# Optimal Tracking Performance of NCSs with Time-delay and Encodingdecoding Constraints

Jun-Wei Hu, Xi-Sheng Zhan\* **D**, Jie Wu, and Huai-Cheng Yan

Abstract: In this paper, the tracking performance of networked control systems (NCSs) under energy constraints with time-delay and encoding-decoding is studied. Through spectral factorization and partial decomposition techniques, we can obtain the explicit representation of the optimal performance. It is shown that the optimal performance is affected by non minimum phase (NMP) zeros, unstable poles and other multiple communication constraints such as time-delay, encoding-decoding and additive white Gaussian noise (AWGN). At the same time, the obtained result shows that a two-parameter compensator is superior to a one-parameter compensator. In addition, it is found that time-delay, encoding-decoding and AWGN affected the tracking capability of NCSs. Finally, an example is given for verifying the correctness of the conclusions.

Keywords: Encoding-decoding, networked control systems, NMP zeros, performance, unstable poles.

# 1. INTRODUCTION

<span id="page-0-0"></span>Automation technology is widely used in the process of production, living, and management [\[1–](#page-8-0)[5\]](#page-8-1). NCSs offer many to people's lives  $[6–10]$  $[6–10]$ . In NCSs, data is passed through a communication network, and both the bandwidth of the communication network and channel capacity are limited, which can produce a packet loss and delay problems. These cause and even lead to system instability. Faced with new problems in NCSs, many scholars have conducted work regarding their solutions. The problem of stability analysis was considered in [\[11–](#page-8-4)[19\]](#page-8-5). Paper [\[20\]](#page-8-6) designed symbolic controllers for NCSs. The stability analysis of NCSs has been well studied, but tracking performance limitation is still a problem for consideration.

The research on performance of NCSs is another hot issue, and the research on performance of NCSs is getting more and more attention [\[21](#page-8-7)[–27\]](#page-9-0). The performance of NCSs is a worthy question investigated in [\[28\]](#page-9-1). Paper [\[29\]](#page-9-2) investigated the tracking performance limitations of linear time-invariant NCSs with two-channel constraints for a random reference signal. The optimal performance and optimal modified performance with multi-parameter constraints were studied in [\[30,](#page-9-3) [31\]](#page-9-4), respectively. Paper [\[32\]](#page-9-5) investigated the performance limitations of NCSs

with quantization and packet-dropouts. The optimal tracking performance of NCSs with communication delay and AWGN was investigated in [\[33\]](#page-9-6). The above studies did not consider the impact of time-delay, encoding-decoding with power constraint. Channel coding and decoding technology can check and correct errors, it is an important part of network communication. In the design of NCSs, the channel input power cannot be infinite. The NCSs can be applied to many fields, including safety control of large power grid. However, in the application process, due to the characteristics of the communication network, NCSs have factors like delay, packet loss, AWGN and coding, which inevitably affect the performance control of the system. In this paper we will analyze the the optimal tracking performance between NCSs and these factors. In general, the communication delay is time varying, uncertain or random. The optimal performance analysis of NCS is difficult because of the time varying delay. When there are no relay devices such as gateway and router in the communication network, the delay in the communication channel can be fixed, or can be converted into a constant delay by some measures. For the convenience of analysis, the optimal performance of the NCSs with a constant time delay, encoding-decoding and two-channel AWGN constraints is studied in this paper.

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The main contributions of this paper are as follows: (i) It is considered with two-channel constraints, the timedelay, encoding-decoding and AWGN. (ii) Relationships between the optimal tracking performance of the NCSs. (iii) The optimal tracking performance of the NCSs is achieved by applying a one-parameter or two-parameter structure. The accurate mathematical expression of the optimal tracking performance is achieved by using the frequency domain method. That the optimal tracking performance depends on the NMP zeros, the unstable poles of a given plant, the characteristics of the reference signal, the channel input power, the time-delay, encoding-decoding, two-channel AWGN.

This paper is structured as follows: A description of the problem is covered in Section 2. Section 3 shows the performance of NCSs with a one-parameter compensator with time-delay consideration, encoding-decoding and AWGN under channel input power constraints. Section 4 studies the performance of NCSs under a twoparameter compensator. Section 5 illustrates the results through some typical examples. Section 6 provides the conclusion and future directions.

## 2. PROBLEM FORMULATIONS

In this article, unified standard notation is used.  $\bar{z}$ denotes complex conjugate of any complex number *z*.  $X(t)$  is the Laplace transform of any continuous-time signal  $x(t)$ .  $C_$  = {s : Re(s) < 0} is the open left-half plane, and  $C_+ = \{s : \text{Re}(s) > 0\}$  indicates the right correspondingly.  $L_2$  is the Hilbert spaces,  $\langle f, g \rangle :=$  $\frac{1}{2\pi} \int_{-\infty}^{+\infty} \text{tr} \left[ f^H(jw) g(jw) \right]$ , for any  $f, g \in L_2$ . From [\[34\]](#page-9-7),  $\ddot{H}_2$  and  $H_2^{\perp}$  are decomposed from *L*<sub>2</sub>, then:  $\mathcal{H}_2 := \{P :$  $P(s)$  *analytic in*  $C_+$ ,  $||P||_2^2$ :  $=\sup_{s}$ σ>0  $\frac{1}{2\pi}\int_{-\infty}^{\infty}||P(\sigma+jw)||^2 dw <$  $\infty$ }, and  $\mathcal{H}_2^{\perp} := \{ P : P(s) \text{ analytic in } C_{-}, ||P||_2^2 =$ sup σ<0  $\frac{1}{2\pi}\int_{-\infty}^{\infty}||P(\sigma+jw)||^2dw < \infty$ .

The NCSs are constructed base on time-delay, encoding-decoding, and AWGN shows in Fig. 1.

In Fig. 1, *G* acts as the plant, and *K* denotes the oneparameter compensator.  $G(s)$  and  $K(s)$  represents transfer functions, which are shorthand for *G* and *K*. The characteristics of communication channel have three parameters: encoding-decoding are represented by *A* and  $A^{-1}$ ;  $\tau$ 



Fig. 1. NCSs with a one-parameter compensator controller.

is time-delay; and  $n_1$  is the AWGN of the feedback channel, while  $n_2$  represents the AWGN of the forward channel. The time-delay is a common problem in practical application of the system.

