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Iterative Learning Control for Fractional Order Linear Systems with Time Delay Based on Frequency Analysis

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Abstract: To overcome the deficiencies of time delay in the repetitive control of fractional-order linear systems, PD^{α}-type iterative learning control (ILC) law and P & convolution-type ILC law are designed for input and state time delay, respectively. Convergence conditions are derived in frequency domain via contraction mapping principle. Besides, the convergence frequency domain of proposed feedback controllers is obtained over a finite frequency range to design the controllers effectively. Then, the effectiveness of the proposed theoretical schemes is demonstrated using two numerical examples. The influence of time delay is eliminated, and output trajectory convergence to the desired one is guaranteed. Moreover, the Nyquist diagram of transfer function $G(s)$ and time delay variation are analyzed in frequency domain to reveal the influence of convergence on the system.

Keywords: Fractional order linear system, frequency domain analysis, iterative learning control, time delay.

1. INTRODUCTION

Fractional calculus was originated in the 17th century, whereas it has been applied in control field for only a few decades [1, 2]. Many practical energizing systems, such as viscoelastic systems, system identification and colored noise, can be described accurately with the help of fractional calculus [3,4]. Fractional control systems frequently suffer from time delay frequently, as reported by Wang *et al*. [5], Zhu *et al*. [6] and Lan *et al*. [7]. The controller design (e.g., fractional order state predictor [8]; algebraic and linear matrix inequality forms [9] and reduced-order observer and output feedback controller [10]) and stability analysis [11] have been widely investigated. However, these previous methods and designed controllers are unsuitable for time-delayed, repetitive, fractional-order control frameworks.

Iterative ideas and theories, which existed in estimation algorithm [12,13], identification [14] and other areas [15], can resolve the aforementioned problem. Iterative learning control (ILC), which was first introduced by Uchiyama [16], is an effective control method that iteratively improves the performance of a process that is repetitive in nature. Arimoto *et al*. [17] further developed and presented ILC in English. Compared with traditional control methods, such as optimal control, adaptive control and sliding mode control [18], ILC can track the desired trajectory in repetitive systems more precisely via an unknown model during a finite duration [19, 20]. For a repetitive fractional order time delayed system, Lan *et al*. [7, 21–23] proposed D^{α}-type, P-type and second-order P-type ILC for fractional order linear and nonlinear time delay systems and derived the convergence conditions. Yan *et al*. [24–26] designed P-type and D^{α}-type ILC that focused on time delay in fractional order linear and nonlinear systems, respectively. Lazarević *et al*. [27, 28] presented PD^{α}-type ILC for fractional order singular time delay system. Meanwhile, the convergence condition of closed-loop PD^{α}-type ILC for fractional nonlinear systems with time delay was described by Chenchen and Jing in [29]. The asymptotic stability of error-tracking ILC for nonlinearly parametric time delay systems with initial state errors was investigated by Yan *et al*. in [30].

Notably, aforementioned ILC laws for time delayed fractional order systems were all studied in terms of time domain. However, frequency responses and frequency characteristics, which could be determined experimentally without prior knowledge of the model and transfer function of system [31–33], play a crucial part in practical applications. Analysis of ILC in frequency domain has received increasing attention in recent years. In [34, 35], the relationships of convergence condition of ILC in time

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and frequency domain were established clearly. Chen and Moore. recently proved the convergence analysis of D^α-type ILC [36] and discussed generalized fractional order controllers $PI^\lambda D^\mu$ recently in frequency domain [37], However, other uncertainties and properties were not considered in these studies. Ye *et al*. [38] analyzed all-pass filtering in ILC, unit-gain *D*-type ILC characteristics were studied in frequency domain with no attenuation of learning speed. Ge *et al*. [39] considered the norm-optimal ILC framework and study its robust monotonic convergence in single-input-single-output integer linear time-invariant systems in frequency domain. Li *et al*. [40] designed D^α-ILC for integer order system without time delay in S-domain, all-pass phase shifter was deployed in a unit-gain *D*-type ILC, while the influence of frequency respond was ignored for bounded time delays. Moreover, Tao *et al*. [41] designed dynamic and static ILC for linear discrete systems with multiple time-delays and polytopic uncertainty. Zhai *et al*. [42] discussed the ILC applied in integer order networked control system with iterative varying references and time-delayed states in frequency domain. Both studies only considered the integer-order system.

Given the advantages of analysis in frequency domain and time delay issue, two types(PD^α-type and *P* & convolution-type) of ILC laws were designed for a repetitive Fractional Order Linear System(FOLS) with time delay in this study. The convergence conditions were derived in frequency domain on the basis of the contraction mapping principle. The results showed that the desired trajectory tracking can be achieved for any bounded time delays. In addition, the design procedure was explicitly outlined, the corresponding convergence domain of the feedback controllers was analyzed. From these, the magnitudes of learning gain were selected to satisfy the convergence condition. Finally, the Nyquist diagram of $G_1(s)$ and other frequency characteristics were analyzed through simulation examples to eliminate the influence of time delay.

