coefficient of controller (42) (43) are the same as that of the proposed controller, with $k_{i,1} = 5$, $k_{i,2} = 5$. Due to bi-limit homogeneity, the parameters are chosen as $\alpha_{i,1} = 0.667$, $\alpha_{i,2} = 0.8$, $\alpha'_{i,1} = 1.33$, $\alpha'_{i,2} = 1.143$. The simulation results of single-link robotic arm are shown in Fig. 10 and Fig. 11.

From Fig. 10, the position and velocity of all followers converge from their respective initial states to leader's states with convergence time 8.913s. From Fig. 11, we can obtain that each state observer estimates agents' states with convergence time 6.308s. Comparing Figs. 10 and 11 with Figs. 5 and 6, it can be seen that the proposed algorithm in this paper has better control performance than the controller (42). Note that the controller (42) needs the agent to continuously acquire the control inputs of its neighbors, and the information of controller updates for each agent is regarded as private and may be inaccessible to other agents, which not only causes an increase in communication burden, but also is not easy to implement in practical applications.

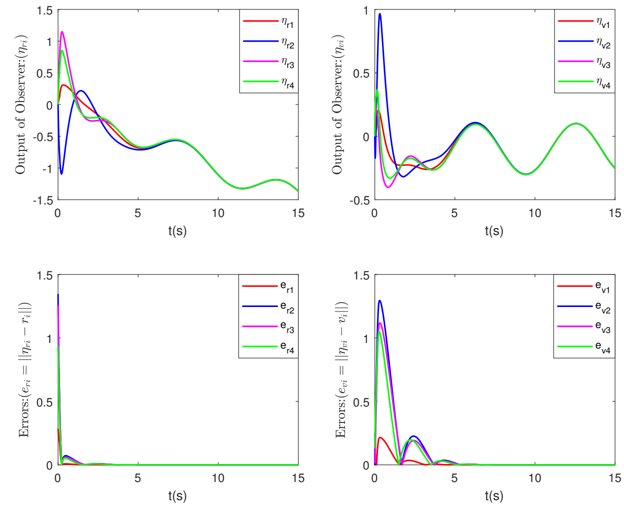

Fig. 11. Output of state observer and observer errors by using controller (42): (a) η_{ri}; (b) η_{vi}; (c) $e_{ri} = \|\eta_{ri} - r_i\|$; (d) $e_{vi} = \|\eta_{vi} - v_i\|$.

6. CONCLUSION

Fixed-time consensus problem with disturbance and unknown velocity has been discussed. First, a fixed-time convergent state observer has been proposed such that the velocity information and the disturbance can be estimated within fixed time. Second, a consensus algorithm has been established based on integral sliding mode such that fixed-time consensus can be achieved and the settling time can be theoretically estimated. Finally, the efficiency of the proposed control scheme has been verified by simulation results.

Noting that the connection topology of networks is invariant in this paper. However, as pointed out in [43], under some undesirable networked environments, the changes of connection topology are randomly occurring and obey the Markov process. The time-varying topology is more realistic relying on the fact that the implemented topology in practice often varies according to the different situations [44]. Future topics will focus on fixed-time consensus control for Markov Jump Systems.

APPENDIX A

It follows from fixed-time state observer (4) that the estimation errors φ_{ri}, φ_{vi}, $\varphi_{\Delta i}$ converge to zero in fixed time, which implies that φ_{ri}, φ_{vi}, $\varphi_{\Delta i}$ are always bounded.

To examine the boundedness of the closed-loop system within T_1, we first analysis the Lyapunov function (8).

When $t \le T_u$, differentiating $V_1(t)$ versus time yields

$$\begin{aligned}\dot V_1(t) =& S_i^T \dot S_i = S_i^T\left(-\gamma_1 \operatorname{sgn}(S_i) - \gamma_2 sig(S_i)^q \right.\\ &\left. + \eta_{\Delta i} - c_2 sig^{\beta_2}(\varphi_{ri})\right)\\ =& S_i^T\left(\Delta_i - \gamma_1 \operatorname{sgn}(S_i) - \gamma_2 sig(S_i)^q\right.\\ &\left. + \eta_{\Delta i} - \Delta_i - c_2 sig^{\beta_2}(\varphi_{ri})\right)\\ \le& -(\gamma_1 - \Delta)\|S_i\| - \gamma_2 m^{(1-q)/2}\|S_i\|^{q+1}\\ &+ \|S_i\|\|\phi_{\Delta i}\| + c_2\|S_i\|\|\phi_{ri}\|^{\beta_2}. \qquad \text{(A.1)}\end{aligned}$$