The signals *r*, *y*, and *u* are defined as reference inputs, the system output and the system input, whose transfer functions are  $R(s)$ ,  $Y(s)$  and  $U(s)$ , which can be simply expressed as *R*, *Y* and *U*. The reference signal *r* is regarded as Brownian motion, and  $E\{|r(t)|\} = 0$ ,  $E\left\{|r(t)|^2\right\} = \sigma_r^2$ . We denote the signals *r*, *n*<sub>1</sub> and *n*<sub>2</sub> by the variances  $\sigma_r^2$ ,  $\sigma_{n_1}^2$  and  $\sigma_{n_2}^2$ .

We assume that the reference signal and AWGN are unrelated to each other. For signal *r*, a tracking error of NCS is defined as:

<span id="page-1-2"></span>
$$
E = R - Y.\tag{1}
$$

From Fig. 1, we get:

<span id="page-1-0"></span>
$$
U = KR - K(Y + n_1)e^{-\tau s},\tag{2}
$$

<span id="page-1-1"></span>
$$
Y = GA^{-1}(AU + n_2).
$$
 (3)

According to [\(2\)](#page-1-0) and [\(3\)](#page-1-1), we have:

<span id="page-1-3"></span>
$$
Y = \frac{GKR}{1 + e^{-\tau s}GK} - \frac{e^{-\tau s}GKn_1}{1 + e^{-\tau s}GK} + \frac{GA^{-1}n_2}{1 + e^{-\tau s}GK}.
$$
 (4)

From  $(1)$  and  $(4)$ , we can obtain:

<span id="page-1-4"></span>
$$
E = \left(1 - \frac{GK}{1 + e^{-\tau s}GK}\right)R - \left(\frac{e^{-\tau s}GK}{1 + e^{-\tau s}GK}\right)n_1
$$

$$
+ \left(\frac{G A^{-1}}{1 + e^{-\tau s}GK}\right)n_2.
$$
(5)

To obtain the performance of NCSs, which considers the channel input power constraint, we define its performance index as follows:

<span id="page-1-5"></span>
$$
J \stackrel{\Delta}{=} (1 - \varepsilon) \{ ||E(s)||^2 \} + \varepsilon \left( E \{ ||Y(s)||^2 \} - \Gamma \right), \tag{6}
$$

where  $0 \le \varepsilon \le 1$ , if  $\varepsilon = 0$ , then there is no power constraint in the channel input.

When selecting the controller (*K* represented) among all possible stable controllers, we can express the optimal performance of the NCSs as:

<span id="page-1-8"></span>
$$
J^* = \inf_{K \in \kappa} J. \tag{7}
$$

For *G*, the factorization is as follows:

<span id="page-1-6"></span>
$$
G = \frac{N}{M}, N, M \in \mathfrak{R} \mathcal{H}_{\infty}.
$$
 (8)

From the Bezout's identity [\[30\]](#page-9-3),

<span id="page-1-7"></span>
$$
MX - e^{-\tau s}NY = 1, X, Y \in \mathfrak{R}H_{\infty}.
$$
 (9)

From [\[35\]](#page-9-8), *K* can be described by Youla parameterization:

$$
\kappa := \left\{ K := -\frac{Y - MQ}{X - e^{-\tau s} N Q}, \ Q \in \Re H_{\infty} \right\}.
$$
 (10)

From [\[36\]](#page-9-9) that any NMP transfer function can be decomposed two parts:

$$
N(s) = L_z(s) N_m(s), \nM(s) = B_p(s) M_m(s),
$$
\n(11)

where  $L_z(s)$  and  $B_p(s)$  are all pass factors, and  $N_m(s)$  and  $M_m(s)$  are minimum phase parts, and it can factor as:

$$
L_z(s) = \prod_{j=1}^n \frac{s - z_i}{s + \bar{z}_i},
$$
  
\n
$$
B_p(s) = \prod_{j=1}^m \frac{s - p_i}{s + \bar{p}_i}.
$$
\n(12)

## 3. OPTIMAL TRACKING PERFORMANCE WITH ONE-PARAMETER COMPENSATOR

From  $(4)$ ,  $(5)$  and  $(6)$ , we get:

$$
J = (1 - \varepsilon) \left\{ E \left\| \frac{\left( R + e^{-\tau s} K G R - K G R + e^{-\tau s} K G n_1 \right)}{-G A^{-1} n_2} \right\|^2}{1 + e^{-\tau s} K G} \right\}
$$
  
+ 
$$
\left\| N A^{-1} \left( X - e^{-\tau s} N Q \right) \right\|_2^2 \sigma_{n_2}^2
$$

$$
+ \varepsilon \left\| N \left( Y - M Q \right) \right\|_2^2 \sigma_r^2 - \varepsilon \Gamma. \tag{13}
$$

According to  $(8)$ ,  $(9)$ ,  $(10)$  and  $(13)$ , we can obtain:

$$
J = (1 - \varepsilon) \|1 - N(Y - MQ)\|_2^2 \sigma_r^2
$$
  
+ 
$$
||e^{-\tau s} N(Y - MQ)||_2^2 \sigma_{n_1}^2
$$
  
+ 
$$
||NA^{-1} (X - e^{-\tau s} NQ)||_2^2 \sigma_{n_2}^2
$$
  
+ 
$$
\varepsilon ||N(Y - MQ)||_2^2 \sigma_r^2 - \varepsilon \Gamma.
$$
 (14)

From [\(7\)](#page-1-8) and [\(14\)](#page-2-2), we can write  $J^*$  as follows:

$$
J^* = \inf_{Q \in \Re H_{\infty}} \left( \left\| \left[ \sqrt{1 - \varepsilon} \left( 1 + N(Y - MQ) \right) \right] \right\|_2^2 \delta_1^2 + \left\| e^{-\tau s} N(Y - MQ) \right\|_2^2 \sigma_2^2 + \left\| N A^{-1} \left( X - e^{-\tau s} N Q \right) \right\|_2^2 \sigma_3^2 \right) - \varepsilon \Gamma. \tag{15}
$$

Theorem 1: For the NCSs shown in Fig. 1, it is assumed that a plant has NMP zeros  $z_i \in C_+$ ,  $i = 1, \dots, n$ , unstable poles  $p_j \in C_+$ ,  $j = 1, \dots, m$ , and satisfy equations [\(8\)](#page-1-6), [\(9\)](#page-1-7), and [\(10\)](#page-2-0), then the optimal tracking performance <span id="page-2-0"></span>of the NCSs with the time-delay, encoding-decoding, twochannel AWGN constraints can be expressed as:

$$
J^* \geq (1 - \varepsilon) \sum_{i=1}^n 2\text{Re}(z_i) \delta_i^2
$$
  
+ 
$$
\sum_{j \in N} \frac{4\text{Re}(p_j) \text{Re}(p_i)}{\bar{p}_j + p_i} \frac{\lambda \lambda^H}{\bar{b}_j b_i} \delta_i^2
$$
  
