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Asynchronously Input-output Finite-time Control of Positive Impulsive Switched Systems

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Abstract: This paper considers asynchronously input-output finite-time control of positive impulsive switched systems(PISS). Firstly, the definition of input-output finite-time stability(IO-FTS) is introduced. By using the Lyapunov functional and average dwell time(ADT) approach, a state feedback controller is designed and new sufficient conditions are obtained to guarantee the corresponding closed-loop system is IO-FTS under asynchronous switching. Such conditions can be solved by linear programming. Finally, a practical example is provided to show the effectiveness of the proposed method.

Keywords: Asynchronous control, average dwell time, input-output finite-time stability, positive impulsive switched systems.

1. INTRODUCTION

Positive switched systems which consist of a finite number of positive subsystems and a switching rule orchestrating the switching between the subsystems have been applied in many practical systems, such as communication networks [1], viral mutation [2], formation flying [3], and so on. Many remarkable results have been presented, see [4–11] and references therein.

However, most results mentioned above focus on the asymptotic stability rather than finite-time stability (FTS). FTS requires that the states do not exceed a certain bound during a special time interval. Some remarkable results about FTS of positive switched systems have been published, see [12–14]. [15] firstly defined the definition of IO-FTS, which was fully consistent with the definition of FTS. IO-FTS means that given a class of norm bounded input signals over a specified time interval $[0,T]$, the outputs of the system do not exceed an assigned threshold during such time interval. For positive switched systems, some meaningful results about IO-FTS analysis have been presented, see [16–19] and references therein.

However, all of the results mentioned above are obtained under the assumption that the controllers are switched synchronously with the switching of system modes. In actual operation, it inevitably takes some time to identify the system modes and apply the matched controller, so there exists asynchronous switching, which means the switching instants of the controllers exceed or lag behind those of the systems. So it is necessary to consider asynchronous switching for efficient controller design. At present, some results on switched systems under asynchronous switching have been proposed, see [20–22]. [20] investigated the problem of asynchronous finite-time dynamic output feedback control problem for switched time-delay systems with non-linear disturbances. [21] addressed the problem of finite-time stabilization under asynchronous switching for a class of switched time-delay systems with nonlinear disturbances. [22] considered the problem of asynchronous finite-time stabilisation for a class of switched systems with average dwell time. It is worth noting that [20–22] are involved in non-positive switched systems. Recently, [23] investigated the stability and asynchronous L_1 control problems for a class of switched positive linear systems with time-varying delays. [24] studied the problem of dwell time(DT) stability, L_1-gain performance analysis and asynchronous L_1-gain controller design of uncertain switched positive linear systems. But, [23] and [24] are involved in asymptotic stability rather than IO-FTS. In practical applications, the impulse has great influence on the performance of the systems. So it should be taken into account in the performance analysis of positive switched systems. Though [25] firstly studied the problem of FTS of a class of discrete im-

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pulsive switched positive time-delay systems under asynchronous switching, the effect of disturbance was ignored. To the best of our knowledge, the IO-FTS problem of continuous PISS under asynchronous switching is still open.

In this paper, we consider the problem of IO-FTS of PISS under two types of disturbances by constructing the Lyapunov functional with ADT technique. Firstly, the concept of IO-FTS is extended to PISS. Secondly, a state feedback controller under asynchronous switching is designed and sufficient conditions are obtained to guarantee that the closed-loop system is IO-FTS. Some sufficient conditions are obtained by linear programming.

The rest of the paper is organised as follows: Section 2 gives some necessary preliminaries and problem statements. In Section 3, the main results are given. In Section 4, a practical examples are provided. Section 5 concludes the paper.