The rest of this paper is organized as follows: Related mathematical definitions are recalled in Section 2. In Section 3, two types of ILC laws are designed in frequency domain for repetitive time delayed FOLS. The corresponding convergence conditions and convergence domain of the feedback controllers are derived in Section 4 and Section 5. In Section 6, simulation examples are presented to illustrate the effectiveness of proposed results. Conclusion and future work are summarized in Section 7.

2. PRELIMINARIES

In this section, some mathematical definitions used in later chapters are presented.

2.1. The operator norm

$$\|e(t)\|_\infty = \max_{1\le k<m} |e_k(t)|,$$

$$\|G\|_\infty = \max_{1\le k<m}\left\{\sum_{j=1}^{m}\left|g^{(i,j)}\right|\right\}, \tag{1}$$

where $e_k(t)$ presents the k^{th} element of $e(t)\in R^m$, $g^{(i,j)}$ denotes the $(i,j)^{th}$ element of $G\in R^{m\times m}$.

2.2. Fractional calculus

Fractional calculus plays an increasingly important role in recent engineering science. Actually, The Caputo definition is most commonly used in engineering because it takes the same form as integer-order differential equations in initial conditions. Therefore, the Caputo fractional definition was adopted as the main tool in this study, and the definition of function f(t) was similar to that in [2]

$$_{t_0}D_t^\alpha f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_{t_0}^{t}\frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}\mathrm{d}\tau, \\ \qquad\qquad n-1<\alpha<n, \\ \dfrac{\mathrm{d^n}}{\mathrm{dt^n}}f(t), \qquad \alpha=n, \end{cases} \tag{2}$$

where t_0 is initial time, $_{t_0}D_t^\alpha$ presents fractional order integral operator in $[t_0,t]$, $\Gamma(\cdot)$ is Gamma function. Especially, when $0<\alpha<1$, it has

$$_{t_0}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{f'(\tau)}{(t-\tau)^{\alpha}}\mathrm{d}\tau. \tag{3}$$

2.3. Laplace transform

The laplace transform of Caputo fractional derivative is described as follows [2]:

$$L(f_t^\alpha(t);s) = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1}f^{(k)}(0), \tag{4}$$

where $n-1<\alpha\le n$, $k\in N^+$. Specifically, when $f(0)=0$, it obtains

$$L(f_t^\alpha(t);s) = s^\alpha F(s). \tag{5}$$

3. PROBLEM FORMULATION

Some physical systems or processes that can be modeled by the FOLS, such as Maxwell model, the generalized Kelvin-Voigt model [43], the voltage-current relation of a semi-infinite lossy resistance-capacitance line or the diffusion of heat in a semi-infinite solid [44, 45]. In this study, repetitive FOLS with time delay in $t\in[0,T]$ is introduced. Moreover, ILC laws are designed and listed to track the reference trajectory.

Definition 1: 1) Let $y_d(t)$ be desired output trajectory of system;

2) $y_d(t)$ is continuous differentiability in $[0,\ T]$;

3) System satisfies same initial conditions, initial value is $x_k(0)=x_d(0)$, initial control input is $u(0)=0$.

3.1. Case 1: Input time delay

For this case, repetitive FOLS with input time delay is characterized by

$$\begin{aligned} x_k^{\alpha}(t) &= Ax_k(t) + Bu_k(t-h_1), \\ y_k(t) &= Cx_k(t), \end{aligned} \tag{6}$$

where A, B and C are appropriate matrixes, $\alpha \in [0,1]$ denotes the α^{th}-order Caputo derivative respect to t. $u, y \in R^r$ and $x \in R^n$ are control, output and state, respectively. k indicates the number of iterations, $h_1 < t \in [0,\ T]$ indicates input time delay.

Taking Laplace transform of system (6), it is described as following form

$$\begin{aligned} s^{\alpha} X_k(s) &= AX_k(s) + Be^{-h_1 s} U_k(s), \\ Y_k(s) &= CX_k(s). \end{aligned} \tag{7}$$

PD^{α}-type ILC law is considered to track the reference trajectory $y_d(t)$,

$$u_{k+1}(t) = u_k(t) + \Upsilon e_k(t) + \Phi e_k^{\alpha}(t), \tag{8}$$

where $e_k(t) = y_d(t) - y_k(t)$ denotes the tracking error, Υ and Φ are appropriate positive learning gain. Taking Laplace transform of (8), it yields

$$U_{k+1}(s) = U_k(s) + \Upsilon E_k(s) + \Phi s^{\alpha} E_k(s). \tag{9}$$

3.2. Case 2: State time delay

For this case, repetitive FOLS with state time delay is characterized by

$$\begin{aligned} x_k^{\alpha}(t) &= Ax_k(t) + A_h x_k(t-h_2) + Bu_k(t), \\ y_k(t) &= Cx_k(t), \end{aligned} \tag{10}$$

where A_h is dimensional system parameters, $h_2 < t \in [0,T]$, which indicates state time delay, Other parameters are same as in Case 1.