+ 
$$
\varepsilon \frac{e^{\tau p_j} L_z^{-1}(\bar{p}_j)}{\bar{b}_j} \delta_i^2 \varepsilon \frac{e^{\tau p_j} L_z^{-1}(\bar{p}_i)}{\bar{b}_i} \delta_i^2 - \varepsilon^2 \delta_i^2
$$
  
+ 
$$
\sum_{j \in N} \frac{4\text{Re}(p_j) \text{Re}(p_i)}{\bar{p}_j + p_i} \frac{\lambda \lambda^H}{\bar{b}_j b_i} \sigma_{n_1}^2
$$
  
+ 
$$
\sum_{i \in N} \frac{4\text{Re}(z_j) \text{Re}(z_i)}{\bar{z}_j + z_i} \frac{\omega \omega^H}{l_i \bar{l}_j} \sigma_{n_2}^2 - \varepsilon \Gamma.
$$
 (16)

<span id="page-2-7"></span><span id="page-2-4"></span>where  $\lambda = e^{\tau p_j} L_z^{-1}(p_j)$ ,  $\omega = e^{\tau z_i} A^{-1}(z_i) N_m(z_i) M^{-1}(z_i)$ , *z*  $b_j = \prod_{\substack{i,j\in\mathbb{N}\\i\neq j}}^m$  $\frac{p_i - p_j}{\bar{p}_i + p_j}$ , and  $l_j = \prod_{\substack{i,j \in N \\ i \neq j}}^n$  $\frac{z_i-z_j}{\bar{z}_i+z_j}$ .

From Theorem 1, the result shows that the optimal performance of NCSs is not only affected by the intrinsic attributes of the system, such as unstable poles and NMP zeros, but also affected by channel input power, AWGN, time-delay and decoding. We can get that the communication constraint is a factor may reduce the tracking capability of NCSs.

Proof: According to [\(15\)](#page-2-3) and *J*<sup>∗</sup>, we can define:

<span id="page-2-6"></span>
$$
J_1^* = \inf_{Q \in \Re H_\infty} \left\| \begin{bmatrix} \sqrt{1 - \varepsilon} \left( 1 + N(Y - MQ) \right) \\ \sqrt{\varepsilon} N(Y - MQ) \end{bmatrix} \right\|_2^2 \delta_r^2, \quad (17)
$$

<span id="page-2-5"></span>
$$
J_2^* = \inf_{Q \in \mathfrak{R} H_{\infty}} ||e^{-\tau s} (Y - MQ) N_m||_2^2 \sigma_{n_1}^2,
$$
 (18)

$$
J_3^* = \inf_{Q \in \Re H_{\infty}} \left\| N A^{-1} \left( X - e^{-\tau s} N Q \right) \right\|_2^2 \sigma_{n_2}^2. \tag{19}
$$

<span id="page-2-1"></span>Because  $L_z(s)$ ,  $B_p(s)$  and  $e^{-\tau s}$  are all-pass factors, from [\(11\)](#page-2-4) and [\(18\)](#page-2-5), based on partial fractions, we can obtain:

<span id="page-2-2"></span>
$$
J_2^* = \inf_{Q \in \Re H_{\infty}} \left\| \left( B_p^{-1} Y N_m - M_m Q N_m \right) \right\|_2^2 \sigma_{n_1}^2
$$
  
= 
$$
\inf_{Q \in \Re H_{\infty}} \left\| e^{-\tau s} \left( \sum_{j \in N} \left( \frac{s + \bar{p}_j}{s - \bar{p}_j} \right) \frac{Y(p_j) N_m(p_j)}{b_j} \right) \right\|_2^2
$$
  
+ 
$$
R_1(s) - M_m Q N_m \right) \left\|_2^2 \sigma_{n_1}^2,
$$

where  $R_1 \in \Re H_{\infty}$  and  $b_j = \prod_{\substack{i \in N \\ i \neq j}}^m$ *pi*−*p<sup>j</sup>*  $\frac{p_i-p_j}{p_j+\bar{p}_i}$ . From [\(9\)](#page-1-7) and  $M(p_i) = 0$ , we get:

$$
Y(p_j) = -e^{\tau p_j} N_m^{-1}(p_j) L_z^{-1}(p_j),
$$

<span id="page-2-3"></span>then  $N_m(p_j)Y(p_j) = -e^{\tau p_j}L_z^{-1}(p_j).$ Therefore,

$$
J_2^* = \inf_{Q \in \Re H_{\infty}} \left\| \sum_{j \in N} \left( \frac{s + \bar{p}_j}{s - \bar{p}_j} - 1 \right) \frac{-e^{\tau p_j} L_z^{-1} (p_j)}{b_j} + R_2 \left( s \right) \right\|_2^2
$$

Optimal Tracking Performance of NCSs with Time-delay and Encoding-decoding Constraints 1015

$$
- M_m Q N_m)\Bigg\|_2^2 \sigma_{n_1}^2,
$$

where  $R_2 \in \Re H_{\infty}$  and  $R_2 = \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j}$  $\frac{L_z(P_j)}{b_j} + R_1(s)$ . Note that ∑ *j*∈*N*  $\int \frac{s + \bar{p_j}}{g}$  $\frac{s+\bar{p_j}}{s-\bar{p_j}}-1$ )  $\frac{-e^{\tau p_j}L_z^{-1}(p_j)}{b_j}$  $\frac{L_z \cdot (p_j)}{b_j} \in H_2^{\perp}, \quad (R_2(s) -$ 

 $M_m Q N_m$ )∈*H*<sub>2</sub>, and an appropriate  $Q$  can be selected such that  $\inf_{Q \in \Re H_{\infty}} ||R_2(s) - M_m Q N_m||_2^2 \sigma_2^2 = 0$ , and then

$$
J_2^* = \left\| \sum_{j \in N} \frac{2 \operatorname{Re}(p_j)}{s - \bar{p}_j} \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j} \right\|_2^2 \sigma_{n_1}^2
$$
  