Notations: The representation $A \succ 0$ $(\succeq 0, \prec 0, \preceq 0)$ means that $a_{ij} > 0$ $(\geq 0, < 0, \leq 0)$, which is also applying to a vector. $A \succ B$ $(A \succeq B)$ means that $A - B \succ 0$ $(A - B \succeq 0)$. R_+^n is the n-dimensional non-negative (positive) vector space. $R^{n\times n}$ denotes the space of $n\times n$ matrices with real entries. I represents the n-dimensional identity matric. A^T denotes the transpose of matrix A. 1-norm $\|x\|$ is defined by $\|x\| = \sum_{k=1}^{n} |x_k|$. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2. PRELIMINARIES AND PROBLEM STATEMENTS

Consider the following PISS:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t_i)}x(t) + B_{\sigma(t_i)}u(t) + C_{\sigma(t_i)}\omega(t), \\ \qquad t \in (t_i, t_{i+1}], \ \ i \in N, \\ \triangle x(t) = E_{\sigma(t_i)}x(t), \ \ t = t_i, \ \ i \in N^+, \\ y(t) = G_{\sigma(t)}x(t), \\ x(t_0^+) = x_0, \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ and $y(t) \in R^s$ represent the control input and output. $\sigma(t)$ is the system switching signal, and take value at $S = 1, 2, ..., M$. For all $t \in [t_0, T)$, $\triangle x(t_i) = x(t_i^+) - x(t_i^-) = x(t_i^+) - x(t_i)$, $x(t_i^+) = \lim_{h\to 0^+} x(t_i + h)$, $x(t_i^-) = \lim_{h\to 0^-} x(t_i + h) = x(t_i)$, it implies system (1) is left continuous. For a switching sequence $0 < t_0 < t_1 < \cdots < t_i < t_{i+1} < \cdots$, $\sigma(t)$ is a left continuous form, when $t \in (t_i, t_{i+1})$, $\sigma(t_i)$ subsystem is active. For $\sigma(t_i)$, $\sigma(t_i) \to \{1, 2, \cdots, M\}$. $\omega(t) \in R^l$ is the exogenous disturbance and defined as

$$W_1 = \left\{ \omega(\cdot) \in L_{1,[0,T_f]} : \int_0^{T_f} \|\omega(t)\| dt \leq d \right\} \tag{2}$$

with a known scalar $d > 0$.

Next, we consider state feedback controllers $u(t) = K_{\sigma(t_i)}x(t)$, $K_{\sigma(t_i)} \in R^{m\times n}$, we call $u(t)$ as matched controller. However, in actual operation, it inevitably takes some time to identify the system modes and apply the matched controller, so there exists asynchronous switching, which means the switching instants of the controllers exceed or lag behind those of the systems. Without loss of generality, we define $\hat{u}(t) = K_{\sigma(t_{i-1})}x(t)$, $K_{\sigma(t_{i-1})} \in R^{n\times m}$. So we consider the following closed-loop system:

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t_i)} + B_{\sigma(t_i)}K_{\sigma(t_{i-1})})x(t) + C_{\sigma(t_i)}\omega(t), \\ \qquad t \in (t_i, \bar{t}_i] \\ \dot{x}(t) = (A_{\sigma(t_i)} + B_{\sigma(t_i)}K_{\sigma(t_i)})x(t) + C_{\sigma(t_i)}\omega(t), \\ \qquad t \in (\bar{t}_i, t_{i+1}], \\ \triangle x(t) = E_{\sigma(t_i)}x(t), \ \ t = t_i, \\ y(t) = G_{\sigma(t)}x(t), \\ x(t_0^+) = x_0, \end{cases} \tag{3}$$

where $i \geq 1$, and $\bar{t}_i (t_i < \bar{t}_i < t_{i+1})$ represents the switching point of the controllers. If $i = 0$, then the closed-loop system is

$$\begin{aligned} \dot{x}(t) =& (A_{\sigma(t_0)} + B_{\sigma(t_0)}K_{\sigma(t_0)})x(t) + C_{\sigma(t_0)}\omega(t), \\ & t \in (t_0, t_1]. \end{aligned} \tag{4}$$

It implies that impulse and asynchronous switching are no influence at initial constant.

Next, we will give some definitions and lemmas for the PISS.

Definition 1: System (3) is said to be positive if for any switching signals $\sigma(t)$, control input $u(t) \succeq 0$ and any disturbance input $\omega(t) \succeq 0$, the corresponding trajectory satisfies $x(t) \succeq 0$ and $y(t) \succeq 0$ for all $t \geq 0$.