Taking Laplace transform of system (10), it is described in the following format

$$\begin{aligned} s^{\alpha} X_k(s) &= AX_k(s) + A_h e^{-h_2 s} X_k(s) + BU_k(s), \\ Y_k(s) &= CX_k(s). \end{aligned} \tag{11}$$

In order to track the reference trajectory $y_d(t)$ as accurately as possible when k goes to infinity for all $t \in [0,T]$, P and convolution-type ILC law is applied

$$u_{k+1}(t) = u_k(t) + \Omega e_k(t) + \vartheta(t-\tau) * e_{k+1}(t-\tau), \tag{12}$$

where $e_k(t) = y_d(t) - y_k(t)$, Ω is an appropriate positive learning gain; $\vartheta(t-\tau)$ is an appropriate positive feedback learning gain; τ denotes the maximum time delay in this form. Taking Laplace transform of (12), it has

$$U_{k+1}(s) = U_k(s) + \Omega E_k(s) + \vartheta(s) e^{-\tau s} E_{k+1}(s). \tag{13}$$

4. PROOF

4.1. Case 1

Theorem 1: For FOLS (7) and ILC law (9), given $U_0(s) = 0$ and $Y_d(s)$, if $\rho_1 < 1$, we obtain $\|E_{k+1}(s)\|_\infty \leq \|E_k(s)\|_\infty$, hence $\|E_k(s)\|_\infty \to 0$, i.e., $Y_k(s) \to Y_d(s)$ as $k \to \infty$ uniformly, where ρ_1 is defined in the following proof:

Proof: Comparing with $E_{k+1}(s)$ and $E_k(s)$, it has

$$\begin{aligned} & E_{k+1}(s) - E_k(s) \\ &= (Y_d(s) - Y_{k+1}(s)) - (Y_d(s) - Y_k(s)) \\ &= Y_k(s) - Y_{k+1}(s) \\ &= C(X_k(s) - X_{k+1}(s)) \\ &= C(s^{\alpha} I - A)^{-1} Be^{-h_1 s}(U_k(s) - U_{k+1}(s)) \\ &= C(s^{\alpha} I - A)^{-1} Be^{-h_1 s}(U_k(s) - \\ &\quad U_k(s) - \Upsilon E_k(s) - \Phi s^{\alpha} E_k(s)) \\ &= C(s^{\alpha} I - A)^{-1} Be^{-h_1 s}(-\Upsilon E_k(s) - \Phi s^{\alpha} E_k(s)). \end{aligned} \tag{14}$$

It yields

$$\begin{aligned} & E_{k+1}(s) \\ &= C(s^{\alpha} I - A)^{-1} Be^{-h_1 s}(-\Upsilon E_k(s) - \Phi s^{\alpha} E_k(s)) + E_k(s) \\ &= [1 - C(s^{\alpha} I - A)^{-1} Be^{-h_1 s}(\Upsilon + \Phi s^{\alpha})] E_k(s). \end{aligned} \tag{15}$$

Taking the norm on both sides of (15), it simplifies

$$\|E_{k+1}(s)\|_\infty \leq \rho_1 \|E_k(s)\|_\infty, \tag{16}$$

where $\rho_1 = \left\| 1 - G_1(s) e^{-h_1 s} \Upsilon - G_1(s) \Phi s^{\alpha} \right\|_2$. $G_1(s)$ is defined as $G_1(s) = C(s^{\alpha} I - A)^{-1} B$.
Therefore, if $\rho_1 < 1$, it obtains $\lim\limits_{k\to\infty} \|E_{k+1}(s)\|_\infty = 0$, the convergence of system (7) is conducted.

Theorem 1 is proved. □

4.2. Case 2

Theorem 2: For fractional order linear system (10) and ILC scheme (12), given $U_0(s) = 0$ and $Y_d(s)$, if

$$\rho_2 = \left\| \frac{I - G_2(s)\Omega}{I + G_2(s)\vartheta(s) e^{-\tau s}} \right\|_2 < 1,$$

it has $\|E_{k+1}(s)\|_\infty \leq \|E_k(s)\|_\infty$,hence $\|E_k(s)\|_\infty \to 0$, i.e., $Y_k(s) \to Y_d(s)$ as $k \to \infty$ uniformly, where the related parameters are defined in the following proof.