= 
$$
\sum_{j \in N} \frac{4 \operatorname{Re}(p_j) \operatorname{Re}(p_i)}{\bar{p}_j + p_i} \frac{\lambda \lambda^H}{\bar{b}_j b_i} \sigma_{n_1}^2.
$$

From  $(17)$ , we get:

$$
J_1^* = \inf_{Q \in \Re H_\infty} \left\| \left[ \frac{\sqrt{1-\varepsilon} \left( L_Z^{-1} - 1 + 1 + N(Y - MQ) \right) \right]}{\sqrt{\varepsilon} N (Y - MQ)} \right] \right\|_2^2 \delta_r^2.
$$

Because  $(L_2^{-1} - 1) \in H_2^{\perp}$ ,  $(L_2^{-1} - 1) \in H_2^{\perp}$ , we can obtain:

$$
J_1^* = \left\| \begin{array}{l} \sqrt{1-\varepsilon} \left( L_2^{-1} - 1 \right) \right\|_2^2 \delta_r^2 \\ 0 + \inf_{Q \in \Re H_\infty} \left\| \left[ \begin{array}{l} \sqrt{1-\varepsilon} \left( 1 + N_m (Y-MQ) \right) \\ \sqrt{\varepsilon} N_m (Y-MQ) \end{array} \right] \right\|_2^2 \delta_r^2. \end{array}
$$

Definition:  $J_2^* = J_{11}^* + J_{12}^*$ , where  $J_{11}^* =$  $\sqrt{1-\epsilon} (L_z^{-1} - 1)$ 0  $\parallel$ 2 2  $\delta_r^2$ , and  $J_{12}^*$  = inf *Q*∈ℜ*H*<sup>∞</sup>  $\sqrt{1-\varepsilon}(1+N_m(Y-MQ))$ <br>  $\sqrt{\varepsilon}N_m(Y-MQ)$  2 2  $\delta_r^2$ . According to [\[37\]](#page-9-10), we can obtain:

$$
J_{11}^* = \left\| \frac{\sqrt{1-\epsilon} (L_Z^{-1} - 1)}{0} \right\|_2^2 \delta_1^2
$$
  
=  $(1-\epsilon) \sum_{i=1}^n 2 \operatorname{Re} (z_i) \delta_1^2$ .

Because  $B_p$  is the all pass factors, we can obtain:

$$
N_m Y B_{p}^{-1} = \sum_{j \in N} \left( \frac{s + \bar{p}_j}{s - \bar{p}_j} \right) \frac{Y(p_j) N_m(p_j)}{b_j} + R_3 \left( s \right).
$$

where  $R_3 \in \Re H_{\infty}, b_j = \prod_{\substack{i \in N \\ i \neq j}}^m$ *pi*−*p<sup>j</sup>*  $\frac{p_i-p_j}{p_j+\bar{p}_i}$ .

From  $(10)$  and  $(12)$ , then

$$
J_2^* = \inf_{Q \in \Re H_{\infty}} \left\| \sum_{j \in N} \left( \frac{s + \bar{p}_j}{s - \bar{p}_j} \right) \frac{Y(p_j) N_m(p_j)}{b_j} \right\|_2^2
$$
  
+  $R_1(s) - M_m Q N_m \left\|_2^2 \sigma_{n_1}^2$ 

$$
+\inf_{Q\in\mathfrak{R}H_{\infty}}\left\|\begin{bmatrix}\sqrt{1-\varepsilon}\\0\end{bmatrix}+\begin{bmatrix}\sqrt{1-\varepsilon}\\ \sqrt{\varepsilon}\end{bmatrix}R_4\right\|_2-\begin{bmatrix}\sqrt{1-\varepsilon}\\ \sqrt{\varepsilon}\end{bmatrix}N_mQM_m\right\|_2^2\delta_r^2.
$$

where  $R_4 \in \Re H_{\infty}$  and  $R_4 = \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j}$  $\frac{L_z (P_j)}{b_j} + R_3.$ Define:

$$
J_{12m}^{*} = \left\| \begin{bmatrix} \sqrt{1-\epsilon} \\ \sqrt{\epsilon} \end{bmatrix} \sum_{j \in N} \frac{2\text{Re}(p_j)}{s-\bar{p}_j} \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j} + \begin{bmatrix} \sqrt{1-\epsilon} \\ 0 \end{bmatrix} \sum_{j \in N} \frac{2\text{Re}(p_j)}{s-\bar{p}_j} \right\|_2^2 \delta_r^2,
$$
  

$$
J_{12n}^{*} = \inf_{Q \in \Re H_{\infty}} \left\| \begin{bmatrix} \sqrt{1-\epsilon} \\ 0 \end{bmatrix} + \begin{bmatrix} \sqrt{1-\epsilon} \\ \sqrt{\epsilon} \end{bmatrix} R_4 - \begin{bmatrix} \sqrt{1-\epsilon} \\ \sqrt{\epsilon} \end{bmatrix} N_m Q M_m \right\|_2^2 \delta_r^2.
$$

By a simple calculation as  $J_2^*$ , we get:

$$
J_{12m}^{*} = \sum_{j \in N} \frac{4 \text{Re}(p_j) \text{Re}(p_i)}{\bar{p}_j + p_i} \frac{\lambda \lambda^H}{\bar{b}_j b_i} \delta_r^2 + \varepsilon \frac{e^{\tau p_j} L_z^{-1}(\bar{p}_j)}{\bar{b}_j} \delta_r^2
$$
  
+ 
$$
\varepsilon \frac{e^{\tau p_j} L_z^{-1}(\bar{p}_i)}{\bar{b}_i} \delta_r^2 - \varepsilon \delta_r^2,
$$
  

$$
J_{12n}^{*} = \varepsilon (1 - \varepsilon) \delta_r^2.
$$

Because  $L_z(s)$ ,  $B_p(s)$  and  $e^{-\tau s}$  are all-pass factors, we have:

$$
J_3^* = \inf_{Q \in \mathfrak{R} H_{\infty}} ||NA^{-1} (X - e^{-\tau s} NQ)||_2^2 \sigma_{n_2}^2
$$
  
= 
$$
\inf_{Q \in \mathfrak{R} H_{\infty}} \left\| \frac{e^{\tau s} A^{-1}(s) N_m(s) X(s)}{B_p(s)} - A^{-1}(s) N_m^2(s) Q \right\|_2^2 \sigma_{n_2}^2.
$$