Lemma 1 [23]**:** System (3) is positive if and only if $A_i + B_iK_i$, $\forall i \in N$ are Metzler matrices and $\forall i \in N$, $C_i \geq 0$, $E_i \geq 0$ and $G_i \geq 0$.

Definition 2: For any switching signal $\sigma(t)$ and any $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the switching numbers, over the interval $[t_1, t_2)$. For given $T_\alpha > 0$ and $n_0 > 0$, if the inequality

$$N_\sigma(t_1, t_2) \leq n_0 + \frac{t_2 - t_1}{T_\alpha} \tag{5}$$

holds, then T_α is called an average dwell time, and n_0 is called a chatting bounding. Generally, we choose $n_0 = 0$.

Definition 3 (Finite-Time Stability (FTS))**:** For a given constant T_f, disturbances signals W_1 defned by (2), and a vector $\delta \succ 0$; system (3) is said to be IO-FTS with respect to $(\delta, T_f, d, \sigma(t))$, if

$$\omega(t) \in W_1 \Rightarrow y^T(t)\delta < 1, \ \ \forall t \in [0, T_f]. \tag{6}$$

If the disturbance $\omega(t)$ satisfies $\omega(t) \in W_2$, W_2 is defined as

$$W_2 = \{\omega(\cdot) \in L_{\infty,[0,T_f]} : max_{t\in[0,T_f]} \|\omega(t)\| \leq d\}, \tag{7}$$

then we give Definition 4:

Definition 4: For a given constant T_f, disturbances signals W_2 defined by (7), and a vector $\delta \succ 0$; system (3) is said to be IO-FTS with respect to $(\delta, T_f, d, \sigma(t))$, if

$$\omega(t) \in W_2 \Rightarrow y^T(t)\delta < 1,\ \forall t \in [0, T_f]. \tag{8}$$

The aim of this paper is to design a state feedback controller $u(t)$ and find a class of switching signals $\sigma(t)$ for system (3) such that the corresponding closed-loop system is IO-FTS.

3. MAIN RESULTS

3.1. Asynchronously input-output finite-time controller design

In this subsection, we will focus on the problem of IO-FTS for PISS (3). Firstly, we consider the situation of $\omega(t) \in W_1$. The following theorem gives sufficient conditions of IO-FTS for system (3).

Let $T_\downarrow(t_{i+1}, t_i) = t_{i+1} - \bar{t}_i$ and $T_\uparrow(t_{i+1}, t_i) = \bar{t}_i - t_i$ represent the running time of the matched controller and unmatched controller in the time interval $(t_i, t_{i+1}]$, respectively. $T^-(t, t_0)$ and $T^+(t, t_0)$ represent the total running time of $T_\downarrow(t, t_0)$ and $T_\uparrow(t, t_0)$.

Theorem 1: Consider the PISS (3), for a given time constant T_f, ξ_1, ξ_2, γ, λ, $\mu > 1$, vectors $\delta \succ 0$, if there exist a set of positive vectors ν_i, ν_j, $i \neq j$, $i, j \in I$ and controller gain K_i such that the following inequalities hold:

$$(A_i + B_i K_j)^T \nu_i - \xi_1 \nu_i \prec 0, \tag{9}$$

$$C_i^T \nu_i \prec \gamma, \tag{10}$$

$$(A_i + B_i K_i)^T \nu_i - \xi_2 \nu_i \prec 0, \tag{11}$$

$$(I + E_i)^T \nu_i - \mu \nu_j \prec 0, \tag{12}$$

$$G_i^T \delta \preceq \nu_i, \tag{13}$$

where $A_i + B_i K_i$ and $A_i + B_i K_j$ are Metzler matrices, then the system (3) is IO-FTS under the following ADT scheme

$$\frac{T^-(t,t_0)}{T^+(t,t_0)} \geq \frac{\ln \xi_2 - \ln \lambda}{\ln \lambda - \ln \xi_1},\quad \xi_2 > \lambda > \xi_1 > 1, \tag{14}$$

$$T_\alpha > T_\alpha^* = \frac{T_f \ln \mu}{-\ln(e^{\lambda^{T_f}} \gamma d)}. \tag{15}$$