Noting that $\varphi_{ri}, \varphi_{vi}, \varphi_{\Delta i}$ are all bounded, which makes $\dot V_1(t)$ bounded. As a result, Lyapunov function $V_1(t)$ as well as S_i is bounded in time interval $[0, T_u]$. Similarly, when $T_u < t \le T_1$, we have

$$\begin{aligned}\dot V_1(t) \le& -(\gamma_1 - \Delta)\|S_i\| - \gamma_2 m^{(1-q)/2}\|S_i\|^{q+1}\\ &+ \|S_i\|\|\phi_{\Delta i}\| + c_2 m^{(1-\alpha_2)/2}\|S_i\|\|\phi_{ri}\|^{\alpha_2}. \qquad \text{(A.2)}\end{aligned}$$

It follows that $V_1(t)$ as well as S_i is bounded in time interval $(T_u, T_1]$. We can obtain that $V_1(t)$ and S_i are bounded when $t \le T_1$.

Next, we show that the closed-loop system are bounded when $t \le T_1$. Then, the closed-loop system can be rewritten as

$$\begin{cases}\dot\varsigma = \kappa,\\ \dot\kappa = -B\psi(t) - Bsig(\psi(t))^p - Bsig(\psi(t))^{\frac{1}{p}}\\ \qquad - \gamma_1 B\operatorname{sgn}(S) - \gamma_2 Bsig(S)^q + B\Delta(t),\\ \psi(t) = k_1\varsigma + k_2\kappa,\end{cases} \qquad \text{(A.3)}$$

where $\operatorname{sgn}(S) = (\operatorname{sgn}(S_1), \cdots, \operatorname{sgn}(S_n))^T$, $sig(S)^q = (sig(S_1)^q, \cdots, sig(S_n)^q)^T$, $\Delta = (\Delta_1, \cdots, \Delta_n)^T$. Consider the Lyapunov function (13), differentiating $V_2(t)$ versus time yields

$$\begin{aligned}\dot V_2(t) \le& -k_1^2\varsigma^T\varsigma - \left(k_2^2 - k_1\lambda_{\max}(B^{-1})\right)\kappa^T\kappa\\ &- \|\psi\|^{p+1} - \sqrt{nm}^{1-\frac{1}{p}}\|\psi\|^{\frac{1}{p}+1}\\ &- \psi^T\left(\gamma_1\operatorname{sgn}(S) + \gamma_2 sig(S)^q - \Delta(t)\right). \qquad \text{(A.4)}\end{aligned}$$

Noting that S and Δ are bounded, so there is a positive constant ϖ such that $\|\gamma_1\operatorname{sgn}(S) + \gamma_2 sig(S)^q - \Delta(t)\| \le \varpi$. Define $o = \sqrt{k_2^2 - k_1\lambda_{\max}(B^{-1})}$. Utilizing the inequality $\|\psi\| = \|k_1\varsigma + k_2\kappa\| \le k_1\|\varsigma\| + k_2\|\kappa\|$, we have

$$\begin{aligned}\dot V_2(t) \le& -k_1^2\|\varsigma\|^2 - o\|\kappa\|^2 - \|\psi\|^{p+1}\\ &- \sqrt{nm}^{1-\frac{1}{p}}\|\psi\|^{\frac{1}{p}+1} + \varpi\left(k_1\|\varsigma\| + k_2\|\kappa\|\right)\\ =& \frac{k_2^2\varpi^2}{4o^2} + \frac{\varpi^2}{4} - \|\psi\|^{p+1} - \sqrt{nm}^{1-\frac{1}{p}}\|\psi\|^{\frac{1}{p}+1}\\ &- \left(k_1\|\varsigma\| - \frac{\varpi}{2}\right)^2 - \left(o\|\kappa\| - \frac{k_2\varpi}{2o}\right)^2. \qquad \text{(A.5)}\end{aligned}$$

Obviously, $\dot V_2(t)$ is bounded, it is easy to obtain that $V_2(t)$ as well as $\|\varsigma\|, \|\kappa\|$ are also bounded when $t \le T_1$. Thus, the states of the closed-loop system are bounded in time interval $[0, T_1]$.

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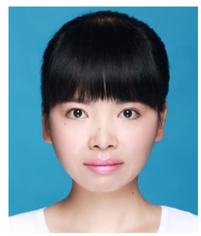

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