According to partial factorization, we get:

$$
\frac{e^{\tau s}A^{-1}(s)N_m(s)X(s)}{B_p(s)} = \sum_{i \in N} \left( \frac{s + \bar{z}_i}{s - \bar{z}_i} \right) \frac{e^{\tau z_i}A^{-1}(z_i)N_m(z_i)X(z_i)}{l_i} + R_5(z_i),
$$

where,  $R_5 \in \Re H_{\infty}$  and  $l_j = \prod_{\substack{i \in \mathbb{N} \\ i \neq j}}^m$  $\frac{z_i-z_j}{\bar{z}_i+z_j}$ . From  $M(p_j) = 0$ , [\(10\)](#page-2-0) and [\(12\)](#page-2-7), we get:  $N_m(z_i)X(z_i) =$  $N_m(z_i)M^{-1}(z_i).$ 

$$
J_{3}^{*} = \inf_{Q \in \mathfrak{R} H_{\infty}} \left\| \sum_{i \in N} \left( \frac{s + \bar{z}_{i}}{s - \bar{z}_{i}} - 1 \right) \frac{e^{\tau z_{i}} A^{-1}(z_{i}) N_{m}(z_{i}) M^{-1}(z_{i})}{l_{i}} \right\|_{2}^{2} + R_{5}(z_{i}) - A^{-1}(z_{i}) N_{m}^{2}(z_{i}) Q \left\| \frac{1}{2} \sigma_{n_{2}}^{2},
$$

 $\text{where} \sum_{i \subset N} \left( \frac{s + \bar{z}_i}{s - \bar{z}_i} - 1 \right) \frac{e^{\bar{z}z_i} A^{-1}(z_i) N_m(z_i) M^{-1}(z_i)}{l_i}$ *i*∈*N*  $\frac{d}{dx} \cdot \frac{d}{dx} \cdot \frac{d}{dx} \in H_2^{\perp}$ , and  $(R_5(z_i))$  $-A^{-1}(z_i)N_m^2(z_i)Q \in H_2$ , we can get  $\inf_{Q \in \Re H_\infty} ||R_5(z_i) A^{-1}(z_i)N_m^2(z_i)Q\|_2^2\sigma_2^2=0$ . Therefore,

$$
J_3^* = \sum_{i \in N} \frac{4 \text{Re} (z_j) \text{Re} (z_i)}{\bar{z}_j + z_i} \frac{\omega \omega^H}{l_i \bar{l}_j} \sigma_{n_2}^2.
$$

# 4. OPTIMAL TRACKING PERFORMANCE WITH TWO-PARAMETER COMPENSATOR

We focus on the system shown in Fig. 2. A twoparameter compensator controller is applied to the NCSs.  $[K_1K_2]$  represents a two-parameter compensator, and  $[K_1(s)K_2(s)]$  is the transfer function. The other variables are the same as in Fig. 2. Then, the set of all stable function is as follows:

$$
\kappa := \{ K : K = [K_1 K_2] = [QY - RM](X - e^{-\tau s} RN)^{-1}, R, Q \in \Re H_{\infty} \}.
$$
\n(20)

From Fig. 2, we can obtain:

$$
U = K_1 R + K_2 (Y + n_1) e^{-\tau s}, \tag{21}
$$

$$
Y = GA^{-1}(AU + n_2).
$$
 (22)

From  $(21)$  and  $(22)$ , we can obtain:

$$
Y = \frac{K_1GR}{1 - e^{-\tau s}K_2G} + \frac{e^{-\tau s}K_2Gn_1}{1 - e^{-\tau s}K_2G} + \frac{GA^{-1}n_2}{1 - e^{-\tau s}K_2G}.
$$
\n(23)

From  $(1)$  and  $(23)$ , we get:

$$
E(s) = \left(1 - \frac{K_1 G}{1 - e^{-\tau s} K_2 G}\right) R(s) + \left(\frac{e^{-\tau s} K_2 G}{1 - e^{-\tau s} K_2 G}\right) n_1 + \left(\frac{G A^{-1}}{1 - e^{-\tau s} K_2 G}\right) n_2.
$$
 (24)

According to  $(4)$ ,  $(5)$  and  $(23)$ , we get:

$$
J = (1 - \varepsilon)E\left\{\left\|\frac{(R - e^{-\tau s}K_2GR - K_1GR)}{-e^{-\tau s}K_2Gn_1 - GA^{-1}n_2)}\right\|^2}{1 - e^{-\tau s}K_2G}\right\}
$$



Fig. 2. NCSs with a two-parameter compensator controller.

<span id="page-4-4"></span>
$$
+\varepsilon\left(E\left\{\left\|\frac{(K_1GR+e^{-ts}K_2Gn_1)}{+GA^{-1}n_2}}{1-e^{-ts}K_2G}\right\|-\Gamma\right). (25)
$$

From [\(8\)](#page-1-6), [\(9\)](#page-1-7), [\(20\)](#page-4-3) and [\(25\)](#page-4-4), we get:

<span id="page-4-5"></span>
$$
J = (1 - \varepsilon) \|1 - NQ\|_2^2 \sigma_r^2 + \|e^{-\tau s}(Y - RM)\|_2^2 \sigma_{n_1}^2
$$
  
+ 
$$
\|NA^{-1}(X - e^{-\tau s}RN)\|_2^2 \sigma_{n_2}^2
$$
  
+ 
$$
\varepsilon \|NQ\|_2^2 \sigma_r^2 - \varepsilon \Gamma.
$$
 (26)

From  $(7)$  and  $(26)$ , we get:

<span id="page-4-6"></span>
$$
J^* = \inf_{Q \in \Re H_{\infty}} \left( \left\| \begin{bmatrix} \sqrt{1 - \varepsilon} (1 - NQ) \\ \sqrt{\varepsilon} NQ \end{bmatrix} \right| \right\|_2^2 \sigma_r^2 + \left\| e^{-\tau s} N (Y - RM) \right\|_2^2 \sigma_{n_1}^2 + \left\| N A^{-1} (X - e^{-\tau s} R N) \right\|_2^2 \sigma_{n_2}^2 \right) - \varepsilon \Gamma. \tag{27}
$$

<span id="page-4-3"></span><span id="page-4-1"></span><span id="page-4-0"></span>Obviously, to get  $J^*$ ,  $Q$  and  $R \in \Re H_{\infty}$  must be chosen appropriately.