Proof: Construct the Lyapunov functional for the system (3) as follows:

$$V_i(x(t)) = x^T(t)\nu_i. \tag{16}$$

When $t \in T_\uparrow(t_{i+1}, t_i)$, from (3) we can get

$$\dot{V}_i(t) = x^T(t)(A_i + B_i K_j)^T \nu_i + \omega^T(t) C_i^T \nu_i \tag{17}$$

from (9) and (10), we obtain

$$\dot{V}_i(t) \leq \xi_1 V_i(t) + \gamma \|\omega(t)\|. \tag{18}$$

When $t \in T_\downarrow(t_{i+1}, t_i)$, from (3) we can get

$$\dot{V}_i(t) = x^T(t)(A_i + B_i K_i)^T \nu_i + \omega^T(t) C_i^T \nu_i \tag{19}$$

from (10) and (11), we obtain

$$\dot{V}_i(t) \leq \xi_2 V_i(t) + \gamma \|\omega(t)\|. \tag{20}$$

When $t = t_i$, from (3) we can get

$$V_i(t_i^+) = x^T(t_i^+) = [(I + E_i)x(t_i)]^T \nu_i. \tag{21}$$

From (12), we have

$$V_i(t_i^+) \leq \mu V_j(t_i). \tag{22}$$

So, for all $\forall t \in (t_i, t_{i+1}]$, from (18), (20) and (22), we obtain

$$\begin{aligned}
&V_{\sigma(t)}(t) \\
&\leq e^{\xi_1 T_\downarrow(t,t_i) + \xi_2 T_\uparrow(t,t_i)} \mu V_{\sigma(t_{i-1})}(t_i) \\
&\quad + \gamma \int_{t_i}^{t} e^{\xi_1 T_\downarrow(t,t_i) + \xi_2 T_\uparrow(t,t_i)} \|\omega(s)\| ds \\
&\leq e^{\xi_1 T_\downarrow(t,t_i) + \xi_2 T_\uparrow(t,t_i) + \xi_1 T_\downarrow(t_i,t_{i-1}) + \xi_2 T_\uparrow(t_i,t_{i-1})} \mu^2 V_{\sigma(t_{i-2})}(t_{i-1}) \\
&\quad + \gamma \int_{t_i}^{t} e^{\xi_1 T_\downarrow(t,t_i) + \xi_2 T_\uparrow(t,t_i)} \|\omega(s)\| ds \\
&\quad + \mu\gamma \int_{t_{i-1}}^{t_i} e^{\xi_1 T_\downarrow(t,t_i) + \xi_2 T_\uparrow(t,t_i) + \xi_1 T_\downarrow(t_i,t_{i-1}) + \xi_2 T_\uparrow(t_i,t_{i-1})} \|\omega(s)\| ds \\
&\leq \cdots \leq \\
&\leq \mu^m e^{\xi_1 T^-(t,t_0) + \xi_2 T^+(t,t_0)} ((V_{\sigma(t_0)}(t_0))) + \gamma d).
\end{aligned} \tag{23}$$

It follows from (13) that

$$\begin{aligned}
(T^-(t,t_0) + T^+(t,t_0)) \ln \lambda \geq & T^-(t,t_0) \ln \xi_1 \\
& + T^+(t,t_0) \ln \xi_2,
\end{aligned} \tag{24}$$

which can be rewritten as

$$\xi_1^{T^-(t,t_0)} \xi_2^{T^+(t,t_0)} \leq \lambda^{(T^-(t,t_0) + T^+(t,t_0))} = \lambda^{t - t_0}. \tag{25}$$

From (25) and $x(t_0) = 0$, we have

$$V_{\sigma(i)}(t) \geq x^T(t)\nu_i, \tag{26}$$

$$V_{\sigma(i)}(t) \leq \mu^{\frac{t}{T_\alpha}} (e^{\lambda^t} \gamma d). \tag{27}$$

From $t \in [0, T_f]$, we obtain

$$\begin{aligned}
y^T(t)\delta = & x^T(t) G_{\sigma(t)}^T \delta \leq \mu^{\frac{t}{T_\alpha}} (e^{\lambda^t} \gamma d) \\
& \leq \mu^{\frac{T_f}{T_\alpha}} (e^{\lambda^{T_f}} \gamma d).
\end{aligned} \tag{28}$$

Substituting (15) into (28), one has

$$y^T(t)\delta < 1,\quad t \in [0, T_f]. \tag{29}$$

According to Definition 4, we conclude that system (3) is IO-FTS. □

Algorithm 1:

Step 1: Inputting matrices A_i, B_i, C_i, E_i, G_i and δ.