Proof: Comparing with $E_{k+1}(s)$ and $E_k(s)$, it can be easily shown that

$$\begin{aligned} & E_{k+1}(s) - E_k(s) \\ &= (Y_d(s) - Y_{k+1}(s)) - (Y_d(s) - Y_k(s)) \\ &= Y_k(s) - Y_{k+1}(s) \\ &= C(X_k(s) - X_{k+1}(s)) \\ &= C(s^{\alpha} I - A - A_h e^{-h_2 s})^{-1} B(U_k(s) - U_{k+1}(s)) \end{aligned}$$

$$
\begin{aligned}
&= C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B \cdot \\
&\quad (U_k(s) - U_k(s) - \Omega E_k(s) - \vartheta(s)e^{-\tau s}E_{k+1}(s)) \\
&= C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B \cdot \\
&\quad (-\Omega E_k(s) - \vartheta(s)e^{-\tau s}E_{k+1}(s)) \\
&= -C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\Omega E_k(s) - \\
&\quad C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\vartheta(s)e^{-\tau s}E_{k+1}(s). \qquad (17)
\end{aligned}
$$

It yields

$$
\begin{aligned}
&E_{k+1}(s) + C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\vartheta(s)e^{-\tau s}E_{k+1}(s) \\
&= E_k(s) - C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\Omega E_k(s). \qquad (18)
\end{aligned}
$$

Transporting the related contents of (18), it has

$$
\begin{aligned}
&[I + C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\vartheta(s)e^{-\tau s}]E_{k+1}(s) \\
&= [I - C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\Omega]E_k(s). \qquad (19)
\end{aligned}
$$

$E_{k+1}(s)$ can be simplified as follows:

$$
\begin{aligned}
&E_{k+1}(s) \\
&= \frac{I - C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B\Omega}{I + C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-}B\vartheta(s)e^{-\tau s}]}E_k(s). \qquad (20)
\end{aligned}
$$

From above inequality it yields

$$
E_{k+1}(s) = \frac{I - G_2(s)\Omega}{I + G_2(s)\vartheta(s)e^{-\tau s}}E_k(s), \qquad (21)
$$

where $G_2(s) = C(s^{\alpha}I - A - A_h e^{-h_2 s})^{-1}B$.
Taking the norm on both sides of (21), it obtains

$$
\|E_{k+1}(s)\|_{\infty} \le \rho_2 \|E_k(s)\|_{\infty}, \qquad (22)
$$

where, $\rho_2 = \left\| \frac{I - G_2(s)\Omega}{I + G_2(s)\vartheta(s)e^{-\tau s}} \right\|_2$.
Therefore, if $\rho_2 < 1$, it can be obtained $\lim_{k\to\infty} \|E_{k+1}(s)\|_{\infty} = 0$, the convergence of system (10) is conducted.
Proof of Theorem 2 is completed. □

5. THE CONVERGENCE DOMAIN OF THE FEEDBACK CONTROLLER

The input-output relation described in the frequency domain can be expressed as

$$
G(j\omega) = N(\omega)e^{(j\theta(\omega))}, \qquad (23)
$$

where $G(j\omega)$ is the input-output transfer function; $N(\omega)$, denotes magnitude characteristics, is defined as $|G(j\omega)|$. $\theta(\omega)$, presents phase characteristics, is defined as $\angle G(j\omega)$. ω is the corresponding frequency.

5.1. The convergence domain of the feedback controller with input time delay

In Case 1, convergence condition is expressed

$$
\rho_1 = \left\| 1 - G_1(s)e^{-h_1 s}\Upsilon - G_1(s)\Phi s^{\alpha}) \right\|_2 < 1.
$$

The convergence condition for the frequency components to converge is equivalent to

$$
\rho_1 = |1 - G_1(j\omega)e^{-h_1 j\omega}\Upsilon - (j\omega)^{\alpha}G_1(j\omega)\Phi| < 1. \qquad (24)
$$

It follows from (23) and (24), expression is obtained

$$
\begin{aligned}
&G_1(j\omega)e^{-h_1 j\omega}\Upsilon \\
&= \Upsilon N_{G_1}(\omega)e^{j(\theta_{G_1}(\omega) - h_1\omega)} \\
&= \Upsilon N_{G_1}(\omega)(j\sin(\theta_{G_1}(\omega) - h_1\omega) \\
&\quad + \cos(\theta_{G_1}(\omega) - h_1\omega)). \qquad (25)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&\Phi G_1(j\omega) \\
&= \Phi N_{G_1}(\omega)e^{j\theta_{G_1}} \\
&= \Phi N_{G_1}(\omega)[j\sin(\theta_{G_1}(\omega)) + \cos(\theta_{G_1}(\omega)], \qquad (26)
\end{aligned}
$$

where $N_{G_1}(\omega)$ presents magnitude characteristics of $G_1(s)$, $\theta_{G_1}(\omega)$ denotes phase characteristics of $G_1(s)$.