<span id="page-4-2"></span>Theorem 2: For the NCSs shown in Fig. 2, it is assumed that a plant has NMP zeros  $z_i \in C_+$ ,  $i = 1, \dots, n$ , unstable poles  $p_j \in C_+$ ,  $j = 1, \dots, m$ , and satisfy equations [\(8\)](#page-1-6), [\(9\)](#page-1-7), and [\(20\)](#page-4-3), then the optimal tracking performance of the NCSs by applying two-parameter compensator can be expressed as:

$$
J^* \geq (1 - \varepsilon) \sum_{i=1}^n 2\text{Re} (z_i) \delta_r^2 + \varepsilon (1 - \varepsilon) \sigma_r^2
$$
  
+ 
$$
\sum_{j \in N} \frac{4\text{Re} (p_j) \text{Re} (p_i)}{\bar{p}_j + p_i} \frac{\lambda \lambda^H}{\bar{b}_j b_i} \sigma_{n_1}^2
$$
  
+ 
$$
\sum_{i \in N} \frac{4\text{Re} (z_j) \text{Re} (z_i)}{\bar{z}_j + z_i} \frac{\omega \omega^H}{l_i \bar{l}_j} \sigma_{n_2}^2 - \varepsilon \Gamma,
$$
 (28)

where  $\lambda = e^{\tau p_j} L_z^{-1}(p_j), \omega = e^{\tau z_i} A^{-1}(z_i) N_m(z_i) M^{-1}(z_i),$  $b_j = \prod_{\substack{i,j\in\mathbb{N}\\i\neq j}}^m$ *pi*−*p<sup>j</sup>*  $\frac{p_i-p_j}{\bar{p}_j+p_i}$ , and  $l_j = \prod_{\substack{i,j\in\mathbb{N} \\ i\neq j}}^n$ *zi*−*z<sup>j</sup>*  $\frac{z_i-z_j}{\bar{z}_j+z_i}$ .

**Proof:** According to [\(27\)](#page-4-6), we define:

<span id="page-4-7"></span>
$$
J_4^* = \inf_{Q \in \Re H_{\infty}} \left\| e^{-\tau s} N (Y - RM) \right\|_2^2 \sigma_{n_1}^2, \tag{29}
$$

<span id="page-4-8"></span>
$$
J_5^* = \inf_{Q \in \Re H_{\infty}} \left\| \begin{bmatrix} \sqrt{1 - \varepsilon} \left( 1 + NQ \right) \\ \sqrt{\varepsilon} NQ \end{bmatrix} \right\|_2^2 \delta_r^2, \tag{30}
$$

<span id="page-4-9"></span>
$$
J_6^* = \inf_{Q \in \Re H_\infty} \left\| N A^{-1} \left( X - e^{-\tau s} R N \right) \right\|_2^2 \sigma_{n_2}^2. \tag{31}
$$

Because  $e^{-\tau s}$  is all-pass factors, we may obtain:

$$
J_4^* = \inf_{Q \in \mathfrak{R} H_\infty} \left\| N \left( Y - RM \right) \right\|_2^2 \sigma_{n_1}^2.
$$

According to [\(29\)](#page-4-7) and the same method of  $J_1^*$ , we can get:

$$
J_4^* = \sum_{j \in N} \frac{4 \text{Re}(p_j) \text{Re}(p_i)}{\bar{p}_j + p_i}
$$
  
 
$$
\times \frac{e^{\tau p_j} L_{z}^{-1}(p_j)^{-1} (e^{\tau p_j} L_{z}^{-1}(p_j))^H}{\bar{b}_j b_i} \sigma_{n_1}^2
$$
  

$$
= \sum_{j \in N} \frac{4 \text{Re}(p_j) \text{Re}(p_i)}{\bar{p}_j + p_i} \frac{\lambda \lambda^H}{\bar{b}_j b_i} \sigma_{n_1}^2.
$$

From [\(30\)](#page-4-8), we can get:

$$
J_5^* = \inf_{Q \in \Re H_\infty} \left\| \left[ \frac{\sqrt{1-\varepsilon} \left( L_Z^{-1} - 1 + 1 + N_m Q \right)}{\sqrt{\varepsilon} N_m Q} \right] \right\|_2^2 \delta_r^2.
$$

Because  $(L_Z^{-1} - 1) \in H_2^{\perp}$ ,  $(1 - N(Y - MQ)) \in H_2$ , we have:

.

$$
J_5^* = \left\| \begin{array}{cc} \sqrt{1-\varepsilon} \left( L_2^{-1} - 1 \right) \Big|_2^2 \delta_r^2 \\ 0 \\ + \inf_{Q \in \Re H_\infty} \left\| \left[ \begin{array}{cc} \sqrt{1-\varepsilon} \left( 1 + N_m Q \right) \\ \sqrt{\varepsilon} N_m Q \end{array} \right] \right\|_2^2 \delta_r^2 \end{array}
$$

Definition:  $J_5^* = J_{51}^* + J_{52}^*$ , where

$$
J_{51}^* = \left\| \begin{matrix} \sqrt{1-\varepsilon} \left( L_2^{-1} - 1 \right) \Big|_2^2 \delta_r^2, \\ 0 \end{matrix} \right\|_2^2 \delta_r^2,
$$
  

$$
J_{52}^* = \inf_{Q \in \Re H_\infty} \left\| \begin{bmatrix} \sqrt{1-\varepsilon} \left( 1 + N_m Q \right) \\ \sqrt{\varepsilon} N_m Q \end{bmatrix} \right\|_2^2 \delta_r^2.
$$

Through simple calculation,  $J_{52}^*$  can be expressed as follows:

$$
J_{52}^* = \inf_{Q \in \Re H_{\infty}} \left\| \begin{bmatrix} \sqrt{1-\epsilon} \\ 0 \end{bmatrix} + \begin{bmatrix} -\sqrt{1-\epsilon} \\ 0 \end{bmatrix} N_m Q \right\|_2^2 \delta_r^2.
$$

According to [\[37\]](#page-9-10), the internal and external factors are decomposed  $\begin{bmatrix} -\sqrt{1-\varepsilon} \\ 0 \end{bmatrix}$ ε  $N_m = \Delta_i \Delta_0.$ 