Step 2: By adjusting the parameters ξ_1, ξ_2, λ, γ, μ and letting $f_i = K_j^T B_i^T \nu_i$, $l_i = K_i^T B_i^T \nu_i$, solving (9)-(13) via linear programming, positive vectors ν_i, f_i, l_i can be obtained.

Step 3: Substituting ν_i and f_i into $f_i = K_j^T B_i^T \nu_i$, K_j can be obtained. Then, substituting K_i and ν_i into $\tilde{l}_i = K_i^T B_i^T \nu_i$.

Step 4: If $\tilde{l}_i \leq l_i$ are satisfied and $A_i + B_i K_i$, $A_i + B_i K_j$ are Metzler matrices, then K_i are admissible. Otherwise, return to Step 2.

Next, we consider $\omega(t) \in W_2$. The following theorem gives sufficient conditions of IO-FTS for system (3).

Theorem 2: Consider the PISS (3), for a given time constant T_f, ξ_1, ξ_2, γ, λ, $\mu > 1$, vectors $\delta \succ 0$, if there exist a set of positive vectors ν_i, ν_j, $i \neq j$, $i, j \in I$ and controller gain K_i, (9)-(14) hold, then the system (3) is IO-FTS under the following ADT scheme:

$$T_\alpha > T_\alpha^* = \frac{T_f ln\mu}{-ln(e^{\lambda^{T_f}}\gamma d)}. \tag{30}$$

Proof: Replacing $\omega(t) \in W_1$ with $\omega(t) \in W_2$, we have $\int_0^{T_f} \|w(t)\| dt \leq T_f d$. Similar to the proof of Theorem 1, then (23) can be transformed into

$$\begin{aligned}
&V_{\sigma(i)}(t) \\
&\leq e^{\xi_1 T_\downarrow(t,t_i)+\xi_2 T_\uparrow(t,t_i)} \mu V_{\sigma(i-1)}(t_i) \\
&\quad + \gamma \int_{t_i}^{t} e^{\xi_1 T_\downarrow(t,t_i)+\xi_2 T_\uparrow(t,t_i)} \|\omega(s)\| ds \\
&\leq e^{\xi_1 T_\downarrow(t,t_i)+\xi_2 T_\uparrow(t,t_i)+\xi_1 T_\downarrow(t_i,t_{i-1})+\xi_2 T_\uparrow(t_i,t_{i-1})} \mu^2 V_{\sigma(i-2)}(t_{i-1}) \\
&\quad + \gamma \int_{t_i}^{t} e^{\xi_1 T_\downarrow(t,t_i)+\xi_2 T_\uparrow(t,t_i)} \|\omega(s)\| ds \\
&\quad + \mu\gamma \int_{t_{i-1}}^{t_i} e^{\xi_1 T_\downarrow(t,t_i)+\xi_2 T_\uparrow(t,t_i)+\xi_1 T_\downarrow(t_i,t_{i-1})+\xi_2 T_\uparrow(t_i,t_{i-1})} \|\omega(s)\| ds \\
&\leq \cdots \leq \\
&\leq \mu^m e^{\xi_1 T^-(t,t_0)+\xi_2 T^+(t,t_0)} ((V_{\sigma(t_0)}(t_0))) + \gamma T_f d).
\end{aligned} \tag{31}$$

It follows from (14) that

$$\begin{aligned}
(T^-(t,t_0) + T^+(t,t_0)) \ln\lambda \geq & T^-(t,t_0) \ln\xi_1 \\
& + T^+(t,t_0) \ln\xi_2,
\end{aligned} \tag{32}$$

which can be rewritten as

$$\xi_1^{T^-(t,t_0)} \xi_2^{T^+(t,t_0)} \leq \lambda^{(T^-(t,t_0)+T^+(t,t_0))} = \lambda^{t-t_0}. \tag{33}$$