Applying the characteristics of $N(\omega)$ and $\theta(\omega)$, the stability condition is shown as

$$
\begin{aligned}
\rho_1^2 =& \{1 - G_1(j\omega)e^{-h_1 j\omega}\Upsilon - (j\omega)^{\alpha}G_1(j\omega)\Phi\}^2 \\
=& 1 + N_{G_1}^2\Upsilon^2 + N_{G_1}^2\Phi^2\omega^{2\alpha} \\
& - 2N_{G_1}\Upsilon\cos(\theta_{G_1} - h_1\omega) \\
& - N_{G_1}\Phi\omega^{\alpha}\cos(\frac{\alpha\pi}{2} + \theta_{G_1}) \\
& + 2N_{G_1}^2\Upsilon\Phi\omega^{\alpha}\cos(\frac{\alpha\pi}{2} + 2\theta_{G_1} - h_1\omega). \qquad (27)
\end{aligned}
$$

Taking into account (24)-(27), the convergence domain of the feedback controller is derived

$$
\begin{aligned}
&N_{G_1}^2\gamma^2 + N_{G_1}^2\Phi^2\omega^{2\alpha} \\
&\le 2N_{G_1}\Upsilon\cos(\theta_{G_1} - h_1\omega) + N_{G_1}\Phi\omega^{\alpha}\cos(\frac{\alpha\pi}{2} + \theta_{G_1}) \\
&\quad - 2N_{G_1}^2\Upsilon\Phi\omega^{\alpha}\cos(\frac{\alpha\pi}{2} + 2\theta_{G_1} - h_1\omega). \qquad (28)
\end{aligned}
$$

From (28), it concludes that small enough magnitudes of learning gain can be found to satisfy the convergence condition.

5.2. The convergence domain of the feedback controller with state time-delay

In Case 2, convergence condition is expressed

$$
\rho_2 = \left\| \frac{I - G_2(s)\Omega}{I + G_2(s)\vartheta(s)e^{-\tau s}} \right\|_2 < 1.
$$

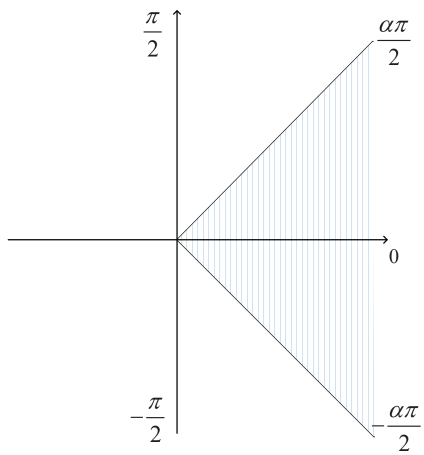

Fig. 1. The learnable band of fractional order linear system.

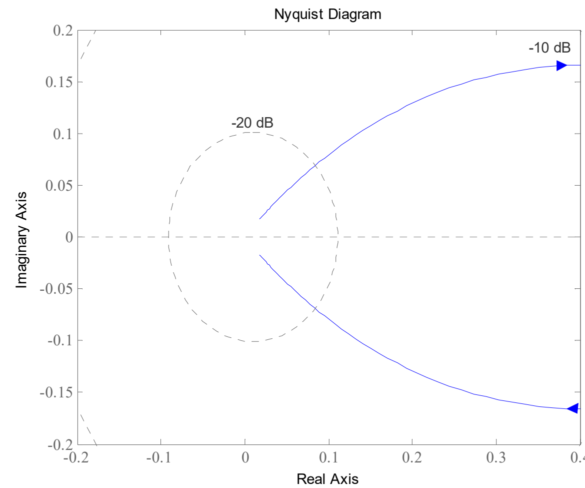

Fig. 2. The Nyquist diagram of $G_1(s)$.

Correspondingly, the convergence condition for frequency components to converge is expressed

$$\rho_2 = \left| \frac{I - G_2(j\omega)\Omega}{I + G_2(j\omega)\vartheta(j\omega)e^{-\tau j\omega}} \right| < 1. \tag{29}$$

Same as Case 1, the convergence domain of the feedback controller is derived

$$\begin{aligned} &N_{G_2}^2\Omega^2 - N_{G_2}^2 N_\vartheta^2 \\ &\le 2N_{G_2}\Omega\cos(\theta_{G_2}) + 2N_{G_2}N_\vartheta\cos(\theta_{G_2} + \theta_\vartheta - \tau\omega), \end{aligned} \tag{30}$$

where N_{G_2} and N_ϑ present magnitude characteristics of $G_2(s)$ and $\vartheta(s)$, respectively. θ_{G_2} and θ_ϑ denote phase characteristics of $G_2(s)$ and $\vartheta(s)$, respectively.

Similarly, from (30), it can obtained that small enough magnitudes of learning gain can be calculated to satisfy the convergence domain.