To find the best *Q*, we introduce  $\Psi \Delta = \begin{bmatrix} \Delta_i^T (-s) \\ I & \Delta_i \Delta_i^T \end{bmatrix}$  $I - \Delta_i \Delta_i^T (-s)$  . We know that  $\Psi_i^T \Psi_i = I$ , and then

$$
J_{52}^{*} = \inf_{Q \in \mathfrak{R} H_{\infty}} \left\| \Psi \left( \begin{bmatrix} \sqrt{1 - \varepsilon} \\ 0 \end{bmatrix} + \begin{bmatrix} -\sqrt{1 - \varepsilon} \\ \varepsilon \end{bmatrix} N_{m} Q \right) \right\|_{2}^{2}
$$
  
\n
$$
= \inf_{Q \in \mathfrak{R} H_{\infty}} \left\| \Delta_{i}^{T} \begin{bmatrix} \sqrt{1 - \varepsilon} \\ 0 \end{bmatrix} + \Delta_{0} Q
$$
  
\n
$$
+ (I - \Delta_{i} \Delta_{i}^{T}) \begin{bmatrix} -\sqrt{1 - \varepsilon} \\ 0 \end{bmatrix} \Big|_{2}^{2} \sigma_{r}^{2}
$$
  
\n
$$
= \inf_{Q \in \mathfrak{R} H_{\infty}} \left\| \Delta_{i}^{T} \begin{bmatrix} \sqrt{1 - \varepsilon} \\ 0 \end{bmatrix} + \Delta_{0} Q \right\|_{2}^{2} \delta_{r}^{2}
$$
  
\n
$$
+ \left\| (I - \Delta_{i} \Delta_{i}^{T}) \begin{bmatrix} -\sqrt{1 - \varepsilon} \\ \varepsilon \end{bmatrix} \right\|_{2}^{2} \delta_{r}^{2}.
$$

Because  $Q \in \Re H_{\infty}$ ,  $Q$  can choose appropriately, so inf *Q*∈ℜ*H*<sup>∞</sup>  $\begin{array}{c} \hline \end{array}$  $\Delta_i^T$  $\int \sqrt{1-\epsilon}$  $\boldsymbol{0}$  $\left\| +\Delta_{0}Q\right\|$ 2 2  $\delta_r^2 = 0$ , we can obtain:  $J_{52}^* = \varepsilon (1 - \varepsilon) \sigma_r^2$ . By  $J_{21}^*$ , we have:  $J_{51}^* = \Bigg\|$  $\sqrt{1-\varepsilon}\left(L_{Z}^{-1}-1\right)$ 0  $\begin{array}{c} \hline \end{array}$ 2 2  $\delta_r^2$  $=(1-\varepsilon)$ *n* ∑ *i*=1  $2\text{Re}(z_i)\delta_r^2$ .

According to [\(31\)](#page-4-9), we get:

$$
J_6^* = \sum_{i \in N} \frac{4 \text{Re}(z_j) \text{Re}(z_i)}{\bar{z}_j + z_i} \frac{\omega \omega^H}{l_i \bar{l}_j} \sigma_{n_2}^2.
$$

Theorem 2 shows that the performance of NCSs under a two-parameter compensator is affected by inherent properties of the system, such as NMP zeros, instability, input power constraints, communication delay, AWGN, and encoding-decoding. The result shows that the constraint may reduce the tracking capability of the NCSs. According to Theorems 1 and Theorems 2, the performance can be improved by using the two-parameter compensator control scheme.

Remark 1: Theorem 2 gives a complete expression on how plant NMP zeros, unstable poles of a given plant and time-delay and AWGN degrade the performance with the NCSs. It can also be seen from the Theorem 2 that the time-delay and AWGN of a communication channel will in general degrade the performance. The following corollary can be obtained by Theorem 2 directly.

**Corollary 1:** In Theorem 2, if the time-delay  $\tau = 0$ , then we can obtain:

$$
J^* \geq (1 - \varepsilon) \sum_{i=1}^n 2 \operatorname{Re}(z_i) \delta_r^2 + \varepsilon (1 - \varepsilon) \sigma_r^2
$$
  
+ 
$$
\sum_{j \in N} \frac{4 \operatorname{Re}(p_j) \operatorname{Re}(p_i)}{\bar{p}_j + p_i} \frac{\beta \beta^H}{\bar{b}_j b_i} \sigma_{n_1}^2
$$
  
+ 
$$
\sum_{i \in N} \frac{4 \operatorname{Re}(z_j) \operatorname{Re}(z_i)}{\bar{z}_j + z_i} \frac{\xi \xi^H}{l_i \bar{l}_j} \sigma_{n_2}^2 - \varepsilon \Gamma,
$$

where  $\beta = L_z^{-1}(p_j)$ ,  $\xi = A^{-1}(z_i)N_m(z_i)M^{-1}(z_i)$ ,  $b_j =$  $\prod_{\substack{i,j\in\mathbb{N}\\i\neq j}}^m$ *pi*−*p<sup>j</sup>*  $\frac{p_i - p_j}{\bar{p}_j + p_i}$ , and  $l_j = \prod_{\substack{i,j \in \mathbb{N} \\ i \neq j}}^n$ *zi*−*z<sup>j</sup>*  $\frac{z_i-z_j}{\bar{z}_j+z_i}$ .

**Corollary 2:** In Theorem 2, if the AWGN  $n_1 = 0$ , we can obtain:

$$
J^* \geq (1 - \varepsilon) \sum_{i=1}^n 2 \operatorname{Re}(z_i) \delta_r^2 + \varepsilon (1 - \varepsilon) \sigma_r^2
$$
  
+ 
$$
\sum_{i \in N} \frac{4 \operatorname{Re}(z_j) \operatorname{Re}(z_i)}{\overline{z}_j + z_i} \frac{\omega \omega^H}{l_i \overline{l_j}} \sigma_{n_2}^2 - \varepsilon \Gamma,
$$

where  $\omega = e^{\tau z_i} A^{-1}(z_i) N_m(z_i) M^{-1}(z_i)$ , and  $l_j = \prod_{\substack{i,j \in N \\ i \neq j}}^n$ *zi*−*z<sup>j</sup>*  $\frac{z_i-z_j}{\bar{z}_j+z_i}$ .

It can be observed from Corollary 1 that the optimal tracking performance only depends on the NMP zeros, the unstable poles of a given plant, encoding-decoding and AWGN. The result presented in Corollary 2 is similar to those previously demonstrated in [\[33\]](#page-9-6).