From (31) and $x(t_0) = 0$, we have

$$V_{\sigma(i)}(t) \geq x^T(t)\nu_i, \tag{34}$$

$$V_{\sigma(i)}(t) \leq \mu^{\frac{t}{T_\alpha}} (e^{\lambda^t} \gamma T_f d). \tag{35}$$

From $t \in [0, T_f]$, we obtain

$$\begin{aligned}
y^T(t)\delta = & x^T(t) G_{\sigma(t)}^T \delta \leq \mu^{\frac{t}{T_\alpha}} (e^{\lambda^t} \gamma T_f d) \\
\leq & \mu^{\frac{T_f}{T_\alpha}} (e^{\lambda^{T_f}} \gamma T_f d).
\end{aligned} \tag{36}$$

Substituting (30) into (36), one has

$$y^T(t)\delta < 1, \quad t \in [0, T_f]. \tag{37}$$

According to Definition 3, we conclude that system (3) is IO-FTS.

4. NUMERICAL EXAMPLE

Example 1: In [26], a water quality system is described by a positive switched system. But, in actual life, affected by the discharge and pollution of industrial waste, the water-quality constituents always abrupt jumps at certain instants, which can be regarded as an impulsive behavior. The parameters of a water quality system with impulsive behavior and disturbance signals $\omega(t) \in W_1$ are given as:

$$A_1 = \begin{bmatrix} -2 & 2.5 & 1 \\ 4 & 1 & 1 \\ 2 & 2 & -3 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 & 0.4 & 0.2 \\ 0.6 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2 \\ 0.5 \\ 0.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & 3 & 3 \\ 4 & -3 & 1 \\ 2 & 2 & -3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.5 & 0.4 & 0.2 \\ 0.6 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.1 \\ 0.6 \\ 0.3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.2 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.5 \\ 0.2 & 0.1 & 0.2 \end{bmatrix},$$

$$\delta = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}.$$

Choosing $T_f = 10$, $\xi_1 = 1.1$, $\xi_2 = 1.3$, $\lambda = 1.16$, $\mu = 3$, $\gamma = 0.05$, $\bar{t}_i - t_i = 0.5$, $\omega(t) = 0.001e^{-t}$, $d = 0.01$. Solving the inequalities in Theorem 1 by linear programming, we have

$$\nu_1 = \begin{bmatrix} 0.0978 \\ 0.1900 \\ 0.1100 \end{bmatrix}, \quad \nu_2 = \begin{bmatrix} 0.1300 \\ 0.1300 \\ 0.1024 \end{bmatrix},$$

$$f_1 = \begin{bmatrix} -0.8000 \\ -0.8000 \\ -0.1500 \end{bmatrix}, \quad f_2 = \begin{bmatrix} -0.8000 \\ -0.8000 \\ -0.1000 \end{bmatrix},$$

$$l_1 = \begin{bmatrix} -0.9000 \\ -0.5000 \\ -0.1000 \end{bmatrix}, \quad l_2 = \begin{bmatrix} -0.5000 \\ -0.1000 \\ -0.1000 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -6.6100 & -6.6100 & -0.8260 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -5.4000 & -5.4000 & -1.0169 \end{bmatrix},$$

$$\tilde{l}_1 = K_1^T B_1^T v_1 = \begin{bmatrix} -0.9754 \\ -0.9754 \\ -0.1219 \end{bmatrix},$$

$$\tilde{l}_2 = K_2^T B_2^T v_2 = \begin{bmatrix} -0.6573 \\ -0.6573 \\ -0.1238 \end{bmatrix},$$

$$A_1 + B_1 K_1 = \begin{bmatrix} -3.3220 & 1.1780 & 0.8348 \\ 0.6950 & -2.3050 & 0.5870 \\ 0.0170 & 0.0170 & -3.2478 \end{bmatrix},$$

$$A_2 + B_2 K_1 = \begin{bmatrix} -2.6610 & 2.3390 & 2.9174 \\ 0.0340 & -6.9600 & 0.5044 \\ 0.0170 & 0.0170 & -3.2478 \end{bmatrix},$$

$$A_2 + B_2 K_2 = \begin{bmatrix} -2.5400 & 2.4600 & 2.8983 \\ 0.7600 & -6.2400 & 0.3899 \\ 0.3800 & 0.3800 & -3.3051 \end{bmatrix},$$

$$A_1 + B_1 K_2 = \begin{bmatrix} -3.0800 & 1.4200 & 0.7966 \\ 1.3000 & -1.7000 & 0.4916 \\ 0.3800 & 0.3800 & -3.3051 \end{bmatrix}.$$