5.3. Learnable band of feedback controller

Notably, (28) and (30) hold for all $\omega \in [0, \infty)$. They satisfy the condition that all learnable bands belong to $[-\frac{\alpha\pi}{2}, \frac{\alpha\pi}{2}]$, which is the blue line area in Fig. 1.

In this study, the frequency range within which the convergence conditions hold is called the learnable band. Fig. 1 shows that the frequency range of most FOLSs is smaller than that of the conventional integer-order linear system. By considering the convergence conditions in two cases, the learning gains with an appropriate magnitude can be selected to satisfy the convergence domain.

6. NUMERICAL SIMULATIONS

In order to verify the effectiveness of the proposed convergence conditions and analysis frequency characteristics, the previously presented ILC schemes are applied to the FOLS with time delay. Examples are designed and listed in this section. All simulations are implemented in MATLAB/SIMULINK.

Example 1: Consider the following repetitive FOLS with input time delay

$$\begin{aligned} &x_k^\alpha(t) = -x_k(t) + 0.8u_k(t - h_1), \\ &y_k(t) = x_k(t), \end{aligned} \tag{31}$$

where, $h_1 = 0.05$ stands for input time delay, $\alpha = 0.5$. The desired trajectory is generated as $y_d(t) = \sin(3\pi t)$, initial state $x_k(0) = 0$.

Taking Laplace transform of (31), it expressed

$$\begin{aligned} &s^\alpha X_k(s) = -X_k(s) + 0.8e^{-h_1 s}U_k(s), \\ &Y_k(s) = X_k(s). \end{aligned} \tag{32}$$

For the above system, PD^α-type ILC updating law is applied

$$u_{k+1}(t) = u_k(t) + 2e_k(t) + e_k^{0.5}(t). \tag{33}$$

Taking Laplace transform of (33), it yields

$$U_{k+1}(s) = U_k(s) + 2E_k(s) + s^{0.5}E_k(s). \tag{34}$$

According to the aforementioned information, the transfer function $G_1(s)$ is calculated

$$G_1(s) = C(s^\alpha I - A)^{-1}B = \frac{0.8}{s^{0.5} + 1}. \tag{35}$$

1) The Nyquist diagram of $G_1(s)$ shows in Fig. 2.

Fig. 2 shows that the frequency domain result of $G_1(s)$ is not necessarily considered for asymptotic stability of system.

2) The tracking performance of system output under PD^α-type ILC is shown in Fig. 3. The values of trajectory

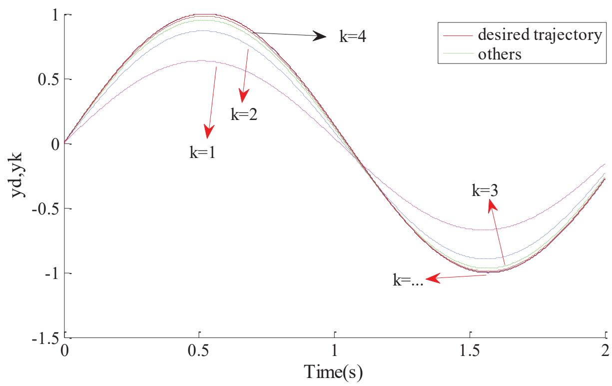

Fig. 3. The tracking performance of the system output.

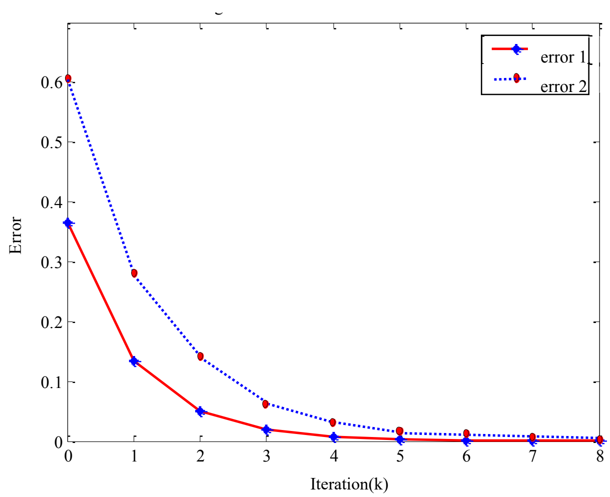

Fig. 4. The norm of the tracking errors in each iteration.

error (error 1) are exhibited in Fig. 4 by numerical iterations, where $k = 8$ and $t \in [0, 2]$.

Figs. 3 and 4 (error 1) show that the system output approaches the desired trajectory accurately within a few iterations. The tracking error at the sixth iteration is very small, which theoretically shows that the PD^{α}-type ILC scheme can be applied to the tracking control of FOLS with input time delay h_1. After 8 times iterations, the tracking error converges to 0.