### 5. ILLUSTRATIVE EXAMPLE

Example: Consider the transfer function of the unstable system to be:

$$
G(s) = \frac{s - k}{(s - 2)(s + 3)}, \ \ k \in (1, 5).
$$

The unstable pole is  $p = 2$ , and  $z = k$  is the NMP zeros value. We choose the signals *r*,  $n_1$ , and  $n_2$ , as  $\sigma_r^2 = 0.5$ ,  $\sigma_{n_1}^2 = 0.2$ , and  $\sigma_{n_2}^2 = 0.1$ . Choose  $\Gamma = 2$ ,  $\varepsilon = \frac{1}{2}$ ,  $\tau = 0.2$ , and  $A(s) = \frac{s+1}{s+2}$  is used to represent the encoding, where *s* = *z*<sub>*i*</sub></sub>, so *A*(*z*<sub>*i*</sub>) =  $\frac{z_i+1}{z_i+2}$  and *A*<sup>-1</sup>(*z*<sub>*i*</sub>) =  $\frac{z_i+2}{z_i+1}$  are obtained.

According to Theorem 1:

$$
J^* = 0.5k + 2.8e^{0.8} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+2}{(k+1)(k-2)}\right)^2
$$

$$
+ 2e^{0.8} \frac{k+2}{k-2} - 0.875.
$$

According to Theorem 2:

$$
J^* = 0.5k + 0.8e^{0.8} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+2}{(k+1)(k-2)}\right)^2 - 0.875
$$

Fig. 3 shows different NMP zeros values have effect on performance of the NCSs. In the case of a one-parameter compensator with a circle line, the performance of the NCSs with AWGN, time-delay, and encoding-decoding is optimized by considering the channel input power constraints. As shown in Fig. 3, the performance of timedelay, AWGN, and coding fail to perform optimal tracking. At the same time, under the one-parameter compensator structure, the performance is closely related to NMP zeros and unstable poles. However, the performance has been greatly improved under the two-parameter compensator structure. Also, from Fig. 3, when the NMP zeros and the unstable zeros position are close, the performance is degraded seriously.

We can obtain the optimal tracking performance of NCSs with different non-minimum phase zeros, as shown in Fig. 4. The paper [\[38\]](#page-9-11) studied the optimal tracking problem of SISO networked systems with considering only communication delays. The optimal tracking performance of NCSs with communication delay and channel input power constraints was studied in paper [\[33\]](#page-9-6). On the basis of [\[33\]](#page-9-6), the tracking performance of NCSs under energy constraints with time-delay, encoding-decoding and



Fig. 3. The optimal tracking performance with different values of no-minimum phase zeros.



Fig. 4. The optimal tracking performance with different values of no-minimum phase zeros.

AWGN constraints is studied in this paper. Through spectral factorization and partial decomposition techniques, we can obtain the explicit representation of the optimal performance as Theorem 1 and Theorem 2. It is shown that the optimal performance is affected by NMP zeros, unstable poles and other multiple communication constraints such as time-delay, encoding-decoding and AWGN. In the Theorem 2, we assume  $\sigma_{n_1}^2 = 0$ ,  $\sigma_{n_2}^2 = 0.2$ , all other values remain the same. From Fig. 4, communication delay and AWGN damage the optimal tracking performance, but coding can improve the optimal tracking performance.

Consider taking different values of  $\tau$ . When we take  $\tau_1 = 0.2$ ,  $\tau_2 = 0.4$ , and  $\tau_3 = 0.8$ , by Theorem 2, and get:

$$
J_{\tau_1}^* = 0.5k + 0.8e^{0.8} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+2}{(k+1)(k-2)}\right)^2
$$



Fig. 5. The optimal tracking performance with different values of time-delay.



As shown in Fig. 5, the performance of different values varies with the value of  $\tau$ . In addition, the delay increases and the optimal tracking performance decreases. This provides some guidance between the time delay and the best optimal tracking performance limitations.

Consider taking different values of  $\varepsilon$ , we take  $\varepsilon_1 = 0.2$ ,  $\varepsilon_2 = 0.4$ , and  $\varepsilon_3 = 0.8$ , at this point  $\Gamma = 5$ , and by Theorem 2, and can get:

$$
J_{\varepsilon_1}^* = 0.8k + 0.8e^{0.8} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+2}{(k+1)(k-2)}\right)^2
$$
  
\n
$$
-0.92,
$$
  
\n
$$
J_{\varepsilon_2}^* = 0.5k + 0.8e^{1.6} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+2}{(k+1)(k-2)}\right)^2
$$
  
\n
$$
-2.375,
$$
  
\n
$$
J_{\varepsilon_3}^* = 0.2k + 0.8e^{3.2} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+2}{(k+1)(k-2)}\right)^2
$$
  
\n
$$
-3.875.
$$

We can see the same result in Fig. 6, which shows the performance for different values of  $\varepsilon$ . As the value changes, we can also observe that the communication constraint increases and the performance decreases. This clearly provides some guidance between the best tradeoffs of the channel input power limit and the best tracking performance.



Fig. 6. The optimal tracking performance with different values of tradeoffs.



Fig. 7. The optimal tracking performance with different values of NMP zeros and encoding-decoding.

Consider taking different values of  $A(z_i)$ . When we take  $A(z_i) = \frac{z_i+1}{z_i+A}$ , and  $A^{-1}(z_i) = \frac{z_i+A}{z_i+1}$ , at this point  $\Gamma = 2$ , by Theorem 2, we get:

$$
J^* = 0.5k + 0.8e^{0.8} \left(\frac{k+2}{k-2}\right)^2 + 0.2k \left(\frac{k+A}{(k+1)(k-2)}\right)^2 - 0.875.
$$

As shown in Fig. 7, when the value of the encoding decreases, the performance is better. In addition, the performance of the system becomes worse when the value of the unstable zeros value and the NMP zeros are similar.

#### 6. CONCLUSION

In this paper, the optimal performance of NCSs under both one-parameter and two-parameter compensators with communication constraints. The network channel constraint taken into consideration of the AWGN, the encoding-decoding of forward network channel, and in the feedback network channel with time-delays and AWGN. The explicit expressions of the optimal performance obtained by applying the  $H_2$  norm and spectral factorization technique. The optimal performance is influenced by the non-minimum phase zeros and the unstable poles.

The complex network with packet loss, time-delay, quantization error, encoding-decoding, and other constraints in the network channel, which have practical application. The proposed method in this paper can be applied too the literature [\[39–](#page-9-12)[42\]](#page-9-13) such as signal modeling and system identification [\[43](#page-9-14)[–48\]](#page-9-15).

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<span id="page-10-0"></span>

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