It is easy to firm that $A_i + B_i K_i$ and $A_i + B_i K_j$ are Metzler matrices and $\tilde{l}_i \leq l_i$ are satisfied, then K_i are admissible. According to (14) and (15), we get $\frac{T^-(t,t_0)}{T^+(t,t_0)} \geq \frac{ln\xi_2 - ln\lambda}{ln\lambda - ln\xi_1} = 1.1$, $T_\alpha^* = 3.5$.

The simulation results are shown in Figs. 1-6, where the initial conditions of system (1) are $x(0) = [0,0,0]^T$. The switching signal $\sigma(t)$ is depicted in Fig. 1. The state trajectories of the open-loop system are shown in Fig. 2. The state trajectories of the closed-loop system are shown in Fig. 3. Fig. 4 shows the controller signal. Fig. 5 plots the evolution of $y^T(t)\delta$, which implies that the corresponding closed-loop system is IO-FTS with respect to (δ T_f, d, $\sigma(t)$). The state trajectories of the closed-loop system under synchronous control are shown in Fig. 6. From Fig. 3 and Fig. 6, we also conclude that the proposed method is effective.

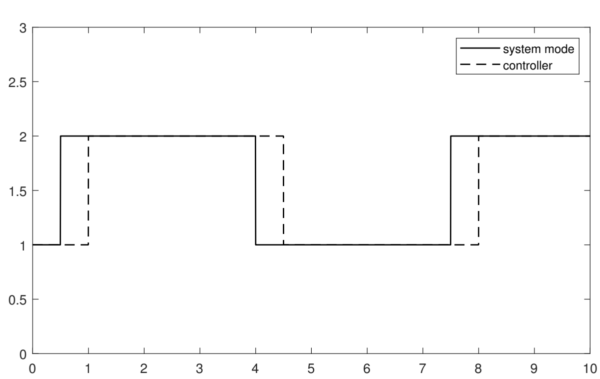

Fig. 1. Switching signal.

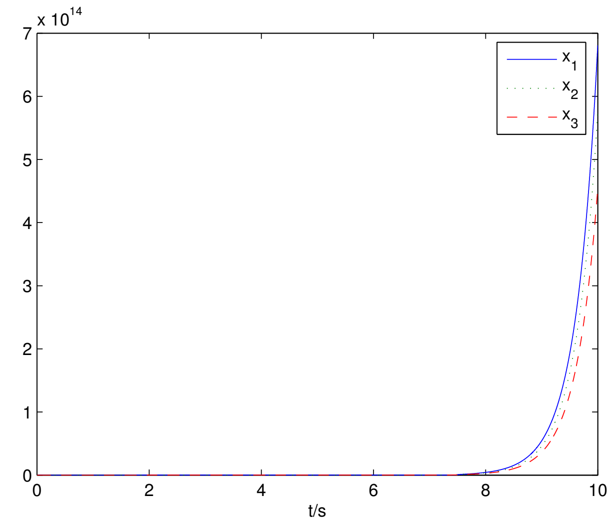

Fig. 2. State trajectories of open-loop system (1).

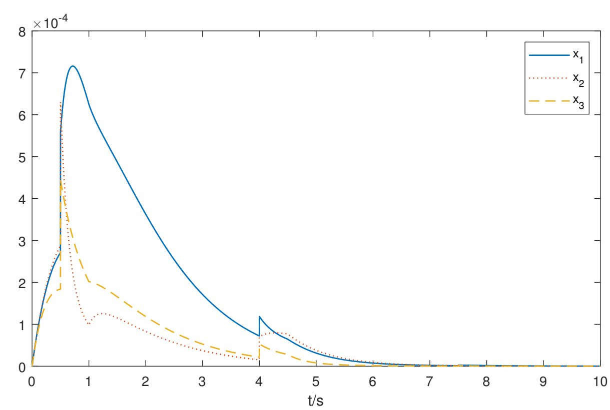

Fig. 3. State trajectories of closed-loop system (1).