Furthermore, for comparison, let $U_{k+1}(s) = U_k(s) + \Phi s^{\alpha} E_k(s)$ to design a D^{α}-type ILC (Yan *et al.* [20]). The tracking errors show in Fig. 4 (error 2). It concludes that PD^{α}-type ILC updating law performs better in convergence rate during the learning process.

Define $Max\{\theta(\omega)\} = Max\{(\theta_{G_1} - h_1\omega), (\frac{\alpha\pi}{2} + \theta_{G_1}), (\frac{\alpha\pi}{2} + 2\theta_{G_1} - h_1\omega)\}$ in (27). In order to show the frequency characteristics of PD^{α}-type ILC applied in FOLS with input time delay, the relationship of the value of $Max\{\theta(\omega)\}$ and frequency is shown in Fig. 5, where time delay h_1 is selected from 0 to 0.05 by 0.01.

From Fig. 5, it shows that when time delay h_1 selected, the $Max\{\theta(\omega)\}$ first decreases to negative and then increases due to the increasing compensation effect of ω increases. The learnable band domain belongs to $[-\frac{\alpha\pi}{2}, \frac{\alpha\pi}{2}]$. In this case, the learnable band is $[-45°, 45°]$.

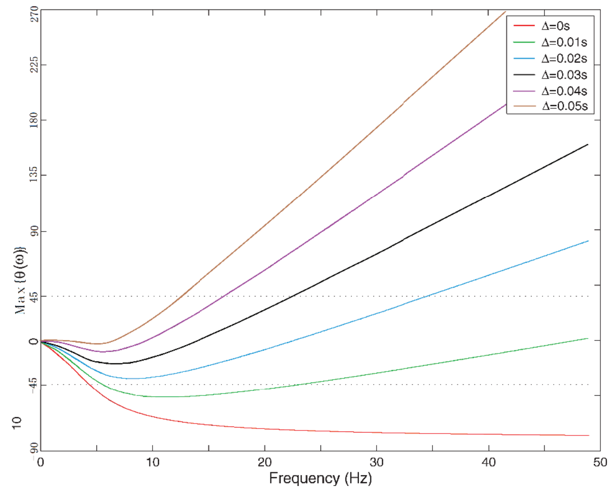

Fig. 5. Time delay h_1 selection for system.

Example 2: Consider the following repetitive FOLS with state time delay.

$$x_k^{\alpha}(t) = -x_k(t) - \frac{1}{2}x_k(t-h_2) + \frac{7}{10}u_k(t),$$
$$y_k(t) = x_k(t), \tag{36}$$

where $h_2 = 0.1$ stands for state time delay, $\alpha = 0.5$.

Taking Laplace transform of system (36), it has

$$s^{0.5}X_k(s) = -X_k(s) - \frac{1}{2}e^{-0.1s}X_k(s) + \frac{7}{10}U_k(s),$$
$$Y_k(s) = X_k(s). \tag{37}$$

For the above system, P and convolution-type ILC updating law is applied.

$$u_{k+1}(t) = u_k(t) + 1.03e_k(t) + 2(t-\tau) * e_{k+1}(t-\tau), \tag{38}$$

where $\tau = 0.1$, which denotes the maximum time delay in this form.

According to (17), the updating law can be written simply in Laplace domain

$$U_{k+1}(s) = U_k(s) + 1.03E_k(s) + 2e^{-0.1s}E_{k+1}(s). \tag{39}$$

Let the reference trajectory be $y_d(t) = 12t^2(1-t)$, initial state $x_k(0) = 0$. The transfer function of $G_2(s)$ is calculated

$$G_2(s) = C(s^{0.5}I - A - A_h e^{-0.1s})^{-1}B$$
$$= \frac{0.7}{s^{0.5} - 1 - 0.5e^{-0.1s}}. \tag{40}$$

1) The Nyquist diagram of $G_2(s)$ is shown in Fig. 6. Fig. 6

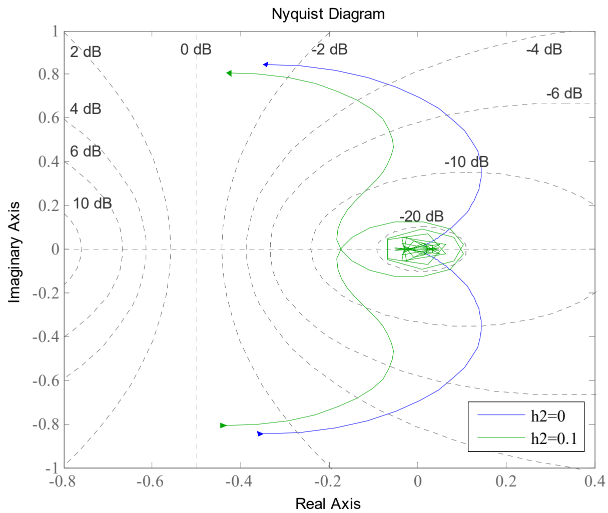

Fig. 6. The Nyquist diagram of $G_2(s)$.

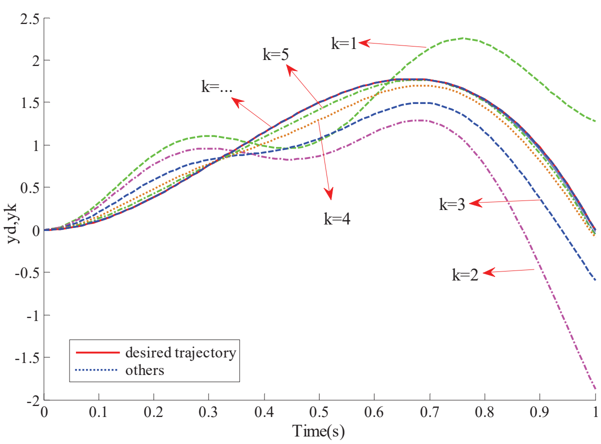

Fig. 7. The tracking performance of the system output.

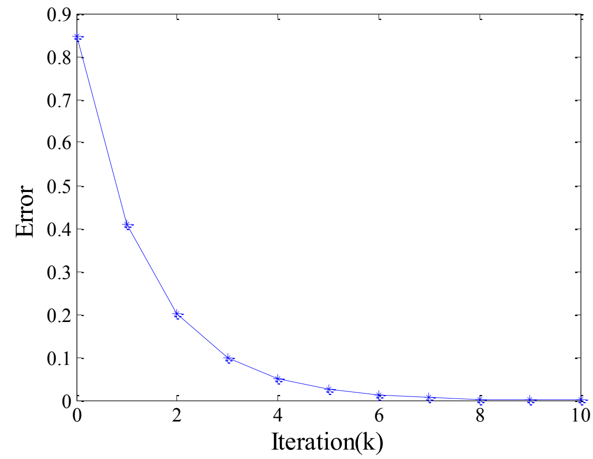

Fig. 8. The norm of the tracking errors in each iteration.

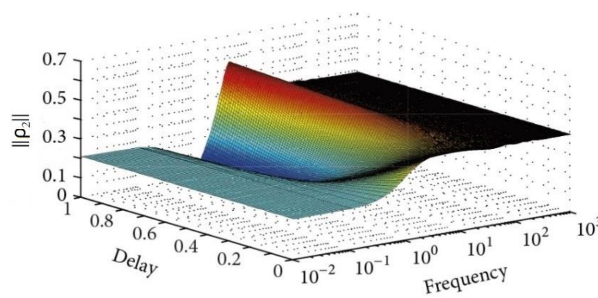

Fig. 9. The relationship of time delay h_2, frequency ω and $\|\rho_2\|$

shows that when FOLS has a state time delay, the Nyquist diagram of $G_2(s)$ (green line) is different from that with no time delays(blue line). From maximum time delay τ in (38) and Fig. 6, we can conclude that the delay link can be replaced by a small inertia link if the delay time is small. Moreover, the convergence condition of system should be considered in this case.

2) The tracking performance of the system output and the norm of tracking error under P and convolution-type ILC are shown in Figs. 7 and 8, respectively, where $k = 10$ and $t \in [0,\ 1]$.

Figs. 7 and 8 indicate that the system output approaches the desired trajectory accurately only after 10 times iterations. It theoretically shows that P and convolution-type ILC updating scheme can be successfully applied. In addition, from Fig. 8, when $k = 10$, $\|y_d(t) - y_{10}(t)\|_2 = 1.86 \times 10^{-3}$, it shows that the uniform convergence of the tracking errors is guaranteed.

3) The relationship of time delay h_2, frequency ω and $\|\rho_2\|$ are shown in Fig. 9. Fig. 9 shows the effect of state time delay on the convergence rate of the system at different frequencies. In the low frequency case, time delay h_2 is larger and its convergence speed is higher, while in high frequency, the results are on the contrary.

7. CONCLUSION AND FUTURE WORK

In this study, the convergence conditions of the time-delayed FOLS based on ILC schemes were derived and analyzed in the frequency domain. The convergence frequency domain of the feedback controller was obtained in consideration of the magnitude and phase characteristics. Two numerical examples were presented to illustrate that the proposed theoretical schemes can be applied to the tracking control of FOLS with time delay, and perform well in terms of convergence results. In addition, the Nyquist diagram of transfer function $G(s)$ and several other properties in the frequency domain were analyzed in detail.

Future work could investigate the properties of related ILC schemes for FOLS in the frequency domain. The application and analysis of ILC schemes for FOLS in engineering practice are other possible research tasks.

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