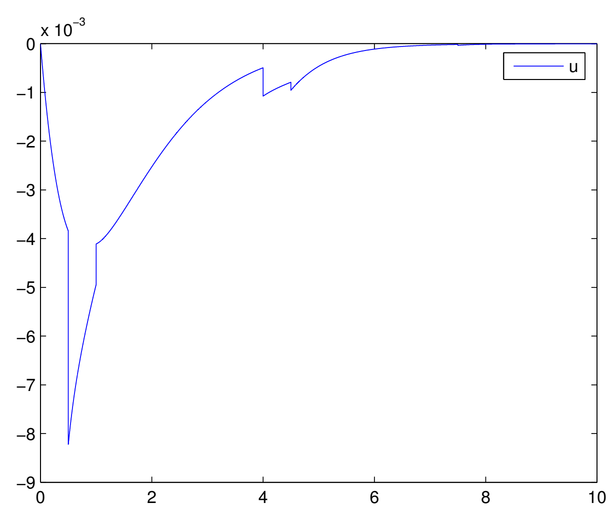

Fig. 4. Control signal.

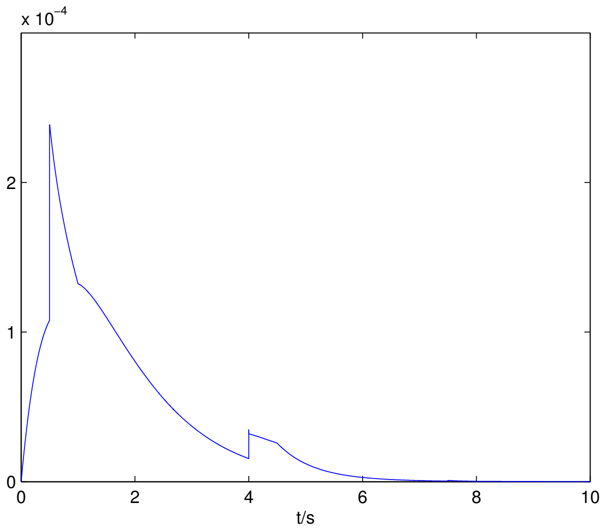

Fig. 5. The evolution of $y^T(t)\delta$ of system (1).

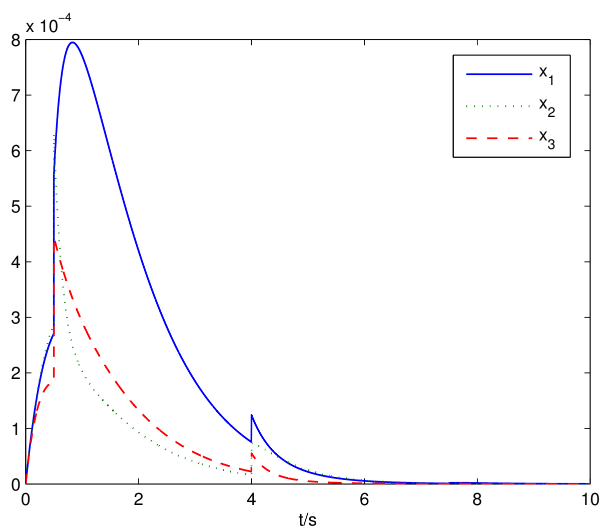

Fig. 6. State trajectories of closed-loop system (1) under synchronous control.

5. CONCLUSIONS

In this paper, we have considered the issue of asynchronously IO-FTS for PISS. Based on the ADT approach, a state feedback controller is constructed to guarantee that the closed-loop system is IO-FTS. Finally, a practical example is given to illustrate the effectiveness of the proposed method.

As we know, time delays arise quite naturally in many dynamical systems and are frequently a source of instability and poor performance [27–29]. Therefore, the problem of asynchronously IO-FTS of positive switched systems with time delay may be interesting topics to study in the future.

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